Application of a Unified Procedure for Continuous-Time and Discrete-time Compensator Design to DC Motor Control

Tooran Emami*[1], John M. Watkins[1] and Richard T. O'Brien[2]

[1]Department of Electrical and Computer Engineering, College of Engineering
[2]Department of System Engineering, U.S. Naval Academy,.

I. Introduction

In last year's GRASP [3], we presented a simple unified design approach, which was independent of the nominal system information, for most standard continuous-time and discrete-time classical compensators. This unified approach is based on a standard root locus design procedure for a proportional-derivative (PD) compensator. In this paper, we demonstrate how the procedures in [3] can be applied to a real system. In particular, we will show how they can be used to design continuous-time and discrete-time lead compensators for shaft position control of a DC motor [2]. In this application, we have used the delta operator [1] to design lead compensators from continuous-time and discrete time frequency responses. Both the continuous-time and discrete-time compensators achieved the design specifications. Procedures for standard compensators (lead, proportional-integral (PI), proportional-integral-derivative (PID), and PI-lead compensators) have been developed, but due to space considerations only the lead (practical PD) procedure is applied in this paper. This paper is organized as follows. In Section II, generalized magnitude and phase criteria are presented. Lead compensator design and an example are presented in Section III. Conclusions are presented in Section IV.

II. Compensator Design

The integrated design procedure using time or frequency domain plant data requires a generalization of the angle criterion from root locus design. The standard closed-loop system is shown in Figure 1 where K is the control gain, $G_c(\gamma)$ is the compensator and $G_p(\gamma)$ represents the plant dynamics. The variable γ has the following relation with the continuous time domain, $\gamma = \begin{cases} s, & \Delta = 0 \\ (e^{s\Delta} - 1)/\Delta, & \Delta \neq 0 \end{cases}$, where s the Laplace is transform variable and Δ is the sampling period [1].

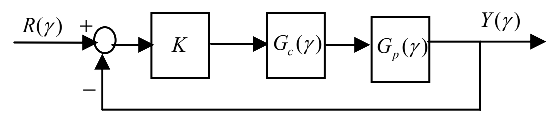

Fig. 1. Closed-loop block diagram

From [3] the generalized angle and magnitude constraints are given by,

$$\begin{aligned} &\angle G_c(\gamma_0) + \angle G_p(\gamma_0) = \phi \\ &K \left| G_c(\gamma_0) G_p(\gamma_0) \right| = 1 \end{aligned} \tag{1}$$

where γ_0 is the unified design point and the desired angle in the angle constraint is $\varphi = \begin{cases} \pm 180°, & \text{root locus} \\ \pm 180° + \text{PM}, & \text{Bode} \end{cases}$, where PM is the desired phase margin for Bode design. The design point γ_0, is given by Table , for our different design procedures, where s_0 is the design point that would be used for continuous time root locus design and ω_{gc} is the desired gain crossover frequency for Bode design.

Table:
Unified design point (γ_0)

	$\Delta=0$	$\Delta \neq 0$
Root Locus	s_0	$(e^{s_0\Delta}-1)/\Delta$
Bode	$j\omega_{gc}$	$(e^{j\omega_{gc}\Delta}-1)/\Delta$

Using the angle constraint in (1), the desired compensator angle θ_c can be computed from the plant information and the design point without knowledge of the compensator type.

$$\angle G_c(\gamma_0) = \phi - \angle G_p(\gamma_0) =: \theta_c . \tag{2}$$

In root locus methods, θ_c determines a geometric relationship between the design point and the compensator poles and zeros. In Bode methods, θ_c is the phase that must be added at the gain crossover frequency.

III. Lead Compensator

The lead compensator in our unified notation has a transfer function $G(\gamma)=\dfrac{\gamma+\alpha}{\gamma+\beta}$, where $\alpha<\beta$. The angle of lead compensator at the design point γ_0 is $\angle G_c(\gamma_0)=\angle(\gamma_0+\alpha)-\angle(\gamma_0+\beta)=\theta_z-\theta_p$ and the lead pole and zero must be selected to satisfy the angle constraint (2), or equivalently, $\theta_c=\theta_z-\theta_p$.

The lead compensator design has three unknowns and only two constraints. As in the continuous-time case, the lead compensator zero is chosen to the right side of PD compensator zero.

$$0\le\alpha_{lead}\le\alpha_{pd}\,. \qquad (3)$$

After the lead zero is chosen the lead pole is computed from

$$\beta=\sigma_0+\frac{\omega_0}{\tan(\theta_p)}\,. \qquad (4)$$

In order to demonstrate the applicability of this method to an experimental system continuous-time and discrete-time compensators were designed to regulate the shaft position of DC motor [2]. The sampling rate for discrete-time compensator was 50 samples/sec ($\Delta=0.02$ sec). The closed-loop step responses of both designs were required to have an overshoot of less than 5%, a settling time of less than 0.175 seconds and no steady state error. From standard second order assumption these specifications correspond to a phase margin of $PM=64.6^o$ and a gain crossover frequency of $\omega_{gc}=25.2$ rad/sec. From (2), the desired compensator angle, θ_c, was found to be 3.6° for the continuous-time compensator and 18.6° for the discrete compensator. A compensator angle greater than zero indicated that a compensator was necessary. The open loop frequency response of the DC motor indicated that the plant was type 1[2]. Thus the steady state error requirement could be met with a PD or lead compensator. As the continuous-time PD compensator is not physically realizable without velocity feedback, a lead compensator was selected for both continuous-time and discrete-time. The compensator design itself was straight forward. The lead zero, which must satisfy (3), was chosen as $\alpha=26$ for both the continuous-time and discrete-time designs. For each design, the pole location β and the control gain K were computed from (4) and (1), respectively. Interestingly enough, the compensator parameters were identical for both designs with the continuous-time and the discrete-time compensators given by $0.445\dfrac{s+26}{s+43.6}$ and $0.445\dfrac{\gamma+26}{\gamma+43.6}$, respectively.

The experimental closed-loop step responses are shown in Figure 2 for a 60° step command. The step response with the continuous–time compensator had an overshoot of 4.8% and a settling time of 0.17 seconds. The step response with the discrete-time compensator had no overshoot and a settling time of 0.12 seconds. As both compensators meet the design specifications, no redesign was necessary.

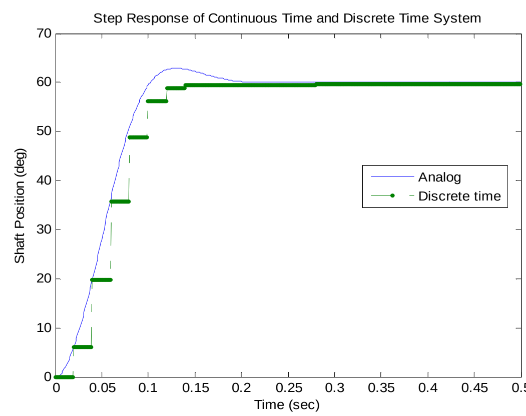

Fig. 2. Continuous-time and discrete-time step responses of compensated DC motor.

IV. Conclusion

The design procedures used in this paper were developed from a PD design procedure [3]. We have shown how our unified procedures can easily apply to a real life systems by designing directly form the experimental frequency responses lead compensators. This example also demonstrates similarity of discrete time design with continuous time when the γ operator is used.

References

[1] R. H. Middleton and G.C. Goodwin, Digital Control and Estimation A Unified Approach, Englewood Cliffs, NJ: Prentice-Hall, 1990.

[2] R.T. O'Brien, Jr. and J.M. Watkins, "A Unified Procedure for Discrete-Time Root Locus and Bode Design," Proceeding of the American Control Conference, Portland, OR, Jun 2005.

[3] T. Emami, j. M. Watkins, "A Unified Procedure for Continuous-Time and Discrete-time Root-Locus and Bode Design," Graduate Research and Scholarly Projects at Wichita State University, 2006.