CONNECTION PROBABILITY IN REFERENCE POINT GROUP MOBILITY MODEL

A Thesis by

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To

My Beloved Parents
ABSTRACT

In ad hoc wireless mobile networks, the mobility of wireless nodes plays a significant role in evaluating the network’s performance. As the nodes are mobile there is a frequent transition between the connections of nodes from up to down state. It is essential to know the likeliness of having a connection between different nodes and the parameters on which the connection probability depends in order to have a better performance of the mobile networks.

This thesis presents such an approach to evaluate the likeliness of having connections amongst the nodes in ad hoc mobile network. In this study a mathematical model is proposed for evaluating one hop connection probability between nodes of different groups. The mathematical model is devised for a specific group mobility model, known as Reference Point Group Mobility model (RPGM) which can be used to simulate the networks where node co-ordinate with each other.

The mathematical model is based on the results of ongoing research in the field of mobile ad hoc networks. It provides a relationship between different network parameters which sufficiently determine the connection probability in reference point group mobility model. This work demonstrates the impact of different parameters on connection probability. Furthermore, the graphical results are used to illustrate the behavior of connection probability in RPGM.

It is found that connection probability of directly connected nodes of two different groups’ increases with decrease in the difference between mean locations of the two groups. Second observation is that as the mobile nodes move with higher velocity, the nodes break their intergroup connections quickly which result in lower connection probability and vice versa. The study also suggests that as range of nodes increases, the connection probability increases.
The mathematical model developed in this thesis work can be used to determine the values of various network parameters for required connection probability in a mobile wireless network. The practical use of this thesis work can be found in determining the connection probability in real world scenarios like disaster recovery where two or more groups need to interact with each other. Another useful application is in military scenario where member are divided in different groups and they interact with each other. In such scenarios the devised model can be used to set values of different network parameters such as velocity of mobile nodes, transmission range etc. to have required connection probability in the network.
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Chapter 1

Introduction

A mobile ad hoc network is an infrastructure-less network. In such networks, wireless devices discover other wireless devices in their range and communicate with them without the use of central access point. Ad hoc wireless networks are a good choice to build a small, all-wireless, inexpensive LAN quickly. Their readily deployable nature has made ad hoc wireless networks quite significant in real world. In the mobile ad-hoc wireless networks, there are frequent transitions in terms of failures and activation of wireless links. As a result the connections are broken and then re-established consistently. These frequent changes pose serious challenges to the performance of the network. As number of devices increase in ad-hoc networks, performance suffers a great deal and measures are required to manage the networks by selecting appropriate network parameters.

Research on mobile ad hoc networks is gaining grounds, and as not many ad hoc networks are deployed, research is mostly simulation based. These simulations have several parameters including those that describe mobility traffic patterns. There have been various mobility patterns developed to predict the future availability of wireless links. These patterns are based on various network parameters which help...
to analyze the network performance. These mobility models simulate ad hoc wireless networks that represent mobile nodes and have several parameters which can be used to evaluate the performance of ad hoc networks before actual implementation.

This research involves the study of behavior of links or connections between different nodes in a network and to predict probability of having stable links between the nodes. As in mobile networks, the location of nodes keep changing as they move, their links with other nodes also keep changing resulting in transitions of links from being up and down. The mobility model being used in this research is Reference Point Group Mobility Model (RPGM). The RPGM model is a group mobility model which simulates the movement of nodes in a group where nodes co-ordinate with each other. The behavior of node movement in RPGM model is studied and a mathematical model is developed to predict the connection probability between nodes of different groups in RPGM model.

1.1 Thesis Contribution

The contribution of this research is in determining the probability of having a connection between nodes where these nodes belong to two different groups in the Reference Point Group Mobility Model. A mathematical model is developed to predict a direct connection between the nodes in RPGM model on selecting appropriate network parameters. The model helps in analyzing the impact of mobility in a network which is illustrated with graphical representations. Another significance of this research is in determining appropriate network parameter values using this mathematical model. These values can be used in designing a mobile ad hoc network, with required connection probability between the nodes before actually implementing the network.
1.2 Thesis Overview

Research in finding the connection probability in Reference Point Group Mobility Model includes the following stages:

• In RPGM model, the relationship of lead node with member nodes is used to find link distance between nodes of different groups. It is assumed that node locations have Gaussian distribution.

• The second stage involves the development of mathematical model for connection probability between nodes that belong to two different groups with the assumption that the two nodes have a direct connection.

• The last stage involves analyzing the results of the proposed model and conclusions of the study of connection probability in RPGM.

The thesis is organized into different chapters where Chapter 2 deals with the literature review of related work in ad hoc research. It explains the significance of mobility models in ad hoc networks and different network parameters that have been studied to evaluate the performance of these ad hoc networks. The analytical model has also been described which is used in this study, as a basis to estimate the connection probability in RPGM. Chapter 3 describes the mathematical model developed to find the connection probability in RPGM. Chapter 4 contributes to the results and evaluation of the proposed mathematical model. Chapter 5 concludes the thesis and suggests future work.
Chapter 2

Related Work

In this chapter, details of previous research in ad hoc networks and how this work has inspired this thesis work has been discussed. The chapter begins with discussion of significance of mobility models and explanation of a specific mobility model, Reference Point group Mobility model which is the main model studied in this research. The next section describes several network parameters used to evaluate the impact of mobility in ad hoc networks. The next section is dedicated to connection probability in a random mobile network and its role played in this research. The last section explains the analytical model for path duration of mobility models and how it has inspired in developing a mathematical model in this thesis work.

2.1 Mobility Models

The mobility models have been developed to simulate the real world ad hoc mobile networks to determine if the proposed solutions will be useful when implemented. The mobility models are used to simulate different network scenarios before actually implementing them. Many mobility models have been developed to simulate the
behavior of mobile nodes. The two types of mobility models are traces and synthetic models:

- Traces: Traces are those mobility patterns that are observed in real life. They provide accurate information especially when a large number of participants are involved and observation period is large.

- Synthetic models: These attempts to realistically represent the behavior of mobile nodes without the use of traces.

All mobility models for ad hoc networks are synthetic mobility models. They can be further divided into two categories:

- Entity mobility models: These models represent multiple mobile nodes whose actions are completely independent of each other. In [1] different entity mobility models have been discussed.

- Group mobility models: In these the mobile nodes’ decision on movement depends on the other mobile nodes in the group. In [1] various group mobility models have been discussed.

In [1], Tracy et al. conclude that to evaluate the performance of ad hoc network, the mobility model should be chosen such that it matches the expected real world scenario because the performance of the ad hoc networks vary significantly with different mobility models. The performance even changes substantially when the same mobility model is used with different parameters. The paper compares network performance of different mobility models and researchers recommends using the Reference Point
Group Mobility model if group mobility model is required and recommends either Random Way Point Mobility model or Random Walk Mobility model in case entity model is required. This section describes different mobility models that have been developed.

- **Entity mobility models**: These are the mobility models which simulate the behavior of individual nodes of group.

- **Random walk mobility model**:
  Random walk mobility model is a model to simulate a mobile network with random directions and speeds. The mathematical description of Random walk mobility model was first given by Einstein [2]. In this mobility model, a mobile node moves from its current location to a new location chosen from predefined ranges of speed and direction, \([\text{speedmin, speedmax}]\) and \([0,2]\) respectively. A mobile node moves for a constant time \(t\) or constant distance \(d\) and at the end of each movement the node changes its speed and direction and then continues along this new path.

  There are many derivatives of random walk mobility model which have been developed including 1-D, 2-D, 3-D, and d-D walks. The 2-D random walk mobility model is of special interest as earth’s surface is modeled using a 2-D representation. The random walk mobility model is a widely used mobility model and is also referred as Brownian Motion. It is a memory less mobility pattern as it retains no knowledge of past locations and speed values. This characteristic generates unrealistic movements such as sudden stops and sharp turns. The similar model developed is random way point mobility model with a difference of including pauses between changes in speed and direction.
• Random waypoint mobility model:

The Random waypoint mobility model includes pause times between changes in direction and/or speed [3]. In this model a mobile node moves from its current location to a destination and then stops at the destination for certain period of time which is the pause time. The mobile node again selects a random destination and moves towards it with a selected speed from range [minspeed, maxspeed]. Here the node again stops for a pause time before starting the processes again.

In most of the performance investigation scenarios the mobile nodes are initially distributed randomly around the simulation area which is not similar when nodes start moving. There exits a high variability in average mobile node neighbor percentage which produces high variability in performance results. The random waypoint model has been used prominently in many studies of ad hoc wireless networks. It can be used to simulate the realistic pattern of movement for example: the conference setting or a museum. The one concern with this mobility model is the straight movement pattern created by mobile node to the next chosen destination.

• Random Direction:

To overcome the problem of density waves in average number of neighbors produced by the random waypoint mobility model, a model called random direction mobility model was developed [4]. In this mobility model, a mobile node chooses a random direction in which it travels similar to random waypoint model. A mobile node travels to the border of the simulation area in that direction and on reaching the boundary, mobile node pauses for specified time, choose another
angular direction and continues the process.

The random direction model is an unrealistic model because it is unlikely that
the people spread themselves in an evenly manner throughout an area like a
building or in a city. It is also unlikely that people will pause only at the
edges of an area. So the modified random direction mobility model [4] has been
developed which allows the mobile nodes to pause and change directions before
reaching the simulation boundary. This is similar to random walk mobility
model with pause times.

- The boundless simulation area:

In the boundless simulation area, a mobile node has a relationship between
its previous direction of travel and velocity with its current direction of travel
and velocity [5]. The boundary of simulation area is handled in a different
way in boundless simulation area mobility model. In all the previous mobility
models the mobile nodes reflect off or stop moving on reaching the boundary of
simulation boundary. In boundless simulation mobility model, a mobile node
on reaching the boundary continues traveling and reappears on the opposite
side of the simulation area.

The boundless simulation area mobility model generates realistic movement
pattern. This is the only mobility model that does not have simulation edge
effects from the performance evaluation as mobile nodes are allowed to travel
unobstructed. The concern of this mobility model is the undesired side effects
due to mobile nodes moving in a torus.

- City section mobility model:
City section mobility model simulates a section of city where ad hoc networks exist and street network is the simulation area [6]. The city being simulated determines the streets and speed limits of those streets. As traveling of the mobile nodes is severely restricted in city section mobility model, it generates realistic movement for a section of a city. In order to simulate the real network, all mobile nodes should be enforced to follow predefined paths which increase the average hop count in simulations compared to other mobility models. City section mobility model needs improvements like including pause times at certain intersections and destinations, incorporate acceleration and deceleration etc. In addition, the model needs further developments like larger simulation area, larger number of streets, other novel path finding areas etc.

- Group mobility models: These are the mobility models which simulate the behavior of nodes which co-ordinate with each other.

- Exponential correlated random mobility model:

One of the first group mobility model is the exponential correlated random (ECR) model[15]. In this mobility model, a motion function is used to create mobile node movements. Given a position (mobile node or group) at time $t$, $b(t)$ is a function to define the next position at time $t+1$, $b(t + 1)$:

$$b(t + 1) = b(t) \exp -\frac{\tau}{\sigma} + (\sigma \sqrt{1 - (\exp -\frac{\tau}{\sigma})^2}) r$$

Where $\tau$ is rate of change of mobile node’s previous location to its new location and $r$ is a random Gaussian variable with variance. The problem with this mobility model is that it is not easy to create given movement pattern by
selecting appropriate values for \((\tau, \sigma)\). There are other mobility models available to improve on this drawback.

- Column mobility model:
  The column mobility model is useful for searching purposes [2]. This model represents the set of mobile nodes that move in a given line or column, which is moving in a forward direction. Initially, a reference grid is defined and all mobile nodes are placed on this grid in relation to its reference point. Each mobile node is allowed to move around its reference point via an entity mobility model. The reference vector is shifted periodically based on an advance vector:

  \[
  \text{New reference point} = \text{old reference point} + \text{advance vector}
  \]

  Where advance vector is a predefined offset that moves the reference grid. It moves using a random angle.

- Nomadic community mobility model:
  As ancient nomadic societies moved from location to location, nomadic community mobility model represents a group of mobile nodes that collectively move from one point to another [7],[2]. Within each community or group of mobile nodes, individuals maintain their personal space. Each mobile node uses an entity model to roam around a given reference point. When reference point changes, all mobile nodes travel to the new area defined by reference point and then begin roaming around the new reference point. Compared to the column mobility model, the mobile nodes in the nomadic community mobility model share a common reference point in column, thus mobile nodes are less restrained in their movement around the defined reference point.
Pursue mobility model:

Pursue mobility model is also defined in [7],[2]. The pursue mobility model attempts to represent mobile nodes tracking a particular target. Example of a scenario where this model can be used is a scenario representing police officers attempting to catch an escaped criminal. The pursue mobility model consists of a single update equation for the new position of each mobile node:

New position = old position + acceleration (target old position) + random vector

Acceleration (target old position) is information on the movement of the mobile node being pursued. Random vector is a random offset for each mobile node being pursued. Random vector is obtained via an entity mobility model.

The column, nomadic community, and pursue mobility models are useful group mobility models for specific realistic scenarios. The movement patterns provided by these three mobility models can be obtained by changing the parameters associated with the reference point group mobility explained in next section.

The next section throws light on the group mobility model, reference point group mobility model which is the basis of this work.

2.2 Reference Point Group Mobility Model

There are various synthetic mobility models available to be used for simulating ad-hoc networks in which the action of multiple mobile nodes is completely independent
of each other like random waypoint mobility model. In some situations it is required to model the co-operative behavior of nodes when they move together in a group for example: in realistic ad hoc mobile network application scenarios such as in a military scenario where group of soldiers are assigned to work together or disaster recovery where the mobile nodes need to interact with each other and exhibit collaborative group behavior. To simulate such scenarios there are different group mobility models available and reference point group mobility model is one such mobility model.

The RPGM model is defined by Xiaoyan Hong et al. in [8]. In RPGM, each group has a logical center whose motion defines the entire group’s motion behavior, including location, speed, acceleration etc. The path of the center determines the trajectory of the group and nodes are uniformly distributed in a group. Each node has a reference point and this reference point follows the group movement. The reference point motion determines the behavior of each node’s independent motion while the motion of the center determines the group motion. RPGM can be used to model various mobility applications. In [8], different uses of RPGM are illustrated:

- **In-place mobility model**: This model illustrates the battlefield situation in which the entire area is divided into different partitions and each group is in charge of one partition.

- **Overlap mobility model**: This model describes an overlap operation in which there are different groups performing different operations within same region. For e.g. disaster recovery area.

- **Convention mobility model**: This models the interaction between exhibitors and attendees. In convention, different groups give their research presentations
in different but connecting rooms and the attendees move from one room to another.

In [9], the group membership of RPGM model is defined using more fundamental characteristic in reference velocity group mobility model. As in RPGM model, the logical reference point is defined for each mobility group whose movement is followed by all the other member nodes in the group. The RPGM model’s group membership is defined by a mobile node’s physical displacement from the group reference center. At time t, the location of the ith node in the jth group is given by:

Reference location: \( Y_j(t) \)
Local displacement: \( Z_{j,i}(t) \)
Local displacement: \( X_{i,j}(t) = Y_j(t) + Z_{j,i}(t) \)

A more fundamental characteristic of a mobility group is given by the similarity of the member nodes’ motion. The membership of the ith node in the jth group is then defined by:

Group velocity: \( W_j(t) \approx P_{i,j}(w) \)
Local velocity deviation: \( U_{i,j}(t) \approx Q_{i,j}(u) \)
Node velocity: \( V_{i,j}(t) = W_j(t) + U_{j,i}(t) \) where \( W_j(t) \) and \( U_{i,j}(t) \) are random variables each drawn from the distribution \( P_{i,j}(w) \) and \( Q_{i,j}(u) \) respectively. In reference velocity group mobility (RVGM) model, the characteristic group velocity serves as a reference velocity for the nodes in the group. RVGM model represents the time derivative of the displacement-based group representation in RPGM model.
This study uses the RPGM model representing the overlap mobility model and the work is discussed in Chapter 3. The next section describes different parameters that are used in simulating the mobility models.

2.3 Network Parameters

The mobility models are simulated to evaluate the network performance by varying different network parameters. The impact of mobility on network performance is analyzed quantitatively and qualitatively by using different network parameters and performance parameters. The various network parameters are number of nodes in a group, transmission range of nodes, number of hops, velocity with which nodes are moving etc. which determine network’s performance. [10] discusses different parameters summarized below:

- Number of link changes: For any pair of nodes i and j, number of link change is number of times the link between i and j changes from down to up.

- Link duration: It is the average duration of link existing between any pair of nodes i and j.

- Path availability: It is the fraction of time during which a path is available between two nodes i and j.

Another parameter which characterizes the effects of mobility in ad hoc networks is average path duration. In simple terms, the average path is defined as sum of links between the two nodes and the research has already been conducted to find the duration for which that path exists. The contribution in this field is made by Fan
Bai et al. [11]. The average path duration is a metric useful to characterize mobility independent of the protocols. Fan Bai et al. [11] defines average path duration as a fraction of time for which path is available between nodes i and j. The path duration for a path $P = (n_1, n_2, ..., n_k)$, consisting of k nodes, at time $t_1$, is the length of the longest time interval $[t_1, t_2]$, during which each of k-1 links between the nodes exist. Specifically at time $t_1$, path duration is the minimum of the durations of the k-1 links $(n_1, n_2)(n_2, n_3)(n_{k-1}, n_k)$ at time $t_1$. Formally,

$$PD(P, t_1) = \min_{1 \leq z \leq k-1} LD(n_z, n_{z+1}, t_1)$$

The average path duration parameter was used by Fan Bai [11] to develop an analytical model for path duration in a mobile network explained in section 2.5. The model was developed with assumption that the path duration for the mobility model is exponentially distributed. The model was generic to all mobility models and in the present work, the effort has been made to apply this analytical model specific to Reference Point Group Mobility model. Another interesting contribution is studied in next section which inspires the use of connection probability in this study.

### 2.4 Connection probability in Random Mobile Network

Leonard E. Miller et al. in [12] find connection probability in a random mobile network where he states that probability that a connection between two nodes has an acceptable transmission quality is given by the probability of link distance $r$ being less than transmission range. In Miller’s research, they find connection probability of nodes for a network deployment where $x$ and $y$ co-ordinates of the mobile locations
have Gaussian distributions. In [12], Miller assumes that x and y co-ordinates of mobile locations have independent zero-mean Gaussian distributions with $\sigma_1$ and $\sigma_2$ as standard deviations of the x and y co-ordinates respectively. Thus the link distance $r$ between the nodes is given by $r = \sqrt{d_x^2 + d_y^2}$ where x and y components of link distances. The results of [12] have been applied in this work to find the connection probability in Reference Point Group Mobility model which is described in the next chapter. The following section discusses the analytical model for path duration from which the idea to develop mathematical model specific to RPGM has been inspired.

2.5 Analytical Model for Path Duration

The study has already been conducted by Fan Bai et al.[12] which tells about the parameters that affect the path duration and Fan Bai assumes that the path duration has exponential distribution with parameter $\lambda_{\text{path}}$. The parameter $\lambda_{\text{path}}$ further depends on different network parameters such as number of hops (h) between the nodes, average speed (V), transmission range (R) of nodes in a network. The $\lambda_{\text{path}}$ between any two nodes increases if the number of hops (h) between nodes increases. This is because the probability of breaking a connection increases with increase in number of hops and hence the path duration decreases. Also with increase in the speed (V) of the nodes in a network, $\lambda_{\text{path}}$ increases and hence the probability of nodes being in each others range decreases and thus probability to have a connection decreases. This analytical model also suggests that with increase in transmission range, $\lambda_{\text{path}}$ decreases but the probability of nodes being connected to each other increases and thus path duration increases. To sum up, the average path duration
given by inverse of $\lambda_{path}$ and

$$\lambda_{path} = \lambda_0 \frac{hV}{R}$$  \hspace{1cm} (2.5.1)

where $\lambda_0$ is a constant of proportionality.

The above model is a generic model for any mobility model which gives the path distribution to be exponential. Inspiring from this analytical model, an effort as been made to devise a mathematical model specific to Reference Point Group Mobility model. This new mathematical model gives the probability of having a one hop connection among the nodes of different groups in a network. In the previous work, the parameter studied is path duration and analysis is performed to evaluate the probability that a particular path (connection probability) exists in the network. In the present work, the parameter selected is connection (link) between nodes and a relation is developed to predict connection probability and its dependence on different network parameters are used. The next chapter discusses the mathematical model developed in this research.
Chapter 3

Connection Probability in RPGM

In this chapter, a mathematical model is developed by using the ideas and concepts of earlier research as explained in Chapter 2. The chapter begins with a discussion of RPGM model used in this study. The mathematical expression is derived for node locations in RPGM model. Next, is the modification of previous analytical model [11] and derivation of distribution of link distance between the nodes. This is followed by discussion of developing a mathematical model which predicts the one hop connection probability between nodes of different groups in RPGM.

In this study the RPGM model used comprises of two distinct groups interacting with one another where each group has a lead node (logical center). The lead nodes’ motion determines the motion of the whole group. The member nodes in a group follow the lead node and their location can be determined from the lead nodes’ location. As mentioned earlier, the node displacement of ith node in group j is given by:

\[ X_{i,j}(t) = Y_j(t) + Z_{j,i}(t) \]

where \( Y_j(t) \) and \( Z_{j,i}(t) \) are Gaussian random variables defined as follows:

\[ Y_j(t) = \eta(\mu_j, \sigma_j^2) \]

which is a Gaussian distribution for displacement of lead node
of jth group and $Z_{i,j}(t) = \eta(\mu_{ij}, \sigma_{ij}^2)$ which is a Gaussian distribution for local displacement of ith node of jth group.

In RPGM model, the location of any node is given by the sum of lead node location (logical center of the group) and the node’s local displacement (displacement of reference point of that node). Therefore, x component of location of ith member node in jth group is given by:

$$x_{ij} = \eta(\mu_{Yxj}, \sigma_{Yxj}^2) + \eta(\mu_{Zxij}, \sigma_{Zxij}^2) \quad (3.0.1)$$

$$x_{ij} = \eta(\mu_{xij}, \sigma_{xij}^2) \quad (3.0.2)$$

and y component of location of ith member of jth group is given by:

$$y_{ij} = \eta(\mu_{Yyj}, \sigma_{Yyj}^2) + \eta(\mu_{Zyij}, \sigma_{Zyij}^2) \quad (3.0.3)$$

$$y_{ij} = \eta(\mu_{yij}, \sigma_{yij}^2) \quad (3.0.4)$$

The idea in this study is to use the analytical model given in section 2.5. The average path duration is found for RPGM by modifying the model with the usage of parameters specific to RPGM which are discussed in section 2.2.

In the average path duration equation from section 2.5, we have

$$\lambda_{path} = \lambda_0 \frac{hV}{R} \quad (3.0.5)$$
The relative speed \( V \) of nodes is replaced by displacement of nodes per unit time, using the convention of Reference Velocity Group mobility model (RVGM). The modified equation is as follows:

\[
\lambda_{\text{path}} = \lambda_0 \frac{h \Delta d_{12}}{\Delta T R} \tag{3.0.6}
\]

\( \Delta d_{12} \) where \( \Delta d_{12} \) represents the distance between the two mobile nodes of group 1 and group 2 for every \( \Delta T = 1 \) second. So the equation changes to the following:

\[
\lambda_{\text{path}} = \lambda_0 \frac{h \Delta d_{12}}{R} \tag{3.0.7}
\]

For this study \( \lambda_0 \) is ignored, which is a constant determined by map layout, node density etc. Now the distribution of link distance is determined between two nodes assuming that node locations have Gaussian distribution and then relation is developed based on the following condition. When \( \Delta d_{12} < R \), it is assumed that the nodes have a direct connection (\( h=1 \)) with each other. This condition is used to determine one hop connection probability in this mathematical model.

To find the distribution for \( \Delta d_{12} \), we find \( d_{x12} \) which is \( x \) component of link distance between one node of group 1 and other node of group 2 and also find \( d_{y12} \) which is a \( y \) component of link distance between a node of group 1 and other node of group 2. The value of \( d_{x12} \) is given by:

\[
d_{x12} = x_{i2} - x_{i1} \tag{3.0.8}
\]

\[
= \eta(\mu_{x12}, \sigma_{x2}^2) - \eta(\mu_{x1}, \sigma_{x1}^2) \tag{3.0.9}
\]

\[
= \eta(\mu_x, \sigma_x^2) \tag{3.0.10}
\]
where

\[ \mu_x = \mu_{x_2} - \mu_{x_1} \tag{3.0.11} \]

and

\[ \sigma^2_x = \sigma^2_{x_2} + \sigma^2_{x_1} \tag{3.0.12} \]

Similarly,

\[ d_{y12} = y_{i2} - y_{i1} \tag{3.0.13} \]

\[ = \eta(\mu_{y12}, \sigma^2_{y2}) - \eta(\mu_{y1}, \sigma^2_{y1}) \tag{3.0.14} \]

\[ = \eta(\mu_y, \sigma^2_y) \tag{3.0.15} \]

where

\[ \mu_y = \mu_{y2} - \mu_{y1} \tag{3.0.16} \]

and

\[ \sigma^2_y = \sigma^2_{y_2} + \sigma^2_{y_1} \tag{3.0.17} \]

Now, \( \Delta d_{12} \) is distance between two nodes of group 1 and 2 given by:

\[ \Delta d_{12} = \sqrt{d^2_{x12} + d^2_{y12}} \]

Using [13], we get that \( \Delta d_{12} = z \) follows a rician distribution where \( z \) is a rician random variable representing \( \Delta d_{12} \). Then the PDF of \( z \) is given by

\[ f(z) = \frac{z}{\sigma^2} e^{-\frac{(z^2 + \mu^2)}{2\sigma^2}} I_0 \left( \frac{z \mu}{\sigma^2} \right) \tag{3.0.18} \]

where, \( I_0 \left( \frac{z \mu}{\sigma^2} \right) \) is called the zero-order modified Bessel function of the first kind ; \( \mu = \sqrt{\mu^2_x + \mu^2_y} \);
and \(
\sigma^2 = \sigma^2_x + \sigma^2_y
\)

This PDF is a Rician probability density function. When \( z\mu >> \sigma^2 \), then the approximation is used where \( I_0 \left( \frac{z\mu}{\sigma^2} \right) \approx \frac{e^{z\mu/\sigma^2}}{(2\pi z\mu/\sigma^2)^{1/2}} \). As \( z \) represent the distance, we replace \( z \) by \( vt \) where \( v \) is speed and \( t \) is time.

\[
z = vt
\]

Differentiating both sides by \( t \) we get

\[
dz = v \, dt
\]

So (3.0.18) modifies as follows:

\[
f(z)dz = (vt)e^{-((vt)^2+\mu^2)/2\sigma^2} \frac{e^{vt\mu/\sigma^2}}{(2\pi vt\mu/\sigma^2)^{1/2}} vdt
\]

(3.0.19)

To find the one hop connection probability eqn is integrated as follows

\[
\int_0^R f(z)dz = \int_0^{R/v} v(\sqrt{vt})^{1/2} e^{-(vt-\mu)^2/2\sigma^2} \frac{e^{vt\mu/\sigma^2}}{(2\pi vt\mu/\sigma^2)^{1/2}} dt
\]

(3.0.20)

The next chapter discusses the results of the equations developed for connection probability using different set of parameters and explains the behavior of connection probability with different parameters.
Chapter 4

Results

This chapter discusses the results of the mathematical model developed in the previous chapter. Various parameters are substituted to analyze the impact of different parameters on the connection probability. This chapter is divided into two sections, the first section describes different parameters and the values that have been used to analyze their impact. The second section describes the impact of those parameters.

4.1 Graphical Representations

• Transmission range (R): The transmission range is the acceptable distance between two nodes to have a connection. The transmission range (R) in this model is measured in meters. The transmission range is varied over wide range to observe the effect in different scenarios as explained in the next section.

• Velocity: Relative velocity is the group velocity of one group relative to the second group. The velocity is measured in meters/sec in his study. These velocity values are varied to analyze their impact on connection probability. The higher velocity values indicate that the groups, and the nodes in the groups
are moving faster while the lower velocity values indicate that the nodes in the
group are moving slower.

• Mean: Resultant mean is the difference in locations of lead nodes of the two
groups and the resultant mean is measured in meters. The variation of resultant
mean indicates the distance between the groups, the higher value of resultant
mean indicates greater distance between the groups and vice versa.

4.2 Impact of different parameters

This section discusses the impact of each parameter used in the mathematical model
on connection probability in RPGM. The results are supported with graphical rep-
resentations and there are different sets of values of parameters are used to analyze
their impact:

• Impact of velocity: To analyze the impact of velocity, the following values are
used. The transmission range is taken as R=800m while the maximum z is
taken as 1000m which means that the maximum distance that the groups can
be apart is 1000m while they have the connection if the distance between them
is less or equal to 800m. The resultant mean is taken as 100m, which means
that at the starting the groups are placed in such a way that average distance
between lead nodes of the two groups is 100 m. The velocity is varied from
5m/sec to 25 m/sec to observe the effect of velocity on connection probability.

Figure 4.1 shows variation of connection probability with velocity. With velocity
5m/sec, the probability value is 0.7998 and it decreases with increase in velocity and these results are consistent with the behavior of Reference point group mobility model. The increase in velocity indicates that the nodes are moving fast and their likelihood of nodes of one group to be in range of the nodes of the second group decreases while if the nodes are moving slower then the likelihood of the nodes to be in range increases. So the results of the mathematical model are as expected behavior of RPGM. As velocity increases the one hop connection probability of the nodes of different groups decreases.

- Impact of mean: To analyze the impact of resultant mean location, the values used are as follows: Three sets of values are used with three different velocity values: v=5m/sec, 10m/sec and 15m/sec. Transmission range(R) is 800m, the maximum boundary z=100m. The mean locations are varied from 100m to
900m. The plots are as shown below:

![Figure 4.2: Connection probability variation with mean with velocity 5m/sec](image)

In Figure 4.2, the plot shows the variation of one hop connection probability with mean. The velocity is taken as 5m/sec, the one hop connection probability with mean 100m is 0.7992, the plot suggests that as the probability decreases as the mean increases. The results can be explained from the way reference point group mobility model behaves. The resultant mean location is the distance between two lead nodes of different groups and the member nodes motion’ are related to lead node in RPGM model. The higher value of mean depicts the distance between the lead nodes is more and so is the distance between the nodes of different groups. Now when the nodes move the likelihood of the nodes to be in each others range depends on the distance they are apart. So higher the resultant mean location the lesser the likelihood of nodes to have a connection.
The one hop connection probability decreases with increase in resultant mean location. The following two graphs have been plotted with different velocities:

The plots in Figures 4.3 and 4.4 show connection probability variation with mean for velocity 10m/sec and 15m/sec respectively, the plots show the variation of one hop connection probability with mean. As explained earlier the one hop connection probability decreases with resultant mean location in RPGM model.

The two plots in figure 4.3 and 4.4 also suggest that as velocity increases the one hop connection probability decreases.
Figure 4.4: Connection probability variation with mean with velocity 15m/sec

The plot in figure 4.3 with velocity 10m/sec shows that as mean increases, the one hop connection probability decreases from 0.7981 to 0.6714 with mean increasing from 100 m to 900 m.

In Figure 4.4 with velocity 15m/sec, the one hop connection probability decreases from 0.5675 to 0.2546 and the values for connection probability are lower with velocity 15m/sec than velocity 10m/sec.

- Impact of transmission range (R) : The following figure shows the impact of transmission range on one hop connection probability. The graph is plotted with velocity 5m/sec, mean is 100m and the transmission range varies from 5m to 450 m.
Figure 4.5: Connection probability variation with range

Figure 4.5 shows that as the transmission range increases, the one hop connection probability increases and this observation is in consistence with the behavior of reference point group mobility model. As the transmission range is higher so the likelihood of the nodes of being in each others node would be higher. As even when the groups are apart the nodes of different groups would have higher probability to have one hop connection with each other.

In summary, it is observed that as the velocity increases, one hop connection probability decreases. With increase in resultant mean location, the probability of having one hop connection between the nodes of two different groups’ increases. The increase in transmission range results in higher one hop connection probability in reference point group mobility model.
Chapter 5

Conclusion and Future Work

The work in this thesis suggests a mathematical model which is useful in determining the connection probability by simulating the ad hoc wireless network in Reference point group mobility model. The mathematical model can also be used to determine different parameter values required in ad hoc networks before actually implementing it. The results of this mathematical model fit well with the characteristics of RPGM model. The variation of connection probability with mean, velocity and transmission range is analyzed and the reasoning is provided how the behavior revealed by this mathematical model is in consistence with expected behavior RPGM model.

The mathematical model gives a relation to evaluate the connection probability of directly connected nodes. This mathematical model could also be extended to evaluate the connection probability of nodes with relay of nodes in between. The parameters selected are characteristic to RPGM, using similar approach, the idea can be incorporated in finding connection probability specific to other mobility models.
REFERENCES
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