# A GENERIC BAYESIAN APPROACH USING LAPLACE APPROXIMATION FOR MODEL-BASED FAILURE PROGNOSIS


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DEDICATION

To
My Parents
and all the friends in Wichita

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ABSTRACT

A generic Bayesian framework using Laplace approximation for model-based remaining useful life prognosis is presented in this thesis. The developed generic Bayesian prognosis approach models and updates the remaining useful life distributions by incorporating timely evolving sensory data using a general Bayesian inference mechanism, and employs an efficient Bayesian updating approach using Laplace approximation (LA) method. The developed Bayesian prognosis approach eliminates the dependency of evolutionary updating process on a selection of distribution types for the parameters for a given system degradation model. Furthermore, with the developed LA method, the Bayesian updating process can be carried out efficiently which makes the proposed approach possible for real-time prognosis applications. The proposed Bayesian prognosis methodology is generally applicable for different degradation models without prior distribution constraints as faced by conjugate or semi-conjugate Bayesian inference models. Electric resistor prognosis application is employed in this study to demonstrate the efficacy of the proposed prognosis methodology.

TABLE OF CONTENTS


Chapter Page

1 INTRODUCTION ... 1

1.1 Background ... 1
1.2 Related Work ... 3
1.2.1 Model-Based Approach ... 4
1.2.2 Data Driven Approach ... 5
1.2.3 Hybrid Approach ... 5
1.2.4 Health Prognosis ... 6
1.2.5 Problems and Challenges ... 7

2 A GENERIC BAYESIAN FRAME FOR MODEL-BASED FALIURE PROGNOSIS ... 9

2.1 A General Bayesian Inference Model ... 9
2.2 Remaining Useful Life Prognosis ... 11

3 BAYESIAN UPDATING TECHNIQUE ... 15

3.1 Conjugate Updating for Bayesian Posterior Distribution ... 15
3.2 Non-conjugate Updating for Bayesian Posterior Distribution ... 15
3.3 Laplace Approximation for Bayesian Posterior Distribution ... 17
3.3.1 Laplace approximation ... 17
3.3.2 Example ... 20

4 PROGNOSIS APPLICATIONS ... 22

4.1 Experimental Setup and Degradation Signals of Resistors ... 22
4.2 Remaining Useful Life Prognosis ... 23
4.3 Compare the Result with MCMC Simulation ... 24

5 CONCLUSIONS AND FUTURE WORK ... 27

5.1 Conclusion ... 27
5.2 Future Work ... 27

6 REFERENCES ... 29

LIST OF FIGURES

Figure Page

1 A Generic Bayesian Prognosis Framework ........................................ 11
2 Simples from Markov Chain Monte Carlo Simulation ........................................ 16
3 Looking for the Local Maximum ........................................ 20
4 Poisson Distribution with $\lambda$=10 ........................................ 21
5 Laplace Approximation for Poisson Distribution ........................................ 21
6 Experiment Setup for Resistor Degradation Testing ........................................ 22
7 Degradation Voltage Signal for 26 Tested Resistors ........................................ 23
8 Real-Time Prognosis Results at Three Different Times ........................................ 24

LIST OF TABLES

Table Page

1 Procedure of the Proposed Bayesian Prognosis Approach ........ 12
2 MCMC Method for Non-Conjugate Bayesian Model ........ 17
3 Procedure for the Laplace Approximation Method ........ 20
4 Failure Times Evaluate for Different Resistors ........ 25
5 Example Table for Demonstration ........ 26
6 Means and Standard Deviations of the Actual Life ........ 26

LIST OF ABBREVIATIONS / NOMENCLATURE

| | |
|---|---|
| RUL | Remaining Useful Life |
| LA | Laplace Approximation |
| RBDO | Reliability-based Design Optimization |
| NoFE | Number of function evaluation |
| MCMC | Markov Chain Monte Carlo |
| MPFS | Matrix Post Failure Stiffness |
| MCT | Multi-Continuum Theory |
| OHT | Open Hole tension |
| OHC | Open Hole Compression |
| RVE | Representative Volume Element |
| SDV | Solution Dependent Variable |

## LIST OF SYMBOLS

| | |
|---|---|
| $H(.)$ | Hessian Matrix |
| $J(.)$ | Jacobi Matrix |
| $L(t)$ | Logged Degradation Signal at Time t |
| $p(x)$ | Prior Distribution |
| $p(L\|x)$ | Likelihood Function in Bayesian Inference |
| $\theta$ | Model Parameters for Bayesian Inference |
| $S(t)$ | Degradation Signal at Time t |
| $T_i$ | Degradation Signal at Time t |
| $\delta, \alpha, \beta$ | Stochastic Model Parameters |
| $\varepsilon$ | Random Error Term |
| $\sigma^2$ | Standard Deviation |
| $\Theta$ | Random Variable |
| $D$ | Sensor Data |
| $\delta'$ | $\ln\delta - \sigma^2/2$ |
| $t_1$ | Stochastic Time Point |

LIST OF SYMBOLS (continued)

| | |
|---|---|
| $N$ | Number sets of parameters data |
| $\mathbf{x}^*$ | Maximum Point for Distribution |
| $\Sigma$ | Covariance Matrix |

# CHAPTER 1

# INTRODUCTION

## 1.1 Background

Probabilistic inference has been extensively used in the modern engineering problems, due to the high reliability demanded problems. To support critical decision-making processes such as maintenance replacement and product design, engineering systems composed of multiple components, complex joints, and various materials, such as distributed manufacturing facilities, electronic devices, advanced military systems, require sensory health monitoring and prognosis [1]. While interpreting data acquired by smart sensors and distributed sensor networks, and utilizes these data streams in making critical decisions, research on real-time diagnosis and prognosis which provides significant advancements across a wide range of application. Maintenance and life-cycle management is one area that is positioned to significantly benefit in this regard due to the pervasive nature of design and maintenance activities throughout the manufacturing and service sectors. Maintenance and life-cycle management activities constitute a large portion of overhead costs in many industries [2]. These costs are likely to increase due to the rising competition in today's global economy.

In the manufacturing and service sectors, unexpected breakdowns can be prohibitively expensive since they immediately result in substantial risks (e.g., lost production, substantial maintenance cost, and poor customer satisfaction). In order to reduce and possibly eliminate such risks, it is necessary to accurately assess the current state of system degradation and precisely evaluate the remaining life of degrading components. Two major research areas have tried to address these challenges: reliability analysis and condition monitoring. Although reliability analysis and condition monitoring are seemingly related, reliability analysis focuses on population-wide characteristics while condition monitoring deals with component-specific properties. Furthermore, both fields of research have evolved separately.

Research in the reliability analysis area can be broadly classified into two subcategories. One category focuses on quantification of reliability and statistical analysis of time-to-failure data, such as [3–5] while the other deals with the development of physics-based models (e.g., fatigue, wear, corrosion) and finite-element methods aimed at reliability-based design optimization (RBDO) [6-9]. In contrast, condition monitoring research uses sensory information from functioning system to assess their degradation states. Some of the applications of condition monitoring include condition monitoring of bearings [10–12], machine tools [13], transformers [14], engines [15], and turbines [16] among many others. Most of the research in this field focuses on system diagnosis and fault classification. Some research utilizes condition monitoring information for performing prognosis [17–21]. However, these efforts focus on the characterizing individual components with little or no utilization of reliability information.

This thesis presents a generic Bayesian framework using Laplace approximation for model-based remaining useful life prognosis. The proposed Bayesian prognosis methodology will benefit both maintenance and product design decisions by providing highly confident lifetime and reliability information. It models and updates remaining useful life distributions by incorporating timely evolving sensory data using a general Bayesian inference mechanism and employs an efficient Bayesian updating approach using a Laplace approach (LA) method. The developed Bayesian prognosis approach eliminates the dependency of evolutionary updating process on a selection of distribution types for the parameters for a given system degradation model. Furthermore, with the developed LA method, the Bayesian updating process can be carried out efficiently which makes the proposed approach possible for real-time prognosis applications. While accounting for variability in loading conditions, material properties, and manufacturing tolerances over the population of system samples, different reliabilities will be identified for different samples. Thus, reliability distribution for an engineering system can be also obtained and updated in a Bayesian format. The proposed Bayesian prognosis methodology is generally applicable for different degradation models without prior distribution constraints as faced by conjugate or semi-conjugate Bayesian inference models. Two practical prognosis applications are employed in this study to demonstrate the efficacy of the proposed Bayesian prognosis methodology. The rest of the thesis is organized as follows. Chapter 2 surveys the literature for remaining useful life and reliability prognosis. The developed Bayesian prognosis methodology will be introduced in detail in chapter 3 and the prognosis case study will be presented in chapter 4.  A brief summary of the presented work is provided in chapter 5.

## 1.2 Related Work

In this chapter, relevant literature related to system degradation modeling and real-time life and reliability prognosis will be reviewed. The goal of real-time health prognosis is to predict the remaining life of a system (or component) based on a real time health condition. For that purpose, a stochastic degradation model must be developed to model system degradation and predict the remaining life in a statistical manner. Prognostic techniques in single time-scale can be classified into three categories: model-based, data-driven and hybrid approach. The first one builds on model-based techniques, where physics-base, statistical probabilistic, and Bayesian updating method are used to simulate the fatigue and fault prorogation model. The second approach based on the previous failure data and uses the knowledge form the machine learning technique, where neural-network, neuro-fuzzy, and other algorithms are employed to construct a surrogate model for the complex failure process.

### 1.2.1 Model-Based Approach

The degradation model contains a multiple of stochastic parameters and they will be continuously updated in real time. In the literature, different stochastic degradation models have been developed to model various degradation phenomena of systems or components. Doksum and Hoyland [22], Lu [23], and Whitmore *et al*. [24] used a Wiener diffusion process for a degradation model. In this approach, it is assumed that one key degradation measure governs failure and takes the statistical model to be a Wiener diffusion process $\{W(t)\}$ with statistical parameters, mean and variance. Bagdonavicius and Nikulin [25] modeled the degradation by a Gamma Process and include possibly time-dependent covariates. In their study, the influence of covariates on degradation was modeled and estimation of reliability and degradation characteristics from data with covariates was considered. Lu and Meeker [26], Boulanger and Escobar [27], Tseng, Hamada and Chiao [28], Hamada [29], Chiao and Hamada [30], and Meeker, Escobar and Lu [31] considered degradation path models. Degradation in these models is modeled by the process $S(\mathbf{u},t)$, where $t$ is time and $\mathbf{u}$ is a multidimensional random vector. By fitting the model parameter vector $\mathbf{u}$ with the degradation data measured, the lifetime and reliability of the systems or components were then studied. Suzuki, Maki and Yokogawa [32] used linear degradation models to study the increase in a resistance measurement over time. Dowling [33], Meeker and Escobar [34] used convex degradation models to study the growth of fatigue cracks. Meeker and Escobar [34] also used concave degradation models to study the growth of failure-causing conducting filaments of clorine-copper compound in printed-circuit boards. Carey and Koenig [35] used similar models to describe degradation of electronic components. Meeker et, al. [34] developed an exponential pattern model to study the life distribution over a population of components, and recently, Gebraeel et, al. [36] applied similar exponential pattern degradation with stochastic process of modeling random error term to study the residual life of single operating device of ball bearings.

### 1.2.2 Data Driven Approach

Besides the stochastic degradation models, some models are developed based on the artificial intelligent technique. Chinnam [37] presents a neural network based model for online estimation of component reliability. Gebraeel et, al. [38] also used the neural network based approach to study the component remaining life distribution and applied on the case study of rolling bearing prognosis. Brotherton et al.[39, 40] combined neural nets with rule extractors and applied it to gas turbine engine prognostics. Their system first fits a dynamically linked ellipsoidal basis function neural network to the vibration data, and fits decision tree to the neural net, which could be used to help realize the model. The new system has been applied to vibration data from seeded fault test stand operation of a jet engine. Parker et al. [41] used polynomial neural network, and trained them using vibration data from seeded faults in helicopter gearboxes system. Wegerich et al [42]. used a similarity based method for data driven prognostic. Their method makes predictions using an average of the training data, weighted based on a similarity measure. They assert that their system is applicable to any multivariate data stream in which a consistent relationship between individual streams exists, but the paper focuses on vibration data from rotating machinery. They use data obtained from a laboratory mechanical test system with induced faults. They demonstrate that their methods can help to detect certain induced faults, but leave estimation of useful life remaining as future work.

### 1.2.3 Hybrid Approach

Atlas et al [43]. present an architecture that combines the model-based and data-driven approaches to fault detection, diagnosis, and prognostics for aircraft it includes a prognostic reasoned that takes as input the output from a variety of specialized prognostic algorithms for different systems within the aircraft, and the prioritizes the most probable failure modes.

### 1.2.4 Health Prognosis

Once the degradation model is obtained for a given degradation signal, health prognosis of systems or components can be further studied. Lu and Meeker [26] considered the case in which the life distribution of a population of devices is to be computed using degradation information obtained from a randomly selected set of devices and illustrated various methods for computing life distributions with various random coefficient models. Tseng *et al.* [28] combined random coefficient models for luminosity degradation with experimental design to identify manufacturing settings that provide slow rates of luminous degradation of fluorescent lamps. Doksum and Hoyland [22] developed a maximum likelihood estimator of inverse Guassian life models for units subject to accelerated stress testing. In their study, accumulated decay is modeled as a Wiener process with drift and diffusion dependent on and changing with the stress level. They illustrated how to use test data to estimate the mean life under normal stress levels. Whitmore and Schenkelberg [44] used a Wiener process to model degradation data collected from accelerated testing, developed methods for estimating the parameters of time and stress transformations and apply their methods on a case study on self-regulating heating cables. Lu *et al.* [45] presented methods for forecasting system performance reliability for systems with multiple failure modes. In their investigation, time series forecasting was used to develop a joint density function for the performance measures. This joint density is then integrated to obtain a reliability function that can be used to assess the system reliability. Lu *et al.* [45] developed a method for real-time estimating the conditional performance reliability for an individual operating component. In their study, sampled measurements are treated as a realization of a stochastic process, and exponential smoothing is used to develop a conditional distribution of the performance variable.

### 1.2.5 Problems and Challenges

Although different methods have been developed for the purpose of prognosis as listed above, they are all model dependent and only applicable for certain or few degradation models being used. In this thesis, a generic Bayesian framework for decision-centered lifetime and reliability prognosis is developed, which employs the non-conjugate Bayesian updating technique and is generally applicable to any stochastic degradation models with random model parameters. For demonstrative purpose, we employ a quadratic exponential pattern degradation model modified from Gebraeel *et al.* [38] and Lu *et al*. [26]. In the study of Gebraeel *et al.* [38], the Bayesian semi-conjugate updating model is used for updating the model parameters and the predicted remaining life distribution has its closed analytical form due the intentionally constructed conjugate properties of the Bayesian updating model. However, there are two obvious drawbacks of this approach. First, prior distributions for the model parameters must be specified as certain distribution types, for example, lognormal distribution and normal distributions, to guarantee the conjugate property to be held. This specification of prior distributions will substantially limit the application of this approach. Second, since the posterior distribution of the model parameters have strong correlation, the updating mechanism can only be applied once and thus cannot be applied for real-time applications.

To make the Bayesian updating technique generally applicable to a broad range of engineering problems, the generic Bayesian framework employs a general Bayesian inference mechanism with a Laplace approximation technique. The proposed methodology is able to predict the remaining life distribution at any time of early degradation stage and to update the distribution in real time with continuously evolving signals. The reliability of the system or components considering the population variability is evaluated at an early degradation stage and updated with more testing specimens involved. Details of the proposed method will be discussed in the next chapter with the modified exponential pattern model as an example.

# CHAPTER 2

# A GENERIC BAYESIAN FRAME FOR MODEL-BASED FALIURE PROGNOSIS

To consistently model diverse degradation signals from different engineering applications, a generic Bayesian prognosis framework is proposed in this chapter. This framework uses a general Bayesian inference model to eliminate dependency of evolutionary updating process on a selection of distribution types in the sensory degradation data model. A Laplace approximation method will be developed and employed for the updating process of a general Bayesian prognosis model.

## 2.1 A General Bayesian Inference Model

As mentioned previously, the degradation signals can continuously be obtained either through embedded sensor network or online monitoring facilities. To effectively extract the valuable information about the health condition of the monitored components or systems, the generic Bayesian framework employs the Bayesian technique for updating the model parameters with evolving sensory degradation signals. Bayesian inference can be defined as a process of fitting a probability model to a set of data, and summarizing the result by a probability distribution on the parameters of the model and on unobserved quantities. This chapter gives a brief introduction of the Bayesian updating technique.

Let $X$ be a random variable with probability density function $f(x, \theta)$, $\theta \in \Omega$. According to the Bayesian point of view, $\theta$ is interpreted as a realization of a random variable $\Theta$ with a probability density $f_{\Theta}(\theta)$. The density function expresses the prior knowledge of $\Theta$ before any future observation of $X$ is taken, that is, a prior distribution. Based on the Bayes' theorem, the posterior distribution of $\Theta$ given a new observation $X$ can be expressed as

$$f_{\Theta|X}(\theta \mid x) = \frac{f_{X,\Theta}(x,\theta)}{f_X(x)} = \frac{f_{X|\Theta}(x \mid \theta) \cdot f_{\Theta}(\theta)}{f_X(x)} \tag{1}$$

The Bayesian approach is used for updating information about the parameter $\theta$. First, a prior distribution of Θ must be assigned before any future observation of *X* is taken. Then, the prior distribution of Θ is updated to the posterior distribution as the new data for *X* is employed. The posterior distribution is set to a new prior distribution and this process can be repeated with evolution of data sets. The difference between the prior distribution and the posterior is the later one with an additional information x.

To update the degradation model parameters using the Bayesian updating technique, the likelihood function $f_{X|\Theta}(x|\theta)$, which combines a new degradation signal with the prior information of model parameters, is quite essential. However, for algebraic convenience, existing researches mostly focus on seeking the conjugate or semi-conjugate models [22, 23]. Conjugate models of Bayesian updating are quite valuable for uncertainty modeling with continuously evolving signals due to its closed form of the posterior distribution. However, only limited conjugate or semi-conjugate models are available and, thus, updating results strongly depend on selection of the models. To overcome such difficulty, non-conjugate Bayesian updating framework must be developed but the computational algorithm is quite complicated. In order to calculate the posterior, simulation method has been involved, which make the updating process computational expensive and time consuming. So this research work is the overcome this, and implement a quick updating process can make the whole updating process more efficient, and maintain the accuracy for the real engineering case.

A Laplace approximation method will be developed in this study and used for the non-conjugate Bayesian updating procedure, which will be introduced in later.

## 2.2 Remaining Useful Life Prognosis

As shown in Figure 1, the generic Bayesian prognosis framework contains three critical steps: (1) updating the model parameters based on the prior information and a new sensory degradation signal; (2) updating the lifetime distribution; (3) updating the reliability distribution. The framework is illustrated in Table 1.

Note that Step 2 to Step 4 can be repeated until the lifetime distribution can be continuously updated with the evolving degradation signals. In the following, the detail procedure will be explained with an exponential degradation family. For the demonstration purpose, a quadratic exponential degradation model can be used as

$$S(t_i) = S_0 + \delta \cdot \exp(\alpha t_i^2 + \beta t_i + \varepsilon(t_i) - \frac{\sigma^2}{2}) \quad (2)$$

where $S(t_i)$ represents the degradation signal at time $t_i$; $S_0$ is a known constant; $\delta$, $\alpha$, and $\beta$ are stochastic model parameters and $\varepsilon$ represents the random error term which follows normal distribution with zero mean and $\sigma^2$ deviation.

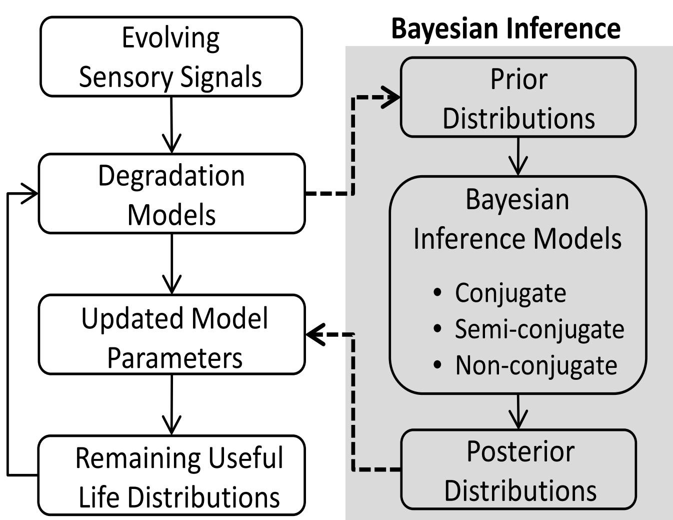



**Figure 1. A Generic Bayesian Prognosis Framework**

**Table 1. Procedure of the Proposed Bayesian Prognosis Approach**

| | |
|---|---|
| STEP1 | Selecting an appropriate degradation model and specifying model parameters and prior distributions |
| STEP2 | Building a likelihood function with the prior distributions and new sensory signal |
| STEP3 | Updating the joint probability distributions of the model parameters with a prior model information and degradation signals by non-conjugate Bayesian updating technique |
| STEP4 | Updating the lifetime distribution based on the updated degradation model parameters |

**Step 1:** Prior distributions are required to be specified for the model parameters $\delta$, $\alpha$ and $\beta$. Gebraeel *et al*. [36] successfully applied the Bayesian updating technique to the similar model by constructing a semi-conjugate Bayesian updating model. By assuming that $\delta$ and $\beta$ follow lognormal and normal prior distributions, respectively, normal posterior distributions for both $\delta'$ and $\beta$ with a certain correlated coefficient will be obtained through the Bayesian updating process, where $\delta'=\ln\delta - \sigma^2/2$. As the model parameters being updated, the degradation signal $S(t_i)$ is proven to be a normal distribution and the lifetime distribution can then be obtained if a failure threshold $D$ is predetermined.

Although the semi-conjugate Bayesian updating model constructed by Gebraeel *et al*. [36] can simplify the computation for the posterior distribution and thus obtain a closed form remaining life distribution, however, there are two obvious drawbacks of this approach. First, prior distributions for the model parameters must be specified as certain distribution types, for example, lognormal distribution and normal distributions, to guarantee the conjugate property to be held. This specification of prior distributions will substantially limit the application of this approach. Second, since the posterior distribution of the model parameters have strong correlation, the updating mechanism can only be applied once and thus cannot be applied for real-time applications. To make it generic to update the degradation models, different prior distributions may be preferred for different model parameters. In such situations, non-conjugate Bayesian updating framework is more desirable because it can update parameter distributions for any given prior distributions.

**Step 2:** For the purpose of demonstration, the model in Eq. (2) is considered. It is more convenient to work with the logged signal at time $t_i$ which will be denoted as $L(t_i)$, as

$$L(t_i) = \ln(S(t_i) - S_0) = \ln\delta - \frac{\sigma^2}{2} + \alpha t_i^2 + \beta t_i + \varepsilon(t_i) = \delta' + \alpha t_i^2 + \beta t_i + \varepsilon(t_i) \tag{3}$$

where $\delta'=\ln\delta - \sigma^2/2$. Suppose a new sensory signal, $L_i = L(t_i)$, be observed as $L_1, L_2, \ldots, L_k$ at times $t_1, t_2, \ldots, t_k$. Since the error terms, $\varepsilon(t_i)$, i= 1,2, …, $k$, are iid normal random variables, the following likelihood function can be obtained for given observations as

$$f\left(L_1, L_2, \cdots, L_k \mid \delta', \alpha, \beta\right) \sim \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^k \cdot \exp\left\{-\sum_{i=1}^{k}\left(\frac{\left(L_i - \delta' - \beta t_i - \alpha t_i^2\right)^2}{2\sigma^2}\right)\right\} \tag{4}$$

**Step 3:** If the prior distributions for $\delta'$, $\alpha$ and $\beta$ are provided, for example, $\pi_0(\delta', \alpha, \beta)$, the joint posterior distribution for these parameters can be expressed as

$$f\left(\delta', \alpha, \beta \mid L_1, L_2, \cdots, L_k\right) \sim \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^k \cdot \exp\left\{-\sum_{i=1}^{k}\left(\frac{\left(L_i - \delta' - \beta t_i - \alpha t_i^2\right)^2}{2\sigma^2}\right)\right\} \times \pi_0\left(\delta', \alpha, \beta\right) \tag{5}$$

As shown in Eq.(5), the posterior joint distribution of model parameters $\delta'$, $\alpha$, and $\beta$ strongly depends on selection of the joint prior distributions $\pi_0(\delta', \alpha, \beta)$. Thus it is not reasonable to use a conjugate or semi-conjugate Bayesian model, which gives a closed form posterior distribution for each parameter. In this circumstance, non-conjugate Bayesian updating framework is more desirable because it can update parameter distributions for any given prior distributions. However, with non-conjugate Bayesian model it is extremely difficult to compute the exact analytical form of the posterior distribution for the model parameters since the normalization factor of the posterior distribution, which is the denominator in Eq.(1), requires complicated and multi- dimensional integration. Although it is hard to obtain the posterior distribution directly, it is feasible to estimate the parameter distributions of the posterior distribution. In this study, a Laplace approximation method that is detailed in the next section is employed for efficient Bayesian posterior distribution approximation.

**Step 4:** The life distribution information can be obtained when a failure threshold $D$ is specified for the degradation signal. By plugging $N$ sets of parameters data extracted from the model parameters' posterior distributions, the life data can be determined by solving the $N$ sets of quadratic equations as

$$T_i = \text{roots}\left(\alpha T^2 + \beta_i T + \delta'_i + \varepsilon_i - D = 0\right), \;\; where\ i = 1, 2, ..., N. \tag{6}$$

When predicting the system life using Eq. (6), the random error term, $\varepsilon$, should not be ignored. Moreover, the life $T_i$ is the positive real root of the quadratic equation. The sample size, $N$, can be increased because solving the quadratic equation in Eq. (6) $N$ times is trivial. The life data, $T_1$, $T_2$, …, $T_N$, can be used for creating the life distribution. As sensory signal evolves over time, the life distribution can be updated in real-time by repeating Step 2 to Step 4.

# CHAPTER 3

# BAYESIAN UPDATING TECHNIQUE

In this chapter, the traditional updating process will be introduced, which includes conjugate updating, non-conjugate updating, and Laplace approximation. For this research work, the Laplace approximation has been applied, and hence more detailed information on this shall be provided.

## 3.1 Conjugate Updating for Bayesian Posterior Distribution

In Bayesian inference, if the posterior distributions are in the same family as the prior probability distribution, the prior and posterior are then called conjugate distribution, and the prior is called the conjugate prior to the likelihood function. For example, the Gaussian family is conjugate to itself which can be called self-conjugate with respect to the likelihood function in a Gaussian form. Therefore, that means if we choose a certain Gaussian probability density distribution as a prior, with a Gaussian likelihood function, the posterior can be reasonable calculated as another Gaussian with different mean and stand deviation.

A conjugate prior is an algebraic convenience, giving a closed-form expression for the posterior: otherwise a difficult numerical integration may be necessary. Further, conjugate priors may give intuition, by more transparently showing how a likelihood function updates a distribution.

## 3.2 Non-conjugate Updating for Bayesian Posterior Distribution

For the non-conjugate pair, the updating process is quite complicated, because the normalizing constant involves multiple integration that make the process numerical infeasible. Even if the expression of posterior is available as a product of likelihood and prior, the shape of distribution can only be estimated by calculating it values at different points. A wise way is to compute the values of PDF at a grid of points after identifying the effective range at each grid point. In addition, it becomes computationally expensive with the number of updating parameters increases. So in order to simulate the accurate posterior distribution, simulation method will be used. Markov Chain Monte Carlo (MCMC) has been extensively used in the non-conjugate Bayesian Updating.

The MCMC method provides a mechanism based on constructing a Markov Chain to draw samples from the complicated posterior distribution as its equilibrium distribution. The states of the chain then approximate the desired posterior distribution after a large number of steps, as shown in Figure 2. Moreover, the quality of the samples improves as a function of the number of the steps. In this study, the Metropolis-Hastings algorithm is implemented for the MCMC method to be applied to the resistor prognosis case study.

The algorithm generates a Markov chain in which each state Xt+1 for δ' , α and β depends only on the previous state $X^t$. The algorithm also uses a proposal density Q($X'$, $X^t$), which depends on the current state $X^t$, to generate a new state $X'$. This proposal is 'accepted' as the next value ($X^{t+1}$ = X') if $u$ drawn from $U(0,1)$ is satisfied. The MCMC method for non-conjugate Bayesian model updating using the Metropolis-Hastings sampling algorithm is summarized in Table 2. More details regarding the MCMC method can be found in [46-49].

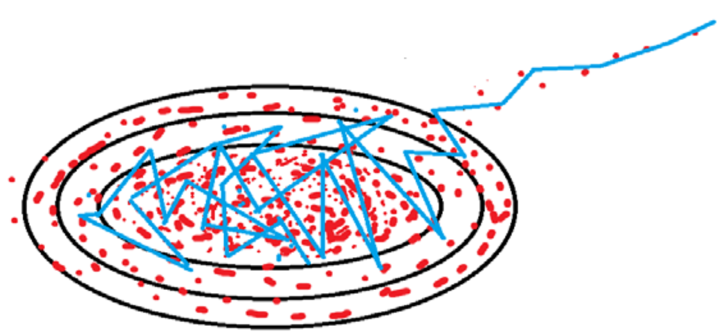


**Figure 2. Simples from Markov Chain Monte Carlo Simulation**

**Table 2. MCMC Method for Non-Conjugate Bayesian Model**

| **Algorithm** A Metropolis-Hastings sampler for the quadratic exponential pattern model |
|---|
| (1) Initialization of Parameters, $X^1 = [\delta_0, \alpha_0, \beta_0]$, set $t$ =1. |
| (2) Propose $X' = Q(X', X^t)$ |
| (3) Draw $u \sim U(0, 1)$ |
| (4) Calculate $a = \{P(X')\cdot Q(X^t\| X')\}/ \{P(X^t)\cdot Q(X'\|X^t)\}$ |
| (5) If $a > u$, $X^{t+1} = X'$; *else*, $X^{t+1} = X^t$ |
| (6) *If* $t<N$, set $t = t+1$, go Step (2); else , stop. |

### 3.3 Laplace Approximation for Bayesian Posterior Distribution

This section details the Laplace approximation method for efficient Bayesian posterior distribution approximation.

#### 3.3.1 Laplace approximation

Let **x** be the unknown variables $\delta$, $\alpha$, $\beta$, and $L$ represent the given data $L_1, L_2 \ldots L_k$, the Bayesian inference model as introduced in the subchapter 3.1 can be generally rewritten as:

$$p(\mathrm{x} \mid L) \propto p(L \mid \mathrm{x}) \cdot p(\mathrm{x}) \tag{7}$$

where $p(\mathbf{x})$ is the prior distribution, $p(L|\mathbf{x})$ is the likelihood function as shown in Eq.(3), and $p(\mathbf{x}|L)$ is the posterior distribution. For the non-conjugate Bayesian inference models, analytical forms for the posterior distributions of unknown variables **x** may not available thus different approximation techniques are needed for the posterior distribution estimation. Laplace approximation is one of these methods that can used to estimate the posterior distribution in Bayesian inference. By using Laplace approximation, the posterior distribution can be considered as a multivariate normal distribution [50, 51]. The following of this subsection details the Laplace approximation method for posterior distribution estimation. Consider the natural logarithm of the posterior distribution

$$\ln p(\mathrm{x} \mid L) \propto -\sum_{i=0}^{\mathrm{k}} \left( \frac{(L_i - \delta^{'} - \beta t_i - \alpha t_i^2)^2}{2\sigma^2} \right) + \ln \pi_0 \left( \delta^{'}, \alpha, \beta \right) \tag{8}$$

Let us expand this natural logarithm to the second order Taylor series at a point $x^*$ as

$$\begin{aligned} &\ln p(\mathrm{x} \mid L) = \ln p(\mathrm{x}^* \mid L) + (\mathrm{x} - \mathrm{x}^*)^T \cdot J(\mathrm{x}^* \mid L) + \frac{1}{2}(\mathrm{x} - \mathrm{x}^*)^T \cdot H(\mathrm{x}^* \mid L) \cdot (\mathrm{x} - \mathrm{x}^*) + O(\mathrm{x}^* \mid L) \\ &where\ J(\mathrm{x}^* \mid L) = \Delta \ln p(\mathrm{x}^* \mid L)\ \ and\ \ H(\mathrm{x}^* \mid L) = \Delta^2 \ln p(\mathrm{x}^* \mid L) \end{aligned} \tag{9}$$

where J(.) and H (.) are the Jacobian matrix and Hessian matrix of $ln\ p(\mathbf{x}^* \mid L)$ evaluated at $x^*$ respectively. $O$ (.) stands for higher order terms in the Taylor series expansion. With the intention of balance the computation cost and approximation accuracy, the higher order terms $O$(.) could be ignored which lead to

$$\ln p(\mathrm{x} \mid L) \approx \ln p(\mathrm{x}^* \mid L) + (\mathrm{x} - \mathrm{x}^*)^T \cdot J(\mathrm{x}^* \mid L) + \frac{1}{2!}(\mathrm{x} - \mathrm{x}^*)^T \cdot H(\mathrm{x}^* \mid L) \cdot (\mathrm{x} - \mathrm{x}^*) \tag{10}$$

If we let the expansion point $\mathbf{x}^*$ to be a local maximum of the ln $p(\mathbf{x} \mid L)$, the Jacobian matrix J(.) will be vanished. Thus, we can obtain a further simplified Tayor series of ln $p(\mathbf{x} \mid L)$ as

$$\ln p(\mathrm{x} \mid L) \approx \ln p(\mathrm{x}^* \mid L) + \frac{1}{2!}(\mathrm{x} - \mathrm{x}^*)^T \cdot H(\mathrm{x}^* \mid L) \cdot (\mathrm{x} - \mathrm{x}^*) \tag{11}$$

After exponentiation of the Taylor series in Eq. (11), the poster distribution $p\ (x|d)$ can be transformed to

$$\begin{aligned} p(\mathrm{x} \mid L) = \exp(\ln p(\mathrm{x} \mid L)) &\approx p(x^* \mid L) \cdot \exp\left\{ \frac{1}{2}(\mathrm{x} - \mathrm{x}^*)^T \cdot H(\mathrm{x}^* \mid L) \cdot (\mathrm{x} - \mathrm{x}^*) \right\} \\ &\propto \exp\left\{ -\frac{1}{2}(\mathrm{x} - \mathrm{x}^*)^T \cdot \Sigma^{-1} \cdot (\mathrm{x} - \mathrm{x}^*) \right\} \end{aligned} \tag{12}$$

where

$$\Sigma = [-H(\mathrm{x}^* \mid L)]^{-1} \tag{13}$$

From Eq. (12), the poster distribution can be approximated with a multivariate normal distribution with a mean value vector $x^*$ and a covariance matrix of $[-H\ (x^*|L)]^{-1}$. According to Eq. (4), the normalizing constant for the posterior distribution can be approximated as

$$Z = \int \exp\left\{\ln p(\mathrm{x} \mid L)\right\} d\mathrm{x} \sim p(\mathrm{x}^* \mid L) \int \exp\left\{-\frac{1}{2}(\mathrm{x}-\mathrm{x}^*)^T \Sigma^{-1} (\mathrm{x}-\mathrm{x}^*)\right\} d\mathrm{x} \tag{14}$$

Taking advantage of the fact that integration of a multivariate normal distribution over the entire variable space equals to 1, the approximation of the normalizing constant Z can be further simplified as

$$\begin{aligned} Z &\approx p(\mathrm{x}^* \mid L) \int \left\{-\frac{1}{2}\left(\mathrm{x}-\mathrm{x}^*\right)^T \Sigma^{-1}(\mathrm{x}-\mathrm{x}^*)\right\} d\mathrm{x} \\ &= p(\mathrm{x}^* \mid L)\sqrt{(2\pi)^n \left|\Sigma^{-1}\right|} \int \frac{1}{\sqrt{(2\pi)^n \left|\Sigma^{-1}\right|}} \exp\left\{-\frac{1}{2}\left(\mathrm{x}-\mathrm{x}^*\right)^T \Sigma^{-1}(\mathrm{x}-\mathrm{x}^*)\right\} d\mathrm{x} \\ &= p(\mathrm{x}^* \mid L)\sqrt{(2\pi)^{\mathrm{n}} \mid \Sigma^{-1} \mid} \end{aligned} \tag{15}$$

where $n$ is the dimension of the unknown variables **x**, which equals to 3 here. $|\Sigma^{-1}|$ is the determinant of $\Sigma^{-1}$. With the normalizing constant $Z$, the posterior distribution $p(\mathbf{x}^*|L)$ can now be obtained as a multivariate normal distribution

$$p(\mathrm{x} \mid L) \approx \frac{1}{\sqrt{(2\pi)^n \left|\Sigma^{-1}\right|}} \exp\left\{-\frac{1}{2}(\mathrm{x}-\mathrm{x}^*)^T \Sigma^{-1}(\mathrm{x}-\mathrm{x}^*)\right\} \tag{16}$$

Equation (16) is a multivariate normal distribution, with a mean value vector $\mathbf{x}^*$ and a covariance matrix Σ. Table 3 summarizes the procedure of using the Laplace approximation method to estimate the posterior distribution in Bayesian updating.

**Table 3. Procedure for the Laplace Approximation Method**

| | |
|---|---|
| STEP 1: | Transform the posterior distribution to a natural logarithm ln $p$ (x\|$L$). |
| STEP 2: | Find the local maxima point $x^*$ of ln $p$ (x\|$L$). Numerical algorithms, such as show in Figure 3. Newton's method as an optimization method has been used for this purpose. |
| STEP 3: | Evaluate the Hessian matrix H(.) at the local maxima point $x^*$. |
| STEP 4: | Use Equation (16) to get the normalized posterior distributions. |

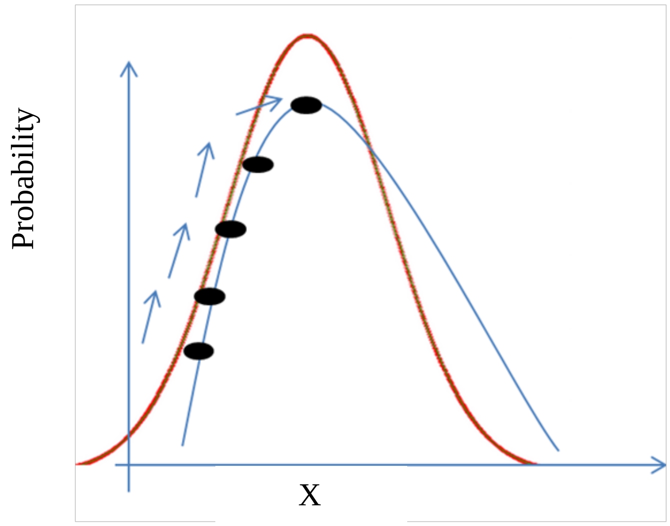



**Figure 3. Looking for the local maximum**

### 3.3.2 Illustration of Example

In this chapter, one example will be present to demonstrate how to use this method. For the demonstration propose, the Poisson distribution will be treated as the unknown distribution, and the Laplace approximation is going to make a reasonable assumption according to the limited information. The formula of Poisson distribution is:

$$f(x;\lambda)=\frac{\lambda^{x}e^{-\lambda}}{x!} \tag{17}$$

Let λ=10 and the patter of the distribution is shown in the Figure 4. The Laplace approximation will be used here to calculate the approximated distribution. The max value and the hessian matrix can be calculated by the finite difference and optimization method. The red line in the Figure 5 is the approximated distribution.

From the Figures, we can find the error involved during the approximation. The main reason of this error because this method ignore the higher order terms in the Tyler serious expansion. However, for the propose of balance the computational efficiency and the accuracy,

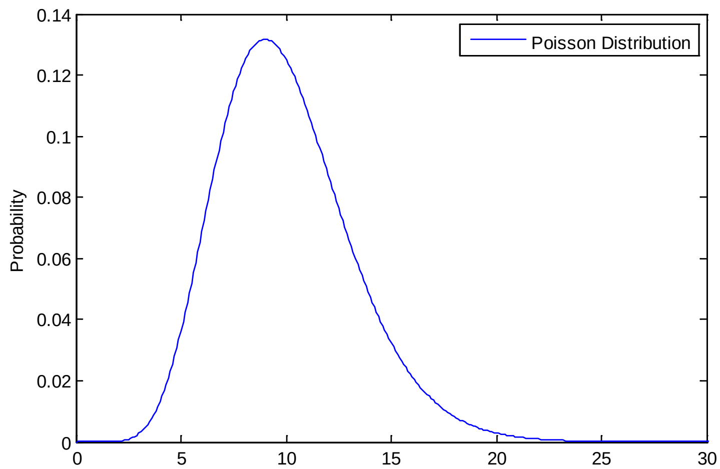



**Figure 4. Poisson Distribution with λ=10**

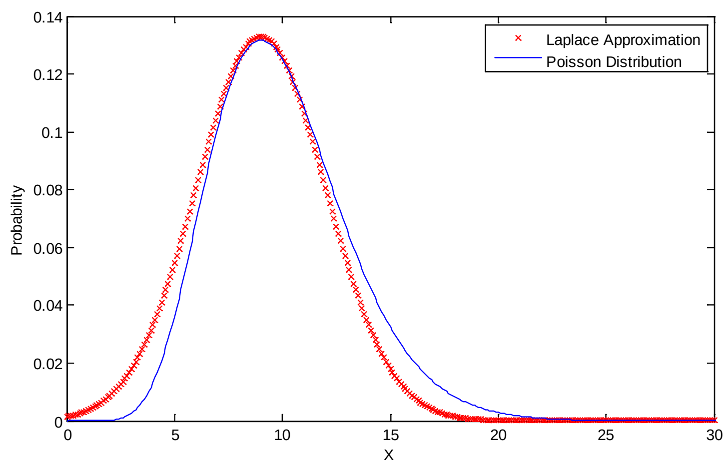



**Figure 5. Laplace Approximation for Poisson Distribution**

# CHAPTER 4

# PROGNOSIS APPLICATIONS

In this chapter, we apply the proposed Bayesian prognosis methodology with the quadratic exponential degradation model presented above to the degradation signals of 26 resistors. They have run to failure under accelerated testing conditions. We first briefly describe our experimental setup and experimental data obtained. We then describe how the proposed Bayesian prognosis methodology is implemented to predict remaining useful life distributions based on sensory signals, as developed in chapter 3. We also evaluate the predictive ability of these models and discuss the results of these experiments.

## 4.1 Experimental Setup and Degradation Signals of Resistors

In this study, 26 identical resistors are employed in the accelerated life testing. Figure 6 shows the experiment setup. In this experiment, regulated power supply is used to provide fixed current to the tested resistor and the voltage is measured and recorded by a DAQ system from National Instruments. Voltage signal is measured at frequency of 100 Hz. Sensory data for these 26 tested resistors are shown in Figure 7.

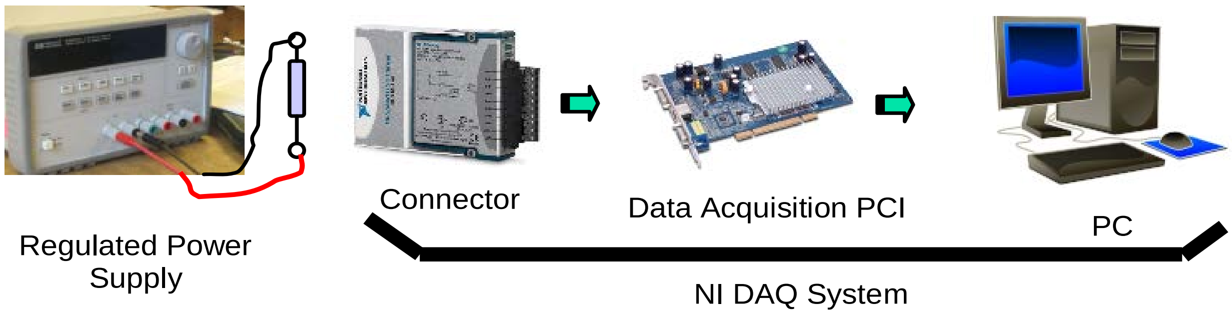



**Figure 6. Experiment Setup for Resistor Degradation Testing**

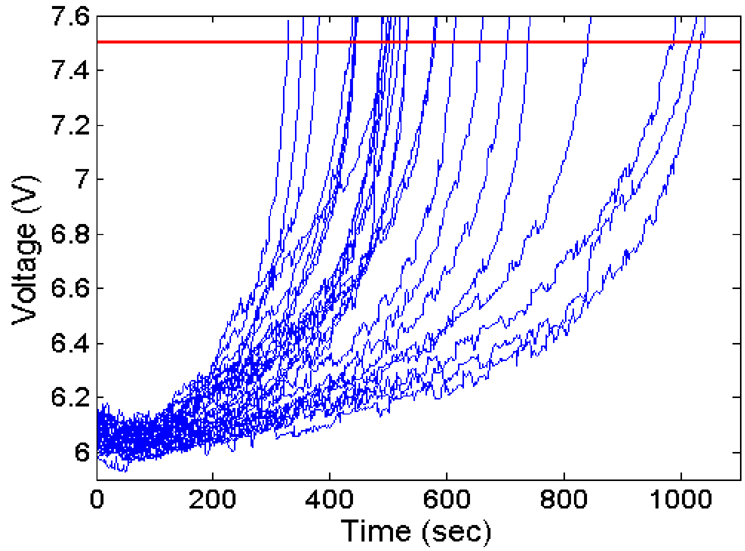
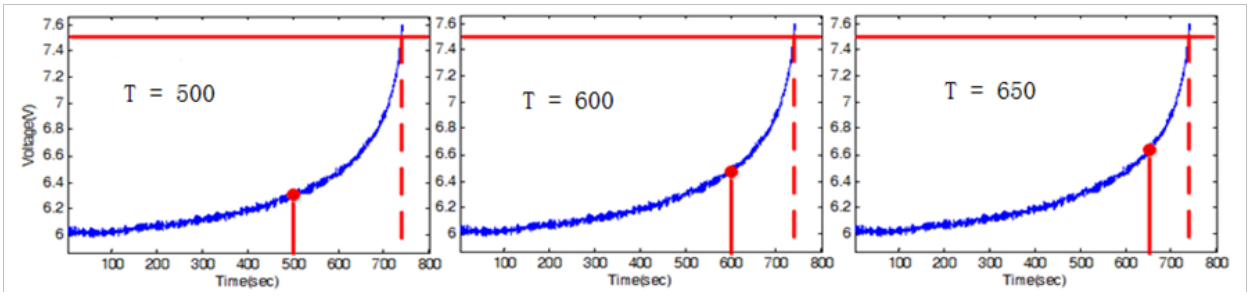



**Figure 7. Degradation Voltage Signal for 26 Tested Resistors**

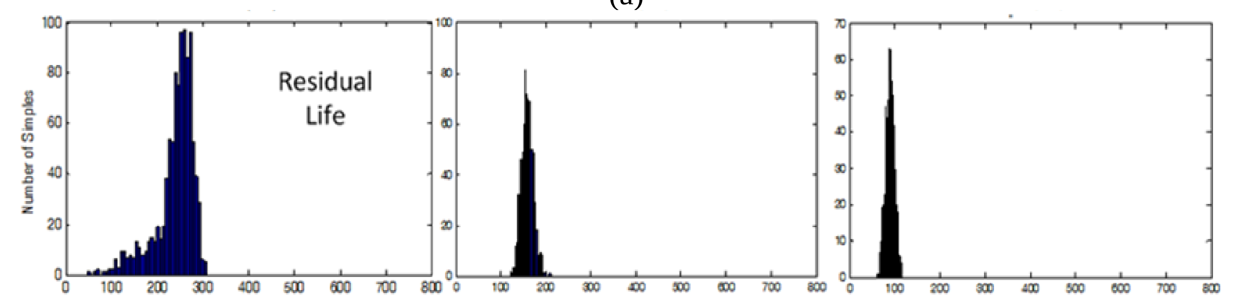
### 4.2 Remaining Useful Life Prognosis

For the prognosis of the resistors, the proposed generic Bayesian approach is applied to a single resistor. The voltage is used for a degradation signal. The threshold of resistor failure is defined to be 7.5 V, as shown in Figure 7. Prognosis for this resistor is carried out at around 500, 600 and 650 seconds. Figure 8 shows the prognosis results for resistor 6 with two sets of subfigures. Three subfigures on the first row show the sensory degradation signals and three different prognosis times, while three subfigures on the second row present the predicted remaining useful life distributions at these three prognosis times.

It is apparent that (1) the predicted remaining life distribution is centered at around the true remaining useful life of the resistor; (2) the confidence of the life prediction is enhanced as more degradation signals get involved in prognosis, as shown in the figure that the distribution of predicted remaining useful life gets narrower. There are 26 resistors in total in the experimental degradation signal database. In order to evaluate the performance of the remaining useful life prognosis, the mean value and the standard deviation for the predicted life for each resistor are compared with the true life obtained directly from the experiment.




(a)




(b)

**Figure 8. Real-Time Prognosis at Three Different Times**
(a) Prediction time. (b) Life time distribution

Table 4 lists the prognosis results for all 26 resistors. As shown in the results, the proposed Bayesian prognosis approach using the Laplace approximation method yields very accurate remaining useful life (RUL) predictions in general.

### 4.3 Comparing the Result with MCMC Simulation

To further demonstrate the effectiveness of the developed generic Bayesian prognosis framework using Laplace approximation method, the Markov Chain Monte Carlo (MCMC) method [52] commonly used for Bayesian updating is also applied for the resistor prognosis case study and the results are compared.

With the aim of compare the developed approach for prognosis accuracy and computational efficiency with the MCMC method, this case study conducted the comparison at the two different prognosis times, 400 and 600 seconds respectively, and 50000 samples have been used to simulate the benchmark result for the MCMC approach. Based on the information given in the previous chapter, the posterior estimations are computed and compare results are shown in the Tables 5 and 6. From the Table 5, the required number of function evaluations of the posterior distribution is much smaller than tradition method and as a result the required computational time is much less compared with the MCMC methods. Efficiently conducting the failure prognosis is extremely important for real time applications, and as indicated by the comparison results, the proposed Bayesian model-based prognosis using the Laplace approximation outperforms the commonly used MCMC method in the prognosis efficiency. On the other hand, the mean and standard deviation values obtained using the proposed method and the MCMC method are shown in table 6, where the proposed method yields very accurate RUL prediction results compared with the MCMC method and the true RUL values.

**Table 4. Failure Times Evaluate for Different Resistors**

| Resistor | Prediction Time | Actual Life | Predicted Life | | Resistor | Prediction Time | Actual Life | Predicted Life | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Mean | STD | | | | Mean | STD |
| 1 | 400 | 506.74 | 470.89 | 11.38 | 14 | 400 | 485.24 | 471.42 | 6.51 |
| 2 | 400 | 432.76 | 435.36 | 3.71 | 15 | 400 | 494.35 | 490.07 | 4.66 |
| 3 | 800 | 1016.22 | 1082.74 | 19.01 | 16 | 400 | 485.38 | 483.79 | 10.98 |
| 4 | 250 | 351.53 | 322.64 | 4.48 | 17 | 400 | 441.53 | 437.16 | 10.28 |
| 5 | 400 | 494.29 | 479.75 | 10.37 | 18 | 400 | 440.44 | 457.23 | 10.44 |
| 6 | 500 | 738.53 | 690.37 | 11.17 | 19 | 250 | 327.23 | 325.15 | 1.42 |
| 7 | 400 | 529.95 | 498.65 | 5.76 | 20 | 700 | 837.90 | 8.55 | 5.98 |
| 8 | 200 | 256.36 | 246.08 | 3.65 | 21 | 400 | 443.14 | 4.48 | 11.54 |
| 9 | 400 | 574.35 | 569.19 | 20.09 | 22 | 400 | 499.55 | 517.63 | 13.72 |
| 10 | 400 | 611.74 | 609.11 | 11.53 | 23 | 800 | 1032.94 | 1041.37 | 8.73 |
| 11 | 200 | 243.66 | 240.45 | 1.02 | 24 | 400 | 531.35 | 484.99 | 3.35 |
| 12 | 800 | 983.07 | 933.06 | 19.11 | 25 | 400 | 576.22 | 535.13 | 6.29 |
| 13 | 250 | 378.06 | 353.65 | 7.52 | 26 | 400 | 655.03 | 643.27 | 4.34 |

**Table 5. Computational Efficiency Comparison**

| Time | Method | NoFE[a] | Time[b] |
|---|---|---|---|
| 400 | Laplace Approximation | 76 | 0.116s |
| | MCMC | 50000 | 33.64s |
| 600 | Laplace Approximation | 95 | 0.138s |
| | MCMC | 50000 | 34.70s |

[a] Number of Function Evaluation; [b] Based on a 2.40 GHz CPU

**Table 6. Means and Standard Deviations of the Actual Life**

| Time | Laplace Approximation | | MCMC | | True RUL |
|---|---|---|---|---|---|
| | Mean | STD | Mean | STD | |
| 400 | 258.16 | 14.74 | 275.28 | 13.19 | 383.53 |
| 600 | 115.03 | 10.97 | 130.32 | 10.65 | 183.53 |

# CHAPTER 5

# CONCLUSIONS AND FUTURE WORK

## 5.1 Conclusion

A generic Bayesian framework using Laplace approximation for model-based remaining useful life prognosis was presented in this thesis.[1] The desire and need for accurate prognosis capability have been around for as long as human beings have operated complex and expensive engineering system. The developed generic Bayesian prognosis approach models incorporating timely evolving sensory data can update the remaining useful life distribution by using a general Bayesian inference mechanism. It also employs an efficient Bayesian updating approach using a Laplace approach method to improve the computational efficiency. The developed Bayesian prognosis approach can eliminate the dependency of evolutionary updating process on a selection of distribution types for the degradation model parameters. The Bayesian updating process can be carried out efficiently with the Laplace approximation method, which enables the proposed approach to be applied for real-time prognosis applications. The efficacy of the proposed Bayesian prognosis methodology has been successfully demonstrated with 26 resistors for the prognosis of their remaining useful lives.

## 5.2 Future Work

Laplace approximation as an efficient analytical method has been used in this thesis, which can reasonable approximates the posterior distribution as a Gaussian distribution. However this method faces problems in several aspects: 1) the accuracy will be questionable

[1] The bulk of this MS thesis was presented and published at the 2013 IIE Industrial and Systems Engineering Research Conference (ISERC), San Juan, Puerto Rico, May 18-22, 2013.

when incorporate small data; 2) optimization method, and sensitivity information are needed, especially for the problem with multiple local maximum points, Laplace approximation can leads large errors; 3) the accuracy of the approximation highly depends on how likely the posterior with the Gaussian distribution, we cannot improve the approximation by anyway. For the asymmetric distribution and distribution with multi-mode, Laplace is not a proper choice. So, the future work is to develop a method to handle all these three questions above. The desired method should without optimization and sensitivity analysis and the accuracy can be improved by some technique. Joseph [52, 53] proposed a new method which is called design of experiment-based interpolation technique for updating the Bayesian posterior. This technique treats the posterior as a performance function, and built a Kriging model for the posterior. Without using the optimization process and sensitivity information, the accuracy can be improved by adding more simple points. Comparing with the Laplace Approximation, this method produces a more advanced way to explore the Bayesian posterior. As the future work, this method should be used in the quick Bayesian updating problem.

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