Blind Carrier Frequency Offset Estimator for Discrete Multi Tone System

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***Abstract*— In this paper, a new blind Carrier Frequency Offset (CFO) estimation algorithm for Discrete Multi Tone (DMT) system is obtained by introducing the Propagator Method (PM) in conjunction with the well-known MUSIC based high resolution searching algorithm. Furthermore, the PM does not require the eigen value decomposition (EVD) or singular value decomposition (SVD) of the covariance matrix of the received signals; computer simulations are also included to demonstrate the effectiveness of the proposed method in comparison with other conventional methods.**

I. INTRODUCTION

The principles of the Discrete Multi Tone (DMT), which is known as orthogonal frequency division multiplexing (OFDM) modulation [1] have been in existence for several decades. Also, it is expected to be used in multiple standards for wireless broadband multimedia communications. Even though the OFDM system is showing excellent performance against multipath fading, it is very sensitive to the carrier offset. Such offset estimation error leads to severe distortion in subcarrier orthogonality and causes inter channel interference (ICI) [2]. Hence Carrier Frequency Offset (CFO) must be estimated and compensated at the receiver to improve system performance. Consequently, there has been considerable work done in the area of CFO estimation. Several approaches have been proposed to estimate the CFO either by using periodic pilot tones [3] or blindly [4]-[5].

In this paper, we proposed a novel algorithm for estimating the CFO in an OFDM receiver without using reference symbols, pilot carriers or extra cyclic prefix. We employed the Propagator Method (PM) [6]-[7] in conjunction with the well-known multiple signal classification (MUSIC) algorithm [8] for estimating the carrier offset in the received signal. We have used the existing structure of OFDM system to form a propagator to explore the presence of carrier offset. The propagator is a linear operator which only depends on steering vectors and which can be easily extracted from the direct data set.

II. PROBLEM FORMULATION

We consider an OFDM system implemented by inverse discrete Fourier transform (IDFT) and discrete Fourier transform (DFT) each of size N for modulation and demodulation respectively. Only P subcarriers of total N subcarriers are used to avoid aliasing effect at the edge of the transmission spectrum. The N samples of IDFT output are given by $\boldsymbol{x}(k) := \boldsymbol{W}_P\boldsymbol{s}(k)$, where $\boldsymbol{W}_P$ consist of the first P columns of the $N \times N$ IDFT matrix and $\boldsymbol{s}(k) = [s_0(k)\ s_1(k) \dots \dots s_{P-1}(k)]^T$ is a QPSK or QAM data symbol to be transmitted through the kth block. An OFDM symbol is denoted as $\{x_{N-G}, \dots \dots x_{N-1}, x_0, x_1 \dots \dots x_{N-1}\}$ of which the first G samples are guard samples to cancel ISI. The removal of the guard samples at the receiver end makes the received sequence the circular convolution of the transmitted sequence with the channel impulse response $h(l),\ l = 0,1, \dots L_c - 1$, where L_c is the channel length. Inside the kth block only the guard portion of the signal will be distorted since the channel length $L_c < G$. The receiver input for the kth block given by

$$\boldsymbol{y}(k) = \boldsymbol{E}\boldsymbol{W}_P\boldsymbol{H}\boldsymbol{s}(k)e^{j(k-1)\varphi(N+G)} + \boldsymbol{z}(k) \quad (1)$$

where $\boldsymbol{H} = diag[H(0), H(1), \dots H(P-1)]$, the $H(i) = \sum_{l=0}^{L_c-1} h(l)\omega^{-il}$, $\boldsymbol{E} = diag(1, e^{j\varphi}, \dots e^{j(N-1)\varphi})$, and φ is the carrier offset. To maintain orthogonality among the sub-channel carriers and to avoid ICI, the matrix $\boldsymbol{E}$ must be estimated and compensated before applying the DFT to (1). The task now is to estimate φ assuming that the k received noisy data blocks are the only measurements available.

III. DEVELOPMENT OF PROPOSED METHOD

The K blocks of the noiseless received data are collected in matrix $\boldsymbol{Y}$ of size $(N \times K)$

$$\boldsymbol{Y} = [\boldsymbol{y}(1)\ \ \boldsymbol{y}(2) \dots \dots \boldsymbol{y}(K)] \quad (2)$$

where the kth block of the received signal in (2) is given by $\boldsymbol{y}(k) = [y_0(k)\ y_1(k) \dots \dots y_{N-1}(k)]^T$. Constructing $(N - M + 1)$ sub-matrices from $\boldsymbol{Y}$, each of size $M \times K$ such as $M \geq P$, the ith sub matrix is given by

$$\boldsymbol{Y^i} = [\boldsymbol{y}^i(1)\ \ \boldsymbol{y}^i(2)\ \ \boldsymbol{y}^i(3) \dots \dots \boldsymbol{y}^i(K)] \quad (3)$$

Where $\boldsymbol{y}^i(k) = [y_{i-1}(k), y_i(k), \dots y_{i+M-1}(k)]^T$

$$i = 1,2, \dots \dots N - M\ ,\ k = 1,2, \dots \dots K$$

Collecting $(N - M + 1)$ sub matrices calculated in (3) each of size $M \times K$ to form matrix $\mathbf{X}$ such that

$$\boldsymbol{X} = [\boldsymbol{Y^1}\ \ \boldsymbol{Y^2} \dots \boldsymbol{Y^{N-M+1}}] = [\boldsymbol{A\Phi^0}\ \ \boldsymbol{A\Phi^1} \dots \ \boldsymbol{A\Phi^{N-M}}].\boldsymbol{S}$$

where $\boldsymbol{A} := \boldsymbol{E}_M\boldsymbol{W}_M$, $\boldsymbol{W}_M$ consists of the first M rows of $\boldsymbol{W}_P$, $\boldsymbol{E}_M = diag(1, e^{j\varphi}, \dots e^{j(M-1)\varphi})$ and $\boldsymbol{\Phi} = diag(e^{j\varphi}, e^{j(\omega+\varphi)}, \dots e^{j(\omega(P-1)+\varphi)})$ including the information of carrier offset, with $\omega = 2\pi/N$. Partitioning $\boldsymbol{A}$ into two sub-matrices $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$. We defined a propagator matrix $\boldsymbol{P}$ satisfying $\boldsymbol{P^H A_1}$=$\boldsymbol{A_2}$.

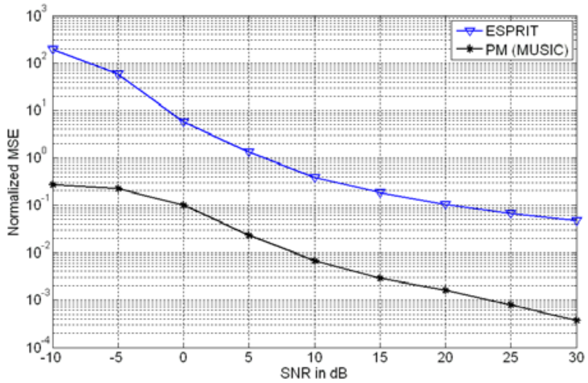

Fig.1. Estimator performance versus SNR

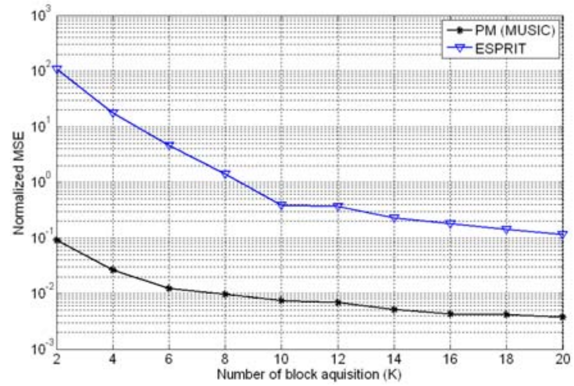

Fig.2. Estimator performance versus number of blocks

The dimension of matrix $\boldsymbol{P^H}$ is $(M-P)\times P$. Similarly we can partition the received data matrix $\boldsymbol{X}$ into two sub-matrices $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$. The Propagator matrix $\widehat{\boldsymbol{P}}$ can be estimated by

$$\widehat{\boldsymbol{P}} = (\boldsymbol{X_1}\boldsymbol{X_1^H})^{-1}\boldsymbol{X_1}\boldsymbol{X_2^H} \tag{4}$$

Matrix $\boldsymbol{E}$ can be defined as

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{P} \\ -\boldsymbol{I} \end{bmatrix} \tag{5}$$

where $\boldsymbol{I}$ is the identity matrix of size. In the noisy channel the basis of $\boldsymbol{E}$ are not orthonormal. Therefore we will use the projection $\boldsymbol{Q} = \boldsymbol{E}(\boldsymbol{E^H}\boldsymbol{E})^{-1}\boldsymbol{E^H}$, Appling MUSIC [8] like search algorithm to estimate the frequency offset using

$$\hat{P}_{MU}(\varphi) = \frac{1}{A(\varphi)^H \boldsymbol{Q} A(\varphi)} \tag{6}$$

IV. Simulation Results

Extensive computer simulations are done to validate our proposed method. In the first experiment, we considered OFDM system with N=64 carriers, of which P=40. Transmitted symbols are drawn from a QPSK constellation. The CP length was eleven symbols and frequency offset φ is assumed to be 0.1ω. The experiment was run under AWGN environment with $N_t = 1000$ independent monte-carlo realizations. We considered blocked data of length 10 and the structure parameter M considered 60 for ESPRIT algorithm while it is 41 for the proposed PM based algorithm. The first figure plots the normalized MSE of the CFO as a function of SNR for the proposed algorithm and compares it with [5]. The experiment is evaluated with OFDM frame size is considered to be N = 64 while the number of block is assumed to be K=10. The structure parameter M is chosen to be 41 for the proposed method and 61 for the ESPRIT algorithm [5]. It is obvious that an achievement of approximately15 dB SNR with the proposed algorithm over [5] would obtain. The second figure evaluates the performance of both CFO estimators as a function of the number of block acquisition. Under 10 dB constant SNR and following same assumptions in experiment one, we can say our proposed method can achieve the same performance at K=2 while ESPRIT algorithm requires almost ten times data block collection.

V. Conclusion

A novel propagator based method in conjunction with the MUSIC based search algorithm or root-MUSIC based algorithm for estimating CFO for OFDM systems is projected. For the same experiment set up, almost 15 dB is achieved in SNR for the proposed PM based method over the ESPRIT one. The proposed method is showing equivalent performance in comparison with the well known ESPRIT type estimator at one tenth of the block acquisitions.

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