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THE REJECTION OF THE HYPOTHESIS OF COMPLETE INDEPENDENCE PRIOR TO CONDUCTING A FACTOR ANALYSIS

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ABSTRACT

Two tests of complete independence were compared in terms of relative power efficiency on 50 equi-correlation matrices and 26 published data sets. Λ_1 a test based on Fisher's $\tanh^{-1}(r)$ transformation had greater power efficiency than Λ_0 a test based on $-\log|R|$ with finite sample corrections. Consequently, sample size requirements for rejecting P=I should be determined from $\Lambda_1$instead of Λ_0. In addition, Baggaley's estimate of $Q=-\log|R|/k$ from the average absolute off-diagonal correlation was compared with a root mean square estimate of Q. The root mean square estimate was preferable but also demonstrated considerable bias. Because of the bias in estimating Q from the off-diagonal correlations, because $\Lambda_1$has greater relative power efficiency for P=I than Λ_0, and because the test statistic for Λ_1 can be approximated from the root mean square off-diagonal correlation, it is preferable to tabulate power results for Λ_1 as a function of the average correlation instead of Λ_0.

Baggaley (1982) reported that when testing whether a k-variate correlation matrix R is from a population characterized by an identity correlation matrix (P=I), $Q=-\log|R|/k$ could be approximated from the average absolute off-diagonal correlation. This result was used by Baggaley to analytically obtain power results for the distribution of $-\log|R|$ in testing P=I. Subsequently, Reddon and Jackson (1984) evaluated the bias in estimating Q for various equi-correlation matrices and also tabulated power results from the asymptotic distribution of $-\log|R|$. Reddon and Jackson (1984) demonstrated that there was considerable bias in Baggaley's estimate of Q, as a function of the equi-correlation and number of variates. In response Baggaley (1986) evaluated six contrived correlation matrices with heterogeneous correlations based on a 12 variate, 2 factor model. These results indicated that Baggaley's estimate of Q may be reasonable for correlation matrices with heterogeneous elements.

In the present paper a set of equi-correlation matrices were analyzed and power results for testing P=I were compared for -log|R| with finite sample corrections for bias (Reddon, Jackson, & Schopflocher, 1985) to a test based on Fisher's $\tanh^{-1}$ transformation of the correlation coefficient (Reddon, 1987). These two tests of complete independence were also compared on 17 previously analyzed data sets (Hakstian & Muller, 1973) and on the 9 normative age samples for the Wechsler Adult Intelligence Scale-Revised (WAIS-R; Wechsler, 1981). For the 26 example data sets Baggaley's estimate of Q was compared with the root mean square off-diagonal correlation as an estimate of Q.

METHOD

Equi-correlation matrices with constant off-diagonal correlation of .10(.10).50 (i.e., .10 to .50 in increments of .10) and with variables numbering 10(10)100 were analyzed. Twenty-six other data sets were also analyzed. The cumulative distribution for -log|R| with finite sample corrections for bias was obtained by evaluating

$$\Lambda_0 = P(-\rho \log|R| < z) = P(\chi^2_m \leq z) - \gamma_2 \{P(\chi^2_{m+4} \leq z) - P(\chi^2_m \leq z)\} - \gamma_3 \{P(\chi^2_{m+6} \leq z) - P(\chi^2_m \leq z)\}$$

where the multiplier $\rho = (N-1-(2k+5)/6)$, γ_2 and γ_3 are negative weights that converge asymptotically to zero, and $m = k(k-1)/2$ is the degrees of freedom. The cumulative distribution for the test based on Fisher's $\tanh^{-1}$ transformation of the correlation coefficient was obtained by evaluating

$$\Lambda_1 = P\left((N-3) \sum_{i=1}^{k} \sum_{j=1}^{k} \tanh^{-1}(r_{ij})^2 < z\right) = P(\chi^2_m \leq z), \quad i < j$$

where $m = k(k-1)/2$ is the degrees of freedom. A test size of .05 was used for both tests and the smallest sample size considered for rejecting P = I was the number of variates plus one so that R would be of full rank.

RESULTS

Results for the 50 equi-correlation matrices that were analyzed are presented in Table 1.

The finite sample corrections for -log|R|, while providing more protection against Type I error rates than without, produced power results that were comparable to those reported by Reddon and Jackson (1984). Baggaley's estimate of Q was biased as a function of the size of the eqi-correlation and number of variates. The regression of Q on r and k resulted in $Q = -.155 + 1.326r + .001k$, $R^2 = .98$. The test of complete independence Λ_1 based on $\tanh^{-1}(r)$ had greater power efficiency than the test Λ_0 based on -log|R| with finite sample corrections.

Results for the 17 data sets analyzed by Hakstian and Muller (1973) are presented in Table 2.

Table 1
RATIO OF OBSERVATIONS (N) TO VARIATES (k) REQUIRED TO REJECT THE HYPOTHESIS OF COMPLETE INDEPENDENCE FOR 50 EQUI-CORRELATION MATRICES (α = .05)

r	Q	k	Λ_0 N/k	Λ_1 N/k
.10	.03	10	20.70	14.00
.10	.05	20	12.35	6.00
.10	.06	30	9.97	3.80
.10	.06	40	8.80	2.78
.10	.07	50	8.10	2.20
.10	.07	60	7.65	1.80
.10	.07	70	7.33	1.54
.10	.08	80	7.08	1.34
.10	.08	90	6.88	1.18
.10	.08	100	6.73	1.06
.20	.10	10	6.90	3.70
.20	.13	20	4.65	1.60
.20	.15	30	3.97	1.03
.20	.16	40	3.65	1.03
.20	.17	50	3.46	1.02
.20	.18	60	3.32	1.02
.20	.18	70	3.23	1.01
.20	.19	80	3.16	1.01
.20	.19	90	3.10	1.01
.20	.19	100	3.06	1.01
.30	.19	10	3.80	1.80
.30	.24	20	2.75	1.05
.30	.27	30	2.43	1.03
.30	.28	40	2.28	1.03
.30	.29	50	2.18	1.02

Table 1 (continued)
RATIO OF OBSERVATIONS (N) TO VARIATES (k) REQUIRED TO REJECT THE HYPOTHESIS OF COMPLETE INDEPENDENCE FOR 50 EQUI-CORRELATION MATRICES (α = .05)

r	Q	k	Λ_0 N/k	Λ_1 N/k
.30	.30	60	2.12	1.02
.30	.31	70	2.07	1.01
.30	.31	80	2.04	1.01
.30	.32	90	2.00	1.01
.30	.32	100	1.98	1.01
.40	.31	10	2.60	1.10
.40	.38	20	1.95	1.05
.40	.41	30	1.77	1.03
.40	.43	40	1.65	1.03
.40	.44	50	1.60	1.02
.40	.45	60	1.55	1.02
.40	.46	70	1.53	1.01
.40	.46	80	1.50	1.01
.40	.47	90	1.48	1.01
.40	.47	100	1.47	1.01
.50	.45	10	2.00	1.10
.50	.54	20	1.50	1.05
.50	.58	30	1.37	1.03
.50	.60	40	1.30	1.03
.50	.61	50	1.26	1.02
.50	.62	60	1.23	1.02
.50	.63	70	1.20	1.01
.50	.64	80	1.19	1.01
.50	.64	90	1.18	1.01
.50	.65	100	1.16	1.01

Table 2

RATIO OF OBSERVATIONS (N) TO VARIATES (k) REQUIRED TO REJECT THE HYPOTHESIS OF COMPLETE INDEPENDENCE FOR 17 DATA SETS (α = .05)

Source Study	k	n	r	RMS(r)	Q	Λ_0 N/k	Λ_1 N/k
Ahmavaara & Markkanen	25	293	.21	.26	.42	1.76	1.04
Bechtoldt (sample 1)	17	212	.31	.36	.55	1.53	1.06
Bechtoldt (sample 2)	17	213	.32	.36	.53	1.59	1.06
Chapman	7	329	.42	.43	.33	2.71	1.57
Davis	9	421	.50	.53	.58	1.78	1.11
Denton & Taylor	13	170	.21	.26	.44	1.85	1.31
Emmett	9	211	.45	.47	.52	1.89	1.11
Fleishman & Hempel	26	197	.38	.42	.77	1.15	1.04
Green et al.	34	283	.25	.27	.38	1.85	1.03
Harman	8	305	.52	.56	.87	1.38	1.13
Harman	13	145	.33	.36	.46	1.85	1.08
Harman	24	145	.30	.33	.48	1.63	1.04
Karlin	8	163	.31	.33	.27	3.00	1.88
Karlin	33	200	.14	.17	.28	2.30	1.15
McLeish	6	100	.25	.28	.18	4.50	3.83
Pemberton	25	154	.29	.33	.54	1.48	1.04
Rimoldi	19	138	.19	.23	.27	2.58	1.21

Note. n = sample size in source study, r = average absolute off-diagonal correlation, RMS(r) = root mean square off-diagonal correlation. See Hakstian & Muller (1973) for source study references.

Baggaley's estimate of Q based on the average absolute off-diagonal correlation was inferior to the root mean square off-diagonal correlation in all but 3 of the 17 data sets. In the 3 cases where Baggaley's estimate was better there were only a small number of variates (Chapman = 7, Karlin = 8, McLeish = 6). The regression of Q on RMS(r) and k resulted in $Q = -.341 + 1.754r + .011k$, $R^2 = .74$. For all 17 data sets Λ_1 had greater power efficiency than Λ_0.

Results for the 9 normative WAIS-R data sets are presented in Table 3.

Table 3

RATIO OF OBSERVATIONS (N) TO VARIATES (k) REQUIRED TO REJECT THE HYPOTHESIS OF COMPLETE INDEPENDENCE FOR 9 WAIS-R DATA SETS (α = .05)

Age	k	n	r	RMS(r)	Q	Λ_0 N/k	Λ_1 N/k
16-17	11	200	.45	.47	.52	1.73	1.09
18-19	11	200	.45	.47	.52	1.73	1.09
20-24	11	200	.45	.46	.51	1.82	1.09
25-34	11	200	.52	.53	.62	1.55	1.09
35-44	11	200	.55	.56	.66	1.55	1.09
45-54	11	200	.53	.53	.64	1.55	1.09
55-64	11	160	.52	.53	.63	1.55	1.09
65-69	11	160	.57	.58	.72	1.46	1.09
70-74	11	160	.48	.49	.57	1.64	1.09

Note. n = sample size in source study, r = average absolute off-diagonal correlation, RMS(r) = root mean square off-diagonal correlation.

For all 9 data sets the root mean square estimate of Q was equal to or superior to Baggaley's estimate. The regression of Q on RMS(r) resulted in Q = -.271+1.694r, R^2 = .98. The Λ_1 test of complete independence had greater power efficiency than Λ_0 for all 9 data sets.

DISCUSSION

Regression analysis indicated that Q = -log|R|/k was not simply equal to the average correlation. That is, Baggaley's estimate of Q or the root mean square estimate of Q were biased as a function of the average correlation and number of variates.

Given that the test of complete independence Λ_1 based on the $\tanh^{-1}(r)$ transformation has acceptable Type I error rates (Reddon, 1987) and given that it has substantially better power than Λ_0, it is the preferred method of determining sample size requirements for rejecting P = I. Sample size requirements previously reported by Baggaley (1982) and Reddon and Jackson (1984) are over estimates. Because, with $i < j$, $\Sigma_{i=1}^{k}\Sigma_{j=1}^{k}\tanh^{-1}(r_{ij})^2$ can be approximated by $k(k-1)/2\tanh^{-1}(\text{RMS}(r))^2$, and also because Λ_1 is a more powerful test than Λ_0, it would be preferable to tabulate power results for Λ_1 instead of Λ_0 as a function of the average off-diagonal correlation.

If the complete independence hypothesis cannot be rejected then there is no justification for conducting a factor analysis because any dependence between the variates may be attributed to sampling error. If the hypothesis of complete independence can be rejected it may be concluded that there is a least one factor.

The number of factors and the stability or reliability of these factors is another issue (cf. Cliff, 1988; Guadagnoli & Velicer, 1988; Krzanowski, 1987). In exploratory factor analytic applications where more than one factor is used Λ_0 may actually be preferable to Λ_1 because Λ_0 will be more conservative and less likely to capitalize on chance.

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