

Sum-Rate Distortion for a Multiantenna Broadcast Channel due to Limited Feedback

Amitav Mukherjee, Neelu Bijukchhe* and H. M. Kwon

Department of Electrical and Computer Engineering, College of Engineering

Abstract. Realization of large capacity gains offered by multiple antenna systems depends upon the presence of channel state information (CSI) at the transmitter. However, very recently the overall impact of feedback on system capacity has been examined in order to highlight the hidden aspect of data rate loss due to feedback in general. In this paper, we extend the system model, called a duplex model that was introduced for a single-user point to point system, to a system for a multiple user scenario as seen in a typical downlink of cellular communications system. A distortion function based on prior results for the Gaussian vector broadcast channel and feedback sum-rate optimization design is subsequently presented. This system model is used to derive bounds on the amount of feedback that can be supported under various antenna, channel and signal-to-noise ratio scenarios. Simulation results are presented to show the impact of feedback on the sum-rate for multi-antenna wireless systems with multiple users.

I. INTRODUCTION

A number of sub-optimal transmission strategies such as zero-forcing dirty paper coding (ZFDPC) [4] and channel inversion (also known as zero-forcing beamforming) [5] have been proposed to approach the optimal capacity region. These schemes carry a lower computational burden but require accurate channel state information at the transmitter (CSIT) to achieve the multiplexing gains associated with MIMO broadcast channels. The CSIT has largely been considered perfect (complete downlink channel knowledge) in most literature [2] [4] [6], but partial CSIT schemes [7] have gained attention recently.

To provide partial CSIT, limited feedback from the receivers (as opposed to full or analog feedback) has been examined in [8], [9] and [10] by means of instantaneous, zero-error feedback links. The partial CSIT provided by limited feedback has been shown to enable throughput asymptotically approaching optimal sum-rates in conditions of high SINR [8],[9] or as the number of users goes to infinity [10]. Limited feedback for broadcast channels can be classified into channel direction information (CDI) [11] and channel quality information (CQI). CDI represents the spatial direction

of the channel, whereas CQI is a quantized form of the channel vector itself.

In this paper, we consider the loss in sum-rate capacity due to reverse feedback overhead and propose a distortion model called the broadcast channel duplex model. This is motivated by the fact that the feedback overhead is non-negligible even for a small number of receivers in order to maintain partial CSIT with a required level of distortion from complete CSIT. Originally, a duplex model was proposed in [12] for the single-user MIMO scenario, based on previous MIMO rate-distortion analysis [13]. Our analysis is restricted to the broadcast channel where the number of receivers is less than or equal to the number of transmit antennas at the base station (BS), therefore user scheduling is not considered in this paper. We show effects of fixed and dynamic-rate limited feedback on the GBC sum-rate for ZFDPC and channel inversion transmission, and derive optimal feedback constraints with respect to the sum-rate offered by no CSIT.

II. SYSTEM MODEL

The baseband signal received by user $i \in \{1, \dots, K\}$ at a

given time instant can be represented as $y_i = \mathbf{h}_i^T \mathbf{x} + n_i$ (1),

where $\mathbf{h}_i^T \in C^{M \times 1}$ is the channel gain vector, $\mathbf{x} \in C^{M \times 1}$ is the transmitted signal vector, and n_i is the complex white Gaussian noise with zero mean and unit variance.

B. Feedback Design

Every channel coherence interval, the i^{th} receiver computes its CDI as $\hat{\mathbf{h}}_i = \mathbf{h}_i / \|\mathbf{h}_i\|$, which has the advantage of varying much more slowly than the magnitude of the channel vector [16]. The transmitter uses the quantized CSI from the receivers to reassemble the estimated channel matrix as $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \dots, \hat{\mathbf{h}}_K]^T$. We define the system feedback sum-rate in bits/(sec-Hz) for a given channel realization with

GBC system bandwidth W as $F = \frac{\left(\sum_{i=1}^K B_i \right)}{WT_C}$. (6)

Further treatment of the feedback design, including the choice of CDI and/or CQI along with codebook design optimization is seen in Sections IV and V.

C. Transmission Strategies

$$R_{no-CSIT}(P) = E_{\mathbf{h}} \left[\log \left(1 + \frac{P}{M} \|\mathbf{h}\|^2 \right) \right]. \quad (7)$$

Observation 1: The duplex sum-rate distortion is an increasing function of F , in sharp contrast to a simplex sum-rate distortion model which is a decreasing function of the feedback sum-rate as shown by [12, eq. (6)] where F is not exclusively included as in (14) but implicitly in R^{CSIT} . The duplex sum-rate distortion for the multipoint-to-point system in (14) is consistent with the single point-to-point system in [12, eq. (9)] in the sense that both are increasing functions of F .

Lemma 1: The infimum of the duplex sum-rate, i.e., a greatest lower bound, is given by the GBC sum-rate with no CSIT and perfect CSIR assuming isotropic fading as [18]

$$\inf_P R_{dup}^{sum}(P) = \sum_{i=1}^K R_i = \sum_{i=1}^K \frac{1}{2} \log \left(1 + \frac{\alpha_i P/M}{n_i + \sum_{j \neq i} \alpha_j P/M} \right), \quad (15)$$

where α_i is the power allocated to user i .

Proof: The lower bound of duplex sum-rate is the GBC sum-rate without CSIT and fading characteristic knowledge (7). However, if the fading distribution is assumed to be isotropic (such as Rayleigh) in nature, then the vector GBC capacity region with no CSIT and perfect CSIR is upper-bounded by the capacity region of a scalar ($M = 1$) broadcast channel with the transmit power scaled down by a factor of $1/M$ [18]. The corresponding scalar broadcast channel (degraded) capacity region is well known [19].

The channel coherence time is the key parameter in determining if feedback can be supported in the duplex broadcast channel. For a point-to-point system it was shown that feedback cannot be supported in a fast fading channel ($WT_C \leq 1$) according to simulation results in [12]. Similarly, even for a multipoint-to-point system, we conjecture from simulation results that the duplex sum-rate will be more degraded than the GBC with no CSIT for a fast fading channel (e.g., $WT_C \leq 5$). The exact bounds on the channel coherence time that can support feedback for a given value of W , P and K may be calculated from (16). Therefore, it is surmised that *no* amount of feedback can be supported for a fast fading scenario.

IV. SIMULATIONS

Fig. 1 illustrates the duplex sum-rate for fixed feedback sum-rate with a codebook of $B=20$ bits for a variety of fading channels with $WT_C = 5, 10, 25, 50$ and 100 . The duplex sum-rate with limited feedback saturates with increasing SNR for all cases, a trend that agrees with simplex sum-rates with fixed feedback in [9], [16]. However, it is evident that the channel coherence time

has a major impact on the distortion from the simplex rate. Even for slow fading channels (i.e., $WT_C \geq 50$), the duplex rate with fixed feedback is 85% of the simplex rate at 20 dB of SNR. More importantly, feedback cannot be supported at *any* SNR for fast fading channels, e.g., $WT_C \leq 5$, even if very low feedback per receiver is adopted. And, even if $WT_C = 25$, the throughput is negative unless SNR exceeds 15 dB. This is why we assume $WT_C = 50$ in Fig. 3 for the dynamic feedback rate discussion.

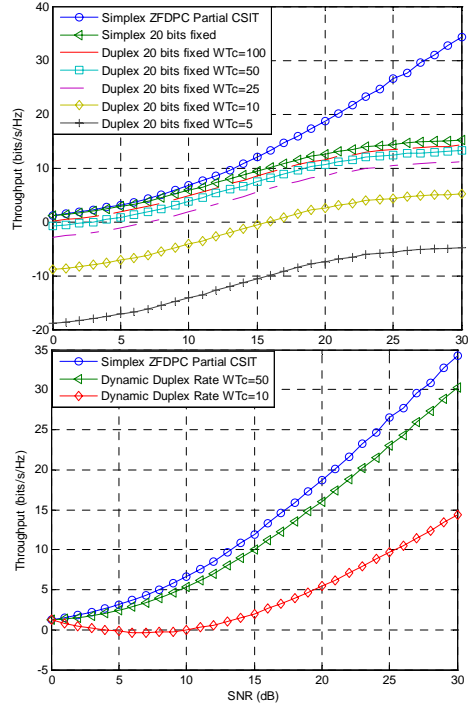


Fig. 1: Partial CSIT duplex sum-rates for ZFDPC with fixed feedback overhead of $B = 20$ bits/receiver when $WT_C = 5, 10, 25, 50$ and 100 , and $M=K=5$.

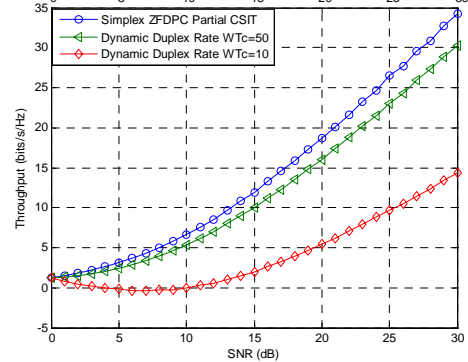


Fig. 2: Optimal, partial CSIT and duplex sum-rates for ZFDPC with dynamic feedback overhead at $WT_C = 10$ and 50 , and $M = K = 5$

The duplex rate for the CI scheme operating with dynamic feedback for a 5×5 GBC assuming $WT_C = 10$ and 50 is shown in Fig. 2. Compared to the simplex rate, the multiplexing gain shown by the slope of the dynamic feedback duplex rate is seen to be tailing off with increasing SNR, indicating that the feedback overhead loss is not adequately compensated by the reduction in mutual interference enabled by the feedback. Once again we note that lower coherence times severely degrade the duplex sum-rate even with dynamic feedback.

VII. CONCLUSION

The consideration of data rate loss due to feedback overhead in broadcast channels casts a new light on the implementation of partial CSIT schemes. The additional overhead due to training for channel estimation is not considered, as are the effects of estimation error at the receivers and feedback corrupted by noise and fading. The combination of these factors and their impact on sum-rate distortion is a topic for future study, for example, by allotting a SNR threshold for feedback.