A Unified Procedure for Continuous-Time and Discrete-time Root-Locus and Bode Design

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I. Introduction

As an alternative to the numerous distinct controller design algorithms in continuous-time and discrete-time classical control textbooks, a simple, unified design approach is presented for all standard continuous-time and discrete-time, classical compensators independent of the form of the system information. This approach is based on a simple root locus design procedure for a proportional-derivative (PD) compensator [2, 3, 4]. The delta operator, which serves as a link between the continuous-time and discrete-time procedures, offers improved numerical properties to the traditional discrete–time shift operator. With this proposed approach, designers can concentrate on the larger control system design issues, such as compensator selection and closed-loop performance, rather than the intricacies of a particular design procedure [2, 4].

Procedures for standard compensators (lead, proportional-integral (PI), proportional-integral-derivative (PID), and PI-lead compensators) have been developed, but due to space considerations only the PD procedure is presented in this paper. This paper is organized as follows. In Section II, generalized magnitude and phase criteria are presented. PD compensator design and an example are presented in Section III. Conclusions are presented in Section IV.

II. Compensator Design

The integrated design procedure using time or frequency domain plant data requires a generalization of the angle criterion from root locus design. The standard closed-loop system is shown in Figure 1 where K is the control gain, $G_c(\gamma)$ is the compensator and $G_p(\gamma)$ represents the plant dynamics. There is a close connection between the forward shift operator q and the Z-transform variable z. The delta operator defines an alternative discrete time operator, $\delta = \frac{q-1}{\Delta}$, where Δ is the sampling period. Similarly, we define a new transform variable γ associated with δ as $\gamma = (z-1)/\Delta$ [1]. The new variable γ has the following relation with continuous time domain, $\gamma = \begin{cases} s & \Delta = 0 \\ \frac{e^{s\Delta}-1}{\Delta} & \Delta \neq 0 \end{cases}$, where s is the Laplace transform variable.

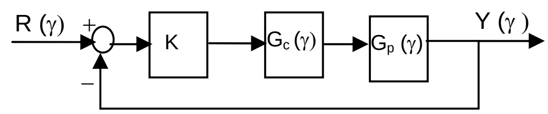

Fig. 1. Closed-loop block diagram

The design point in continuous-time root locus is given by $s_0 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$ where ζ is the damping ratio and ω_n is the normal frequency. In root locus design, the compensator must satisfy the well-known angle and magnitude criteria

$$\begin{aligned} &\angle G_c(\gamma_0) + \angle G_p(\gamma_0) = \pm 180^\circ \\ &K\left| G_c(\gamma_0) G_p(\gamma_0) \right| = 1 \end{aligned} \quad (1)$$

where,

$$\gamma_0 = \begin{cases} s_0 & \Delta = 0 \\ \frac{e^{s_0\Delta} - 1}{\Delta} & \Delta \neq 0 \end{cases} \quad (2)$$

In Bode design methods, the specifications are incorporated through the desired phase margin PM and gain crossover frequency ω_{gc} and result in another set of angle and magnitude constraints

$$\angle G_c(\gamma_0) + \angle G_p(\gamma_0) = \pm 180° + PM$$
$$K\left| G_c(\gamma_0) G_p(\gamma_0) \right| = 1$$
(3)

where,

$$\gamma_0 = \begin{cases} j\omega_{gc} & \Delta = 0 \\ \dfrac{e^{j\omega_{gc}\Delta} - 1}{\Delta} & \Delta \neq 0 \end{cases}$$
(4)

Equations (1) and (3) can be combined to get the generalized angle and magnitude constraints

$$\angle G_c(\gamma_0) + \angle G_p(\gamma_0) = \varphi$$
$$K\left| G_c(\gamma_0) G_p(\gamma_0) \right| = 1$$
(5)

where the desired angle in the angle constraint is, $\varphi = \begin{cases} \pm 180°, & \text{root locus} \\ \pm 180° + PM, & \text{Bode} \end{cases}$. Using the angle constraint in (5), the desired compensator angle θ_c can be computed from the plant information and the design point without knowledge of the compensator type.

$$\angle G_c(\gamma_0) = \varphi - \angle G_p(\gamma_0) =: \theta_c$$
(6)

In root locus methods, θ_c determines a geometric relationship between the design point and the compensator poles and zeros. In Bode methods, θ_c is the phase that must be added at the gain crossover frequency.

III. PD Compensator

As in the continuous-time case, the design procedures for all compensators are based on the PD design procedure. The PD compensator in our unified notation has a transfer function, $G_c(\gamma) = \dfrac{\gamma + \alpha}{\Delta\gamma + 1}$, the angle of the PD compensator at the design point γ_0 is, $\theta_c = \angle G_c(\gamma_0) = \angle(\gamma_0 + \alpha) - \angle(\Delta\gamma_0 + 1)$, therefore

$$\theta_z = \angle(\gamma_0 + \alpha) = \theta_c + \angle(\Delta\gamma_0 + 1)$$
(7)

must hold and the compensator zero is given by

$$\alpha = \sigma_0 + \frac{\omega_0}{\tan(\theta_z)}$$
(8)

where $\gamma_0 = -\sigma_0 + j\omega_0$ is the design point in the unified notation.

As an example, we will use root locus to design a PD compensator for $G_p(\gamma) = \dfrac{0.005\gamma + 0.995}{\gamma^2 + 0.995\gamma}$ with a sampling period of $\Delta = 0.01$ seconds. The closed-loop step-response is specified to have an overshoot of less than 4% and a settling time of less than 4 seconds.

Using standard second and assumptions in the continuous domain, these specifications should be met by a design point of $s_0 = -1 + j\,0.5$. From (2), this corresponds to $\gamma_0 = -0.9963 + j\,0.4950$ for delta-domain root locus. The desired compensator angle θ_c is computed from (6), the compensator zero is computed from (7) and (8), and the gain K is calculated from the magnitude criterion in (5). The root locus results are shown in Table 1. As can be seen from Table 1, the design point is achieved and the design specifications are met.

TABLE 1
PD COMPENSATOR RESULTS

Compensator angle (θ_c)	1.097 rad
PD zero (α_{PD})	1.2391
K(gain)	0.9987
Closed-loop poles	-0.9963 ± j 0.4950
Settling time $(T_s)_{measured}$	2.60 sec
Percent overshoot $(P.O.)_{measured}$	0.77 %

IV. Conclusion

The design procedures for four compensators: lead, PI, PID and PI-lead were developed from a PD design procedure. Only the PD compensator is presented here. These procedures are analogous to the continuous-time design procedures presented in [2, 3]. This common design approach helps to bridge the gap between the more intuitive continuous-time design and the practical direct discrete-time design.

References

[1] R. H. Middleton and G.C. Goodwin, Digital Control and Estimation A Unified Approach, Englewood Cliffs, NJ: Prentice-Hall, 1990.
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