An Efficient Carrier Offset Estimator for Multicarrier Modulation System

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***Abstract*— In this paper, we proposed a closed-form solution for blind Carrier Frequency Offset (CFO) estimation employing the Rank-Revealing QR triangular factorization Method (RRQR) for Multicarrier Modulation system. The advantage of using the RRQR it gives precious information about numerical rank and efficiently separates the signal space from the noise space. Computer simulations are showing the superior performance of RRQR compared with the method employing ESPRIT Algorithm.**

I. Introduction

Multicarrier Modulation (MCM) technique [1] is used in data delivery systems over the phone line, digital radio and television, and wireless networking systems. It has already been accepted for the wireless local area network standards IEEE 802.11a, High Performance LAN type 2 (HIPERLAN/2), and Mobile Multimedia Access Communication (MMAC) Systems. Even though MCM technique like Orthogonal Frequency Division Multiplexing (OFDM) is showing excellent performance against multipath fading, it is very sensitive to Carrier Frequency Offset (CFO), that leads to a severe distortion in subcarrier orthogonality and causes inter channel interference (ICI) [2]. Hence CFO must be estimated and compensated either by using periodic pilot tones [3]-[4] or blindly [5]-[7].

We proposed a novel algorithm for estimating the CFO in an OFDM receiver without using reference symbols, pilot carriers or extra cyclic prefix. We employed the Rank-Revealing QR triangular factorization (RRQR) [8]-[9] for estimating the carrier offset in the received signal. The RRQR is a good alternative of conventional subspace decomposition techniques like SVD, EVD [10] etc. with a lower computational cost. Moreover, it is quite supportive in rank deficient least square problems.

II. Problem Formulation

We consider an OFDM system implemented by inverse discrete Fourier transform (IDFT) and discrete Fourier transform (DFT) each of size N for modulation and demodulation respectively. Only P subcarriers of total N subcarriers are used to avoid aliasing. The N samples of IDFT output are given by $\boldsymbol{x}(k) := \boldsymbol{W}_P\boldsymbol{s}(k)$, where $\boldsymbol{W}_P$ consist of the first P columns of the $N \times N$ IDFT matrix and $\boldsymbol{s}(k) = [s_0(k)\ s_1(k) \dots\dots s_{P-1}(k)]^T$ is a QPSK or QAM data symbol to be transmitted through the k-th block. An OFDM symbol is denoted as $\{x_{N-G}, \dots\dots x_{N-1}, x_0, x_1 \dots\dots x_{N-1}\}$ of which the first G samples are guard samples to cancel ISI.

The receiver input for the k-th block given by

$$\boldsymbol{y}(k) = \boldsymbol{E}\boldsymbol{W}_P\boldsymbol{H}\boldsymbol{s}(k)e^{j(k-1)\varphi(N+G)} + \boldsymbol{z}(k) \quad (1)$$

where $\boldsymbol{H} = diag[H(0), H(1), \dots\dots H(P-1)]$, $H(i) = \sum_{l=0}^{L_c-1} h(l)\omega^{-il}$.

$\boldsymbol{E} = diag(1, e^{j\varphi}, \dots\dots e^{j(N-1)\varphi})$ and φ is the carrier offset. To maintain orthogonality among the sub-channel carriers and to avoid ICI, the matrix $\boldsymbol{E}$ must be estimated and compensated. The task now is to estimate φ assuming that the k received noisy data blocks are the only measurements available.

III. Development of Proposed Method

By collecting the K blocks of the received data in matrix $\boldsymbol{Y}$ of size $(N \times K)$

$$\boldsymbol{Y} = [\boldsymbol{y}(1)\ \ \boldsymbol{y}(2) \dots\dots \boldsymbol{y}(K)] + \boldsymbol{Z} \quad (2)$$

where the k-th block of the received signal in (2) is given by $\boldsymbol{y}(k) = [y_0(k)\ y_1(k) \dots\dots y_{N-1}(k)]^T$, and the $\boldsymbol{Z}$ is the corresponding additive white gaussian noise matrix. Constructing $(N-M+1)$ sub-matrices from $\boldsymbol{Y}$, each of size $M \times K$ such as $M \geq P$. The i-th sub matrix is given by

$$\boldsymbol{Y^i} = [\boldsymbol{y}^i(1)\ \ \boldsymbol{y}^i(2)\ \ \boldsymbol{y}^i(3) \dots\dots \boldsymbol{y}^i(K)] + \boldsymbol{Z}^i \quad (3)$$

$$\boldsymbol{y}^i(k) = [y_{i-1}(k), y_i(k), \dots\dots y_{i+M-1}(k)]^T + \boldsymbol{z}^i(k)$$
$$i = 1,2, \dots\dots N-M,\ \ k = 1,2, \dots\dots K \quad (4)$$

Collecting $(N-M+1)$ sub matrices calculated in (4) to form a $LM \times K(N-M-L+2)$ matrix $\mathbf{X}$

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{S} = \begin{bmatrix} \boldsymbol{Y}^1 & \boldsymbol{Y}^2 & \dots & \boldsymbol{Y}^{N-M-L+2} \\ \boldsymbol{Y}^2 & \boldsymbol{Y}^3 & \dots & \boldsymbol{Y}^{N-M-L+3} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{Y}^L & \boldsymbol{Y}^{L+1} & \dots & \boldsymbol{Y}^{N-M+1} \end{bmatrix}$$

Also, it can be easily shown that

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \\ \vdots \\ \boldsymbol{X}_L \end{bmatrix} = \begin{bmatrix} \boldsymbol{A\Phi}^0 & \boldsymbol{A\Phi}^1 & \dots & \boldsymbol{A\Phi}^{N-M-L+1} \\ \boldsymbol{A\Phi}^1 & \boldsymbol{A\Phi}^2 & \dots & \boldsymbol{A\Phi}^{N-M-L+2} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A\Phi}^{L-1} & \boldsymbol{A\Phi}^L & \dots & \boldsymbol{A\Phi}^{N-M} \end{bmatrix} \quad (5)$$

Where, $\boldsymbol{A} := \boldsymbol{E}_M\boldsymbol{W}_M$, $\boldsymbol{W}_M$ consists of the first M rows of $\boldsymbol{W}_P$, $\boldsymbol{E}_M = diag(1, e^{j\varphi}, \dots e^{j(M-1)\varphi})$ and the matrix $\boldsymbol{\Phi} = diag(e^{j\varphi}, e^{j(\omega+\varphi)}, \dots e^{j(\omega(P-1)+\varphi)})$ including the information of carrier offset, with

$\omega = 2\pi/N$. The matrix $\boldsymbol{X}$ can be partitioned into two subgroups of same size $\boldsymbol{X}^e$ and $\boldsymbol{X}^o$, where group matrices $\boldsymbol{X}^e$ and $\boldsymbol{X}^o$ are given by even and odd sub-matrices of matrix $\boldsymbol{X}$. It can be noticed that the matrices $\boldsymbol{X}^o$ and $\boldsymbol{X}^e$ are related by $X^e = \boldsymbol{X}^o\boldsymbol{\Phi}$. Applying RRQR

$$\boldsymbol{X} = \boldsymbol{Q}\boldsymbol{R} = \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \\ \vdots & \vdots \\ \boldsymbol{Q}_{L1} & \boldsymbol{Q}_{L2} \end{bmatrix} \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} \\ \boldsymbol{0} & \boldsymbol{R}_{22} \end{bmatrix} \quad (6)$$

where the $\boldsymbol{L}$ sub-matrices $\boldsymbol{Q}_{11}, \boldsymbol{Q}_{21} \ldots \boldsymbol{Q}_{L1}$ are of dimensions $M \times P$ and collectively forming signal sub-space in matrix $\boldsymbol{Q}$. The sub-matrix $\boldsymbol{R}_{11}$ is upper triangular square full rank matrix while $\boldsymbol{R}_{12}$ is holding remaining important information with dimensions $P \times K(N - M - L + 2)$. Because of rank-revealing QR-factorization re-write (6) as

$$\boldsymbol{X}^o \cong \boldsymbol{Q}^o[\boldsymbol{R}_{11} \;\; \boldsymbol{R}_{12}] \quad (7)$$

$$\boldsymbol{X}^e \cong \boldsymbol{Q}^e[\boldsymbol{R}_{11} \;\; \boldsymbol{R}_{12}] \quad (8)$$

where group matrices $\boldsymbol{Q}^e$ and $\boldsymbol{Q}^o$ are given by even and odd sub-matrices of signal subspace, from (7), we get

$$[\boldsymbol{R}_{11} \;\; \boldsymbol{R}_{12}] = \boldsymbol{Q}^{o\dagger}\boldsymbol{X}^o \quad (9)$$

where the operator $[\blacksquare]^\dagger$ is the pseudo inverse of the matrix. Substituting the above equation into (7)

$$\boldsymbol{X}^o\boldsymbol{\Phi} = \mathbf{Q}_0\boldsymbol{X}^o \quad (10)$$

where the matrix $\mathbf{Q}_0 = \boldsymbol{Q}^e\boldsymbol{Q}^{o\dagger}$. Re-write (10) as

$$\boldsymbol{\Phi}_{ii}\boldsymbol{X}_i^o = \mathbf{Q}_0\boldsymbol{X}_i^o, \; i = 1,2,\ldots\ldots P \quad (11)$$

Equation (11) is a classical eigenvalue problem with the eigenvector $\boldsymbol{X}_i^o$ and the eigenvalue $\boldsymbol{\Phi}_{ii}$. The eigenvector $\boldsymbol{X}_i^o$ is the i-th column of the matrix $\boldsymbol{X}^o$ and the $\boldsymbol{\Phi}_{ii}$ is the i-th diagonal element of the diagonal matrix $\boldsymbol{\Phi}$. Clearly here P eigenvalues of the matrix $\mathbf{Q}_0$ correspond to the P diagonal elements of the diagonal matrix $\boldsymbol{\Phi}$. Hence, the CFO can be estimated as

$$\exp(j\varphi) = \frac{trace(\mathbf{Q}_0)}{\sum_{k=0}^{P-1} e^{jk\omega}} \quad (12)$$

IV. Simulation Results

We considered OFDM system with N=64 carriers, of which P=40 are used carriers. Transmitted symbols are drawn from equiprobable QPSK constellation. The cyclic-prefix (CP) length is eleven symbols, the matrix structure parameter L is assumed to be two and the frequency offset is assumed to be 0.1ω. The experiment is verified under AWGN environment with $N_t = 1000$ independent monte-carlo realizations.

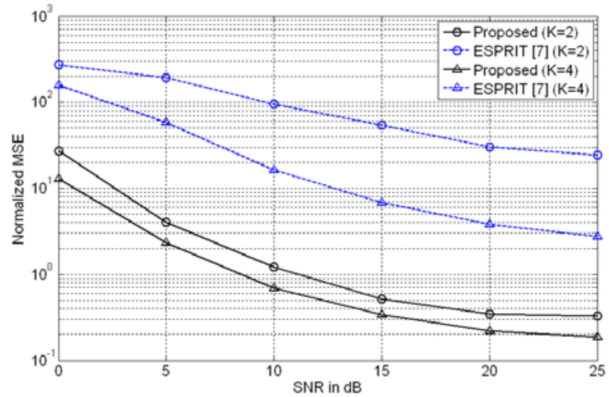

Fig.1. Normalized MSE versus SNR at $\varphi = 0.1\omega$

The normalized MSE is compared with two different number of blocks (K=2 and K=4) acquisition. Even for a small number of block acquisition our algorithm performs much better than the classical ESPRIT [7] type algorithm. For example, to achieve the same MSE performance with just K=4, the reference algorithm requires an approximately 20 dB of additional SNR.

V. Conclusion

We proposed a low complexity blind OFDM CFO Estimation algorithm. The main advantage with the proposed algorithm is that it does not use any training symbols and therefore saving transmission bandwidth. Also, it is equipped with a closed-form formula which alleviates the problem of searching entire subspace for required parameter of interest.

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