Mapping a controller from the s-domain to z-domain using Magnitude Invariance Method (MIM).

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Abstract- We are presenting a new approach for mapping a continuous time controller to a discrete time controller. Methods traditionally used for this mapping include forward rectangular rule, bilinear rule and zero-pole matching. This approach, unlike these other methods, produces a discrete time transfer function with a magnitude response exactly same as its analog prototype. To achieve this objective we are using the Magnitude Invariance Method (MIM) which was recently developed in the field of signal processing. The step and frequency responses of the closed loop systems obtained using this approach will be systematically investigated to evaluate the effectiveness of this mapping.

1. Introduction.

Use of digital computers for controlling physical systems has become more and more popular due to the many advantages of digital control. As a result it sometimes becomes necessary to replace the existing continuous-time controllers with discrete-time controllers. To achieve this we often map the continuous-time controller to discrete-time using a suitable mapping technique. Various methods have been suggested for mapping a controller from the s-domain (continuous-time) to z-domain (discrete-time). The most popular methods used for this type of mapping are [1] [2]: Backward difference, Forward difference, Matched-z, Impulse-invariance method and Bilinear transform. In this paper we are making use of Magnitude Invariance Method (MIM) [2] to map a controller from the s-domain to z-domain. This method uses a mapping technique such that:

$$\left|H(e^{j\omega})\right| = \left|H_c(j\Omega)\right|_{\Omega=\omega/T} = \left|H_c(j\omega/T)\right|, \left|\omega\right| \le \pi$$

where, H_c is the continuous-time transfer function, H is the discrete-time transfer function, and T is the sampling time.

Thus the magnitude of the frequency response of the discrete-time controller is exactly equal to that of the continuous-time controller. This condition does not hold for the traditional mapping techniques. This property becomes very helpful in certain cases where the magnitude response of the controller is an important selection criterion. The magnitude response of the discrete-time controller can be directly related to that of its analog prototype.

2. Experiment, Results, Discussion, and Significance.

In this method cepstral processing is used to obtain the unit impulse response h[n]. Cepstral processing is used to achieve the decorrelation of the autocorrelation function of h[n]. This in fact is the critical step in this algorithmic mapping procedure. The starting point of this algorithm is to sample the magnitude squared frequency response of the analog controller. Cepstral processing is then implemented to obtain the complex cepstrum. A phase lifter sequence is used to separate the minimum phase part and the maximum phase part. In the cepstral processing step we make use of Discrete Fourier Transform (DFT) as well as Inverse Discrete Fourier Transform (IDFT). The length of the DFT and IDFT determines how accurately the impulse response is obtained. If the length of the transforms is increased the impulse response will be more accurate. The algorithm is implemented as follows:

- $R[k] = |H[k]|^2$
- $\hat{R} = \ln(R[k])$
- $\hat{r}[n] = DFT^{-1}\left(\hat{R}[k]\right)$
- $\hat{r}_{mn} = \hat{r}[n] \times l_{mn}[n]$ where, $l_{mn} = \begin{Bmatrix} 0 & n<0 \\ 0.5 & n=0 \\ 1 & n>0 \end{Bmatrix}$
- $\hat{R}_{mn}[k] = DFT\left(\hat{r}_{mn}[n]\right)$
- $R_{mn}[k] = \exp\left(\hat{R}_{mn}[k]\right)$
- $h[n] = DFT^{-1}(R_{mn}[k])$

Once the unit impulse response h[n] is obtained, we make use of the algorithm proposed by Graupe, Krause and Moore [3], to obtain the rational transfer function H(z). A general expression for the transfer function is given by:

$$H(z) = \frac{\sum_{i=0}^{M} b_i z^{-i}}{1 + \sum_{i=1}^{M} a_i z^{-i}}$$

The transfer function parameters are calculated using the following matrix calculations:

$$\begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_M \end{bmatrix} = -\begin{bmatrix} \beta_M & \beta_{M-1} & . & . & \beta_1 \\ \beta_{M+1} & \beta_M & . & . & \beta_2 \\ . & . & . & . & . \\ . & . & . & . & . \\ \beta_{2M-1} & \beta_{2M-2} & . & . & \beta_M \end{bmatrix}^{-1} \begin{bmatrix} \beta_{M+1} \\ \beta_{M+2} \\ . \\ . \\ \beta_{2M} \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_M \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ \beta_M \end{bmatrix} + \begin{bmatrix} \beta_0 & 0 & . & . & 0 \\ \beta_1 & \beta_0 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ \beta_{M-1} & \beta_{M-1} & . & . & \beta_0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_M \end{bmatrix}$$

Where, β_0=h[0], β_1=h[1] and so on.

3. Conclusions.

In this paper we are using a new method, MIM, to map a continuous time controller to a discrete-time controller. The step and frequency responses of the closed loop systems obtained using this approach will be carefully investigated to determine the effectiveness of this mapping.

References:

[1] Gene F. Franklin, J. David Powell, Michael Workman, Digital control of Dynamic Systems, Pearson education, 2005.

[2] Larry D. Paarmann, Mapping from the s-domain to the z-domain via the magnitude –invariance method, ELSEVIER, Signal Processing, 69 (1998) 219-228.

[3] D. Graupe, D.J. Krause, J.B. Moore, Identification of autoregressive moving average parameters of time series, IEEE Trans. Auto. Cont. AC-20 (1) (February 1975) 104-107.