Isolating a Single Qubit in a Multi Qubit System

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1. Introduction

Quantum computing, a whole new paradigm to computing, is based on using Quantum Mechanics for computation. The basic element in a quantum computer is a "qubit" which is a two-state physical system. Unlike a classical bit which can exist in one of two states, 0 or 1, a qubit can not only exist in its two basis states, but also in any linear combination of these states. A quantum computer comprises several qubits interacting with each other. The states of qubits are manipulated by performing single- and multi-qubit gate operations on them wherein one or more parameters of the system are varied via classical control lines. Since every control line opens the quantum system to a noisy environment, in any design of a quantum computer, it is desirable that the number of control lines be minimal.

Just like the NAND gate is a universal gate in classical computing, there are a set of gates universal to quantum computing. It has been shown that single qubit gates together with a two-qubit Controlled-NOT (CNOT) gate is universal for quantum computing, that is, any operation within a quantum computer can be decomposed into a series of single qubit and CNOT gates [1]. However, in a quantum computer having nearest-neighbor architecture, each qubit is coupled to several qubits adjacent to it. Therefore, to perform a gate operation on a single qubit, it needs to be isolated from the qubits adjacent to it by switching off the coupling. This greatly increases the complexity of the control circuitry. We show here how this problem can be overcome by designing a method whereby a single qubit can be isolated in a multi-qubit system without having to switch off the coupling.

2. Isolating a Qubit Without Shutting off Coupling

The Hamiltonian describing the evolution of a single qubit system is a 2 × 2 matrix given as:

$$\mathbf{H}_1 = \Delta\ \boldsymbol{\sigma}_X + \varepsilon\ \boldsymbol{\sigma}_Z \qquad (1)$$

Here, Δ is the tunneling parameter, ε is the bias acting on the qubit and $\boldsymbol{\sigma}_X$ and $\boldsymbol{\sigma}_Z$ are the 2 × 2 Pauli spin matrices. Now consider a system of two such qubits interacting through a coupling term "ξ". The Hamiltonian of the two-qubit coupled system is given by a 4 × 4 matrix:

$$\mathbf{H}_2 = \begin{pmatrix} \varepsilon_A+\varepsilon_B+\xi & \Delta_B & \Delta_A & 0 \\ \Delta_B & \varepsilon_A-\varepsilon_B-\xi & 0 & \Delta_A \\ \Delta_A & 0 & -\varepsilon_A+\varepsilon_B-\xi & \Delta_B \\ 0 & \Delta_A & \Delta_B & -\varepsilon_A-\varepsilon_B+\xi \end{pmatrix} \qquad (2)$$

where Δ_A, Δ_B, ε_A and ε_B are the tunneling and bias for qubits *A* and *B*, respectively. Suppose we want to perform a unitary operation on qubit B only, without performing any operation on qubit A. Since qubit A is coupled to qubit B, the evolution of qubit B will depend on the state of qubit A. In order to isolate qubit B from qubit A, we need to switch off the coupling between the two qubits, whereby we can treat qubit B as a single qubit system. However, as previously mentioned, this is not desirable from a quantum computing point of view. The question then to ask is whether or not the 4 × 4 Hamiltonian describing the two-qubit system can be reduced to a single qubit system of the form of Eq. (1) describing the evolution of qubit B only in the presence of the coupling term?

The answer to this question is yes. By maintaining a high bias on qubit A, we can freeze its state. Under this condition, the evolution of the two qubit system reduces to the evolution of qubit B only, taking place in two different subspaces in which the state of qubit A is $|0\rangle$ and $|1\rangle$. In each of these subspaces, the Hamiltonian of qubit B is of the form of Eq. (1) with the bias term, ε, replaced by the effective bias ($\varepsilon_B \pm \xi$). The coupling term, ξ, adds to the bias acting on qubit B, ε_B, in the subspace where qubit A is in the $|0\rangle$ state and it subtracts from the bias term in the subspace where qubit A is in the $|1\rangle$ state. This is because the expectation values of the $\boldsymbol{\sigma_{ZA}}$ operator are +1 and -1 in the subspaces where the states of qubit A are $|0\rangle$ and $|1\rangle$, respectively. Single qubit rotations can now be performed on qubit B by controlling the parameters of qubit B.

Since the reduced Hamiltonian approach described by us is based on an approximation, we next show the effect of this approximation on the solution by comparing the evolution of a two-qubit system using the unreduced (exact) and reduced Hamiltonians. Suppose the system initially starts out in the partially entangled state:

$$\frac{\sqrt{3}}{2}|00\rangle + \frac{\sqrt{3}}{4}|10\rangle + \frac{1}{4}|11\rangle \qquad (3)$$

We choose the values of the parameters as follows: $\Delta_A = \Delta_B = 50$ MHz, $\varepsilon_A = 10$ GHz, $\varepsilon_B = 30$ MHz, $\xi = 12.5$ MHz, and t = 10 ns. Figure 2 shows the evolution of the entangled state under the 4 × 4 Hamiltonian as given by Eq. (2) using the chosen parameters. We next study the evolution of the system under the reduced Hamiltonians in each subspace. The initial state of qubit B is $\sqrt{(3)}/2\ |0\rangle$ and $\sqrt{(3)}/4\ |0\rangle + \frac{1}{4}\ |1\rangle$, respectively, in the subspaces where qubit A is in the $|0\rangle$ and $|1\rangle$ states. Figure 3 shows the probabilities of the $|0\rangle$ and $|1\rangle$ states of qubit B which are identical to the plots for the $|00\rangle$ and $|01\rangle$ states in Fig. 2. Figure 4 shows the probabilities of the $|0\rangle$ and $|1\rangle$ states of qubit B which are identical to the plots for the $|10\rangle$ and $|11\rangle$ states in Fig. 2. We can, therefore, see that the reduced Hamiltonian approach can be used to isolate a single qubit in a coupled system without having to shut off the coupling. The method generalizes to an n-qubit system. By fixing the states of the qubits adjacent to the qubit we wish to perform an operation on, we can effectively isolate a qubit in a multi-qubit system.

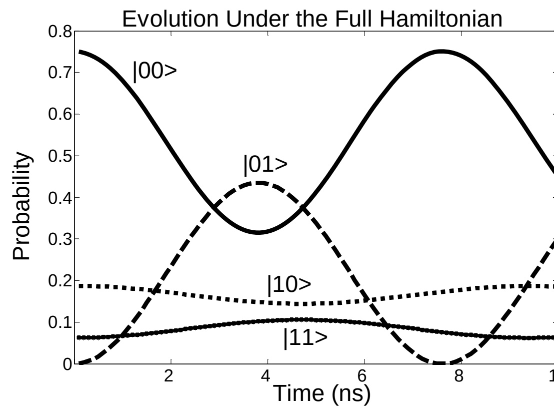

Fig. 2. Evolution of the probabilities of the 4 basis states of the 2-qubit system starting in the initial state given by (3).

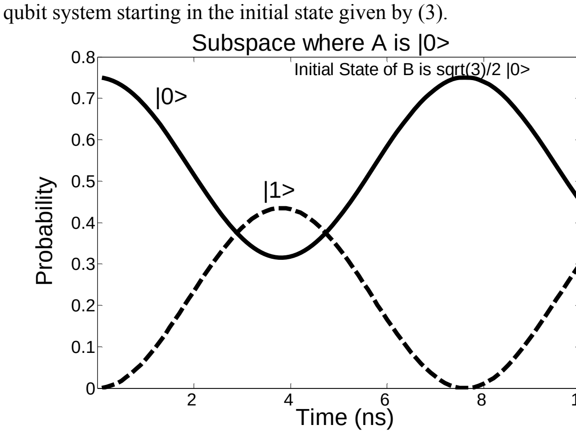

Fig. 3. Evolution of the probabilities of the states $|0\rangle$ and $|1\rangle$ of qubit B in the where qubit A is in the $|0\rangle$ state.

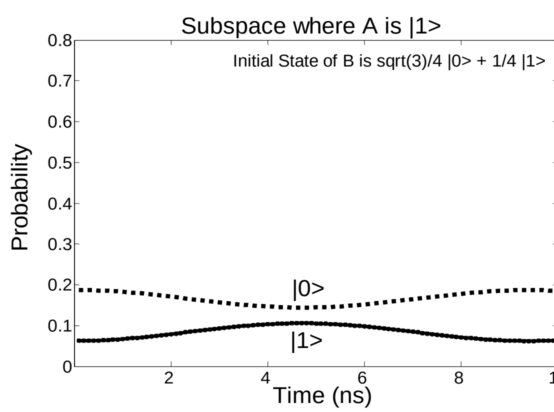

Fig. 4. Evolution of the probabilities of the states $|0\rangle$ and $|1\rangle$ of qubit B in the subspace where qubit A is in the $|1\rangle$ state.

[1] A. Barenco, "Elementary gates for quantum computation," *Phys. Rev. A*, vol. 52, pp. 3457-3467, 1999