SOURCE CHANNEL SEPARATION IN ENERGY CONSTRAINED MULTI-TERMINAL SOURCE-CHANNEL COMMUNICATIONS SCHEMES

A Thesis by

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I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

Dr. Kamesh Namuduri, Committee Chair

We have read this thesis and
recommend its acceptance:

Dr. Ravi Pendse, Committee Member

Dr. Krishna Krishnan, Committee Member

DEDICATION

To

My Beloved Parents

ACKNOWLEDGEMENTS

I express my sincere gratitude to my advisor Dr. Kamesh Namuduri for sharing his vast knowledge with me and for guiding me with my thesis work at every step. He has been a constant source of inspiration and extremely co-operative. I would also like to thank Dr. Ravi Pendse and Dr. Krishna Krishnan for sparing time to review my thesis work.

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ABSTRACT

Source coding and channel coding are two important parts of a communications system. Source coding deals with the compression of source data, while channel coding adds some redundancy to circumvent the channel errors. Source coding and channel coding can be done either jointly or separately depending on the design issues like complexity of the system, rate, power, distortion tradeoffs. For optimal performance, all the parameters should be carefully chosen and optimized.

This thesis looks at the scenarios of multi-terminal communications model where source and channel separation will not degrade the performance of the system in terms of rate power and distortion tradeoffs. Specifically, we investigate the transmission of data from correlated sources over an orthogonal multiple access channel under a total power constraint.

It is observed that source-channel separation in this particular scenario is possible without any loss in the performance of the system. For the correlated sources case of the multi-terminal communications, we provide an alternate approach for achieving source-channel separation.

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Chapter 1

Introduction and Preview

1.1 Significance of the Problem

Design of a communication system involves issues related to source coding, channel coding, rate, power and distortion tradeoffs. Design of these parameters involve source and channel characteristics. The design parameters can be chosen either jointly or separately for source and channel, leading to joint source-channel coding and separate source-channel coding respectively. Joint source-channel coding is a well known problem and has been addressed by several researches for various communications model [1] [2] [3]. Separation of source-channel is of significant importance, as it reduces the complexity of the system without degrading the performance.

Claude Shannon in 1948 [4] proved that source coding and channel coding processes can be separated for stationary memoryless sources and channels without any performance degradation in terms of rate and distortion of the communication system. The separation theorem states that if the minimum coding rate of the source that can be achieved is strictly below the capacity of the channel, then that source can be transmitted reliably over the channel. There is a converse part to this theorem that states that - reliable transmission of the source data over a channel is impossible if the source coding rate is strictly greater than the channel capacity.

Since then much work has been done to extend the results of the theorem to a more general class of sources and channels [5] [6] [7]. Separation of source and channel coding makes it easier to implement the communications system with reduced complexity. This is more relevant for a sensor network model as there are a number of sensors which send a huge amount of data to one or more destinations. Hence the complexity of the system increases as the number of sensors (sources) increase.

1.2 Sensor Network Model

A sensor network is a group of sensors that observe a certain physical phenomenon and send their observations to an intended receiver. In a sensor network the sensors are usually employed in large numbers and hence the data associated with them is large. Since the power and bandwidth available for the sensors is limited, source and channel coding become very crucial in sensor networks. Much work has been done in this context, and approaches in which source coding and channel coding can be done jointly or separately have been presented in the literature [1] [3] [8].

The block diagram of a typical sensor network is shown in Figure 1.1. In this illustration, a group of sensors observe a certain physical phenomenon and transmit their observations to a local base station. The base station is located closely to the sensors and it itself can be a sensor making its own observations. The base station then transmits this data to an intended receiver, which usually is located relatively far. The reason for this two step communication between the sensors and the receiver is, the power level at the sensors is very low and sensors cannot afford the cost of transmission to the receiver on their own. The base station, that has more power

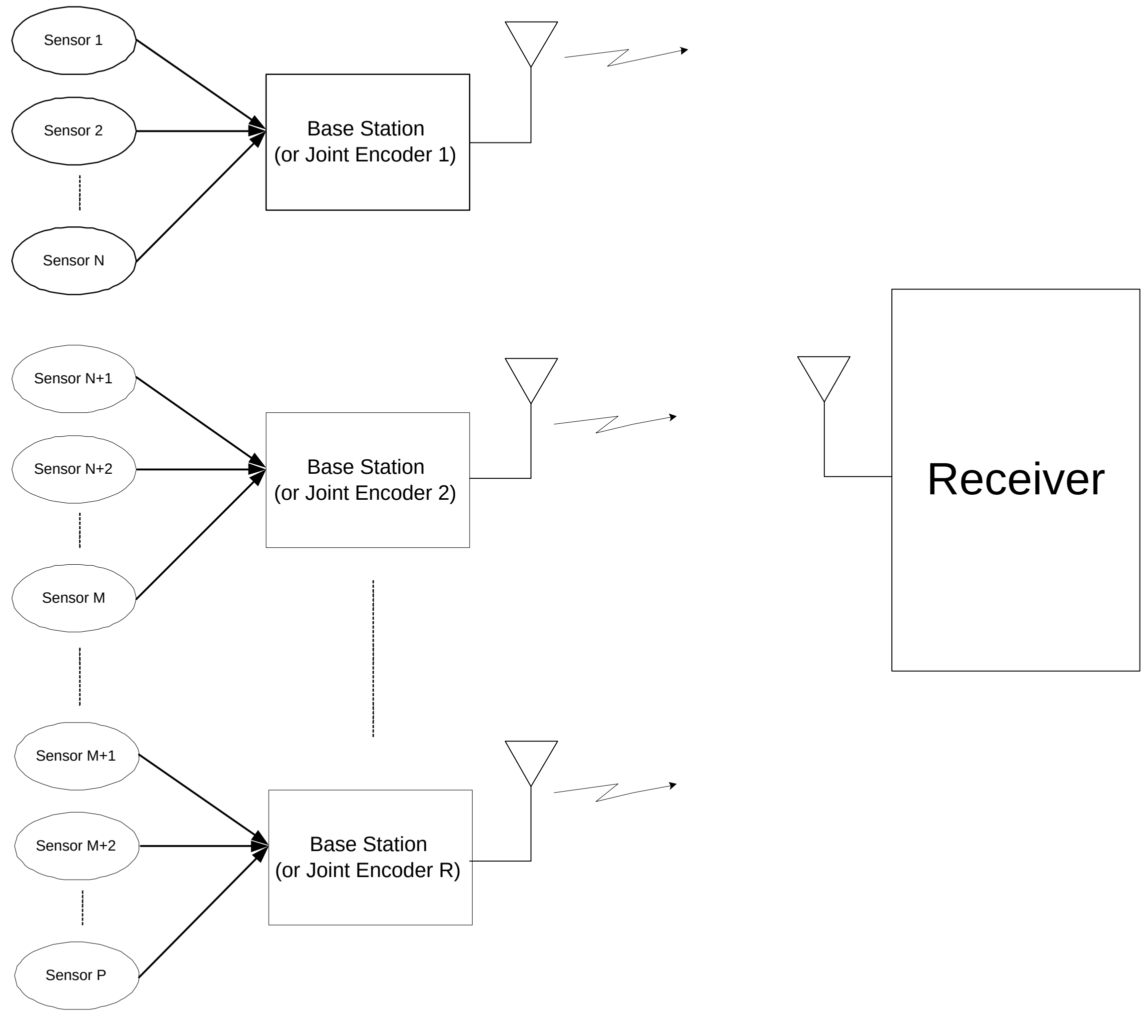

Figure 1.1: Block Diagram of a typical Sensor Network.

than the rest of the sensors can afford the communication cost of all the sensors, collects all the observations and retransmits them to the receiver. Since the sensors are observing the same physical phenomenon, the data from the sensors is highly correlated and hence is highly redundant. To reduce the redundancy, the sensors have to coordinate with one another and exploit the correlation to reduce data. Since the distance between the sensors and the base station is usually very small, the channel between them can be assumed to be lossless, i.e., there is no noise, fading or interference present in the channel and that the observations from the sensors are received without any loss at the base station. Hence in source coding, the channel characteristics do not play any significant role. Therefore in this case source coding can be done separately from channel coding.

The communication link between the base station and the receiver is relatively very long and the noise level, fading and/or interference in the channel cannot be neglected in this case. If the base station employs orthogonal communication and the channel is noise is additve white Gaussian noise (AWGN) channel without any fading, source coding can be separated from channel coding [9]. By doing so the system outperforms the joint source-channel coding system for this case. If on the other hand the communication scheme is not orthogonal, it has been shown in [6] that the system performs better without any source coding or channel coding.

In this thesis, cases when separation doesn't degrade the performance of the system are investigated. Specifically, various issues related to source-channel separation in a multi-terminal communication system in presented.

1.3 Contributions of the Thesis

The following contributions to the multi-terminal source coding are made in this thesis.

1. The problem of source-channel separation is investigated in a multi-terminal communications system.

2. Scenarios in which source-channel separation will not degrade the performance of the system are presented.

3. Tradeoffs among communication system parameters such as rate, power and distortion are investigated for the multi-terminal communications model and some experimental results supporting our observations are presented.

4. For the correlated case, an alternate approach for separation based on the Karhunen-Loeve Transform is presented.

1.4 Thesis Organization

The remainder of the report is organized as follows. In Chapter 2 background work is presented. In Chapter 3 the theoretical model and key results are presented. In Chapter 4 experimental results are presented. Conclusions and future work are discussed in Chapter 5.

Chapter 2

Literature Review

One of the objectives behind separation of source and channel coding is to reduce the design complexity of the communication system. By separating source coding from channel coding, one can implement them as separate processes. Source-channel separation should ensure that the performance of the system does not degrade.

Significant research has been done in this field. Shannon presented mathematical definitions for sources and channels, and discussed source coding and channel coding for discrete as well as continuous cases. He defined entropy (of a source), channel capacity, and mutual information. Shannon presented several theorems related to source coding, channel coding and joint source-channel coding [4]. One of the fundamental theorems is the source coding theorem, which tells that a source can be transmitted over a channel reliably only if the entropy of the source is less than or equal to the capacity of the channel.

Vembu et al. revisited the source-channel coding theorem [10]. They give a practical example where theorem fails to hold. The example shows that the knowledge of the minimum source coding and the maximum capacity of the channel is not sufficient (as was earlier proposed) for reliable transmission. They propose a new set of necessary and sufficient conditions that are required for reliable source-channel communication. They do not impose restrictions on the nature of sources and channels such as memorylessness, stationarity, ergodicity, causality, information stability, etc,. Also they characterize a class of channels for which the separation theorem holds regardless the nature of the sources.

Issues related with allocation of the available bandwidth for a CDMA system are discussed in [11]. Allocation of more bits for source coding increases the quality of the source, but it reduces the number of bits available for channel coding, hence reducing the reliability of transmission over the noisy channel. Allocation of more bits for channel coding, increases the reliability of transmission, but at the cost of reducing the number of bits available for source coding and hence the quality of the source data. They come up with an optimal allocation strategy that involves a tradeoff for source coding and the channel coding rates. The performance of the system with the proposed bandwidth allocation strategy is compared for two systems - one system employing QPSK modulation, with linear block channel codes and minimum mean square error (MMSE) receiver at the decoder, and the other system employing a BPSK modulation scheme, with rate compatible punctured convolution codes (RCPC), and a soft decision Viterbi decoder at the decoder. In both the models the channel is modeled as an additive white Gaussian noise (AWGN) with a flat Rayleigh fading.

Acharya et al. consider a multi-terminal source channel communications system with correlated sources to come up with optimal allocation of power under a total rate and power constraints [12]. They consider a sensor network model where the sensors observe a common phenomenon and send their observations to a central node. Since all the sensors observe the same common phenomenon the data from the sensors is highly correlated. The sensors transmit using a unique codeword allocated to them (like in a CDMA setup). They find out the optimal set of code-words, that minimize the end to end TMSE of the overall system under total rate and power constraints. It is shown that the optimal code-words are the eigen vectors of the source correlation matrix and the optimal power allocation strategy is to waterfill the eigen values of the correlation matrix over the total available power. This is of significance as it shows that for correlated sources the optimal power allocation strategy and the optimal codeword set depends upon the correlation structure of the sources or the statistics of the sources.

Source-channel coding problem in a multi-terminal source-channel communications system where the sources may or may not be correlated is considered in [9]. They propose a source-channel separation theorem which states that separate source and channel coding performs better than uncoded schemes in multi-terminal source channel systems with orthogonal multiple access channel. The paper discusses the various tradeoffs between rate-power (cost functions) and distortion, for a generalized pair of sources and channels. They consider an example of a quadratic Gaussian sensor network with orthogonal multiple access channel and show that separation of source and channel coding outperforms the uncoded scheme.

Chapter 3

Theoretical Model

3.1 Communications system overview

Source coder/decoder and channel coder/decoder are essential blocks of a communications system. The function of the source coder is to map the source symbols (that may be binary or non-binary) to the minimum possible binary data. The theoretical limit for the minimum rate is given by the entropy of that source. The channel encoder's role on the other hand is to provide reliability for the data to be transmitted over a noisy channel. The channel encoder adds error correcting code (ECC) bits, that can be used to provide reliability to the transmitted data.

In the case of multiple sources and multiple channels, the source encoder and the channel encoder have other roles to play. The source encoder allocates the available coding rate among the sources subject to a given end-to-end distortion. The channel encoder allocates the total power available at the transmitter subject to a total power constraint. The power allocation strategy depends on the channel conditions. A source-channel communication scheme requires the design of rate and power allocation strategies under a total power constraint at the transmitter and the total allowable distortion at the receiver. Figure 3.1 shows the basic elements of a communications model. Referring to this Figure the symbol S represents the source data that is encoded and transmitted at the transmitter and $\hat{\mathrm{S}}$ represents the data received at the receiver. If mean square error is the distortion measure, the end-to-end distortion for the system is given by $(\mathrm{S} - \hat{\mathrm{S}})^2$.

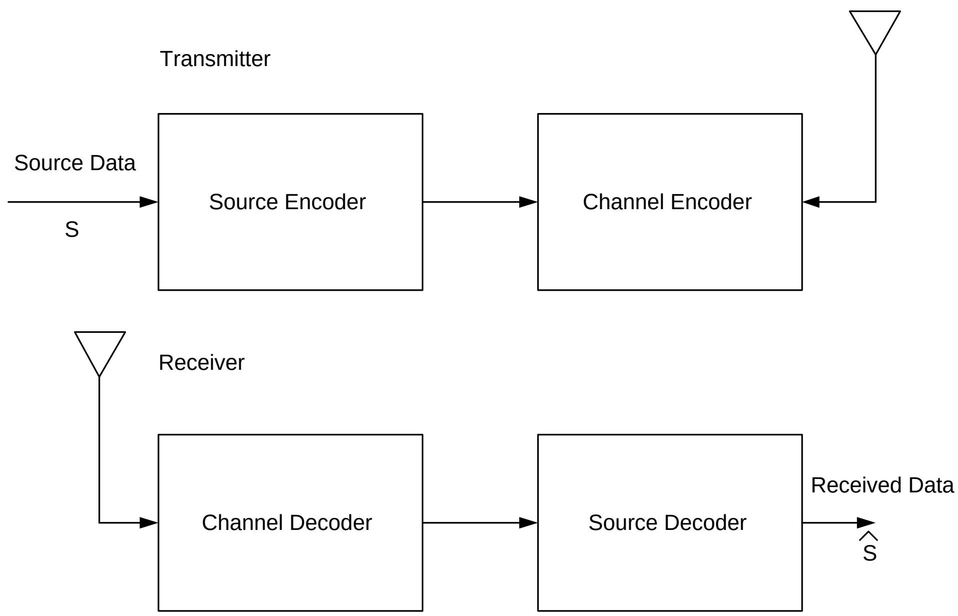

Figure 3.1: Communications system model

The next section begins with the definitions of key concepts in the multi-terminal communication scheme. Then we present the rate and power allocation schemes for different scenarios of multi-terminal communication system.

3.2 Definitions

3.2.1 Mutual Information

Mutual information between two random variables is the measure of the mutual dependencies between them. In other words, it gives a measure of the common information shared between the two random variables. Mutual information is defined in [13] as follows. Let X and Y be two random variables and $I(X;Y)$ denote the mutual information between X and Y. The mutual information is then given by

$$I(X;Y) \;=\; \sum_{y\epsilon Y}\sum_{x\epsilon X} p(x,y)\log_2\frac{p(x,y)}{p(x)p(y)}, \tag{3.2.1}$$

where $p(x)$ and $p(y)$ are the probability density functions of the random variables X and Y respectively and $p(x,y)$ represents the joint probability density function of X and Y. If X and Y are independent then the mutual information between the two would be zero.

3.2.2 Multiple Access Channel

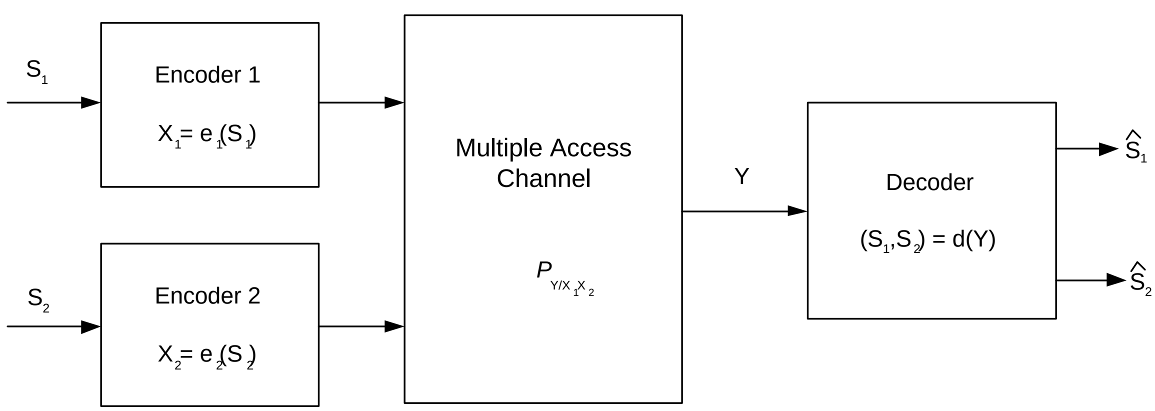

Figure 3.2: Communications model for multiple source and multiple access channel.

Multiple access channel is a channel in which more than one user accesses the channel. The channel can be shared in terms of time, frequency or bandwidth. The multiple access channel is shown in Figure 3.2. In this Figure, S_1 and S_2 are the sources, e_1 and e_2 are the encoding functions and d is decoding function. The multiple access channel is described by

$$(X_1, X_2, Y, P_{Y/X_1,X_2}) \tag{3.2.2}$$

where X_1 and X_2 are inputs to the channel and Y is the output of the channel.

3.2.3 Rate

Rate or coding rate can be defined as the number of bits required to represent one source symbol. The units for rate is bits per symbol.

3.2.4 Capacity

Channel capacity can be defined as the amount of information that can be transmitted reliably over the channel. Figure 3.3 illustrates a single source S transmitting over and noisy channel denoted by N. In this Figure M is the input random variable to the encoder and X is encoded symbol to be transmitted over the channel. The output of the channel is denoted by Y and $\hat{M}$ is the decoded output. The random variable $\hat{S}$ is the reconstructed source symbol at the receiver. The channel capacity in this case is defined as

$$C = \sup_f I(X;Y), \tag{3.2.3}$$

where f is the probability distribution of the random variable X.

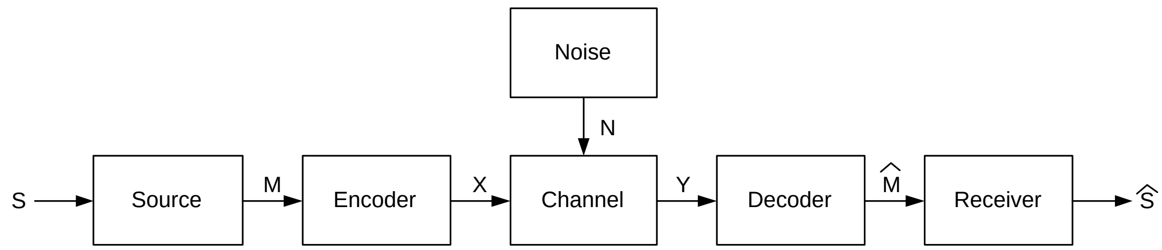

Figure 3.3: An end to end communications system model with AGWN noise.

3.3 Multi-terminal Source Channel Communications Model

In the following discussion all vectors are denoted by bold lower case letters, and are assumed to be column vectors. Scalars are denoted by lower case letters. Matrices are denoted by bold uppercase letters and random variables by capital italicized letters. Also the expression $\mathrm{E}\{X\}$ denotes the expectation (or the averaging) operator over the random variable X. The covariance of X and Y is denoted by $cov(X,Y)$ and is defined by $cov(X,Y) \triangleq \mathrm{E}\{X.Y^T\} - \mathrm{E}\{X\}\mathrm{E}\{Y\}^T$

3.3.1 System Model

The multi-terminal source channel communication model is described as follows. There are 'M' correlated (continues) sources $(U_1, U_2, \ldots, U_M)$ whose sample values at time instance t are given by $\mathbf{u} = (u_1(t), u_2(t), \ldots, u_M(t))$. For simplicity, it is assumed that the sources are spatially correlated, and there is no temporal dependence between them. Hence the time index t can be dropped. The source samples can be written as $\mathbf{u} = (u_1, u_2, \ldots, u_M)$. It is assumed that the samples follow a Gaussian distribution with zero mean and covariance matrix $\mathbf{B}$, i.e., the pdf of $\mathbf{u}$ is given by $\mathcal{N}(\mathbf{0}_M, \mathbf{B})$

where the normalized cross-correlation matrix is given by

$$\mathbf{B} \;=\; E[\mathbf{b}\mathbf{b}^T] = \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1M} \\ \rho_{21} & 1 & \dots & \rho_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{M1} & \rho_{M2} & \dots & 1 \end{bmatrix}. \tag{3.3.1}$$

The variables ρ_{ij} represent the correlation coefficient between i^{th} and the j^{th} sensors.

Each of the sample values $(u_i's)$ is quantized and modulated using a binary phase shift keying (BPSK) modulator. It is assumed that a scalar quantizer (Q) having 2^q quantization levels, denoted by the set $\mathrm{Q}_L = \{1, 2, \dots, 2^q\}$. Each sample of the continuous source is quantized and is approximated by q bits. The quantizer output is represented by the vector of indices denoted by $\mathbf{I} = (\mathrm{I}_1, \mathrm{I}_2, \dots, \mathrm{I}_M)$. The BPSK modulator maps these indices I_k's into q bit channel input words $X_k = (\mathrm{X}_{k,1}, \mathrm{X}_{k,2}, \dots, \mathrm{X}_{k,q})$ where $\mathrm{X}_{k,q} \epsilon \{-1, +1\}$. The multi-terminal source channel communication model considered in this paper is as shown in 3.4.

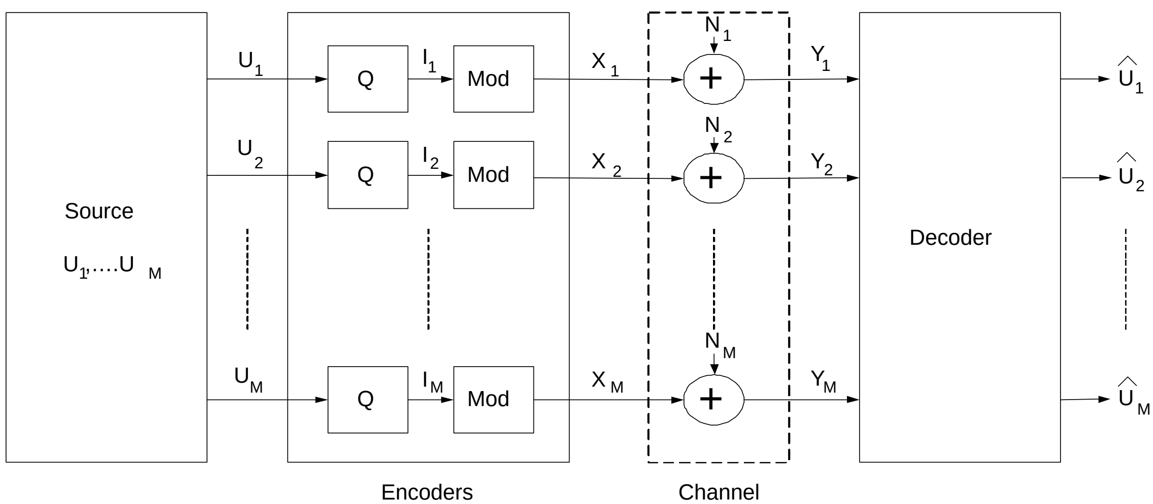

Figure 3.4: Multi-terminal source channel communications model

Since the sensors access the medium at the same time, we need a reservation based medium access protocols such as time division multiple access (TDMA) or frequency division multiple access (FDMA), so that we can assume no interference between transmitters. Each of the modulator outputs get into the decoder through channel, so we can model the channel as an array of M independent additive white Gaussian noise channels with equal average noise power N_0. For each channel k and input word X_k the channel output signal is given by $Y_k = X_k + \eta_k$ and $Y_k = (\mathrm{Y}_{k,1}, \mathrm{Y}_{k,2}, \ldots, \mathrm{Y}_{k,q})$. The noise samples considered in this thesis are i.i.d Gaussian with pdf $p(\eta_{k,q}) = N(0, \mathbf{R})$, where $\mathbf{R}$ is a diagonal matrix.

3.4 Separation of Source and Channel Coding

The Shannon's coding theorem for point to point communication for a noiseless channel says that a source (U) can be reliably transmitted over a channel with arbitrarily low probability of error if

$$H(U) \; < \; \mathrm{C} \; = \sup_{P_X} I(X;Y) \tag{3.4.1}$$

where X is the channel input and Y is the channel output. The result of this theorem can be extended to a multiple access channel case where there are more than one sources transmitting over a common noiseless channel. In this case the sources can be reliably transmitted over the channel if

$$H(U,V) \; < \; \mathrm{C}_{joint} \quad = \quad \sup_{P_{X_1 X_2}} I(X_1, X_2; Y) \tag{3.4.2}$$

$$H(U,V) \; < \; \mathrm{C}_{separate} \quad = \quad \sup_{P_{X_1} P_{X_2}} I(X_1, X_2; Y) \tag{3.4.3}$$

where U, V are the sources, X_1, X_2 are the channel inputs and Y is the channel output. For the multiple access model, the source coding and channel coding can be done either jointly as shown in equation (3.4.2) or separately as in equation (3.4.3). The block diagrams for joint and separate source channel coding are shown in the Figures 3.5 and 3.6 respectively.

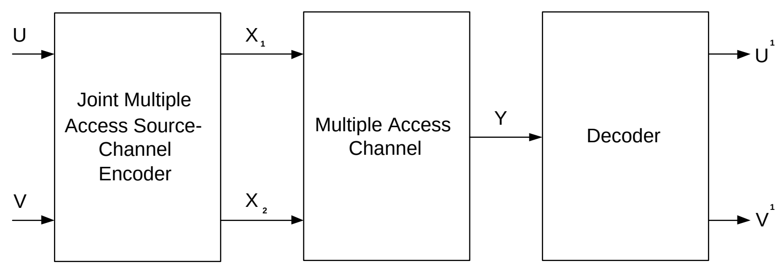

Figure 3.5: Multi-terminal source channel communications model with joint source channel coding.

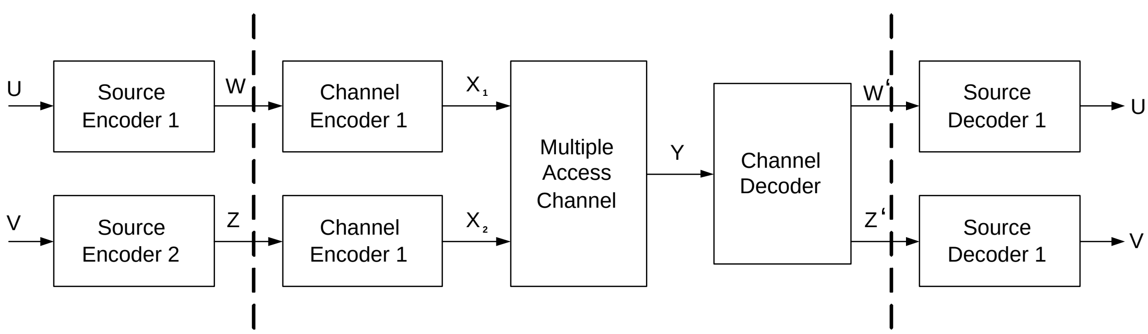

Figure 3.6: Multi-terminal source channel communications model with separate source channel coding.

In Figure 3.6, for separation to hold good it is required that the performance of the system does not degrade due to separation as compared with the joint source channel coding. It has been shown in [8] that for separation to hold good in a multiple access channel, the necessary and sufficient condition is

$$\mathbf{C}_{separate} = \mathbf{C}_{joint}. \tag{3.4.4}$$

Typically source-channel separation is considered in terms of separating source coding from channel coding, i.e., the rate at which the source is being coded and the maximum rate at which the sources can be transmitted over the channel without significant loss of data. However, in communications systems, source-channel separation impacts design issues beyond source and channel coding. For instance, the way power and channel bandwidth are allocated to the sources. In the following section two cases of the multi-terminal source channel model is considered. In the first scenario, uncorrelated sources are transmitted over an orthogonal multiple access channel and in the second scenario, correlated sources are transmitted over orthogonal multiple access channel. It is shown that the performance of the system is not degraded by separation of source and channel coding.

3.4.1 Uncorrelated sources over orthogonal multiple access channel

When the sources are uncorrelated the covariance and the correlation matrices of **??** and 3.3.1 would be diagonal matrices. The noise in the channel is assumed to be additive white Gaussian with uniform two sided power spectral density of $\frac{N_0}{2}$. Power allocation in this case is uniformly done. Each bit is allocated the same power level, while the rate is allocated as per the variances of the sources as discussed below.

The optimal bit allocation problem for i.i.d. Gaussian $\mathcal{N}(0, \sigma_i^2)$ has been discussed in [13]. The optimal bit allocation scheme known as the reverse water-filling scheme, in which sources whose variances cross a certain threshold (λ) are transmitted, and the ones below the threshold are not transmitted. This water-filling scheme is illustrated in Figure 3.7. The threshold value (λ) depends on the available sum rate (R) and total distortion (D).

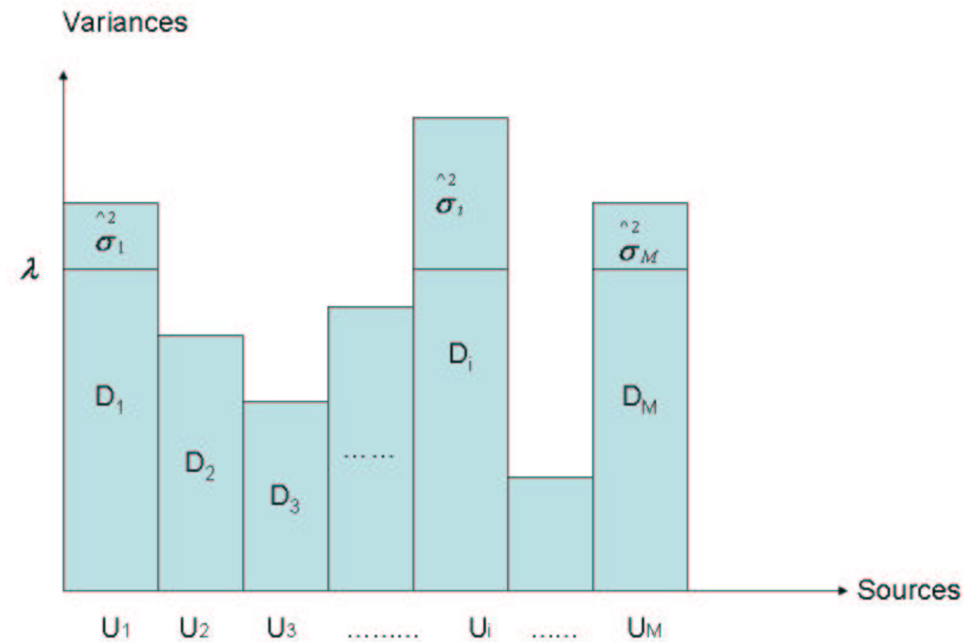

Figure 3.7: Rate allocation using reverse water-filling of the source variances.

In the case of uncorrelated sources, $U_i \backsim \mathcal{N}(0, \sigma_i^2)$, i = 1, 2, ..., M are identically distributed gaussian sources, D_i, i = 1, 2, ..., M is the allowable distortion for each source. The rate distortion function for this case can be written as

$$R(D) = \sum_{i=1}^{M} \frac{1}{2} \log_2 \left(\frac{\sigma_i^2}{D_i} \right) \tag{3.4.5}$$

where

$$D_i = \begin{cases} \lambda & \text{if } \lambda < \sigma_i^2 \\ \sigma_i^2 & \text{if } \lambda \geqslant \sigma_i^2 \end{cases} \tag{3.4.6}$$

The strategy for source-channel coding is as follows. First λ is chosen based on the available rate and allowable distortion. Then starting from the sources with maximum variance is allocated bits above the threshold λ till the available rate is exhausted.

In this case, allocation of rate to the sources and hence source coding depends only upon the source statistics. They do not depend on the channel properties. Also, power allocation and channel coding is done independent of the source statistics. Thus, source coding and channel coding are separable.

3.4.2 Correlated sources over orthogonal multiple access channel

This case has been investigated to some extent in [6] [9] [13] [**?**]. In this case the sources are assumed to be correlated and they are transmitted over an orthogonal multiple access channel. The optimal power allocation strategy has been shown to be dependent on the source statistics. Let us assume 'M' transmitters, transmitting symbols using unit norm codewords of length 'L' over an AWGN channel with uniform two sided power spectral density of $\frac{N_0}{2}$ dB.

The received signal at the receiver can then be written as

$$\mathbf{r} = \mathbf{S}\mathbf{P}^{\frac{1}{2}}\mathbf{b} + \eta. \tag{3.4.7}$$

The matrix $\mathbf{P}$ is a diagonal matrix given by $diag(p_1, p_2, \ldots, p_M)$ and p_i is transmitted power of the i^{th} transmitter. The variable $\mathbf{S}$ represents the L×M signature waveform matrix, $\mathbf{b}_{M\times 1}$ is a vector of input symbol to be transmitted, η represents the additive white Gaussian noise (AWGN) component, with uniform two sided power spectrum density of $\frac{N_0}{2}$ dB. Let $\mathbf{B}$ represent the normalized cross-correlation matrix defined by $\mathbf{B} = E\{\mathbf{b}\mathbf{b}^T\}$. The total mean square error (TMSE) is then shown to be [12]

$$\begin{aligned} \text{TMSE} &= \sum_{i=1}^{M} \text{MSE}_i && (3.4.8) \\ &= tr(\mathbf{C}^T\mathbf{S}\mathbf{P}^{\frac{1}{2}}\mathbf{B}\mathbf{P}^{\frac{1}{2}}\mathbf{S}^T\mathbf{C} + \sigma^2\mathbf{C}^T\mathbf{C} - 2\mathbf{C}^T\mathbf{S}\mathbf{P}^{\frac{1}{2}}\mathbf{B} + \mathbf{I}_M). && (3.4.9) \end{aligned}$$

The variable $\mathbf{C}$ represents the receiver filter and $\mathbf{I}_M$ is an identity matrix of dimension $M \times M$.

The optimal signature sets in this case are shown to be the eigen-vectors of the normalized cross correlation matrix $\mathbf{B}$. This is obtained by minimizing TMSE over $\mathbf{S}$, $\mathbf{P}$ and $\mathbf{C}$ (the receiver filter) under a total power constraint, i.e., the optimization problem in this case is

$$\min_{\mathbf{S},\mathbf{P},\mathbf{C}} \{\text{TMSE}\} \text{ subject to } tr(\mathbf{P}) = \mathbf{P}_{tot} \tag{3.4.10}$$

The optimal $\mathbf{C}$ which minimizes the TMSE is the MMSE receiver. Using this the optimization problem is reduced to

$$\min_{\mathbf{A}\epsilon\mathbf{A}} tr\{(\sigma^2\mathbf{B}^{-1} + \mathbf{A}^T\mathbf{A})^{-1}\} \tag{3.4.11}$$

where, $\mathbf{A} = \mathbf{S}\mathbf{P}^{\frac{1}{2}}$ is a set of all L×M matrices such that $tr(\mathbf{A}^T\mathbf{A}) = \mathbf{P}_{tot}$.

The optimal signature sets and the power allocation scheme are then achieved by majorizing the eigen values of the $\mathbf{B}^{-1}$ matrix and by water-filling the eigen-values of the $\mathbf{B}^{-1}$ with those of the $\mathbf{A}^T\mathbf{A}$. In this process some of the eigen values of the A matrix are set to zero in order to meet the total power constraint, as shown in the Figure 3.8.

Thus power allocation is achieved by water-filling of the eigen-values of the B^{-1} matrix. Once the power is allocated to each source we allocate the rate to each one (rate allocation done by jointly encoding of the sources as per joint entropy or other means since the sources are correlated). This approach is an example of joint source-channel coding, where power allocation and channel coding are done based on the source statistics.

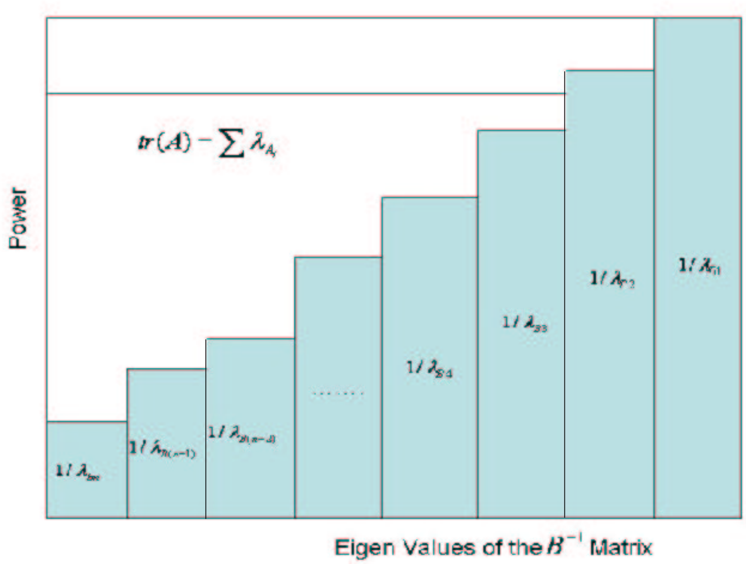

Figure 3.8: Water-filling of the eigen values of the B^{-1} matrix over the total power

3.4.3 Separate source-channel coding

In this section we suggest an alternative separate source channel coding approach to the joint source channel coding method discussed in section 3.4.2. When the symbols from the sources are correlated we can make use of the KL transform to de-correlate the sources. The cross-correlation matrix **B** can be split using singular value decomposition as

$$\mathbf{B} = \mathbf{U}\Lambda\mathbf{U}^{-1}, \tag{3.4.12}$$

where **U** is a matrix composed of eigen-vectors of **B** along its columns, Λ is a diagonal matrix $diag(\lambda_1, \lambda_2, \ldots, \lambda_M)$, where ($\lambda_i$ is the i^{th} eigen value, and $\mathbf{U}^{-1}$ is the matrix inverse of **U**. Now consider the transformation

$$\hat{\mathrm{b}} \;=\; \mathrm{U}^{-1}\mathrm{b}. \tag{3.4.13}$$

Then the new cross-correlation matrix after the transform becomes

$$\begin{aligned}
\hat{\mathrm{B}} &= E\{\hat{\mathrm{b}}\hat{\mathrm{b}}^T\} \\
&= E\{\mathrm{U}^{-1}\mathrm{b}\mathrm{b}^T(\mathrm{U}^{-1})^T\} \\
&= \mathrm{U}^{-1}E\{\mathrm{b}\mathrm{b}^T\}(\mathrm{U}^{-1})^T \\
&= \mathrm{U}^{-1}\mathrm{B}(\mathrm{U}^{-1})^T \\
&= \Lambda. \qquad (3.4.14)
\end{aligned}$$

That is, the sources are uncorrelated of one another and the transformed data is majorized (since $\lambda_i > \lambda_j$ for $i > j$). Source and channel coding, in this case can be done independently following the procedure in section 3.4.1.

Chapter 4

Experimental Results

In this section we give the results of the experiments that were carried out for optimal rate allocation and power allocation strategies for the cases discussed in section 3.4.1, 3.4.2 and 3.4.3.

In section 3.4.1, for the case of uncorrelated sources the optimal rate allocation strategy is to allocate the rate to the various sources depending upon their variances, i.e., the source with the maximum variance is given more rate as compared to the source with a lower variance. This scheme was compared with an arbitrary scheme in which we allocate rate uniformly among the sources irrespective of their variances. The results of the two schemes are plotted in 4.1. From the Figure it can be seen that the optimal rate allocation scheme discussed in section 3.4.1 performs better than the arbitrary scheme.

In order to simulate the approach discussed in section 3.4.2, we consider arbitrary correlated sources. The power allocation is then obtained by water-filling of the B^{-1} matrix and the eigen vectors form the signature sets. This is compared with an arbitrary power allocation scheme with orthogonal signatures, and arbitrary signatures. The results are given in Figures 4.2 and 4.3.

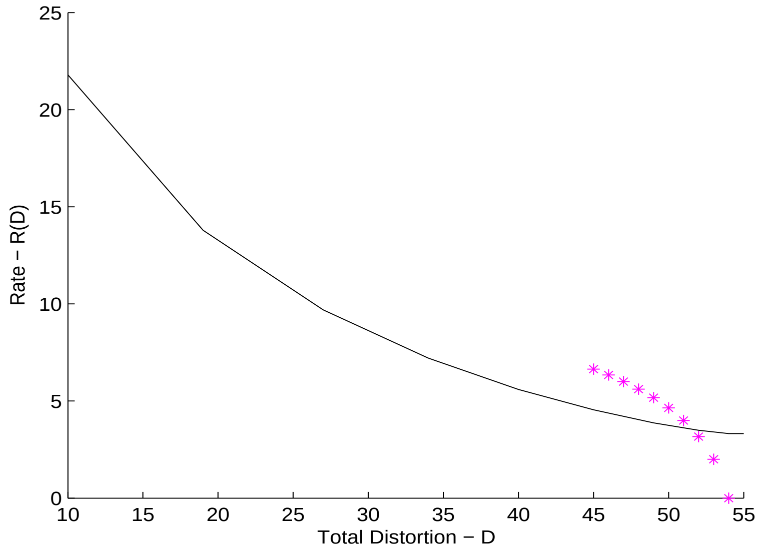

Figure 4.1: Rate-Distortion plots: Comparison of rate distortion regions for optimal and arbitrary rate allocation strategies. The optimal rate allocation is based on the reverse-water filling scheme and the arbitrary rate allocation scheme considers uniform rate allocation for all the sources.

Figure 4.2 compares MSE values obtained for scheme mentioned above, while Figure 4.3 gives the PSNR (peak signal to noise power ratio) values in dB for the three schemes. From the results it can be seen that the optimal power allocation scheme results in the least mean square error and gives best peak signal to noise ratio levels as compared with the other schemes. Figures 4.4 and 4.5 compares the performance of the joint coding scheme discussed in section 3.4.2 with that of the separate source channel coding scheme described in section 3.4.3. It is seen that both the systems perform equally good and there is no performance loss in the system by separating source coding from channel coding.

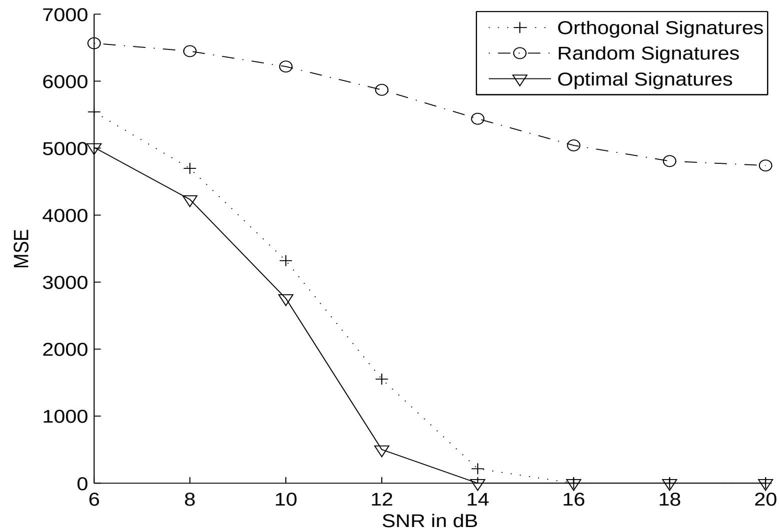

Figure 4.2: Comparison of MSE for the optimal and arbitrary schemes.

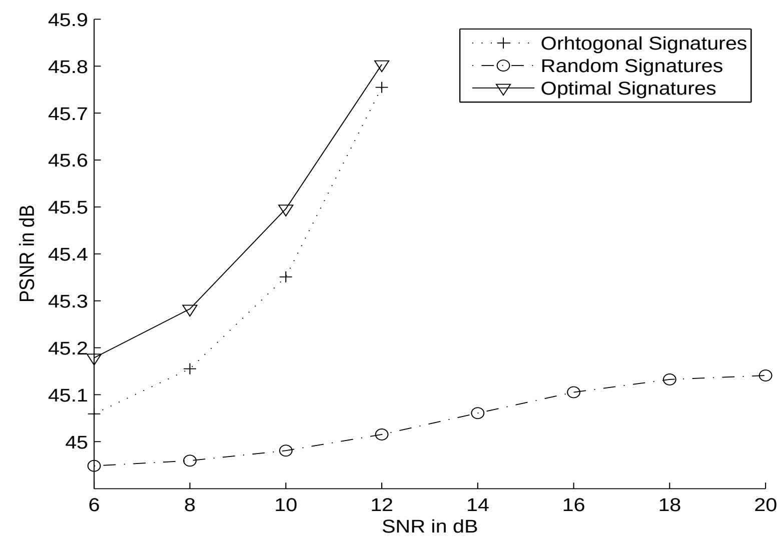

Figure 4.3: Comparison of PSNR for the optimal and arbitrary schemes.

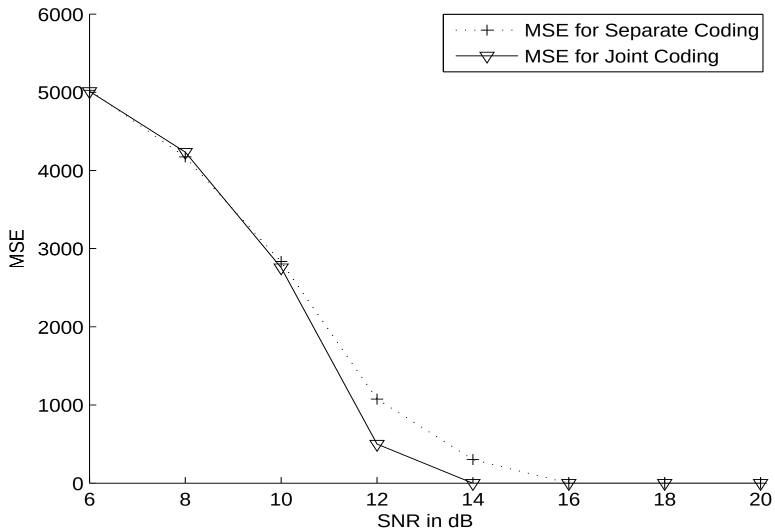

Figure 4.4: MSE comparison for joint and separate source-channel coding strategies.

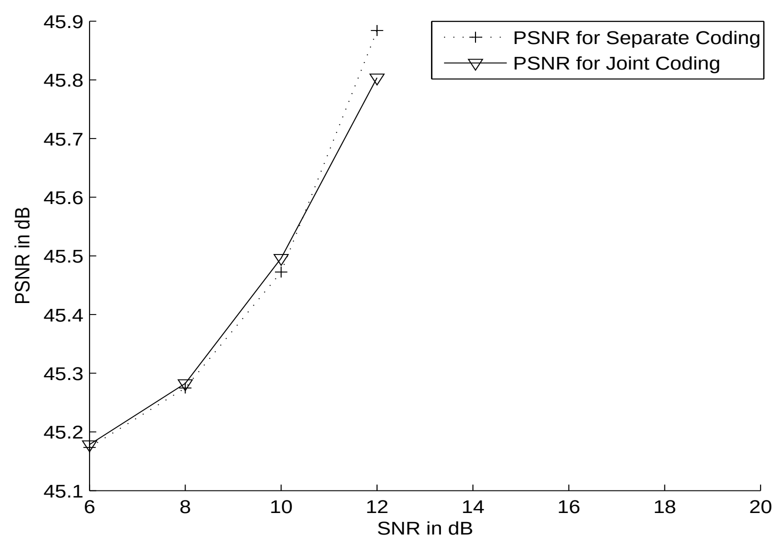

Figure 4.5: PSNR comparison for joint and separate source-channel coding strategies.

Chapter 5

Conclusion and Future work

In this thesis we investigated the scenarios in a sensor network model where source coding can be separated from channel coding. Specifically the multi-terminal communications model is considered, and the scenarios where source-channel separation is feasible are investigated. The advantage with this approach is that it simplifies the communication system. The future work for this thesis would be to extend the results obtained in this work to a more generalized case of sensor networks with different channel models.

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