

**OPERATIONAL TOLERANCE ALLOCATION AND MACHINE  
ASSIGNMENT UNDER PROCESS CAPABILITY AND PRODUCT VALUE  
CONSTRAINTS**

A Dissertation by

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I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirement for the degree of Doctor of Philosophy with a major in Industrial Engineering.

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## **DEDICATION**

To whom I owe it all—my great parents

## **ACKNOWLEDGMENTS**

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## ABSTRACT

Process planning is an activity within the production process that translates design requirements into a detailed description of instructions for transforming a raw stock of material into a completed product. Instructions contain a sequence of operations that should be followed in order to arrive at a final product that satisfies design requirements. Over the years, many researchers have examined the modeling and analysis of process plans for the production of discrete parts. As a result, a number of mathematical models have been proposed. Input to these models is usually a sequence of operations required to complete the part, each associated with processing equipment having a certain capability. These models can be used to check the feasibility of the process plan or to calculate operational tolerances so that the final product is produced within design specifications.

Existing models allocate operational tolerances under specific assumptions. For example, when these models consider the process capability, they formulate it as a single fixed value that represents the worst case performance of a process capability. None of the existing models consider the stochastic nature of process capability. In addition, the current tolerance allocation research does not place emphasis on the value of the product under consideration. It is logical that high-value products should be assigned to highly capable processes in order to increase the confidence level in producing them to design specifications. Furthermore, the existing tolerance allocation methods typically associate single processing equipment (machine) with certain capability to each operation in the process plan. No option exists for automatically choosing a different machine in case the assigned machine results in an infeasible plan. Actually, assigning a single machine to

each operation can result in suboptimality in terms of allocated tolerances and may lead to scheduling inflexibility.

This research proposes a model that simultaneously allocates operational tolerances and assigns the most economical machines. Two versions of the model are developed—stochastic and fuzzy. The stochastic version of the model captures the stochastic nature of the process capability in which a stochastic distribution is used to represent the capability of a process. Alternately, the fuzzy version evaluates the process capability utilizing experts' knowledge. Both proposed versions introduce flexibility and optimality to the modeling of the production process by considering multiple machines available for each type of operation. This helps in selecting the machine with the lowest possible capability of making the process plan feasible while allocating maximum tolerances to each operation. Furthermore, a formula that determines the product's value and considers the change of product value through the different stages of the production process is presented and integrated with the proposed models. Both versions of the proposed model can be easily solved by common off-the-shelf software. The models are implemented and analyzed using literature-application examples. Experimental results confirm the effectiveness of the proposed model.

This research also generated a novel approach to evaluating process capability based on readily available information and knowledge of experts. Currently, the 6-sigma spread in the distribution of the product quality characteristics is used as a measure of process capability. This value represents the collective contribution of many factors (machine, tool, setup, labor, etc.) involved in the production process. Measuring the process capability, however accurately done, only represents a single combination of

these factors. Any change in one of these factors would affect capability of the resultant process. Moreover, the existing methods for evaluating process capability are tedious and time consuming and require possible interruption and production stoppage. The suggested approach considers a number of different factors that may have an effect on process capability. It utilizes the concept of fuzzy logic to transfer experts' evaluations of these factors into a value representing process capability. Initial results indicate that the proposed technique is very effective.

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## LIST OF ABBREVIATIONS

AI	Artificial intelligence
ANN	Artificial neural network
ANOVA	Analysis of variance
ASD	Actual standard deviation
ASME	American Society of Mechanical Engineering
ASP	Average selling price
ATP	Average constraint throughput
BP	Blueprint tolerance or stock removal requirement
$C_M$	Manufacturing cost
CPV	Constraint product value
CS	Constraint occupation time for one product unit
CT	Cycle time
D	Demand
$DC_j$	Dimensional chain for a BP tolerance or a stock removal requirement
DM	Decision maker
DOE	Design of experiments
FLP	Fuzzy linear programming
FPC	Fuzzy process capability
FPCE	Fuzzy process capability evaluation
FR	Fuzziness range
FSD	Fuzzy standard deviation
GA	Genetic Algorithms

GD&T	Geometric dimensioning and tolerancing
IP	Integer Programming
LP	Linear programming
LPC	Lower process capability limit
LSL	Lower design specification limits
LTB	Larger-the-better
m	Process/operation mean
M	Machine
$m_c$	Number of machines available
MC	Material cost
MC	Machine capacity
MRC	Minimum required capability
NLP	Nonlinear programming
NTB	Nominal-the-best
OBJ	Value for the objective function of the non-fuzzy version of the model
OS	Operator skill
PC	Product criticality
PC	Process capability
PCR	Process capability ratio
PV	Product value
r	Rejection rate
R	Revenue
RSS	Root of sum squares



S	Number of process sets in a process plan
SA	Simulated annealing
SR	Stock removal
STB	Smaller-the-better
t	Operational tolerance
TC	Tool condition
TOC	Theory of constraints
UPC	Upper process capability limit
USL	Upper design specification limits
VAP	Value added processes
WC	Working conditions
WCM	Worst case model

# CHAPTER 1

## INTRODUCTION

Design engineers continuously encounter the dilemma of whether to design for better performance or lower cost. Better-performing products are generally more expensive to produce, while lower-cost products may result in bad performance. Specification of tolerances at the design stage is an example of this dilemma. According to the American Society of Mechanical Engineers (ASME, 1982), tolerance is the total amount by which a specific dimension is permitted to vary from a nominal value. Although the ideal amount of variation is zero, it is neither practical nor economical to meet the ideal. Determining the acceptable amount of dimensional variation at the design stage impacts both the cost incurred at the manufacturing stage and the performance of the product. In general, loose tolerances results in low manufacturing cost while tight tolerances result in higher cost. However, loose tolerances may result in poor functionality.

Through out the design phase for a product, the designer must identify the tolerances for both the individual components and the final assembly using tolerance allocation, or tolerance synthesis. In tolerance allocation, the assembly tolerance is specified ahead of time and then the designer distributes tolerances for the individual components. The opposite of tolerance allocation is tolerance analysis in which the components' tolerances are predetermined and the final assembly tolerance is evaluated accordingly. For the production of a single component, tolerance allocation can be defined as distributing the component's design-specified tolerances among the various operations required for the production of the component. These operational tolerances, or

process tolerances, represent the limits with which each operation should comply in order for the final accumulated dimension of a component to meet blueprint specifications

### **1.1 Design Versus Operational Tolerances**

Conventionally, tolerance allocation is carried out in two stages: design and manufacturing. Design engineers allocate tolerances to individual components while satisfying assembly tolerance requirements, which, in turn, are dictated by functionality and quality requirements. The tolerances found on drawings are of this type.

Operational tolerances are related to, but different from, design tolerances. Through the production of individual components, manufacturing engineers must typically work within the tolerance limits set by design engineers for that specific component. However, there is no direct correspondence between design tolerances and operational tolerances. For example, all the dimensions of a part formed by stamping have the same operational tolerances since they are formed by the same operation. However, they do not have the same design tolerances, which are dependent on how the part will be used.

Before production begins, normally the manufacturing engineer plans the sequence of operations to convert a workpiece from its initial form to its final shape while satisfying design requirements. This is known as the process plan. A key feature of process planning is the determination of operational tolerances using tolerance allocation.

### **1.2 Tolerance Allocation Methods**

Every tolerance allocation model in the process planning stage should fulfill the requirements of the designed-specified tolerances, also called blueprint (BP) tolerances. In fact, one or many operational tolerances may stack up to one BP tolerance. Tolerance

stack-up is handled by the identification of the tolerance chains. The tolerance chain identification relates to the recognition of the relationship between the operations based on their sequence and the determination of the effect of each operation on the dimension of the produced component. Tolerance charting is a graphical, heuristic tool that helps in determining this relationship. It shows the machined dimensions, the blueprint tolerances, the stock removals, and the actual machine capabilities. It is used to perform process tolerance allocation/analysis on machined parts. Moreover, the tolerance chart ensures that a specific processing plan is appropriate for producing a part per specifications.

Besides fulfilling the BP requirements, process capability is the other important factor that must be considered when allocating operational tolerances. For a particular operation to be feasible, its assigned operational tolerance must be wider than the associated process capability. It is customary to use the 6-sigma ( $\pm 3\sigma$ ) spread in the distribution of product quality characteristics as a measure of process capability (Montgomery, 1996), where  $\sigma$  is the standard deviation of the process under statistical control. The boundaries of the 6-sigma spread are the upper and lower process capability limits (UPC and LPC). UPC and LPC are 3-sigma to the right and to the left of the process mean ( $m$ ), respectively. Obviously, the process capability is a stochastic entity by nature. It has a mean and a variance, and it must be accurately represented as a stochastic distribution.

Historically, allocating tolerances at the design stage has attracted more attention than allocating tolerances at the process plan stage. However, research in both areas has developed in the same direction. Several tolerance allocation methods have been proposed over the years. Tolerances were assigned based on the experience and intuition

of the engineer. This intuitive approach does not consider the cost-tolerance relationship. Examples of non cost-driven methods include proportional scaling and constant precision methods.

Opposite of the intuitive methods, the majority of published articles on tolerance synthesis are actually based on optimization, most of which are based on the cost-tolerance models. Speckhart (1972) and Spotts (1973) are the initial attempts in the literature to assign an optimal set of tolerances that would fulfill design functional requirements and minimize manufacturing cost. Both methods use the Lagrange multiplier method to solve the problem. The Lagrange method is a closed-form optimal solution for the least-cost component tolerances. However, it has certain limitations. For example, it cannot treat discontinuous cost tolerance functions because it requires a continuous first derivative. Also, it cannot handle cost-tolerance functions where the process is limited to a specified tolerance range, i.e., where preferred tolerance limits are specified (Chase et al., 1990). Many other optimization methods were investigated to overcome these limitations, including the geometric programming method, the linear programming method, and the nonlinear programming method.

A new direction of research in the tolerance allocation area started to appear in the mid-1990s. Quality engineering aims to produce a product that is robust with respect to all noise factors. Taguchi et al. (1989) stated that one of the objectives of optimizing a design should be to minimize the effect of variation in the design parameters on system performance. This implies that tolerances should be considered when determining nominal values for design dimensions. Gadallah and Elmaraghy (1994, 1995) applied quality engineering principles to the tolerance synthesis problem. They used the analysis

of variance (ANOVA) technique in order to help in allocating tolerances. Other researchers used the design of experiments (DOE) approach to solve the tolerance synthesis problem (Kusiak and Feng, 1995).

In spite of the large amount of research in tolerance allocation at the process planning stage, the following drawbacks are noticed:

- Current tolerance allocation research neglects the stochastic nature of process capability. It assumes centered processes and uses the worst case performance of a process, represented by its LPC value, as the restriction with which a process should comply to be considered capable.
- Existing tolerance allocation methods do not place much emphasis on the value of the product under consideration. A product's value is a function of its revenue, the percentage of completed cycle time while in production, the value-added processes through which it has gone, and its criticality. High-value products should be assigned to more capable processes in order to increase the confidence level in their production to design specifications.
- Existing tolerance allocation methods typically associate single processing equipment (machine) with certain capability to each operation in the process plan. This approach of assigning a single machine to each operation results in suboptimality. It also leads to inflexibility since the machines are fixed and there is no option for automatically choosing a different machine in case the assigned machine results in an infeasible process plan. Traditionally, the process plan is made feasible by assigning a more capable machine to these operations in an iterative manner. Otherwise, the current process plan is

declared infeasible and a new process plan is selected. This whole process is tedious and computationally time consuming.

### **1.3 Research Focus and Objectives**

The objectives of this research are defined as follows:

1. To Model, implement, and analyze an operational tolerance allocation and machine assignment model for the production of individual components, considering the following:

- Process capability as a stochastic variable. Alternately, the process capability will be analyzed as a linguistic variable with fuzzy values.
- The value of the produced part.

2. To generate a novel approach to the evaluation of process capability based on readily available information and knowledge of experts.

### **1.4 Dissertation Organization**

This dissertation has been organized into six chapters. The first chapter is the introduction. The second chapter provides the required background on topics related to this research and reviews the related literature. The third chapter presents two versions of the proposed operational tolerance allocation and machine assignment model. The proposed model is implemented and tested in the fourth chapter. The fifth chapter presents a new approach for process capability evaluation referred to as fuzzy process capability evaluation (FPCE). Finally, the sixth chapter presents the conclusions and future work recommendations.

## **CHAPTER TWO**

### **BACKGROUND AND LITERATURE REVIEW**

#### **2.1 Background**

This section introduces a brief background on the following topics related to the subject of this research: cost-tolerance relationships, tolerance charting, tolerance allocation techniques, fuzzy logic, and the process capability issue.

##### **2.1.1 Cost-Tolerance Functions**

From a manufacturing standpoint, tolerances are intended to save money. In fact, a lot of research has been done to identify the cost-tolerance relationship. The existing tolerance allocation research considers the cost in two major ways: manufacturing cost and quality loss cost. Recent research has considered both costs simultaneously.

###### **2.1.1.1 Manufacturing Cost-Tolerance Functions**

Manufacturing cost are those costs required to produce a product to function according to specifications. The manufacturing cost-tolerance relationship emphasizes that tight tolerance results in higher manufacturing cost while loose tolerance results in lower manufacturing cost. The higher cost is usually associated with additional setup, extra operations of precision and measuring devices, the need for more expensive machines and/or tools, and the need to lower the production rate. Figure 2.1 shows a typical manufacturing cost-tolerance curve.



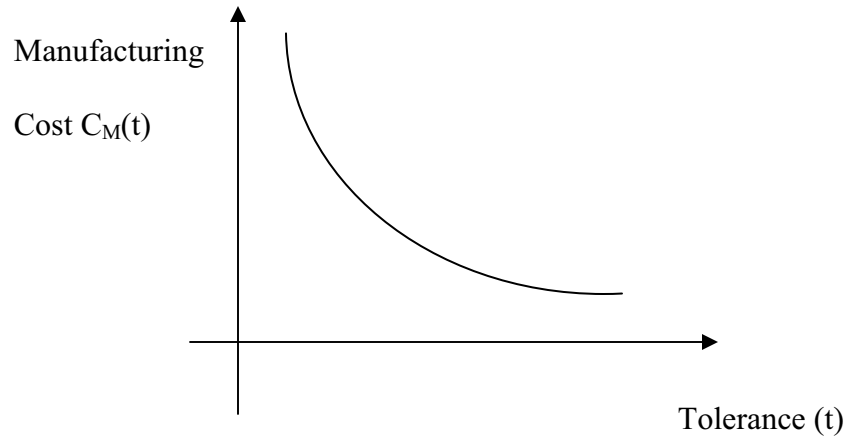


Figure 2.1. A typical manufacturing cost-tolerance curve.

An explicit mathematical expression for the manufacturing cost-tolerance relationship is highly dependent on field data. Therefore, many functions have been introduced to capture this relationship. Section 2.2.1 below reviews these functions.

#### 2.1.1.2 Quality Loss Cost Function

According to Taguchi, the quality loss function is an expression estimating the cost of quality value versus target value. In addition, it evaluates product variability in terms of monetary loss due to product failure in the eyes of the consumer (Taguchi et al., 1989). Therefore, a continuous function, rather than a step function, is more suitable to map the quality loss. Figure 2.2 shows a typical symmetric quadratic quality loss function,  $L(y)$ , shown as:

$$L(y) = K (y-m)^2 \tag{2.1}$$

where

$y$  denotes the measurement of quality characteristic

m is the target value for y

K is a constant

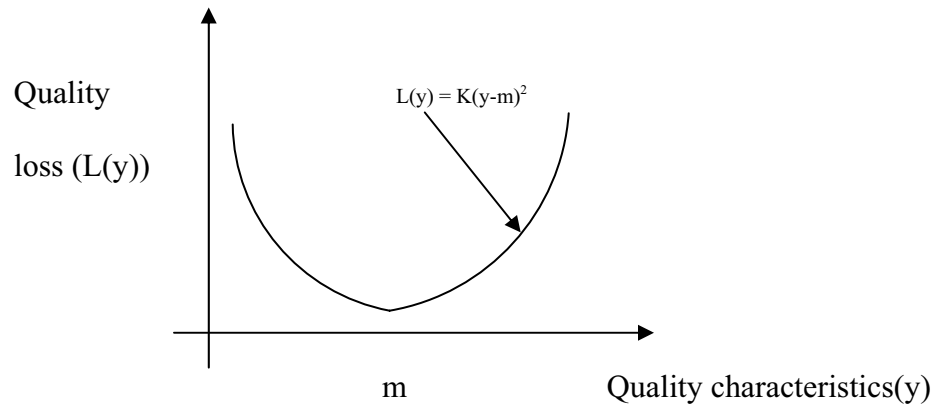


Figure 2.2. A symmetric quadratic loss function.

If one particular part has a deviation  $\Delta$  from m and causes a loss of A, the constant K in equation (2.1) can be determined as:

$$K = A / \Delta^2 \quad (2.1)$$

As shown, the quality loss given by equation (2.1) is larger than zero when the functional characteristics fall far from the target value, regardless of how far from the target they are.

Equation (2.1) is suitable where oversized and undersized parts cause the same losses, i.e., the above equation is appropriate for situations where the nominal value is the best (NTB case). However, there are instances in which it is more harmful for a product's performance to be off-target in one direction than in the other direction. For this

asymmetric case, two distinct quality-loss coefficients are required, one for each direction from the target. Figure 2.3 shows this case along with its quality loss functions.

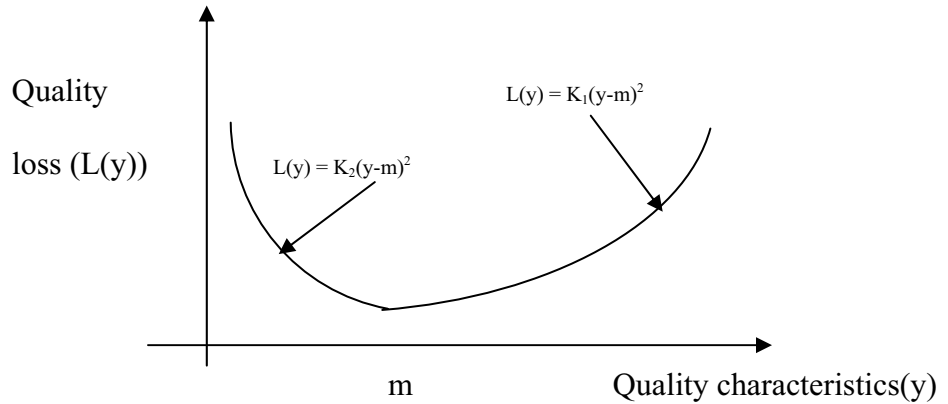


Figure 2.3. Asymmetric quadratic loss function.

For some quality characteristics, as the value of  $y$  moves far from zero, the performance becomes worse and the loss begins to increase. The target value in this case is ideally zero and is referred to as the smaller-the-better (STB). Automotive exhaust pollution and metal corrosion are examples of this case. On the other hand, some products have better performance while their target grows. For example, it is favorable to have a very high value for the strength of a welded joint. In this the larger-the-better (LTB) case, as the target value approaches infinity, the quality loss approaches zero. Figures 2.4 and 2.5 show these two situations associated with their quality loss functions.

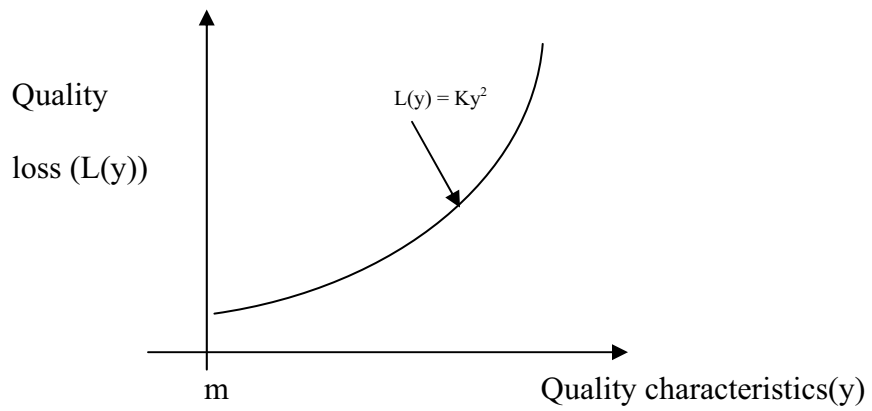


Figure 2.4. STB quadratic loss function.

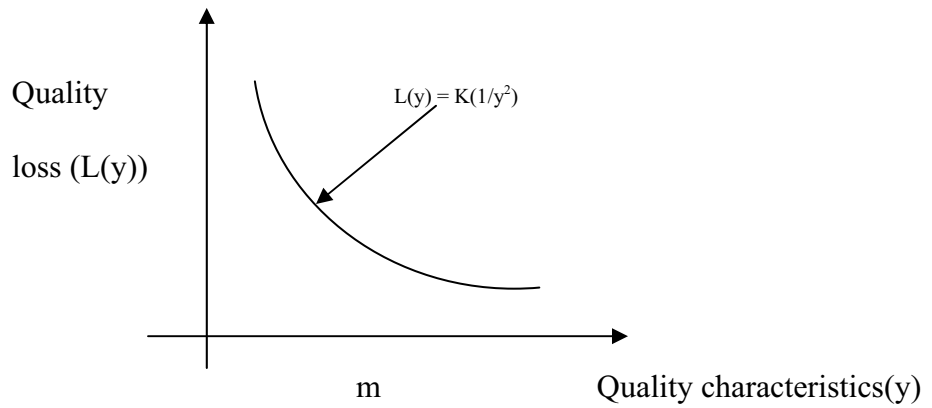


Figure 2.5. LTB quadratic loss function.

As a result of the introduction of Taguchi's quadratic loss function a new direction for tolerance allocation research was launched.

### 2.1.2 Tolerance Charting

Historically, one of the problems which faced the mass production strategy is that final products tolerances used to run out of control. This promotes research to search for a

planning tool to ensure that working dimensions and tolerances will not violate blueprint requirements. A tolerance chart was therefore developed to validate a process plan.

Tolerance charting is a graphical representation of a sequence of machining operations on a workpiece. It shows the machined parts, the blueprint tolerances, the stock removals, and the machine capabilities. The major function of tolerance charting is to ensure that a specific processing plan is appropriate for producing a product according to specifications.

Tolerance charts have been manually used for dimensional control in manufacturing since the 1950s. With the development of CAD/CAM technologies, many attempts have been made to computerize the tedious manual tolerance charting procedure. Regardless of whether tolerance charting is manual or computerized, it seeks to fulfill three universal principals:

1. The maximum possible tolerance should be assigned to each process without violating the specified blueprint tolerance.
2. Tolerance values assigned to each process should be consistent with the process capability.
3. Minimum and maximum stock removal constraints for each process should be met when considering the machine, the cutting tool, and the work material system.

In 1983, Wade introduced a new approach for constructing and using tolerance charting (Wade, 1983). Later, this approach became the basis for further developments in this area. Wade's approach is briefly described here and shown in Figures 2.6 and 2.7.

1. The blueprint and strip layout data are set up in the tolerance chart framework.
2. Schematics are constructed. Considerations are given to both the process capabilities and the occurrence of one or more machined cuts in the schematics for several BP dimensions. The user seeks to optimize the tolerance on cuts after several interactive processes.
3. All the cut tolerances, balance dimension lines, lines involving data and BP mean values are entered into the chart. A tracing method is used to identify the lines involved.
4. For all the remaining cuts that contribute to stock removal tolerance stack-up, tolerances are assigned and the balance stock removal tolerance buildups are computed.
5. The mean stock removal values are then determined by careful consideration of the relevant machine tool/cutting tool/work material for each constraint cut in the schematics.
6. The remaining cuts and balance mean dimensions are computed through inspection and backtracking until all unknown mean dimensions are obtained.

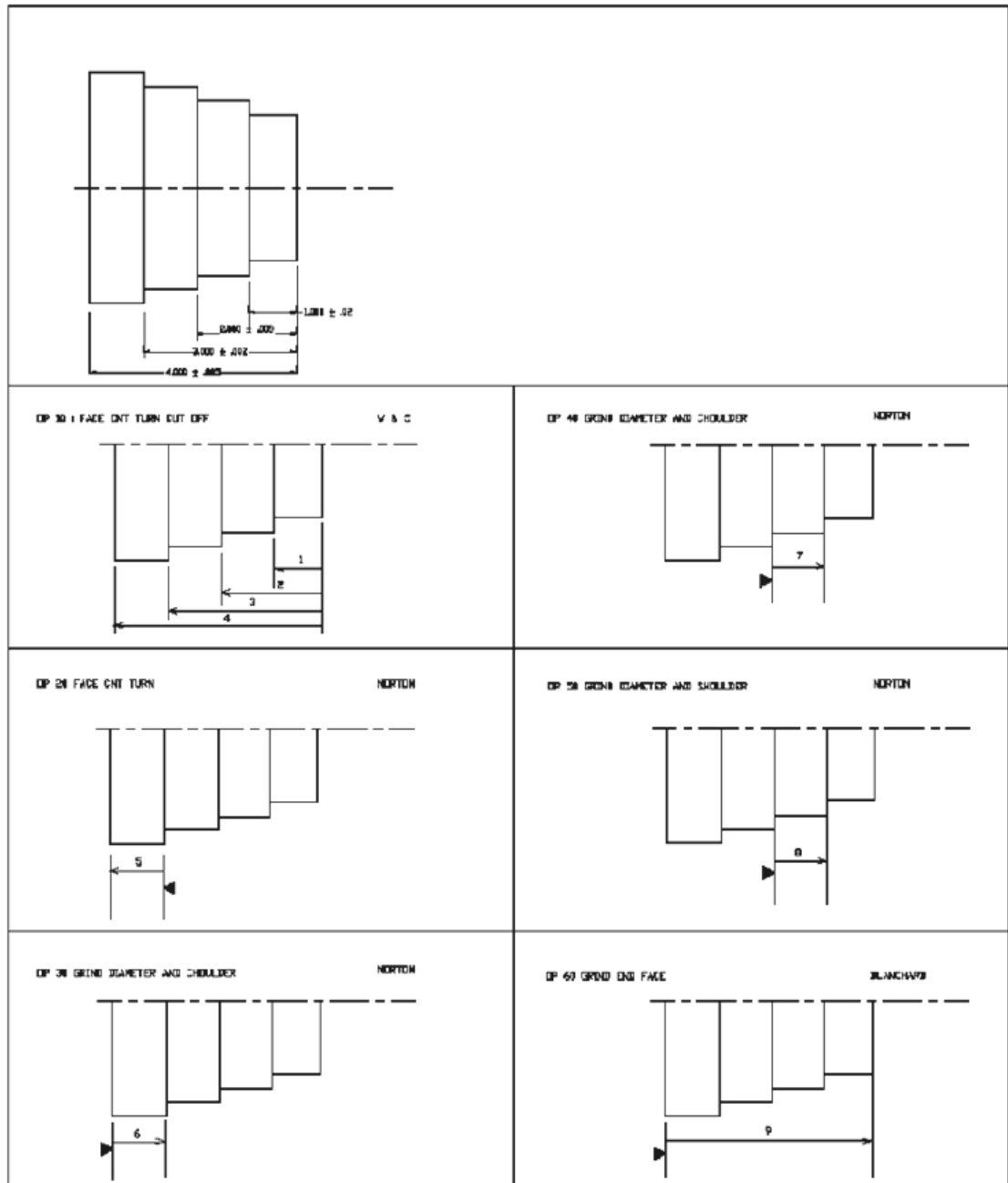


Figure 2.6. A manual tolerance chart for steel plug example-BP and process plan.

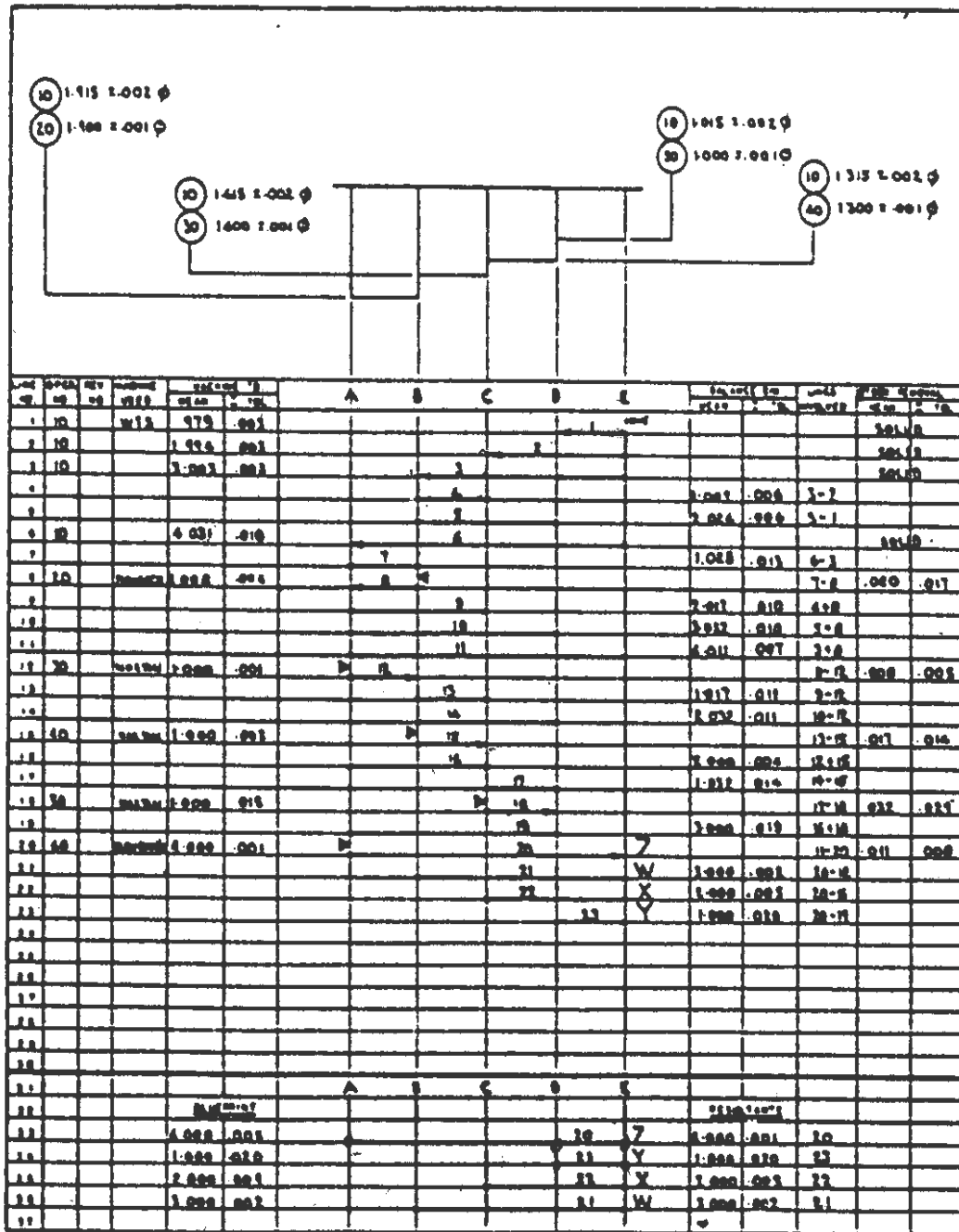


Figure 2.7. A manual tolerance chart for steel plug example.



From a broader perspective, tolerance charting combines two aspects: tolerance chain identification, and tolerance control and optimization (Ngoi and Seow, 1996). Tolerance chains define the effect of each individual operation and the sequence of operations on the final dimension of a BP specification. Several dimensional chain identifications have been developed over the last two decades including the following: (1) root tree graph, (2) branch and link, (3) matrix, (4) graph representation, and (5) surface tag. Associated with the tolerance chain identification is a procedure used to check the accumulation of tolerances at resultant dimensions and the stock-removal calculations. This procedure is called the trace method.

The tolerance control aspect of the tolerance charting is related to the distributing of a BP tolerance among the required production operations using a tolerance allocation technique. Furthermore, a tolerance analysis model is usually used to verify allocated tolerances. For more details on different versions of tolerance charting, Lehtihet et al. (2000) is an appropriate reference.

### **2.1.3 Tolerance Allocation Techniques**

Traditionally, tolerances were assigned based on experience and the capability of processes. Proportional scaling and constant precision factor methods are two examples of experience-based methods.

#### **2.1.3.1 Proportional Scaling Method**

In this method, the individual tolerances are, firstly, intuitively determined. If the sum of the individual tolerances exceeds the required functional tolerance  $T$ , then each individual tolerance  $t_i$  is reduced relative to its corresponding dimension  $d_i$ , such that (Chase and Greenwood, 1987)

$$d_1/t_1 = d_2/t_2 = \dots = d_n/t_n \quad (2.3)$$

### 2.1.3.2 Constant Precision Method

This method is based on the rule of thumb which says that the tolerance of a component increases as the cubic root of its nominal size, whereby (Chase and Greenwood, 1987):

$$t_i = P (d_i)^{1/3} \quad (2.4)$$

where the precision factor P is calculated as:

$$P = T/\sum (d_i)^{1/3} \text{ (worst-case method) or}$$

$$P = T/\sum [(d_i)^{2/3}]^{1/2} \text{ (statistical method)} \quad (2.5)$$

Both of the methods do not consider the cost-tolerance relationship and are non-optimization methods. Conversely, the majority of tolerance allocation techniques are optimization-based.

### 2.1.3.3 Optimization-Based Tolerance Allocation techniques

The basic optimization problem can be stated as follows:

Minimize (or maximize)  $f(X)$

Such that

$$g_j(X) \leq 0 \quad \text{for } j = 1 \dots m$$

$$X_i \geq 0 \quad \text{for } j = 1 \dots n \quad (2.6)$$

The value of the objective function  $f$  must be made as small (or as large) as possible. The constraints functions  $g_i$  define a subspace of the decision variable  $X$  called a feasible region. The solution to the problem (the optimum) must lie within this feasible region.

In a tolerance synthesis problem, most researchers use the manufacturing cost as the objective function. The goal is to minimize this cost, subject to a set of constraints consisting mainly of the tolerance stackup constraint, the machine capability constraint, and other constraints related to applied assumptions and optimization method.

A clear understanding of how individual tolerances stack up to produce the overall tolerance is gained by careful analysis. Two models have been widely used for calculating a tolerance stackup: the worst-case model (WCM) (called sometimes a sure-fit model) and the statistical model (also called the root sum squares (RSS) model).

Using the WCM approach, the functional tolerance is calculated as the arithmetic summation of individual tolerances as:

$$t_y = \sum t_i \quad (\text{One dimension}) \quad (2.7)$$

$$t_y = \sum \left| \frac{\partial g}{\partial X_i} \cdot t_i \right| \quad (\text{Multidimensions}) \quad (2.8)$$

where

$t_y$  is the functional tolerance

$t_i$  is the individual feature tolerance

$\frac{\partial g}{\partial X_i}$  is the derivative of the  $g_i$ -constraint function

This linear summation of tolerances assumes that all individual dimensional tolerances occur at their worst limit at the same time. Hence, the WCM model usually assigns tight individual tolerances, which means high manufacturing costs.

In the RSS method, the individual dimensions are usually assumed to obey a normal distribution. Therefore, the functional tolerance is given by

$$\text{One dimension: } t_y = (\sum t_i^2)^{0.5} \quad (2.9)$$

$$\text{Multidimensions: } t_y = (\sum ((\delta g / \delta X_i) \cdot t_i)^2)^{0.5} \quad (2.10)$$

Apparently, the RSS approach allows for looser individual tolerances than the WCM approach for the same stackup limits, which leads to a lower manufacturing cost.

Other methods also can be used to relate the functional tolerance with the individual tolerances including the modified statistical method, Spotts's modified method, the mean shift model, and the Monte-Carlo model (Wu et al., 1988).

The following are brief description of some optimization-based tolerance allocation methods:

### **Lagrange Multiplier Method**

Speckhart (1972) and Spotts (1973) are the initial attempts in the literature to assign an optimal set of tolerances to fulfill the design functional requirements and minimize the manufacturing cost. Both methods used the Lagrange multiplier method to solve the problem.

This method has a closed-form solution for the least-cost component tolerances. Assuming a cost-tolerance function of the form  $C_M(t) = a_i + b_i/t^{K_i}$ , for each component,

the derivation begins by combining the cost-minimization function and the assembly constraint into an augmented system of equations to be minimized by setting the derivatives to zero:

$$\frac{\partial}{\partial t_i}(\text{cost function}) + \lambda \frac{\partial}{\partial t_i}(\text{constraint}) = 0 \quad (i = 1, \dots, n)$$

where  $\lambda$  is the Lagrange-multiplier. After derivation, the assembly tolerance  $T_{ASM}$  is given by

$$T_{ASM}^2 = t_1^2 + \sum \left( \frac{K_i b_i}{K_1 b_1} \right)^{2/(K_i+2)} t_1^{2(K_i+2)/(K_i+2)} \quad (\text{RSS model}) \quad (2.11)$$

$$T_{ASM}^2 = t_1 + \sum \left( \frac{K_i b_i}{K_1 b_1} \right)^{1/(K_i+1)} t_1^{(K_i+1)/(K_i+1)} \quad (\text{WCM}) \quad (2.12)$$

where  $t_i$  is given by

$$t_i = \left( \frac{K_i b_i}{K_1 b_1} \right)^{1/(K_i+2)} t_1^{(K_i+2)/(K_i+2)} \quad (2.13)$$

The Lagrange multiplier method, as presented above, is an efficient tolerance allocation method because it provides a true optimum solution. Also, it allows for different cost-tolerance functions to be used. However, it has certain limitations (Chase et al., 1990):

- It cannot treat discontinuous cost tolerance functions because it requires a continuous first derivative.

- It cannot handle cost-tolerance functions where the process is limited to a specified tolerance range, i.e., where preferred tolerance limits are specified.
- It has difficulty when applied to assemblies with interdependent tolerance loops or chains, that is, assemblies that are described by more than one assembly function with shared dimensions.

Many other optimization methods were investigated to overcome these limitations, including the geometric programming method, the linear programming method, and the nonlinear programming techniques.

### **Geometric Programming Method**

For an exponential cost-tolerance function of the form  $C_M(t) = a_i \cdot \exp.^{(-bt_i)}$ , the geometric programming method is described by (Wilde and Prentice, 1975)

$$\begin{aligned}
 A &= \sum \left( \frac{1}{b_i} \right) \\
 R &= \prod (a_i)^{1/b_i A}, \prod_{i=1}^n a_i = a_1 \cdot a_2 \dots a_n \\
 G &= \prod \left( \frac{1}{b_i} \right)^{1/b_i A} \\
 t_i &= \frac{T / A + \ln [(a_i b_i G)] / R}{b_i}
 \end{aligned} \tag{2.14}$$

Geometric programming yields the same tolerances derived by the Lagrange method. However, this method is only adaptable for the exponential cost-tolerance function.

### **Linear Programming**

In this method the nonlinear cost-tolerance curve is divided into different segments. Inside each segment, the corresponding cost-tolerance curve is linearized, and the total cost becomes (Patel, 1980)

$$C_M(t) = \sum a_i t_i + b$$

where

$a_i$  is the slope of the linearized cost-tolerance line

$b$  is a constant

The simplex linear programming method is used to allocate individual tolerances. Obviously, this is an approximation technique. It approaches the continuous cost-tolerance function, as the segment size gets smaller.

### **Nonlinear Programming**

Many nonlinear optimization algorithms have been used in tolerance allocation research, including: the Hooke-Jeeves direct search method, the random adaptive search method, and the reduced gradient method (Siddall, 1982).

## **2.1.4 Introduction to Fuzzy Set Theory**

### **2.1.4.1 Crisp and Fuzzy Sets**

In 1965, L.A. Zadeh published his famous paper “Fuzzy Sets,” providing a new mathematical tool to describe and handle vague or ambiguous notations. Since then, fuzzy set theory has been rapidly developed by Zadeh himself and numerous researchers, and an increasing number of successful real applications of this theory in a wide variety of unexpected fields have been appearing.

Fuzzy sets are considered one of the most efficient means of handling imprecision, vague parameters, and ill-defined relationships, which characterize engineering problems. Fuzzy set theory is a generalization of classical set theory. In normal set theory, an object either belongs to the set or does not belong to the set. Such a

set is termed a crisp set. However, in the case of fuzzy sets, objects can enjoy partial membership in the set. Fuzzy set is defined as a set “A” in a universe of discourse X and is characterized by a membership function  $\mu_A(x) \in [0, 1]$ . A membership function provides a measure of the degree of similarity of an element in X to the fuzzy set. It is represented as a set of ordered pairs of element x and its grade of membership function as

$$A = \{(x, \mu_A(x)) / x \in X\} \quad (2.15)$$

A membership value of zero means that the object is not a member of the set, while a membership value of one means that the object is definitely a member of the set. Values between zero and one denote partial membership in the set.

Fuzzy sets can represent imprecise quantities as well as linguistic terms. Hence, even design constraints specified linguistically, such as “accurate machine is required” can be written mathematically by making use of the fuzzy set notation. A linguistic variable is defined as a variable that does not have a definite quantitative value and instead has a qualitative description in words. For example, the term “machine accuracy” is a linguistic variable if it is defined as not accurate, accurate, and very accurate, instead of being assigned specific numbers.

#### **2.1.4.2 Membership Functions**

A membership function is a graphical representation of the information about a fuzzy set. It represents a smooth and gradual transition between two regions of a particular set. Membership function has three basic regions, namely core, support, and boundaries, (Yen, J., and Langari). An example for membership function is shown in Figure 2.8.



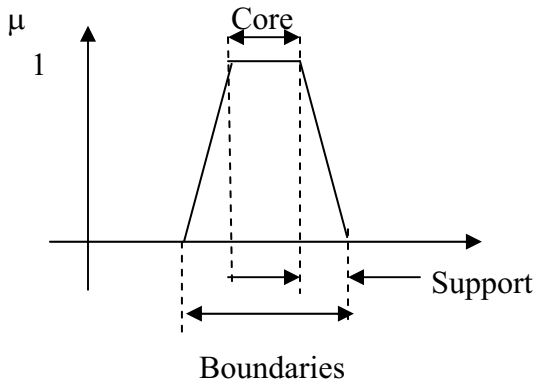


Figure 2.8. Fuzzy set regions

The core of a fuzzy set “A” represents the region that has a full membership value, which is “1”, and comprises all the elements  $x$  of the universe such that

$$\mu_A(x) = 1 \quad (2.16)$$

The support of a membership function of a fuzzy set ‘A’ is defined as the region that has a partial membership value and is represented as

$$0 < \mu_A(x) < 1 \quad (2.17)$$

The boundaries of a membership function of a fuzzy set “A” are defined as the region containing all elements that have a non-zero membership function.

Membership functions come in different shapes. The following is a brief description of the main types of membership functions.

## Triangular Membership Function

This function is represented as a triangle with the apex as its center with a full membership value. It has left and right edges that gradually slope down to a degree of zero. Figure 2.9 shows a triangular membership function, which is defined by three parameters {a, b, c} and is represented as

$$\text{Triangle } (x : a, b, c) = \begin{cases} 0 & x < a \\ \frac{x - a}{b - a} & a \leq x \leq b \\ \frac{c - x}{c - b} & b \leq x \leq c \\ 0 & x > c \end{cases} \quad (2.18)$$

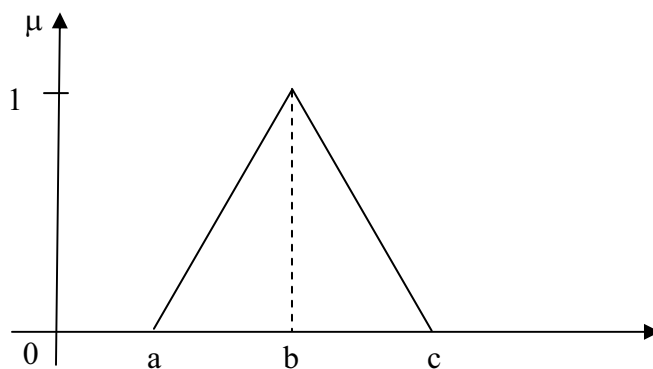


Figure 2.9. Triangular membership function

The terms a, b, and c define the shape of the triangular membership function. The triangular membership function is the simplest type of membership function and is most commonly used when there is insufficient information about the linguistic terms.

### Trapezoidal Membership Function

A trapezoidal membership function is defined using four parameters  $\{a, b, c, d\}$  as illustrated in Figure 2.10. Mathematically, it is defined as

$$\text{Trapezoid} \quad (x : a, b, c, d) = \begin{cases} 0 & x < a \\ \frac{x - a}{b - a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d - x}{d - c} & c \leq x \leq d \\ 0 & x > d \end{cases} \quad (2.19)$$

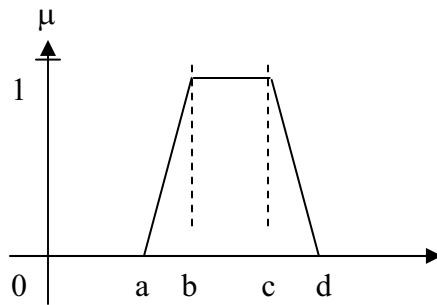


Figure 2.10. Trapezoidal membership function

It is clear that the triangular function is a special case of the trapezoidal function when  $b = c$ . The trapezoidal membership function is the most usable type of membership functions as it can be easily implemented with different mathematical operations, (Pan and Yuan, 1997).

### Bell-Shaped Membership Function

A bell-shaped membership function is defined by three parameters  $\{a, b, c\}$  as shown in Figure 2.11. It is defined by the relation

$$Bell(x: a, b, c) = \frac{1}{1 + \left| \frac{x-c}{a} \right|^{2b}} \quad (2.20)$$

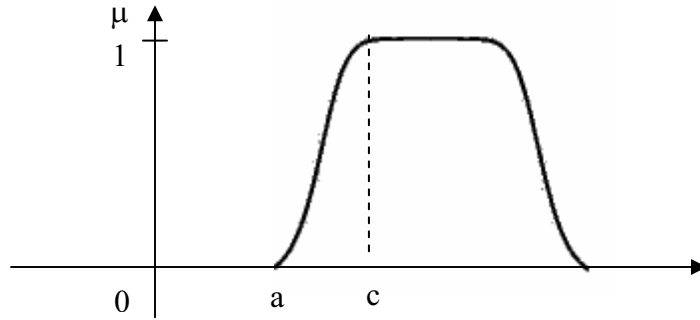


Figure 2.11. Bell-shaped membership function

The shape of a bell-shaped membership function can be altered by changing the values of  $a$ ,  $b$ , and  $c$ . The center and width can be altered by changing  $c$  and  $a$  respectively and  $b$  is used to alter the slope of crossover points.

#### 2.1.4.3 $\alpha$ -Level Set

The  $\alpha$ -level set of a fuzzy set  $A$  is defined as an ordinary set  $A_\alpha$  for which the degree of its membership function exceeds the level  $\alpha$

$$A_\alpha = \{x | \mu_A(x) \geq \alpha\}, \alpha \in [0,1] \quad (2.21)$$

Since the  $\alpha$ -level set  $A_\alpha$  is an ordinary set, it can be defined by the characteristic function:

$$c_{A_\alpha} = \begin{cases} 1 & \text{if } \mu_A(x) \geq \alpha \\ 0 & \text{if } \mu_A(x) < \alpha \end{cases}$$

Actually, an  $\alpha$ -level set is an ordinary set whose elements belong to the corresponding fuzzy set to a certain degree  $\alpha$ . It is clear that the following evident property holds for the  $\alpha$ -level set

$$\alpha_1 \leq \alpha_2 \Leftrightarrow A\alpha_1 \supseteq A\alpha_2$$

Moreover, from the definition of the  $\alpha$ -level sets, it can be easily understood that the following basic properties hold

$$\begin{aligned} (A \cup B)_\alpha &= A_\alpha \cup B_\alpha \\ (A \cap B)_\alpha &= A_\alpha \cap B_\alpha \end{aligned}$$

Using the concept of  $\alpha$ -level sets, the relationship between ordinary sets and fuzzy sets can be characterized by the decomposition theory which states fuzzy set “A” can be represented by

$$A = \bigcup_{\alpha \in [0,1]} \alpha A_\alpha \tag{2.22}$$

where  $\alpha A_\alpha$  denotes the algebraic product of a scalar  $\alpha$  with the  $\alpha$ -level set  $A_\alpha$ , i.e., its membership function is given by

$$\mu_{\alpha A_\alpha}(x) = \alpha \mu_{A_\alpha}(x), \forall x \in X \tag{2.23}$$

The decomposition theorem states that a fuzzy set A can be decomposed into a series of  $\alpha$ -level sets  $A_\alpha$  by which A can be reconstructed. Thus, any fuzzy set can be viewed as a family of ordinary sets.

#### 2.1.4.4 Bellman-Zadeh Fuzzy Decision

In 1970, Bellman and Zadeh (1970) introduced an approach for handling decision making under fuzzy conditions. This approach forms the basis of an overwhelming majority of fuzzy decision-making related models. Bellman and Zadeh introduced three basic concepts: fuzzy goal, fuzzy constraint, and fuzzy decision and explored the application of these concepts to decision-making processes under fuzziness. In this approach, if  $X$  is a set of possible alternatives that contains the solution of a decision-making problem under consideration, then a fuzzy goal  $G$  is a fuzzy set on  $X$  characterized by its membership function

$$\mu_G : X \rightarrow [0, 1]$$

A fuzzy constraint  $C$  is a fuzzy set on  $X$  characterized by its membership function

$$\mu_C : X \rightarrow [0, 1]$$

Knowing that both the fuzzy goal and the fuzzy constraint are desired to be satisfied simultaneously, the fuzzy decision  $D$  would result from the intersection of the  $G$  and  $C$ . Mathematically, the fuzzy decision is the fuzzy set  $D$  on  $X$  defined as

$$D = G \cap C$$

and is characterized by its membership function

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)) \quad (2.24)$$

The maximizing decision is then defined as

$$\max_{x \in X} \mu_D(x) = \max_{x \in X} \min(\mu_G(x), \mu_C(x)) \quad (2.25)$$

More generally, the fuzzy decision  $D$  resulting from  $k$  goals  $G_1, G_2, G_k$  and fuzzy constraints  $C_1, C_2, \dots, C_m$  is defined as

$$D = G_1 \cap \dots \cap G_k \cap C_1 \dots C_m$$

and the corresponding maximizing decision is defined as

$$\max_{x \in X} \mu_D(x) = \max_{x \in X} \min(\mu_{G_1}(x), \dots, \mu_{G_k}(x), \mu_{C_1}(x), \dots, \mu_{C_m}(x))$$

#### 2.1.4.5 Fuzzy Linear Programming (FLP)

In 1976, H. J. Zimmerman (1976) first introduced the fuzzy set theory into conventional linear programming problems. He considered linear programming problems with a fuzzy objective and fuzzy constraints. Since then, fuzzy linear programming has been developed in a number of directions with many successful applications. This section presents an overview of the fuzzy linear programming as proposed by Zimmerman.

A linear programming problem can be written as

$$\begin{aligned} \text{Minimize} \quad & z = cx \\ \text{Subject to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

where

$c$  is an  $n$ -dimensional row vector  $= (c_1, c_2, \dots, c_n)$

$x$  is an  $n$ -dimensional column vector  $= (x_1, x_2, \dots, x_n)^T$

$b$  is an  $m$ -dimensional column vector =  $(b_1, b_2, \dots, b_n)^T$

$A$ : is an  $m \times n$  matrix =  $[a_{ij}]$

Zimmerman (1976) proposed to soften the rigid requirements of the decision maker (DM) to strictly minimize the objective function and strictly satisfy the constraints. Namely, by considering the impression or fuzziness of the DM's judgment, he softened the usual linear programming problem into the following fuzzy version

$$\begin{aligned} Bx < b' \\ x \geq 0 \end{aligned} \quad (2.26)$$

where:

$<$  is a relaxed or fuzzy version of the inequality " $\leq$ "

$$B = \begin{bmatrix} c \\ A \end{bmatrix}$$

$$b' = \begin{bmatrix} z \\ b \end{bmatrix}$$

For treating the  $i^{\text{th}}$  fuzzy inequality  $(Bx)_i < b'_i$ ,  $i = 0, \dots, m$ , Zimmerman proposed the following linear membership function

$$\mu_i((Bx)_i) = \left\{ \begin{array}{ll} 1 & ; (Bx)_i \leq b'_i \\ 1 - \frac{(Bx)_i - b'_i}{d_i} & ; b'_i \leq (Bx)_i \leq b'_i + d_i \\ 0 & ; (Bx)_i \geq b'_i + d_i \end{array} \right\} \quad (2.27)$$

where  $d_i$  is a constant expressing the limit of the admissible violation of the  $i^{\text{th}}$  inequality, called the fuzziness range. Figure 2.12 shows such a linear membership function.



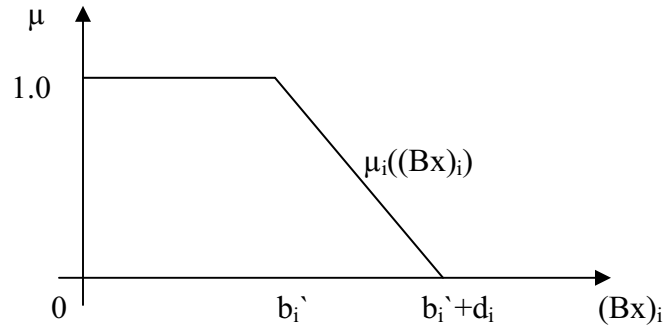


Figure 2.12. Linear membership function

Following the fuzzy decision of Bellman and Zadeh (1970) together with the linear membership functions, the problem of finding the maximum decision is to choose  $x^*$  such that

$$\mu_D(x^*) = \max_{x \geq 0} \min_{i=0, \dots, m} \{ \mu_i((Bx)_i) \}$$

In other words, the problem is to find  $x^* > 0$ , which maximizes the minimum membership function values.

Substituting

$$b_i'' = b_i' / d_i, \quad (B'x)_i = (Bx)_i / d_i \quad (2.28)$$

the problem is rewritten as:

$$\mu_D(x^*) = \max_{x \geq 0} \min_{i=0, \dots, m} \{ 1 + b_i'' - (B'x)_i \} \quad (2.29)$$

By introducing the auxiliary variable  $\lambda$ , this problem can be transformed into the following equivalent conventional linear programming problem

$$\begin{aligned}
& \text{Maximize } \lambda \\
& \text{subject to } \lambda \leq 1 + b_i'' - (B'x)_i, i = 0, 1, \dots, m \\
& x \geq 0
\end{aligned} \tag{2.30}$$

### 2.1.5 Process Capability and Process Capability Ratio Analysis

Generally, a repeatable process should be used for the production of products. That is, the process must be capable of operating with acceptable variability around the target or nominal dimensions of the product quality characteristics. Regardless of how well a process is designed and maintained, a certain amount of natural variability always exists. This variability is the result of small unavoidable causes. In statistical quality control, when variability in a system consists only of this natural variability, the system is called a stable system. Unfortunately, assignable causes do occur, typically at random, resulting in an “out-of-control” system.

It is customary to take the 6-sigma spread in the distribution of the product quality characteristics (i.e.,  $\pm 3\sigma$ ) as a measure of process capability (Juran and Gryna, 1988), where  $\sigma$  is the standard deviation of the process under statistical control.

The above definition of process capability demonstrates the spread of a production process. To utilize this information it must be related to design specifications. Therefore, a process capability ratio (PCR or  $C_p$ ) rather than 6-sigma is used to measure the capability of a process.

PCR is a measure of the capability of a process to manufacture products with respect to design specifications. This concept of PCR was first proposed by Gryna in Juran's *Quality Control Handbook* (Juran and Gryna, 1988), originally published in 1962. The original form of process capability ratio is

$$C_p = \text{tolerance width} / \text{process capability} = \frac{USL - LSL}{6\sigma} \quad (2.32)$$

where USL and LSL are the upper and lower design specification limits.

This definition applies to bilateral equal design specifications where process average coincides with the mean of design specifications. It measures only the spread of the specifications relative to the  $6\sigma$  spread of the process. Hence, this index cannot give sufficient information about the process capability when the process mean does not coincide with the design mean. To more accurately reflect the actual process capability, a new ratio was introduced by Montgomery (1996)

$$C_{pk} = \min(C_{pU}, C_{pL}) = \min\left\{ C_{pU} = \frac{USL - \mu}{3\sigma}, C_{pL} = \frac{\mu - LSL}{3\sigma} \right\} \quad (2.33)$$

This formula assumes that  $LSL \leq \mu \leq USL$ . As illustrated by Montgomery (1996),  $C_{pk}$  is still an inadequate measure of process centering. That is, for a fixed  $\mu$  value,  $C_{pk}$  depends inversely on  $\sigma$  and becomes large as  $\sigma$  approaches zero. This makes  $C_{pk}$  unsuitable for center measuring because large values of  $C_{pk}$  do not tell any thing really about the

location of  $\mu$  in the LSL-USL interval. One way to overcome this shortage is by adopting the  $C_{pm}$  version of process capability ratio

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - \mu_0)^2}} = \frac{C_p}{\sqrt{1 + \psi^2}} \quad (2.34)$$

where

$$\mu_0 = \frac{LSL + USL}{2}$$

$$\psi = \frac{\mu - \mu_0}{\sigma}$$

## 2.2 Literature Review

Tolerance assignment has been the research focus for decades. Early literature published on this topic goes back to the 1950s (Marks, 1953; Pike and Silverberg, 1953). According to Hong and Chang (2002), the existing tolerance research can be classified into seven distinct categories: tolerance schemes, tolerance modeling and representation, tolerance specification, tolerance analysis, tolerance allocation or synthesis, tolerance transfer, and tolerance evaluation. This section surveys the literature in the following areas: cost-tolerance relationships, tolerance allocation techniques, and the process capability issue in tolerance allocation research.

### 2.2.1 Manufacturing Cost-Tolerance Functions

Chen and Maghsoodloo (1995) summarized five cost-tolerance functions: reciprocal, reciprocal square, reciprocal power, exponential, and exponential-reciprocal power. These functions are shown in Table 2.1 with associated literature citations and possible application areas. Clearly, all approaches emphasize that the manufacturing cost is an explicit function of the tolerance.

TABLE 2.1

VARIOUS MANUFACTURING COST-TOLERANCE FUNCTIONS

Model	Function	Possible application area	Author
Reciprocal	$C_M(t) = a + b/t$ ,	Network circuit Shaft and bearing assembly	(Pinel and Roberts, 1972) (Chase and Greenwood, 1988)
Reciprocal-Squared	$C_M(t) = b/t^2$ , $C_M(t) = a + b/t^2$	Mechanical assembly	(Hillier, 1966), (Spotts, 1973), (Parkinson, 1985)
Reciprocal-Power	$C_M(t) = b/t^c$ , $C_M(t) = a + b/t^c$ ,	Airframe component Mechanism	(Bennett and Gupta, 1970), (Lee and Woo, 1990), (Sutherland and Roth, 1975)
Exponential	$C_M(t) = a + b/\exp.^{ct}$ ,	Shaft and bearing Helical spring	(Speckhart, 1972) (Wilde and Prentice, 1975)
Exponential/Reciprocal Power	$C_M(t) = b/\exp.^c t^c$ ,	Cylindrical surface	(Michael and Siddall, 1981)

$C_M(t)$  = total manufacturing cost.

t = tolerance.

a = cost factor including setup, tooling, material and prior operations.

b = cost of producing a single piece dimension to a specified tolerance.

c = data-dependent constant.

Wu et al. (1988) compared the above models based on their ability to capture the cost-tolerance relation of real-field data. They concluded that the most accurate model to fit field data is the exponential/reciprocal power model. This model has the minimum curve fitting errors. The exponential cost-tolerance function ranked second in terms of minimizing curve fitting errors. The models with reciprocal relation came in third. In addition, they indicated that all five models showed good approximation to the empirical data curve at loose tolerances ( $\geq 0.35$  mm) but had larger errors at tight tolerances ( $\leq 0.1$  mm).

Dong et al. (1994) presented six new models for the manufacturing cost-tolerance association: (1) combined reciprocal power and exponential function, (2) combined linear, (3) B-Spline curve, (4) cubic polynomial, (5) fourth-order polynomial, and (6) fifth-order polynomial. They showed that the new proposed models fit the empirical cost-tolerance data with considerably less fitting error than the existing functions. Therefore, these models provide more reliable results for tolerance synthesis.

A new approach was adopted by Chen (2001) and Lin and Chang (2002) to map the manufacturing cost-tolerance relationship. They used an artificial neural network (ANN) to capture the manufacturing cost-tolerance relationship. The cost-tolerance experimental data was used as training/testing sets to build the network. Lin and Chang (2002) concluded that the application of ANN to map the cost-tolerance relation yields “better performance in controlling the average fitting error than all conventional fitting models.” Moreover, Chen (2001) illustrated that using ANN would avoid making assumptions about the used form of regression equation along with assumptions about its parameters.

## **2.2.2 Optimization-Based Tolerance Allocation Techniques**

Tolerance allocation is applied at two stages of product development: design and process planning. The literature in this section is divided based on this classification.

### **2.2.2.1 Optimization-Based Design Tolerance Allocation Techniques**

Speckhart (1972) and Spotts (1973) are the early literature that assigned optimal tolerances while minimizing the manufacturing costs. Both researchers utilized the Lagrange multiplier to solve the suggested nonlinear programming models. Speckhart minimized the exponential manufacturing cost-tolerance function subject to both deterministic (WC model) and stochastic (RSS model) stackup constraints. Spotts applied the Lagrange multiplier method to minimize the manufacturing cost of an inverse square mathematical function. Dresner and Barken (1993) used then numerical Lagrange multiplier for the optimization of allocated tolerances. Chen (1995, 1996) optimized the tolerance allocation problem utilizing the Kuhn-Tucker necessary condition and Lagrange multiplier method for nonlinear multiple constraints. Wilde and Prentice (1975) solved the problem formulated by Speckhart (1972) using geometric programming. Although this method is limited to the sure-fit case (WC model), it presents a closed form, noniterative solution for the exponential cost-tolerance problem presented in Speckhart's paper.

Chen et al. (1984) adopted an interactive linear programming-based design algorithm to solve the tolerance synthesis problem. In this method, the objective function and all the constraints are approximated by linear functions based on one point in the state space. The problem is then solved using LP techniques. The new optimum point then becomes the basis for the next linear approximation.

Ostwald and Huang (1977) presented a technique for optimal tolerance allocation choosing one of many process alternatives. They solved the model by 0-1 integer programming, selecting cost as the objective function and design requirements as the constraints. A probability and sensitivity analysis was also conducted to study the relationship between manufacturing cost and scrap rate when the design constraints are violated. Kim and Knott (1988) proposed several modifications to Ostwald and Hogan's method in an effort to facilitate computations. The objective function was changed from minimizing manufacturing cost to maximizing savings and the constraints were changed from functional tolerances to excess tolerances.

Chase et al. (1990) proposed four methods to solve the tolerance allocation problem: exhaustive search, univariate search, the 0-1 method, and sequential quadratic programming. They concluded that the proposed univariate search method, based on Lagrange multipliers and a procedure for reducing the set of process combination tested, is the most efficient among the four evaluated methods. The exhaustive method is a reliable method for problems with fewer than 25 or 30 variables. They also showed that the 0-1 method is too inefficient to be of practical value. The sequential quadratic method, based on nonlinear programming, proved to be capable of treating multiple loop assembly functions. It is worth noting that none of these methods guarantee finding the global value. Actually, the exhaustive search method presented by Chase et al. (1990) can be considered a variation of the integer programming method initially introduced by Ostwald and Huang (1977).

Lee and Woo (1989, 1990) formulated the tolerance allocation problem as a probabilistic optimization problem in which each dimension with its tolerance is linked to



a random variable. By introducing the idea of reliability index, they were able to transform the problem into a deterministic integer programming problem and solve it by suitable procedures.

Li et al. (1998) and Kao (2000) applied a sequential quadratic programming algorithm embedded with a Monte Carlo simulation to optimize tolerance allocation having multivariate normal distribution. When compared with some previous studies, the proposed method results in less manufacturing cost and higher yield.

The majority of the existing tolerance synthesis optimization models set the objective function as minimizing the manufacturing cost. The difference is in what type of manufacturing cost-tolerance relationship they used. After the introduction of Taguchi's quadratic quality loss function, some researchers chose to minimize the quality loss cost while allocating tolerances, (Kapur and Cho, 1994; Kapur et al. 1990). Furthermore, another group of researchers simultaneously minimized both quality loss cost and manufacturing cost in the tolerance allocation problem, (Soderberg, 1994; Krishnaswami and Mayne, 1994).

Wei and Lee (1998) proposed a nonlinear programming model that minimizes the total manufacturing loss. In order to connect produced tolerances with the process capability, the manufacturing loss was formulated as a function of a proposed standardized process tolerance. The standardized process tolerance is equal to the operational tolerance of an operation divided by its standard deviation.

In their economic model, Vasseur et al. (1993) allocated tolerances based on profit maximization. They incorporated a manufacturing cost function into an economic model that estimates the consumer's demand for the product as a function of both the

price and quality level. The output of this model is an optimum production mode that maximizes both the profit and quality level.

Ngoi and Seow (1996) used a multi-criteria objective function. They formulated the objective function as minimizing the manufacturing cost and maximizing the working dimensions.

#### **2.2.2.2 Optimization-Based Operational Tolerance Allocation Techniques**

With the development of CAD/CAM technologies, many attempts were made to computerize the tedious manual tolerance charting procedure. One of the most significant works was made by Irani et al. (1989). They proposed a graph theoretic representation for the tolerance chart. A tracing algorithm was introduced to identify dimensional chains from this graph. A goal programming model was adopted to allocate blueprint tolerances among related working dimensions. Similarly, Mittal et al. (1990) used a graph theoretic representation method to represent the process plan. They formulated a linear programming model with the objective function of achieving minimum slack or residual tolerances for the blueprint dimensions and/or the stock removal. No cost considerations were taken into account in the presented method and residual tolerances of all processes had an equal probability to be minimized.

Chen and Cheraghi (1996) modified the Irani et al. model and implemented a procedure for linking the process plan generation with the CAD environment. The suggested method could extract data on operations from the CAD model and could generate a sequence of operations using graphic representations.

Ngoi (1992) presented a mathematical model for the tolerance chart balancing process and made use of linear programming to solve the problem. By implementing a

weighting system, Ngoi reduced the objective function into a deterministic linear one. However, no explanation was given to how weights should be assigned. In 1993, Ngoi and Chua (1993) presented a matrix approach for deriving the resultant dimensions and tolerances during tolerance charting. Following this, Ngoi and Ong (1993) extended the idea and produced an algorithm to reduce the unknown working dimensions and blueprint dimensions into a system of linear equations. These equations were subsequently solved using Gaussian elimination techniques.

Ji (1993, 1994) used a linear programming method for the assignment of tolerances to working dimensions with the main objective of maximizing the cumulative tolerance of each working dimension. He introduced a proportional scaling smoothing approach that combines simplex method and the direct smoothing approach.

Jeang (1998) introduced a mathematical model for tolerance chart balancing during machining process planning. He proposed a graphic rooted tree representation technique in describing the sequence of the machining process in process planning. The criteria considered were based on the combined effect of manufacturing cost and quality loss.

Chen et al. (1997) proposed an online process control system to allocate operational tolerances and validate the process plan based on actual measurements. The model aims to maximize allocated tolerances to each individual machining operation. It updates the operational tolerances allocated to machining cuts based on produced dimension after every operation. Simultaneously with tolerance calculation, the model also calculates the mean working dimensions. Cheraghi et al. (1999) expanded the model by developing a dynamic feed forward process control system. The core of this system is

the process adjustment routine (PAR). The PAR analyzes the gathered on-line information and accordingly validates the process plan. If any actions are needed, PAR advises the machine operator to make adjustments to the subsequent operations so that the final part is produced per specifications.

Adopting a similar on-line feedback idea, in 1997 Fraticelli et al. (1997) introduced the concept of sequential tolerance control, an approach that uses real-time measurement information at the completion of stage  $j$  to do any needed adjustments for stage  $j+1$ . The procedure is repeated at appropriate locations along the  $n$  stages. The study did not mention how frequently the measurements should be taken. Wheeler et al. (1999) combined the sequential tolerance control technique with an implicit enumeration approach to select an optimal set of operations when several operations are available for each required process in the process plan. A formulation was given to the probability of producing good parts as a function of process precision and cost. An algorithm was developed to select the set of operations that minimize manufacturing cost and maintain acceptable proportion of good parts.

### **2.2.3 Non-Optimization Tolerance Allocation Techniques**

The non-optimization methods in tolerance synthesis research can be categorized into three groups: quality engineering methods, knowledge-based and expert methods, and intelligent computing methods such as simulating annealing, genetic algorithms, artificial neural networks, and fuzzy logic.

#### **2.2.3.1 Quality Engineering Methods**

One of the major drawbacks of the aforementioned optimization models is that their applicability tends to decline dramatically as the complexity of mechanical

assemblies increases. Therefore, another new stream of research based on a new principle began to appear in the 1990s. “Quality engineering is a discipline that aims at an integrated system of overall quality in which every activity involved in production is controlled to produce the products with minimum deviation from a target value” (Taguchi et al., 1989). This is also called robust engineering. Robust engineering enforces the applicability of a three-step procedure during the product design and production phases: system design, parameter design, and tolerance design. Gadallah and Elmaraghy (1994, 1995) were among the first researchers to apply the parameter design technique to the tolerance optimization model. They used analysis of variance (ANOVA) and design of experiments (DOE) techniques to select the critical dimensions to be controlled. Then, they minimized the manufacturing costs while tightening these tolerances. The suggested algorithm does not use the cost-tolerance function, which avoids any fitting errors. However, the manufacturing cost is optimized indirectly by tightening the important design dimensions to the overall function of the product. Kusiak and Feng (1995) formulated the synthesis of discrete tolerances as a combinatorial optimization problem and solved it by integer programming (IP), DOE, and the Taguchi method. Their goal was to compare the three methods based on minimizing the manufacturing cost. They concluded that DOE is more appropriate than IP for nonlinear problems and that it can be used to solve probabilistic problems. In contrast, Gupta and Feng (2000) used the fractional factorial method to concurrent design of parameters and tolerances. Parkinson (2000) applied a similar way of robust parameter design to tolerance.

### **2.2.3.2 Knowledge-Based and Expert Systems-Based Techniques**

Manivannan et al. (1989) used a knowledge-based system to assign the dimensions and tolerances of two cylindrical mating parts. The ISO standards for limits and fits are referenced according to different types of fits, namely interference, clearance, or transition. The advantage of this system is that the rules can be easily modified as needed. However, the system requires the user to identify the mating parts that are to be toleranced. Janakiram et al. (1989) developed an expert system to select the best tolerance specifications from alternative manufacturing processes based on the assembly functional requirements and the accumulated manufacturing tolerances. Alternative processes are generated from a knowledge base. The alternatives are ranked based on their functionality and cost, and the best alternative is selected based on these two factors. The method was tested on a simple shaft and hub example. To be implemented in a CAD environment, Panchal et al. (1992) presented a method to assign tolerances based on feature extraction, feature inference, and a rule-based tolerance allocation approach. The method is limited to cylindrical parts assemblies.

Lu and Wilhelm (1991) proposed a different approach for tolerance synthesis based on artificial intelligence (AI). They used the geometry of the assembly to define relationships between the assembly dimensions. The designer identifies dimensions for some key assembly characteristics. Then, an AI-based system computes matched dimensions for the rest of the assembly. Finally, tolerances are identified as intervals about the nominal values.

Although applying the knowledge-based and expert systems principles in tolerance allocation gives a new direction for the research in this area, these approaches

are often limited to cylindrical mating components. Another major drawback of these systems is the inconsideration of the cost factor.

### **2.2.3.3 Intelligent Computing Techniques**

Another group of efforts to overcome the impracticality of optimization methods are based on intelligent computing techniques such as genetic algorithms (GA), simulated annealing (SA), artificial neural networks (ANN), and fuzzy logic. Lee and Johnson (1993) are the first researchers to adopt GA in tolerance allocation. They solved the same problem introduced by Lee and Woo (1990) using GA and the Monte Carlo simulation method. Their goal was to minimize the manufacturing cost-tolerance function while maintaining a specified yield. By comparing their results with Lee and Woo findings, they demonstrated the capability and efficiency of applying GA in tolerance synthesis area. The genetic algorithm approach was also adopted by Li et al. (2000) to find simultaneously the optimal machining datum set and tolerances for rotational parts. The machining datum set was connected to tolerances through a tolerance chart and a proposed dimensional chain tolerance method. They built a mixed discrete nonlinear optimization model that was solved by the GA search method. Their computational results indicated that the proposed GA-based methodology is capable and robust in finding the optimal machining datum set and tolerances.

Zhang and Wang (1993) formulated the tolerance allocation problem as a nonlinear optimization model that allows the selection between several processes to produce a single dimension. A SA algorithm was used to solve the model. They found that SA was robust in that it always converges to a solution. Al-ansary and Deiab (1997) solved the same problem with GA. Their genetic engineering approach showed better

results than the simulated annealing results. Chen (2001) used a backpropagation network to map the cost-tolerance relationship and SA to solve the generated optimization model. A real-life tolerance allocation example illustrated the effectiveness and efficiency of the proposed method.

Kopardekar and Anand (1995) applied ANN to allocate tolerances. They used the backpropagation paradigm to train and test the proposed network. By using ANN in tolerance allocation, machine capability issue and mean shift problem were well handled.

Dupinet et al. (1996) exploited fuzzy logic to estimate the coefficients of the cost-tolerance function based on linguistic variables. The tolerance allocation problem was solved using a SA algorithm. The authors concluded that the proposed hybrid fuzzy logic-SA approach is well suited for the tolerance allocation problem and that it allows for considering some important factors that are difficult to evaluate quantitatively. Ji et al. (2000a, b) used fuzzy logic to evaluate the machinability of a part. Then they established a mathematical model for tolerance allocation by combining the fuzzy-based machinability factors and the functional sensitivity factor of a part. A GA was used to optimize this model. The feasibility of the model was validated using a practical example.

### **2.3.2 Process Capability Analysis**

Process capability measures that have been introduced in the past years can be generally classified into three categories: process capability measures for coordinate tolerance specifications, process capability measures for geometric tolerance specifications, and process capability measures for multivariate processes (Meyappan, 1999). The process capability measures for co-ordinate tolerances can be divided into measures for normal distributions and measures for non-normal distributions. Bothe



(1997) explained several capability indices, like  $C_p$ ,  $C_{pk}$ ,  $C_{pu}$ ,  $C_{pl}$ , and  $C_{pm}$ , to assess the capability of normally distributed processes. For non-normal distributed processes, Clements (1989) presented an approach based on the use of the Pearson system of curves. To calculate a process capability ratio for these types of processes, estimates for the mean, variance, skewness, and kurtosis for the process are required. For non-normal processes, Rogowski (1994) proposed to transfer the data into normal and then to use the ordinary  $C_p$  of the normally distributed process. The type of transformation is obtained by goodness-of-fit tests.

Geometric Dimensioning and Tolerancing (GD&T) is the engineering standard that provides a unified terminology and approach to describe both the geometry of the features and the associated tolerance of the product. GD&T contains different types of tolerances such as form, orientation, profile, position, and run-out. Gruner (1991) explained how to calculate a PCR for a specified variable tolerance (tolerance varies as the actual size of the feature varies) using GD&T. Methods for generating the PCR for a spherical or circular tolerance zone (Davis et al. 1993) and concentricity (Sakar and Pal, 1997) have also been studied.

The multivariate characteristics of the processes that specify geometric dimensions have led to much research and new capability indices for analyzing the multivariate processes. Multivariate analysis deals with the correlation of variables involved. It differs from univariate and bivariate analysis since it used concepts of covariance and correlation, thus reflecting the relationship among three or more variables (Johnson and Wicher, 1992). Developed methods are based on the principle that accounts for all the variables involved in the process but gives a single value from which process

conditions can be analyzed. Nuehard (1987) proposed a method for calculating capability indices for multivariate processes in which the variance is adjusted for correlation by multiplying it by a factor, and then the adjusted variance is used to calculate the indices. Hubele et al. (1991) discussed the disadvantage of using univariate capability index and the advantage of the bivariate process capability vector. They considered the bivariate normal distribution and analyzed the process for its capability.

### **2.3.2.1 Process Capability in Tolerance Allocation Research**

The primary consideration in the tolerance allocation research area is the relationship between assembly function and components tolerances. Consideration of cost is another important issue. A third important factor, which does not receive appropriate attention in research, is the machining requirements needed to perform specific production jobs. Only in the last few years have some tolerance allocation models been used the machine capability ratios to reflect the need for specific machining requirements.

To consider the process capability issue, Wei and Lee (1995) formulated the tolerance allocation problem as an LP model based on a standardized process tolerance. The standardized tolerance for operation  $i$  is equal to the process tolerance of operation  $i$  divided by the estimated standard deviation of operation  $i$ . This model generates higher operational tolerances than Ngoi's (1992) LP tolerance allocation model, which reflects the importance of process capability consideration. Adopting the same idea of standardized tolerance, Wei and Lee (1998) proposed a nonlinear programming model composed of a cumulative standard normal probability function and manufacturing cost to allocate process tolerances. The proposed model minimized the total manufacturing loss.

A comparison between the results obtained by this new formulation and other methods indicated that this model could obtain higher allocated operational tolerances.

Feng and Balusu (1999) compared the application of various PCRs ( $C_p$ ,  $C_{pk}$ , and  $C_{pm}$ ) based on a numerical example of non-linear tolerance synthesis. They used the Taguchi quality loss function together with the manufacturing cost as the objective function. Different numerical values of the customer quality loss constant (variable A in equation (2.2) above) were also used. The combined results of A and the PCR indicated that the value of A has a larger impact on the tolerance design than the PCR. However, this conclusion is based only on one numerical example. Further case studies are needed in order to generalize this statement.

## CHAPTER 3

### THE MATHEMATICAL MODELING

#### 3.1 Problem Definition

Analysis of the process plan for discrete part manufacturing has been traditionally carried out using the tolerance charting technique. This is a manual and cumbersome process making it infeasible for applications involving a large number of operations. Researchers have attempted to overcome this problem by modeling the production process as an optimization model. Different models have been developed under various objective functions and constraints. Typically, these models are used to calculate operational tolerances such that the final product is produced within design specifications. Input to these models is usually a sequence of operations required to complete the part with each operation having associated processing equipment with certain capability.

All the existing operational tolerance allocation models incorporate specific assumptions under which they are applicable. For example, these models assume that processes are centered on their nominal values all the time. Therefore, to allocate bilateral operational tolerances, these models require an allocated tolerance of operation  $i$  ( $t_i$ ) to be higher than the lower process capability (LPC) value of operation  $i$ , i.e., these models use  $t_i \geq LPC_i$  as the capability process constraint. In fact, representing the process capability by a single LPC value disregards the stochastic nature of the process capability. The LPC value represents only the worst-case performance of a process. Generally, adopting the LPC single value to represent the process capability ignores the possibility of having a

shifted-mean process, forces the models to select more accurate machines resulting in higher cost, and may increase the type I error with respect to the feasibility of process plans.

Another deficiency with existing tolerance allocation models is the lack of product value consideration. None of the existing models place emphasis on the value of the product that is being produced. A product value is a function of the product's revenue, the completed percentage of the product's cycle time, the value-added processes through which the product has gone, and the criticality of the product characteristics. High-value products should be assigned to more capable processes in order to increase the confidence level in their production to design specifications.

A third issue with the traditional operational tolerance allocation models results from the assigning of a single machine to each operation required to complete a part. Usually, the assigned machines are those with the lowest capability so that maximum tolerance can be allocated for each operation. This approach results in suboptimality. In addition, it results in inflexibility since the machines are fixed and there is no provision for choosing a different machine in case the assigned machine results in an infeasible process plan or the machine is not available due to scheduling conflicts.

In this research, a simultaneous model for allocating operational tolerances and assigning machines is presented. Two versions of the model are introduced, stochastic and fuzzy. The stochastic version of the model captures the actual stochastic nature of process capability. In this method, a stochastic distribution, characterized by its mean and standard deviation, is used to represent the capability of a process. Alternately, the fuzzy version of the model evaluates the process capability utilizing the expert's knowledge. In

this version, based on the expert opinion, each process capability is formulated as a linguistic variable that has a fuzzy nature. Membership functions are used to represent the various levels of the linguistic process capability. Both proposed versions introduce flexibility and optimality to the mathematical model for the production process by considering all available machines for each type of operation. This helps in selecting the machine with the lowest capability possible to make the process plan feasible while allocating maximum tolerances to each operation. Additionally, the effects of product value on operational tolerance allocation and machine assignment are integrated with the proposed models.

The next section explains the probabilistic process capability concept, the expert-based process capability evaluation, and the product value definition. The two stochastic and fuzzy versions of the operational tolerance allocation and machine assignment model are presented in section three.

### **3.2 Description of the Model's Components**

#### **3.2.1 Probabilistic Process Capability**

The proposed model uses a stochastic distribution to represent the capability of a process. The normal distribution is used here; however, the methodology is generic and can be adapted to any other stochastic distribution.

The probability that a continuous normal random variable ( $x$ ) is higher than a value  $x_1$  and lower than a value  $x_2$  can be obtained by the cumulative normal distribution function as

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx \quad (3.1)$$

For a process with multiple operations, assume that the operations outputs follow a normal distribution with mean ( $m$ ) and standard deviation ( $\sigma$ ). Then, the probability for an operation to produce the required operational tolerance  $t_i$  is used in the process capability constraint for that operation. For example, if the required probability for operation  $i$  to generate the specified operational tolerance  $t_i$  is 95%, then the constrained area under the cumulative standard normal distribution for operation  $i$  should be higher than or equal to 0.95. For this reason, the risk level  $\alpha$  is used as a tool to assign higher preference for some operations based on the expert's decision. That is, a small  $\alpha$ -value would be assigned for high critical operations while a large  $\alpha$ -value would be assigned for less critical operations. As a result, the process capability constraint for any operation can be formulated as

$$\int_{-\infty}^{t_i} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx \geq 1 - \alpha_i \quad (3.2)$$

According to equation (3.2), assume that for a particular operation the  $\alpha_i$  value is set equal to 0.01, then the probabilistic process capability means that the probability of that operation to satisfy the required tolerance  $t_i$  is equal to or higher than 0.99. As can be seen, to increase the confidence that an operation will meet the requirements, a low  $\alpha$  should be assigned to that operation. On the other hand, low critical operations can be coupled with higher  $\alpha$  values. How critical an operation is and, accordingly, the assigned  $\alpha$ -value for that operation can be determined based on the expert's opinion.

The above probabilistic process capability representation is more accurate than the current LPC-approach used in the existing tolerance allocation models. Hence, it is anticipated that this new formulation would produce more realistic results than those

generated by the existing models. In fact, higher operational tolerances are expected from the suggested formulation since the process capability is not any more represented by its worst case performance. Furthermore, since the mean value ( $m$ ) is a factor in the suggested formulation, the actual process performance including any shift, if existed, can be implemented.

### **3.2.2 Experts-Based Fuzzy Process Capability**

Both the single-value process capability method and the probabilistic process capability method need a considerable amount of previous data. In many cases the process of data collection can be very difficult. Moreover, it can contain various types of errors since it relies on operator experience and measuring device accuracy. It also requires time and adds to the cost of the process capability analysis. Analyzing the process capability by means of fuzzy logic will overcome these disadvantages. Fuzzy logic is used to capture the expert's knowledge, and offers an easier and cheaper of analyzing process capability.

Typically, an expert verbally evaluates the process capability. For example, the capability of a process to perform a particular task can be fully evaluated by one of three descriptions: capable, semi-capable, or quite capable. At the borders of each evaluation, the process would be partially capable, partially semi-capable, or partially quite capable.

Membership functions are used to present the expert's verbal evaluations in a solvable way. Figure 3.1 shows an example of a trapezoidal process capability membership functions. The x-axis represents a dimensionless evaluation scale, while the y-axis represents the value of the membership function. The scale ranges from 1 to 100. Value 1 represents the highest process capability level, while a value of 100 represents



the lowest process capability level. A fuzzy process capability (FPC) evaluation of 15, for example, means a fully capable process, while an evaluation of 65 means a partial capable/semi-capable process.

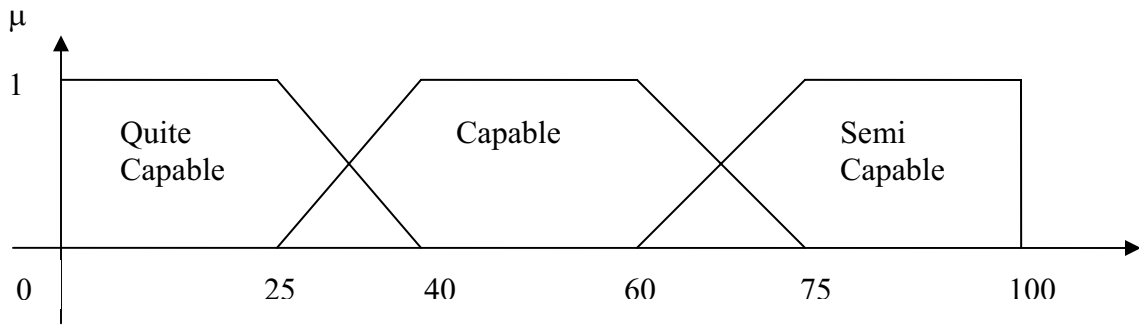


Figure 3.1. A trapezoidal process capability membership functions.

Based on Zimmerman’s fuzzy programming approach explained in section 2.1.4.5, the fuzzy process capability constraint will look as

$$\lambda \leq 1 + \frac{FPC_i}{FR_i} - \frac{t_i}{FR_i} \quad (3.3)$$

where

$FPC_i$  is the expert-based fuzzy process capability

$FR_i$  is the fuzziness range

In this formulation an experts assessed value for the process capability, which is FPC, is adopted. Furthermore, a fuzziness range (FR) is associated with each FPC. The expert can be very sure about his evaluation, i.e., he would provide an evaluation with a full membership function. Hence, the FR associated with this evaluation would be very

low. On the other hand, if a partial membership evaluation is provided, the associated FR value would be equal to the distance from the point where the fuzzy membership function has a full membership value to the point where the value of the membership function becomes zero. That is, instead of using the ordinary LPC to represent the process capability constraint, a value based on the expert's assessment, which is FPC, is adopted.

The above fuzzy-process capability representation provides a valid alternative for the ordinary LPC approach. In fact, this representation eliminates the drawbacks that exist with both the LPC and probabilistic process capability analysis. Yet, the efficiency of this approach highly depends on the accuracy of the acquisition and representation of the expert's knowledge.

### **3.2.3 "In-production" Products Value**

The value of a product can be viewed from various perspectives. Accordingly, it can be defined differently. For example, in marketing, the value of a product is the consumer's expectations of product quality in relation to the actual amount paid for it. It is often expressed by the following equations:

$$\text{Value} = \text{Quality received} / \text{Price, or}$$

$$\text{Value} = \text{Quality received} / \text{Expectations}$$

On the other hand, in neoclassical economics, the value of an object or service is often seen as nothing but the price it would bring in an open and competitive market (Stretton, 1999). This is determined primarily by the demand for the object relative to supply. In classical economics, price and value are not seen as equal. According to Keen (2001), value refers to "the innate worth of a commodity, which determines the normal ratio at

which two commodities exchange.” Ludwig von Mises (1949) asserted that "value" is always a subjective quality. That there is no value implicit in objects or things and that value is derived entirely from the psychology of market participants. Neap and Celik (1999) emphasized that value reflects the owner(s)/buyer(s) desire to retain or obtain a product, and this introduces subjective aspects to the value of a product.

The above product’s value definitions are associated with the final shape of a product in which the customer is the one who perceived the product and evaluated it. The current research views the product’s value from another perspective, that is, the in-production value of a product. According to these two perspectives, the factors that affect the value of a product can vary. For example, the final customer perception of product value is the combined result of the product price, delivery time, and product quality—considering the eight dimensions of quality (performance, features, reliability, conformance, durability, serviceability, aesthetics, and perceived quality) (Garvin, 1984). In contrast, the factors that contribute to the product value while the product is still in production can be summarized as follows: the product’s revenue, the percentage of the product’s completed-cycle time, and the product criticality. The definitions and formulation of these factors follow.

#### **3.2.3.1 Product’s Revenue (R)**

The net profit generated by a product is essential in making the production decision for this product. The profit is a function of the total production cost of a product and the selling price for that product. Traditionally, a product’s revenue can be formulated either by subtracting the total cost from the selling price or by using the ratio of the selling price/total cost.

Another way to analyze the revenue factor is by using the thought process of the Theory of Constraints (TOC) (Goldratt, 1990). The TOC proposes that the total throughput of a system is only affected by the throughput produced by the system's constraint. Because the production time of the constraint is limited, job scheduling for the constraint should be carefully performed in order to generate maximum throughput per the constraint's unit time. In addition, products which consume a large portion of constraint time should be assigned a higher value than products which use less constraint time.

Constraint Product Value (CPV) is a way to compare products based on their revenue contribution generated by a factory constraint. It combines the product's material cost and the product's throughput rate (revenue/unit time) on the constraint (Witte and Ashline, 1997). CPV for a given product can be defined as

$$CPV_{\text{product}} = \left( CS_{\text{product}} * \left( \frac{ATP}{CS} \right)_{\text{constraint}} \right) + MC_{\text{product}} \quad (3.4)$$

where:

$CPV_{\text{product}}$  is the constraint product value for a product (\$)

$CS_{\text{product}}$  is the constraint's occupation time for one unit of a product (sec/unit)

$\left( \frac{ATP}{CS} \right)_{\text{constraint}}$  is the average throughput per constraint time for all products on the constraint (\$/sec)

$MC_{\text{product}}$  is the material cost of the product (\$/unit)

To find the time required for one yielded product on the constraint ( $CS_{\text{product}}$ ), the production rate of the product in the constraint, typically given in units per hour, is needed. For a product that visits the constraint one time,  $CS_{\text{product}}$  in seconds is given by

$$CS_{\text{product}} = \frac{3600}{\text{Production rate (in units per hr)}} \text{ (sec/unit)} \quad (3.5)$$

For the calculation of the average throughput per constraint time for the constraint being analyzed, i.e.,  $\left(\frac{ATP}{CS}\right)_{\text{constraint}}$ , the  $ATP_{\text{constraint}}$  and the  $CS_{\text{constraint}}$  are calculated as follows:

$$ATP_{\text{constraint}} = \sum_i^k (ASP_i - MC_i) \times D_i \quad (3.6)$$

where

$ATP_{\text{constraint}}$  is the average throughput for the constraint (\$/unit)

$k$  is the number of products produced by the analyzed constraint

$MC_i$  is the material cost of the product (\$/unit)

$ASP_i$  is the average selling price for a product I (\$/unit)

$D_i$  is the sold demand of product i

Also,  $CS_{\text{constraint}}$  can be found as

$$CS_{\text{constraint}} = \sum_i^k CS_{\text{product}(i)} \times D_i \quad (3.7)$$

Once the  $ATP_{\text{constraint}}$  and the  $CS_{\text{constraint}}$  are calculated separately, the  $\left(\frac{ATP}{CS}\right)_{\text{constraint}}$  can be found as

$$\left(\frac{ATP}{CS}\right)_{\text{constraint}} = \frac{ATP_{\text{constraint}}}{CS_{\text{constraint}}} \quad (3.8)$$

Finally, a revenue ratio using the CPV method can be written as

$$PV_R = \frac{ASP}{CPV} \quad (3.9)$$

Essentially, this ratio represents a tool for a product's value considering the profit generated by the product with respect to its CPV.

### 3.2.3.2 Percentage of Completed Cycle Time (%CT)

The cycle time of a given product, also called flow time, refers to the average time a product spends as work in process (WIP), or it is the average time from the release of a job at the beginning of a product's route until it reaches the point of finished product inventory (Hopp and Spearman, 1999). Considering the percentage of finished cycle time toward defining a product's value is very reasonable. For example, for the same product, a unit with a 90% finished cycle time is more valuable than a unit with a 10% finished cycle time. However, for two different products, 10%-finished cycle time of product A could have a higher value than a 90%-finished cycle time of product B. Therefore, in a situation where multiple products are considered for production, both the percentage of finished cycle time and the product's total cycle time should be considered. For the total cycle time consideration, a normalization factor of the considered product with respect to all other produced products is presented in the formula for the product's value.

During the trip from raw material to a finished product the product, undergoes many processes. Although all the process times are counted in the cycle time, many of these stages do not add any extra value to the product. In the lean manufacturing terminology these stages are labeled as waste. WIP inventory is an example of a non-value added process. Processes which add to the value of a product are called value-added processes. As a result, only value-added processes should be considered in the calculation of a product's value.

Finally, a formula that considers the contributions of percentage of completed cycle time, total cycle time, and value-added process to the product's value is suggested as

$$PV_{CT} = \left( 1 + \frac{CT_C}{CT_p} \times \%VAP \times R_p \right) \quad (3.10)$$

where:

$CT_C$  is the amount of completed product's cycle time

$CT_p$  is the product's total cycle time

$\%VAP$  is percentage of value added processes in  $CT_C$

$R_p$  is a linear normalization factor for the considered product.  $R_p$  can be calculated from

$$R_p = \frac{CT_p - CT_{\min}}{CT_{\max} - CT_{\min}} \quad (3.11)$$

where

$CT_{\min}$  is the minimum cycle time among all produced products

$CT_{\max}$  is the maximum cycle time among all produced products

### 3.2.3.3 Product Criticality (PC)

A product criticality is related to the classifications of product characteristics. A characteristics classification is applied usually during the first stages of product and process design but because of manufacturability, quality, or handling considerations, the classifications may be upgraded by suppliers, the internal process, installation, or quality planners. A typical product characteristics classification can be as follows (Gaplan, 1990):

- Critical: when a small deviation will produce or lead to a substantial safety hazard or a complete performance loss.
- Major: when a small deviation will produce or lead to some safety hazard, significant performance or reliability reduction, or complete loss of further manufacturability.
- Minor: when a small deviation may produce or lead to minimal safety hazard, some performance or reliability reduction, or substantial manufacturability problems.
- Incidental: when a small deviation cannot produce or lead to any safety hazard, performance, or reliability reduction but may cause minimal manufacturability problems.

These classifications are often distinguished by reference to measures of process capability, such as  $C_p$  or  $C_{pk}$ . A typical approach might have the critical characteristic classes assigned to a higher than 1.33 PCR processes, major characteristics assigned to



1.33 PCR processes, minor characteristics assigned to 1 to 1.33 PCR processes, and incident characteristics products assigned to a 1.00 PCR processes. In the current research, it is assumed that the criticality classification of the analyzed product is known. Furthermore, the associated PCR ratio of this product is used as the criticality contribution to its value, notated by  $PV_{PC}$ .

### 3.2.3.4 The Proposed PV Formula

Based on the previous discussion, a formula for in-production product value that summarized the effects of the revenue, percentage of completed cycle time, and product's criticality can be given as

$$\text{Product Value (PV)} = [w_1 \ w_2 \ w_3] \cdot \begin{bmatrix} PV_R \\ PV_{CT} \\ PV_{PC} \end{bmatrix} \quad (3.12)$$

where  $w_1$ ,  $w_2$ , and  $w_3$  are weights of the revenue, percentage of completed cycle time, and product's criticality, respectively.

It is worth mentioning here that the suggested PV formula would return a dimensionless number. Actually, this will facilitate the integration of this value with the suggested tolerance allocation/machine assignment model, as shown in the next section.

## 3.3 The Integrated Mathematical Model

The proposed mathematical model is a non-linear mixed integer model which has been developed with the main objective of allocating maximum tolerances to each of the operations by assigning the most economical machine possible to the operations. Two versions of the model are introduced—stochastic and fuzzy. The stochastic version of the model considers a probabilistic process capability, while the fuzzy version considers a

fuzzy process capability. Both proposed versions integrate the effects of the product value.

The mathematical models are developed for the following two cases:

1. One machine per process set: All operations in a process set (defined as the type of processes that a product may go through) are carried out by one machine. For example, a set of grinding operations are carried out by a single machine. This procedure minimizes the amount of loading/unloading and hence the setup time. Yet, since a single machine is used for all the operations, assigned tolerances can be tighter.

2. One machine per operation: Operations within a process set can be carried out on different machines. Therefore, the multiple operations of a grinding set may be carried out by different machines. Since multiple machines are available for a set of operations, larger tolerances than the “one machine per process set” case can be assigned.

Before introducing the mathematical formulation, the notations used are defined as follows:

- S: number of process sets in a process plan
- f: index of process sets;  $f = 1, \dots, s$
- $mc_f$ : number of machines available for operations in process set f
- g: index of machines available,  $g = 1, \dots, mc_f$
- $H_f$ : number of operations in process set f
- h: index of operations in process set f;  $h = 1, \dots, H_f$
- $ASD_{fg}$ : actual standard deviation of machine g assigned to process set f
- k: number of blueprint tolerances and stock removal requirements
- $DC_j$ : dimensional chain for BP tolerance or stock removal requirement j
- j: index of blueprint tolerances and stock removal requirements

- $t_{fh}$ : operational tolerance of operation h in process set f
- $BP_j$ : blueprint tolerance or stock removal requirement j
- $m_{fh}$ : mean of operation h in process set f
- $\sigma_{fh}$ : required standard deviation of operation h in process set f
- $\alpha_{fh}$ : risk factor of operation h in process set f
- $PV_{fh}$ : product value at operation h in process set f
- $\lambda$ : dummy variable
- OBJ: value for the objective function of the non-fuzzy version of the model
- $FR_{OBJ}$ : fuzziness range for the OBJ
- $FR_{BP_j}$ : fuzziness range for the  $BP_j$
- $FSD_{fg}$ : fuzzy standard deviation
- $FR_{FSD_{fg}}$ : fuzziness range for the  $FSD_{fg}$

### 3.3.1 Stochastic Version of the Model

#### Case I: One Machine per Process Set

The objective function consists of the following two components:

$$1. \quad \text{Max} \left\{ \text{Min}_f \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg} \right\} \quad (3.13)$$

where

$$X_{fg} = \begin{cases} 1, & \text{if machine } g \text{ is selected for operations in process set } f \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{g=1}^{mc_f} X_{fg} = 1; \quad f=1, \dots, S \quad (3.14)$$

The first part of the objective function selects, among available machines, those with the highest spread (standard deviation) that can successfully complete the part. The result would be the allocation of larger operational tolerances. A machine with the highest standard deviation means a machine with the lowest process capability. Assigning machines with lower capability implies lower manufacturing cost since the manufacturing cost is higher on a machine with a lower capability than on a machine with a higher capability. Furthermore, for this assignment, machines are assumed to be identical in terms of their state or in terms of scheduling, except that some are more capable in holding tolerances than others. Other parameters like speed, feed rate, or depth of cut are not considered, nor are fixtures requirements/conflicts.

The above objective function can be re-stated in as

$$\text{Maximize } \sum_{f=1}^S a_f \quad (3.15)$$

subject to:

$$\sum_{g=1}^{mc_f} ASD_{fg} * X_{fg} \geq a_f ; f = 1, \dots, S \quad (3.16)$$

2. The second part of the objective function treats standard deviation as a decision variable. It tries to determine the minimum required standard deviation ( $\sigma_{fh}$ ) that is needed if operation h in process set f is to produce a dimension within specifications. Once calculated,  $\sigma_{fh}$  values can be compared to the actual standard deviation ( $ASD_{fg}$ ) of the machines available for process set f to determine whether or not they are capable of holding the specified tolerances. An index defined as the ratio of the minimum required

$\sigma_{fh}$  over the  $ASD_{fg}$  is introduced to achieve this comparison. The objective is to maximize this ratio as

$$\text{Max} \left[ \text{Min} \left( \frac{\sigma_{fh}}{\sum_{g=1}^{mc_f} ASD_{fg} * X_{fg}} \right) \right] ; f = 1, \dots, S; h = 1, \dots, H_f \quad (3.17)$$

An equivalent representation of (3.17) is

$$\text{Maximize } Z \quad (3.18)$$

subject to:

$$\left[ \frac{\sigma_{fh}}{\sum_{g=1}^{mc_f} ASD_{fg} * X_{fg}} \right] \geq Z ; \quad f = 1, \dots, S; h = 1, \dots, H_f$$

or

$$\sigma_{fh} - (Z) * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg} \geq 0 ; f = 1, \dots, S; h = 1, \dots, H_f \quad (3.19)$$

After defining the objective function, the constraints are as described as follows:

1. The tolerance chain identification constraint for the BP tolerances and stock removals requirements written as

$$\sum_{t_m \in DC_j} t_{fh} \leq BP_j, \quad j = 1, 2, \dots, k; \quad f = 1, \dots, S; \quad h = 1, \dots, H_f$$

It is not the intention of this research to introduce a new method for the tolerance chain identification, therefore, the tolerance chain identification approach presented in Cheraghi et al. (1999) will be used here.

2. The probabilistic process capability constraint is assumed to follow the normal distribution. This constraint is written as a function of the mean and standard deviation of each operation, which will be among the model outputs. This constraint is written as

$$\int_{-\infty}^{t_{fh}} \frac{1}{\sigma_{fh} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - m_{fh}}{\sigma_{fh}} \right)^2} dx > 1 - \alpha_{fh} \quad f = 1, \dots, S; h = 1, \dots, H_f$$

3. The product value effect constraint. High-value products should be assigned to more capable processes in order to increase the confidence level in their production to design specifications. Hence, it is rational for any operational tolerance allocation model to consider the product value. The next constraint provides a way for the product value to be integrated with the proposed model. This constraint is written as

$$\frac{t_{fh}}{3 * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg}} \geq PV_{fh} \quad f = 1, \dots, S; h = 1, \dots, H_f$$

The  $\frac{t_{fh}}{3 * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg}}$  ratio is actually a process capability ratio (PCR) whose value in

the above constraint is based on the product value. It is known that as the PCR value increases, the expected rejection rate decreases. As a result, this constraint assures lower rejection rate probability as the product value increases. Furthermore, since the model is considering the change of product value through the different stages of the production

process, it is expected that more capable processes would be assigned to the product while it has higher values.

Ultimately, the full model can be written as

$$\text{Maximize } \sum_{f=1}^S a_f + Z \quad (3.20)$$

subject to

$$\sum_{g=1}^{mc_f} ASD_{fg} * X_{fg} \geq a_f \quad (3.21)$$

$$X_{fg} = \begin{cases} 1, & \text{if machine } g \text{ is selected for operations in process set } f \\ 0, & \text{otherwise} \end{cases} \quad (3.22)$$

$$\sum_{g=1}^{mc_f} X_{fg} = 1 \quad (3.23)$$

$$\sigma_{fh} - (Z) * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg} \geq 0 \quad (3.24)$$

$$\sum_{t_{fh} \in DC_j} t_{fh} \leq BP_j \quad (3.25)$$

$$\int_{-\infty}^{t_{fh}} \frac{1}{\sigma_{fh} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - m_{fh}}{\sigma_{fh}} \right)^2} dx > 1 - \alpha_{fh} \quad (3.26)$$

$$\frac{t_{fh}}{3 * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg}} \geq PV_{fh} \quad (3.27)$$

$$t_{fh} > 0 \quad (3.28)$$

$$\sigma_{fh} > 0 \quad (3.29)$$

$$j = 1, \dots, k ; f = 1, \dots, S ; h = 1, \dots, H_f$$

Equation (3.20) states the objective of the model. The selection of the least capability machines is expressed as “ $a_f$ ” and written as constraint in equation (3.21).  $X_{fg}$  is a decision variable with a binary value that is used to choose the most feasible machine possible for each operation. Only one machine must be selected for each operation set. This requirement is built into equations (3.22) and (3.23). The ratio of the minimum process capability required to the actual process capability is expressed as “ $Z$ ” as shown in constraint (3.24). Equation (3.25) restricts the tolerance stack-up in the dimension chains for the BP tolerance requirements and stock removal limits. Equation (3.26) represents the probabilistic nature of the process capability. Finally, equation (27) considers the product value effect.

### **Case II: One Machine per Operation**

In the second case, each operation in a set can be assigned to any one of the machines available in that set. The advantage of this is that higher tolerances can be assigned to the operations since the most economical machine is chosen for each operation. On the other hand, a higher number of setups is required for this method than in case I.

A mathematical model has been developed for this case. The notations used are the same as in the previous case, except that the decision variable  $X_{fg}$  is replaced by the decision variable  $X_{fhg}$  in the mathematical model. The reason behind using an additional subscript h is that the selection of a machine g is done for operation h in set f unlike the previous case where the selection of a machine g was done for all the operations in set f. The mathematical model for this case is written as



$$\text{Maximize } \sum_{f=1}^S \sum_{h=1}^{H_f} a_{fh} + Z \quad (3.30)$$

subject to

$$\sum_{g=1}^{mc_f} ASD_{fg} * X_{fhg} \geq a_{fh} \quad (3.31)$$

$$X_{fhg} = \begin{cases} 1, & \text{if machine } g \text{ is selected for operation } h \text{ in process set } f \\ 0, & \text{otherwise} \end{cases} \quad (3.32)$$

$$\sum_{g=1}^{mc_f} X_{fhg} = 1 \quad (3.33)$$

$$\sigma_{fh} - (Z) * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fhg} \geq 0 \quad (3.34)$$

$$\sum_{t_{fh} \in DC_j} t_{fh} \leq BP_j \quad (3.35)$$

$$\int_{-\infty}^{t_{fh}} \frac{1}{\sigma_{fh} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - m_{fh}}{\sigma_{fh}} \right)^2} dx > 1 - \alpha_{fh} \quad (3.36)$$

$$\frac{t_{fh}}{3 * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fhg}} \geq PV_{fh} \quad (3.37)$$

$$t_{fh} > 0 \quad (3.38)$$

$$\sigma_{fh} > 0 \quad (3.39)$$

$$j = 1, \dots, k ; f = 1, \dots, S ; h = 1, \dots, H_f$$

### 3.3.1.1 Analysis of Model Outputs

Outputs from the presented mathematical model are values for  $Z$ ,  $X_{fg}$  ( $X_{fhg}$ ),  $t_{fh}$ ,  $m_{fh}$ , and  $\sigma_{fh}$ . Analysis of these values facilitates decision making in the following ways:

1. Deciding if the process plan is feasible can be done by analyzing the value of  $Z$ , which will be discussed later.
2. If the process plan is feasible,  $X_{fg}$  ( $X_{fhg}$ ), values determine the machine to be used to carry out each operation.
3. If the process plan is feasible,  $t_{fh}$  values determine operational tolerances.
4. If the process plan is not feasible, then the following apply:
  - a. Comparison of  $\sigma_{fh}$  and ASD values identify the operations/machines that cause the plan to be infeasible.
  - b.  $\sigma_{fh}$  values indicate the minimum machine standard deviation that is needed to make the plan feasible.
  - c.  $m_{fh}$  values indicate the maximum machine shift that can be used without violating the feasibility of the process plan.

### 3.3.1.2 Analysis of the Z Index

The ratio “Z” is referred to as the “feasibility index” since its value determines whether a process plan is feasible or not. The following set of rules is followed to determine if a process plan is feasible:

- a. If  $Z \geq 1$ , from equation (3.24),

$$\sigma_{fh} - (Z) * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg} \geq 0 ; f = 1, \dots, S; h=1, \dots, H_f;$$

when the minimum standard deviation requirement, determined by the model for each operation, is greater or equal to  $(Z) * \sum_{g=1}^{mc_f} ASD_{fg} * X_{fg}$ , and since  $Z \geq 1$ , the minimum standard deviation required is greater than or equal to the actual standard deviation of the machine. This means that the machines selected have higher accuracy than the required accuracy value. Since the minimum standard deviation requirements are based on the tolerance requirements of the operations, the  $Z$  values determine that the tolerance requirements are satisfied by the machines selected and hence the process plan is feasible.

b. If  $Z < 1$ , then according to equation (3.24), the ASD value for at least one operation is higher than the minimum required standard deviation, which means that a more accurate machine is needed and the current plan is infeasible. The machines/operations that have caused the infeasibility can be identified as follows:

- For every operation  $h$  in process set  $f$ , compare  $\sigma_{fh}$  with the  $ASD_{fg}$  value of the machine selected for that process set.
- If for a given process set  $i$ ,  $\sigma_{fh} < ASD_{fg}$  for some  $h=h^*$  and  $g=g^*$ , then machine  $g^*$  is not capable of meeting the specification for operation  $h^*$  in process set  $i$ .

In order to obtain a satisfactory process plan, the machine/s that had have an actual standard deviation that is greater than the minimum standard deviation required ( $\sigma_{fh}$ ) should be replaced with a machine whose capability is at least equal to the minimum standard deviation required. Otherwise, one or more of the following actions should be taken.

- The current process plan should be replaced by a new one.
- The BP specifications should to be revised.

- In-production product values ( $PV_{fh}$ ) should be re-evaluated.
- Different preferences for operations ( $\alpha_{fh}$ ) should be assigned.

### 3.3.2 Fuzzy Version of the Model

As discussed in section 2.1.4.5, Zimmerman (1976) proposed to soften the rigid requirements of the decision maker (DM) to strictly minimize the objective function and strictly satisfy the constraints. Namely, by considering the impression or fuzziness of the DM's judgment, he softened the usual linear programming problem into a fuzzy linear programming problem. It was also shown in section 2.1.4.5 that an ordinary maximization LP problem with fuzzy objective and constraints can be expressed as the following (see section 2.1.4.5 for notation definitions):

$$\text{maximize } \lambda \tag{3.40}$$

subject to

$$\lambda \leq 1 + b_i - (B'x)_i, i = 0, 1, \dots, m$$

$$x \geq 0$$

The same procedure is adopted here to modify the two models presented in the previous section. That is fuzzy versions of the above two models are presented based on Zimmerman's approach. In this approach both the objective function and the constraints should be considered as fuzzy variables. Yet, for our application, the fuzziness is only related to the process capability constraints. Therefore, except for the process capability constraints, all other components in the suggested model will be allowed to vary in a very narrow range. This enables the use of Zimmerman's procedure with only one fuzzy variable, which is the process capability.

The mathematical formulation for “one machine per process set” and “one machine per operation” are given below.

**Case I: One Machine per Process Set**

The fuzzy version of the model presented in equations (3.20 to 3.29) is given as

$$\text{Maximize } \lambda \tag{3.41}$$

subject to

$$\lambda \leq 1 + \frac{\text{OBJ}}{\text{FR}_{\text{OBJ}}} - \frac{\sum_{f=1}^S a_f + Z}{\text{FR}_{\text{OBJ}}} \tag{3.42}$$

$$\lambda \leq 1 - \frac{\sum_{g=1}^{\text{mc}_f} \text{FSD}_{fg} * X_{fg}}{\text{FR}_{\text{FSD}_{fg}}} + \frac{a_f}{\text{FR}_{\text{FSD}_{fg}}} \tag{3.43}$$

$$\lambda \leq 1 + \frac{\sum_{g=1}^{\text{mc}_f} \text{FSD}_{fg} * X_{fg}}{\text{FR}_{\text{FSD}_{fg}}} - \frac{\sigma_{fh}/Z}{\text{FR}_{\text{FSD}_{fg}}} \tag{3.44}$$

$$\lambda \leq 1 + \frac{3 * \sum_{g=1}^{\text{mc}_f} \text{FSD}_{fg} * X_{fg}}{\text{FR}_{\text{FSD}_{fg}}} - \frac{t_{fh}/\text{PV}_{fh}}{\text{FR}_{\text{FSD}_{fg}}} \tag{3.45}$$

$$\lambda \leq 1 + \frac{\text{BP}_j}{\text{FR}_{\text{BP}_j}} - \frac{\sum_{t_{fh} \in \text{DC}_j} t_{fh}}{\text{FR}_{\text{BP}_j}} \tag{3.46}$$

$$\sum_{g=1}^{\text{mc}_f} X_{fg} = 1 \tag{3.47}$$

$$X_{fg} = \begin{cases} 1, & \text{if machine } g \text{ is selected for operations in process set } f \\ 0, & \text{otherwise} \end{cases} \tag{3.48}$$

$$t_{fh} > 0 \tag{3.49}$$

$$\tag{3.50}$$

$$\sigma_{fh} > 0$$

$$j = 1, \dots, k ; f = 1, \dots, S ; h = 1, \dots, H_f$$

In this formulation, the objective function of maximizing  $\lambda$  is equivalent to maximizing the objective function shown in equation (3.20). The OBJ variable, shown in equation (3.41), is equal to the objective function value that resulted from solving the model presented in equations (3.20 to 3.29). This OBJ value is formulated to be fuzzy with a range of  $FR_{OBJ}$ . The fuzzy range (or range of fuzziness) is defined as the distance from the point where a fuzzy membership function has a full membership value (which is the point of the OBJ value in this model) to the point where the value of the membership function becomes zero. Similar to the objective function formulation, BP requirements are presented as fuzzy variables with a fuzzy range of  $FR_{BPj}$ , as shown in equation (3.45). In solving this model, both  $FR_{OBJ}$  and the  $FR_{BPj}$  are assigned a very small value. That is, their fuzziness range is very small. In other words, they are formulated as fuzzy, but behaving as crisp variables.

Equations (3.42 to 3.44) include the experts-based FSD. As was discussed in section 3.2.2, a fuzzy process capability (FPC) can be generated based on the expert knowledge. Then the FSD is obtained as one sixth the FPC. According to the expert, the FSD is allowed to vary with a fuzzy range of  $FR_{FSD}$ . The value of the  $FR_{FSD}$  depends on the selected membership function along with its characteristics of core and boundaries.

Equations (3.44 to 3.49) have not changed from the original version. Finally, this model can be solved by using any LP optimization software package.

## Case II: One Machine per Operation

The fuzzy version of the model presented in equations (3.30 to 3.39) is given as

$$\text{Maximize } \lambda \quad (3.51)$$

subject to

$$\lambda \leq 1 + \frac{\text{OBJ}}{\text{FR}_{\text{OBJ}}} - \frac{\sum_{f=1}^S \sum_{h=1}^{H_f} a_{fh} + Z}{\text{FR}_{\text{OBJ}}} \quad (3.52)$$

$$\lambda \leq 1 - \frac{\sum_{g=1}^{\text{mc}_f} \text{FSD}_{fg} * X_{fhg}}{\text{FR}_{\text{FSD}}} + \frac{a_{fh}}{\text{FR}_{\text{FSD}}} \quad (3.53)$$

$$\lambda \leq 1 + \frac{\sum_{g=1}^{\text{mc}_f} \text{FSD}_{fg} * X_{fhg}}{\text{FR}_{\text{FSD}}} - \frac{\sigma_{fh}/Z}{\text{FR}_{\text{FSD}}} \quad (3.54)$$

$$\lambda \leq 1 + \frac{\text{BP}_j}{\text{FR}_{\text{BP}}} - \frac{\sum_{t_{fh} \in \text{DC}_j} t_{fh}}{\text{FR}_{\text{BP}}} \quad (3.55)$$

$$\lambda \leq 1 + \frac{3 * \sum_{g=1}^{\text{mc}_f} \text{FSD}_{fg} * X_{fhg}}{\text{FR}_{\text{FSD}}} - \frac{t_{fh}/\text{PV}_{fh}}{\text{FR}_{\text{FSD}}} \quad (3.56)$$

$$\sum_{g=1}^{\text{mc}_f} X_{fhg} = 1 \quad (3.57)$$

$$X_{fhg} = \begin{cases} 1, & \text{if machine } g \text{ is selected for operation } h \text{ in process set } f \\ 0, & \text{otherwise} \end{cases} \quad (3.58)$$

$$t_{fh} > 0 \quad (3.59)$$

$$\sigma_{fh} > 0 \quad (3.60)$$

$$j = 1, \dots, k; f = 1, \dots, S; h = 1, \dots, H_f$$

Except for the “one machine per operation” instead of “one machine per operation set,” this model is similar to the one presented in (3.41 to 3.50). Similarly, any LP software package can be used to solve this model.

### 3.3.2.1 Analysis of Model Outputs

Outputs from the presented fuzzy models are values for  $Z$ ,  $X_{fg}$  ( $X_{fhg}$ ),  $\sigma_{fh}$ , and  $t_{fh}$ . This version of the model lacks the  $m_{fh}$  output, which resulted from the stochastic version. Yet, the model is capable of deciding whether a process plan is feasible or not based on the value of the  $Z$  index. In fact, the part of the  $Z$  index analysis presented in section 3.3.1.2, where  $Z \geq 1$ , is also applicable to this fuzzy version of the model. Moreover, if  $Z \geq 1$ , i.e., the process plan is feasible,  $X_{fg}$  ( $X_{fhg}$ ) values determine the machine to be used to carry out each operation, and  $t_{fh}$  values determine the allocated operational tolerances. Finally, for a  $Z < 1$  output, one or more of the following actions can be taken:

- The current process plan should be replaced by a new one.
- The BP specifications need to be revised.
- In-production product values ( $PV_{fh}$ ) should be re-evaluated.
- The expert’s process capability evaluations need to be checked for any miss-estimations.



## CHAPTER 4

### IMPLEMENTATION AND TESTING

The applicability of the proposed modeling methodology is tested by applying it to an example part (steel sleeve) which is described in the next section. Testing is conducted in two stages. In the first stage, introduced in section two, the separate effects of probabilistic process capability, fuzzy process capability, and product value on the tolerance allocation problem are presented. The third section introduces the second testing stage in which both the stochastic and fuzzy versions of the proposed integrated model are implemented for the “one machine per operation” and the “one machine per operation set” cases. The chapter ends with conclusive remarks in section four.

#### **4.1 The Workpiece**

Figure 4.1 shows a steel sleeve example part which is used to test the proposed modeling methodology, (Ji et al., 1995). In addition, Figure 4.2 shows this same steel sleeve example with all its intermediate surfaces. Ten operations are carried out on the part. They are divided into five process sets: rough boring, rough turning, finish boring, finish turning, and grinding, as shown in Table 4.1. The term  $O_{fh}$  in the table represents operation  $f$  in process set  $h$ . The blueprint and the stock removal requirements are given in Table 4.2.

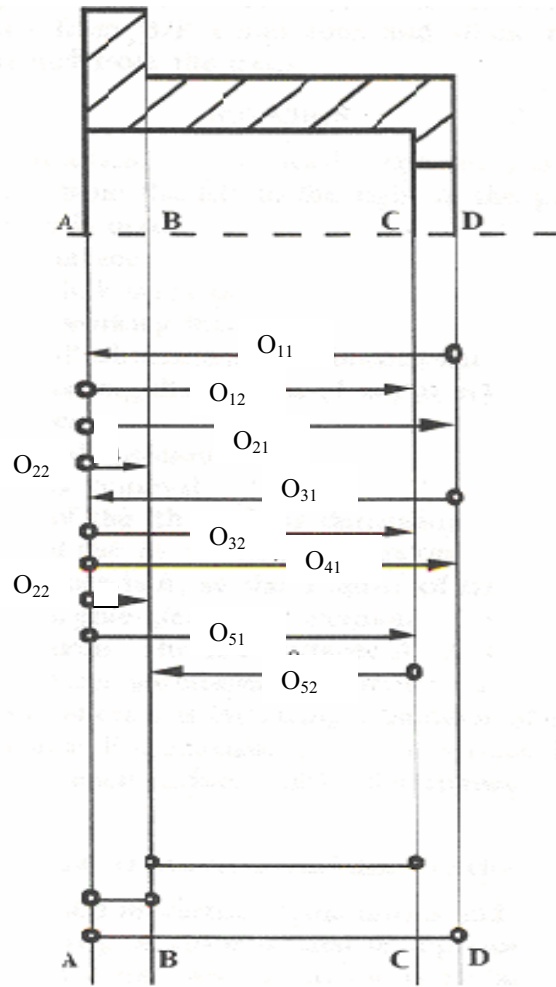


Figure 4.1. The workpiece (Ji et al., 1995).

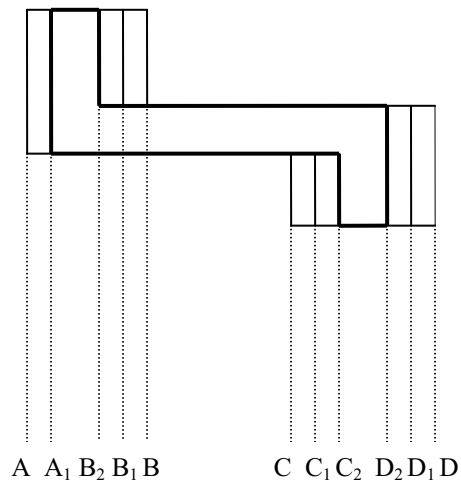


Figure 4.2. The workpiece with its intermediate surfaces

TABLE 4.1

OPERATION SETS FOR STEEL SLEEVE EXAMPLE

Set # (i)	Operations Set	Operation Number
1	Rough Boring	O <sub>11</sub>
		O <sub>12</sub>
2	Rough Turning	O <sub>21</sub>
		O <sub>22</sub>
3	Finish Boring	O <sub>31</sub>
		O <sub>32</sub>
4	Finish Turning	O <sub>41</sub>
		O <sub>42</sub>
5	Grinding	O <sub>51</sub>
		O <sub>52</sub>

TABLE 4.2

BP AND STOCK REMOVAL REQUIREMENTS FOR STEEL SLEEVE EXAMPLE

	End Surface	Mean	± Tolerance
Blue Print	B <sub>1</sub> C <sub>1</sub>	50.0	0.025
	A <sub>1</sub> B <sub>1</sub>	10.0	0.075
	A <sub>1</sub> D <sub>2</sub>	70.0	0.050
Stock Removal	D <sub>1</sub> D	3.00	0.300
	AA <sub>1</sub>	1.00	0.250
	CC <sub>1</sub>	1.00	0.500
	D <sub>2</sub> D <sub>1</sub>	1.00	0.150
	B <sub>1</sub> B	0.60	0.400
	C <sub>1</sub> C <sub>2</sub>	0.30	0.150
	B <sub>2</sub> B <sub>1</sub>	0.30	0.125

Execution of the suggested mathematical modeling requires the identification of a tolerance chain for each blueprint and stock removal (BP/SR) dimension. To derive the

tolerance chains, a method developed by Cheraghi et al. (1999) is utilized. The results of this method are given in Table 4.3.

Table 4.3  
TOLERANCE CHAIN IDENTIFICATIONS

		Tolerance Chain
Blue Print	$B_1C_1$	$t_{10}$
	$A_1B_1$	$t_9 + t_{10}$
	$A_1D_2$	$t_7$
Stock Removal	$D_1D$	$t_1 + t_3$
	$AA_1$	$t_3 + t_5$
	$CC_1$	$t_2 + t_3 + t_5 + t_6$
	$D_2D_1$	$t_5 + t_7$
	$B_1B$	$t_3 + t_4 + t_5 + t_8$
	$C_1C_2$	$t_6 + t_9$
	$B_2B_1$	$t_8 + t_9 + t_{10}$

## 4.2 Single Effect Testing

The objective of this section is to introduce the separate effect of probabilistic process capability, fuzzy process capability, and product value on the operational tolerance allocation problem. Although these factors are combined in an integrated tolerance allocation/machine assignment model in chapter three, each can significantly contribute to the operational tolerance allocation problem, as shown in the next subsections.

### 4.2.1 Operational Tolerance Allocation Considering Probabilistic Process Capability

A tolerance allocation model with probabilistic process capability model can be written as follows

$$\text{Maximize } \sum t_{fh} \quad (4.1)$$

$$\text{Subject to } \sum_{i \in DC_j} t_{fh} \leq BP_j \quad (4.2)$$

$$\int_{-\infty}^{t_{fh}} \frac{1}{\sigma_{fh} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - m_{fh}}{\sigma_{fh}} \right)^2} dx > 1 - \alpha_{fh} \quad (4.3)$$

$$t_{fh} \geq 0 \quad (4.4)$$

$$j = 1, \dots, k ; f = 1, \dots, S ; h = 1, \dots, H_f$$

For the workpiece shown in Figure 4.1, it is assumed that the considered operations are centered, i.e.,  $m_{fh} = 0$ . Furthermore, it is also assumed that these operations have the standard deviations ( $\sigma_{fh}$ ) and risk levels ( $\alpha_{fh}$ ) values as given in Table 4.4. The  $\sigma_{fh}$  values are hypothetical values, which are used for demonstration purposes. Conversely, the  $\alpha_{fh}$  values are experts-based values. The  $\alpha_{fh}$  values are used as a tool to assign higher preferences for critical operations. For instance, the first four operations in the current example have indirect effects on BP tolerances. As a result, a relatively large  $\alpha_{fh}$  value is assigned to them. The next four operations are finishing operations; hence, they are considered to be more critical than the first four and, accordingly, granted lower  $\alpha_{fh}$  values. Finally, the grinding operations have the lowest  $\alpha_{fh}$  values since they are the most critical.

TABLE 4.4

$\sigma_{fh}$ ,  $\alpha_{fh}$ , AND  $LPC_{fh}$  VALUES FOR THE TEN OPERATIONS SHOWN IN FIGURE 4.1

Operation Number	$\sigma_{fh}$	$\alpha_{fh}$	$LPC_{fh}$
O <sub>11</sub>	0.050	0.500	0.150
O <sub>12</sub>	0.050	0.500	0.150
O <sub>21</sub>	0.030	0.400	0.090
O <sub>22</sub>	0.030	0.400	0.090
O <sub>31</sub>	0.020	0.200	0.060
O <sub>32</sub>	0.020	0.200	0.060
O <sub>41</sub>	0.010	0.100	0.030
O <sub>42</sub>	0.010	0.100	0.030
O <sub>51</sub>	0.005	0.010	0.015
O <sub>52</sub>	0.005	0.010	0.015

In order to evaluate the proposed probabilistic process capability approach, it is compared with the ordinary LP method. In developing the LP model, objective function and BP/SR constraints that are similar to those shown in equations 4.1 and 4.2 are used. However, the LP model uses the  $LPC_{fh}$  values for the process capability constraint. A process capability constraint based on the LP method is written as

$$t_{fh} \geq LPC_{fh} \quad f = 1, \dots, S; h = 1, \dots, H_f \quad (4.5)$$

$LPC_{fh}$  values are given in Table 4.4.

The operational tolerance allocation problem is solved for the workpiece shown in Figure 4.1 based on two methods. LINGO is used to solve the two models. Table 4.5 shows a comparison between these two approaches based on the minimum allocated operational tolerance, the resulted rejection rates, and the related manufacturing cost, which are discussed in details below. The numbers in Table 4.5 are rounded up to three digits after the decimal point.

TABLE 4.5  
PROBABILISTIC VS. LP TOLERANCE ALLOCATION METHODS

Operation number	The allocated operational tolerances ( $t_{fh}$ )		The rejection rate ( $r_{fh}$ ),		The manufacturing cost ( $C_M$ ), \$x	
	The prob. method	The LP method	The prob. method	The LP method	The prob. method	The LP method
O <sub>11</sub>	0.200	0.250	zero	zero	51.359	46.301
O <sub>12</sub>	0.200	0.350	zero	zero	51.359	41.897
O <sub>21</sub>	0.100	0.050	0.001	0.096	93.506	262.094
O <sub>22</sub>	0.148	0.220	zero	zero	62.966	48.921
O <sub>31</sub>	0.094	0.030	zero	0.317	100.909	661.710
O <sub>32</sub>	0.106	0.070	zero	0.134	87.324	151.996
O <sub>41</sub>	0.050	0.050	zero	zero	262.094	262.094
O <sub>42</sub>	0.058	0.090	zero	zero	204.361	106.688
O <sub>51</sub>	0.042	0.015	zero	0.003	355.882	2534.910
O <sub>52</sub>	0.025	0.020	zero	zero	936.446	1442.210
				Sum.	2206.206x	5558.812x

### Minimum Allocated Operational Tolerance

As it can be seen from Table 4.5, the minimum allocated operational tolerance that resulted from the probabilistic approach is 0.025. This is higher than the 0.015 associated with the LP approach. Larger minimum allocated tolerance means that less capable machine or less manufacturing cost. Hence, according to this criterion the probabilistic approach has a slight edge over the LP approach. Yet it is difficult to conclude that it will always perform better. Therefore, the next criterion is used to further compare these models.

### Rejection Rates

Table 4.5 also shows the rejection rates based on the probabilistic and the LP approaches. The rejection rate is calculated as the probability of producing parts outside the  $\pm 3\sigma$  range, assuming that the operations outputs are normally distributed.

As shown, the probabilistic approach resulted in zero rejection rates for nine of the ten operations. Only the third operation may generate rejected parts with a 0.001 rate. This is a significant improvement over the LP method, which resulted in four operations with failure rates. The rejection rate for the entire process based on the LP approach is:

$$\text{Rejection rate}_{\text{Total}} = 1 - \prod_{h=1}^{H_f} (1 - r_{fh}), f = 1, \dots, S$$

where  $r_{fh}$  is the rejection rate of operation  $h$  in process set  $f$

Therefore, the total rejection rate resulted from the LP approach is 0.467.

As seen above, the comparison results of the rejection rates illustrate the superiority of the proposed probabilistic approach over the LP approach. The third criterion further emphasizes this result furthermore.

### **Manufacturing cost**

As shown in Table 4.5, the lowest allocated tolerance based on the probabilistic approach is higher than the one that resulted from the LP approach (0.025 vs. 0.015). This means that a less accurate machine is needed or, in other words, less manufacturing cost. To illustrate this point, assume the cost-tolerance function for the considered operations has the following form (Ngoi and Seow, 1996)

$$C_M = \left( 37.31 + \frac{5.6196}{t_i^2} \right) x \quad (4.6)$$

where  $x$  is the cost/tolerance conversion



Table 4.5 shows the related manufacturing cost for the ten considered operations based on the two approaches. The superiority of the probabilistic approach over the LP is obvious from the summation of the manufacturing cost. The probabilistic approach costs only 39.69% of the LP approach cost.

To summarize, based on the above three criteria, adopting the suggested probabilistic approach for the operational tolerance allocation problem resulted in larger allocated operational tolerances than with the LP approach. Accordingly, less manufacturing cost and higher acceptance production rate would be generated.

The proposed probabilistic process capability approach has one more key advantage over the LP approach. That is, its ability to handle shifted mean processes. Since the LP approach uses a single LPC for the process capability representation, it actually assumes centered processes. Yet, this assumption may not always be valid. On the other hand, the probabilistic approach characterizes the process capability by a stochastic distribution that is defined by a mean and a standard deviation. Hence, the probabilistic approach is capable of catching the actual performance of a process, whether it is centered, i.e., has a zero mean value, or shifted, i.e., has a mean value that does not equal zero.

In order to demonstrate the above advantage, the operational tolerance allocation problem for the steel sleeve example shown in Figure 4.1 is resolved, based on a shifted processes assumption. Two shifted scenarios are considered: one standard deviation shift and one and a half standard deviation shift. Values of  $\sigma_{fh}$  and  $\alpha_{fh}$  are the same as shown in Table 4.4. Table 4.6 shows the assigned tolerances that resulted from the two shift scenarios. The results of the centered case are also tabulated for comparison purposes.

Analysis of the results shown in Table 4.6 exemplifies a general trend that can be stated as follows: tighter operational tolerances are allocated as the process shift increases. In fact, this is an anticipated outcome since the probability of a process to allocate high tolerances decreases as its shift increases. Actually, infeasible output resulted from solving the current steel sleeve example when a shift of two standard deviations was assigned to the required processes. In other words, with a shift as high as  $2\sigma_{fh}$ , the existed process can not fulfill the BP/SR requirements, and the required preferences resulted from the assigned  $\alpha_{fh}$ .

TABLE 4.6  
CENTERED AND SHIFTED PROCESSES COMPARISON

Operation Number	Allocated Operational Tolerances ( $t_{fh}$ )		
	Centered Processes ( $m_{fh}=0$ )	Shifted Processes ( $m_{fh}=\sigma_{fh}$ )	Shifted Processes ( $m_{fh}=1.5\sigma_{fh}$ )
O <sub>11</sub>	0.200	0.150	0.125
O <sub>12</sub>	0.200	0.150	0.125
O <sub>21</sub>	0.100	0.118	0.103
O <sub>22</sub>	0.148	0.118	0.103
O <sub>31</sub>	0.094	0.087	0.077
O <sub>32</sub>	0.106	0.087	0.077
O <sub>41</sub>	0.050	0.050	0.050
O <sub>42</sub>	0.058	0.062	0.058
O <sub>51</sub>	0.042	0.037	0.034
O <sub>52</sub>	0.025	0.025	0.025

#### 4.2.2 Operational Tolerance Allocation Considering Fuzzy Process Capability

A tolerance allocation model with fuzzy process capability can be written as

$$\begin{aligned} &\text{Maximize } \lambda && (4.7) \\ &\text{subject to} \end{aligned}$$

$$\lambda \leq 1 + \frac{\text{OBJ}}{\text{FR}_{\text{OBJ}}} - \frac{\sum t_{\text{fh}}}{\text{FR}_{\text{OBJ}}} \quad (4.8)$$

$$\lambda \leq 1 + \frac{\text{BP}_j}{\text{FR}_{\text{BP}_j}} - \frac{\sum_{t_{\text{inDC}_j}} t_{\text{fh}}}{\text{FR}_{\text{BP}_j}} \quad (4.9)$$

$$\lambda \leq 1 + \frac{\text{FPC}_{\text{fh}}}{\text{FR}_{\text{fh}}} - \frac{t_{\text{fh}}}{\text{FR}_{\text{fh}}} \quad (4.10)$$

$$\begin{aligned} &t_{\text{fh}} \geq 0 && (4.11) \\ &j=1, \dots, k; f=1, \dots, S; h=1, \dots, H_f \end{aligned}$$

where  $\text{FPC}_{\text{fh}}$  is an expert-based fuzzy process capability evaluation.

In this formulation, the objective function of maximizing  $\lambda$  is equivalent to maximizing the summation of the operational tolerances typically used in an LP formulation. The OBJ variable, shown in equation (4.7), is equal to the objective function value that resulted from an LP model solution. This OBJ value is formulated to be fuzzy with a range of  $\text{FR}_{\text{OBJ}}$ . In the same way, each of the constraints related to the tolerance chain identifications of the BP/SR, as shown in equation (4.8), is formulated as a fuzzy variable with a fuzzy range of  $\text{FR}_{\text{BP}_j}$ . Since fuzziness is only associated with process capability,  $\text{FR}_{\text{OBJ}}$  and  $\text{FR}_{\text{BP}_j}$  are assigned very small values, which makes both objective function and the BP/SR requirements crisp in reality but with fuzzy formulation.

Equation (4.9) represents the process capability constraint based on the experts opinion. For the generation of the  $FPC_{th}$  for the example shown in Figure 4.1, the following assumptions are used:

- According to the expert, an operation can be fully evaluated by one of three descriptions: capable, semi-capable or quite capable. At the border of these evaluations, the operation would be partially capable, partially semi-capable, or partially quite capable. Hence, the trapezoidal fuzzy membership function is adopted to represent the expert's processes capabilities evaluations.
- Figure 4.4 shows the process capability-membership functions for the ten operations required for the production of the workpiece that is shown in Figure 4.1. The y-axis represents the value of the membership function. The upper x-axis represents a dimensionless scale, which is used to extract the expert's evaluations. The scale ranges from 1 to 100. Value 1 represents the highest process capability level while a value of 100 represents the lowest level. With the expert's assistance, an equivalent unit-based scale is generated, as shown in the lower x-axis of Figure 4.3. The unit-based evaluation is needed for the formulation and solution of the fuzzy process capability model.
- Table 4.7 shows the expert evaluation for the considered ten operations along with their fuzziness ranges.

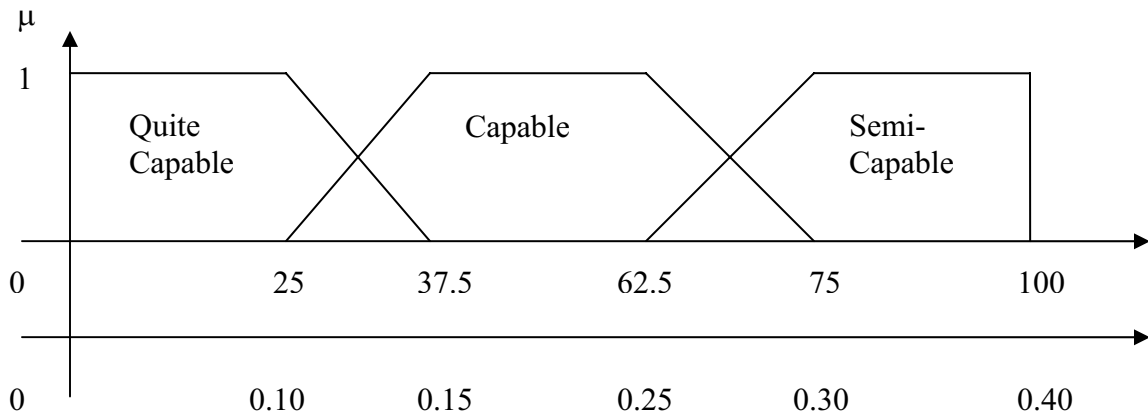


Figure 4.3. The process capability-membership functions for the considered operations.

TABLE 4.7  
EXPERT PROCESS CAPABILITY ASSESSMENTS FOR THE TEN OPERATIONS  
OF FIGURE 4.1

Operation No.	Experts Assessment (Linguistic)	Experts Evaluation (Dimensionless)	Experts Evaluation (FPC <sub>fh</sub> ), (Unit-Based), in	Fuzziness Range (FR <sub>fh</sub> )
O <sub>11</sub>	Semi-capable	90	0.360	0.001
O <sub>12</sub>	Semi-capable	90	0.360	0.001
O <sub>21</sub>	Capable or semi-capable (not sure)	70	0.280	0.050
O <sub>22</sub>	Capable or semi-capable (not sure)	70	0.280	0.050
O <sub>31</sub>	Capable or quite capable (not sure)	30	0.12	0.050
O <sub>32</sub>	Capable or quite capable (not sure)	30	0.12	0.050
O <sub>41</sub>	Quite capable	20	0.080	0.001
O <sub>42</sub>	Quite capable	20	0.080	0.001
O <sub>51</sub>	Quite capable	10	0.040	0.001
O <sub>52</sub>	Quite capable	10	0.040	0.001

Based on the  $FPC_{fh}$  given in Table 4.6, the fuzzy model shown in equation (4.7 to 4.11) is solved. The resulting operational tolerances using LINGO are given in Table 4.8 along with the associated rejection rates. Also, the LP results are represented in the table for comparison purposes.

The fuzzy approach resulted in sound improvement over the LP approach, generally allocating higher operational tolerances than the LP method. Consequently, lower rejection rates are expected, as shown by the results in Table 4.8.

TABLE 4.8  
FUZZY VS. LP APPROACHES

Operation No.	Fuzzy Model		LP Model	
	Allocated $t_{fh}$	Rejection Rate	Allocated $t_{fh}$	Rejection Rate
O <sub>11</sub>	0.260	zero	0.250	zero
O <sub>12</sub>	0.300	zero	0.350	zero
O <sub>21</sub>	0.050	0.096	0.050	0.096
O <sub>22</sub>	0.220	zero	0.220	zero
O <sub>31</sub>	0.080	zero	0.030	0.317
O <sub>32</sub>	0.080	zero	0.070	0.134
O <sub>41</sub>	0.060	zero	0.050	zero
O <sub>42</sub>	0.060	zero	0.090	zero
O <sub>51</sub>	0.041	zero	0.015	0.003
O <sub>52</sub>	0.034	zero	0.020	zero

#### 4.2.3 Operational Tolerance Allocation Considering Product Value

The following is a tolerance allocation model which considers the effect of the product value:

$$\text{Maximize } \sum t_{fh} \quad (4.12)$$

Subject to

$$\sum_{t_{ieDC_j}} t_{fh} \leq BP_j \quad (4.13)$$

$$\int_{-\infty}^{t_{fh}} \frac{1}{\sigma_{fh} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - m_{fh}}{\sigma_{fh}} \right)^2} dx > 1 - \alpha_{fh} \quad (4.14)$$

$$\frac{t_{fh}}{3\sigma_{fh}} \geq PV_{fh} \quad (4.15)$$

$$t_{fh} \geq 0 \quad (4.16)$$

$$j = 1, \dots, k; f = 1, \dots, S; h = 1, \dots, H_f$$

The difference between this tolerance allocation model and the probabilistic process capability model shown in equations (4.1 to 4.4) is in the consideration of the product value effect shown by the constraint in equation (4.15). The  $\frac{t_{fh}}{3\sigma_{fh}}$  ratio is a process capability ratio (PCR) of which the value is decided based on the value of the product. It is known that as the PCR value increases, the expected rejection rate decreases. As a result, according to this constraint, as the  $PV_{fh}$  increases the rejection rate probability of the related operation, i.e.,  $O_{fh}$ , decreases.

This NLP model can be solved using any nonlinear optimization software package. Since the model is considering the change of product value through the different stages of the production process, it is expected that more capable operations, i.e., operations that can generate tighter tolerances, would be assigned to the product throughout its high-value phases. Before introducing a solution for the model shown in equations (4.12 to 4.16),  $PV_{fh}$  calculations are presented.

#### 4.2.3.1 Product Value Calculations

The procedure explained in section 3.2.3 is followed here to find the product value at the various stages of the production process. The three components of product value are calculated first, and then the  $PV_{fh}$  is found.

1.  $PV_R$  for the considered workpiece can be found as follows:

- Assume that the machine of operation  $O_{31}$  is the system's constraint. Also, assume that five products use this machine, including the workpiece shown in Figure 4.1.
- Table 4.9 lists the simplex mill machine-production rates,  $CS_{product}$ ,  $MC_{product}$ ,  $ASP_i$ ,  $D_i$ , and total cycle time for each of the five products.
- $CS_{product}$  is calculated using equation (3.5). For example,  $CS_{Product\#1} = 3600/10 = 360$  sec/unit.
- Using equation (3.6),  $ATP_{constraint} = \$88075$ .
- Using equation (3.7),  $CS_{constraint} = 78600$  sec.
- From equation (3.8),  $\left(\frac{ATP}{CS}\right)_{constraint} = 1.121$  \$/sec.
- Table 4.10 shows the resulting CPV and  $PV_R$  values for the five products based on equations (3.4) and (3.9), respectively.



TABLE 4.9

PRODUCTION RATES,  $CS_{\text{product}}$ ,  $MC_{\text{product}}$ ,  $ASP_i$ ,  $D_{il}$ , AND TOTAL CYCLE FOR ALL FIVE PRODUCTS

Product No.	Production Rate on $O_{31}$ Machine, (unit/hr)	$CS_{\text{product}}$ , (sec/unit)	Material Cost ( $MC_{\text{product}}$ ), (\$)	Average Selling Price ( $ASP_i$ ), (\$)	Demand ( $D_i$ ), (units)	Total Cycle Time (sec)
1	10	360	50	400	30	4520
2	5	720	30	350	50	6900
3	8	450	60	800	10	6180
4	12	300	35	700	75	7380
5	15	240	35	100	20	6060

TABLE 4.10

CPV AND  $PV_R$  VALUES FOR THE FIVE PRODUCTS

Product No.	CPV (\$)	$PV_R$
1	439.7	0.910
2	809.3	0.433
3	547.1	1.462
4	359.7	1.946
5	294.8	0.339

2.  $PV_{CT}$  calculations can be performed as follows

- Assume that the cycle time for each operation of the ten operations required to produce the workpiece in Figure 4.1, is as shown in Table 4.11.
- Assume that only the value-added processes are considered. Hence, the %VAP in equation (3.10) is equal to 100% or 1.0.

TABLE 4.11  
CT FOR THE CONSIDERED OPERATIONS

Operation No.	Cycle Time (sec)
O <sub>11</sub>	420
O <sub>12</sub>	420
O <sub>21</sub>	300
O <sub>22</sub>	240
O <sub>31</sub>	240
O <sub>32</sub>	360
O <sub>41</sub>	1200
O <sub>42</sub>	1200
O <sub>51</sub>	900
O <sub>52</sub>	900

- Based on the total cycle times for each of the considered five products, given in Table 4.9,  $R_p$  for each product can be found as shown in Table 4.12.

For example, using equation (3.11),  $R_{\text{product\#2}} = \frac{6900 - 4520}{7380 - 4520} = 0.832$

TABLE 4.12  
RP FOR THE FIVE PRODUCTS

Product No.	$R_p$
1	0.000
2	0.832
3	0.580
4	1.000
5	0.539

- Using equation (3.10),  $PV_{CT}$  for each operation with respect to each product is shown in Table 4.13.

TABLE 4.13

PV<sub>CT</sub> FOR THE CONSIDERED OPERATIONS

Operation No.	PV <sub>CT_prod#1</sub>	PV <sub>CT_prod#2</sub>	PV <sub>CT_prod#3</sub>	PV <sub>CT_prod#4</sub>	PV <sub>CT_prod#5</sub>
O <sub>11</sub>	0	0.889	0.620	1.068	0.575
O <sub>12</sub>	0	0.945	0.659	1.136	0.612
O <sub>21</sub>	0	0.986	0.688	1.185	0.638
O <sub>22</sub>	0	1.018	0.710	1.223	0.659
O <sub>31</sub>	0	1.050	0.733	1.262	0.680
O <sub>32</sub>	0	1.099	0.766	1.320	0.711
O <sub>41</sub>	0	1.260	0.879	1.515	0.816
O <sub>42</sub>	0	1.422	0.992	1.709	0.920
O <sub>51</sub>	0	1.543	1.076	1.854	0.999
O <sub>52</sub>	0	1.664	1.161	2.000	1.077

3. PV<sub>PC</sub> are assigning assuming that the operations-dimensions are classified as shown in Table 4.14.

TABLE 4.14

PV<sub>PC</sub> FOR THE CONSIDERED OPERATIONS

Dimension from Operation no.	Dimension classification	PV <sub>PC</sub>
O <sub>11</sub>	Incidental	1.000
O <sub>12</sub>	Incidental	1.000
O <sub>21</sub>	Incidental	1.000
O <sub>22</sub>	Incidental	1.000
O <sub>31</sub>	Incidental	1.000
O <sub>32</sub>	Major	1.333
O <sub>41</sub>	Major	1.333
O <sub>42</sub>	Major	1.333
O <sub>51</sub>	Minor	1.250
O <sub>52</sub>	Minor	1.250

Finally, it is assume that the weights for the revenue, percentage of fished cycle time, and product criticality are 0.4, 0.3, and 0.3, respectively. Then, using equation (3.12), the product value assigned to each of the ten operations for each product would be as shown in Table 4.15. As shown, the five products can be ranked with respect to their product value in a descending manner as: product #4, product #3, product #2, product #5, and product #1.

TABLE 4.15  
PV<sub>FH</sub> VALUES

Operation No.	PV <sub>fh(prod#1)</sub>	PV <sub>fh(prod#2)</sub>	PV <sub>fh(prod#3)</sub>	PV <sub>fh(prod#4)</sub>	PV <sub>fh(prod#5)</sub>
O <sub>11</sub>	0.664	0.604	1.071	1.399	0.608
O <sub>12</sub>	0.664	0.689	1.083	1.420	0.619
O <sub>21</sub>	0.664	0.760	1.091	1.434	0.627
O <sub>22</sub>	0.664	0.778	1.098	1.445	0.633
O <sub>31</sub>	0.664	0.788	1.105	1.457	0.640
O <sub>32</sub>	0.764	0.903	1.215	1.574	0.749
O <sub>41</sub>	0.764	0.951	1.249	1.637	0.780
O <sub>42</sub>	0.764	1.000	1.282	1.691	0.812
O <sub>51</sub>	0.739	1.011	1.283	1.710	0.810
O <sub>52</sub>	0.739	1.047	1.308	1.753	0.834

After the PV<sub>fh</sub> is found, the model shown in equations (4.11 to 4.15) can be solved, as described in the next subsection.

#### 4.2.3.2 Product-Value Model Solution

In order to demonstrate the effect of product value on tolerance allocation, the model shown in equations (4.11 to 4.15) is solved five times, each time with a PV<sub>fh</sub> value based on different product. Table 14.6 shows the resulting allocated operational tolerances based on PV<sub>fh</sub> values from products 1, 2, 3, and 5. The general observation

from this table is that as the used  $PV_{fh}$  values increase the resulting  $t_{fh}$  for the considered operations becomes tighter. For example, product #3 has the highest  $PV_{fh}$  values among the four analyzed products (1, 2, 3, and 5); therefore, the tolerance allocation model that has  $PV_{fh}$  from product #3 has resulted in tighter or equal operational tolerances for eight and nine operations compared to the models which have  $PV_{fh}$  values from products #1, #2, and product # 5, respectively.

TABLE 4.16

ALLOCATED TOLERANCES USING FOUR DIFFERENT PRODUCT VALUES

Operation No.	Allocated Tolerance ( $t_{fh}$ ), using:			
	$PV_{fh(\text{prod}\#1)}$	$PV_{fh(\text{prod}\#2)}$	$PV_{fh(\text{prod}\#3)}$	$PV_{fh(\text{prod}\#5)}$
O <sub>11</sub>	0.240	0.231	0.202	0.243
O <sub>12</sub>	0.228	0.228	0.228	0.228
O <sub>21</sub>	0.060	0.069	0.098	0.056
O <sub>22</sub>	0.178	0.178	0.178	0.178
O <sub>31</sub>	0.096	0.081	0.066	0.097
O <sub>32</sub>	0.115	0.122	0.108	0.118
O <sub>41</sub>	0.050	0.050	0.050	0.050
O <sub>42</sub>	0.065	0.072	0.057	0.068
O <sub>51</sub>	0.035	0.028	0.042	0.032
O <sub>52</sub>	0.025	0.025	0.025	0.025

When  $PV_{fh}$  values from the fourth product were used, the model resulted in an infeasible solution. That is, the current processes are not capable enough to meet the product value requirements for  $PV_{\text{prod}\#4}$ . Hence, since the fourth product has high  $PV_{fh}$  values, more capable processes are required for the production of this product based on the current process plan.

### **4.3 Integrated-Model Testing**

This section introduces the testing and validation of the stochastic and fuzzy versions of the proposed integrated model that is shown in section 3.3. Testing is applied to the steel sleeve part shown in Figure 4.1. The tolerance chain identifications and  $\alpha_{fh}$  values are as given in Tables 4.3 and 4.4, respectively. In addition, for the testing application of sections 4.3.1 and 4.3.2, values of the second product ( $PV_{fh(\text{prod}\#2)}$ ) shown in Table 4.14 are used.

#### **4.3.1 Stochastic Integrated Model**

In order to test this model, data on the machines available for each process set and their actual standard deviation (ASD) are assumed to be as given in Table 4.17. The term  $M_{fg}$  in the table refers to machine  $g$  available for process set  $f$ . As shown, it is assumed that the same set of three machines (machines with same capabilities) are available for the rough boring, rough turning, finish boring, and finish turning operations. Three machines are also available for grinding operations.

TABLE 4.17

AVAILABLE MACHINES WITH THEIR ASD<sub>FG</sub>

Set #	Operation Set	Machines Available	ASD <sub>fg</sub>
1	Rough boring	M <sub>11</sub>	0.100
		M <sub>12</sub>	0.050
		M <sub>13</sub>	0.005
2	Rough turning	M <sub>21</sub>	0.100
		M <sub>22</sub>	0.050
		M <sub>23</sub>	0.005
3	Finish boring	M <sub>31</sub>	0.100
		M <sub>32</sub>	0.050
		M <sub>33</sub>	0.005
4	Finish turning	M <sub>41</sub>	0.100
		M <sub>42</sub>	0.050
		M <sub>43</sub>	0.005
5	Grinding	M <sub>51</sub>	0.050
		M <sub>52</sub>	0.010
		M <sub>43</sub>	0.001

The next subsections present testing results for the “one machine per operation set” and the “one machine per operation” scenarios.

#### 4.3.1.1 One Machine per Process Set

Using the identified dimensional chains,  $\alpha_{fh}$ , and  $(PV_{fh(\text{prod}\#2)})$ , the mathematical model shown in equations (3.20 to 3.29) is then generated and solved using the optimization software package LINGO. Outputs from the model are operational tolerances, minimum required machine standard deviation, maximum machine shift, and the machines selected for each operation set, as tabulated in Table 4.18.

TABLE 4.18

MODEL OUTPUTS FOR “ONE MACHINE PER PROCESS SET” CASE

Z=1.432					
Operation Number	Operational Tolerance	Machine Selected	Standard Deviation		Maximum Machine Shift
			Minimum Required	Actual for Selected Machine	
O <sub>11</sub>	0.1582	M <sub>12</sub>	0.0844	0.050	0.0500
O <sub>12</sub>	0.2083		0.1235		
O <sub>21</sub>	0.1418	M <sub>22</sub>	0.0716	0.050	0.0500
O <sub>22</sub>	0.1495		0.0776		
O <sub>31</sub>	0.0513	M <sub>33</sub>	0.0106	0.005	0.0400
O <sub>32</sub>	0.0924		0.0319		
O <sub>41</sub>	0.0500	M <sub>43</sub>	0.0122	0.005	0.0300
O <sub>42</sub>	0.0513		0.0129		
O <sub>51</sub>	0.0488	M <sub>53</sub>	0.0127	0.001	0.0100
O <sub>52</sub>	0.0250		0.0065		

The following remarks can be drawn from the solution of the model:

- Since  $Z=1.432 > 1$ , the process plan is feasible.
- A high Z value means more accuracy for the produced product. However, this may require more capable machines. On the other hand, a close to one Z value means that higher operational tolerances are allocated.
- Machine 2 is selected for operation sets 1 and 2, while machine 3 is selected for operation sets 3, 4, and 5. Note that these machines have the lower capabilities that can feasibly fulfill both the design and product value requirements.
- The maximum allowable shift for each machine, without violating the feasibility of the current process plan, is found.



- The model provides flexibility to the planning and scheduling practices. For example, the model has selected machine 2 for operation set 1. This machine is the second capable machine among the three machines available for that set of operations. This means that more accurate machines in that set (i.e., machine 3) could replace machine 2, in case of scheduling conflict or other restrictive conditions.
- Flexibility is also embedded in the machines' shifts values resulting from the model. For instance, for any machine selection of operation set 2, either a centered machine or a non-centered machine, with a shift up to 0.05 in, can be selected without violating the feasibility of the current process plan.
- $PV_{fh}$  and  $\alpha_{fh}$  values significantly influence machine assignment, which affects the allocated operational tolerances. Therefore, both  $PV_{fh}$  and  $\alpha_{fh}$  values should be carefully calculated/assigned.

#### **4.3.1.2 One Machine per Operation**

This section presents the results of “one machine per operation” case in which a different machine can be used for each operation. Table 4.19 shows these results for  $Z > 1$ . This case has generally assigned higher operational tolerances than the “one machine per operation set” case. This is because the “one machine per operation” case allows each operation to use a machine among a number of existing machines.

TABLE 4.19

THE MODEL OUTPUTS FOR “ONE MACHINE PER OPERATION” CASE

Z=1.008					
Operation Number	Operational Tolerance	Machine Selected	Standard Deviation		Maximum Machine Shift
			Minimum Required	Actual for Selected Machine	
O <sub>11</sub>	0.1853	M <sub>11</sub>	0.1056	0.100	0.0500
O <sub>12</sub>	0.2667	M <sub>11</sub>	0.1672	0.100	0.0512
O <sub>21</sub>	0.1146	M <sub>22</sub>	0.0504	0.050	0.0500
O <sub>22</sub>	0.1766	M <sub>22</sub>	0.0988	0.050	0.0500
O <sub>31</sub>	0.0575	M <sub>33</sub>	0.0106	0.005	0.0400
O <sub>32</sub>	0.0600	M <sub>33</sub>	0.0114	0.005	0.0412
O <sub>41</sub>	0.0500	M <sub>43</sub>	0.0122	0.005	0.0300
O <sub>42</sub>	0.0513	M <sub>43</sub>	0.0129	0.005	0.0300
O <sub>51</sub>	0.0570	M <sub>53</sub>	0.0202	0.001	0.0100
O <sub>52</sub>	0.0167	M <sub>53</sub>	0.0029	0.001	0.0100

#### 4.3.2 Fuzzy Integrated Model

In order to test this model, the expert’s evaluation for the machines available for each operation/operation set is required. The expert normally presents fuzzy process capability (FPC). Yet, the fuzzy standard deviation (FSD) is needed for the solution of this model. Hence, FSD is obtained as one sixth the FPC. Table 4.20 includes these FSD<sub>fg</sub> along with their fuzziness ranges (FR<sub>FSD</sub>) for the ten operations needed to produce the part shown in Figure 4.1.

TABLE 4.20

MACHINES AVAILABLE AND THEIR  $FSD_{FG}$ 

Set #	Operation Set	Machines Available	$FSD_{fg}$	$FR_{FSD}$
1	Rough Boring	$M_{11}$	0.120	0.0100
		$M_{12}$	0.040	0.0050
		$M_{13}$	0.006	0.0010
2	Rough Turning	$M_{21}$	0.120	0.0100
		$M_{22}$	0.040	0.0050
		$M_{23}$	0.006	0.0010
3	Finish Boring	$M_{31}$	0.120	0.0100
		$M_{32}$	0.040	0.0050
		$M_{33}$	0.006	0.0010
4	Finish Turning	$M_{41}$	0.120	0.0100
		$M_{42}$	0.040	0.0050
		$M_{43}$	0.006	0.0010
5	Grinding	$M_{51}$	0.040	0.0050
		$M_{52}$	0.015	0.0010
		$M_{43}$	0.003	0.0005

The next subsections present the testing results for the “one machine per operation set” and the “one machine per operation” scenarios.

#### 4.3.2.1 One Machine per Process Set

Using the identified dimensional chains,  $\alpha_{fh}$ , and  $(PV_{fh(\text{prod}\#2)})$ , the mathematical model shown in equations (3.40 to 3.49) is then generated and solved using the optimization software package LINGO. Outputs from the model are operational tolerances, minimum required machine standard deviation, and the machines selected for each operation/operation set, as tabulated in Table 4.21.

TABLE 4.21

THE FUZZY MODEL OUTPUTS FOR “ONE MACHINE PER PROCESS SET” CASE

Z=1.250				
Operation Number	Operational Tolerance	Machine Selected	Standard Deviation	
			Minimum Required	Expert-Based
O <sub>11</sub>	0.1826	M <sub>12</sub>	0.050	0.040
O <sub>12</sub>	0.1093		0.050	
O <sub>21</sub>	0.1173	M <sub>22</sub>	0.050	0.040
O <sub>22</sub>	0.1318		0.050	
O <sub>31</sub>	0.1327	M <sub>33</sub>	0.010	0.006
O <sub>32</sub>	0.1407		0.010	
O <sub>41</sub>	0.0173	M <sub>43</sub>	0.010	0.006
O <sub>42</sub>	0.0182		0.010	
O <sub>51</sub>	0.0092	M <sub>53</sub>	0.005	0.003
O <sub>52</sub>	0.0095		0.005	

The following remarks can be drawn from the solution of the model:

- Since  $Z=1.250>1$ , the process plan is feasible.
- Machine 2 is selected for operation sets 1 and 2, while machine 3 is selected for operation sets 3, 4, and 5. Note that these machines have the lower capabilities that can feasibly fulfill both the design and product value requirements.
- Accurate expert evaluations are needed in order for the model to produce reliable results.

#### 4.3.2.2 One Machine per Operation

This section presents the results of the “one machine per operation” case in which different machine can be used for each operation. Table 4.22 shows these results for  $Z>1$ .

This case has generally assigned higher operational tolerances than the “one machine per operation set” case. This is because the “one machine per operation” case allows each operation to use a machine among a number of existing machines.

TABLE 4.22

THE FUZZY MODEL OUTPUTS FOR “ONE MACHINE PER OPERATION” CASE

Z=1.25				
Operation Number	Operational Tolerance	Machine Selected	Standard Deviation	
			Minimum Required	Expert-Based
O <sub>11</sub>	0.2196	M <sub>12</sub>	0.050	0.040
O <sub>12</sub>	0.3683	M <sub>22</sub>	0.050	0.040
O <sub>21</sub>	0.1382	M <sub>22</sub>	0.050	0.040
O <sub>22</sub>	0.3495	M <sub>22</sub>	0.050	0.040
O <sub>31</sub>	0.0184	M <sub>33</sub>	0.010	0.006
O <sub>32</sub>	0.0994	M <sub>33</sub>	0.010	0.006
O <sub>41</sub>	0.0499	M <sub>43</sub>	0.010	0.006
O <sub>42</sub>	0.0182	M <sub>43</sub>	0.010	0.006
O <sub>51</sub>	0.0505	M <sub>53</sub>	0.005	0.003
O <sub>52</sub>	0.0244	M <sub>53</sub>	0.005	0.003

### 4.3.3 Model Validation

In 2001, Thirtha presented an original model for the simultaneous allocation of operational tolerances and machine assignment. The model introduced practical aspects to the modeling of the production process by considering all available machines for each type of operation. In order to validate the applicability of the proposed model, it is compared to the results of Thirtha’s model. Furthermore, results for the proposed model under changing product values are compared with equal-product value results.

The next two subsections show these comparisons for the “one machine per operation set” case.

#### 4.3.3.1 Comparison of Proposed Model with Thirtha’s Model

The example in Figure 4.1 is solved by Thirtha’s model and two versions of the proposed model. Yet, Thirtha’s model does not consider the effect of product value. Instead it uses a fixed PCR ratio usually set as 1.33. Therefore, for the solution of proposed models, an equal product value of 1.33 is assigned to each of the ten considered operations.

Table 4.23 shows the results of both proposed models and Thirtha’s model for the “one machine per process set” case. Note that the actual outputs of Thirtha’s model are the minimum required capability (MRC), the allocated operational tolerances, and the assigned machines. For comparison purposes, instead of MRC, the required standard deviation, calculated as  $MRC/6$ , is shown in Table 4.23.

TABLE 4.23

#### OUTPUTS OF THE PROPOSED STOCHASTIC ( $Z = 1.42$ ), FUZZY ( $Z= 2.60$ ) AND THIRTHA’S ( $Z=1.25$ ) MODELS

Op. No.	Operational Tolerance			Machine Selected			Standard Deviation					
							Minimum Required			Actual		
	Stc.	Fuz.	Thi.	Stc.	Fuz.	Thi.	Stc.	Fuz.	Thi.	Stc.	Fuz.	Thi.
O <sub>11</sub>	0.246	0.057	0.185	M <sub>11</sub>	M <sub>12</sub>	M <sub>13</sub>	0.150	0.050	0.064	0.100	0.040	0.005
O <sub>12</sub>	0.246	0.151	0.210				0.150	0.050	0.013			
O <sub>21</sub>	0.054	0.126	0.115	M <sub>21</sub>	M <sub>22</sub>	M <sub>23</sub>	0.150	0.050	0.013	0.100	0.040	0.005
O <sub>22</sub>	0.151	0.100	0.050				0.150	0.050	0.013			
O <sub>31</sub>	0.100	0.100	0.050	M <sub>32</sub>	M <sub>33</sub>	M <sub>33</sub>	0.071	0.010	0.013	0.050	0.006	0.005
O <sub>32</sub>	0.100	0.150	0.125				0.071	0.010	0.013			
O <sub>41</sub>	0.050	0.024	0.050	M <sub>43</sub>	M <sub>43</sub>	M <sub>43</sub>	0.016	0.010	0.013	0.005	0.006	0.005
O <sub>42</sub>	0.050	0.050	0.050				0.016	0.010	0.013			
O <sub>51</sub>	0.050	0.050	0.025	M <sub>53</sub>	M <sub>53</sub>	M <sub>53</sub>	0.007	0.005	0.007	0.001	0.001	0.001
O <sub>52</sub>	0.025	0.024	0.025				0.007	0.005	0.006			

As shown in Table 4.23, the stochastic model has generally resulted in larger allocated tolerances than both the fuzzy and Thirtha models. In fact, the stochastic model has resulted in eight operations with higher or equal operational tolerance than the fuzzy and Thirtha models. Likewise, comparing the assigned machines results from the three models support the superiority result of the stochastic model. The stochastic model resulted in less accurate machine assignment for three operations sets against Thirtha's model assignment, and two operations set against the fuzzy model. As explained previously, less accurate machine assignment means lower manufacturing cost and higher scheduling/planning flexibility. In addition, the stochastic model has one more extra advantage over the other two, that is, it can estimate the maximum shift that a machine may have without violating the current process plan feasibility.

The fuzzy model has, in general, a slight advantage over Thirtha's model. Yet, as mentioned before, the efficiency/reliability of the fuzzy model is highly dependable on the accuracy of the expert's knowledge.

In summary, the above comparison results illustrate the validity and effectiveness the proposed model.

#### **4.3.3.2 Equal vs. Unequal Product Value Comparison**

This section shows the effect of considering the product value effect on the allocated tolerances and assigned machines. The example in Figure 4.1 is solved using the two versions of the proposed model under equal and changing product values. For "equal product value" scenario, a 1.33 value is assigned to each of the ten considered operations. Yet, product values of the fourth product shown in Table 4.15 are used for the "changing product value" scenario. Note that all  $PV_{fh(prod\#4)}$  are higher than 1.33, i.e., the

“changing product value” scenario is assigned higher product values than the 1.33-value used in the “equal product value” scenario.

Tables 4.24 and 4.25 show the solutions of both the “equal product value” and “changing product value” scenarios solved based on the stochastic and fuzzy versions of the proposed model, respectively. The general observation of these two tables is tighter tolerances and higher accurate machines are allocated/assigned in the “changing product value” scenario. This observation holds for both versions of the model. Indeed, this is a key conclusion of the proposed models where products with high values are assigned more accurate machines and allocated tighter tolerances. As a result, the probability of these products to be produced per specifications is increasing.

TABLE 4.24

OUTPUTS OF THE ‘EQUAL PRODUCT VALUE’ (Z=1.42) AND ‘CHANGING PRODUCT VALUE’ (Z=1.46) SCENARIOS BASED ON THE STOCHASTIC MODEL VERSION

Opt. No.	Operational Tolerance		Machine Selected		Standard Deviation			
	Equal PV	Changing PV	Equal PV	Changing PV	Minimum Required		Actual	
					Equal PV	Changing PV	Equal PV	Changing PV
O <sub>11</sub>	0.246	0.246	M <sub>11</sub>	M <sub>11</sub>	0.150	0.146	0.100	0.100
O <sub>12</sub>	0.246	0.260			0.150	0.150		
O <sub>21</sub>	0.054	0.053	M <sub>21</sub>	M <sub>23</sub>	0.150	0.016	0.100	0.005
O <sub>22</sub>	0.151	0.065			0.150	0.060		
O <sub>31</sub>	0.100	0.091	M <sub>32</sub>	M <sub>33</sub>	0.071	0.043	0.050	0.005
O <sub>32</sub>	0.100	0.095			0.071	0.059		
O <sub>41</sub>	0.050	0.050	M <sub>43</sub>	M <sub>43</sub>	0.016	0.016	0.005	0.005
O <sub>42</sub>	0.050	0.035			0.016	0.016		
O <sub>51</sub>	0.050	0.025	M <sub>53</sub>	M <sub>53</sub>	0.007	0.007	0.001	0.001
O <sub>52</sub>	0.025	0.025			0.007	0.007		



TABLE 4.25

OUTPUTS OF THE ‘EQUAL PRODUCT VALUE’ (Z=1.25) AND ‘CHANGING PRODUCT VALUE’ (Z=1.25) SCENARIOS BASED ON THE FUZZY MODEL VERSION

Opt. No.	Operational Tolerance		Machine Selected		Standard Deviation			
	Equal PV	Changing PV	Equal PV	Changing PV	Minimum Required		Actual	
					Equal PV	Changing PV	Equal PV	Changing PV
O <sub>11</sub>	0.057	0.150	M <sub>12</sub>	M <sub>12</sub>	0.050	0.050	0.040	0.040
O <sub>12</sub>	0.151	0.151			0.050	0.050		
O <sub>21</sub>	0.126	0.150	M <sub>22</sub>	M <sub>22</sub>	0.050	0.050	0.040	0.040
O <sub>22</sub>	0.100	0.030			0.050	0.050		
O <sub>31</sub>	0.100	0.100	M <sub>33</sub>	M <sub>33</sub>	0.010	0.010	0.006	0.006
O <sub>32</sub>	0.150	0.099			0.010	0.010		
O <sub>41</sub>	0.024	0.037	M <sub>43</sub>	M <sub>43</sub>	0.010	0.010	0.006	0.006
O <sub>42</sub>	0.050	0.050			0.010	0.010		
O <sub>51</sub>	0.050	0.050	M <sub>53</sub>	M <sub>53</sub>	0.005	0.005	0.001	0.001
O <sub>52</sub>	0.024	0.024			0.005	0.005		

#### 4.4 Conclusions

This chapter presented the implementation and testing of the proposed modeling methodology presented in Chapter 3. Testing results of “single factor” implementation illustrated the advantages of including the probabilistic process capability, fuzzy process capability, and product value on the operational tolerance allocation problem. Moreover, analysis of the proposed integrated model, considering its stochastic and fuzzy versions, indicated superior performance of the model in its related activities.

## CHAPTER 5

### FUZZY PROCESS CAPABILITY EVALUATION (FPCE)

#### 5.1 Introduction

Generally, a repeatable process should be used for the production of products. That is, the process must be capable of operating with acceptable variability around the target or nominal dimensions of the product quality characteristics. Regardless of how well a process is designed and maintained, a certain amount of natural variability always exists. This variability is the result of unavoidable common causes. In statistical quality control, when variability in a system consists only of this natural variability, the system is called a stable system. Process capability studies are used to analyze the variability in stable systems.

It is customary to take the 6-sigma spread in the distribution of product quality characteristics as a measure of process capability ( $\pm 3\sigma$ ), where  $\sigma$  is defined as the standard deviation of the process under statistical control (Juran, 1988). In order to link 6-sigma with design requirements, the process capability ratio (PCR) is used. PCR is a measure of the process used to manufacture products with respect to the specification limits. It has been widely used in the manufacturing industries to evaluate process performance. However, the use of the PCR ignores sampling errors, which exist since data must be collected in order to calculate these ratios. A great degree of uncertainty may be introduced into capability assessment owing to sampling errors (Pearn and Chen, 1999). In addition, the process of data collection is a tedious process during which time many factors need to be controlled. Error is hardly avoidable in the process of data

collection. Even if data was accurately collected, it is a time consuming process requiring possible interruption and stoppage of production.

Looking at a process capability from a broader perspective reveals that process capability represents a collective contribution of many factors (such as machine, tool, setup, labor, etc.) involved in the production process. Any changes in one of these factors would affect the resultant process capability. The information about these factors can be easily extracted based on expert's knowledge. In reality, people who have worked with a process or a machine for long time have a very good sense of its capability. The knowledge of these expert people is usually expressed in a linguistic format. Hence, a tool is needed to translate the judgmental nature of experts' knowledge into a mathematical form. Fuzzy logic theory can be used to do this.

This chapter presents a novel approach for the evaluation of process capability based on readily available information and knowledge of experts. It is a flexible system that considers a number of different factors that may have an effect on process capability. It utilizes the concept of fuzzy logic to transfer experts' evaluation of the factors into a value representing the process capability.

The fuzzy process capability evaluation (FPCE) methodology is introduced in the next section. The third section presents the validation of the proposed methodology and compares it with the ordinary process capability ratio analysis. The chapter ends with discussion and conclusions in the fourth and fifth sections.

## **5.2 FPCR Methodology**

Fuzzy logic principles have been extensively used in transferring human knowledge into intelligent information and formulated mathematically. The nature of

uncertainty in human knowledge is handled by selecting appropriate methods and representing vagueness properly, which is difficult to comprehend and analyze otherwise. In the current research, fuzzy logic is used to formulate the human knowledge about four factors that primarily affect the capability of a process. Each factor is divided into various levels, and experts' opinion is used to evaluate these levels. Different weights are assigned to the factors based on their impact on the process capability. Finally, a numerical value representing process capability is generated. The following steps describe the FPCE approach.

### **Step 1: Defining the Fuzzy Set**

Many factors affect the capability of a process. This research considers the following four important factors which are believed to be the main items that affect the capability of a process: machine capacity (MC), operator skill (OS), tooling condition (TC), and working condition (WC). MC is related to the technical characteristics of a machine, for example, the speed (rpm) range in which the machine can perform well, the vibration level of the machine while it is working, and whether the machine is computer-programmability, etc. All these elements are combined under the MC factor, which is considered as an important factor that influences the capability of a process. In addition, the experience and skill of the operator is another factor that affects the capability of a process. The third factor is the TC. For instance, a carbonized-steel tool has better performance characteristics than a cast-iron tool. Similarly, a brand new tool produces less rejected parts than a worn out tool. Finally, WC is chosen as a fourth factor that affects process capability. All the environmental circumstances of the workplace are summarized under this factor, for example, heating, air conditioning, and ventilation,

lighting intensity, suitable space between machines, etc. Based on the above discussion, the fuzzy set can be defined as

$$U = \{u_1, u_2, \dots, u_m\} = \{MC, OS, TC, WC\} \quad (5.1)$$

where

$u_i$  represents the  $i^{\text{th}}$  fuzzy factor

$m$  is the number of factors

Although only four factors are considered in this research, the technique is not restricted to these four factors. The group of factors can be modified to comply with different operations circumstances and different experts opinions.

### **Step 2: Factor Levels**

According to the experts opinion, each of the four factors discussed above is divided into three levels ( $n_i=3, i=1, \dots, 4$ ). The expert classifies the factor levels based on the rejection rate a process will generate as a result of the status for a specific factor. For example, for a specific task that has given design requirements, the expert evaluates the MC factor as follows, given that all the other factors are at their best evaluation: quite capable if the process rejection rate is less than 1 ppm, capable if its rejection rate is between 50 and 3,000 ppm, and incapable if its rejection rate is higher than 300,000 ppm. Table 5.1 shows expert-level classifications for the four factors. It is clear that there are ranges in the rejection rate scale where the expert can not evaluate the factors crisply. For example, if the rejection rate is 20 ppm, the expert can not precisely evaluate the MC

factor as capable or very capable. Hence, for any rejection rate that lies between the specified brackets, the expert evaluates a factor partially by two levels.

TABLE 5.1  
FACTORS LEVELS

Rejection Rate (ppm)	Factor			
	MC	OS	TC	WC
$\geq 300,000$	Incapable	Insufficient	Bad	Not Suitable
3000-50000	Capable	Sufficient	Good	Suitable
$\leq 50$	Very Capable	Above Sufficient	Excellent	Quite Suitable

In order to formulate the expert judgmental evaluations, fuzzy membership functions must be assigned to the different levels of each factor. The trapezoidal membership function is selected to represent this fuzzy nature. The reason for this selection is that for each level there is a range over which the factor can be described by only one level. In this range, the level membership value is one, i.e., full membership value. The area between two levels is considered fuzzy since it has partial membership values from the two levels. Figure 5.1 shows the fuzzy membership function for the three levels of the MC factor. Notice that for any MC evaluation higher than 80%, the process is expected to generate  $\leq 1$  ppm as rejection, given that all the other three factors are at their 100% performance. Similarly, a 40% to 60% evaluation means that a process is expected to generate 50 to 3,000 ppm as rejection. The membership functions for the OS, TC, and WC factors are shown in Figures 5.2, 5.3, and 5.4, respectively.

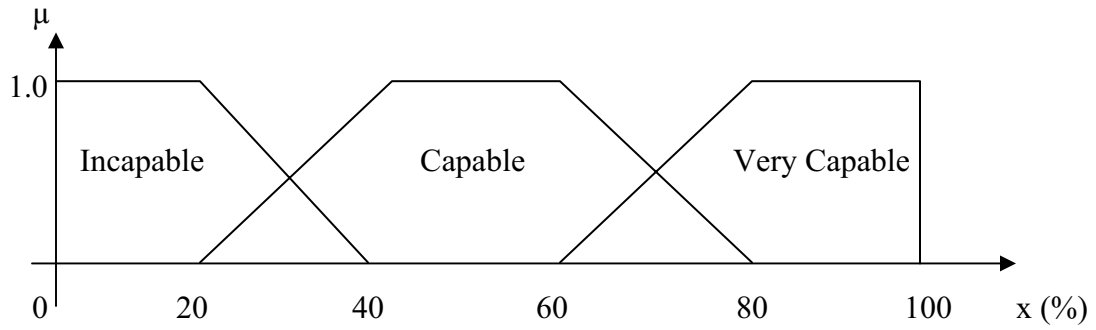


Figure 5.1: Fuzzy membership function of the machine capacity levels

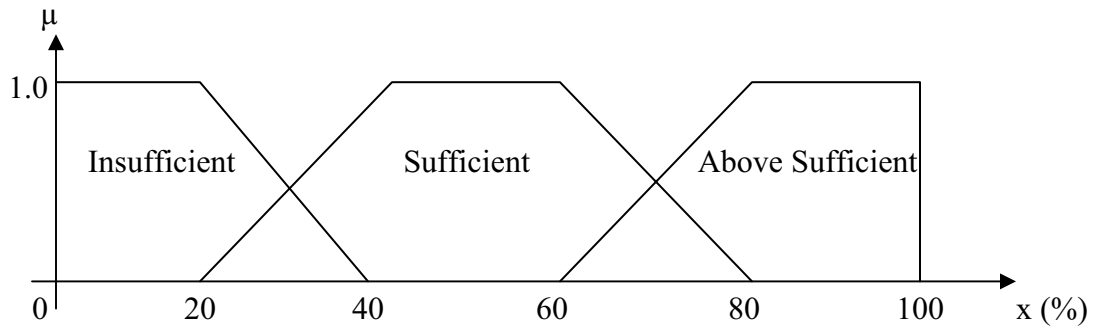


Figure 5.2: Fuzzy membership function of the operator skill levels

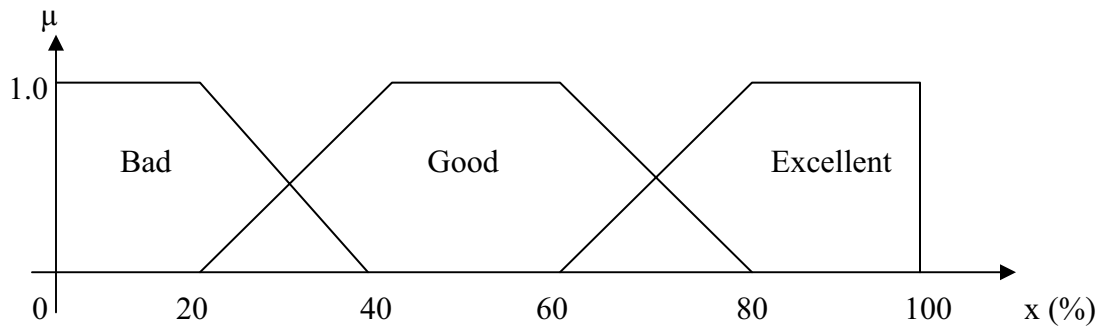


Figure 5.3: Fuzzy membership function of the tooling condition levels

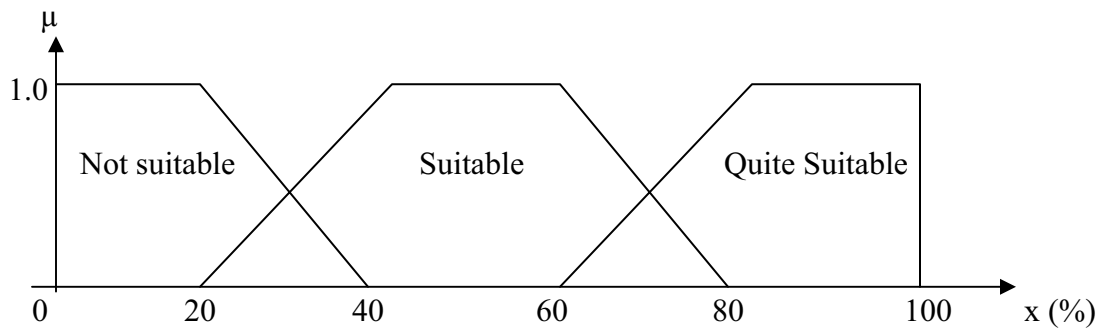


Figure 5.4: Fuzzy membership function of the working condition levels

Based on the above discussion, the fuzzy factors can be written such that

$$u_i = \{u_{i1}, u_{i2}, \dots, u_{ij}\}, i = 1, 2, \dots, m; j = 1, 2, \dots, n_i \quad (5.2)$$

where

$u_{ij}$  denotes the  $j^{\text{th}}$  fuzzy level of the  $i^{\text{th}}$  factor

$n_i$  represents the number of levels for the  $i^{\text{th}}$  factor

### **Step 3: Establishing the Evaluation Set**

In order to impose a numeric measure for process capability evaluation, process capability is divided into ten equally spaced grades, namely

$$\xi = \{\xi_1, \xi_2, \dots, \xi_p\} = \{10\%, 20\%, \dots, 100\%\}, p = 10 \quad (5.3)$$

where:

$\xi$  is the fuzzy evaluation set

$p$  is the number of process capability grades

Note that  $\xi$  measures the degree that a process belongs to a specific category of capability.

### **Step 4: First-Order Fuzzy Evaluation Matrix**

Based on the expert's experience, the first-order fuzzy evaluation matrix for each factor can be given as



$$R_i = \begin{bmatrix} r_{i11} & r_{i12} & \dots & r_{i1p} \\ r_{i21} & r_{i22} & \dots & r_{i2p} \\ r_{i31} & r_{i32} & \dots & r_{ijp} \end{bmatrix}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n_i, \quad p = 10 \quad (5.4)$$

where  $r_{ijp}$  is the fuzzy membership value for level  $j$  in grade  $p$  for factor  $i$ .

Therefore, the first-order matrices for the four factors are as follows:

$$R_1 = \begin{bmatrix} 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 \end{bmatrix}$$

Note that these matrices are correlated with the fuzzy membership functions shown in Figures 5.1 to 5.4. They represent the fuzzy degree assignments to the three grades of the four factors defined in Table 5.1. For example, fuzzy representation for the first level in the MC factor is given as: (1.0, 1.0, 0.5, 0.2, 0, 0, 0, 0, 0, 0). This means that the fuzzy membership value for the first level of factor is full (equal to one) in the first grade of process capability grades (at  $p=1$ ). Similarly, this membership value is equal to: 1, 0.5, 0.2, and 0.0 at  $p=2, 3, 4,$  and  $5, \dots, 10,$  respectively.

It is worth mentioning here that all classifications shown in Table 5.1 and the fuzzy representations given in Figures 5.1 to 5.4 are somewhat empirical and can be modified to comply with different expert assessments.

### Step 5: Establishing Weight Vector of Each Grade

In the case where a factor has some levels with no full membership value in its membership function, a weight vector of for this factor is given as:

$$A_i = (a_{i1}, a_{i2}, \dots, a_{in_i}) \quad (5.5)$$

$$\text{where } a_{ij} = \frac{r_{ijp}}{\sum_{j=1}^{n_i} r_{ijp}} \quad (5.6)$$

Note that for factor  $i$ ,  $a_{ij}$  is a weighted membership value for grade  $j$  with respect to other grades in the same factor.

### Step 6: First-Order Fuzzy Evaluation

When the fuzzy evaluation is made for every grade of the  $i^{\text{th}}$  factor, the first-order fuzzy evaluation set for that factor is obtained by

$$B_i = A_i \cdot R_i = (a_{i1}, a_{i2}, \dots, a_{in_i}) \begin{bmatrix} r_{i11} & r_{i12} & \dots & r_{i1p} \\ r_{i21} & r_{i22} & \dots & r_{i2p} \\ r_{i31} & r_{i32} & \dots & r_{i3p} \end{bmatrix} = (b_{i1}, b_{i2}, \dots, b_{ip}), \quad (5.7)$$

By applying the same procedure for all the factors, the first-order fuzzy evaluation matrix of a process can be given as

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \dots \\ B_m \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mp} \end{bmatrix} \quad (5.8)$$

**Step 7: Determining Factors Weight Vector**

In practice, not all factors (MC, OS, TA, and WC) are equally important. Hence, the second weighting vector, W, is introduced as

$$W = (w_1, w_2, \dots, w_m) \quad (5.9)$$

From a mathematical point of view, W represents the degree of importance of the factors and can be determined by the expert.

**Step 8: Second-Order Fuzzy Evaluation**

By combining W and B, the second-order fuzzy evaluation is obtained as

$$F = W.B = (w_1, w_2, \dots, w_m) \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mp} \end{bmatrix} = (f_1, f_2, \dots, f_p) \quad (5.10)$$

**Step 9: Determining Process Capability**

Finally, by combining equations (5.3) and (5.10), the process capability is determined using the weighted average method as

$$\varepsilon = \frac{\sum_{p=1}^{10} f_p \xi_p}{\sum_{p=1}^{10} f_p} \quad (5.11)$$

According to the above steps, a dimensionless percentile numerical evaluation ( $\varepsilon$ ) can be obtained for a process capability. The value of  $\varepsilon$  is associated with the implemented membership functions for the four factors. Therefore, for  $\varepsilon$  with 0% to 20% value, the rejection rate is expected to be  $\geq 300,000$  ppm. Similarly, for  $\varepsilon$  with 40% to 60% value, the rejection rate is expected to be between 50 and 3,000 ppm, and for  $\varepsilon$  with 80% to 100% value, the rejection rate is expected to be  $\leq 1$  ppm. For any  $\varepsilon$  value that lies between these brackets, linear interpolation can be used to find the expected rejection rate.

### **5.3 FPCE Validation**

In order to validate the applicability of the presented FPCE approach, the simple workpiece shown in Figure 5.5 was produced. This workpiece was produced based on two scenarios. In the first scenario, two technicians (A and B) produced forty units of this workpiece, twenty units for each. Other than different operators, all other production inputs (machine, tool, and working conditions) remained identical. The objective of this scenario was to test the affect of the operator skills on the process capability. The second scenario was used to test the tool affect on the process capability. In this scenario, one technician produced forty parts using two different tools (I and II), twenty for each tool. Similar to the first scenario, all other factors than the tool factor were identical. After the production, dimension X was measured. Table 5.2 shows the mean and standard deviation for the collected data. The USL and LSL for X are 1.01'' and .99'', respectively. Next, both the PCR and FPCE approaches were applied to the X dimension.

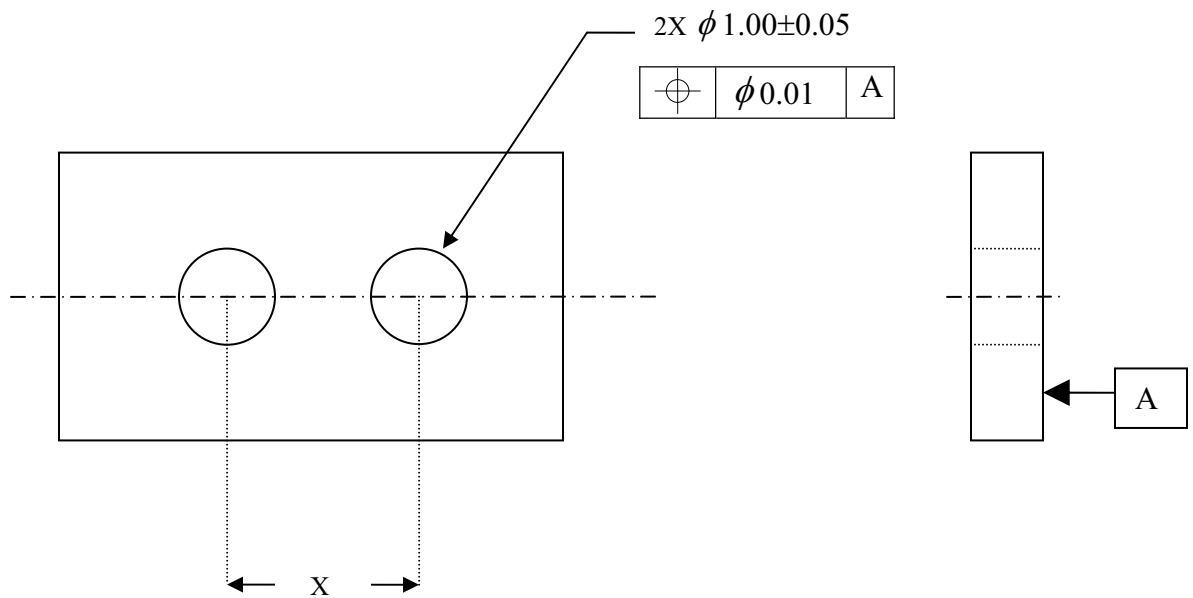


Figure 5.5. The workpiece

TABLE 5.2

RESULTS FOR DIMENSION X

	First Scenario		Second Scenario	
	Operator A	Operator B	Tool I	Tool II
Mean (m)	0.9988	0.9983	0.9988	0.9975
Standard Deviation (s)	0.0029	0.0036	0.0029	0.0043

### 5.3.1 PCR Estimation

Three steps are needed for the estimation of the PCR (Chandra, 2001):

- Collect data: this step has been already conducted. Moreover, the control chart was drawn to ensure that the process is in statistical control during the period of data collection.
- Analyze data: tests were conducted to ensure that the collected data is free from outliers and that the normality assumption is valid.
- Calculate process capability ratio: As shown in Table 5.2, the process is not centered. Hence, the  $C_{pk}$  is used to calculate the PCR.  $C_{pk}$  is given as

$$C_{pk} = \min(C_{pU}, C_{pL}) = \min\left\{ C_{pU} = \frac{USL - \mu}{3\sigma}, C_{pL} = \frac{\mu - LSL}{3\sigma} \right\} \quad (5.11)$$

In this expression, both the population mean and variance ( $\mu$  and  $\sigma$ ) are to be estimated. The unbiased estimators for these parameters are  $\bar{x}$  and  $S/c_4$ , respectively.

Therefore, the estimate of  $C_{pk}$  is

$$C_{pk} = \min(C_{pU}, C_{pL}) = \min\left\{ C_{pU} = \frac{USL - \bar{x}}{3 \frac{S}{c_4}}, C_{pL} = \frac{\bar{x} - LSL}{3 \frac{S}{c_4}} \right\} \quad (5.12)$$

where  $c_4$  is a constant that can be obtained from a typical control chart constant's-tables.

For a sample size of 20,  $c_4 = 0.9870$ . Therefore, using equation (5.12),  $C_{pk}$  values are as shown in Table 5.3.

TABLE 5.3  
THE  $C_{pk}$  FOR DIMENSION X UNDER DIFFERENT CONDITIONS

	$C_{pk}$
First Scenario, Operator A	0.9769
First Scenario, Operator B	0.7556
Second Scenario, Tool I	0.9769
Second Scenario, Tool II	0.5669

Under the normality assumption, the rejection rates for the four cases can also be found. For example, for the “first scenario, operator A”, the rejection rate is equal to the following

$$\text{Rejection rate} = \text{Prob. } (X > \text{USL}) + \text{Prob. } (X < \text{LSL})$$

Using the means and standard deviation values shown in Table 5.2, the rejection rates for the four cases can be found, as shown in Table 5.4.

TABLE 5.4  
THE REJECTION RATES (R) FOR DIMENSION X UNDER DIFFERENT CONDITIONS

	Rejection Rate (r), ppm
First Scenario, Operator A	5172
First Scenario, Operator B	5998
Second Scenario, Tool I	5172
Second Scenario, Tool II	7961

### 5.3.2 FPCE Approach

The FPCE methodology described in section 5.2 is applied here for the production of the dimension X shown in Figure 5.5. The FPCE approach is applied for the two scenarios: different operators and different tools.

#### **First Scenario (Different Operators)**

In order to test the proposed method, the expert's assessments for MC, OS, TC, and WC were acquired as follows:

Operator A case: {50%, 90%, 70%, 90%}

Operator B case: {50%, 60%, 70%, 90%}

Since all production parameters are identical except the operator, only the operator evaluation is different in the above assessments. Next, using the fuzzy membership functions shown in Figures 5.1 through 5.4, the membership values for the expert assessment, for the two cases, are shown in Table 5.5. For example, the 70% expert assessment for the MC factor lies exactly in the middle between the second and third (II and III) levels. Hence, a partial 0.5 membership value for each level is assigned to both level II and level III.



TABLE 5.5

MEMBERSHIP VALUES OF THE EXPERT'S ASSESSMENT FOR THE TWO CASES

Factor	Levels of Each Factor, Operator A			Levels of Each Factor, Operator B		
	I	II	III	I	II	III
MC	0.5	0.5	0.0	0.5	0.5	0.0
OS	0.0	0.0	1.0	0.0	1.0	0.0
TC	0.0	0.5	0.5	0.0	0.5	0.5
WC	0.0	0.0	1.0	0.0	0.0	1.0

Note that the definition of the fuzzy set, the factor levels, and the evaluation set is identical to the first three steps discussed above. Also, the evaluation matrices given in step four above will be used here. Because all fuzzy membership functions are assumed to have a full membership value at some point in their scale, step five is not necessary here. Then, using equation (5.7), the first-order fuzzy evaluation (step 6) is generated for both cases.

For operator A case,

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.25 & 0.10 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.80 & 1.00 & 1.00 \\ 0.00 & 0.00 & 0.25 & 0.40 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.80 & 1.00 & 1.00 \end{bmatrix},$$

For example,

$$B_1 = A_1 \cdot R_1 = [0.5 \ 0.5 \ 0.0] \cdot \begin{bmatrix} 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 & 0.5 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.8 & 1.0 & 1.0 \end{bmatrix}$$

$$= [0.50 \ 0.50 \ 0.50 \ 0.50 \ 0.50 \ 0.50 \ 0.25 \ 0.10 \ 0.00 \ 0.00]$$

In the same way, for operator B case,

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.25 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.50 & 0.80 & 1.00 & 1.00 & 0.50 & 0.20 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.25 & 0.40 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.80 & 1.00 & 1.00 \end{bmatrix},$$

Then, assuming that the four factors have the following weights:

$W = (0.40, 0.25, 0.20, 0.15)$ , the second-order fuzzy comprehensive evaluation

using equation (5.10) for operator A case will be as follows:

$$F = W.B = [0.40 \ 0.25 \ 0.20 \ 0.15] \cdot \begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.25 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.50 & 0.80 & 1.00 & 1.00 & 0.50 & 0.20 & 0.00 & 0.00 \\ 0.70 & 0.70 & 0.50 & 0.38 & 0.30 & 0.30 & 0.15 & 0.06 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.80 & 1.00 & 1.00 \end{bmatrix}$$

$$F = (0.20, 0.20, 0.25, 0.28, 0.30, 0.30, 0.40, 0.46, 0.50, 0.50)$$

In the same way, for operator B case,

$$F = (0.200, 0.200, 0.375, 0.480, 0.550, 0.550, 0.400, 0.310, 0.250, 0.250)$$

Finally, using equation (5.11), the FPCE value,  $\varepsilon$ , is 64.16% and 55.33% for both “operator A” and “operator B” cases, respectively. That is, the considered process is 64.41% capable of accomplishing the considered job if operator A does the work and 55.33% capable if operator B does the work.

Using the implemented membership functions, 64.14% and 55.33% capable processes mean that these processes are expected to generate 2,389 and 13,975 ppm as rejection rates, respectively. For example, a 60% capable process corresponds to a

3,000 ppm rejection rate. Also, 80% capability corresponds to a 50 ppm rejection rate. By interpolation, 64.14% capability corresponds to 2,389 ppm rejection rate.

### **Second Scenario (Different Tools)**

For this scenario, the expert's assessments were acquired as follows:

Operator A case: {70%, 90%, 70%, 95%}

Operator B case: {70%, 60%, 30%, 95%}

Adopting the same procedure shown in the first scenario, the FPCE value,  $\varepsilon$ , is 64.14% and 58.23% for both "tool I" and "tool II" cases, respectively. These percentages correspond to 2,389 and 7,160 rejection rates.

### **5.4 Discussion**

Comparing the rejection rates resulting from the FPCE approach to the rejection rates of the PCR analysis shows that they are not quite identical. However, both approaches indicated the same trend, that is if operator A executes the job, then less rejection is expected than if operator B does it. Similarly, both approaches indicate the superiority of tool A. It is worth mentioning that the FPCE approach highly depends on the expert's assessment of the considered factors. Hence, accurate assessments are crucial for the efficiency of the approach.

### **5.5 Conclusions**

A new method for evaluating process capability has been introduced. The proposed method avoids the errors that existed in the traditional process capability assessment. Fuzzy logic tools are used to formulate the expert's opinion regarding factors

that affect the performance of a process. In order for the FPCE to produce reliable results, an accurate expert assessment is crucial.

## CHAPTER 6

### CONCLUSIONS AND FUTURE WORK

#### 6.1 Summary and Conclusions

Research in tolerance allocation is carried out in two stages: process planning and design. In the last twenty years, process planning-tolerance allocation has received higher emphasis as industry recognizes that operational tolerance management is a key element in programs for improving quality and reducing overall costs. Yet, the existing methods for allocating tolerances in the process planning stage still incorporates many drawbacks: they use the worst-case performance of a process capability which neglects the stochastic nature of the process capability, they do not consider the value of the product to which tolerances are assigned, and they assume a predetermined single processing equipment for the execution of each manufacturing operation specified in the process plan. This research proposed an integrated tolerance allocation and machine assignment model in which all these issues are considered. Alternately, for the formulation of process capability, another modeling approach was presented in which the process capability was analyzed as a linguistic variable where expert's knowledge is acquired using fuzzy logic tools. All suggested modes were analyzed and applied to a literature-based application example.

Also in this research, a novel approach for the evaluation of process capability based on readily available information and knowledge of experts was generated. The suggested approach considers a number of different factors that may have an effect on the capability of a process. It utilizes the concept of fuzzy logic to transfer the experts' evaluation of these factors into a value representing the process capability. The proposed

technique was validated by comparing its results with the outputs of a typical process capability analysis. The comparison showed the effectiveness of the proposed approach.

## **6.2 Significant Contributions**

In accomplishing its research objectives, this dissertation made several contributions:

1. Proposed a probabilistic approach for treating the process capability issue within the operational tolerance allocation problem. This approach adopted the stochastic nature of process capability. A normal distributed-process capability was presented as an example. The proposed model is able to consider both centered and shifted processes.
2. Proposed a fuzzy approach for treating the process capability issue within the operational tolerance allocation problem. This approach evaluated the capability of a process based on experts' experience. Fuzzy logic tools were used to convert the linguistic expert knowledge into a solvable form.
3. Generated a procedure for in-production product value estimation. Factors that were found to affect this value are: the product's revenue, the product's percentage of completed cycle, the value-added processes through which a product has gone, and the criticality of the product characteristics.
4. Proposed an operational tolerance allocation-machine assignment model that adopted the stochastic nature of process capability, integrated the effect of product value, and considered the availability of multiple machines for the execution of each operation in the process plan. The model introduced a feasibility index, which acts as an indicator of the feasibility of the selected process plan. The

model also could specify the source of infeasibility as well as the required action to be taken, in the event that the process plan is designated as infeasible.

5. Proposed an operational tolerance allocation-machine assignment model that adopted the fuzzy process capability evaluation, integrated the effect of product value, and considered the availability of multiple machines for the execution of each operation in the process plan.
6. Generated a novel approach for the process capability evaluation based on readily available information and the knowledge of experts. The proposed method avoids the errors that existed in the traditional process capability assessment. Fuzzy logic tools were used to formulate the expert-based knowledge of the factors that affect the performance of a process.

### **6.3 Future Work**

This dissertation has solved important issues in the operational tolerance allocation problem. Yet, a large number of problems in this area remain unsolved. Following are two sets of recommendations for future work. The first is directly related to this research work; the second has a broader spectrum.

#### **6.3.1 Directly-Related Future Work Recommendations**

1. Examine process capability with stochastic distributions other than normal distribution.
2. Further develop the fuzzy version of the proposed model using other fuzzy programming methods such as multi-objective fuzzy linear/non-linear programming and interactive multi-objective linear/non-linear with fuzzy parameters programming.

3. Build a simulation model to compare the proposed models that were generated for the “one machine per operation” and “one machine per operation set” cases considering the machine’s parameters, such as setup, speed, and cost.
4. Integrate the proposed FPCE technique into the tolerance allocation problem by using the FPCE for assigning different weights for the process plan operations, for example.
5. Automate the proposed model by implementing the entire procedure using “C” or Java language and connect the system to a CAD environment.

### **6.3.2 Broader-Future Work Recommendations**

1. Integrate the two typical phases of tolerance allocation, design and process planning—a worthwhile area of research examination.
2. Investigate tolerance allocation for geometric tolerances, rather than dimensional tolerances which are the majority of tolerance allocation research.
3. Expand the study of tolerance allocation, which is mostly limited to the design and manufacturing of mechanical parts, to include the electronics manufacturing and nano-manufacturing areas.



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