LINEAR QUADRATIC REGULATOR DESIGN FOR DOUBLY FED INDUCTION GENERATOR USING SINGULAR PERTURBATION TECHNIQUES

A Thesis by

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The following faculty members have examined the final copy of this dissertation for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Electrical Engineering.

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John Watkins, Committee Chair

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Edwin Sawan, Committee Member

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Gamal Weheba, Committee Member
DEDICATION

To my beloved parents, brother, sister, my husband and my dearest son
I would like to extend my deepest appreciation to my advisors, Drs. Edwin Sawan and John Watkins for their guidance during last three years. I would like to thank Dr. Edwin Sawan for his encouragement and support which helped me to complete my research. His continuous and convincing direction not only as my advisor but also as a spirited teacher motivated me to accomplish my goals during my graduate studies. Mentoring from Dr. John Watkins helped me to complete my graduate studies as I planned. I would like to thank both of them for enlightening me about the many concepts of control systems.

My sincere gratitude goes to my dissertation committee member Dr. Gamal Weheba who helped me to improve my technical writing skills. Thank you for your time and support with my research and graduate studies at various stages.

My friends Shilpa Kauthekar, Frederick Segbefia, John Doffing, Zhuuxing Hu, Trevor Hardy, Siva Veeramachaneni and Dr. Geethalakshmi S. L. made my studies enjoyable and educational. They were always willing to share their professional knowledge and help and advise me whenever needed.

I would like to thank my parents, my brother and my sister for their unconditional love, encouragement and continuous support, which were instrumental in this endower.

Last but not least, I extend my sincere acknowledgement to everyone who helped me during my graduate studies at Wichita State University. I would like to apologize for not thanking everyone personally.
ABSTRACT

Doubly fed induction generators (DFIG) are widely used in wind power generation because of their ability to be operated at varying rotational speeds while producing power output at a constant frequency. Electrical dynamics of a DFIG is modeled using field oriented control and represented as fourth order system. This fourth order dynamics exposes a two-time scale behavior. Using singular perturbation techniques the time-scales can be separated as slow and fast subsystems. Feedback control schemes can be designed and the closed-loop stability of each model can be compared.

In this work, a linear quadratic feedback controller is designed for the DFIG electrical dynamics using exact, reduced order and composite models. The performances of the closed loop models are compared based on the system cost. The robustness and reliability of the control schemes are analyzed for each controller designs based on the nominal system.

Based on the analysis and results, the reduced order controller performance is equally as good as the exact and composite designs during steady state operations.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Chapter 1 INTRODUCTION</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Doubly-fed Induction Generator (DFIG)</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Literature Review</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Motivation</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Contribution of this Thesis</td>
<td>5</td>
</tr>
<tr>
<td>1.5 Summary &amp; Organization of the thesis</td>
<td>5</td>
</tr>
<tr>
<td><strong>Chapter 2 THEORETICAL BACKGROUND</strong></td>
<td>6</td>
</tr>
<tr>
<td>2.1 System State Space</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Feedback Control</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Model linearization via state feedback (input-state linearizable)</td>
<td>7</td>
</tr>
<tr>
<td>2.4 Similarity Transformation: State transformation to Jordan block for systems with pairs of complex conjugate roots</td>
<td>7</td>
</tr>
<tr>
<td>2.5 Linear Quadratic (LQ) Problem</td>
<td>8</td>
</tr>
<tr>
<td>2.5.1 Infinite Time Horizon – Linear Quadratic Regulator (LQR)</td>
<td>9</td>
</tr>
<tr>
<td>2.5.2 LQR with Cross-Product term</td>
<td>10</td>
</tr>
<tr>
<td>2.6 Singular Perturbation approach</td>
<td>10</td>
</tr>
<tr>
<td>2.6.1 Slow subsystem (Reduced order model)</td>
<td>11</td>
</tr>
<tr>
<td>2.6.2 Composite system</td>
<td>12</td>
</tr>
<tr>
<td>2.7 LQR for singularly perturbed systems</td>
<td>13</td>
</tr>
<tr>
<td>2.7.1 LQR for reduced order model</td>
<td>13</td>
</tr>
<tr>
<td>2.7.2 LQR for composite model</td>
<td>14</td>
</tr>
<tr>
<td><strong>Chapter 3 DOUBLY-FED INDUCTION GENERATOR CONTROL</strong></td>
<td>15</td>
</tr>
<tr>
<td>3.1 LQR Design</td>
<td>22</td>
</tr>
<tr>
<td>3.2 Parameter Variation Analysis</td>
<td>33</td>
</tr>
<tr>
<td><strong>Chapter 4 CONCLUSIONS AND FUTURE WORK</strong></td>
<td>51</td>
</tr>
<tr>
<td>4.1 Conclusions</td>
<td>51</td>
</tr>
<tr>
<td>4.2 Future Work</td>
<td>52</td>
</tr>
</tbody>
</table>

LIST OF REFERENCES 54


LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: LQR Feedback using Singular Perturbation (Exact Model)</td>
<td>23</td>
</tr>
<tr>
<td>2: LQR Feedback using Singular Perturbation (Reduced Model)</td>
<td>25</td>
</tr>
<tr>
<td>3: LQR Feedback using Singular Perturbation (Composite Model)</td>
<td>27</td>
</tr>
<tr>
<td>4: Stability Analysis for Exact Model ($0 &lt; \varepsilon &lt; 0.3$)</td>
<td>48</td>
</tr>
<tr>
<td>5: Stability Analysis for Reduced Model ($0 &lt; \varepsilon &lt; 0.3$)</td>
<td>49</td>
</tr>
<tr>
<td>6: Stability Analysis for Composite Model ($0 &lt; \varepsilon &lt; 0.3$)</td>
<td>50</td>
</tr>
</tbody>
</table>


LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LQ design for linearized DFIG using singular perturbation</td>
</tr>
<tr>
<td>2</td>
<td>Step response for system states and outputs when, $U_1(t) = 1$, $U_2(t) = 0$ and $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>3</td>
<td>Step response for system states and outputs when, $U_1(t) = 0$, $U_2(t) = 1$ and $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>4</td>
<td>Step response for system states and outputs when, $U_1(t) = 1$, $U_2(t) = 0$ and $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>5</td>
<td>Step response for system states and outputs when, $U_1(t) = 0$, $U_2(t) = 1$ and $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>6</td>
<td>Plot of singular perturbation parameter vs. the cost for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>7</td>
<td>Plot of singular perturbation parameter vs. the cost for $Q/R = 10^{-4}$</td>
</tr>
<tr>
<td>8</td>
<td>Plot of singular perturbation parameter vs. the cost for $Q/R = 7.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>9</td>
<td>Zoomed plot of singular perturbation parameter vs. cost, $Q/R = 7.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>Plot of singular perturbation parameter vs. the cost for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>11</td>
<td>Robustness: changed $R_s$ by -4%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>12</td>
<td>Robustness: changed $R_s$ by -3%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>13</td>
<td>Robustness: changed $R_s$ by -2%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>14</td>
<td>Robustness: changed $R_s$ by -1%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>15</td>
<td>Robustness: changed $R_s$ by +1%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>16</td>
<td>Robustness: changed $R_s$ by +2%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>17</td>
<td>Robustness: changed $R_s$ by +3%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>18</td>
<td>Robustness: changed $R_s$ by +4%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>19</td>
<td>Robustness: changed $R_s$ by -4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>20</td>
<td>Robustness: changed $R_s$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>21</td>
<td>Robustness: changed $R_s$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>22</td>
<td>Robustness: changed $R_s$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>23</td>
<td>Robustness: changed $R_s$ by +1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>24</td>
<td>Robustness: changed $R_s$ by +2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<tr>
<td>25</td>
<td>Robustness: changed $R_s$ by +3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<tr>
<td>26</td>
<td>Robustness: changed $R_s$ by +4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>27</td>
<td>Reliability: reduced $B(3,1)$ by 10%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>28</td>
<td>Reliability: reduced $B(3,1)$ by 20%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>29</td>
<td>Reliability: reduced $B(3,1)$ by 30%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>30</td>
<td>Reliability: reduced $B(3,1)$ by 40%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>31</td>
<td>Reliability: reduced $B(3,1)$ by 50%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>32</td>
<td>Reliability: reduced $B(3,1)$ by 60%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>33</td>
<td>Reliability: reduced $B(3,1)$ by 70%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>34</td>
<td>Reliability: reduced $B(3,1)$ by 80%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>35</td>
<td>Reliability: reduced $B(3,1)$ by 90%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>36</td>
<td>Reliability: reduced $B(3,1)$ by 100%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>37</td>
<td>Reliability: reduced $B(3,1)$ by 10%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>38</td>
<td>Reliability: reduced $B(3,1)$ by 20%, other parameters kept constant, $Q/R = 10^{-2}$</td>
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<tr>
<td>39</td>
<td>Reliability: reduced $B(3,1)$ by 30%, other parameters kept constant, $Q/R = 10^{-2}$</td>
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<tr>
<td>40</td>
<td>Reliability: reduced $B(3,1)$ by 40%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>41</td>
<td>Reliability: reduced $B(3,1)$ by 50%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>42</td>
<td>Reliability: reduced $B(3,1)$ by 60%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>43</td>
<td>Reliability: reduced $B(3,1)$ by 70%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>44</td>
<td>Reliability: reduced B(3,1) by 80%, other parameters kept constant, Q/R = 10^{-2}</td>
</tr>
<tr>
<td>45</td>
<td>Reliability: reduced B(3,1) by 90%, other parameters kept constant, Q/R = 10^{-2}</td>
</tr>
<tr>
<td>46</td>
<td>Reliability: reduced B(3,1) by 100%, other parameters kept constant, Q/R = 10^{-2}</td>
</tr>
<tr>
<td>47</td>
<td>Robustness: changed $L_S$ by -4%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>48</td>
<td>Robustness: changed $L_S$ by -3%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>49</td>
<td>Robustness: changed $L_S$ by -2%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>50</td>
<td>Robustness: changed $L_S$ by -1%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>51</td>
<td>Robustness: changed $L_S$ by +1%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>52</td>
<td>Robustness: changed $L_S$ by +2%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>53</td>
<td>Robustness: changed $L_S$ by +3%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>54</td>
<td>Robustness: changed $L_S$ by +4%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>55</td>
<td>Robustness: changed $R_R$ by -4%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>56</td>
<td>Robustness: changed $R_R$ by -3%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>57</td>
<td>Robustness: changed $R_R$ by -2%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>58</td>
<td>Robustness: changed $R_R$ by -1%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>59</td>
<td>Robustness: changed $R_R$ by +1%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>60</td>
<td>Robustness: changed $R_R$ by +2%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>61</td>
<td>Robustness: changed $R_R$ by +3%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>62</td>
<td>Robustness: changed $R_R$ by +4%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>63</td>
<td>Robustness: changed $L_R$ by -4%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>64</td>
<td>Robustness: changed $L_R$ by -3%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>65</td>
<td>Robustness: changed $L_R$ by -2%, all other parameters kept constant for Q/R = 10^{-6}</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>66</td>
<td>Robustness: changed $L_R$ by -1%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>67</td>
<td>Robustness: changed $L_R$ by +1%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>68</td>
<td>Robustness: changed $L_R$ by +2%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>69</td>
<td>Robustness: changed $L_R$ by +3%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>70</td>
<td>Robustness: changed $L_R$ by +4%, all other parameters kept constant for $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>71</td>
<td>Reliability: reduced B(4,2) by 10%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>72</td>
<td>Reliability: reduced B(4,2) by 20%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>73</td>
<td>Reliability: reduced B(4,2) by 30%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>74</td>
<td>Reliability: reduced B(4,2) by 40%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>75</td>
<td>Reliability: reduced B(4,2) by 50%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>76</td>
<td>Reliability: reduced B(4,2) by 60%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>77</td>
<td>Reliability: reduced B(4,2) by 70%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>78</td>
<td>Reliability: reduced B(4,2) by 80%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>79</td>
<td>Reliability: reduced B(4,2) by 90%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>80</td>
<td>Reliability: reduced B(4,2) by 100%, other parameters kept constant, $Q/R = 10^{-6}$</td>
</tr>
<tr>
<td>81</td>
<td>Robustness: changed $L_S$ by -4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>82</td>
<td>Robustness: changed $L_S$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>83</td>
<td>Robustness: changed $L_S$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>84</td>
<td>Robustness: changed $L_S$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>85</td>
<td>Robustness: changed $L_S$ by +1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>86</td>
<td>Robustness: changed $L_S$ by +2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<tr>
<td>87</td>
<td>Robustness: changed $L_S$ by +3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>88</td>
<td>Robustness: changed $L_S$ by +4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>89</td>
<td>Robustness: changed $R_R$ by -4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>90</td>
<td>Robustness: changed $R_R$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<tr>
<td>91</td>
<td>Robustness: changed $R_R$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>92</td>
<td>Robustness: changed $R_R$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>93</td>
<td>Robustness: changed $R_R$ by +1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>94</td>
<td>Robustness: changed $R_R$ by +2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>Robustness: changed $R_R$ by +3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>96</td>
<td>Robustness: changed $R_R$ by +4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>97</td>
<td>Robustness: changed $L_R$ by -4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>Robustness: changed $L_R$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<td>99</td>
<td>Robustness: changed $L_R$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<tr>
<td>100</td>
<td>Robustness: changed $L_R$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
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<tr>
<td>101</td>
<td>Robustness: changed $L_R$ by +1%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>102</td>
<td>Robustness: changed $L_R$ by +2%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>103</td>
<td>Robustness: changed $L_R$ by +3%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>104</td>
<td>Robustness: changed $L_R$ by +4%, all other parameters kept constant for $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>105</td>
<td>Reliability: reduced $B(4,2)$ by 10%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>106</td>
<td>Reliability: reduced $B(4,2)$ by 20%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>107</td>
<td>Reliability: reduced $B(4,2)$ by 30%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>108</td>
<td>Reliability: reduced $B(4,2)$ by 40%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
<tr>
<td>109</td>
<td>Reliability: reduced $B(4,2)$ by 50%, other parameters kept constant, $Q/R = 10^{-2}$</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES (cont.)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>Reliability: reduced $B(4,2)$ by 60%, other parameters kept constant, $Q/R = 10^{-2}$</td>
<td>79</td>
</tr>
<tr>
<td>111</td>
<td>Reliability: reduced $B(4,2)$ by 70%, other parameters kept constant, $Q/R = 10^{-2}$</td>
<td>79</td>
</tr>
<tr>
<td>112</td>
<td>Reliability: reduced $B(4,2)$ by 80%, other parameters kept constant, $Q/R = 10^{-2}$</td>
<td>80</td>
</tr>
<tr>
<td>113</td>
<td>Reliability: reduced $B(4,2)$ by 90%, other parameters kept constant, $Q/R = 10^{-2}$</td>
<td>80</td>
</tr>
<tr>
<td>114</td>
<td>Reliability: reduced $B(4,2)$ by 100%, other parameters kept constant, $Q/R = 10^{-2}$</td>
<td>80</td>
</tr>
</tbody>
</table>
Chapter 1

INTRODUCTION

The concepts of modern control theory are applied to control physical systems. Physical systems are described using mathematical models in order to analyze system behavior and to design the control strategy. The dynamic order of the developed mathematical model should not complicate the design process. The presence of small scale dynamics such as capacitance inductance et. al. can increase the dynamic order of the mathematical models. Simplified model designs can ignore these parameters to reduce the complexity of the design process. The desired response of the physical system based on these simplified models may or may not be acceptable. But the techniques of singular perturbation give the developer a legitimate procedure to incorporate these fast scale dynamics in the modeling without making the design overly complicated. The concepts of singular perturbation are simply extended for systems which expose slow and fast dynamics but not limited to parasitic parameters only.

The concepts of singular perturbation techniques can be used to control systems which exhibit two timed scale dynamics or for systems with distinct clusters of eigenvalues. The system can be asymptotically expanded into slow and fast modes. The slow phenomenon of the full system is dominant in system response. The fast phenomena (small scaled dynamics) can be neglected and a reduced order model is obtained. The control design based on the reduced model must ensure system stability while achieving the desired actual system performance. To incorporate the full system dynamics, a composite model consisting of slow and fast subsystems are developed, independent control designs are implemented, and the full system behavior with the two designs combined is analyzed.
1.1. Doubly-fed Induction Generator (DFIG)

Induction generators can be classified as singly-fed and doubly-fed machines. In singly-fed machines only the stator windings are connected to the electric grid to supply the induced voltage at a constant frequency. If the generator is operated above rated synchronous speed, the excess power generated is wasted in the resistance placed in the rotor circuit. In contrast, for doubly-fed machines the stator and rotor both are connected to the electric-grid. At speeds below the rated synchronous speed only stator windings are active and above rated synchronous speeds both the stator and rotor are active [1]. The rotor circuit of the doubly-fed machines has a convertor instead of resistance; hence the excess power generated is also transmitted to grid. This increases the power generation efficiency.

The DFIG rotor circuit is coupled to the electric-grid via two back to back voltage source convertors (VSC). The two convertors are known as the machine-side and grid-side convertors. The two-convertors are linked with a dc-link capacitor. The machine-side convertor controls the real and reactive power production of the generator whereas grid-side convertor controls dc-link voltage and reactive power absorbed from the grid [4].

Wind speeds are highly fluctuating and for maximum aerodynamic efficiency of wind power generation, the wind turbines have to track the varying wind speed. DFIG’s can be operated at varying rotational speeds therefore they are widely used in wind power generation. Also in variable speed DFIG wind turbines, the mechanical stresses and acoustic noise are less compared to other available designs. And precise control of torque and speed is achieved using DFIG [4].

DFIGs have two operating conditions, under normal operation DFIGs have to react to the system variations, such as adding a new power load (step input). The controller is expected to return to its normal operation within few cycles. This is known as steady state operation. The
presence of abnormality such as a grid fault could cause the system to react much faster. If the DFIG fails to react quickly, it could disconnect from the grid and cause more distress. This is known as the transient operation of DFIG [3].

1.2. Literature Review

When the DFIGs are connected to the power grid, utilities require the DFIGs to follow the real and reactive power reference values at steady state operations. Real power is controlled to ensure that the load is served as desired and the reactive power is controlled to ensure that the voltage is regulated in the presence of fluctuating wind velocity [5]. This can be achieved through controlling the stator and rotor currents [6].

Due to complexity of the DFIG dynamics most of the literature utilize $d-q$ axis to model using a synchronous rotating frame or stationary frame [4]-[9]. To simplify the computation the three phase system is converted into a two phase orthogonal time invariant system, which is known as $d-q$ axis. This transformation is computed using the Parks transformation [10].

Field oriented control (FOC) or vector control is used to control the electromagnetic torque and rotor excitation current components independently in the $d-q$ axis of the generator [7], [11]. Active and reactive power output of the generator can be decoupled for control purpose [11].

Typically DFIG dynamics are represented as a fifth order system [12]-[14] that describes the stator, rotor and the rotor shaft dynamics. The time scale responses of these three components are different. To simplify the controllers, control schemes are developed for the electrical dynamics [5], [7], [12], and [14] or mechanical dynamics [14] separately. Based on the current literature the electrical system can be modeled as fourth order system. The fourth order system includes stator resistance, stator inductance, rotor resistance, rotor inductance, and synchronous rotating reference frame. Ekanayake et. al. showed that stator dynamics are vital only in transient
dynamics as their time response is faster than rotor. This has led to several works in literature considering only rotor dynamics of DFIG assuming that stator dynamics are constant [6], [15].

Most of the present literature uses proportional integral (PI) controllers for active power and reactive power control [6], [16] – [19]. Compared to a classical PI controller, modern controllers such as linear quadratic regulator [6], internal model control [17] and $H_\infty$ robust control [20] are more robust and reliable. Improved controllers for wind power generation using DFIG are being investigated currently by both industry and academia. Controllers which are more economical, reliable and robust are a necessity for a large penetration of distributed wind generation and integration to the power systems.

1.3. Motivation

Using reduced order controllers for DFIG is economical compared to full order controllers [21], but controllers must ensure system reliability and robustness during both steady-state and transient operations. The DFIG machine model possesses a two time scale nature. Therefore, using high-gain feedback for control, the system can be controlled using singularly perturbed techniques [15].

For systems with a two-timed scale nature, singular perturbation techniques can be used to obtain three different system models and feedback control for each model can be developed. The three models are: (1) exact model, which will consider the full order of the system, (2) reduced order model, using the slow dynamics of the given system and (3) composite model, which considers the slow and fast part of the system dynamics independent of each other. This will give a common platform to analyze the system performance [22]-[25].

The fourth order DFIG Machine model has two-complex conjugate poles closer to the origin of the s-plane. This will influence the rotor current dynamics [26]. Furthermore, stator current
dynamics are faster than the rotor current dynamics. This property will enable controllers for DFIG using singular perturbation techniques.

1.4. Contribution of this Thesis

This work utilizes the DFIG model with two complex conjugate pole pairs which exhibit a two timed-scale behavior. An LQR feedback controller is designed for exact (the fourth order model), reduced (the rotor side dynamics/slow part of the machine model) and composite models (the slow/rotor-side and fast/stator-side separately) for steady state operation.

Feedback controllers are designed based on the performance of the three models for the nominal system parameters. This is determined by ensuring that all the three models satisfy the necessary conditions for singular perturbation technique to be applied.

To ensure that the controller is reliable and robust, stability of the closed loop was analyzed for the three different designs by using parameter variations in the given system. Based on this analysis, reliability and robustness of the reduced order controller is validated.

1.5. Summary and Organization of the Thesis

An approach for controller design for DFIG under steady state operations is investigated in this work. The primary objective is to design a reduced order controller and compare the cost, stability, reliability and robustness with the full order and the composite models.

Chapter 1 provides an introduction to this thesis and describes the work done in the area of DFIG control. Chapter 2 provides a detailed theoretical background of singular perturbation techniques. Chapter 3 investigates the controller design for DFIG using the singular perturbation techniques and discusses the results, and finally chapter 4 concludes the thesis with recommended future work.
Chapter 2
THEORETICAL BACKGROUND

2.1. System State Space

The state space representation of linear time-invariant (LTI) system can be written as

\[
\dot{x}(t) = Ax(t) + Bu(t) \quad (1.1)
\]

\[
y(t) = Cx(t) + Du(t) \quad (1.2)
\]

where \( A \) is a \( n \times n \) constant state matrix, \( B \) is a \( n \times l \) constant input matrix, \( C \) is a \( p \times n \) constant output matrix, \( D \) is a \( p \times l \) constant transmission matrix, \( x(t) \) is a \( n \times 1 \) state vector, \( u(t) \) is a \( l \times 1 \) input vector, and \( y(t) \) is a \( p \times 1 \) output vector.

The determinant of \((sI - A)\) is known as the characteristic polynomial of the LTI system in (2.1), where \( s \) is the Laplace argument. The eigenvalues of the state matrix \( A \) or the roots of the characteristic polynomial correspond to open-loop poles of the system.

2.2. Feedback Control

A feedback control can be designed using state feedback or output feedback. To implement feedback control for a given mathematical model, any change in the input signal must impact all the system states (state feedback) or the system output (output feedback). To determine the controllability of a given system model, the controllability matrix, \( P_c \), must have full rank. The controllability matrix is defined as

\[
P_c = \begin{bmatrix} B & AB & A^2B & \ldots & A^{n-1}B \end{bmatrix} \quad (1.3)
\]

Model controllability is indicated by stating, system matrices ‘\((A, B)\) are controllable’.
2.3. Model Linearization via State Feedback (Input-State Linearizable)

For nonlinear system modeling and designing, the system nonlinearity is linearized and the concepts of linear system theory are applied for controlling. If \((A, B)\) is controllable and the nonlinear system state space has the following form

\[
\dot{x} = Ax + B\gamma(x)[u - \alpha(x)],
\]

then the system can be linearized using a state feedback [27], given by

\[
u = \alpha(x) + \gamma^{-1}(x)v
\]

In (2.4) the additive and multiplicative state nonlinearities are given by \(\alpha(x)\) and \(\gamma(x)\) respectively. A state feedback of the form (2.5) can be chosen if \(\gamma(x)\) is nonsingular. Substituting (2.5) into (2.4) yields a linearized system of the form

\[
\dot{x} = Ax + Bv
\]

For stabilizing system the feedback \(v = -G\dot{x}\) is chosen such that \(A - BG\) is Hurwitz [27].

2.4. Similarity Transformation: State transformation to Jordan Block for Systems with Pairs of Complex Conjugate Roots

For the linear system in (2.1) a coordinate transformation from \(x(t)\) to \(z(t)\) is defined using an \(n \times n\) non-singular matrix \(T\) such that,

\[
x(t) = Tz(t)
\]

\[
\Rightarrow \dot{z}(t) = \hat{A}z(t) + \hat{B}u(t)
\]

\[
y(t) = \hat{C}z(t) + \hat{D}u(t)
\]

where \(\hat{A} = T^{-1}AT\), \(\hat{B} = T^{-1}B\), \(\hat{C} = CT\) and \(\hat{D} = D\).

To obtain the state transformation to block Jordan form for a system of order \(2 \times 2\) with a pair of complex conjugate root, the steps are given below:
• The eigenvalues of the state matrix are given by

\[
eig(A) = \left[ \left( p_1 \pm jp_2 \right) \right]
\]  
(1.10)

• The corresponding eigenvectors for these eigenvalues are defined as

\[
eigenvectors = \begin{bmatrix} (a_1 \pm jb_1) \\ (a_2 \pm jb_2) \end{bmatrix},
\]

(1.11)

• The transformation matrix \( T \) from a given model with a pair of complex roots to Jordan block is given by,

\[
T = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}
\]

(1.12)

• The transformed Jordan block will be of the form,

\[
\hat{A} = T^{-1}AT = \begin{bmatrix} p_1 & p_2 \\ -p_2 & p_1 \end{bmatrix}
\]

(1.13)

2.5. Linear Quadratic (LQ) Problem

Feedback control of dynamic systems is designed to achieve a desired system performance. In optimal control the control law minimizes a performance index (or cost function) of the given system based on the initial conditions. The performance index is often formulated using quadratic terms of the system states, the admissible input and terminal value of the states. By finding the minimum of the cost function the energy of the system input and internal (states) signals are minimized. The feedback design of an LQ problem guarantees a stable closed loop system. The cost function for a general case is defined as

\[
J = \frac{1}{2} x^T(t_f) S x(t_f) + \frac{1}{2} \int_0^{t_f} \left\{ x(t)^T Q x(t) + u(t)^T R u(t) \right\} dt
\]

(1.14)
This function is minimized for $x(t_f) = x_0$ such that the constraint of (2.1) is satisfied. $S$ and $Q$ are semi-positive definite and $R$ is positive definite. The matrices $S$, $Q$ and $R$ are symmetric and chosen on a trial and error basis in order to obtain desired system performances.

The optimal feedback, $u^*(t)$, is defined as

$$u^*(t) = -G(t)x^*(t) \quad (1.15)$$

where the feedback gain, $G$, is given by

$$G(t) = -R^{-1}B^T K(t)x^*(t) \quad (1.16)$$

and $K(t)$ is a positive definite symmetric matrix that is the solution to the Matrix Riccati equation given by

$$\dot{K}(t) = -A^T K(t) - K(t)A - Q + K(t)BR^{-1}B^T K(t) \quad (1.17)$$

After substituting (2.15) into (2.1), the closed loop system is given by,

$$\dot{x}(t) = \left( A - BR^{-1}B^T K(t) \right)x(t) \quad (1.18)$$

When optimal feedback control law is implemented the optimal cost of the given system is

$$J(t_f) = \frac{1}{2}x^*(t_f)^T K(t_f)x^*(t_f) \quad (1.19)$$

The feedback design of LQR controller assures a stable closed loop system and the weighting matrices $Q$ and $R$ can be chosen in a trial and error method such that the desired performance specifications are achieved.

2.5.1. Infinite Time Horizon – Linear Quadratic Regulator (LQR)

If $t_f \to \infty$, then in equation (2.14) the terminal cost $x^T(t_f)Sx(t_f) = 0$. Then the Riccati equation becomes the ‘Algebraic Riccati Equation (ARE),’ that is $\dot{K}(t) = 0$. 

9
2.5.2. LQR with Cross-Product term

A cost function which contains the cross-product term of $x(t)$ and $u(t)$, using completing the squares method, the function is rearranged to obtain the optimal control

$$J = \frac{1}{2} \int_0^\infty \left\{ x^T Q x + 2 x^T M u + u^T R u \right\} dt$$  \hspace{1cm} (1.20)

For a system in (2.1) the cost function (2.20) is minimized for the initial conditions $x(t_o) = x_o$, with an optimal feedback gain given by

$$u^* = - R^{-1} (B^T K + M^T) x$$  \hspace{1cm} (1.21)

The Riccati equation for the above function has the form

$$0 = K \left(A - BR^{-1} M^T \right) + \left(A - BR^{-1} M^T \right)^T K + Q - MR^{-1} M^T - KBR^{-1} B^T K$$  \hspace{1cm} (1.22)

2.6. Singular Perturbation Approach

In a state space realization of the form (2.1), open-loop poles of the state matrix $A$ can have different groups of eigenvalues. If the difference between the real part of the poles is greater than a factor of 10, then the system possesses a time scale separation. For models of these systems, the concepts of singular perturbation can be applied. The slow and fast modes of the system can be analyzed individually for feedback design if and only if the fast modes are strictly stable. The system in (2.1) can be rewritten showing the slow and fast modes explicitly.

$$\dot{x} = A_{11} x + A_{12} z + B_1 u$$  \hspace{1cm} (1.23)

$$\varepsilon \dot{z} = A_{21} x + A_{22} z + B_2 u$$  \hspace{1cm} (1.24)

$$y = C_1 x + C_2 z$$  \hspace{1cm} (1.25)

where $x \in \mathbb{R}^m$ is the slow mode and $z \in \mathbb{R}^r$ is the fast mode of the system and $m + r = n$. To apply the concepts of linear system theory for singularly perturbed systems, the state matrix and
input matrix must be in standard form, that is

\[
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\text{ and } B = \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix}
\tag{1.26}
\]

To analyze the dominant slow mode during stable operation, the system fast mode is considered that it had reached steady state, therefore \( \dot{z} = 0 \). The system in (2.23) and (2.24) is reduced to

\[
\varepsilon \dot{z} = A_{21}x + A_{22}z + B_2u = 0
\tag{1.27}
\]

\[
\Rightarrow z = -A_{22}^{-1}(A_{21}x + B_2u)
\tag{1.28}
\]

The system in (2.23)-(2.25) is reduced to

\[
\dot{x}_s = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_s + (B_1 - A_{22}^{-1}B_2)u_s
\tag{1.29}
\]

\[
y_s = (C_1 - C_2A_{22}^{-1}A_{21})x_s - C_2A_{22}^{-1}B_2u_s
\tag{1.30}
\]

### 2.6.1. Slow Subsystem (Reduced Order Model)

The reduced order model for the system expressed in (2.23) – (2.25) is given as

\[
\dot{x}_s = A_o x_s + B_o u_s
\tag{1.31}
\]

\[
y_s = C_o x_s + D_o u_s
\tag{1.32}
\]

where \( A_o = A_{11} - A_{12}A_{22}^{-1}A_{21}, B_o = B_1 - A_{22}^{-1}B_2, C_o = C_1 - C_2A_{22}^{-1}A_{21}, D_o = -C_2A_{22}^{-1}B_2 \)

A feedback control of the form \( u_s = -G_o x_s \) can be implemented to the slow subsystem to modify the subsystem performances. To analyze the full system behavior in (2.23) – (2.25) the controller gain has to be augmented with zeros to comply with system dimension. That is, the feedback gain, \( F \in \mathbb{R}^n \) is chosen to analyze the full system behavior.
2.6.2. Composite System

The composite system analyzes the system in (2.23) – (2.25) as two separate systems: slow and fast modes, and combines them together as a composite control. The slow subsystem is given in (2.31) – (2.32). And the fast subsystem is given by

\[
\dot{e} = A_{22}e + B_2u_f
\]

\[
y_f = C_z z_f
\]

The control law for slow and fast modes can be designed independently and the two feedback control design can be combined for the actual full order system in (2.23)-(2.25) system in operation. For the slow subsystem the control is given by

\[
u_s = -G_o x_s
\]

For the fast subsystem the feedback control is given by

\[
u_f = -G_z z_f
\]

Combining the two feedback designs in order to control the actual full order system in (2.23)-(2.24), the composite control is given by

\[
u_c = -[G_o \quad G_z] \begin{bmatrix} x_s \\ z_f \end{bmatrix} = -[G_1 \quad G_2] \begin{bmatrix} x \\ z \end{bmatrix}
\]

where

\[
G_1 = \left\{ I - G_z A_{22}^{-1} B_2 \right\} G_o + G_z A_{22}^{-1} A_{21}
\]
2.7. LQR for Singularity Perturbed Systems

A general infinite time horizon state feedback regulator cost is given by the following equation

\[ J = \int_0^\infty \{ y^T y + u^T Ru \} dt \]  

(1.40)

For a singularly perturbed system (2.23)-(2.25), the cost function is minimized for a given initial condition by separating the slow and fast modes. The optimal solution for each subsystem is then combined to obtain a composite design [23].

2.7.1. LQR for reduced order model

The cost function in (2.39) is adjusted to incorporate only the slow phenomenon (2.31) – (2.32) of the full system and is given by

\[ J_s = \int_0^\infty \left\{ x_s^T \left( C_o^T C_o \right) x_s + 2u_s^T D_o^T x_s + u_s^T \left( R + D_o^T D_o \right) u_s \right\} dt \]  

(1.41)

Cost function (2.40) has cross-product terms of \( x_s \) and \( u_s \). The Riccati solution \( K_s \) and the optimal feedback \( G_o \) can be obtained using equations (2.22) and (2.21) to minimize the cost function in (2.40).

The cost for the full system based on the reduced order controller design is given by,

\[ J_r = \frac{1}{2} \begin{bmatrix} x_o^T & z_o^T \end{bmatrix} P_r \begin{bmatrix} x_o \\ z_o \end{bmatrix} \]  

(1.42)

where \( P_r \) is the positive definite solution of the Lyapunov equation,

\[ P_r \left( A - BF \right) + \left( A - BF \right)^T P_r = -F^T RF - Q \]  

(1.43)

Feedback gain \( F \) is determined by using (2.33).
2.7.2. LQR for Composite Model

The cost function in (2.40) is modified to the fast dynamics in (2.34) and (2.35).

\[ J_f = \frac{1}{2} \int_0^\infty \{ y_f^T y_f + u_f^T R u_f \} dt \]  \hspace{1cm} (1.44)

For this performance index, Riccati solution \( K_f \) and the optimal feedback gain \( G_2 \) can be obtained using equations (2.17) and (2.16) with appropriate system matrices. The composite control \( G_c = [G_1 \quad G_2] \) is applied to the full order system in (2.23) – (2.25) to minimize the performance index in (2.40). The cost of the composite control is given by

\[ J_c = \frac{1}{2} \begin{bmatrix} x_o^T & z_o^T \end{bmatrix} P_c \begin{bmatrix} x_o \\ z_o \end{bmatrix} \]  \hspace{1cm} (1.45)

where \( P_c \) is the positive definite solution of the Lyapunov equation given by

\[ P_c (A - BG_c) + (A - BG_c)^T P_c = -G_c^T R G_c - Q \]  \hspace{1cm} (1.46)
A nonlinear state space realization of a DFIG machine model is given in Tang et. al.[14]. The state equations describe the open loop model of the stator and rotor electrical dynamics only. The state space model is obtained using a $d - q$ synchronous rotating frame.

\[
\begin{bmatrix}
\dot{\varphi}_{ds} \\
\dot{\varphi}_{qs} \\
\dot{\varphi}_{dr} \\
\dot{\varphi}_{qr}
\end{bmatrix} =
\begin{bmatrix}
-\frac{R_s}{\sigma L_s} & \omega_s & \frac{R L_m}{\sigma L_s L_r} & 0 \\
-\omega_s & -\frac{R_r}{\sigma L_r} & 0 & \frac{R L_m}{\sigma L_s L_r} \\
\frac{R L_m}{\sigma L_s L_r} & 0 & -\frac{R_r}{\sigma L_r} & \omega_s \\
0 & \frac{R L_m}{\sigma L_s L_r} & -\omega_s & -\frac{R_r}{\sigma L_r}
\end{bmatrix}
\begin{bmatrix}
\varphi_{ds} \\
\varphi_{qs} \\
\varphi_{dr} \\
\varphi_{qr}
\end{bmatrix} +
\begin{bmatrix}
v_{ds} \\
v_{qs} \\
v_{dr} - \omega_s x_s \\
v_{qr} + \omega_r x_s
\end{bmatrix}
\] (2.1)

\[
y(t) =
\begin{bmatrix}
\frac{i_{ds}}{\sigma L_s} \\
\frac{i_{qs}}{\sigma L_s} \\
\frac{i_{dr}}{\sigma L_r} \\
\frac{i_{qr}}{\sigma L_r}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & -\frac{L_m}{\sigma L_s L_r} & 0 \\
0 & 1 & 0 & -\frac{L_m}{\sigma L_s L_r} \\
-\frac{L_m}{\sigma L_s L_r} & 0 & 1 & 0 \\
0 & -\frac{L_m}{\sigma L_s L_r} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\varphi_{ds} \\
\varphi_{qs} \\
\varphi_{dr} \\
\varphi_{qr}
\end{bmatrix}
\] (2.2)

where the states $\varphi(t) = [\varphi_{ds} \ \varphi_{qs} \ \varphi_{dr} \ \varphi_{qr}]^T$. $\varphi_{ds}$ and $\varphi_{qs}$ are the $d$ and $q$ axis stator flux $\varphi_{dr}$ and $\varphi_{qr}$ are the $d$ and $q$ axis rotor flux. $L_m$, $L_s$ and $L_r$ are inductances of mutual, stator, and rotor respectively. $R_s$ and $R_r$ are stator and rotor resistances. $v_{ds}$ and $v_{qs}$ are $d$ and $q$ axis stator voltages. $v_{dr}$ and $v_{qr}$ are $d$ and $q$ axis rotor voltages. $i_{ds}$ and $i_{qs}$ are $d$ and $q$ axis stator current. $i_{dr}$ and $i_{qr}$ are $d$ and $q$ axis rotor current. $\omega_s$ is the rotational speed of the synchronous frame. $\omega_r$ is the rotational speed of the rotor. The leak coefficient is defined as, $\sigma = 1 - \frac{L_m^2}{L_s L_r}$. The
system constants are $R_s = 0.00706 \, pu$, $R_r = 0.005 \, pu$, $L_s = 3.071 \, pu$, $L_r = 3.056 \, pu$, $L_m = 2.9 \, pu$, and $\omega_s = 1 \, pu$ [14].

The given state space representation of the DFIG (3.1)–(3.2) is nonlinear. To apply the concepts of linear time invariant (LTI) system theory, the system is linearized using feedback linearization. The state feedback of the form

$$
\begin{align*}
\dot{v}_{dr} &= \omega_s \varphi_{qr} - G_{r1}'^T \begin{bmatrix} \varphi_{ds} & \varphi_{qs} & \varphi_{dr} & \varphi_{qr} \end{bmatrix}^T + u_1 \\
\dot{v}_{qr} &= -\omega_s \varphi_{dr} - G_{r2}'^T \begin{bmatrix} \varphi_{ds} & \varphi_{qs} & \varphi_{dr} & \varphi_{qr} \end{bmatrix}^T + u_2
\end{align*}
$$

(2.3)

is chosen to eliminate the nonlinear terms in equation (3.1) by Tang et. al [14]. Therefore the linearized system is given by

$$
\begin{bmatrix} 
\dot{\varphi}_{ds} \\
\dot{\varphi}_{qs} \\
\dot{\varphi}_{dr} \\
\dot{\varphi}_{qr}
\end{bmatrix} = 
\begin{bmatrix}
\frac{-R_s}{\sigma L_s} & \omega_s & \frac{R_s L_m}{\sigma L_s L_r} & 0 \\
0 & \frac{-R_s}{\sigma L_s} & \frac{R_s L_m}{\sigma L_s L_r} & 0 \\
\frac{R_s L_m}{\sigma L_s L_r} & 0 & \frac{-R_s}{\sigma L_s} & \omega_s \\
0 & \frac{R_s L_m}{\sigma L_s L_r} & 0 & \frac{-R_s}{\sigma L_s}
\end{bmatrix}
\begin{bmatrix}
\varphi_{ds} \\
\varphi_{qs} \\
\varphi_{dr} \\
\varphi_{qr}
\end{bmatrix} - 
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
G
\begin{bmatrix}
\varphi_{ds} \\
\varphi_{qs} \\
\varphi_{dr} \\
\varphi_{qr}
\end{bmatrix} + 
\begin{bmatrix}
v_{ds} \\
v_{qs} \\
u_1 \\
u_2
\end{bmatrix}
$$

(2.4)

where $G = \begin{bmatrix} G_{r1}'^T \\ G_{r2}'^T \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \end{bmatrix}$ is the linear feedback gain that has to be designed to achieve the desired system performances. In equation (3.4) $[v_{ds} \quad v_{qs} \quad u_1 \quad u_2]^T$ are external inputs to the system. The impact of these inputs is not considered in the rest of the controller design and analysis of the system performance. Substituting the values of system parameters into system matrices

$$
A = 
\begin{bmatrix}
-0.221 & 1 & 0.021 & 0 \\
-1 & -0.221 & 0 & 0.021 \\
0.0149 & 0 & -0.0157 & 1 \\
0 & 0.0149 & -1 & -0.0157
\end{bmatrix}
$$

(2.5)
The eigenvalues of the state matrix $A$ exposes a time scale separation therefore techniques of singular perturbation are used to analyze closed loop system stability. Design and analysis of feedback control is considered in two stages.

In stage one the linearized dynamics of the DFIG is state transformed to Jordan model and then into singularly perturbed form. An LQR feedback design is implemented for three different models: exact, reduced and composite. The closed loop performances are compared for different ratios of $Q/R$ weights. Design procedure of stage one is summarized in Figure 1.

In stage two, robustness and reliability of the closed loop performance is analyzed for each model. The design stability is compared based on the closed loop cost.
Figure 1: LQ design for linearized DFIG using singular perturbation

Stage 1-Step 1: State transformation

To separate the slow and fast modes explicitly and derive the model in singularly perturbed form two transformations are done. The first transformation decouples the slow and fast modes, that is

$$\varphi(t) = T_1 \hat{\varphi}(t) \quad \text{and}$$

(2.8)

The second transformation arrange the slow modes in the upper block diagonal region and the fast modes in the lower block diagonal region, that is

$$\hat{\varphi}(t) = T_2 [x \quad z]^T$$

(2.9)

Combining the two transformations in equations (3.8) and (3.9) yields

$$\varphi(t) = T_1 T_2 [x \quad z]^T$$

(2.10)

where \( \varphi(t) = [\varphi_{ds} \quad \varphi_{qs} \quad \varphi_{dr} \quad \varphi_{qr}]^T \), \( \hat{\varphi}(t) = [\hat{\varphi}_1 \quad \hat{\varphi}_2 \quad \hat{\varphi}_3 \quad \hat{\varphi}_4]^T \), \( x = [x_1 \quad x_2]^T \) and
$z = [z_1 \ z_2]^T$

The linearized model is in the form of

$$\phi(t) = (A - BG) \varphi$$  \hspace{1cm} (2.11)

To transform it to singularly perturbed form,

$$A_{sp} = T_2^{-1}T_1^{-1}AT_1T_2$$  \hspace{1cm} (2.12)

$$B_{sp} = T_2^{-1}T_1^{-1}B$$  \hspace{1cm} (2.13)

$$C_{sp} = CT_1T_2$$  \hspace{1cm} (2.14)

$$G_{sp} = GT_1T_2$$  \hspace{1cm} (2.15)

That is

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = (A_{sp} - B_{sp}G_{sp}) \begin{bmatrix} x \\ z \end{bmatrix}$$  \hspace{1cm} (2.16)

$$y = C_{sp} \begin{bmatrix} x \\ z \end{bmatrix}$$  \hspace{1cm} (2.17)

**Transformation 1: Jordan Form**

The eigenvalues of the state matrix $A$ has two pairs of complex conjugate roots. Using MATLAB

$$\text{eig} \left( A \right) = \begin{bmatrix} p_1 \pm jp_2 \\ q_1 \pm jq_2 \end{bmatrix} = \begin{bmatrix} -0.0369 \pm j \\ -0.001 \pm j \end{bmatrix}$$  \hspace{1cm} (2.18)

The corresponding eigenvectors for roots of complex conjugate pairs are given as,

$$\text{eigenvectors} = \begin{bmatrix} (a_1 \pm jb_1) \\ (a_2 \pm jb_2) \\ (a_3 \pm jb_3) \\ (a_4 \pm jb_4) \end{bmatrix}, \begin{bmatrix} (c_1 \pm jd_1) \\ (c_2 \pm jd_2) \\ (c_3 \pm jd_3) \\ (c_4 \pm jd_4) \end{bmatrix}$$  \hspace{1cm} (2.19)

Therefore the transformation matrix is
\[ T_1 = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} = \begin{bmatrix} 0.5784 & 0 & 0 & 0.4982 \\ 0 & 0.5784 & 0.4982 & 0 \\ -0.4068 & 0 & 0 & 0.5018 \\ 0 & -0.4068 & -0.5018 & 0 \end{bmatrix} \quad (2.20) \]

**Transformation 2: Inter-changing rows and columns**

The rows and columns of the Jordan transformation have to be interchanged to apply singular perturbation concepts. Thus the transformation \( T_2 \) is defined such that

\[ T_2 = T_2^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (2.21) \]

Substituting equations (3.20)-(3.21) into equations (3.12)-(3.15) yields the new transformed system

\[ A_{sp} = \begin{bmatrix} -0.001 & 1 & 0 & 0 \\ -1 & -0.001 & 0 & 0 \\ 0 & 0 & -0.0369 & 1 \\ 0 & 0 & -1 & 0.0369 \end{bmatrix} \quad (2.22) \]

\[ B_{sp} = \begin{bmatrix} 0 & -1.1735 \\ 1.1735 & 0 \\ -1.0109 & 0 \\ 0 & -1.0109 \end{bmatrix} \quad (2.23) \]

\[ C_{sp} = \begin{bmatrix} 0 & 0.0693 & 3.0228 & 0 \\ -0.0693 & 0 & 0 & 3.0228 \\ 0 & 0.0985 & -3.0016 & 0 \\ -0.0985 & 0 & 0 & -3.0016 \end{bmatrix} \quad (2.24) \]
\begin{equation}
G_{sp} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\
G_{21} & G_{22} & G_{23} & G_{24} \end{bmatrix} = \begin{bmatrix} 0.5784 & 0 & 0 & 0.4982 \\
-0.4068 & 0 & 0 & 0.5018 \\
0 & 0.5784 & 0.4982 & 0 \\
0 & -0.4068 & -0.5018 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \end{bmatrix} \tag{2.25}
\end{equation}

**Stage 1- Step 3: Singular perturbation form**

To express the transformed system into standard singular perturbation form the perturbation parameter has to be obtained using the ratio between the real parts of the complex poles. That is

\[ \varepsilon_o = \frac{q_l}{p_1} = \frac{0.001}{0.0369} = 0.0267 \tag{2.26} \]

Rewriting the matrices in standard singular perturbation form,

\begin{equation}
\begin{bmatrix} \dot{x} \\
\varepsilon_o \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} -0.001 & 1 & 0 & 0 \\
1 & 0.001 & 0 & 0 \\
0 & 0 & -1.3874 & 37.6011 \\
0 & 0 & -37.6011 & -1.3874 \end{bmatrix} \begin{bmatrix} 0 & -1.1735 \\
1.1735 & 0 \\
-38.0098 & 0 \\
-38.0098 & 0 \end{bmatrix} \hat{G} \begin{bmatrix} x \\
z \end{bmatrix} \tag{2.27}
\end{equation}

\[ y = \begin{bmatrix} 0 & 0.0693 & 3.0228 & 0 \\
-0.0693 & 0 & 0 & 3.0228 \\
0 & 0.0985 & -3.0016 & 0 \\
-0.0985 & 0 & 0 & -3.0016 \end{bmatrix} \begin{bmatrix} x \\
z \end{bmatrix} \tag{2.28} \]

The following notations are used to proceed with LQR design of the model

\begin{equation}
\begin{bmatrix} \dot{x} \\
\varepsilon_o \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} A_s & A_{12} \\
A_{21} & A_f \end{bmatrix} \begin{bmatrix} x \\
z \end{bmatrix} + \begin{bmatrix} B_s \\
B_f \end{bmatrix} u \quad \text{and} \tag{2.29}
\end{equation}

\begin{equation}
y = \begin{bmatrix} M_1 & M_2 \end{bmatrix} \begin{bmatrix} x \\
z \end{bmatrix} \tag{2.30} \end{equation}

where \( A_{12} = A_{21} = 0_{2 \times 2} \), \( A_{sp} = \begin{bmatrix} A_s & 0 \\
0 & A_f/E_o \end{bmatrix} \), \( B_{sp} = \begin{bmatrix} B_s \\
B_f/E_o \end{bmatrix} \) and \( C_{sp} = [M_1 \quad M_2] \). The poles
of the fast part are stable. The inverse of $A_f$ exists and given by

$$A_f^{-1} = \begin{bmatrix} -0.001 & -0.0266 \\ 0.0266 & -0.001 \end{bmatrix} \quad (2.31)$$

Both $(A_s, B_s)$ and $(A_f, B_f)$ are controllable and eigenvalues of matrix $A_f$ is all stable.

### 3.1. LQR Design

To control the closed loop model in equation (3.16), the gain $\hat{G}$ is designed using LQR infinite time horizon. The performance index is chosen as

$$J = \frac{1}{2} \int \left( x^T Q x + z^T R z + u^T R_{2x2} u \right) dt \quad (2.32)$$

In equation (3.32) the matrices $Q_{4x4} = Q \times \text{eye}(4 \times 4)$ and $R_{2x2} = R \times \text{eye}(2 \times 2)$. $Q$ and $R$ are numerical values chosen on a trial error basis such that the closed is stable for all three controller designs. To find the cost obtained using LQR and LQR based feedback gains an initial condition

$$\begin{bmatrix} x(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} x_o \\ z_o \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (2.33)$$

is chosen.

**Stage 1 – Step 4 – Exact Control**

The exact model is given in equations (3.16) and (3.17). The performance index in (3.32) is minimized for the exact model using the initial conditions in (3.33). To obtain the Riccati solution MATLAB command ‘lqr($A_{sp}, B_{sp}, Q_{4x4}, R_{2x2}$)’ is used. For different each choice of $Q$ and $R$ values the optimal feedback gain and the cost were calculated. Resulting optimal feedback gain and optimal cost for each choice of $Q/R$ are tabulated in Table 1.
For the exact model represented in equations (3.16) and (3.17) the LQR feedback design guarantees a stable closed loop performance. The cost and feedback gains are optimum for each chosen $Q/R$ ratio. Feedback gain, $G$, for the original system in (3.4) can be determined using the coordinate transformation $G = G_{sp}T_2^{-1}T_1^{-1}$.

**Stage 1 – Step 4 – Reduced Order Control**

A reduced order model for the full system in (3.16) is obtained assuming the fast scale dynamics have reached steady state. The reduced model is obtained by setting the singular perturbation parameter $\varepsilon_o = 0$.

$$\varepsilon_o = 0 \Rightarrow [\varepsilon_o \dot{x}] = [A_{21} \ A_f] \begin{bmatrix} x \\ z \end{bmatrix} + [B_f] u$$  \hspace{1cm} (2.34)

$$\Rightarrow z = -A_f^{-1}B_f u$$  \hspace{1cm} (2.35)
The reduced order model is

\[
\begin{align*}
\dot{x}_s &= A_s x_s + B_s u_s \\
y_s &= M_1 x_s - M_2 A_f^{-1} B_f u_s
\end{align*}
\tag{2.36}
\]

The performance index in (3.32) is modified to consider only the slow dynamics of the full system

\[
J = \frac{1}{2} \int_0^\infty \left\{ x_s^T Q x_s + u_s^T R u_s \right\} dt
\tag{2.37}
\]

The performance index in (3.37) is minimized for the system in (3.36). Using MATLAB command \([G_o, K_r, E_r] = \text{lqr}(A_{sp}, B_{sp}, Q, R)\) the reduced order closed loop feedback gain \(G_o\), Riccati solution \(K_r\), and eigenvalues \(E_r\) are obtained for each chosen \(Q/R\).

The optimal feedback gain, \(G_o\) was augmented with zeros to obtain the feedback for the full system in (3.16)

\[
F = \begin{bmatrix} G_o & 0_{2 \times 2} \end{bmatrix}
\tag{2.38}
\]

The closed loop system using the full order feedback gain \(F\) is

\[
A_{cl-f} = A_{sp} - B_{sp} F
\tag{2.39}
\]

The cost \(J_r\) for the full system using the feedback gain \(F\) is obtained by the Lyapunov equation

\[
P_r A_{cl-f} + A_{cl-f}^T P_r + \left( Q + F^T R F \right) = 0
\tag{2.40}
\]

To obtain the Lyapunov solution MATLAB \(P_r = \text{lyap}(A_{cl-f}, (Q + F^T R F))\) was used and the cost \(J_r\) was calculated as

\[
J_r = \frac{1}{2} \begin{bmatrix} x_o^T & z_o^T \end{bmatrix} P_r \begin{bmatrix} x_o \\ z_o \end{bmatrix}
\tag{2.41}
\]

Different \(Q\) and \(R\) values were used to calculate the feedback gain and associated cost of the full order system are tabulated in table 2.
TABLE 2:
LQR FEEDBACK USING SINGULAR PERTURBATION (REDUCED CONTROL)

<table>
<thead>
<tr>
<th>Q/R ratio</th>
<th>Feedback Gain (F)</th>
<th>Cost ($J_r$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-7}$</td>
<td>$\begin{bmatrix} 4.9 \times 10^{-19} &amp; 5.8 \times 10^{-5} &amp; 0 &amp; 0 \ -5.8 \times 10^{-5} &amp; 7.8 \times 10^{-12} &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td>0.25304</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>$\begin{bmatrix} 3.84 \times 10^{-18} &amp; 4.7 \times 10^{-4} &amp; 0 &amp; 0 \ -4.7 \times 10^{-4} &amp; 4.1 \times 10^{-18} &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td>2.0593</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>$\begin{bmatrix} 1.8 \times 10^{-17} &amp; 2.4 \times 10^{-3} &amp; 0 &amp; 0 \ -2.4 \times 10^{-3} &amp; 2.4 \times 10^{-17} &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td>11.0777</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>$\begin{bmatrix} -4.9 \times 10^{-17} &amp; 9.2 \times 10^{-3} &amp; 0 &amp; 0 \ -9.2 \times 10^{-3} &amp; 2.1 \times 10^{-16} &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td>46.9907</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$\begin{bmatrix} 3.1 \times 10^{-16} &amp; 3.1 \times 10^{-2} &amp; 0 &amp; 0 \ -3.1 \times 10^{-2} &amp; 2.2 \times 10^{-16} &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td>222.8687</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>$\begin{bmatrix} 7.8 \times 10^{-16} &amp; 1.0 \times 10^{-1} &amp; 0 &amp; 0 \ -1.0 \times 10^{-1} &amp; 7.8 \times 10^{-16} &amp; 0 &amp; 0 \end{bmatrix}$</td>
<td>1475.7346</td>
</tr>
</tbody>
</table>

The feedback gain in (3.38) will not be optimum for the full order model in (3.16) and the closed loop stability is not guaranteed as in exact model. To determine the closed loop stability the cost is used. The cost equation in (3.41) will be positive if and only if the Lyapunov solution $P_r$ is positive definite. A positive definite Lyapunov solution can be obtained if and only if the closed system is stable. From Table 2, the cost is positive for each chosen $Q/R$ ratio. Therefore the closed loop in equation (3.39) is stable for all choices of $Q/R$.

**Stage 1 – Step 4 – Composite Control**

The composite control considers the slow and fast dynamics individually of the exact model in equation (3.17). Feedback designs for both the subsystems are designed independently and combined as a composite design. The slow subsystem design parameters are obtained in stage 1 – step 4 – reduced order control. The fast subsystem control is explained below.
For the exact model in equation (3.17) fast scale dynamics are

\[ \varepsilon_0 \dot{z}_f = A_f z_f + B_f u_f \]
\[ y_f = M_2 z_f \quad (2.42) \]

The performance index in (3.32) is modified to consider only the fast dynamics

\[ \Rightarrow J_f = \frac{1}{2} \int_0^\infty \left\{ z_f^T Q I_{2 \times 2} z_f + u_f^T R I_{2 \times 2} u_f \right\} dt \quad (2.43) \]

Cost function in (3.43) is minimized using MATLAB command \([G_2, K_f, E_f] = lqr(A_f, B_f, Q I_{2 \times 2}, R I_{2 \times 2})\) where \(G_2, K_f, E_f\) are closed loop optimal feedback gain, Riccati solution and Eigen values for the fast subsystem in equation (3.42).

Combining the two feedback designs, \(G_o\) and \(G_2\) inorder to control the actual full order system in (3.16), the composite control is given by

\[ u_c = -\begin{bmatrix} G_o & G_2 \end{bmatrix} \begin{bmatrix} x_s \\ z_f \end{bmatrix} = -\begin{bmatrix} G_1 & G_2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = -G_c \begin{bmatrix} x \\ z \end{bmatrix} \quad (2.44) \]

where

\[ G_1 = \left\{ I - G_2 A_2^{-1} B_2 \right\} G_o \quad (2.45) \]

The closed loop system using \(G_c\) is

\[ A_{cl,c} = A_{sp} - B_{sp} G_c \quad (2.46) \]

The cost, \(J_c\) for the full system using the feedback gain \(G_c\) is obtained using the Lyapunov equation

\[ P_c A_{cl,c} + A_{cl,c}^T P_c + \left( Q_{4 \times 4} + G_c^T R G_c \right) = 0 \quad (2.47) \]

To obtain the Lyapunov solution MATLAB \(P_r = \text{lyap} \left( A_{cl,r}^T, (Q_{4 \times 4} + G_c^T R G_c) \right)\) was used and the cost \(J_c\) was calculated as
Different Q and R values were used to calculate the cost of the full order system and results plotted in the following figures and the costs for the nominal singular perturbation parameter are compared in Table 3.

**TABLE 3:**

<table>
<thead>
<tr>
<th>Q/R ratio</th>
<th>Feedback Gain (G&lt;sub&gt;C&lt;/sub&gt;)</th>
<th>Cost (J&lt;sub&gt;C&lt;/sub&gt;)</th>
</tr>
</thead>
</table>
| 10<sup>-7</sup> | \[
\begin{bmatrix}
-7.8 \times 10^{-11} & 5.8 \times 10^{-5} & -1.4 \times 10^{-06} & 2.1 \times 10^{-21} \\
-5.8 \times 10^{-5} & -7.8 \times 10^{-11} & -4.3 \times 10^{-21} & -1.4 \times 10^{-06}
\end{bmatrix}
\] | 0.25304 |
| 10<sup>-6</sup> | \[
\begin{bmatrix}
-6.4 \times 10^{-9} & 4.7 \times 10^{-4} & -1.4 \times 10^{-5} & 2.5 \times 10^{-20} \\
-4.7 \times 10^{-4} & 6.5 \times 10^{-9} & -3.9 \times 10^{-20} & -1.4 \times 10^{-5}
\end{bmatrix}
\] | 2.0592 |
| 10<sup>-5</sup> | \[
\begin{bmatrix}
-3.3 \times 10^{-7} & 2.4 \times 10^{-3} & -1.4 \times 10^{-4} & 2.8 \times 10^{-19} \\
-2.4 \times 10^{-3} & -3.3 \times 10^{-7} & -3.6 \times 10^{-19} & -1.4 \times 10^{-4}
\end{bmatrix}
\] | 11.0765 |
| 10<sup>-4</sup> | \[
\begin{bmatrix}
-1.2 \times 10^{-5} & 9.2 \times 10^{-3} & -1.3 \times 10^{-3} & 3.0 \times 10^{-18} \\
-9.2 \times 10^{-5} & -1.2 \times 10^{-5} & -3.3 \times 10^{-18} & -1.3 \times 10^{-3}
\end{bmatrix}
\] | 46.9088 |
| 10<sup>-3</sup> | \[
\begin{bmatrix}
-3.6 \times 10^{-4} & 3.1 \times 10^{-2} & -1.1 \times 10^{-2} & 2.2 \times 10^{-17} \\
-3.1 \times 10^{-2} & 3.7 \times 10^{-4} & -3.4 \times 10^{-17} & -1.2 \times 10^{-2}
\end{bmatrix}
\] | 220.8795 |
| 10<sup>-2</sup> | \[
\begin{bmatrix}
-7.0 \times 10^{-3} & 1.0 \times 10^{-2} & -7.0 \times 10^{-2} & 1.5 \times 10^{-17} \\
-1.0 \times 10^{-2} & -7.0 \times 10^{-3} & -3.1 \times 10^{-16} & -7.0 \times 10^{-2}
\end{bmatrix}
\] | 1497.1186 |

Similar to the reduced order control the feedback gain in (3.45) will not be optimum for the full order model in (3.16) and the closed loop stability is not guaranteed. Using the value of the cost in equation (3.49) the closed loop stability the cost is used. The cost equation in (3.49) is positive if and only if the Lyapunov solution \( P_c \) is positive definite. \( P_c \) is positive definite if and only if the closed system in equation (3.47) is stable. From Table 2, the cost is positive for each chosen Q/R ratio. Therefore the closed loop in equation is stable for all choices of Q/R.
The actual full order system is given in equation (3.17) and (3.18). For each chosen Q/R ratio the closed loop system state response and output response are simulated in MATLAB for a step input. MATLAB output is shown in Figures 2-5 for two different Q/R ratios. The system responses for other Q/R ratios considered are included in the appendix.
Figure 2: Step response for system states and outputs when, $U_1(t) = 1$, $U_2(t) = 0$ and $Q/R = 10^{-6}$
Figure 3: Step response for system states and outputs when, $U_1(t) = 0$, $U_2(t) = 1$ and $Q/R = 10^{-6}$
Figure 4: Step response for system states and outputs when, $U_1(t) = 1$, $U_2(t) = 0$ and $Q/R = 10^{-2}$
Figure 5: Step response for system states and outputs when, $U_1(t) = 0$, $U_2(t) = 1$ and $Q/R = 10^2$
3.2. Parameter Variation Analysis

The closed loop stability depends on nominal system parameters. With time these parameters will change due to environmental conditions. The effect of parameter change in closed loop stability is analyzed in this section using three different approaches.

**Approach 1:**

The time scale difference between the stator and rotor dynamics is indicated by the singular perturbation parameter, $\varepsilon_o$. Parameter uncertainty of the rotor and stator may result in variation of the ratio between the two time scales. This can result in unknown values of the singular perturbation parameter epsilon. To study the impact of such uncertainty, a range of epsilon values $0 < \varepsilon < 0.3$ is considered. When $\varepsilon$ is varied, the closed loop stability is determined by finding the closed cost for each controller design. In Figures 6-10 the system cost for all three control vs epsilon variation is plotted for individual Q/R ratio.

![Figure 6: Plot of singular perturbation parameter vs. the cost for Q/R = 10^{-6}](image-url)
Figure 7: Plot of singular perturbation parameter vs. the cost for $Q/R = 10^{-4}$

Figure 8: Plot of singular perturbation parameter vs. the cost for $Q/R = 7.4 \times 10^{-4}$

Figure 9: Zoomed plot of singular perturbation parameter vs. cost, $Q/R = 7.4 \times 10^{-4}$
**Approach 2**

Robustness of the feedback design is analyzed while the perturbation parameter is changed. The state matrix $A$ has four constant system parameters $R_s$, $R_r$, $L_s$ and $L_r$ which indicates the stator and rotor resistance and inductance. Each of these parameters is changed by a range of $\pm 4\%$ from their nominal value and the system stability is determined by analyzing the cost. Figures 11-18 and Figures 19-26 shows the system cost when $R_s$ is changed from -4\% to 4\% from its nominal value for a $Q/R = 10^{-6}$ and $Q/R = 10^{-2}$ respectively. Simulation diagram for other $Q/R$ ratios considered is attached in annexure 1. Tables 4-6 summarize the system performances for all three control designs for $10^{-6} < Q/R < 10^{-2}$ each parameter change.

In the Figures 11-26 the line indicating ‘Jordan model’ represents the exact model cost only under epsilon variation. The other three lines show the closed system cost under $R_s$ and $\varepsilon$ variation.
Figure 11: Robustness: changed $R_s$ by $-4\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 12: Robustness: changed $R_s$ by $-3\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 13: Robustness: changed $R_s$ by $-2\%$, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 14: Robustness: changed $R_s$ by -1%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 15: Robustness: changed $R_s$ by +1%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 16: Robustness: changed $R_s$ by +2%, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 17: Robustness: changed $R_s$ by $+3\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 18: Robustness: changed $R_s$ by $+4\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 19: Robustness: changed $R_s$ by $-4\%$, all other parameters kept constant for $Q/R = 10^{-2}$
Figure 20: Robustness: changed $R_s$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 21: Robustness: changed $R_s$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 22: Robustness: changed $R_s$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$
Figure 23: Robustness: changed $R_s$ by +1%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 24: Robustness: changed $R_s$ by +2%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 25: Robustness: changed $R_s$ by +3%, all other parameters kept constant for $Q/R = 10^{-2}$
The ratio of $\frac{Q}{R} = 10^{-2}$ then for a particular range of $\varepsilon$ the system cost is negative. This indicates that the closed loop is unstable. For $Q/R = 10^{-6}$ change in stator resistance did not affect the closed loop stability.

**Approach 3**

Reliability of the feedback design is analyzed while the perturbation parameter is also changed. The input matrix $B$ (equation 3.6) has only two elements. Each element is changed from 0-1 at 0.1 steps and the closed loop cost is analyzed. Figures 27-36 shows the system cost for a $Q/R = 10^{-6}$ and Figures 37-46 for $\frac{Q}{R} = 10^{-2}$. Tables 4-6 summarize the system performances for $10^{-6} < Q/R < 10^{-2}$ for each parameter change in each control design.
Figure 27: Reliability: reduced B(3,1) by 10%, other parameters kept constant, Q/R = 10^{-6}

Figure 28: Reliability: reduced B(3,1) by 20%, other parameters kept constant, Q/R = 10^{-6}

Figure 29: Reliability: reduced B(3,1) by 30%, other parameters kept constant, Q/R = 10^{-6}
Figure 30: Reliability: reduced $B(3,1)$ by 40%, other parameters kept constant, $Q/R = 10^{-6}$

Figure 31: Reliability: reduced $B(3,1)$ by 50%, other parameters kept constant, $Q/R = 10^{-6}$

Figure 32: Reliability: reduced $B(3,1)$ by 60%, other parameters kept constant, $Q/R = 10^{-6}$
Figure 33: Reliability: reduced B(3,1) by 70%, other parameters kept constant, Q/R = $10^{-6}$

Figure 34: Reliability: reduced B(3,1) by 80%, other parameters kept constant, Q/R = $10^{-6}$

Figure 35: Reliability: reduced B(3,1) by 90%, other parameters kept constant, Q/R = $10^{-6}$
Figure 36: Reliability: reduced $B(3,1)$ by 100%, other parameters kept constant, $Q/R = 10^6$

Figure 37: Reliability: reduced $B(3,1)$ by 10%, other parameters kept constant, $Q/R = 10^{-2}$

Figure 38: Reliability: reduced $B(3,1)$ by 20%, other parameters kept constant, $Q/R = 10^{-2}$
Figure 39: Reliability: reduced B(3,1) by 30%, other parameters kept constant, Q/R = $10^{-2}$

Figure 40: Reliability: reduced B(3,1) by 40%, other parameters kept constant, Q/R = $10^{-2}$

Figure 41: Reliability: reduced B(3,1) by 50%, other parameters kept constant, Q/R = $10^{-2}$
Figure 42: Reliability: reduced $B(3,1)$ by 60%, other parameters kept constant, $Q/R = 10^{-2}$

Figure 43: Reliability: reduced $B(3,1)$ by 70%, other parameters kept constant, $Q/R = 10^{-2}$

Figure 44: Reliability: reduced $B(3,1)$ by 80%, other parameters kept constant, $Q/R = 10^{-2}$
Closed loop system of the three control designs are all reliable under parameter variation for $Q/R = 10^{-6}$ and $Q/R = 10^{-2}$.
### TABLE 4:
STABILITY ANALYSIS FOR EXACT MODEL \((0 < \varepsilon < 0.3)\)

<table>
<thead>
<tr>
<th>Q/R ratio</th>
<th>Robust</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rs</td>
<td>Ls</td>
</tr>
<tr>
<td>(10^{-7})</td>
<td>±4%</td>
<td>±4%</td>
</tr>
<tr>
<td>(10^{-6})</td>
<td>±4%</td>
<td>±4%</td>
</tr>
<tr>
<td>(10^{-5})</td>
<td>×</td>
<td>−1 to 4%</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>×</td>
<td>±4%</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>(10^{-2})</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

Note: X indicates the system is not stable for the parameter variation.

### TABLE 5:
STABILITY ANALYSIS FOR REDUCED MODEL \((0 < \varepsilon < 0.3)\)

<table>
<thead>
<tr>
<th>Q/R ratio</th>
<th>Robust</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rs</td>
<td>Ls</td>
</tr>
<tr>
<td>(10^{-7})</td>
<td>±4%</td>
<td>±4%</td>
</tr>
<tr>
<td>(10^{-6})</td>
<td>±4%</td>
<td>±4%</td>
</tr>
<tr>
<td>(10^{-5})</td>
<td>×</td>
<td>−1 to 4%</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>×</td>
<td>−1 to 4%</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>×</td>
<td>−1 to −4%</td>
</tr>
<tr>
<td>(10^{-2})</td>
<td>−4 to 0%</td>
<td>±4%</td>
</tr>
</tbody>
</table>
Note: X indicates the system is not stable for the parameter variation

**TABLE 6:**  
STABILITY ANALYSIS FOR COMPOSITE MODEL \((0 < \varepsilon < 0.3)\)

<table>
<thead>
<tr>
<th>Q/R ratio</th>
<th>Robust</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rs</td>
<td>Ls</td>
</tr>
<tr>
<td>(10^{-7})</td>
<td>±4%</td>
<td>±4%</td>
</tr>
<tr>
<td>(10^{-6})</td>
<td>±4%</td>
<td>±4%</td>
</tr>
<tr>
<td>(10^{-5})</td>
<td>×</td>
<td>(-1) to 4%</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>×</td>
<td>(-1) to 4%</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>(10^{-2})</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

Note: X indicates the system is not stable for the parameter variation
CHAPTER 4
CONCLUSION AND FUTURE WORK

4.1. Conclusions

A design using singular perturbation technique for DFIG is presented in this work. Due to the uncertainties associated with the system parameters, reliability and robustness of the reduced order model is compared with the exact and composite models. This work addressed a vital issue with the large penetration of distributed wind turbines, i.e. developing controller using reduced order dynamics compared full order dynamics where similar system response is achieved in both designs. The following outcomes are achieved in this work:

- For the nominal system, the cost of all three designs are positive for the singular perturbation parameter, \( 0 \leq \varepsilon \leq 0.3 \). Closed loop cost is calculated using the Lyapunov solution; a positive cost indicates positive definite Lyapunov solution which guaranties the system stability. Therefore in could be inferred that the system is stable for Q/R ratio less than 0.01.

- From singular perturbation theory, the following condition for cost of the system should be satisfied for the validity of the technique [22].

\[
J_{\text{exact}} \leq J_{\text{composite}} \leq J_{\text{reduced}}
\]

Based on Figures 6 – 10, this condition is satisfied only if Q/R is less than \( 7.4 \times 10^{-4} \). This condition is violated only in a small region of \( \varepsilon \), therefore this violation is due to numerical ill conditioning of the system parameters.

- When the system parameters are varied, from tables 4 – 6, it can be seen that the system is reliable for the region of Q/R is less than \( 10^{-6} \) for all three models. Further it can be
seen that the robustness of the reduced order model equal to the exact and composite model.

- From Tables 4-6 it could be seen that, for all three models system is reliable for the entire range of Q/R considered in this work.

Based on the above discussion it could be concluded that the reduced order model performs similar to the exact and composite model, and the system is robust for parameter variation within 4% and reliable if the Q/R ratio is less than $10^{-6}$.

### 1.6. Future Work

To extend the work towards improving the controller for DFIG the following future work is recommended:

- From Figures 2-5 the closed loop system response $Q/R = 10^{-2}$ is comparatively better than $Q/R = 10^{-6}$. But system robustness and reliability is better if $Q/R < 10^{-6}$. A different weighing method could should be examined to improve both the performance, robustness and reliability.

- Develop a controller for the electromechanical system and combine the proposed controller with this controller for overall DFIG steady state control.

- Include the power electronics converter dynamics and develop a controller for better performance. Investigate the possibility of using a three-time scale model.

- Develop a controller using singular perturbation technique for transient operations of the DFIG.

- Include the rake effect of wind to develop and improve the controller in the wind farm using the large scale control systems concepts.
LIST OF REFERENCES


APPENDIX A

ADDITIONAL SIMULATION RESULTS

Figure 47: Robustness: changed $L_S$ by -4%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 48: Robustness: changed $L_S$ by -3%, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 49: Robustness: changed $L_S$ by -2%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 50: Robustness: changed $L_S$ by -1%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 51: Robustness: changed $L_S$ by +1%, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 52: Robustness: changed $L_S$ by $+2\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 53: Robustness: changed $L_S$ by $+3\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 54: Robustness: changed $L_S$ by $+4\%$, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 55: Robustness: changed $R_R$ by -4%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 56: Robustness: changed $R_R$ by -3%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 57: Robustness: changed $R_R$ by -2%, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 58: Robustness: changed $R_R$ by -1%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 59: Robustness: changed $R_R$ by +1%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 60: Robustness: changed $R_R$ by +2%, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 61: Robustness: changed $R_R$ by +3%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 62: Robustness: changed $R_R$ by +4%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 63: Robustness: changed $L_R$ by -4%, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 64: Robustness: changed $L_R$ by -3%, all other parameters kept constant for $Q/R = 10^{-5}$

Figure 65: Robustness: changed $L_R$ by -2%, all other parameters kept constant for $Q/R = 10^{-5}$

Figure 66: Robustness: changed $L_R$ by -1%, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 67: Robustness: changed $L_R$ by $+1\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 68: Robustness: changed $L_R$ by $+2\%$, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 69: Robustness: changed $L_R$ by $+3\%$, all other parameters kept constant for $Q/R = 10^{-6}$
Figure 70: Robustness: changed $L_R$ by +4%, all other parameters kept constant for $Q/R = 10^{-6}$

Figure 71: Reliability: reduced $B(4,2)$ by 10%, other parameters kept constant, $Q/R = 10^{-6}$

Figure 72: Reliability: reduced $B(4,2)$ by 20%, other parameters kept constant, $Q/R = 10^{-6}$
Figure 73: Reliability: reduced $B(4,2)$ by 30%, other parameters kept constant, $Q/R = 10^{-6}$

Figure 74: Reliability: reduced $B(4,2)$ by 40%, other parameters kept constant, $Q/R = 10^{-6}$

Figure 75: Reliability: reduced $B(4,2)$ by 50%, other parameters kept constant, $Q/R = 10^{-6}$
Figure 76: Reliability: reduced B(4,2) by 60%, other parameters kept constant, Q/R = 10^{-6}

Figure 77: Reliability: reduced B(4,2) by 70%, other parameters kept constant, Q/R = 10^{-6}

Figure 78: Reliability: reduced B(4,2) by 80%, other parameters kept constant, Q/R = 10^{-6}
Figure 79: Reliability: reduced $B(4,2)$ by 90%, other parameters kept constant, $Q/R = 10^{-6}$

Figure 80: Reliability: reduced $B(4,2)$ by 100%, other parameters kept constant, $Q/R = 10^{-6}$

Figure 81: Robustness: changed $L_S$ by -4%, all other parameters kept constant for $Q/R = 10^{-2}$
Figure 82: Robustness: changed $L_S$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 83: Robustness: changed $L_S$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 84: Robustness: changed $L_S$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$
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Figure 89: Robustness: changed $R_R$ by -4%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 90: Robustness: changed $R_R$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$
Figure 91: Robustness: changed $R_R$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 92: Robustness: changed $R_R$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 93: Robustness: changed $R_R$ by +1%, all other parameters kept constant for $Q/R = 10^{-2}$
Figure 94: Robustness: changed $R_R$ by $+2\%$, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 95: Robustness: changed $R_R$ by $+3\%$, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 96: Robustness: changed $R_R$ by $+4\%$, all other parameters kept constant for $Q/R = 10^{-2}$
Figure 97: Robustness: changed $L_R$ by -4%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 98: Robustness: changed $L_R$ by -3%, all other parameters kept constant for $Q/R = 10^{-2}$

Figure 99: Robustness: changed $L_R$ by -2%, all other parameters kept constant for $Q/R = 10^{-2}$
Figure 100: Robustness: changed $L_R$ by -1%, all other parameters kept constant for $Q/R = 10^{-2}$

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Figure 114: Reliability: reduced B(4,2) by 100%, other parameters kept constant, Q/R = 10^-2