

# A Bayesian Approach for Verifying Process Improvement

Nasser Safaie\*

*Department of Industrial and Manufacturing Engineering, College of Engineering*

**Abstract.** This research is an effort to introduce the Bayesian approach as a tool for evaluating process adjustments aimed at causing a shift to the process average. This is usually encountered in scenarios where the process is found to be stable and centered away from the design target. A number of changes is proposed and tested as part of the improvement efforts. As such, it is desired to evaluate the effect of these changes as soon as possible and take appropriate actions. Using simulated data, the performance of the proposed approach is measured in terms of the number of measurements (iterations) needed to estimate the true average. The results indicate that the Bayesian approach can be used to verify process improvement.

## 1. Introduction

Control charts were introduced by Dr. Shewhart in 1920's and involve two phases [1]. In phase I, a set of historical data is analyzed to assess stability and identify special causes. If no special causes are present, the in-control process parameters are estimated and control limits are established. In phase II, the data are sequentially collected over time to assess whether the performance has changed from the estimated value. However, there are some scenarios where the process is found to be centered away from the desired target and changes are made to improve its performance. As such, it would be desirable to quantify the effect of the proposed changes before a full scale implementation. Under these conditions, users of the traditional Shewhart control chart have one of two options; either to assume complete ignorance of the new process parameters (i.e., repeat phase I), or utilize the target as standard value for phase II. The first requires accumulating a new set of data which may be time-consuming, while the second may result in a reject decision short of providing an estimate of the true parameter. The purpose of this paper is to evaluate the Bayesian approach as a third alternative. In this method, earlier practical experience can be taken into account explicitly to help the practitioner adjust the parameters and transfer faster to phase II application of the Shewhart chart.

## 2. Theoretical Modeling

During the transition period, it can be assumed that  $P(\mu)$  represents the prior with mean  $\mu_0$  and standard deviation  $\sigma_0$ . If  $y$  denotes a process measurement with unbiased mean and standard deviation (gage repeatability)  $\sigma_g$ , then, according to Box and Tiao [2], the posterior  $P(\mu|y)$  will follow the normal with mean  $\mu_1$  and standard deviation  $\sigma_1$  given by:

$$\mu_1 = (1 / (w_0 + w_1))(w_0\mu_0 + w_1y), \text{ and } (1 / \sigma_1^2) = w_0 + w_1$$

Where,

$$w_0 = 1 / \sigma_0^2 \text{ and } w_1 = 1 / \sigma_g^2$$

As pointed out in [2], this is an appealing result, since the reciprocal of the variance is a measure of information which determines the weight to be attached to a given observation. The variance of the posterior distribution is the reciprocal of the sum of the two measures of information  $w_0$  and  $w_1$  reflecting the fact that the two sources of information are pooled together.

As an illustration, suppose a process improvement team is considering two levels of performance. The current stable level,  $L_0$  where the prior is approximated by  $N(800, 50^2)$ , and the second level  $L_1$ , represents an improved level which is anticipated to be  $N(950, 50^2)$ . Process changes will be made during the transition period and individual observations collected sequentially. The measuring instrument is known to produce data with  $\sigma_g = 10$ . Here the uncertainty of the measurement is assumed constant but the process average for the prior of  $L_1$  is closer to the design target than that of  $L_0$ . No change is anticipated in the process variance. Table 1 shows posterior distributions after the first observed value of  $y = 950$ . As can be seen after the first observation, the two beliefs  $L_0$  and  $L_1$  about the process average as represented by their posterior distributions are much closer together. The two priors are

influential in deciding the posterior distributions. As measurements accumulate, these two posteriors are expected to converge to the true process average. This would define the measure of performance of the Bayesian approach.

Table1. Prior and posterior distributions of  $\mu$  for Levels  $L_0$  and  $L_1$ .

	Prior distribution	Likelihood from data	Posterior distribution	
$L_1$	$\mu \sim N(950, 50^2)$	$\mu \sim N(950, 10^2)$	$\mu \square N(946, 9.8^2)$	$L_1$
$L_0$	$\mu \sim N(800, 50^2)$		$\mu \square N(940, 9.8^2)$	$L_0$

### 3. Performance Analysis

To assess performance, simulated data from four different distributions are generated using Microsoft Excel. Each represents 100 measurements obtained following a specified shift in the process average. Figure1 shows the number of iterations required for the two prior distributions ( $L_0$  and  $L_1$ ) to converge to the true process mean.

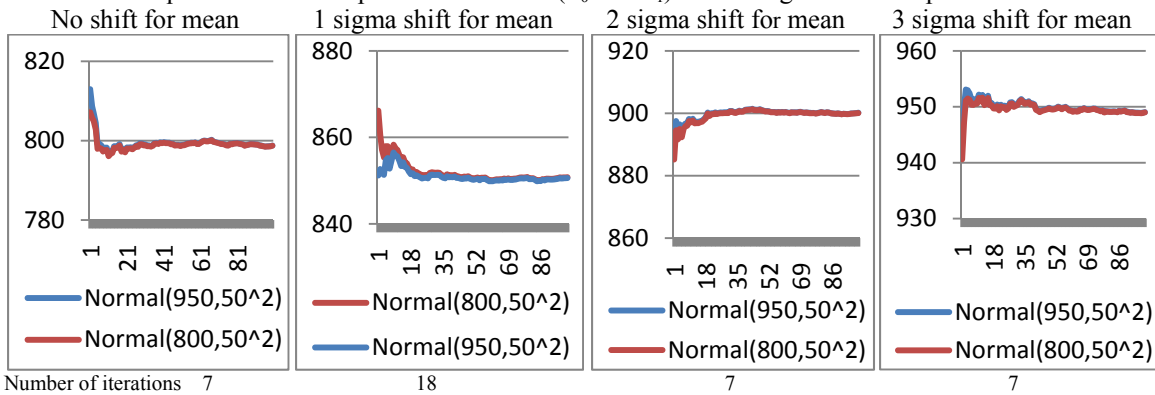


Fig.1. The plots for performance of detecting a shift in  $\mu$  using updating the posterior distribution along with the number of observations in which two priors plots meet each other.

### 4. Conclusions

This paper presented a Bayesian approach for evaluating efforts to shift the process average closer to the target. This would involve testing a number of proposed changes and quantifying their effect. During these transition periods one or more aspects of the Shewhart model are violated, adding difficulties in estimating the new process average. The Bayesian approach is found to make a good bridge between the two options available for users of the traditional Shewhart control charts. The results obtained using simulated data indicate that this approach makes it easier to transition from phase I to phase II.

### 5. Acknowledgement

I am sincerely grateful to my advisor Dr. Weheba for his sustained guidance during the course of my research and his helpful input for preparing this paper.

### References

[1] Montgomery, D. C. (2009). *Introduction to Statistical Quality Control (6th ed.)*. New York: Wiley.  
 [2] Box G. E. P. and Tiao, G. C. (1973). *Bayesian inference in statistical analysis*. Menlo Park: Addison- Wesley.