Robust and Reduced Order H_∞ Filtering via LMI Approach

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1. Introduction

The purpose of filtering is to estimate the state variables of a given system. On the early stage, the study was based on the assumption that the system dynamics are known by a certain model and external noise was white with known statistical properties, that is well known Kalman filtering. But in the practical applications, the disturbances may not be known exactly and the system uncertainties may appear in modelling. In these robustness issues, H_∞ technique is usually used because it is more robust to disturbances.

The model reduction is important issue in many applications, especially when fast data processing is necessary to reduce the complexity and real time computational burden, even though it costs the loss of performance and robustness. For instance the reduced order filter problem was studied in H_∞ setting[1]. More recently, LMI(Linear Matrix Inequality) technique was developed. The important role of LMI technique was already recognized in the early 1960's, but the recent emergence as a powerful computational design tool in system and control engineering was originated from the computational efficiency and flexibility to treat large class of system design problems[2],[3]. The recent research for robust reduced-order filtering problem was studied in H_2 setting via LMI approach in [4]. To the author's best knowledge, until now, these concepts were not applied to H_∞ setting. Thus, the author thinks that a research to this direction is valuable.

2. Preliminary result

At first, the model order reduction in H_∞ setting with certain plant model, i.e., not-robust case, will be studied and the characteristics will be surveyed. The two lemmas and one theorem are playing the key role in this research.

Lemma 1[2]:
Consider a stable linear time invariant system :

$$\begin{cases} \dot{x} = Ax + Bd \\ y = Cx + Dd \end{cases} \qquad (1)$$

where A,B,C,D are appropriate dimensional matrices, d is disturbance vector and its transfer function is:

$$T(s) = C(sI - A)^{-1}B + D \qquad (2)$$

Let γ be a given positive scalar. Then $\|T\|_\infty \prec \gamma$ iff there exists a matrix $P \succ 0$ such that

$$\begin{bmatrix} PA + A^T P & PB & C^T \\ B^T P & -\gamma^2 I & D^T \\ C & D & -I \end{bmatrix} \prec 0 \qquad (3)$$

This lemma is a well-known Bounded Real Lemma.
To include plant model uncertainties, we formulate them in the form of a polytopic model as follows.

$$\begin{bmatrix} A & B \\ C & D \\ L & 0 \end{bmatrix} \in \left\{ \begin{bmatrix} A(\alpha) & B(\alpha) \\ C(\alpha) & D(\alpha) \\ L(\alpha) & 0 \end{bmatrix} = \sum_{i=1}^{s} \alpha_i \begin{bmatrix} A_i & B_i \\ C_i & D_i \\ L_i & 0 \end{bmatrix}, \alpha \in \Xi \right\} \qquad (4)$$

where Ξ is unit simplex such as

$$\Xi \equiv \left\{ (\alpha_1, \cdots, \alpha_n) : \sum_{i=1}^{s} \alpha_i = 1, \alpha_i \geq 0 \right\} \qquad (5)$$

This formulation is convex combination, so it is useful for LMI approach. Another useful lemma is as follows.

Lemma 2[3]:
Let Γ, Λ, and $\Theta = \Theta^T$ be given matrices. There exists a matrix F to solve the matrix inequality

$$\Gamma F \Lambda + (\Gamma F \Lambda)^T + \Theta < 0 \qquad (6)$$

iff the following conditions are satisfied

$$\Gamma^{\perp} \Theta \Gamma^{\perp T} < 0 \qquad (7)$$
$$\Lambda^{T\perp} \Theta \Lambda^{T\perp T} < 0 \qquad (8)$$

where $\perp$ is orthogonal complement operator.

In this case, all solution matrices F are parametrized by

$$F = -R^{-1}\Gamma^T\Phi\Lambda^T\Psi + \Omega^{1/2}L\Psi^{1/2} \qquad (9)$$

where Φ, R and L are free parameters subject to

$$\Phi = (\Gamma R^{-1}\Gamma^T - \Theta)^{-1} > 0,\ R>0,\ \|L\|<1 \qquad (10)$$

where Ω and Ψ are defined by

$$\Omega = R^{-1} - R^{-1}\Gamma^T(\Phi - \Phi\Lambda^T\Psi\Phi)\Gamma R^{-1} \qquad (11)$$

$$\Psi = (\Lambda\Phi\Lambda^T)^{-1} \qquad (12)$$

This lemma shows us that almost all the linear controller design problems can be formulated as (6).

The following theorem shows the necessary and sufficient condition for the existence of filter and filter gain that we wish to find.

Theorem 1[3]:

Consider n* order linear filter F(n*<n: n=model order) :

$$\begin{Bmatrix} \dot{x}_F = A_F x_F + B_F y \\ z^* = L_F x_F \end{Bmatrix} \qquad (13)$$

there exists an n* order filter F to solve the γ-suboptimal H_∞ filtering problem iff there exist matrices X and Y(0<X≤Y) such that the following conditions are satisfied

$$\begin{bmatrix} XA + A^T X & XB \\ B^T X & -\gamma^2 I \end{bmatrix} \prec 0 \qquad (14)$$

$$\begin{bmatrix} C^T \\ D^T \end{bmatrix}^{\perp} \begin{bmatrix} YA + A^T Y + L^T L & YB \\ B^T Y & -\gamma^2 I \end{bmatrix} \begin{bmatrix} C^T \\ D^T \end{bmatrix}^{\perp T} \prec 0 \qquad (15)$$

$$\text{rank}\,(X - Y) \le n^* \qquad (16)$$

and all γ-suboptimal n* order filter F that corresponds to a feasible matrix pair (X,Y) are given by

$$F = \begin{bmatrix} 0 & L_F \\ B_F & A_F \end{bmatrix} = -R^{-1}\Gamma^T\Phi\Lambda^T\Psi + \Omega^{1/2}L\Psi^{1/2} \qquad (17)$$

where Φ, R and L are free matrix parameters subject to (10), (11) and (12).

As we can see, (14) and (15) are uncertainty parameter-α dependent matrix form thus it is nonlinear matix inequality.

Also, (16) is nonconvex rank constraint.

To linearize (14), projection lemma and inverse projection lemma are used[4].

3. Discussion

From the above preliminary result , we can know that in the case of robust and reduced order H_∞ filtering with polytopic model uncertainty, it is formulated as nonconvex nonlinear MI setting. It means that the expected tackling strategy will be more complex and difficult than convex LMI problem. Also we can assume that due to nonconvex formulation, the global solution cannot be found easily.

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5. References

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