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**An investigation of methods of producing,
supporting and spinning thin nickel films**

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**AN INVESTIGATION OF METHODS OF PRODUCING,
SUPPORTING AND SPINNING THIN NICKEL FILMS**

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AN INVESTIGATION OF METHODS OF PRODUCING, SUPPORTING
AND SPINNING THIN NICKEL FILMS

BY

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A THESIS

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CHAPTER I

INTRODUCTION

The determination of the tensile strength for thin metallic films has been carried out using three principal methods:

1. High Speed Rotor Method [1, P.524; 2, P.1657]¹
2. Bulge Method [3, P.183]
3. Tensile Test Method [3, P.115]

The high speed rotor and bulge methods were devised by J.W. Beams; the tensile test was developed by Marsh and Pashley.

In the high speed rotor technique the film is electro-deposited onto the cylindrical surface of a small solid steel rotor. The rotor is then spun around its longitudinal axis, which is vertical, by using an ultra-centrifuge [4, P.886]. During this rotation the rotor is supported in vacuum by a vertical magnetic field; the rotation is produced by a horizontal rotating magnetic field. Assuming the centripetal force required to keep an element of the film, between φ and $\varphi+d\varphi$, in circular motion is furnished by the film tensile strength and the adhesion between film and rotor. It is shown [5, P.27] that the following equation holds:

$$\omega^2 R^2 d = T + AR/h$$

Where ω is the angular speed of the rotor, R is the rotor

¹Numbers in brackets refer to references cited in the list at the end of this thesis.

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radius, d is the density of the deposited film, h is the film thickness, T is the tensile strength and A is the adhesive force per unit area. The main disadvantage of this method is due to the effects of adhesion. Although several methods have been used to reduce adhesion between the film and rotor, it has been found that for film thicknesses below 3×10^{-6} cm the adhesion becomes random and measurements in this range are unreliable [3, P.183] .

Indications are that adhesion begins to increase in this range [3, P.149] .

The bulge technique consists of evaporating the metal onto a film of Zapon which has been cemented over a hole in a substrate. The Zapon which is exposed by the hole is then dissolved using amyl acetate, leaving the metal film over the hole. Air pressure is then applied to one side of the film causing it to bulge and eventually rupture. The height and shape of the bulge are measured by a microscope or by a light interference method. This method has at least two disadvantages; irregularities in the adhesive, and uneven stresses may both affect the fracture behavior [3, P.115] .

The tensile strength is calculated from the relation [3, P.115] ,

$$T = PA^2/4hd$$

Where P is the air pressure required to produce a bulge height d in a film of thickness h and A is the area of the film.

In the tensile test the specimen is in the form of a ribbon and is pulled apart. The simplest machine for this purpose was devised by Pashley. It consists of two grips to which the specimen is attached by a suitable adhesive; one grip is fixed, the other is mounted on a trolley which moves on ball bearings. A thread

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attached to the trolley passes over a pulley to a scale pan onto which weights may be placed to increase the load on the film. The film is wider than the grips so that the stress on the edges is reduced. In this method it is difficult to apply the stress uniformly, and the protrusion of the film over the edge of the grips also introduces error [3, P.115]. A refinement of this technique has been developed by Marsh [3, P.117] but has the same disadvantages as the instrument of Pashley.

Strength measurements on polycrystalline silver films have been made by the high speed rotor and bulge methods [1, P.524; 2, P.1657; 3, PP.186,187]. The results obtained are in general agreement; both indicate a strength about equal to that of bulk silver for film thicknesses greater than 2×10^{-5} cm, and an increase in tensile strength as the film thickness becomes less than 2×10^{-5} cm. The maximum recorded tensile strength is about twenty times that of bulk silver when measured by the high speed rotor method and only four times that of bulk silver when measured by the bulge method [3, P.123]. These values, for very thin films, may be in disagreement due to adhesive effects in the rotor method causing the measured strength to be too large or due to the uneven stress and adhesive effects in the bulge method causing the measured strength to be too small.

Marsh has measured the fracture stress for gold single and polycrystalline evaporated films in the microtesting machine. Pashley, with the simple stress device, found the same general trend of stress for gold films as obtained by Marsh [3, P.124]. Marsh found a fracture stress exceeding that of bulk gold by a

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substantial factor in a range of thicknesses where the bulge technique of Beams indicates the films should exhibit bulk strength. Hence, results by these methods are not in agreement [3, P.123] .

As originally conceived this research was to develop a technique for removing the adhesion in the high speed rotor method and to determine strength for thin films. To be certain that adhesion is completely removed, the substrate must be removed leaving only free films to be spun in the centrifuge. It was found, however, that producing free films in the form of a cylindrical shell presented many problems. This, coupled with the difficulties encountered in spinning free films, made this program too large to be completed by one investigator. Hence, this thesis, covers an investigation of electroplating methods for producing free films in the desired configuration, and methods of supporting and spinning these films in the ultra-centrifuge. The investigation has been concentrated on nickel films since a free film must be ferromagnetic to be supported by a magnetic field. Developments near the conclusion of this phase of the program removed this requirement. Nickel films were successfully produced and the data obtained is included. These films, however, were not of a quality to give reliable results.

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CHAPTER II

INVESTIGATION OF METHODS FOR PRODUCING THIN NICKEL FILMS

Thin films are commonly made by either electroplating or evaporating a metal onto a suitable substrate. Electroplating the film onto a substrate was used in this investigation because an evaporation unit was not available in the physics department. Since the purpose was to produce free films, any substrate which was suitable had to be such that it could be removed after plating without damaging the film. Two types of removal were considered; the first was a difference in melting point between film and substrate, the second was by chemical action on the substrate. All substrates used were in the form of a tube about half an inch long and one-fourth inch in outside diameter. The wall thickness varied.

LEAD AND WOOD'S METAL SUBSTRATE

The melting point of nickel is around 1452 degrees Centigrade, hence lead and Wood's metal (50 Bi, 25 Pb, 12.5Sn, 12.5Cd) with melting points of 327 degrees Centigrade and 65.5 degrees Centigrade respectively were chosen as substrates. These were plated using a nickel bath of the following composition [6, P.367]; Nickel Sulfate 67.5 gram/liter; Ammonium Chloride five and three-tenths gram/liter. The temperature was held at thirty-two degrees Centigrade. Plating time was twenty-five minutes and the current

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was twenty-five milliamperes. This produced films about 10^{-4} cm thick.

Upon completion of plating, the lead cylinder was placed on wire mesh and melted with a Bunsen burner, the Wood's metal cylinder was melted by placing it in a beaker of hot water. The film in both cases was almost completely destroyed when the substrate melted. This was probably due to adhesion between film and substrate, or to the surface tension of the molten metal. No further investigation was made in this area.

ALUMINUM SUBSTRATE

The nickel films produced on the aluminum substrates were under high stress, porous, and a dull gray color. Most of the films produced split while still in the caustic solution used to remove the substrate, but a few were recovered without tearing. These, however, were porous and had been attacked by the caustic solution. The porosity is caused in part by the pits formed in the substrate when applying a zincate film to the outer surface, prior to plating.

The substrates were prepared from aluminum rod which was made tubular by drilling so that the wall thickness was about one millimeter. To further reduce the wall thickness and to produce a smooth surface these substrates were buffed with successively finer abrasives down to crocus cloth. This was accomplished by slipping the tubes over a steel insert, which was then placed in the chuck of the lathe and the cylinder rotated. Prior to plating the following steps were taken [6, P.284] :

1. Cleanse
 - a. Immersed in alcohol for fifteen minutes
 - b. Washed with a household detergent
2. Zinc Immersion Dip - One minute at room temperature
3. Water rinse
4. Acid Dip - 50% Nitric Acid by volume at room temperature -
10 seconds
5. Water rinse
6. Zinc Immersion Dip - One minute at room temperature
7. Water rinse

The zinc immersion dip was prepared using sodium hydroxide 525 grams/liter and zinc oxide 97.5 grams/liter. This zinc immersion dip produced a zincate film on the surface of the substrate. This is needed because nickel will not adhere to aluminum and plating directly is not practical. Electroplating immediately followed this preparation. The plating bath was a high sulfate bath with the following composition

□ 6, P.364 □ :

Nickel sulfate -----	75 grams/liter
Ammonium chloride -----	15 grams/liter
Anhydrous sodium sulfate -----	75 grams/liter
Boric acid -----	15 grams/liter

The temperature was thirty-two degrees Centigrade, plating time was thirty minutes and the current was twenty-five milliamperes.

To remove the substrate the cylinders were placed in a caustic solution of seven and five-tenths grams/liter of sodium hydroxide. One to two hours were required to completely remove the substrate.

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PLASTIC SUBSTRATE

Initial attempts to produce films on plastic substrates were unsuccessful but with a modification of the plating bath nickel films were produced. These were a bright ornamental nickel film and were practically free from holes when 10^{-3} cm or more thick. A residue of silver from the paint used to make the plastic conducting was left on the inner surface of the films. This silver added mass to the film without adding the proper amount of tensile strength and may have caused the films to fail at lower angular velocities than they would otherwise fail. This coupled with the fact that the edges were jagged caused little confidence to be placed in the strength measurements made using these films (see Appendix I). Films less than 10^{-3} cm thick were porous and failed due to swelling of the substrate when placed in the acetone solvent.

The plastic substrates were made from the cartridges of Lindy ball point pens which were drilled out to make the wall thickness as small as possible. The substrates were then painted with a silver conducting paint (SC12; Micro-Circuits Company) while they were revolved about their longitudinal axis in the same manner as described for the aluminum substrate. After drying for about an hour, the silver surface was polished using cerium oxide and all but a small portion of the surface was covered with shellac. This small portion located near the middle of the cylinder was about two millimeters wide and extended completely around the cylinder forming a small ring of exposed silver. This step was necessary because the method of suspending the cylinders in the plating solution would cause the film to be damaged if it were plated over the complete cylindrical

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surface. Although extreme care was exercised when applying the shellac, only about one out of ten films had well defined edges; the others appeared jagged under low magnification.

To prepare the surface for plating, the substrate was immersed in alcohol for a half hour to remove grease, then washed in household detergent and rinsed in distilled water. Two plating solutions were used. The first was a Watts' bath containing the following

[6, P.367] :

Nickel sulfate	-----	330 grams/liter
Nickel chloride	-----	45 grams/liter
Boric acid	-----	37.5 grams/liter

The temperature was maintained at thirty-eight degrees Centigrade; currents from five to twenty-five milliamperes were used with plating times from two and one-half hours to twenty-five minutes. These films were porous, of a dull gray finish and under extremely high stress. Many of these split when placed in the acetone solution used to remove the substrate.

The second bath was also a Watts' bath but commercially available anti-pitters and brighteners were added.² The operating temperature was thirty-eight degrees Centigrade. It was found that a current of twenty-five milliamperes for twenty-five minutes produced films in the range of 1×10^{-3} cm. Lower currents or shorter times or both produced unsatisfactory films.

²These additives were obtained from; Metal Finishing Company; 1423 McLean Blvd. S., Wichita, Kansas. Two Brighteners were used 2RL and 7RL. The anti-pitter was number 22.

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To remove the substrate, the cylinders were placed in a beaker of acetone for about thirty minutes. Following removal of the plastic, the film was immersed in a paint remover (SL11; Micro-Circuits Company) to break down the binder holding the silver particles. This process, however, did not remove the silver particles themselves.

COPPER SUBSTRATE

Thin nickel films in the thickness range of interest are made commercially for nuclear experimentation. These films are flat and are deposited on a copper substrate, about 10^{-4} centimeters thick, which is subsequently removed using a trichloroacetic acid solution.³ Using this method two cylindrical films were produced but were not of a quality for making strength measurements due to difficulties in producing a satisfactory copper substrate.

The first substrate was made from one-quarter inch copper tubing. These tubes were drilled out to reduce wall thickness and then buffed as described under aluminum substrates. In addition to the buffing they were polished using cerium oxide. These substrates were pitted due to imperfections in the copper tubing and hence the films were porous. These substrates were also difficult to remove due to their thickness. Plating was done using the Watt's bath plus anti-pitters and brighteners, operating at fifty-eight degrees Centigrade. The current was twenty-five milliamperes for twenty-five minutes.

³One hundred grams trichloroacetic acid, 500 milliliters twenty-eight percent ammonia and enough water to make a liter.

In an attempt to produce a copper substrate which would be about 10^{-4} cm in thickness and free from pits, the silvered plastic substrates were again used. Copper was plated on the silvered plastic but was not of a quality to polish and use as a substrate for the nickel.

The first copper bath used was a copper striking bath consisting of the following [6, P.324] :

Copper cyanide	-----	15 grams/liter
Sodium cyanide	-----	27.8 grams/liter
Free sodium cyanide	-----	11.3 grams/liter
Sodium hydroxide	-----	3.3 grams/liter

This gave unsatisfactory copper films because the cyanide reacted with the silver paint causing blisters. The second bath was an acid bath of the following composition. 6, P.318 :

Copper sulfate	-----	210 grams/liter
Sulfuric acid	-----	60 grams/liter
Thiourea	-----	.01 grams/liter
Dextrin	-----	.01 grams/liter
Hydrochloric acid	-----	.01 grams/liter

The films produced using this bath were granular and plated only in patches. No further attempts were made to produce copper substrates using plating.

PLATING BATH AND METHOD OF SUSPENSION

To accomplish the plating in this investigation a constant temperature plating bath was constructed. (see Fig. 1). This

consisted of the following parts:

1. Ceramic crock (water container) with plexiglas lid.
2. Motor for circulating water; General Electric 1/20 horsepower, 3450 R.P.M.
3. Circulating element; a flat circular disc of aluminum two and one-half inches in diameter attached to the end of an aluminum shaft on the circulating motor
4. A 600 ml beaker (to contain plating solution) with plexiglas lid.
5. A brass bushing, in the lid of the beaker, which functions as a bearing for the agitator shaft.
6. Agitator shaft for rotating sample to be plated.
7. Synchronis motor, Cramer SXR34F 4 R.P.M. for rotating agitator shaft.
8. 100 watt aquarium heater (not shown).
9. A Finwall cartridge thermostat (not shown).

The beaker containing the plating solution, the thermostat and the heater were immersed in the crock of water through holes in the crock lid. The water was circulated and maintained at constant temperature thus keeping the plating solution at constant temperature.

The substrate was immersed in the bath using a magnet fitted into the rotating shaft. Since the substrates were not of ferromagnetic material they were placed on a steel bolt to provide magnetic coupling. The head and nut of the bolt were painted with black glyptal paint to prevent them from being plated.

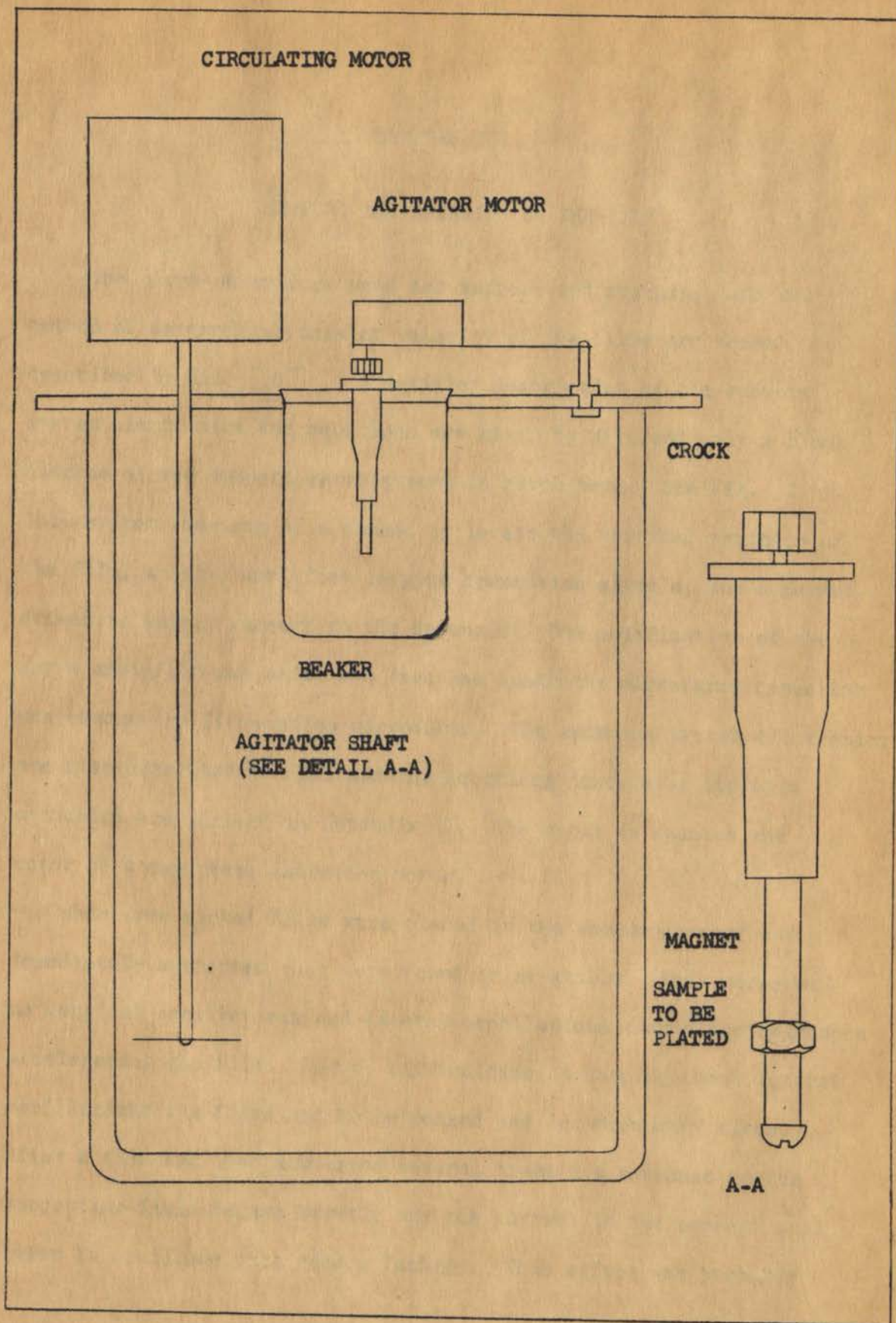


FIG. 1. PLATING BATH

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CHAPTER III

SUPPORT AND SPINNING OF THE FILM

The ultra-centrifuge used for support and spinning, and the method of determining angular velocity of the films are those described by Hill [5]. A detailed description of the support system electronics and equations are given by Hill and only a block diagram of the support servo system is given here. See Fig. 2. This system consists of a sensor to locate the vertical position of the film, a servo amplifier to give correction signals, and a magnet driver to supply current to the solenoid. One modification of the servo amplifier was made; the five one hundredths microfarad capacitor was changed to five-tenths microfarad. The spinning system electronics are also described by Hill and the equations describing the spin mechanics are derived in Appendix II. The rotor is spun as the rotor of a two phase induction motor.

When free nickel films were placed in the vacuum chamber and immediately supported they were found to be stable. They performed no vertical oscillations and lateral oscillations took place only upon accelerating the film. Due to difficulties in damping these lateral oscillations the films had to be raised and let down many times. After a film had been supported several times its response to the supporting field became erratic and the current in the support coil began to oscillate in a random fashion. This effect was probably

due to induced dipoles in the film, but attempts to demagnetize the film proved useless. This effect was found to be permanent (several films with this characteristic were kept for a week, but were found to still exhibit this effect) and the films were discarded. It was discovered that this behavior was caused by having power on the position sensing oscillator when the film was unsupported and in closest proximity to the sensing coil. The time required for this effect to render a film useless varied with the width of the film, being about five minutes for a film one and a half millimeters wide to ten minutes for a three millimeter film. Hence, although the exact nature of this effect was not determined, it could be controlled.

As mentioned previously the films tend to oscillate laterally when accelerated. Various configurations of dampers were investigated to prevent these oscillations. These dampers functioned on the same principle as discussed by Hill. A typical damper consists of a steel rod, about three-sixteenths of an inch in diameter and two and a half inches long, slipped through a cork and anchored to the bottom of a beaker of oil by a chain and weight. The beaker is placed below the vacuum chamber and the rod is lifted by the support field until the chain is tight. Magnetic coupling between the object supported and the rod causes the rod to follow the lateral motion of the object and thus damp these oscillations. The steel rod and chain are much too massive to use when spinning free films; hence, in the first configuration used the rod was replaced by a sewing needle and the chain by a thread.

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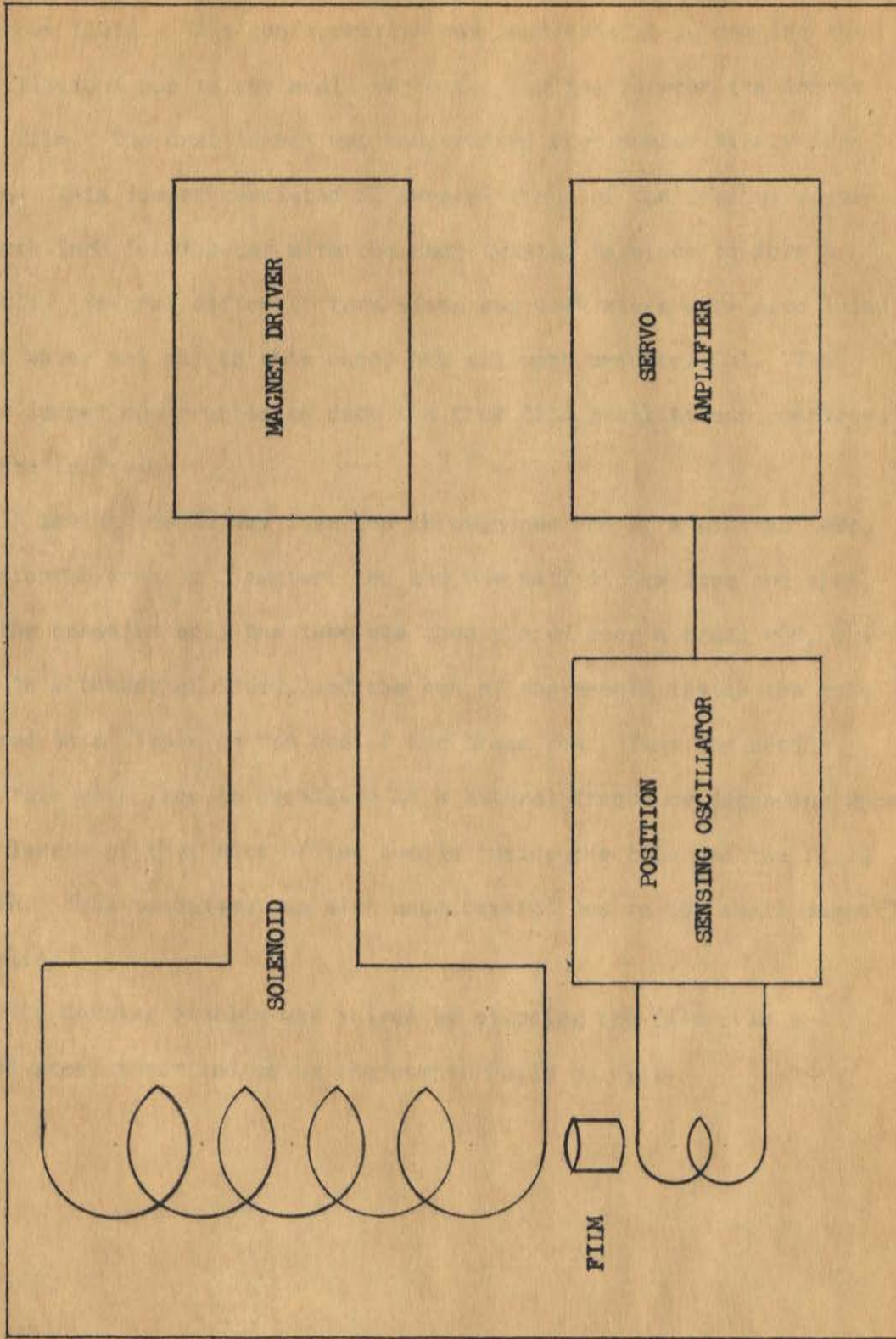


FIG. 2. BLOCK DIAGRAM OF SUPPORT SYSTEM

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Several different sizes of needles and corks were used, and with each cork-needle combination water was tried as well as oil for the viscous fluid. This configuration was unsuccessful in damping the oscillations due to the small magnetic coupling between the needle and film. The next damper was constructed from number thirty iron wire. This damper consisted of several turns of the iron wire one-fourth inch in diameter with the ends twisted together to form a needle. Several different turn sizes and cork sizes were also tried with water and oil in this case, but all were unsuccessful. The last damper constructed to damp the free film oscillations consisted of the following:

A sewing needle was inserted through one end of a plastic tube, one-fourth inch in diameter, two and one-half inches long and open on the opposite end; the tube was then placed over a brass rod, which was in a beaker of fluid, and the end of the needle inside the tube rested in a dimple on the end of the brass rod. Thus the needle and tube were free to oscillate at a natural frequency depending upon the length of that part of the needle inside the tube and the fluid depth. This technique was also unsuccessful due to the small magnetic coupling.

The damping problem was solved by slipping the film over a solid steel rotor and using the damper built by Hill.

CHAPTER IV

RESULTS AND RECOMMENDATIONS

The nickel films which were produced using the plastic substrates were used in the development of the support and accelerating techniques. Fifteen of these of usable quality were produced and accelerated until rupture occurred. The average strength found was 1.5×10^9 dynes/cm with an average deviation of $.6 \times 10^9$ dynes/cm for films in the 1×10^{-3} cm range. This data may be found in Appendix I, page 22. It should be noted, however, that these films were somewhat porous and the edges were jagged. This, coupled with the fact that a silver residue was present on the inner surface of the film, causes question as to the validity of these strength values.

Of the materials used for substrates in this investigation, copper is the most promising for future effort. These substrates may be readily removed with trichloroacetic acid solution without damage to the film. A plating technique was not found which could be used to produce a satisfactory copper substrate; however, future investigation might produce such a technique. It may also be possible to produce these substrates using evaporation techniques.

If evaporation techniques are used it seems possible to use either the Zapon previously described or a sugar aerosol-water solution [6, P.675] to produce a thin soluble film on a solid rotor and

to evaporate a metallic film upon this surface. After evaporation the rotor could be placed in a suitable solvent, amyl acetate for Zapon, water for the aerosal-water, and this intermediate film removed leaving the undamaged metal film free. Whether this method would be effective depends upon the ability of the solvent to penetrate between the metal film and rotor.

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APPENDIX I

DATA ON FILMS

Fifteen films were produced which were of a quality to be spun in the centrifuge. The data obtained are given in table I. The width of the film was determined using a traveling microscope and averaging five values obtained for each film. The inside diameter was determined by measuring the substrate diameter prior to plating and the mass was determined using an analytical balance. From these measurements the film thickness was determined using the formula

where m is the mass of the film, p is the density (8.6 grams/cm³ for nickel), h is the average width and r is the inside radius. The method used to determine the angular velocity is the same as described by Hill [5]. No attempt was made to calibrate accurately the signal generator used to determine the angular velocity of the film, but the generator was calibrated using the sixty cycle line supply.

TABLE I

STRENGTH DATA ON THIN NICKEL FILMS

ROTOR NUMBER	THICKNESS IN CM	TENSILE STRENGTH IN DYNES/CM ²
1	1×10^{-3}	1.3×10^9
2	1.4×10^{-3}	2.5×10^9
3	1.7×10^{-3}	3.8×10^9
4	1.9×10^{-3}	2.5×10^9
5	1.3×10^{-3}	1.1×10^9
6	1.3×10^{-3}	$.78 \times 10^9$
7	2.1×10^{-3}	$.85 \times 10^9$
8	1.6×10^{-3}	1.1×10^9
9	1.3×10^{-3}	1.2×10^9
10	2.2×10^{-3}	$.91 \times 10^9$
11	1.4×10^{-3}	1.8×10^9
12	1.9×10^{-3}	2.3×10^9
13	1.3×10^{-3}	1.7×10^9
14	1.4×10^{-3}	1.4×10^9
15	1.5×10^{-3}	2.3×10^9

APPENDIX II

The purpose of this appendix is to derive an equation for the eddy currents induced in an infinite cylindrical shell by a rotating magnetic field. The power dissipated in a unit length of the cylinder may be calculated from this eddy current equation and assuming heat losses are due only to radiation the equilibrium temperature may then be calculated.

Let the inside radius of the cylinder be a , the outside radius b , and the axis of the cylinder be the z axis of coordinates. The rotating magnetic field will then be parallel to the xy plane. This is shown in Fig. 3.

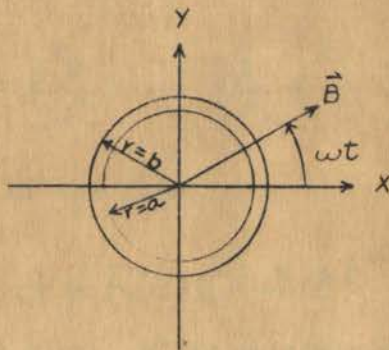


Fig. 3

The magnetic vector potential shall be used to find the eddy current equation. By definition.

$$\nabla \times \vec{A} = \vec{B} \quad (1)$$

Where A is the magnetic vector potential and B is the magnetic induction. The eddy current density is given by

$$\vec{I} = (-1/\tau)(d\vec{A}/dt) \quad (2)$$

Where τ is the resistivity of the material. The equation which \vec{A} must satisfy is

$$\frac{4\pi\mu}{\tau} \frac{d\vec{A}}{dt} = \nabla^2 \vec{A} \quad (3)$$

Where μ is the magnetic permeability. Because of the symmetry of the problem cylindrical coordinates will be used. Equation (3) then becomes

$$\frac{4\pi\mu}{\tau} \frac{d\vec{A}}{dt} = \frac{\partial^2 \vec{A}}{\partial r^2} + \frac{1}{r} \frac{\partial \vec{A}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \vec{A}}{\partial \phi^2} + \frac{\partial^2 \vec{A}}{\partial z^2} \quad (4)$$

Since the magnetic induction does not change as a function of z , A need be only a function of r , ϕ and t . From equation (1) it follows that the potential needs a component only in the z direction. The vector A may then be written as $A = A(r, \phi, t)$. Equation (4) then becomes

$$\frac{4\pi\mu}{\tau} \frac{dA}{dt} = \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A}{\partial \phi^2} \quad (5)$$

Assume a solution to (5) of the form

$$A(r, \phi, t) = R(r) \Phi(\phi) e^{j\omega t} \quad (6)$$

Substituting (6) into (5) gives, after multiplying by r^2 and dividing by $A(r, \phi, t)$

$$\frac{r^2}{R} \frac{d^2 R}{dr^2} + \frac{r}{R} \frac{dR}{dr} + \frac{1}{\Phi} \frac{d^2 \Phi}{d\phi^2} = j\rho_0 r^2 \quad (7)$$

Where $j = \sqrt{-1}$ and $P = 4\pi\mu\omega/\tau$. Using a well known argument from the theory of partial differential equations, (7) may be written as

$$\frac{1}{\Phi} \frac{d^2 \Phi}{d\phi^2} = -n^2 \quad (8)$$

$$\frac{r^2}{R} \frac{d^2 R}{dr^2} + \frac{r}{R} \frac{dR}{dr} - j\rho_0 r^2 = n^2 \quad (9)$$

Equation (8) has solutions of the form

$$\Phi = A \cos(n\varphi) + B \sin(n\varphi) \quad (10)$$

Equation (9) when written as

$$\frac{d^2 R}{dn^2} + \frac{1}{n} \frac{dR}{dn} - \left(j\rho_0 + \frac{n^2}{n^2} \right) R = 0 \quad (11)$$

is seen to be Bessel's equation with solutions of the form,

$$R_n = A J_n [j^{\frac{1}{2}} \rho_0^{\frac{1}{2}} n] + B Y_n [j^{\frac{1}{2}} \rho_0^{\frac{1}{2}} n]$$

Where $J_n(x)$ is called a solution of the first kind and $Y_n(x)$ is called a solution of the second kind. Both solutions are of order n .

The vector potential is thus:

$$\vec{A} = \vec{k} (C J_n [j^{\frac{1}{2}} \rho_0^{\frac{1}{2}} n] + D Y_n [j^{\frac{1}{2}} \rho_0^{\frac{1}{2}} n]) (E \cos(n\varphi) + F \sin(n\varphi)) e^{j\omega t} \quad (12)$$

In vacuum the resistivity becomes infinite and equation (9) reduces to

$$\frac{d^2 R}{dn^2} + \frac{1}{n} \frac{dR}{dn} - \frac{n^2 R}{n^2} = 0 \quad (9^1)$$

This equation has solutions of the form

$$R_n = C n^n + D n^{-n}$$

hence, under this condition, the vector potential becomes

$$\vec{A} = \vec{k} (C n^n + D n^{-n}) (E \cos(n\varphi) + F \sin(n\varphi)) e^{j\omega t} \quad (13)$$

Referring to Fig. 3, page 24, the equation for the rotating field may be written as

$$\vec{B} = B (\vec{i} \cos(\omega t) + \vec{j} \sin(\omega t))$$

where B is the constant magnitude. The x component and the y component are respectively: $B \exp(j\omega t)$ (real part), $B \exp(j\omega t)$ (imaginary part).

The problem of finding the vector potential due to the applied rotating field and the presence of the cylinder is calculated in two parts. The first part is calculated considering only the x component

of the applied field; the second part when only the y component is considered. When these two components of A are added vectorially they give the resultant potential. There are three regions of interest namely, $r \leq a$, $r > b$, and $a < r \leq b$.

When $r \leq a$ the solutions are of the form (13). The curl of \vec{A} is given as

$$\nabla \times \vec{A} = \vec{e}_r \frac{\partial A}{\partial \varphi} - \vec{e}_\varphi \frac{\partial A}{\partial r}$$

since $\vec{A} = \vec{k}A(r, \varphi, t)$. In cylindrical coordinates the x component of the field becomes

$$\vec{B}_x = B [\vec{e}_r \cos(\varphi) - \vec{e}_\varphi \sin(\varphi)] e^{i\omega t} \quad (\text{real part})$$

Equating components of the last two equations gives

$$\begin{aligned} \frac{1}{r} \frac{\partial A}{\partial \varphi} &= B \cos(\varphi) e^{i\omega t} \\ \frac{\partial A}{\partial r} &= B \sin(\varphi) e^{i\omega t} \end{aligned}$$

whose solution is, within an additive constant,

$$A = r B \sin(\varphi) e^{i\omega t} \quad (\text{real part})$$

This then is the part of A due to the x component of the applied field. The solution due to the applied field and presence of the cylinder will be

$$\vec{A}_i = \vec{k} r B C \sin(\varphi) e^{i\omega t} \quad (\text{real part}) \quad (14)$$

No terms from (13) involving r^{-n} can appear in (14) since this region includes the origin.

By a similar argument the solution when $r > b$ is

$$\vec{A}_o = \vec{k} B [D r^{-1} + r] \sin(\varphi) e^{i\omega t} \quad (\text{real part}) \quad (15)$$

When $a < r \leq b$ the solution is of the form

(real part)

The boundary conditions which must be satisfied are

$$\vec{A}_i = \vec{A}_m, \quad \mu \frac{\partial \vec{A}_i}{\partial n} = \frac{\partial \vec{A}_m}{\partial n} \quad ; \quad \vec{A}_o = \vec{A}_m, \quad \mu \frac{\partial A_o}{\partial n} = \frac{\partial A_m}{\partial n}$$

Substituting in these boundary conditions and solving for M and N

gives

$$N = \frac{\mu_1 [\mu J_1(a) - j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} a J_1'(a)]}{[\mu J_1(a) - j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} J_1'(a)] [j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} b Y_1(b) + \mu Y_1(b)] + [\mu Y_1(a) - j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} a Y_1'(a)] [j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} J_1'(a) + \mu J_1'(a)]}$$

$$M = - \frac{[\mu Y_1(a) - j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} a Y_1'(a)]}{[\mu J_1(a) - j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} J_1'(a)]} N$$

where

$$J_1(a) = J_1[j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} a]$$

and similarly for $J_1(b)$, $Y_1(b)$ and $Y_1(a)$. Also

$$J_1'(a) = \left. \frac{dJ_1[j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} r]}{d(j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} r)} \right|_{r=a}$$

and similarly for $J_1'(b)$, $Y_1'(b)$ and $Y_1'(a)$. The component of A due to the x component of the field and presence of the cylinder is thus solved.

The procedure used to determine the part of A due to the y component of the applied field and presence of the cylinder is exactly the same and the solutions are of the form, when $a \leq r \leq b$,

$$\vec{A} = \vec{k} \{ L J_1[j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} r] + S Y_1[j^{\frac{3}{2}} \rho_0^{\frac{1}{2}} r] \} \cos(\varphi) e^{j\omega t} \quad (\text{imaginary part}) \quad (17)$$

It turns out that $S=-N$, and $L=-M$. The two components of A are then

$$(\vec{A})_{\text{due to } x} = \vec{k} \left\{ M J_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] + N Y_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] \right\} \sin(\varphi) e^{j\omega t} \quad (\text{real part})$$

$$(\vec{A})_{\text{due to } y} = -\vec{k} \left\{ M J_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] + N Y_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] \right\} \cos(\varphi) e^{j\omega t} \quad (\text{imaginary part})$$

The eddy currents induced due to each of these components may be found using equation (2), which gives

$$\vec{t}_x = -j \frac{\omega}{\gamma} \vec{k} \left\{ M J_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] + N Y_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] \right\} \sin(\varphi) e^{j\omega t} \quad (\text{real part})$$

$$\vec{t}_y = j \frac{\omega}{\gamma} \vec{k} \left\{ M J_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] + N Y_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] \right\} \cos(\varphi) e^{j\omega t} \quad (\text{imaginary part})$$

The power dissipated per unit length of the cylinder is

$$\bar{P} = \int_V \frac{1}{2} \vec{t} \cdot \vec{t}^* dV = \frac{1}{2} \int_a^b \int_0^{2\pi} \vec{t} \cdot \vec{t}^* n \, dr \, d\varphi$$

where

$$\vec{t} = (\vec{t}_x + \vec{t}_x^*)/2 + (\vec{t}_y - \vec{t}_y^*)/2j$$

Let

$$M J_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] + N Y_1 [j^{\frac{3}{2}} p_0^{\frac{1}{2}} n] = R(n)$$

then

$$\vec{t}_x = -j \frac{\omega}{\gamma} \vec{k} R \sin(\varphi) e^{j\omega t} \quad (\text{real part}) \quad (18)$$

and

$$\vec{t}_y = j \frac{\omega}{\gamma} \vec{k} R \cos(\varphi) e^{j\omega t} \quad (19)$$

The total power may be found by adding the power due to \vec{t}_x to that due to \vec{t}_y . This gives using (18) and (19)

$$\bar{P}_x = \frac{1}{2} \int_a^b \int_0^{2\pi} \frac{\omega^2}{\gamma^2} B^2 R R^* \sin^2(\varphi) n \, dr \, d\varphi$$

$$\bar{P}_y = \frac{1}{2} \int_a^b \int_0^{2\pi} \frac{\omega^2}{\gamma^2} B^2 R R^* \cos^2(\varphi) n \, dr \, d\varphi$$

hence

$$\bar{P}_x = \bar{P}_y$$

and

$$\bar{P} = \bar{P}_x + \bar{P}_y = \frac{\pi \omega^2 B^2}{\gamma} \int_a^b R R^* n \, dn \quad (20)$$

To evaluate the integral

$$\int_a^b R R^* n \, dn$$

the following procedure is used. R and R^* must satisfy the equations

$$n \frac{d}{dn} \left(n \frac{dR}{dn} \right) - (j p_0 n^2 + n^2) R = 0 \quad (21)$$

$$n \frac{d}{dn} \left(n \frac{dR^*}{dn} \right) - (-j p_0 n^2 + n^2) R^* = 0 \quad (22)$$

Multiply (21) by R^* and (22) by R and subtract to obtain

$$R^* n \frac{d}{dn} \left(n \frac{dR}{dn} \right) - R n \frac{d}{dn} \left(n \frac{dR^*}{dn} \right) - 2j p_0 R R^* n^2 = 0$$

Which gives

$$2j p_0 R R^* n = R^* \frac{d}{dn} \left(n \frac{dR}{dn} \right) - R \frac{d}{dn} \left(n \frac{dR^*}{dn} \right)$$

Multiplying by dn and integrating from a to b gives

$$\int_a^b R R^* n \, dn = \frac{1}{2j p_0} \left[R^* \frac{d}{dn} \left(n \frac{dR}{dn} \right) - R \frac{d}{dn} \left(n \frac{dR^*}{dn} \right) \right]_a^b \quad (23)$$

When the integral on the right hand side of (23) is integrated by parts, it becomes

$$\int_a^b R R^* n \, dn = \frac{1}{2j p_0} \left[j^{\frac{1}{2}} p_0^{\frac{1}{2}} n R^* R' - (-j)^{\frac{1}{2}} p_0^{\frac{1}{2}} n R R'^* \right]_a^b \quad (24)$$

where

$$R' = \frac{dR}{d(j^{\frac{3}{2}} p_0^{\frac{1}{2}} \nu)} \quad (24)$$

Using 24 the power becomes

$$\bar{P} = \frac{\pi \omega^2 B^2}{\tau 2j p_0} \left[j^{\frac{3}{2}} p_0^{\frac{1}{2}} \nu R^* R' - (-j)^{\frac{3}{2}} p_0^{\frac{1}{2}} \nu R R^{*'} \right]_a^b \quad (25)$$

Evaluating equation (25) between the limits, $a = .375$ cm and $b = .376$ cm gives:

$$\bar{P} = \frac{\omega B^2}{16\mu} (1.3 \times 10^{-4}) \text{ erg/sec} \quad (26)$$

Where $\tau = 10^4$ in electromagnetic units, $\mu = 10^2$ and $\omega = 10^5$ cps. If the magnitude of the rotating field is assumed to be 20 gauss equation (26) becomes

$$\bar{P} = 2.5 \times 10^2 \text{ erg/sec}$$

To determine the equilibrium temperature of the film it is assumed that heat is lost only by radiation giving

$$2\sigma (T_{ef}^4 - T_2^4) = \bar{P} \quad (27)$$

where σ is the emissivity of the material, σ is a constant equal to 5.67×10^{-5} in cps units, T_2 is the temperature of the surroundings, in degrees Kelven and T_{ef} is the equilibrium temperature of the film in degrees Kelven. From equation (27) it is seen that the heating effect due to the eddy currents is completely negligible.

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