Robust and reduced order fault detection filter design via LMI approach

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1. Introduction

The more science and technology develops, the more demand on reliability and security of complex systems are required. But, in many cases, failures are inevitable. Thus, on-line monitoring of such faults as they occur during operation of a dynamic system is necessary. As a consequence, functionally redundant fault detection schemes were developed [1]. In this study, estimator based fault detection methods will be the focus. The key to estimator based fault detection is to generate a fault indicating signal (residual) using input and output signals from the monitored system. Faults are detected by setting a variable threshold on a residual quantity generated from the difference between real measurements and the estimates of these measurements found using the mathematical model. However, there is always a model-reality mismatch between plant dynamics and the model used for the residual generation. Consequently, the residual generator must be made robust to the modeling uncertainties and disturbances. The residual should be sensitive to faults but insensitive to the modeling uncertainties and disturbance [2]. But, due to inevitable parameter uncertainties and disturbance encountered in a practical application, we will rarely find a situation where the conditions for a perfectly robust residual generation are met. It is therefore necessary to provide sufficient robustness not only in the residual generation stage but also in the decision-making stage. The goal of robust decision-making is to minimize the false and missing alarm rates due to the effects that modeling uncertainties and unknown disturbance will have on the residuals. This can be achieved in several ways [3]. In the adaptive threshold approach, the residual thresholds are varied according to the control activity of the process. Also, the order reduction of the residual generator is researched because it gives us a fast data processing and the reduction of the complexity. The generalized observer is easier to handle than the state observer for the order reduction so, in this study, the generalized observer is used. In the fault detection literature this observer is usually called a fault detection filter to emphasize the relationship with the filtering concept. Recently, the LMI (Linear Matrix Inequality) technique has received much attention. So, we try to design the robust and reduced order fault detection filter (RFDF) via LMI approach.

2. Main Result

Let's consider the following uncertain continuous time linear system described by

\[
\begin{bmatrix}
\dot{x}(t) \\
y(t)
\end{bmatrix} = \begin{bmatrix}
A & B_u & B_d \\
C & D_u & D_d
\end{bmatrix} \begin{bmatrix}
x(t) \\
u(t) + B_f f(t)
\end{bmatrix} + \begin{bmatrix}
B_d d(t)
\end{bmatrix} + \begin{bmatrix}
D_d d(t)
\end{bmatrix}
\]

where \( x(t) \in \mathbb{R}^n \) is the state, \( y(t) \in \mathbb{R}^m \) is the measurable output, \( f(t) \in \mathbb{R}^n \) is the detectable fault signal of the sensor, component and actuator faults, \( d(t) \in \mathbb{R}^n \) is the bounded disturbance and \( u(t) \in \mathbb{R}^p \) is the control signal. To included the plant model uncertainties, we formulate them in the form of a polytopic model as follows

\[
A = \sum_{i=1}^{s} \alpha_i A_i, \quad B_u = \sum_{i=1}^{s} \alpha_i B_{ui}, \quad B_d = \sum_{i=1}^{s} \alpha_i B_{di}, \quad C = \sum_{i=1}^{s} \alpha_i C_i, \quad D_u = \sum_{i=1}^{s} \alpha_i D_{ui}, \quad D_d = \sum_{i=1}^{s} \alpha_i D_{di}, \quad \alpha_i \in \Gamma
\]

where \( \Gamma \) is a unit simplex such as

\[
\Gamma = \left\{ (\alpha_1, \ldots, \alpha_s) : \sum_{i=1}^{s} \alpha_i = 1, \alpha_i \geq 0 \right\}
\]

The former approach is to formulate the RFDF problem as a \( H_{\infty} \) model–matching problem [4]. It can be stated as

\[
\min_{F} \frac{\| F - J \|}{\| F \|}
\]
However, this approach may produce a false alarm in a no fault situation [6]. Thus, we propose new approach which is multi-objective $H_\infty$ optimization [7] based on the model-matching formulation. We can describe the RFDF design problem in multi-objective $H_\infty$ optimization setup as the following

![Figure 1](image-url) Fig. 1. Block diagram for residual error $r(t)$ with reference model and tracking filter

In the figure, $W(s)$ is the reference model and a poorly chosen reference model can result in a residual generator with poor robustness [5]. The suitable choice helps focusing on the robustness properties while keeping the residual sensitive to fault and conforming to the specified performance. With given positive scalar $a_d, a_f$ and $a_u$, the RFDF design problem in multi-objective $H_\infty$ optimization setup is to find $F(s)$ such as (5)

$$\min_{F(s)} \{ a_d * y_d + a_f * y_f + a_u * y_u \}$$

subject to

$$\max_{\alpha \in \Gamma} \left| F(s) \right| \left[ G_{y_d}(s) \right] = \max_{\alpha \in \Gamma} \left| F(s) \right| \left[ G_{y_d}(s) \right] \leq y_d$$

$$\max_{\alpha \in \Gamma} \left| W(s) - F(s) G_{y_f}(s) \right| \leq y_f$$

$$\max_{\alpha \in \Gamma} \left| W(s) - F(s) G_{y_u}(s) \right| \leq y_u$$

where $G_{y_d} = W_{y_f} \cdot G_{y_d}$, and

$$\Gamma = \left\{ \alpha : \tau_0 = F \left[ G_{y_f} \right] - G_{y_d} \right\}$$

Using Finsler’s lemma and congruence transformation, (5) can be transformed to the following LMIs (6). Thus, the reduced-order RFDF can be obtained by solving

$$\min_{v_1, s, y_2, a_d, a_f, a_u} \left\{ \| y_d + a_f * y_f + a_u * y_u \|_2 \right\}$$

where $\tau > 0$ and $a_d, a_f, a_u$ are given as positive scalars.

3. Conclusion

According to the result, we can find that the detection time is not much changed from reducing the filter order. In other

words, the order reduction does not give much degradation on the detecting performance. Thus, our RFDF order reduction is applicable. Also, from this research, the assertion in [8] and [9] that the time window size and the proper threshold selection play an important role on the detecting performance is verified.


