IMPACT DYNAMICS OF MECHANICAL SYSTEMS AND STRUCTURES, AND APPLICATIONS IN CRASH ENERGY MANAGEMENT, IMPULSE MITIGATION, AND IMPACT INJURY BIOMECHANICS

A Dissertation by

Rasoul Moradi

M.Sc., University of Tehran, Iran, 2000

B.Sc., University of Tehran, Iran, 1997

Submitted to the Department of Mechanical Engineering and the faculty of the Graduate School of Wichita State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

May 2012
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The following faculty members have examined the final copy of this dissertation for form and content, and recommended that it is accepted in partial fulfillment of the requirement for the Doctor of Philosophy with a major in Mechanical Engineering.

Hamid Lankarani, Committee Chair

Michael McCoy, Committee Member

Ramazan Asmatulu, Committee Member

Brian Driessen, Committee Member

Krishnan Krishna, Committee Member

Accepted for the College of Engineering

Zulma Toro-Ramos, Dean

Accepted for the Graduate School

J. David McDonald, Dean
DEDICATION

To my parents, Ali and Morvarid; my wife, Ladan; and my little angels, Souren and Sophie
ACKNOWLEDGMENTS

I wish to express my deep and sincere gratitude to Dr. Hamid Lankarani, faculty member of the Department of Mechanical Engineering at Wichita State University. His wide knowledge, logical way of thinking, understanding, and support have been of great value to me. He and his lovely family have been very kind friends to me and my family during the last three years. Without his guidance, this research would not have been possible.

I extend gratitude to my committee members, Dr. Michael McCoy, Dr. Ramazan Asmatulu, Dr. Brian Dirresen, and Dr. Krishnan Krishna, for their valuable comments and suggestions.

I am grateful to my beloved wife, Ladan Lamee, for her editorial comments on this dissertation, as well as her support and understanding, and being with me through good and bad moments of my life.

I take this opportunity to express my sincere gratitude to my father, mother, brothers, and sisters for providing me spiritual and moral support.

I am grateful to all faculty and staff of the Department of Mechanical Engineering for their constant support and guidance throughout the duration of my studies at Wichita State University.

My thanks also go to TASS Americas for supporting parts of this project and providing valuable suggestions.
ABSTRACT

Among the different load conditions on a mechanical system, impact loading and its contribution to the design process require special consideration. The static methods of stress, strain, and deflection analyses are not applicable under impact conditions. The main goal of this study is to address the fundamental aspects of impact and to examine its applications for different design requirements. First, different approaches to the impact phenomena, namely stereomechanics, contact mechanics, stress wave propagation, finite element method, and energy method are investigated in this dissertation. The advantages and disadvantages of each method are pointed out, and the areas of application of each method and the degree of accuracy are examined. Quantification of energy absorption during impact is the most complicated part of impact modeling and is one of the topics of interest addressed in this dissertation.

Application of the impact analysis methodologies in vehicular accidents and protection of occupants are the eventual goals of this research, demonstrated using some case studies and applicable examples. Because occupant safety is a major concern in the automobile and aerospace industries, a crashworthy design must be able to dissipate the kinetic energy of impact in a controlled manner. Four test cases or applications related to impact energy management or dissipation, impulse mitigation, and impact injury biodynamics are thus presented. The application examples include the design of a truck side guard and quantification of its effects on reducing occupant injury in the collision of a small car with a truck; lumbar load attenuation for seated occupants of a rotorcraft; injuries to pedestrians impacted by a sport or utility vehicle equipped with a frontal guard; and investigation of a motorcyclist impact with roadside barriers. For each case, an analysis methodology is developed, and from the modeling and simulations, impact design issues are addressed.
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CHAPTER ONE
INTRODUCTION

1.1 Background

Not long ago, the design of components and machines was based on trial and error and experienced craftsmanship. In today’s industry, there is little room for error, and there is a great need for optimized and cost-effective design of machines with high reliability and durability. The development of mathematical models and related assumptions and approximations in engineering design have taken over the task of trial and error in an effective time and cost-saving manner. The accuracy of the models and results strongly depends on the user’s experience and accurate definition of the parameters and variables in an engineering problem.

Mechanical loadings and interactions between different components in a system are an important part of all mechanical engineering design. It is known that materials have different characteristics for different loading conditions. Four classifications of loading are generally recognized in engineering design: static loading, fatigue loading, dynamic loading, and impact loading. System and component design functions are quite different for these various loadings. The basics of impact loading is the main interest of this study, which will be examined in detail in the first part of this dissertation. Industrial applications for different impact design functions will be demonstrated in the chapters that follow.

Impact is the transmission of a high force or impulse from one body to another within a very short period of time. Colliding bodies experience sudden changes in velocity with little change in spatial position during the collision process. The static methods of stress, strain, and deflection analyses are meaningless under impact conditions due to the propagation, reflection, and interference of elastic/plastic waves traveling within the engineering solid. The study of
impact and its contribution toward the design process has attracted extensive engineering effort during the last few decades.

The “piecewise analysis method” reported in the literature has been used primarily for systems containing particles or unconstrained bodies. A set of linear momentum-balance/impulse equations are formulated and solved at the time of impact in order to evaluate the velocities of the system right after impact.

For many impact problems, since high stress levels are encountered in the affected parts due to the application of impulsive forces, the rigid body assumption is no longer adequate. Different models have been postulated to represent the variations of the force induced at the surfaces of the two bodies in contact. The most well-known contact force model was derived by Hertz in 1895 [1]. The material compliance of the two bodies accounts for the generalized parameters in the Hertz contact force law. A large number of studies have been performed since then to extend the model used by Hertz to the contact between any two surfaces, and extended to mechanical systems impact.

More accurate results can be obtained utilizing the stress wave propagation approach for an impact loading. The variation of local strain/stress levels in the solid can be identified as a function of time and space. On the other hand, stress wave propagation is highly mathematical and requires a large amount of simplification of the impacted mechanical system. In engineering applications where complex geometries are involved, the stress wave method can be captured by the use of explicit finite element analysis (FEA).

The most complicated part of modeling an impact with either approach is the process of energy transfer. The kinetic energy of the impacting bodies is reduced and in the process leads to vibration and plastic deformations in the system. In this dissertation, the energy dissipation for
different methods for the impact dynamics of mechanical systems and structures shall be examined and applicational examples will be investigated.

Two engineering design functions can be recognized concerning impact. The first is to mitigate damage to the impactor. An example of this is in vehicle collisions where the crush zones of the vehicle’s structure, airbags, and interior padding reduce the impact loading on the occupants. Another example is in the design of a side guard for heavy trucks and trailers, which reduces intrusion of the passenger compartment of striking cars to the truck or trailer side, hence reducing the injury level of the car occupant(s). The second design function is to retard the impactor and mitigate damages to the target. An example of this is the impact between the seat pan and the occupant of an aircraft seat during a crash landing, or between the seat pan and occupant of a military vehicle during a mine blast. In both cases, the seat pan produces impulsive loading to the pelvis of the occupant, which may result in severe lumbar injury to the occupant.

The main focus of this dissertation is to investigate different industrial applications of impact theories and methods for each design function. The applicational case studies will involve the most common area of impact theories, namely vehicular crashes. Figure 1.1 shows some examples of impact in mechanical systems and structures.

Studies of crashworthiness, structural impact analysis, and post-crash dynamic behavior of vehicle and aircraft occupants have attracted significant attention from investigators in both industry and academia. Occupant safety is a major concern in the aerospace and automobile industries. Hence, a crashworthy design must be able to dissipate the kinetic energy of impact in a controlled manner. Proper modeling of contact forces and energy dissipation forms the basis for designing crashworthy structures. This has provided motivation for this study to examine and
investigate some impact examples for the proper modeling of contact/impact and for different design functions.

![Diagram of impact examples](image)

**Figure 1.1. Examples of Impact in Mechanical Systems and Structures**

### 1.2 Literature Review

Engineering materials show different responses for different loading conditions. In an attempt to classify an impact condition, four loading classifications in engineering are presented in the literature: static, fatigue, rapid or dynamic, and impact [2-4]. These classifications are categorized by the rate of loading (or rise time) on the mechanical system or components compared with the fundamental period of the system. Static loading occurs when the rise time of the load is at least 3 times greater than the fundamental period of the mechanical system. The usual methods of analysis for stress, strain, and displacements along with static material
properties are utilized to design the system or components. Fatigue loading is a time-varying load where the rise time between two magnitude changes still remains greater than three fundamental periods. Rapid loading is defined as a loading condition where the rise time of the load application ranges between 1.5 to 3.0 times the fundamental period of the mechanical system. Vibration methods of stress and deflections analysis should be used in the loading category. If the time of load application is less than 0.5 times the fundamental period of the mechanical system, then the loading is defined as “impact loading.” The static methods of stress, strain, and deflection analyses are no longer valid under impact conditions. Dynamic methods of analyses such as contact mechanics, energy methods, stress wave propagation, and FEA must be used to estimate the effects of impact on mechanical systems. An overview of impact dynamics analyses according to different methodologies are explained next.

1.2.1 Impact Dynamics

The nature of impact involves a minimum of two-body mechanical system. Impact may be defined as a sudden change in the momentum of each contacting body, without a corresponding change in their positions or configuration. That is, impact is the transmission of a high force or impulse from one body to another within a very short period of time. During this time period, colliding bodies experience sudden changes in velocity with little change in spatial position. Impact phenomena is quite varied depending on the velocity of the impactor and the characteristics of the target or struck system. Zukus et al. [5, 6] have shown that for a high velocity impact, the response of the impacted components is local and highly dependent on the constitution of the material of the impacting body. For low-velocity impacts, both the geometry and material respond to the impactor. Shivaswamy [7] showed that the stiffness of the impacted system greatly influences the contact time and force, and that contact force, contact time, and
system stiffness are inversely related. As the system stiffness increases, the contact force increases and the contact time decreases.

The subject of impact has attracted the interest of scientists and engineers from different areas of expertise from industry to academia. The common goal has been to develop theories that can predict the behavior of colliding objects accurately. In the evolution of impact theory, several major approaches are recognized. The appropriate approach utilized depends on the parameters desired from the analysis: velocities, stresses, deflections, plastic deformation or energy absorption, along with types of simplifying assumptions on the impact event. The duration of the contact period governs the choice of the method used for analyzing the impact [8, 9]. These methodologies are as follows:

- Classical mechanics or stereomechanics
- Contact mechanics
- Stress wave propagation
- Plastic deformation method
- Energy method or equivalent load method
- Finite element method
- Experimental method

“Stereomechanics” is the application of classical Newtonian mechanics to colliding bodies in order to predict pre- and post-impact velocities. This method uses the conservation of energy and momentum laws [10]. The coefficient of restitution is used to relate the effect between the pre- and post-impact velocities as the result of energy dissipation. The advantage of stereomechanics is that it is algebraic and thus easy to apply and accessible to practicing engineers. The disadvantage is the lack of analytical tools to define the coefficient of restitution.
Thus, collisions involving energy dissipation are typically analyzed experimentally. Another disadvantage is that neither the contact duration nor the contact force is predicted by the stereomechanics approach.

The “contact mechanics” approach to the impact of a mechanical system is through the examination of the contact forces in the contained area of deformation between the colliding bodies. Contact mechanics uses force-deformation equations to estimate the local stresses along with elastic/plastic deformations and contact-duration times. This method originated from the Hertz theory of elastic contact for two spheres in contact. Several investigators, including Lankarani, have extended the Hertzian theory to include the effects of plastic deformation, internal damping, and hysteresis [11, 12]. Contact mechanics is a useful and efficient method of introducing the forces developed in impacts and including them in multibody dynamics analysis. The disadvantages are the analytical selection of a force-deformation equation and establishing the parameters, which are required to define the force-deformation relationship.

The “stress wave theory” addresses the phenomenon of when an impactor strikes a solid. At impact, a strain wave is initiated at the contact region and transverses or radiates throughout the solid. As time progresses, these original waves contact the boundary surfaces of the solid and reflect inward. This generates standing or interfering strain waves, which produce larger strains and stresses than that of ordinary static loading. The advantage of the stress wave method is an accurate strain/stress analysis on the impacted elastic solid. Also, the variation of local strain/stress levels in the solid can be identified as a function of time and space. On the other hand, the stress wave propagation is highly mathematical and requires considerable simplification of the impacted mechanical system, which limits it to mainly one-dimensional problems such as a rigid body impactor impacting the end of an elastic cantilever rod.
For impacts where plastic strains occur outside of the contact area, some form of “structural plastic” analysis is required. The perfect elastic-plastic analysis exhibits the most practicality in predicting large plastic strains due to impact loading. Bohnenblust [13], Conroy [14, 15], and Symonds [16] developed methods to analyze beams undergoing plastic deformation using rigid perfectly plastic and elastic perfectly plastic material constituents.

The “energy method” or “equivalent load method” is an approximation technique for the analysis of impact problems. It has a great deal of design practicality because it can be readily used to analyze mechanical components with impact loading. The basis for this method is that mechanical energy is conserved in the elastic compression of the impacted system bodies [4, 5]. The potential and kinetic energies of the impactor are converted into strain energy, which is stored within the struck system or structure. The maximum deflections and stresses of the impacted system occur when the velocity of the impactor becomes zero when all of the impactor’s energy has transferred to the target. Equating the impactor’s energy to the system’s strain energy yields the maximum deformation or deflection of the system. From the known deflections, the stresses of the mechanical system can be approximated as the result of the impact, thus aiding in design of the system. The disadvantage is that the basic energy method ignores the inertial effects of the target and energy conversion. Cox [17] improves on the basic energy method by including inertial effects of the target.

As discussed, these classical methods are quite limited to simple impacts of mechanical systems. The “finite element analysis” has the capability of solving complex systems, unlike the classical methods. However, incompetent use of FEA without knowledge of its inner workings and proper modeling techniques can be disastrous.
1.2.2 Dynamic Analysis of Multibody Systems

A multibody system is defined as a collection of interconnected rigid or flexible bodies by kinematic joints and/or some force elements, each of which may undergo large translational or rotational movements [18]. A multibody system can be used to study the kinematic and dynamic motion characteristics of a wide variety of systems in a large number of engineering fields of application from biomechanics to space systems. Constraint conditions describe the interconnection between a pair of bodies through the mechanical joints. Constraints are typically an algebraic equation that defines the relative translation or rotation between two bodies. More simply, multibody system methodologies include the development of mathematical models of systems and the implementation of computational procedures to perform the simulation, and analysis of the global motion produced [19].

The dynamic analysis of multibody systems, a research area with broad applications in a variety of engineering fields, has attracted significant attention over the last few decades [20-25]. The fundamental approaches of rigid and flexible body dynamics have been discussed in several review papers [26-32]. In turn, many multibody computational programs capable of automatic generation and integration of differential equations of motion have been developed, namely DAP [21], ADAMS [33], and MADYMO [34].

The prediction of the dynamic behavior of multibody mechanical systems involves the formulation of the governing equations of motion and the evaluation of their kinematic and dynamic characteristics. The transient dynamic analysis of constrained mechanical systems may require the solution of a mixed set of algebraic and differential equations [19]. The equations can be described in terms of different sets of coordinates. If the number of coordinates is greater than the number of the system degrees of freedom, then algebraic equations are required to show the
dependency of the coordinates for both open- and closed-loop systems. Assume that the algebraic equations are presented by \( m \) independent algebraic holonomic constraint equations [21]:

\[
\Phi(q,t) = 0
\]  
(1.1)

where \( q \) is a vector of \( n \) coordinates. The constraint equations represented by equation (1.1) are non-linear in terms of \( q \). The kinematic velocity and acceleration equations are obtained by taking the first and the second time derivatives of equation (1.1). Then,

\[
\Phi_{\dot{q}} = \Phi_{\ddot{q}} = 0
\]  
(1.2)

\[
\Phi_{\ddot{q}} \ddot{\dot{q}} = -(\Phi_{\dot{q}})_{\dot{q}} \dot{q} \dot{q} - 2\Phi_{\dot{q}} \ddot{q} - \Phi_{tt} = \gamma
\]  
(1.3)

in which \( \Phi_{\dot{q}} \) is the constraint’s Jacobian matrix, which has a dimension of \( m \times n \), \( \dot{q} \) is the acceleration vector, and \( \gamma \) is the right-hand side of acceleration equations that contain quadratic velocity terms. In case of scleronomous constraints, that is, when \( \Phi \) is not explicitly a function of time, the terms \( \Phi_{t} \), \( \Phi_{qt} \), and \( \Phi_{tt} \) in equation (1.3) vanish.

The kinematic analysis of a multibody system can be carried out by solving equations (1.1), (1.2), and (1.3) by specifying the necessary initial conditions. In addition to kinematic equations of position and velocities, the dynamic analysis of a mechanical system can be described by a set of differential equations of motion. Different methodologies can lead to suitable descriptions of multibody systems, each of them presenting relative advantages and disadvantages. In either case, the resulting equations have the same physics, although a different mathematical appearance in the form of a set of differential equations that may be solved analytically or numerically with some initial conditions on the system coordinates or velocities.
The equations of motion may be derived in terms of the accelerations $\ddot{q}$ as a set of $n$ second-order ordinary differential equations of the following form [37]:

$$ M\ddot{q} = g + g^{(c)} , \quad g^{(c)} = -\Phi_q^T\lambda $$

(1.4)

where $M$ is the system mass matrix, vector $g$ contains the applied and the gyroscopic forces in the Newton-Euler equations of motion, $\lambda$ is a vector of $m$ Lagrange multipliers, and the term $\Phi_q^T\lambda$ represents the constraint forces. When the system is kinematically constrained, the effect of the kinematic joints on the motion of the system can be taken into account by including the forces applied on each pair of bodies connected by a joint in the system vector of forces.

In dynamic analysis, a unique solution is obtained when the constraint algebraic equations are considered simultaneously with the differential equations of motion, for a proper set of initial conditions. Mathematically, the simulation of a constrained multibody system requires the solution of a set of $n$ differential equations coupled with a set of $m$ algebraic equations, where $n$ is the number of degrees of freedom and $m$ is the number of constraints, written as

$$
\begin{bmatrix}
M & \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\lambda
\end{bmatrix}
= 
\begin{bmatrix}
g \\
\gamma
\end{bmatrix}
$$

(1.5)

This system of equations is solved for $\ddot{q}$ and $\lambda$. Then, in each integration time step, the accelerations vector, $\ddot{q}$, together with velocities vector, $\dot{q}$, are integrated using a proper integration scheme [35] in order to obtain the system velocities and positions for the next time step. This procedure is repeated until the final analysis time is reached [21, 35-36].

The direct integration method of equations of motion is prone to integration errors, because the position and velocity constraint are only satisfied at the initial time. The constraint
violations result from accumulated integration errors and become more apparent with stiff
systems. In order to control the constraint violation during the numerical integration, one method
is to use the Baumgarte stabilization method (BSM) [37], and equation (1.5) is modified as

\[
\begin{bmatrix}
M & \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{q}} \\
\dot{\lambda}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{g} \\
\gamma - 2\alpha\dot{\Phi} - \beta^2\Phi
\end{bmatrix}
\]  

(1.6)

where \(\alpha\) and \(\beta\) are prescribed positive constants that represent the feedback control parameters
for the velocities and positions constraint violations [37-38]. In fact, the BSM allows constraints
to be slightly violated before corrective action occurs, in order to force the violation to vanish.

With this methodology, the numerical result oscillates around the exact solution. Magnitude and
frequency of the oscillation depend on the Baumgarte parameter values. Other stabilization
methods for equations of motion include the coordinate partitioning method, and the augmented
Lagrangian method.

Equations (1.5) and (1.6) are referred to as equations of motion in terms of the
“acceleration” or the “forces of constraints.” They are also referred to as the “standard” form of
the equations of motion. The equations of motion can also be derived in terms of the total
momenta of the system. This method is called “canonical” equations or the equations of motion
in terms of momenta of constraints [19, 21]. A study of the numerical solutions for two forms by
Lankarani revealed that the canonical equations provide a more accurate and stable response
[19].

A multibody mechanical system may undergo a discontinuous motion due to an abrupt
change in the applied forces on the system, such as the one shown in Figure 1.2. For those
mechanical systems that undergo an impact, the usual numerical solution of the equations is not
valid. The most common example of this impulsive loading is encountered in the problem of
impact. For a kinematically constrained system, the impulsive effects are not only due to impulsive forces at the surfaces of the colliding bodies, but also to discontinuities in the forces of constraints.

![Figure 1.2. Impact in multibody systems](image)

The continuous contact mechanics method of impact analysis assumes that the motion is not discontinuous, and the contact force acts on the bodies in a continuous manner. The differential equations of motion of the system remain the same as discussed previously. The only difference is that a contact force between the colliding bodies must be added to the system of forces during the contact period.

Despite its long history, multibody system contact mechanics is still an active challenging domain for research as stated by Schiehlen, “more work is required to better understand the micromechanical phenomena influencing the macro-mechanical multibody motion with contact” [39].

### 1.2.3 Multibody Systems with Imperfect Joints

In the dynamic analysis of multibody mechanical systems, it is assumed that the kinematic joints are ideal or perfect. This means that joints are not modeled [40], and the effects of clearance, local elastic/plastic deformations, and wear are neglected. The functionality of the
kinematic joints relies on the relative motion allowed between the connected components, which, in practice, implies the existence of a clearance between the mating parts, leading to surface contact, shock transmission, and the development of different regimes of friction and wear. Regardless of the amount of clearance, this can lead to vibration and fatigue phenomena, lack of precision, or, in the limit, even random overall behavior. If there is no lubricant or other damping materials in the joint, any relative velocity between the connected bodies initiates impact in the system, and the corresponding impulses are transmitted throughout the multibody system. Therefore, it is very important to quantify the effects of both clearance joints and body flexibility on the global system response in order to define the minimum level of suitable tolerances that allow mechanical systems to achieve required performances. An example is the existing clearance between journal and bearing in a revolute joint, as depicted previously in Figure 1.1 (c).

Degradation of the performance of mechanical systems with clearance joints has been recognized for a number of years [41]. It is known that the existence of clearance in joints leads to load amplification, where the magnitude of load rises in proportion to the amount of clearance presents [42]. These impact forces contribute to the failure of components due to shock loading, thus reducing the system life, generating high noise levels, and causing energy dissipation and exciting unwanted vibratory responses [43].

The subject of representation of imperfect joints has attracted the attention of many researchers and has produced considerable theoretical and experimental work devoted to the dynamic simulation of mechanical systems with joint clearances. It has been demonstrated that the dynamic response of mechanical systems with clearance joints can be periodic in some situations and chaotic in others [43-48].
The dynamics of joints with clearance is controlled by contact-impact forces rather than the kinematic constraints of ideal joints. A force model that accounts for the geometric and material characteristics of the clearance joint components describes these impacts and the eventual continuous contact [49]. The energy dissipation effects are introduced in the joints through the contact force model and by friction forces that develop during contact [50]. In the continuous force contact model, it is necessary to calculate the relative velocity between the impacting surfaces. The relative velocity between the contact points is projected onto the tangential line to the colliding surfaces and onto the normal-to-colliding surfaces, yielding a relative tangential velocity and a relative normal velocity. The normal relative velocity determines whether the contact bodies are approaching or separating. The tangential relative velocity determines whether the contact bodies are sliding or sticking.

1.2.4 Energy Dissipation in Impact

The impact phenomena can be evaluated on two different scales, namely the micro-scale and the macro-scale. Accordingly, the impact parameters, such as deformation or penetration in impact, are divided into local and global deformations.

In micro-scale point of view, as mentioned previously, two main methodologies are considered, based on rigid-body collision assumptions. In the continuous approach, based on the Hertzian theory, the hysteresis damping or local plasticity accounts for the energy dissipation. For the piecewise analysis method, the coefficient of restitution accounts for energy dissipation in impact. For both the continuous and piecewise analysis methods, it is assumed that the application of the contact force during the period of contact does not cause any significant general (global) deformation of the bodies. The most complicated part of modeling an impact with either approach is the process of energy transfer. Mathematical models for this physical
phenomenon must be idealized, and the postulated dynamic behavior must be verified by suitable experiments. Contact forces that are produced during an impact act for a small time period, during which time there might be high energy dissipation. The kinetic energy of the impacting bodies is reduced and, in the process, leads to vibration and plastic deformation in the system.

In continuous impact analysis, to represent the energy loss in impact, the variations of the contact force must follow different trends during the two phases of contact. Different models based on the Hertzian theory have been developed to account for the energy loss due to the hysteresis damping or permanent indentation of the two bodies after impact [11]. To verify whether the hysteresis damping model or local plastic penetration model is applicable or not, contact stress at the point of contact was evaluated based on the contact force data.

In the discontinuous method of impact analysis, energy loss in the impact may be expressed in terms of the coefficient of restitution and the relative approach velocity. Lankarani developed a relationship between the hysteresis damping factor and the coefficient of restitution based on the Hertzian contact force model, and hence, the continuous method of impact analysis was suggested in terms of the coefficient of restitution [19]. According to Goldsmith [51] and Lankarani [11], the dissipated energy in impact is proportional to the cubic power of the initial impact velocities of the bodies.

For many impact problems, however, especially for high-speed impacts, large deformations occur on the colliding bodies, and the deformation associated with impact needs to be considered in macro-scale. Hence, in macro-scale, the load applied to the system during the impact is applied in equations of motion, and the resultant stresses are compared with the strength of the structure. Using the appropriate material properties and failure criteria, the resultant strains and stresses can be evaluated, and the global configuration of the system can be
determined. This process is appropriately developed, and the related dissipated energy due to
global deformation is well formulated.

The methodology for both micro- and macro-scale impact energy dissipations will be explained in detail in the following chapters.

1.2.5 Structural Crashworthiness/Safety and Regulations

Probably the most common collisions affecting people are those involving bicycles, cars, and aircraft. Structural crashworthiness is a term used to describe the study that is conducted on the impact performance of a structure when it collides with another object where humans are involved in the collision process. The crashworthiness study uses analytical tools of a structural engineer, such as theories of elasticity and plasticity along with finite element analysis, to assess a vehicle’s structural design. It involves an examination of the following:

- Estimation of collision forces and accelerations
- Damage of structure
- Impact energy absorption
- Survivability of passengers and evaluation of occupant injury

A crashworthy design must be able to dissipate the kinetic energy of impact in a controlled manner. The goal of a crashworthy structure is to protect and minimize occupant injuries in the event of a collision. Whether the vehicle is a car, plane, or train, two basic strategies are used to mitigate injuries to occupants. The first is to reduce the intrusion of striking objects into the occupant area, which is accomplished by a strong compartment area. The second is to absorb the impact energy using structures, both exterior and interior. This increases the ride-down time of occupants and reduces occupant accelerations and thus potential injuries.
Crash severity for car occupants is measured by the level of cabin accelerations generated and the degree of cabin intrusions of the impacting vehicle on the struck vehicle [52]. Crash severity is further measured by injury indexes on the occupants. For vehicle-to-vehicle crashes, the impact energy is shared between the crash partners according to the compatibility of the vehicles. Severity is lower between compatible vehicles in which reduced cabin accelerations, cabin intrusions, and injuries occur as the result of matching load paths, vehicle stiffness, and vehicle masses. A compatible vehicle collision occurs when the vehicles’ masses, collision stiffnesses, and physical geometries of the collision partners closely match one another. An example of an incompatible collision is that of a crash between a sport utility vehicle (SUV) or a light truck or van (LTV) and a passenger vehicle, where stiffnesses, masses, and load paths do not match well.

Both vehicles in an accident event experience the same collision force, which arises from the crushing of the vehicle structures and both vehicles experience the same contact time of the collision. Thus, the collision force as a function of time for each vehicle is the same, so a vehicle’s collision acceleration is governed by its mass. Hence, the lighter vehicle will experience a higher acceleration, thus higher \( \Delta V \), than that of the heavier vehicle in a collision. Using stereomechanics and assuming a zero coefficient of restitution, \( e \), the \( \Delta V \) for colliding vehicles can be estimated as

\[
\begin{align*}
\Delta V_1 &= \frac{m_2}{m_1 + m_2} \left( V_{closing} \right) \\
\Delta V_2 &= \frac{m_1}{m_1 + m_2} \left( V_{closing} \right)
\end{align*}
\]  

(1.6)

If \( m_2 \) is assumed to be the heavier vehicle, then the other will experience a larger \( \Delta V \). Higher \( \Delta V \)’s create greater injury potential.
In a vehicle collision where there is a stiffness mismatch between the collision partners, the softer vehicle will deform significantly before the stiffer vehicle deforms. The stiffer vehicle does not share or participate in the energy dissipation and increases the crash severity of the softer vehicle. An ideal collision event would be where the collision partners equally share in the dissipation of crash energy.

Vehicle geometry affects crash severity. A collision between different makes/sizes of vehicles can involve the misalignment of vehicle structures, thus compounding the crash severity by providing a mismatch in structural load paths. The degree of vehicle geometry inducing load path matching is called “geometric compatibility.” The worst geometric incompatible impacts occur between SUVs and compact passenger vehicles. The impact of a heavy truck with a passenger vehicle exhibits even worse incompatibility. In head-on collisions involving an SUV and a compact car, the bumper of one does not match the bumper of the other, and therefore the SUV overrides the compact structure. Neither of the vehicles’ structures fully participates in energy absorption because their structural load paths are not aligned. A similar situation can occur in the impact of a compact car with the rear of an SUV or truck, whereby the compact car underrides the SUV structure, and crash severity is increased because the vehicle structure absorbs little energy.

Impact mode is the configuration of the collision, namely the frontal, rear, side, or oblique crash modes. In the frontal collision of a passenger vehicle, the front structure of the vehicle provides the energy-absorbing crush of 20 to 30 inches before occupant-cabin deformation occurs. In a frontal crash, most of the crash energy is absorbed by the body structure, and a small portion of the energy by the power train and the fire wall. Relative to the body frame, the crash energy is mostly absorbed by the main longitudinal members. Crush and
folding locations in these members occur at misalignment of the structure or reduced cross section to produce bending and bucking. Side impacts are the most severe for the target occupants because only 8 to 12 inches of crush distance is available before cabin intrusion occurs, and the occupant injury is controlled by structural deformations rather than acceleration. Door side beams must transfer collision loads to A- and B-pillars of the vehicle and limit deflections. Cross sections of the door-sill must be as large as possible to limit deflections. The floor board should be stiffened, and the door should hook into a sill, as well as the A- and B-pillars. Rear collisions are similar to frontal collisions because 20 to 30 inches of crush is available for energy management. For those fuel tanks located between the bumper and rear axle, the rear structure must protect the integrity of the fuel tank.

Federal Motor Vehicle Safety Standards (FMVSS) require vehicles to provide a certain crash response in order to protect occupants. As previously mentioned, vehicle stiffness must vary over the vehicle to provide the acceptable protection in different accident directions. In a crash event, intrusion into the passenger compartment must be minimized, and to protect the occupant, the cabin must not exhibit a collapse failure by using a stiff compartment structure. Other vehicle structures, such as the front and rear zones, must absorb crash energy through controlled crush deformation. In other words, crush zones should be away from the passenger compartment, and a stiff frame should be near the passenger compartment. Door safety beams, which stiffen the door and lessen intrusion during side impacts, is another example of ensuring stiffness in vehicle crashes.

As a vehicle crushes, it dissipates impact/collision energy. Vehicle stiffness controls the energy management of the collision along with peak accelerations in the vehicle cabin, maximum vehicle crush, and time duration of the crash pulse. To reduce injuries in general, a
softer structure lowers cabin accelerations, increases crush distance for more energy absorption, and increases the time duration of the collision. Too soft of a structure allows cabin intrusion. Crashworthiness is a design optimization of stiffness—not to soft and not to stiff.

Current crash standards allow vehicles to decelerate at moderate rates (25-40 G’s) and then depend on the restraint system to “ride down the occupants” during a collision. A heavier car (more collision energy) will require a stiffer structure to pass the requirement, since the available crush distance is relatively the same for each vehicle design.

The crashworthiness of small aircraft and rotorcraft use the same approach, to some extent, to that of cars during survival crashes or impact landings. If the collision dynamics of the aircraft itself are of interest and the aircraft does not break up, the stereomechanics approach can be used to determine the final configuration of the impacting systems [53]. In some cases upon impact, the rear fuselage slaps the ground. In others, owing to extended rigid landing gear, the fuselage remains oriented closer to its pre-crash tail-up heading. With a good approximation, it can be assumed that the normal impulse $P_n$ is

$$P_n = mv_n = mvsin\gamma$$

where $\gamma$ is the flight path angle. Analysis of data by NASA has led to the conclusion that the “triangular” shape of the pulse does the best job of representing the experimental vertical pulse shape [53]. Knowledge of the pulse shape allows for the calculation of peak forces and accelerations.

To protect the aircraft occupant in any survivable crashes, Federal Aviation Regulations (FAR) specify the requirements for crashworthiness of seat structures of general aviation aircraft, transport aircraft, and rotorcraft, according to Part 23, Part 25, and Part 27 respectively [54-56]. Two impact tests define FAR requirements for crashworthiness performance of the seat system.
Both tests address the seat and restraint systems of the aircraft. In the first test, the aircraft seat is oriented 60º with respect to the closing velocity. The 60º rotation provides acceleration loading in both a spinal vertical and spinal transverse direction, simulating a crash-landing event. The deceleration is a triangular pulse. This test provides an assessment of the energy absorption characteristics of the seat to mitigating vertical loading to the occupant and potential spinal injuries. The second test orients the aircraft seat with 10º of yaw rotation with respect to the closing velocity along with 10º floor pitch and 10º roll for seat legs. Again, the deceleration is a triangular pulse. This test assesses the structural performance of the seat structure and restraints in maintaining integrity. These test configurations are detailed in Table 1.1.

TABLE 1.1. Test Conditions and Compliance Criteria for Title 14 CFR, Parts 23, 25, and 27 Category Aircraft Seats

<table>
<thead>
<tr>
<th>Dynamic Test Requirements¹</th>
<th>Part 23</th>
<th>Part 25</th>
<th>Part 27</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test- I</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity (ft/sec)</td>
<td>31 (9.5 m/s)</td>
<td>35 (10.7 m/s)</td>
<td>30 (9.2 m/s)</td>
</tr>
<tr>
<td>Peak Acceleration (G’s)</td>
<td>19²/15³</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td>Time to Peak (sec)</td>
<td>0.05/0.06</td>
<td>0.08</td>
<td>0.031</td>
</tr>
<tr>
<td><strong>Test- II</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity (ft/sec)</td>
<td>42 (12.8 m/s)</td>
<td>44 (13.4 m/s)</td>
<td>42 (12.8 m/s)</td>
</tr>
<tr>
<td>Peak Acceleration (G’s)</td>
<td>26/21</td>
<td>16</td>
<td>18.4</td>
</tr>
<tr>
<td>Time to Peak (sec)</td>
<td>0.05²/0.06³</td>
<td>0.09</td>
<td>0.071</td>
</tr>
</tbody>
</table>

| Compliance Criteria         |         |         |         |
| Lumbar Load (lb)            | 1500 (6675 N) | 1500 (6675 N) | 1500 (6675 N) |

¹all input pulses are triangular
²pilot
³occupant

One industrial application of impact dynamics in mechanical systems is the aircraft seat system crashworthiness, which will be investigated later on in this dissertation, in a manner similar to the effect of clearance joints in mechanical systems. The study involves occupant protection in the event of an aircraft crash where the vertical impulse from the floor is applied to the seat system and the occupant. This is typical of what is observed in a clearance joint of a
mechanical system, resulting in load amplification, where the magnitude rises in proportion to the amount of clearance present.

As can be seen, there are many different aspects of the impact dynamics of mechanical systems and their application to structural crashworthiness and occupant protection of transportation systems. Depending on the nature of the impact investigation, whether basic or applied, a number of studies can be cited. This dissertation examines the different aspects of impact dynamics and crashworthiness, from basics to applications. For each aspect, a separate chapter, literature review, and list of references are provided.

1.3 Objectives of Dissertation

The goal of this research is to contribute to the body of knowledge on the impact analysis of mechanical systems. The specific objectives of this dissertation can be summarized as follows:

- To investigate the impact methodologies in multibody systems and their contribution to impact phenomena.
- To evaluate the advantages and disadvantages of each impact approach and implementation of the methods in different applications.
- To analyze and model the energy-dissipation algorithm in each impact approach.
- To provide insight into how to define the design functions for each impact event and how to approach the impact problem according to design functions.
- To implement the methodologies of impact analysis into different energy-management design functions, and to address some applications of vehicular accidents in case studies.
- To examine four different test cases related to impact energy management, impulse mitigation, and impact injury biomechanics.
1.4 **General Methodology**

Figure 1.3 illustrates the outline of the general methodology for this dissertation.

Figure 1.3. General Methodology for This Study
The details of the methodology followed in this dissertation in order to reach the eventual goals of the study are as follows:

- First, impact dynamics is investigated, and different methodologies for impact analysis are examined. These methodologies include “stereomechanics,” “contact method,” “stress wave propagation,” “plastic deformation,” “energy method,” and “finite element method” (Chapter Two).

- The schemes of accounting for the energy dissipation are investigated, and the analytical models for each impact approach will be detailed (Chapter Three). Structural damping along with local deformation is the source of energy dissipation for low- and medium-impulse impacts. For high-impulse impacts, when the energy of impact is high enough to create stresses beyond the material yield stress, the global deformation, permanent or plastic deformation, is the main source of energy dissipation. For this case, the elastic or damping energy dissipation can be ignored. According to the design function of each vehicular accident in order to protect the occupant, the energy management of the impact scenario will be defined and the system performance will be evaluated.

- Industrial applications with design functions on energy management, impact impulse mitigation, and impact injury reduction shall be investigated in four test cases. Due to the specific design function for each case study, a particular methodology is developed described in detail for each. These test case studies include the following:
  - **Case Study One:** Application to crash energy management—“influence of a truck side underride guard height on cabin intrusion and occupant injury potential of a small car in car/large-truck side crashes.” The goal here is to protect the impactor and hence to reduce the intrusion of the small car to the passenger compartment and also
the injury level sustained by the driver of the small car by proper selection of the side guard material and geometry. In this design function, the goal is reached by a trade-off between crushable and stiff design of the impactor structure so that the injury outcome is minimized (Chapter Four).

- **Case Study Two**: Application to crash energy management and impulse mitigation—“lumbar load attenuation for rotorcraft-seated occupant using an effective seat energy-absorbing system design technique.” The design function here is to utilize a crushable seat system so that the impact between the occupant and the seat pan occur over an extended period of time, hence mitigating the impact force to the occupant and reducing the lumbar load. An analysis of the modeling of contact forces and energy dissipation during impact is conducted in this study (Chapter Five).

- **Case Study Three**: Application to impulse mitigation and impact injury biomechanics—“evaluation of the kinematics and injury potential to different sizes of pedestrians impacted by a utility vehicle with a frontal guard.” A pedestrian who is struck by a LTV with different frontal configurations representing the front end stiffness of the LTV is investigated in this study. The effect of a frontal guard utilized in LTVs on injury sustained by different sizes of pedestrians is presented in this work (Chapter Six).

- **Case Study Four**: Application to impulse mitigation and impact injury biomechanics—“a multibody modeling and design of experiment investigation of a motorcyclist impact on roadside barriers at upright and sliding configurations.” This study deals with the occupant injuries from motorcycle accidents under different conditions (Chapter Seven).
• The final step in this research is to make general conclusions on the different utilized methodologies for impact analysis in mechanical systems and structures, and summarize the results for any general impact scenario. Specific conclusions shall be provided for individual case studies (Chapter Eight).

1.5 References


[34] MADYMO model manual, Release 7.2, TNO Road-Vehicles Research Institute, January 2010.


2.1 Introduction

According to Kozlov [1], the first investigation of impact goes back to 1668, conducted by Wallis, Wren, and Huygens. Newton later referred to Wren's work in his famous work, “Mathematical Foundations of Natural Philosophy,” published in 1687. The subject of impact attracts the interest of scientists and engineers from different areas of knowledge. The mechanical engineer’s interest in impact problems is motivated by the desire to develop valid models for the behavior of mechanical systems where impact is inherent to their function in order to predict the after-impact configuration as well as the applied force and transmitted energy to each impacting body during impact. In the evolution of impact theory, several major approaches are recognized. Depending on the parameters desired from the analysis along with the types of simplifying assumptions of the impact event, an appropriate approach can be utilized. The duration of the contact period governs the choice of the impact analyzing method.

2.2 Stereomechanics Approach—Rigid Body Collision

“Stereomechanics,” also known as the “instantaneous method” or “piecewise method,” is the application of classical Newtonian mechanics to impacting bodies in order to predict post-

---

1 Parts of this chapter have been or will be published in the following sources:

impact velocities, given initial velocities and contact directions. Goldsmith [2] devotes a book chapter to the application of this theory to several problems on impact. Brach [3] uses this approach exclusively to model numerous problems of practical value. The theory is based on the impulse-momentum and kinetic energy conservation equations for rigid bodies. The impulse-momentum for a mass $m$ traveling with linear velocity $v$ is given by the vector equation [4]:

$$mv_f - mv_i = \int_{t_1}^{t_2} F dt$$  \hspace{1cm} (2.1)

which indicates that the change in linear momentum $mv$ of a mass due to an impact is equal to the impulse $\int F dt$ acting upon it, defined by the integration of the contact force over the duration of contact. Impact categories in this method can be categorized as follows:

- **Direct**: momentum vectors of impactors are parallel to one another.
- **Oblique**: momentum vectors of impactors are not parallel to one another.
- **Central**: center of gravity of impactors and contact point are collinear.
- **Eccentric**: center of gravity of impactors and contact point are not collinear.

The most general and predominant type of collision is the oblique eccentric collision, which involves both relative normal and tangential velocities. For the central impact, impactors are treated as point masses. This theory can be extended to rigid body collisions with 2-D and 3-D geometries.

A quantity $e$, called the coefficient of restitution (COR), is a global measure of the energy loss during impact and may incorporate different forms of dissipation such as viscoelastic work performed on the materials of the impacting bodies, plastic deformation of contact surfaces, and vibration in the two bodies. The COR is used to relate the pre- and post-impact velocities. In general, two limiting cases exist in this impact model, with the COR ranging from 0 to 1. The first case, $e = 1$, is the impact of rigid bodies or perfect elastic bodies, and the second case is the
impact of perfect plastic bodies, \( e = 0 \). Brach also notes that the COR can be allowed to take negative values from 0 to -1. This means that some energy has been lost during impact but without velocity reversal. As an example, consider a projectile penetrating through a barrier. The penetration work reduces the velocity of the projectile without reversing it.

The COR is not a material property but varies with the types of material impacted, impact velocities, and surface geometry, such as a sphere-sphere contact or a sphere-plate contact [2, 5]. Experimental testing has shown that increasing the impact velocity tends to decrease the COR. This would be expected because higher velocity impacts would produce more damping and plastic energy dissipation. Table 2.1 shows the coefficient of restitution values for some metallic materials at different impact velocities [2]. At higher impact velocities, the coefficient of restitution is lower, meaning that more energy is dissipated when the colliding bodies are moving faster. At high impact velocities, the energy dissipates due to plastic deformation. At low impact velocities, the effects of phenomena such as adhesion [6] and gravity become significant [7].

**TABLE 2.1. Coefficient of Restitution for Different Material and Velocities at Impact [2]**

<table>
<thead>
<tr>
<th>Impactor Material</th>
<th>Target Material</th>
<th>Impact Velocity (fps)</th>
<th>COR Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardened Steel</td>
<td>Hardened Steel</td>
<td>8.0</td>
<td>0.82</td>
</tr>
<tr>
<td>Hardened Steel</td>
<td>Hardened Steel</td>
<td>12.0</td>
<td>0.79</td>
</tr>
<tr>
<td>Hardened Steel</td>
<td>Hardened Steel</td>
<td>16.0</td>
<td>0.76</td>
</tr>
<tr>
<td>Hardened Steel</td>
<td>Hardened Steel</td>
<td>20.0</td>
<td>0.73</td>
</tr>
<tr>
<td>Soft Steel</td>
<td>Soft Steel</td>
<td>8.0</td>
<td>0.75</td>
</tr>
<tr>
<td>Soft Steel</td>
<td>Soft Steel</td>
<td>12.0</td>
<td>0.67</td>
</tr>
<tr>
<td>Soft Steel</td>
<td>Soft Steel</td>
<td>16.0</td>
<td>0.64</td>
</tr>
<tr>
<td>Soft Steel</td>
<td>Soft Steel</td>
<td>20.0</td>
<td>0.60</td>
</tr>
<tr>
<td>Steel</td>
<td>Brass</td>
<td>12.0</td>
<td>0.40</td>
</tr>
<tr>
<td>Lead</td>
<td>Lead</td>
<td>12.0</td>
<td>0.20</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>Lead</td>
<td>12.0</td>
<td>0.17</td>
</tr>
</tbody>
</table>

During an impact, colliding bodies first deform and then come to a common velocity \( u \) for an instant. If the bodies have some degree of restitution, the bodies will separate as the stored
elastic energy is released. This stored elastic energy separates the bodies and contributes to the post-impact velocities.

The coefficient of restitution may be defined using two different methods:

- Newton’s hypothesis:
  \[ e = -\frac{V_n^+}{V_n^-} \]  
  where \( V_n^- \) and \( V_n^+ \) are the relative normal velocities of the colliding bodies before and after impact, respectively.

- Poisson’s hypothesis:
  \[ e = \frac{\pi_r}{\pi_c} \]
  where \( \pi_r \) is the impulse in the restitution phase, and \( \pi_c \) is the impulse in compression phase of an impact. In the case of no friction, both Newton’s hypothesis and Poisson’s hypothesis give the same result for the COR. Newton’s hypothesis, however, has been shown to result in the violation of energy conservation principles in some cases [8-10]. The use of Poisson’s hypothesis, on the other hand, has provided solutions consistent with the conservation of energy principles.

With the use of the COR and the momentum conservation law, the post-impact velocities \( V_{1f} \) and \( V_{2f} \) of bodies \( m_1 \) and \( m_2 \) from a central (direct) impact can be determined as [3]

\[ V_{1f} = V_{1i} - \frac{(1+e)m_2(V_{1i}-V_{2i})}{m_1+m_2} \]  
\[ V_{2f} = V_{2i} + \frac{(1+e)m_1(V_{1i}-V_{2i})}{m_1+m_2} \]

where \( V_{1i} \) and \( V_{2i} \) are the pre-impact velocities of bodies \( m_1 \) and \( m_2 \), respectively.

For impacts that involve tangential impulses, the coefficient of tangential restitution (COTR), \( \mu \), defines the ratio between the tangential and normal impulse in a collision as in
equation (2.6). Like \( e \), the coefficient of tangential restitution must be experimentally determined in most cases. The COTR controls the changes in velocity and energy loss in the tangential direction.

\[
\mu = \frac{p_t}{p_n}
\]  

(2.6)

Tangential impulses arise from friction and/or plastic deformation at the contact point. For a Coulomb model friction, the dynamic coefficient of friction \( \mu_f \) is used as the magnitude of the COTR \( \mu \). Note that \( \mu \) may not always have the magnitude of \( \mu_f \).

For oblique contacts, Brach proposes the use of the tangential restitution coefficient \( e_t \) to relate the tangential velocities before and after impact [3]. He also shows that \( \mu \) and \( e_t \) are related. Therefore, only two independent coefficients \( e \) and \( \mu \) are required to solve impact problems. The final system energy cannot be zero for a perfectly inelastic and frictionless impact (\( e = 0 \) and \( \mu = 0 \)). For an oblique impact of two point masses, writing the balance of momentum in each direction along with the equations of coefficient of restitution for each direction yields

\[
\begin{align*}
V_{1fn} &= V_{1in} - \frac{(1+e)m_2(V_{1in}-V_{2in})}{m_1+m_2} \\
V_{1ft} &= V_{1it} - \frac{\mu(1+e)m_2(V_{1in}-V_{2in})}{m_1+m_2} \\
V_{2fn} &= V_{2in} + \frac{(1+e)m_1(V_{1in}-V_{2in})}{m_1+m_2} \\
V_{2ft} &= V_{2it} + \frac{\mu(1+e)m_1(V_{1in}-V_{2in})}{m_1+m_2} \\
\end{align*}
\]

(2.7)

Routh used Poisson’s hypothesis to define the coefficient of restitution and developed a graphical technique for calculating the resulting impulses between two bodies [11]. Because of the inconsistencies in Newton’s hypothesis, many studies have used Routh’s method instead and developed computer-oriented solutions for the impact problems. Among these are Han and Gilmore [12], and Wang and Mason [13], who were able to remove the energy gains arising from the use of Newton’s hypothesis in the analysis of two free or unconstrained impacting bodies.
However, their formulation is not suitable for kinematically constrained systems and is restricted to systems of free objects. Kane [8] pointed out an increase in energy of the system while analyzing the impact of the compound pendulum with the ground. This problem of energy gain cannot be addressed by Han and Gilmore’s formulation, since the compound pendulum is a constrained system. Pereira and Nikravesh [14] solved the double pendulum problem utilizing Newton’s hypothesis, but instead of recognizing the correct mode of impact, they established bounds on values of “e” to achieve the energy balance. Glocke and Pfeiffer [15] used Poisson’s hypothesis in conjunction with a complimentarily approach by adding and deleting constraints to obtain normal and tangential impulses at the contact point. Cataldo and Sampaio [16] compared different models of impact with friction, including Newton’s and Poisson’s, utilizing the classical impact process diagrams. Ahmed et al. [17] also developed a Poisson-based canonical formulation for the treatment of impact problems in jointed multibody systems. Lankarani [18] extended the formulation of Ahmed et al. to the analysis of impact problems with friction in any general multibody system including both open- and closed-loop systems. This formulation utilizes a Cartesian momentum-based or canonical form of the equations of motion as well as the impulse-momentum equations. It also utilizes Poisson’s hypothesis for the definition of the coefficient of restitution. It recognizes the modes of impact, i.e., sliding, sticking, and reverse sliding, using an impulse process diagram. Seven different types of impact are identified and the solutions for each type are presented in Tables 2.2 and 2.3. The velocity jumps are then calculated by constructing momentum-balance/impulse equations [18].

Coulomb’s law of friction provides the sliding and sticktion condition by defining the coefficient of friction in terms of the normal and frictional forces. In terms of the associated impulses, the conditions can be described as follows:
Sticktion: $\frac{d\tau}{d\tau} < \mu_s$

Sliding Impending: $\frac{d\tau}{d\tau} = \mu_s$

Sliding: $\frac{d\tau}{d\tau} = \mu_k$

where $d\tau$ is the increment friction force, $d\tau$ is the incremental normal contact force, and $\mu_s$ and $\mu_k$ are the static and kinetic friction coefficients, respectively. In case of the sliding condition, the direction of dpt will always be opposite to the slip velocity $V_t$.

TABLE 2.2. Seven Different Cases of Impact [18]

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Oblique Impact: $V_t^- \neq 0$</th>
<th>Direct Impact: $V_t^- \neq 0 \quad \pi_{ns} \neq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\tau}{d\tau} &lt; \mu_s$</td>
<td>$0 &lt; \pi_{ns} &lt; \pi_{nc}$ or $\pi_{nc} &lt; 0$</td>
<td>Type 1 Type 2</td>
</tr>
<tr>
<td>$\frac{d\tau}{d\tau} \geq \mu_s$</td>
<td>$0 &lt; (1 + e) \pi_{nc} - \pi_{ns}$ or $\pi_{ns} &lt; 0$</td>
<td>Type 3 Type 4 Type 5 Type 6 Type 7</td>
</tr>
</tbody>
</table>

TABLE 2.3. Description of Seven Cases of Impact [18]

<table>
<thead>
<tr>
<th>Impact Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>Sliding and sticking in the compression phase</td>
</tr>
<tr>
<td>Type 2</td>
<td>Sliding and sticking in the restitution phase</td>
</tr>
<tr>
<td>Type 3</td>
<td>Sliding and reverse sliding in the compression phase</td>
</tr>
<tr>
<td>Type 4</td>
<td>Sliding and reverse sliding in the restitution phase</td>
</tr>
<tr>
<td>Type 5</td>
<td>Forward sliding</td>
</tr>
<tr>
<td>Type 6</td>
<td>Sticking with no relative approach tangential velocity</td>
</tr>
<tr>
<td>Type 7</td>
<td>Sliding with no relative approach tangential velocity</td>
</tr>
</tbody>
</table>

The contact force is developed during the contact process between times $t_1$ and $t_2$. A typical shaped pulse for the normal contact force is idealized by a triangular shape. The impulse is the area under the force-time curve. Using the idealized triangular shape, the peak contact force can be estimated as

$$F_p = 2F_{avg} = \frac{2P_x}{(t_2-t_1)} = \frac{2m_1[(1+e)m_2(V_{lin}-V_{lin})]}{(m_1+m_2)(t_2-t_1)}$$  \hspace{1cm} (2.9)
It is clear from the equation (2.9) that increasing the mass of the impactors and the coefficient of restitution and decreasing the time of contact increases the contact force. An elastic collision $e = 1$ will have twice the peak collision force as a plastic impact $e = 0$ for the same contact time.

The work done by an impulse is directly related to the change in the system kinetic energy. The energy loss for an impact with tangential impulses can be determined as the negative of the work done by the normal $P_x$ and tangential $P_y$ impulses. The collision energy loss is related to the effective mass, COR, $e$, COTR, $\mu$, and closing velocity squared. Dissipated energy during the impact can be calculated as

$$T_L = \frac{1}{2} \bar{m}(1 + e)(V_{2inx} - V_{1inx})^2[(1 - e) + 2\mu r - (1 + e)\mu^2]$$

(2.10)

where $\bar{m} = \frac{m_1m_2}{m_1 + m_2}$ is the effective mass and $r = \frac{(V_{2iny} - V_{1iny})}{(V_{2inx} - V_{1inx})}$.

Sometimes the impulses of external forces are not negligible, especially in impacts that involve low-speed impactors. A common example of this type of impact is vehicle collisions or crashes. Care must be taken to assign the correct algebraic sign to the external impulse. This is best accomplished by the use of free-body diagrams. It should be remembered that frictional impulses always oppose the relative sliding motion. Also, in order to evaluate the impulses due to external forces, some knowledge about the duration of the collision is required.

Overall, the advantage of the “stereomechanics” method is that it is algebraic and, thus, easy to use and accessible to practicing engineers. The disadvantage is the lack of analytical tools to effectively determine the COR for any other type of geometry much more sophisticated than sphere-sphere contact. Therefore, collisions involving energy dissipation are typically analyzed experimentally, as evidenced by vehicle crash testing. Also, neither the contact duration nor the contact force maybe predicted using the stereomechanics approach.
2.3 Contact Mechanics

The theory of impact reviewed earlier is based on the simplifying assumption of perfectly rigid bodies. Actual physical objects are compliant; hence, the impact duration is finite and not exactly zero. This more realistic view of impact phenomena has led many researchers to consider the continuous dynamic models of collisions where bodies deform during impact.

When two bodies collide, a contact force is generated and applied between the two bodies for a short period of time. The intermittent motion in this case is caused by an abrupt change in the applied forces. Unlike the piecewise method, the “continuous analysis method” assumes that the motion is not discontinuous, and the contact force acts on the bodies in a continuous manner [19]. The differential equations of motion of the system remain the same as discussed in Chapter One. The only difference is that contact forces between the colliding bodies must be added to the system’s vector of forces during the period of contact. Therefore, the equations of motion are integrated over the period of contact. When two solid bodies are in contact, deformation takes place in the contact zone, and a contact force results. Determination of the relationship between the contact force and the relative deformation is the most important step in continuous analysis.

Contact mechanics uses force-deformation equations to estimate local stresses along with elastic/plastic deformations and contact duration times. This approach to impact analysis is through the examination of the contact force in the contained area of deformation between the colliding bodies. The derivations are generally based on the assumption of static or quasi-static conditions, although it has been extended to approximate solutions when an impact is involved. This assumption means that the impact duration is assumed to be much longer than the fundamental frequencies of the impacting bodies. It has been shown by Rayleigh that when the contact period is long compared to the natural periods of vibration of the system, vibration of the
system can be neglected [20]. This result supports the treatment of an impact problem like a quasi-static problem with a few assumptions.

In general, an impact may be considered to occur in two phases: the “compression phase” and the “restitution phase.” During the compression phase, the two bodies deform in the normal direction to the impact surface, and the two bodies reach the common normal velocity. The end of the compression phase is referred to as the instant of maximum compression. The restitution phase starts at this point and lasts until the two bodies separate. Due to energy dissipation in the process of an impact, it is apparent that the variation of the contact force during the two phases cannot be the same.

In a general form, the normal force-indentation relationship may be written as [21]

\[
F = F_c(\delta) + F_v(\delta, \dot{\delta}) + F_p(\delta, \dot{\delta})
\]  

(2.11)

where \(F_c\) is the elastic (conservative) part of the normal contact force, \(F_v\) is the viscous damping part, and \(F_p\) is the dissipative part due to plastic deformation. Viscous dissipation can be related to the rheological properties of the materials. Plastic dissipation can be determined from the materials’ stress-strain curve. Figure 2.1 shows the different components of the force-indentation relationship.

![Figure 2.1](image)

(a) Elastic Component, (b) Visco-Elastic Component, and (c) Plastic Deformation Component of Force-Indentation Relationship
The best-known contact force law between two spheres of isotropic materials is the result of pioneering work by Hertz, based on the theory of elasticity [22-23]. The Hertz contact theory is restricted to frictionless surfaces and perfectly elastic solids. It relates the contact force $F_N$ as a non-linear power function of penetration depth as

$$F_N = K\delta^n$$  \hspace{1cm} (2.12)

where $K$ is a constant of proportionality, and $\delta$ is the relative normal penetration between the spheres, the exponent $n$ is equal to 1.5 for circular and elliptical contacts, and the parameter $K$ is dependent on the material properties and the shape of the contact surfaces. For two spheres in contact, the generalized stiffness coefficient is the function of the radii of the spheres $R_i$ and $R_j$ and the material properties as [2]

$$K = \frac{4}{\sqrt{3(\sigma_i + \sigma_j)}} \left[ \frac{R_i R_j}{R_i + R_j} \right]^{0.5}$$  \hspace{1cm} (2.13)

where the material parameters $\sigma_i$ and $\sigma_j$ are given by $\sigma_k = \frac{1-v_k^2}{E_k}$, $k = i, j$, in which the quantities $v_k$ and $E_k$ are the Poisson’s ratio and the Young’s modulus, respectively, associated with each sphere. For contact between a sphere body “i” and a plane surface body “j,” the generalized stiffness coefficient can be expressed by

$$K = \frac{4}{\sqrt{3(\sigma_i + \sigma_j)}} \sqrt{R_i}$$  \hspace{1cm} (2.14)

On the basis of the Hertz theory, Dubowsky et al. [24] presented an expression for the penetration as a function of the contact force of an internal pin inside a cylinder:

$$\delta = F_N \left( \frac{\sigma_i + \sigma_j}{L} \right) \left[ \ln \left( \frac{L^{1-b}(R_i-R_j)}{F_N R_i R_j (\sigma_i + \sigma_j)} \right) + 1 \right]$$  \hspace{1cm} (2.15)

where $R_{i,j}$ and $\sigma_{i,j}$ are the same parameters as shown in equations (2.13) and (2.14), $L$ is the length of the cylinder, and the exponent $b$ has a value of 3. Since equation (2.15) is a non-linear
implicit function for $F_N$, with a known penetration depth, $F_N$ can be evaluated using an iterative scheme, such as the Newton-Raphson method.

To better estimate the contact force, the viscous term must be defined to address the damping characteristics of the impact, $F_v(\delta, \dot{\delta})$ in equation (2.11). It has been found that at low impact velocities, the energy dissipation due to hysteresis damping was the primary factor [20]. The simplest model of the contact force between two bodies known as the Kelvin-Voigt model assumes a parallel linear spring and damper connecting the two points on the two bodies [25]. The contact force is evaluated from

$$f = k\delta + d\dot{\delta}$$

(2.16)

where $k$ is the spring stiffness, and $d$ is the damping coefficient. In general, the stiffness and damping coefficients have been assumed to be known parameters, and the analysis has been confined to unconstrained bodies. Hunt and Crossley [26] later showed that the linear spring-damper model does not represent the physical nature of energy transferred during the impact. Instead, they represent the contact force by the Hertz force-penetration law with a non-linear viscous-elastic element. In order to guarantee that the damping force $d\dot{\delta}$ satisfies both boundary conditions at the time of initial contact and at the time of separation, the coefficient $d$ is chosen such that the damping force is in phase with the indentation velocity but proportional to the indentation. The coefficient $d$ is increased from zero at the beginning of impact to $d$ at a certain penetration $\delta$ specified by the user. This prevents the viscous force from being discontinuous. This model is referred to as the “hysteresis damping” model and may be expressed as

$$d = \mu_d \delta$$

(2.17)

where the parameter $\mu_d$ is called the hysteresis damping factor.
On the basis of Hunt and Crossley’s work, Lankarani and Nikravesh [27] developed a contact force model with “hysteresis damping” for impact in multibody systems. The model uses the general trend of the Hertz contact law, in which a hysteresis damping function is incorporated with the intent to represent the energy dissipated during the impact. They suggested separating the normal contact force into elastic and dissipative components as

\[ F_N = K\delta^n + D\dot{\delta} \]  

where D is a hysteresis coefficient. The hysteresis coefficient is written as a function of penetration depth as [27]

\[ D = \mu_D\delta^n \]  

In the discontinuous method, the coefficient of restitution represents the damping and energy loss in the system, and the hysteresis damping factor represents the energy loss in the continuous approach. The following relationship was thus obtained between the two parameters [27]:

\[ \mu_D = \frac{3K(1-e^2)}{4\delta(-)} \]  

where the generalized parameter K can be evaluated by equations (2.13) and (2.14), e is the restitution coefficient, and \( \delta(-) \) is the initial impact velocity. The normal contact force is finally expressed as [27]

\[ F_N = K\delta^n \left[ 1 + \frac{3(1-e^2)}{4} \frac{\dot{\delta}}{\delta(-)} \right] \]  

The energy loss through the work done by the damping force is evaluated as [27]

\[ T_L \approx \frac{2}{3} \frac{\mu_D}{K} \frac{m_i m_j}{m_i + m_j} \left( \frac{\dot{\delta}(-)}{ \delta(-)} \right)^3 \]  

In a later work, Lankarani and Nikravesh [28] proposed a new approach for contact force analysis, in which the permanent indentation is also included. At moderate or high impact velocities, especially in the case of metallic solids, permanent indentations are left behind on the
colliding surfaces. Hence, local plasticity of the surfaces in contact becomes the dominant source of energy dissipation during impact. Barnhart [29] suggests that, due to the plastic effects of impact, the energy is dissipated as

\[
F_N = \begin{cases} 
F_N = K\delta^n & \text{during compression} \\
F_m\left(\frac{\delta-\delta_p}{\delta_m-\delta_p}\right)^n & \text{during restitution}
\end{cases}
\]  

(2.22)

Hence, the two parameters \(F_m\) (maximum contact force) and \(\delta_m\) (maximum indentation) depend only on the relative impact velocity of the two colliding bodies. The parameter \(\delta_p\) (permanent indentation) must be evaluated experimentally for each particular impact situation. By using the following equation for energy dissipation:

\[
T_L = \int_0^{\delta_m} F_{\text{com}} \, d\delta + \int_{\delta_m}^{\delta_p} F_{\text{rest}} \, d\delta
\]

then,

\[
T_L = F_m \frac{\delta_p}{n+1}
\]

By equating the kinetic energy absorbed during the impact from stereomechanics to the energy associated with permanent deformation, \(\delta_p\) can be obtained by [19]

\[
\delta_p = \frac{(n+1) m_1 m_j}{2 F_m m_{i+m_j}} [\delta^{(-)}] \delta (1 - e^2)
\]

(2.23)

Hence, the maximum indentation and maximum contact force between the two bodies depend on the material properties, masses, radii, and velocities of the two bodies right before impact. The permanent indentation is evaluated from the initial approach velocities and a known coefficient of restitution between the bodies. Assuming the same duration for the compression and restitution phases, the contact duration approximated by Lankarani [19] is

\[
\Delta t = 2 \frac{\delta_m}{\delta} \int_0^1 \frac{dz}{(1-z^{n+1})^{0.5}} \approx 2.94 \frac{\delta_m}{\delta^{(-)}} \text{, for } n = 1.5
\]

(2.24)
It is clear that the proposed model assumes that the permanent indentation is the sole factor accounting for energy loss during impact. This is not an unreasonable assumption for most impact problems in stiff systems with an initial relative velocity larger than \(10^{-5}\sqrt{E/\rho}\), where \(\sqrt{E/\rho}\) is the propagation speed of elastic deformation waves in the colliding solids with \(E\) as Young’s modulus, and \(\rho\) is the density of the material. A stiff system is one that does not undergo large gross elastic deformations. However, a stiff system does undergo deformations locally at the site of impact. This system will be left with permanent deformations after impact [20]. The two parameters \(F_m\) and \(\delta_m\) depend only on the relative impact velocity of the two colliding bodies. The permanent indentation left behind accounts for the energy loss during impact.

In reality, the surfaces of contact of the two bodies are not smooth, and the effect of the friction cannot be ignored for many practical impact situations. To model this frictional effect, a friction force must also be applied on the two bodies in the plane tangent to the impact surface. Hence, the total force at the surface of impact would have two components, namely the normal contact force and the tangential friction force. Similarly, the integral of the total force at the impact surface results in two components of impulse: one normal impulse due to the contact force, and one friction impulse due to the friction force.

Friction affects the reaction force at the contact point and may increase or decrease the coefficient of restitution depending on whether sliding persists in the same direction or reverses after the moment of impact. The law of friction is used to evaluate the friction force, and this force is added to the system vector of forces. Luminaries of science such as Coulomb developed
friction studies, but there is still no simple model that can be universally used by designers to calculate the friction force for a given pair of bodies in contact.

The presence of friction in the contact surfaces makes the contact problem more complicated, because friction may lead to different modes, such as sticking, sliding, or reverse sliding. The friction model must be capable of detecting these modes to avoid energy gains during impact.

Coulomb’s friction law can represent the most fundamental and simplest model of friction between dry contacting surfaces. When sliding occurs, the tangential friction force $F_r$ is proportional to the magnitude of the normal contact force $F_N$ at the contact point by introducing a coefficient of friction $c_f$ [30]. The application of the original Coulomb’s friction law in a general purpose computational program may lead to numerical difficulties because it is a highly non-linear phenomenon that may involve switching between sliding and stiction modes.

In the last two decades, a number of articles have been devoted to the issue of friction [30-35]. Coulomb’s friction model formed the base model for most of them, with some modification in order to avoid the discontinuity at zero relative tangential velocity and to obtain a continuous friction force. Dubowsky [32], Rooney and Deravi [33], Threlfall [34], and Ambrosio [35], among others, assume different friction force models to implement in the dynamic equations of motion of a multibody system.

A continuous analysis may now be performed by adding the contact and friction force to the multibody system’s forces and numerically integrating the equations of motion forward in time in conjunction with the developed contact force model. This method does account for changes in the configuration and the velocities of the system during contact.
To verify whether the hysteresis damping model is applicable or not, contact stress at the point of contact is evaluated based on the contact force data. To check for yielding, maximum shear stress is evaluated as a function of contact force. It must be noted that the maximum shear stress does not occur on the contact force, but rather below the surface. The maximum shear stress for cylinders of length $l$ in contact, subjected to force $F$ is given by [36]

$$
\tau_{\text{max}} \approx 0.3 \frac{2F}{\pi l} \sqrt{\frac{\pi l}{2F} \frac{l/d_1 + l/d_2}{(1-v_1^2)/E_1 + (1-v_2^2)/E_2}}
$$

As can be seen, the contact mechanics approach is a powerful method of introducing the forces developed in impacts to be included in multibody dynamics analysis. It can provide contact force and contact time information for an impact event along with local stress information at the contact area. The disadvantages are the analytical selection of a force-deformation equation and establishing the parameters, which are required, to define the force-deformation equation.

### 2.4 Stress Wave Propagation

In the design of mechanical systems under static or quasi-static loading, one should consider that the stress field satisfies the equilibrium equation together with the body forces and the static boundary condition. In these cases, the effects of inertia are entirely neglected. For a body made up of perfectly plastic material, the ultimate loading is the greatest loading under which a solution to the static problem can be found so that the yield criterion is not violated. If a loading beyond the limit load is applied, then the static problem solution will not be appropriate, and the inertia effects must be taken into account. If the time of loading is short, most of the external work may be transformed into kinetic energy so that excessive deformation is prevented.
For example, when a nail is struck by a hammer, it may experience a force that produces a stress wave in excess of its static yield strength without permanent deformation.

Impact is accompanied by a stress wave that propagates in the impacting bodies away from the region of impact. If the energy that is transformed into vibrations becomes an important fraction of the total energy, then the inertia forces in the material must be considered. The classical statics-based approach then becomes an insufficient method to examine an impact problem. The wave propagation approach is covered extensively by Goldsmith [2] and Zukas et al. [25] for a wide variety of problems. In this section, the fundamental theories of stress wave propagation are briefly examined, and then the application methods of utilizing stress wave propagation approach using the finite element method to model impact phenomena is discussed.

Overall, the dynamics behavior of impacted solids may roughly be divided into three classes [37]. For loading conditions that result in stresses below the yield point, materials behave elastically. For metals, the classical elastic Hook’s law is applicable in this case. A number of detailed mathematical solutions have been developed in the literature for different loading conditions in this class. For these problems, both the geometry of the entire structure as well as the material property play a major role in resisting external forces. As the intensity of the applied loading is increased, the material is driven into the plastic range. The response of the system tends to become highly localized and is more affected by the constitutive properties of the material in the vicinity of load application than the geometry of the total structure. The behavior here involves large deformations, heating, and often failure of the colliding solids through a variety of mechanisms. Two stress waves now propagate through the solid: an “elastic wave” traveling at the speed of sound in the solid, followed by a much slower in velocity but more intense “plastic wave.” With still further increases in loading intensity or impact velocity, the
pressures are generated by the impacts that exceed the strength of the colliding solids by several orders of magnitude. These propagate as “shock waves” and behave hydrodynamically like a fluid. Table 2.4 summarizes the material behavior in different impact regimes.

**TABLE 2.4. Dynamic Aspects of Mechanical Testing**

<table>
<thead>
<tr>
<th>Loading Regime</th>
<th>Creep</th>
<th>Static</th>
<th>Rapid</th>
<th>Impact</th>
<th>Ballistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Time Characteristic (sec)</td>
<td>1k</td>
<td>1</td>
<td>20m</td>
<td>10m</td>
<td>10n</td>
</tr>
<tr>
<td>Typical Strain Rate (per sec)</td>
<td>1m</td>
<td>1</td>
<td>50</td>
<td>1K</td>
<td>1M</td>
</tr>
<tr>
<td>Method of Engineering Examination</td>
<td>Creep Rate or Stress Relation</td>
<td>Stress Strain Curve</td>
<td>Vibration</td>
<td>Elastic and Plastic Wave Propagation</td>
<td>Shock Wave Propagation</td>
</tr>
<tr>
<td>Inertial Forces</td>
<td>Ignored</td>
<td>Ignored</td>
<td>Considered</td>
<td>Considered</td>
<td>Considered</td>
</tr>
<tr>
<td>Thermal</td>
<td>Isothermal</td>
<td>Isothermal</td>
<td>Adiabatic</td>
<td>Adiabatic</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>General Stress Levels</td>
<td>Low</td>
<td>Moderate</td>
<td>Moderate</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

The theories of elasticity and plasticity provide the basis for wave analysis in solids. Depending on the type of data available and information required from the stress wave equation, a relationship could be set up for the analysis. If the constitutive equations of material as well as the conditions of impact are available, it is possible to describe the stress history within the interior of the material. The inverse problem, in which material properties are to be found from the experimental impact test, is equally important. In theory, the advantage of stress wave method is an accurate stress analysis on the impacted elastic solid. Also, the variation of local strain/stress levels in the solid can be identified as a function of time and space. On the other hand, stress wave propagation is highly mathematical and requires a large amount of simplification of the impacted mechanical system. This limits its application to mainly one-dimensional problems, such as a rigid body impacting the end of an elastic cantilever rod [38].
engineering applications where complex geometries are involved, the stress wave method can be captured by the use of finite element analysis (FEA). A physical test that uses stress wave mechanics to determine the dynamic properties of an engineering material is the split Hopkinson bar (SHB) test, from which the dynamic stress-strain response of the material can be determined.

2.4.1 Elastic Wave Propagation

When an impact force or impulse is applied to an elastic body, the generated disturbance travels through the solid as stress waves, which are analogous to “sound waves” traveling through air. The particles in a thin layer of material at the contact region are set into motion. The remainder of the body, remote from the loading, remains undisturbed for some finite length of time. As time passes, the thin region of moving particles expands and propagates into the body in the form of an elastic deformation wave. Behind the wave front, the body is deformed and the particles are in motion. Ahead of the wave front, the body remains undeformed and at rest. If the geometry of the body is simple and uniform, and if the applied force is well defined and uniformly applied, equations for wave propagation in an elastic media may be utilized to evaluate stresses and deformations in the body. It has been found that the propagating waves reflect internally from boundaries of the body and interfere with one other. Depending on the boundary conditions, “standing” or “interfering strain waves may produce the local larger strains and stresses associated with impact condition than that of ordinary static loading. If the impact force or impulse exhibits a velocity of less than that of the speed of sound in the impacted solid, the wave propagation will be elastic.

For a material subjected to stresses through some external dynamic loading or testing apparatus, the traditional static solid mechanic approaches cannot be applied. For loadings that cause stresses below the yield limit of the material, “elastic stress waves” are generated. The
theory of elasticity provides the basis for wave analysis in solids. The structure experiences deformations that can be determined from the combinations of equations of motion and can be utilized to obtain an infinitesimal element. The inertia effects of the elements and the material’s constitutive and compatibility relations can be utilized to obtain the displacement equation of motion called Navier’s equation of elasticity, which is the equation of motion for the elastic waves in a solid. For impact, since the intensity or rate of loading is high enough, the inertia forces in the material must be considered.

The solution to problems in impact mechanics requires the application of the basic laws of mechanics and physics, as well as a description of the behavior of the material being considered. The system of equations governing the motion of a homogeneous, isotropic, linearly elastic body consists of the stress equations of motion, the Hook’s law, and the strain-displacement relationships given by the following:

- Strain-displacement relations (kinematics):
  \[ \varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \]  
  \[ (2.26) \]

- Material compatibility conditions (constitutive equations):
  \[ \sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2 \mu \varepsilon_{ij} \]  
  \[ (2.27) \]

- Equations of motion applied to an infinitesimal element, (equilibrium):
  \[ \sigma_{ij,j} + \rho b_i = \rho a_i \]  
  \[ (2.28) \]

where subscripts \( i, j = 1, 2, 3 \), and \( \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \) (Lame’s constant) and \( \mu = \frac{E}{2(1+\nu)} \) (shear modulus) are two independent elastic constants that define all elastic material properties for an isotropic engineering material. Equations (2.26) to (2.28) are vectorial equations, which can be written in three directions to obtain each scalar equation. For example, the equation of
equilibrium is derived from the summation of forces in the x, y, and z directions, and equating these forces to the change in momentum of the element yields the equations of motions, given by

\[
\sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i \quad \text{yields} \quad \begin{cases}
\frac{\partial}{\partial x} \sigma_x + \frac{\partial}{\partial y} \tau_{xy} + \frac{\partial}{\partial z} \tau_{xz} + \rho b_x = \rho \frac{\partial^2}{\partial t^2} u_x \\
\frac{\partial}{\partial x} \tau_{xy} + \frac{\partial}{\partial y} \sigma_y + \frac{\partial}{\partial z} \tau_{yz} + \rho b_y = \rho \frac{\partial^2}{\partial t^2} u_y \\
\frac{\partial}{\partial x} \tau_{xz} + \frac{\partial}{\partial y} \tau_{yz} + \frac{\partial}{\partial z} \sigma_z + \rho b_z = \rho \frac{\partial^2}{\partial t^2} u_z
\end{cases} \tag{2.29}
\]

Equations (2.26) to (2.28) may be combined to obtain the displacement equation of motion called Navier’s equation, which is the equation of motion for the elastic wave in a solid, given by

\[
\mu u_{ij,j} + (\lambda + \mu) u_{ij,j} + \rho b_i = \rho \ddot{u}_i \quad \text{yields} \quad \begin{cases}
(\lambda + \mu) \frac{\partial}{\partial x} \Delta + \mu \Delta^2 (u_x) + \rho b_x = \rho \frac{\partial^2}{\partial t^2} u_x \\
(\lambda + \mu) \frac{\partial}{\partial y} \Delta + \mu \Delta^2 (u_y) + \rho b_y = \rho \frac{\partial^2}{\partial t^2} u_y \tag{2.30} \\
(\lambda + \mu) \frac{\partial}{\partial z} \Delta + \mu \Delta^2 (u_z) + \rho b_z = \rho \frac{\partial^2}{\partial t^2} u_z
\end{cases}
\]

where \(\Delta = \nabla \cdot u = u_{,j} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\) is the divergence of displacement of vector \(u\). For an impact, the body force influence at the short period of impact can be neglected against the high impulse load from impact, and then the equations becomes

\[
\mu u_{ij,j} + (\lambda + \mu) u_{ij,j} + \rho b_i = \rho \ddot{u}_i \tag{2.31}
\]

Taking divergence (\(\nabla\cdot\)) of the above equation gives the “longitudinal wave equation”:

\[
\mu u_{ij,i} + (\lambda + \mu) u_{ij,i} = \rho \ddot{u}_i \quad \text{or} \quad \frac{\partial^2 \Delta}{\partial t^2} = \frac{M}{\rho} \frac{\partial^2 \Delta}{\partial x_i \partial x_i} \tag{2.32}
\]

where

\[
M = (\lambda + 2\mu) = \frac{E(1-v)}{(1+v)(1-2v)} \tag{2.33}
\]

“P-waves” are those waves in which the particle motion induced by the disturbance is normal to the wave front and parallel to the pulse propagation direction, and the strain is pure dilatation. P-waves are also nomenclature for the terms dilatational, longitudinal, primary, or pressure waves. A P-wave is associated with normal stress and can propagate in all types of media. In linear
elasticity, the P-wave modulus $M = \rho c_L^2$, also known as the longitudinal wave modulus, is one of the elastic moduli available to describe isotropic homogeneous materials, where $c_L$ is the velocity of a P-wave in the infinite elastic solid.

Analogous to the method above, one can take the curl ($\nabla \times$) of equation (2.31) to get the “shear wave equation”: $\mu \nabla \times u_{i,ji} + (\lambda + \mu) \nabla \times u_{j,ji} = \rho \nabla \times \ddot{u}_i$. But $\nabla \times u_{j,ji} = 0$ and $\nabla \times u_{i,ji} = (\nabla \times u_i)_j$. Defining $\nabla \times u_i = \psi$, then

$$\mu \psi_{i,ji} = \rho \ddot{u}_i$$  \hspace{1cm} (2.34)

Equation (2.34) is related to the propagation of a shear wave with a velocity of $c_S^2 = \frac{\mu}{\rho}$, where $\mu$ is the shear modulus and $\rho$ is the density of the solid. The distortional or transverse or shear or secondary “S-waves” are those waves wherein material particles move in a plane at right angles normal to the pulse propagation direction in which the wave front propagates at a velocity of $c_S$. An S-wave is associated with shearing stress and can propagate only in media with shear stiffness, that is, in solids and not liquids.

The statement of the “elasto-dynamic” problem will be completed with establishing the second-order partial differential wave equations and initial and boundary conditions, which could be the essential (geometric or Dirichlet) and/or natural (dynamic or Neumann) boundary conditions describing the primary or secondary variables of the second-order partial differential equation. Using the original equation in one, two, and three dimensions for the longitudinal wave equations yields

$$\begin{cases} 
1 - D: & \frac{\partial^2 u}{\partial x^2} - \frac{1}{c_L^2} \frac{\partial^2 u}{\partial t^2} = 0 \\
2 - D: & \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{c_L^2} \frac{\partial^2 u}{\partial t^2} = 0 \\
3 - D: & \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{1}{c_L^2} \frac{\partial^2 u}{\partial t^2} = 0 
\end{cases}$$  \hspace{1cm} (2.35)

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In general, solutions for equations (2.32) and (2.34) in 3-D are highly complicated, and very few closed solutions exist using this method due to its complexity. Stress propagation in slender bars is considered to be P-wave only, if the ratio of the bar’s length to diameter is greater than 10, for which lateral considerations may be ignored. Then, for a bounded media, the equation of wave propagation yields

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c_L^2} \frac{\partial^2 u}{\partial t^2} = 0, \quad c_L^2 = \frac{E}{\rho}$$

(2.36)

If the bar is not slender, the assumptions are not valid; therefore, it is necessary to take the inertia of lateral contraction into consideration:

$$\frac{\partial^2 u}{\partial t^2} = c_L^2 \frac{\partial^2 u}{\partial x^2} + v^2 k^2 \frac{\partial^4 u}{\partial x^2 \partial t^2}, \quad c_L^2 = \frac{E}{\rho}$$

(2.37)

where v is Poisson’s ratio, and k is the radius of gyration. A useful parameter is the ratio $\alpha$ of the S-wave to P-wave speeds:

$$\alpha = \frac{c_s}{c_L} = \sqrt{\frac{(1-\nu)}{2(1-\nu)}}$$

(2.38)

The second-order hyperbolic partial differential equation (2.37) can be solved using two methods: separation of the partial differential equation into two second-order ordinary differential equations, and mapping the coordinates to the new coordinate system, which is called the “d’Alembert’s method.”

For the “separation of variables,” the displacement is defined by $u(x, t) = F(x)G(t)$. The solution of the problem then reduces to the solving of two second-order differential equations:

$$\begin{align*}
F'' + k^2 F &= 0 \\
G + \lambda_n^2 G &= 0
\end{align*}$$

(2.39)
where $\lambda_n^2 = k^2 c_L^2$. By using boundary conditions of the Sturm-Liouville boundary value problem, $F(x) = A\cos(kx) + B\sin(kx)$ and the unknowns $A$ and $B$ can be determined. The initial value problem of the equation yields $G_n(t) = A_n\cos(\lambda_n t) + B_n\sin(\lambda_n t)$. Then,

$$u(x, t) = \sum_{n=1}^{\infty} u_n(x, t)$$  \hspace{1cm} (2.40)

Applying the initial condition yields the unknowns $A_n$ and $B_n$, and the displacement (strain) and stress can be evaluated at different locations and times.

In the d’Alembert’s method, coordinate transformation from $(x, t)$ to $(\zeta, \eta)$ occurs, such that $\zeta = x - c_L t$, $\eta = x + c_L t$ in equation (2.32) yields the canonical form of hyperbolic equations, $\frac{\partial^2 u}{\partial \zeta \partial \eta} = 0$. This means that $u(\zeta, \eta)$ can be written as $u(x, t) = f(x - c_L t) + g(x + c_L t)$, and the wave would be divided by two left- and right-running waves with a constant velocity [39]. In this method, the two wave shapes are functions of $x$ only at any time and move in the positive and negative directions of $x$ at a constant velocity.

The only difference between the propagation of elastic stress disturbances in bounded and unbounded media is geometrical. In theory, the transmission of such disturbances can be treated by solving the equations of small motion with the appropriate boundary conditions. In practice, however, the addition of boundaries introduces immense complexities into the mathematical formulation of the problem so that very few closed solutions exist.

In stress wave calculations, two different velocities must be considered: the velocity of the stress wave traveling at wave speed $c_L$, and the particle’s motion velocity designated by symbol $v$. The particle velocity is the velocity of the material as the stress wave transmits energy through the medium. In the case of an impact, the particle velocity is the striker’s initial impact speed, usually designated as $v_o$. As the stress wave passes, the particle’s velocity changes from zero to $v_o$. All the material behind the stress wave is now at $v_o$. The relationship between these
two velocities \( c_L \) and \( v_o \) is developed from the impulse-momentum equation, where the impulse of force \( F \) changes the velocity \( v_0 \) of the mass \( \rho c_L \Delta t \) resulting in the momentum change of \( p_f - p_i \) in the impacted rod in time period \( \Delta t \):

\[
\int_{t_i}^{t_f} F dt = \int_{t_i}^{t_f} \sigma_0 A dt = \sigma_0 A \Delta t = p_f - p_i = \rho A c_L \Delta t v_0
\]  

(2.41)

Here again, \( \rho \) is the density of the material, and \( c_L \Delta t \) is the length of the material now at velocity \( v_0 \). The initial magnitude of the impact stress is given by \( \sigma_0 = \rho c_L v_0 \). In the same manner as for the transverse (shear) stress pulse, \( \sigma = \rho c_s v_0 \). This analysis assumes that the mass \( M \) of the striker is significant enough to generate a change in velocity of \( v_0 \) in the mass \( m \) of the rod.

Boundary conditions of the solid body affect the response of the stress wave that propagates into it. For most general boundary conditions, the incident wave will be partially reflected and partially transmitted. The degree of reflection and transmission at the boundary condition is dependent upon the mechanical impedance \( \rho c_L \) of the materials at the boundary. The total deflection or stress at any point along the rod is the sum of the \( n^{th} \) propagating wave and \( (n^{th} - 1) \) reflecting wave. At a boundary condition, continuity of displacements and forces \( (\sigma A) \) is required. By designating subscript \( i \) for incident, \( r \) for reflected, and \( t \) for transmitted, the continuity of displacements amplitude \( A \) of the waves are related by \( A_i + A_r = A_t \). Using the d’Alembert’s solution to the boundary conditions, the displacements at stresses at boundary conditions are determined by their “mechanical impedances” as

\[
A_r = \frac{\rho^2 c_L^2}{\rho^2 c_L^2 + \rho_1^2 c_L^2} A_i, \quad A_t = \frac{2}{\rho^2 c_L^2 + \rho_1^2 c_L^2} A_i, \quad \sigma_r = \frac{\rho^2 c_L^2}{\rho^2 c_L^2 + \rho_1^2 c_L^2} \sigma_i, \quad \sigma_t = \frac{\rho^2 c_L^2}{\rho^2 c_L^2 + \rho_1^2 c_L^2} \sigma_i
\]  

(2.42)

Here, subscript 1 represents properties of the first medium carrying the incident wave to the boundary, while subscript 2 represents the properties of the second medium transmitting the wave. The “mechanical impedance” is defined as the product of mass density \( \rho \) and P-wave
propagation velocity \( c_L \). For a free-end boundary condition, the mechanical impedance is \( \rho_2 c_{L2} = 0 \), and at the free boundary condition, it is observed that the displacement is double that of the impacted end’s displacement. The stress at the end of rod will become zero after the passing of the incident stress wave because the reflected stress wave will be the negative of the incident stress wave, and when summed together, they cancel each other. For a fixed-end boundary condition, the mechanical impedance is \( \rho_2 c_{L2} = \infty \) and the fixed-end deflection is zero. The fixed-end stress will be twice the incident stress wave because the reflected stress wave now equals the magnitude of the incident wave. The fixed-end boundary condition transmits twice the incident wave also. Equation (2.42) assumes equal medium areas on either side of the boundary; if not, stresses need to be adjusted accordingly to each area.

An example of a one-dimensional elastic impact is shown in Figure 2.2. Figure 2.2 (a) diagrams an impact of a rigid body of mass \( M \) with a velocity \( v_0 \) into an elastic rod of density \( \rho \) and elastic modulus \( E \). Figure 2.2 (b) shows the compressive elastic stress wave propagating at \( c_L \) through the bar with a compressive initial impact stress of \( \sigma_0 = \rho c_L v_0 = v_0 \sqrt{E \rho} \).

![Figure 2.2. One-Dimensional Stress Wave Propagation Due to Impact Loading](image-url)
As the initial stress $\sigma_0$ propagates toward the right side, the free-end stress diminishes as the impactor starts to rebound away from the target. The diminishing stress is noted as $\sigma_e$ and is governed by the force equilibrium on the impactor with mass $M$ and velocity $v_e$ reacting upon the bar stress of $\sigma_e$ and bar area $A$ such that,

$$ M \frac{dv_e}{dt} + \sigma_e A = 0 \quad \text{where} \quad \sigma_e = \rho c_L v_e = v_e \sqrt{E \rho} \quad (2.43) $$

Substituting $\frac{dv_e}{dt} = \frac{d\sigma_e}{dt} \frac{1}{\sqrt{E \rho}}$, equation (2.43) becomes a first-order ordinary differential equation yielding

$$ \sigma_e = \sigma_0 e^{-\frac{A/\sqrt{E \rho}}{M}}, \quad 0 < t < \frac{2L}{c} \quad (2.44) $$

At time $t_1 = L/c_L$, the indent compressive wave at compressive stress $\sigma_0$ is reflected as a compressive wave with stress $\sigma_0$, accordingly to equation (2.42). At time $t_1$, the end stress is now $2\sigma_0$. The compressive stress is shown in Figure 2.2 (d) by the sum of the $n^{th}$ propagating wave and $(n^{th} - 1)$ reflecting wave. Timoshenko provides a solution for the maximum superposed stress experienced by the rod with a total mass of $m$ when impacted with mass $M$ at some point and time history of the impact as [40]

$$ \sigma_{\text{max}} = \sigma_0 \left( \sqrt{\frac{M}{m}} + 1 \right) \quad (2.45) $$

In a transverse wave, also called a distortional, rotational, secondary, or shear wave, the particle motion is normal to the direction of the pulse, and the strain is a shearing strain. Distortional (or transverse or shear) waves are those waves wherein material particles move in a plane at right angles to that in which the wave front propagates.

2.4.2 Plastic Wave Propagation

For high-energy impacts where plastic strains occur outside of the contact area, some form of plastic analysis is required [41]. When plastic strains go beyond the scale of contained
deformation, the elastic wave propagation model can no longer be applied to analyze impact problems. Elastic theory cannot model the conversion of kinetic energy into heat energy during plastic deformations. Impacts of this nature must be described by relationships that account for large strains and plastic deformations that occur in the process. The perfect elastic-plastic analysis exhibits the most practical way in predicting large plastic strains due to impact loading. Bohnenblust [42], Conroy [43-44], and Symonds [45] developed methods to analyzed beams undergoing plastic deformations using rigid perfect plastic and elastic-perfect plastic material constitutes.

The plastic wave propagation method extends the elastic wave theory. This is the domain of high-velocity impact generally associated with explosives and projectiles. Goldsmith [2] presents an extended study of the subject using the theory of plastic wave propagation. In the theory of plastic strain propagation, the material is considered to be incompressible in the plastic domain. Also, the state equation relating stress, strain, and strain rate is assumed to be independent of temperature. Maugin [46] and Lubliner [47] postulate that where ductile materials are used, the loading is applied over a long period of time, high temperatures are involved or high strain rates occur, and rate dependence cannot be ignored in describing the plastic behavior of materials. Zukas et al. [25] present an extensive treatment of plastic wave propagation using both rate-dependent and rate-independent theories.

As in the case of elastic wave theory, the plastic propagation wave method is too complex to analyze an impact problem with great complexity. Review of the literature has indicated that the only impact problems found to be solved by the plastic wave propagation method have been the tensile impact of a semi-infinite wire and the impact compression of cylinders. A three-
dimensional plastic wave study is out of the scope of this study, and only a brief explanation of the one-dimensional plastic wave is provided here.

In the bar impact analysis, plastic flow near the impact end introduces three-dimensional effects (radial, inertial, heating), so that one-dimensional theory can be applied only at points far away from the point of application of the load. By increasing the striking velocity, a three-dimensional theory is required for complete analysis of experimental results [25]. Plate geometry offers the opportunity to study the behavior of materials at higher loads and shorter times, while offering again the simplicity of one-dimensional analysis—uniaxial strain. In fact, uniaxial strain can be achieved when the plane wave propagating through a material with dimensions and constraints are such that the lateral strains are zero. Similar to the one-dimensional stress wave in bar analysis, plate impact analyses neglect the effects of thermo-mechanical coupling, which can be significant at strains exceeding 30% [48]. Much of the work found in the literature has assumed hydrodynamic behavior of the material. However, an elastic precursor can produce significant volumetric strain. An elastic unloading wave can significantly change the local state of the material before the arrival of the plastic wave, for which finite elastic and plastic effects may need to be accounted.

Following the elastic stress-strain relationship and the fact that plastic strain is incompressible, the stress at the direction of the strain can be calculated by examining the total principle strain, which is the sum of the elastic strain \( \varepsilon^e \) and the plastic strain \( \varepsilon^p \), namely

\[
\varepsilon_i = \varepsilon_i^e + \varepsilon_i^p, \quad i = 1, 2, 3
\]

where \( i \) is the principle strain direction:

\[
\varepsilon_1 = \frac{\sigma_1(1-2\nu)}{E} + \frac{2\sigma_2(1-2\nu)}{E}, \quad \varepsilon_2 = \varepsilon_3 = 0
\]

(2.46)

where \( E \) is the elastic modulus, \( \nu \) is the Poisson’s ratio, and \( \sigma \) is the principle stress.
The plasticity condition according to either von Mises or Tresca failure theory relates the principle stress to the yield strength $\sigma_Y$ as

$$\sigma_1 - \sigma_2 = \sigma_Y$$  \hspace{1cm} (2.47)

Using the definition of the bulk modulus $K$ into equations (2.46) and (2.47) yields

$$\sigma_1 = K\varepsilon_1 + \frac{2\sigma_Y}{3} \text{ where } K = \frac{E}{3(1 - 2\nu)}$$  \hspace{1cm} (2.48)

The most important difference between the uniaxial stress and uniaxial strain is the bulk compressibility term. The stress continues to increase regardless of the yield strength or strain hardening due to the plastic impact. For ballistic impact or other high-rate phenomena where the material does not have enough time to deform laterally, a condition of uniaxial strain is established.

The maximum stress for uniaxial strain in one-dimensional elastic wave propagation is called “Hugoniot elastic limit” $\sigma_{\text{HEL}}$. This is also the dynamic yield stress for the impact. If the Hugoniot elastic limit is exceeded, then a plastic stress wave will develop and progress after the elastic stress wave. The elastic wave will move with speed of $c_E$ followed by a plastic wave moving with speed of $c_p$. As shown, the speed of the plastic wave is a function of the slope of the stress-strain curve at a given value of strain:

$$c_E^2 = \frac{E(1-\nu)}{\rho_0(1-2\nu)(1+\nu)}, \quad c_p^2(\sigma) = \frac{1}{\rho_0} \frac{d\sigma}{d\varepsilon}$$  \hspace{1cm} (2.49)

For impact velocities that are much greater than the speed of sound or elastic propagation velocity $c_E$, “shock waves” form. In this situation, where $c_E < c_p$, the continuous plastic wave front breaks down, and a single discontinuous shock front is formed, traveling at a shock velocity $U$, as illustrated in Figure 2.3.
Across the shock front, there is a discontinuity in stress, density, velocity, and internal energy. Shock waves will be formed under conditions of extremely high impulsive stress and will propagate in a material in a manner similar to the fluid dynamics situation. Using a simplified equation of state analogous to the case of the elastic wave, the shock velocity will be obtained as

\[ U_s^2 = \frac{1}{\rho_0^2} \frac{p_1 - p_0}{\nu_0 - \nu_1}, \quad \nu_0 = \frac{1}{\rho_0} \text{ and } \nu_1 = \frac{1}{\rho_1} \]  

(2.50)

As mentioned previously, to study the propagation of longitudinal stress and strain waves in a thin bar, it is common to represent the problem by a one-dimensional approximation. The wavelengths are assumed to be much longer than the transverse dimensions of the bar. This approximation yields good results at points of the bar far enough away from the bar ends. Near the ends, three-dimensional corrections are necessary. Let \( x \) denote the Lagrangian coordinate along the bar axis and \( u(x, t) \) be the corresponding displacement; the engineering strain \( \epsilon(x, t) \) and velocity \( v(x, t) \) are then given by

\[ \epsilon = \frac{\partial u}{\partial x}, \quad v = \frac{\partial u}{\partial t} \]  

(2.51)

The kinematic compatibility relation yields

\[ \frac{\partial v}{\partial x} = \frac{\partial \epsilon}{\partial t} \]  

(2.52)

The equations of motion for zero-body-force condition would be reduced to

\[ \frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} \]  

(2.53)
A shock front is said to occur at point \( x = \alpha(t) \) of the bar, if the velocity \( v \) is discontinuous at that point. The shock front is moving at a finite speed \( c \) in the positive \( x \)-direction, that is, \( c = \dot{\alpha}(t) > 0 \) in designating the values of \( v \) just to the right (in front) of the shock and just to the left of (behind) the shock by \( v^+ \) and \( v^- \), respectively. The jump in \( v \) is defined as [47]

\[
|v| = v^- - v^+
\]  

(2.54)

Lubliner [47] derived the jump in velocity, stress, and strain relations by treating the shock front as a thin zone in which these quantities change very rapidly with constant rates. If the shock thickness is \( h \), then for a front moving to the left and to the right

\[
|v| \approx \pm \frac{h}{c} \frac{\partial v}{\partial x}
\]  

(2.55)

Since the duration of the shock passage at a given point is \( h/c \), then

\[
|v| \approx \frac{h}{c} \frac{\partial v}{\partial \tau}
\]  

(2.56)

Applying these approximations to equations (2.52) and (2.53), the shock relations are

\[
|\varepsilon| = \pm \frac{1}{c} |v|, \quad |\sigma| = \pm \rho c |v|
\]  

(2.57)

where the + and − signs apply to fronts moving to the right and to the left, respectively.

Eliminating \( |v| \), the shock-speed equation would be

\[
\rho c^2 = \frac{|\sigma|}{|\varepsilon|}
\]  

(2.58)

Equation (2.64) shows the speed of one-dimensional wave in a solid which can be written as

\[
c^2 = \frac{E}{\rho}
\]

By means of some approximating assumptions, Taylor [49] and Lubliner [47] derived a formula for the dynamic yield stress of a rigid-plastic bar impacted into a rigid target in terms of the impact speed and the specimen dimensions before and after impact, utilizing different
approaches. For a bar made of work-hardening material, the problem was treated by Lee and Tupper [50]. If the conventional stress-strain relation is given by \( \sigma = F(\varepsilon) \) and the initial yield stress is \( \sigma_E \), then the material just ahead of the shock front may be assumed to be about to yield, so that \( \sigma = \sigma_E \) there, while immediately behind the front, the stress is \( \overline{\sigma} = F(\overline{\varepsilon}) \). Therefore, the stress jump is \( \sigma = \overline{\sigma} - \sigma_E \).

Assuming the elastic-plastic material property for the bar changes the nature of the problem drastically. In an elastic solid, disturbances cannot be propagated at a speed faster than the elastic wave speed. Assuming a bilinear stress-strain material, as shown in Figure 2.4, for a one-dimensional problem, the velocity of each wave front has its own characteristic speed, dependent on the respective moduli of the elastic and plastic regions, \( E \), and \( E_1 \), resulting in the wave profile shown in Figure 2.5. Donnell theory [51] was extended for a uniaxially loading bar by considering the bilinear elastic-plastic material and independent strain rate [52-54]. Using a Lagrangian coordinate system with the x-axis parallel to the bar axis, the equation of motion in the x direction is given as

\[
\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial x}
\]

(2.59)

Applying the boundary condition for bar impacted at the end, and letting \( \xi = x/t \), the three solutions were obtained as

For \( |x| < c_1 t \): \( \varepsilon = \text{constant} = \frac{\varepsilon_1}{c_1} = \varepsilon_1 \)

(2.60)

For \( c_1 t < |x| < c_0 t \): \( E(\varepsilon) = \frac{x^2}{t^2} \)

(2.61)

For \( c_0 t < |x| \): \( \varepsilon = 0 \)

(2.62)
The solution for strain as a function of $\xi = x/t$ is presented in Figure 2.5, which illustrates the two wave fronts traveling with their own characteristic velocities dependent on the slope of the tangent to the stress-strain curve at that point.

![Figure 2.4. Stress-Strain Relation and Wave Profile for Bilinear Material](image)

Figure 2.4. Stress-Strain Relation and Wave Profile for Bilinear Material

![Figure 2.5. Strain Distributions in Rod Produced by Constant Velocity Impact at End](image)

Figure 2.5. Strain Distributions in Rod Produced by Constant Velocity Impact at End

A much more comprehensive treatment of shock wave can be found in the work of Duvall et al. [55-56], Murri and Wentorf [57], Rinehart [58], and Seigel [59].

2.5 Plastic Deformation of Structures in Dynamic Loading

In previous sections, for both continuous and piecewise analysis methods, it was assumed that the application of the contact force during the period of contact does not cause any significant global deformation of the bodies. For many impact problems, however, especially for high-speed impacts, large global deformations occur on the colliding bodies. A rigid body model of the bodies in contact is no longer adequate for these problems. An obvious example of such problems is the vehicle crash in which the structural deformations are of major concern for the
performance criterion of the mechanical system. In this section, the structural deformations of impacting parts within large-scale mechanical systems are investigated. The methods are, in general, based on the plastic hinge model.

The plastic hinge technique is based on the limit analysis of the structure. When a beam is subjected to load levels larger than those corresponding to yielding, large plastic strains will occur in localized regions called plastic hinges. The plastic hinge concept was previously developed [60] by utilizing generalized spring elements to represent the constitutive characteristics of the plastic deformations of beams. The axial deformation has been modeled by a translational joint and a point-to-point restraint. The bending deformation has been modeled by a revolute joint and a torsional spring, as shown in Figure 2.6.

![Figure 2.6. Plastic Hinge Representing Axial and Bending Deformations: (a) Translational Joint and Translational Restraint to Represent Axial Deformation, (b) Revolute Joint and Torsional Restraint to Represent Bending Deformation [19](a)](image)

Most ductile engineering materials have considerable reserve strength capacity past their initial yield. This reserve strength is termed ultimate capacity, plastic moment, or limit design. The material model for limit design is often elastic-plastic, or simply rigid perfectly plastic, and thus material elasticity and strain hardening are ignored. In the case of perfectly plastic bodies, limit analysis is called unrestricted plastic flow, and the loading state at which it becomes possible is called ultimate or limit loading [47].

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The problem of impact or impulsive loading of structural elements such as beams, plates, and shells has most often been treated within the constitutive framework of limit analysis: rigid perfectly plastic behavior, with the yield criterion in terms of generalized stresses [47]. When the loading exceeds the limit design of the beam, the beam will plastically collapse. Collapse is the occurrence of large displacements and rotations without an increase in load. For a plastic deformation of the beam under impact loading, it is assumed that the impact energy is great enough to produce plastic deformations, thus elastic deformations are ignored because these are much less in magnitude than that of plastic deformations.

From the static plastic response of beams, a collapse load generates a beam that departs from static equilibrium. If this load is maintained (which is not the case in impact), deflections of the beam become excessive. Under impact, a collapse magnitude load occurs over a very short magnitude of time, imparting finite external energy into the beam. A beam will reach its final deformation shape when the impactor’s finite energy imparted to the beam is absorbed through plastic deformation and the beam will return to equilibrium, but in a plastically deformed state. If the impact energy is greater than what the beam can absorb, then collapse will occur.

Dynamic analysis of the structures on the basis of the rigid perfectly plastic model has been conducted, and the solutions have been surveyed by Krajcinovic [61] and Jones [62]. A common feature of the solutions is that if the body is restrained against rigid-body motion, then the separation of variables can be applied and the velocity becomes the product of time-dependent amplitude and a function of position:

\[ v(x,t) = \dot{W}(t)g(x) \]  
(2.63)
The separation of variables implies that all points of the body come to rest at once. If the time at which this occurs is \( t_s \), then the permanent displacement field \( u_p(x) \) can be found by integrating the velocity over time from \( t = 0 \) to \( t = t_s \).

The equations of motion of a beam according to the elementary (Euler-Bernoulli) theory can be obtained by adding inertial forces to the distributed load. Here, only a beam of a doubly symmetric cross section, with the centroidal axis along the x-axis and with bending confined to the xy-plane is considered. If the deflection in the positive y-direction is \( w(x, t) \), then the inertial force per unit length is \(-\bar{\rho} \frac{\partial^2 w}{\partial t^2}\), where \( \bar{\rho} = \rho A \) is the beam mass per unit length, \( \rho \) being the mass density and \( A \) the cross-sectional area. With \( q \) denoting the distributed load per unit length, the equation of motion is

\[
\frac{\partial^2 M}{\partial x^2} = q - \bar{\rho} \frac{\partial^2 u}{\partial t^2} \quad (2.64)
\]

When a plastic hinge occurs at the dynamic load point, the shear force is zero, \( q = 0 \).

Bleich and Salvadori [63] examined the motion of an impulsively loaded elastic perfectly plastic beam by using the natural modes of vibration of elastic beams for the entire beam during the initial elastic phase and for the portions of the beam separated by plastic hinges after the beam yields, provided that the hinges are stationary. Salvadori and DiMaggio [64] studied the development of hinges for various degrees of load concentration, ranging from uniformly distributed to concentrate. The impact loading of a rigid perfectly plastic beam that is built in at both ends was studied by Symonds and Mentel [65], who took into account the axial forces that develop when such a beam deflects.

The method of Lee and Symonds [66] was applied by Parkes [67] to study the impact on a cantilever beam of length \( L \) by an object of mass \( m \) traveling at a velocity \( v_0 \) to its tip. The beam is again assumed to be rigid perfectly plastic, so that the kinetic energy of the striker can be
absorbed only in a plastic hinge. Initially, the hinge is at the tip, but it moves in time toward the built-in end, as shown in Figures 2.7 and 2.8.

The velocity function is given by

\[
\begin{cases} 
  v(x,t) = \dot{w}(x,t) = \dot{W}(t) \left( 1 - \frac{x}{\bar{x}(t)} \right) & \text{for } 0 \leq x \leq \bar{x} \\
  v(x,t) = 0 & \text{for } \bar{x} \leq x \leq l
\end{cases}
\]  

(2.65)

The portion of the beam right under the impact load travels at the velocity of \( v_0 \) at the instant of impact loading, while the remainder of the beam is stationary. Therefore, to maintain
dynamic equilibrium, a disturbance propagates away from the loading point, while the striker is assumed to remain in contact with the beam. A plastic hinge develops under the impact point at \( t = 0 \) and propagates the disturbance away from the beam tip toward the supports during the first phase when \( t < t_1 \). The plastic hinge remains stationary at the support until the beam and the striker come to rest, when all the initial kinetic energy of the striking mass \( mv^2/2 \) is dissipated plastically, and in the second phase when \( t > t_1 \).

For phase-one of the deformation, the deflection function along the beam can be calculated as

\[
w_1(x) = \frac{m^2 v_0^2}{3 M_U \rho} \left[ \frac{1 + \beta}{(1 + \alpha)^2} - \frac{1 + 2\beta}{1 + \alpha} + 2 \ln \left( \frac{1 + \alpha}{1 + \beta} \right) \right]
\]

(2.66)

where \( \beta = \frac{\bar{p} x}{2m} \) and \( \alpha = \frac{\bar{p} x}{2m} \).

Then, maximum deflection occurs at the tip when

\[
t_1 = \frac{m v_0 \bar{p} L^2}{3 M_U 2m + \bar{p} L}
\]

(2.67)

The first phase of motion is completed when the traveling hinge reaches the support but \( \dot{W} \) is not yet zero. The total kinetic energy of the system that remains to be dissipated during the second phase of motion can be calculated by subtraction of the kinetic energy at the end of the first phase of motion and the initial kinetic energy as

\[
KE_{t=t_1} = \frac{1}{2} \int_0^L \bar{p} \dot{w}(x,t) dx + \frac{1}{2} m \dot{W}^2
\]

(2.68)

Then, the total energy to be dissipated at second phase is

\[
mv_0^2 (1 + \frac{\bar{p} L}{3m}) \left/ \left[ 2 \left( 1 + \frac{\bar{p} L}{2m} \right)^2 \right] \right.
\]

(2.69)

On the other hand, since \( w_2 = \frac{W_2}{L} (L - x) \), the energy dissipated by the plastic hinge at the support in second phase can be easily obtained by
Equating the energy left at the end of phase one with the plastic energy dissipated at the hinge yields

\[ w_2(x) = mv_0^2 L \left(1 - \frac{x}{L}\right) \left(1 + \frac{\bar{p} L}{3m}\right)/(2M_U \left(1 + \frac{\bar{p} L}{2m}\right)^2 \right) \]  

The final deflection would be summation of deflection at the end of phase one and the deflection at phase two:

\[ w_f = \frac{m^2 v_0^2}{3M_U \bar{p}} \left[ \frac{1+\beta}{1+\bar{\alpha}} \right] - \frac{1+2\beta}{1+\bar{\alpha}} + \frac{2\beta}{1+\bar{\alpha}} + 2 \ln \left(\frac{1+\alpha}{1+\beta}\right) + \frac{\alpha}{(1+\bar{\alpha})^2} \left(3 + 2\bar{\alpha}\right) \left(1 - \frac{\beta}{\bar{\alpha}}\right) \]  

\[ 0 \leq \beta \leq \bar{\alpha}, \quad \bar{\alpha} = \bar{p} L/2m \]

If the mass of the striker is much greater than that of the beam, that is, if \( \frac{\bar{p} L}{m} \ll 1 \) or \( \bar{\alpha} \to 0 \), then the permanent deflection is approximately

\[ w_f = \frac{mv_0^2 L}{2M_U} \left(1 - \frac{x}{L}\right) \]  

Thus, the beam remains straight, and the deflection is that which is necessary so that all kinetic energy is absorbed by the hinge at the built-in end. If, on the other hand, the mass of the striker is much less than that of the beam or \( \frac{\bar{p} L}{m} \gg 1 \) or \( \bar{\alpha} \geq 1 \), then the final shape is a superposition of a rotation and a local deformation near the tip. The form is approximately given by

\[ w_f(x) = \frac{2m^2 v_0^2}{3\bar{p} M_U} \ln \left(\frac{\bar{p} L}{m} \right) \left(1 + \frac{\bar{p} L}{2m}\right)^2 \right) \]  

\[ w_f(0) = \frac{2m^2 v_0^2}{3\bar{p} M_U} \ln \beta \]  

In general, the energy absorption occurs in two phases: phase-one, where energy absorption occurs as plastic hinges travel transverses through the beam, and phase-two, where
energy absorption occurs as the traveling hinges become stationary hinges. Low-energy impacts involve only a phase-one mechanism, and high-energy impacts involve both phase-one and phase-two mechanisms. For beam structures, the impact energy is absorbed by the combination of plastic hinges that form. A hinge will continue to absorb energy until the strain at these hinges exceeds the fracture strain. Structures that form multiple hinges are capable of plastically absorbing greater impact energy, which means a higher degree of indetermination results in higher energy absorption in the system.

Figure 2.9 shows a traveling plastic hinge for a fully clamped beam impacted by an impactor at its mid-span.

![Figure 2.9. Plastic Hinge Traveling Along Beam [41]](image)

For a fully-clamped beam under plastic impact, the plastic deformation equation for the mid-point of the beam for a given impact energy, assuming there is a total transfer of impact energy into the plastic deformation of the beam, is

\[ w_f = \frac{m^2 v_0^2}{24 M u \tilde{\beta}} \left[ \frac{\bar{\alpha} - \beta}{(1+\bar{\alpha})(1+\beta)} + 2 \ln \left( \frac{1+\bar{\alpha}}{1+\beta} \right) \right], \quad 0 \leq \beta \leq \bar{\alpha} \quad (2.76) \]

where \( \beta = \frac{\bar{v}_m}{m}, \quad \bar{\alpha} = \frac{\rho L}{m}. \)
Extensive study of dynamic loading of structures including beams, plates, and shells has been conducted by Jones [62].

A simple frame is an assemblage of bars or beams that are joined together rigidly so that they cannot rotate with respect to one another. The joints transmit bending moment, and the members resist the applied loads primarily through bending; axial force and shear are considered secondary effects. Collapse is assumed to occur when sufficient plastic hinges have formed to produce a mechanism. In a multistory frame, collapse may be limited to a single story; therefore, the overall degree of static indeterminacy is not a relevant parameter for the determination of the necessary number of hinges.

A one-story, one-bay frame, as shown in Figure 2.10, is statically indeterminate of degree three, and the collapse of the frame as a whole indeed requires four hinges, as shown in Figure 2.10 (a) and (c). Consider, however, Figure 2.10 (d), which illustrates the beam mechanism. This mechanism does not entail collapse in the sense of unlimited displacements; the deflection of the beam is limited by that of the columns. In practice, however, a structure may be said to collapse when its displacements can become significantly greater than those in the elastic range. The only pertinent collapse mechanism for the frame of Figure 2.10 is the beam mechanism (d).

Figure 2.10. One-Story One-Bay Frame: (a–c) Four-Hinge Mechanism, (d) Beam Mechanism [47]
In a frame comprising several stories and bays, the number of possible collapse mechanisms can become quite large. Every transversely loaded member may form a beam mechanism, and each story may produce a panel mechanism. Furthermore, for any joint at which three or more members come together, a plastic hinge may form independently in each member near the joint (if only two members meet, the hinge can form only in the weaker member).

It is convenient to establish a basis of independent mechanisms, called elementary mechanisms, such that all mechanisms may be regarded as superpositions of the elementary ones. These elementary mechanisms, as first discussed by Neal and Symonds [68], consist of all the beam and panel mechanisms, in addition to the joint mechanisms constituted by the formation of plastic hinges, at a joint in every one of the members that come together there, resulting in a rotation of the joint.

The joint mechanisms are not in themselves collapse mechanisms, since the external work rate associated with them is zero (unless an external moment acts at the joint), but they are used in combination with beam and/or panel mechanisms in order to cancel superfluous hinges.

2.6 Material Models at Impact

The topic of the behavior of the materials at high rates of strain has been one of considerable interest since World War-II, when dynamic plasticity and plastic wave propagation first received attention. Materials are often categorized as linear-elastic, nonlinear elastic, viscoelastic, viscoplastic, and so on. Each description refers to a mathematical representation for a class of idealized material behavior. The most general form of a material-constitutive equation should cover the description of material behavior under the total range of strain rates that may be encountered. However, this can be extremely difficult, even for uniaxial stress; therefore, the majority of the constitutive equations generally cover only a narrow range of strain rates. Some
of the considerations in dynamic testing have been summarized in Table 2.4. As higher strain rates are encountered, the strain-stress properties may change, depending on material type. The simplest method for determining characteristics of the strain-rate sensitivity of a material is to increase the speed of a uniaxial tension or compression test.

In general, the constitutive relationship necessary to completely and accurately describe a material’s behavior can be extremely complex and mathematically intractable. Thus, simplified models, restricted classes of material behavior, and mathematical idealizations are utilized in the solution of engineering problems.

The study of impact processes encompasses a wide range of materials responses that cross traditional academic boundaries. Because of the complexity of the subject, no comprehensive solution to impact problems in all velocity regimes exists. Analytical models have very restricted applications because of the simplifying assumptions employed in their derivation. It is now generally accepted that material failure under impact loading is a time-dependent phenomenon, which means that the material model should be such that it has the rate-sensitivity parameters to model the impact and dynamic loadings.

Due to rate-dependency of most metals, an assumed stress-strain relation used in solving dynamic problems is not necessarily the same as the stress-strain relation obtained from static or quasi-static tests [69]. A generalized constitutive equation that includes both rate-independent and rate-dependent models as special cases was proposed in the early 1960s by several researchers [70-72]. This equation has the form of

\[ \frac{\partial \varepsilon}{\partial t} = f(\sigma, \varepsilon) \frac{\partial \sigma}{\partial t} + g(\sigma, \varepsilon) \]  

(2.78)

For loading \( \sigma \frac{\partial \sigma}{\partial t} > 0 \), and for unloading \( f(\sigma, \varepsilon) \) is replaced by \( \frac{1}{E} \), or if finite strains must be taken into account, \( \frac{(1-\varepsilon)^2}{E} \).
Strain rates influence the strength of materials when impacted. In general, as the strain rate increases, both the tensile and compressive strengths increase. Increased impactor velocity will increase the strain rate. Effect of strain rate on the strength of mild steel is shown in Figure 2.11 for uniaxial compressive loading.

Figure 2.11. Stress (klbf/in²)-Strain Curves for Mild Steel at Various Uniaxial Compressive Strain Rates According to Marsh and Campbell [73]

The effect of strain rate on the strength for mild steel is shown in Figure 2.12 for tensile loading [74]. For tension, the upper and lower yield strengths increase with strain rate. Fracture strain decreases with increasing strain rate, and the material becomes more brittle at higher strain rates. The ultimate fracture strength slightly increases. Indeed, it appears that apart from the upper yield stress, this material behaves as a perfectly plastic material with little or no strain hardening at high strain rates.
As mentioned previously, many different constitutive equations for the strain rate sensitive behavior of materials have been proposed in the literature [70-72]; however, it is evident from the experimental literature that there is still considerable uncertainty and lack of reliable data even for some common materials. Empirical relations have been developed to quantify the strain rate sensitive behavior of engineering metals. For uniaxial plastic strain rates, the ratio of dynamic flow strength $\dot{\sigma}_0$ to static yield strength $\sigma_0$ is given by [75]

$$\frac{\dot{\sigma}_0}{\sigma_0} = 1 + \left( \frac{\dot{\varepsilon}}{D} \right)^{1/q} \quad (2.79)$$

where $q$ and $D$ are empirically determined constants for a particular material for its lower dynamic yield point. Some $q$ and $D$ constants for engineering metals are provided in Table 2.5. Equation (2.79) can be written for the effective or equivalent stress-strain rate for any dynamic uniaxial, biaxial, or triaxial state of stress-strain as

$$\dot{\sigma}_e = \left[ (\dot{x} - \dot{x})^2 + (\dot{y} - \dot{y})^2 + (\dot{z} - \dot{z})^2 + 6(\dot{x}_y^2 + \dot{x}_y^2 + \dot{x}_y^2) \right]^{0.5} / \sqrt{2} \quad (2.80)$$

and

$$\dot{\varepsilon}_e = \sqrt{2} \left[ (\dot{x} - \dot{x})^2 + (\dot{y} - \dot{y})^2 + (\dot{z} - \dot{z})^2 + 6(\dot{x}_y^2 + \dot{y}_y^2 + \dot{z}_y^2) \right]^{0.5} / 3 \quad (2.81)$$
TABLE 2.5. Sample Coefficients for Cowper-Symonds Constitutive Equation [75]

<table>
<thead>
<tr>
<th>Material</th>
<th>D (1/sec)</th>
<th>q</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild Steel</td>
<td>40.4</td>
<td>5</td>
<td>Cowper and Symonds (1957)</td>
</tr>
<tr>
<td>High Tensile steel</td>
<td>3200</td>
<td>5</td>
<td>Paik and Chung (1999)</td>
</tr>
<tr>
<td>Aluminum Alloy</td>
<td>6500</td>
<td>4</td>
<td>Bodner and Symonds (1962)</td>
</tr>
<tr>
<td>Titanium Alloy (Ti 50A)</td>
<td>120</td>
<td>9</td>
<td>Symonds and Chon (1974)</td>
</tr>
<tr>
<td>Stainless Steel 304</td>
<td>100</td>
<td>10</td>
<td>Forrestal and Sagartz (1978)</td>
</tr>
</tbody>
</table>

In order to simplify theoretical analyses, Perrone [76] suggests that material strain hardening and strain-rate-sensitive effects could be decoupled and the corresponding constitutive equations expressed in the product form of

\[
\frac{\dot{\sigma}_0}{\sigma_0} = f(\dot{\varepsilon})g(\varepsilon)
\]  

(2.82)

where \( f(\dot{\varepsilon}) \) could be in the form of equation (2.79) in the uniaxial case and \( g(\varepsilon) \) is for strain hardening. However, some coupling is evident in the experimental test data, since strain rate effects decrease with increase in strain.

2.7 Numerical Analysis of Impact Phenomena

Analytical models are quite useful for developing an appreciation for the dominant physical phenomena occurring in a given impact situation. Numerical techniques offer an effective way of obtaining a complete solution to impact problems. This is especially true for oblique impacts or situations where a three-dimensional stress state is dominant, for which there are virtually no models that can deal with such complexity. Excellent results have been obtained for situations where material behavior, especially material failure under high loading rates, is well understood and characterized [25].

It is necessary in a computer analysis to replace a continuous physical system by a discretized system. In the discretization process, the continuum is replaced by a computational mesh. The discretization techniques most commonly used are the finite difference and finite element methods. The finite element method (FEM) is a numerical approximation method, which
is based on the solution of a system of differential equations [77]. These differential equations contain the parameters such as stress-strain that are used to calculate the forces and displacements for a prescribed geometry. The FEM can be viewed as an application of the Rayleigh-Ritz method and is primarily based on a displacement approximation. Extensive references can be found for fundamental and advanced theory and applications of FEM and FEA and is not in the scope of this study. The application of the FEM in structural impact and techniques of accurate modeling of the impact scenario will be investigated briefly here.

The bulk of computer codes used for impact studies fall into two categories of discretization: Lagrangian and Eulerian. Various hybrid schemes have also been developed over the years, such as the arbitrary Lagrangian-Eulerian (ALE), the smooth particle hydrodynamics (SPH), and so forth. In a Lagrangian code, the mesh is attached to bodies and distorts with them. Material surfaces are distinct in aiding application of boundary conditions and loading. For problems in which large distortions predominate or where mixing of materials that are initially separated occur, an Eulerian description of material behavior is necessary. In the Eulerian approach, the computational grid is fixed in space while material passes through it. Since no one computational technique can handle all situations in impact dynamics, hybrid methods have been developed.

When mesh distortions become very large in Lagrangian computations, truncation errors rise to unacceptably high values. Quadrilateral meshes become entangled and may take on shapes resulting in negative volumes, thus rendering computational useless. Impact bodies should be meshed with low-order elements such as three-node, six-degrees-of-freedom triangular elements or four-node, twelve-degrees-of-freedom tetrahedral elements. These lower-ordered elements were found to be more resistant to distortion and reduce computational effort. In order to
capture the wave propagation effects in 2-D and 3-D problems, the element aspect ratio must be less than four. When meshing with triangular elements, four triangular elements per quad shape are used instead of two. Two-element arrangement introduces asymmetry to the analysis. Mesh density should be higher in plastic deformation zones of contact bodies.

Excessive distortion of grids due to shear or fold over onto themselves during impact develops numerical inaccuracies resulting in negative masses. These problems can be overcome to some extent through the use of sliding interfaces and rezoning. When two surfaces are constrained to remain in contact, the tied sliding interfaces are useful to insure the displacement continuity across the interface. One of the sliding surfaces is designated as a master surface, the other a slave surface. As the name implies, the slave-surface motion is dependent on the behavior of the master surface. In theory, the designation of the master and slave surfaces is arbitrary. In practice, however, the choice is highly problem- and code-dependent. A key for successful modeling is the correct usage of the contact or gap elements to enforce the contact boundary conditions when they occur. The contact stiffness should be the Young’s modulus of the softer contacting body. If automatic contact control is used in an FEA analysis package, the user must have a good understanding of how the contact parameters are generated.

Contact detection and computation require rather complex procedures and consume considerable CPU time. In addition, when increasing contact stiffness, in order to improve the satisfaction of contact conditions (and to decrease penetration events between bodies), the computational time step, depending on the biggest stiffness value of all finite elements and contact springs, must be decreased. Other CPU time problems are linked to the detection of all slave and master entities potentially in contact, at each computation step, when bodies are moving fast and deform considerably.
Situations involving impact or impulsive loading excite wide-spectrum frequencies in the affected structure. If the response of the structure is controlled by a relatively small number of low-frequency modes, the problem is said to be in the structural dynamic category. On the other hand, in wave-propagation problems, the high-frequency modes dominate the response throughout the time of interest. In practice, combinations of these conditions are frequently encountered. In many low-velocity impact situations, in accidental collisions for example, initial high-frequency transients gradually decay to the steady-state or free-vibration regime.

Situations wherein some portions of a structure are mechanically stiffer than other portions create difficulties for numerical integration of the equations of motion. If a time step small enough to accurately treat the rapidly varying components is chosen, then it will be excessively small for the remaining components, resulting in excessive computation effort and round-off error in the low-frequency components. On the other hand, if the time step is adjusted to the slowly varying components, instability may result, and the high-frequency components will not be accurately resolved.

When solving dynamic problems with the finite element method, it must be noted that FEM is used only for the spatial discretization, and time steps are conducted by the finite difference method. At each time step, the equilibrium equations are solved and the values of unknowns are determined. Time integration routines are the heart of most structural dynamics problems. Hence, the subject has been extensively studied [78-87]. Two methods can be utilized for time integrations: the implicit method or backward-difference approximation, and the explicit method or forward-difference approximation. In implicit methods, the information at any time step depends on the previous time step, and the current time step and equilibrium are achieved at each time step using an iterative procedure. In the explicit method, information at each time step
can be obtained using data from previous the time step, and there is no dependency on the current time step. This implies that displacements at some time \( t + \Delta t \) in the computational cycle are independent of the accelerations at that time. Explicit analysis is well suited for dynamic simulations such as impact and crash analysis, and implicit analysis is good for structural dynamics with low to moderate frequency content, and static and pseudo-static phenomena like metal spring-back after forming.

Implicit formulation allows for pseudo-static phenomena and a dynamic approach. In the pseudo-static case where acceleration and velocity forces are neglected, the principle of virtual work leads to

\[
[K][u] = \{F\} \tag{2.83}
\]

where \([K]\) is stiffness of the matrix, \([u]\) is the displacement vector, and \([F]\) is the external force vector. The displacement is found directly by inversion of the stiffness matrix, which is a classical inversion problem. It uses Gauss elimination, with its usual matrix conditioning problems.

For the explicit dynamic case, acceleration and velocity forces are carried using the equation of motion as

\[
[M][\ddot{u}] + [C][\dot{u}] + [K][u] = \{F\} \tag{2.84}
\]

where \([M]\) is the mass matrix, and \([C]\) is the damping matrix. Having the initial conditions, that is, initial velocity and displacement at each time step, an explicit formulation can be written on the fundamental differential equation of dynamics as

\[
[M][\ddot{u}] = \{F_{\text{ext}}\} + \{F_{\text{int}}\} \tag{2.85}
\]

The central difference integration with \( \dot{u}_{n+1/2} = \frac{u_{n+1} - u_n}{\Delta t} \) and \( \ddot{u} = \frac{\dot{u}_{n+1} - \dot{u}_n}{\Delta t} \) yields

\[
u_{n+1} = (2u_n - u_{n-1}) + [M]^{-1}(\Delta t)^2(F_{\text{ext}-n} - F_{\text{int}-n}) \tag{2.86}
\]
The computed response may become unstable in explicit integrations unless care is taken to restrict the size of the time step. This problem has been studied rigorously for linear problems by Courant et al. [88], who found that in explicit integration, the computation will be stable if the time step satisfies the relation \( \Delta t \leq 2/\omega \), where \( \omega \) is the highest natural frequency of the mesh. No rigorous stability criterion has been determined for nonlinear problems, but it is customary to determine the time step from \( \Delta t = kl/c \), where \( k \) is a factor of less than unity, \( l \) is the smallest element size, and \( c \) is the stress wave speed along the part. But the local truncation error of most implicit and explicit schemes is of the order \((\Delta t)^3\). While this is insignificant for explicit schemes, it is a matter of concern for implicit methods, where the time step is so much larger. The computational time in the explicit method is dominated by the smallest element or stiffest element. No stiffness matrix of the element assemblage needs to be calculated in the explicit method, so even with small time steps, this method is still effective for high-frequency loads, such as crash dynamics. For explicit methods, since the time step size is controlled by the smallest mesh dimension, as distortions increase, the time step is progressively reduced and approaches zero for large distortions, rendering computations economically impractical.

Mathematically, if \( u_n = u(t) \) is the current system state and \( u_{n+1} = u(t + \Delta t) \) is the state at the later time (\( \Delta t \) is a small time step), then for an explicit method,

\[
u_{n+1} = F(u_n) \tag{2.87}\]

while for an implicit method, to find \( u_{n+1} \), one solves the equation

\[
G(u_n, u_{n+1}) = 0 \tag{2.88}\]

It is clear that implicit methods require extra computation, such as solving equation (2.88), and they can be much harder to implement. Implicit methods are used because many problems arising in practice are stiff, for which the use of an explicit method requires
impractically small time steps to keep the error in the result bounded. For such problems, to achieve given accuracy, it takes much less computational time to use an implicit method with larger time steps, even taking into account that an equation like equation (2.88) must be solved at each time step.

The engineering approach to any modeling process including finite element analysis consists of several main steps:

- Initial analysis of the situation (problem definition)
- Pre-processing
- Computation
- Post-processing

The accuracy of any FEA strongly depends on the experience of the user and an engineering understanding of the problem. The simplification of the geometry (idealization of the structure) needs to be investigated in order to omit the structural details, which are not important for setting the tasks so that the computation time can be saved. The pre-processing begins with the discretization of the existing structure. A meaningful mesh must be developed by the elements linked over discrete places (nodes). Depending upon the concrete problem definition and material used for the structure, the material model must be selected, which is crucial to the success of the computation. Likewise, the definition of the loads and appropriate boundary conditions are given in the preprocessor. Afterwards in the solver, the numerical procedure takes place to solve the defined set of equations, and all interesting parameters (stress, strain) are determined. The postprocessor facilitates a view of the results (graphical analysis). In the postprocessor, the obtained diagrams give an overview of prevailing circumstances immediately. The plausibility of the results is checked, and a comparison with experimental
investigations can be done. Based on the results and an evaluation of the results, it is necessary to repeat the simulation, whereby sometimes the selected structure idealization and load definition may have to be changed.

2.8 References


CHAPTER THREE
ENERGY DISSIPATION IN IMPACT

3.1 Introduction

The study of impact and its contribution in the machine and component design process have attracted extensive engineering efforts. Impact is defined as the collision of two or more structures or mechanical systems. Loading during an impact is different from that of a static loading where a state of equilibrium is reached. The forces generated during a collision are exerted and removed over a very brief period of time. During this brief period of time, the generated forces do not have time to propagate throughout the objects undergoing impact. There are a number of ways kinetic energy can be removed from an object as the result of an impact event. Both objects involved in an impact can be subjected to elastic and/or plastic deformations [1], elastic deformation being temporary and plastic being permanent. During the impact, some energy is stored as elastic strain, and some energy is absorbed through stress wave propagation, plastic deformation, material dampening, or other phenomenon such as sound or heat [2].

Systems undergoing permanent global deformation require complex analytical descriptions using time-dependent analyses [3]. A static analysis cannot be used for this type of impact scenario because of the large plastic deformations experienced by the system during impact. During impact, as local deformations progress, the forces involved in the impact continue to change. To describe such an event, a dynamic model of these changing forces is required.

For normal design under static conditions, the design engineer assesses loads imparted to the mechanical component, and performs stress, deflection, and fatigue analyses to size the component. In impact conditions, the designer must assess the energy imparted into the
component, perform stress and deflection analyses and use these analyses for design performance. The geometry of the resisting structure or component along with its material properties provides the energy-absorbing characteristics of the impact design. The energy of the impact is set equal to the component’s dimensions and material properties to size the component.

In a broad sense, two design goals usually are addressed in an impact design. The first design goal is to “mitigate damage to the impactor.” To mitigate damage to the impactor, soft stiffness, large deflections, and plastic crush are required for the target side. Lower stiffness during the large plastic crush produces a lower contact load and a longer contact time. Many energy-absorbing mechanisms or concepts have been proposed and adapted in operational systems. An example of this impact design is the use of airbags to mitigate injury to drivers’ heads in an accident or to occupants of a small car involved in an impact into the side or rear guard of large trucks or trailers. The second design function is to “retard the impactor.” Hence high stiffness, small deflections, and high material strengths are required for the target side. This produces large contact stress and short contact duration. However, this is not true for all cases. An example of this design function would be a bird strike impact on the leading edge panels of aircraft or the front bull guard of pickup trucks and sport utility vehicles (SUVs).

Human safety is a high priority in the vehicle industry; therefore, vehicle structures are designed for crashworthiness so that they attenuate the occupant or pedestrian injuries in crash events. Crashworthy design optimization leads to relatively soft and crushable front ends of vehicles and stiff and intrusion-resistant passenger compartments.

The concepts of impact energy management and impulse mitigations are evaluated in this dissertation because they are primarily used in structural crashworthiness. The general principles of impact mitigation in elastic and plastic regions as well as the design of an energy-absorbing
device are investigated. Since most impacts in the area of crashworthiness are associated with plastic deformation, this methodology is extended for structural plastic deformation and has been referred to as the “plastic hinge method” in which some assumptions of material models are required. Design and sizing of components subjected to impact loading for some simple examples are also evaluated.

3.2 Designing for Impact-Impulse Mitigation

Impacts produce abrupt changes in motion and velocity, thus producing high acceleration changes in a body for a finite period of time. This acceleration profile is referred to as “shock” or, even better, “shock pulse.” Mechanical shock is a non-periodic disturbance characterized by suddenness and severity. Such extreme disturbances cause significant forces that may damage mechanical components or electronics. Shock impulses are caused by the following:

- A sudden introduction of energy into the system.
- A large force excitation.
- A sudden change in velocity.

Analysis of a shock problem starts with the examination of the shock pulse. This is a complex transient wave shape represented by many sinusoidal amplitudes and frequencies. Maximum amplitude, time duration, and shape of the shock pulse are important. In practice, a complex shock pulse is compared to several simple pulse shapes for analysis. The majority of excitations can be characterized as triangular shock pulse. For vehicle and aircraft industries, the triangular shocks with different peak and time durations reasonably represent the input impulse [4].

One of the main design functions for a mechanical system experiencing impact loading is to provide shock mitigation. In the design of shock mitigation, the item which is desired to be protected is usually the impactor and the body possesses kinetic energy. Examples for impact
shock attenuation are airbags, suspension systems, and packaging protection for fragile components. An aircraft seat with and energy-absorbing system and an appropriate cushion to protect the occupant during a crash landing or seat ejection in case of emergency is another example of a shock-mitigation system using structural plastic deformation.

Generally speaking, the goal of any shock-mitigation and shock-arresting device is to spread out the input impulse shock over time, reducing its magnitude to acceptable levels for the system being protected, or below a human tolerable limit for cases involving shocks or impacts on humans. In other words, the input pulse is absorbed by the shock-arresting device, and the shock energy is released over a broader time, lowering the magnitude of the pulse to protect the system. This is illustrated in Figure 3.1.

![Figure 3.1. Typical Input and Output Acceleration Pulse in Shock-Mitigation Device](image)

From the integration of the shock pulse $G_{in}$ over the time duration $t_0$, the change in velocity $\Delta V$ of the impactor can be obtained, which is the area under the curve, $G_{in}$ versus time. With the change in velocity $\Delta V$ computed from the input shock pulse, and the fragility level $G_{out}$ of the system to be protected known, the required natural frequency $f_n$ of the isolated system can be evaluated as

$$G_{out} = \frac{2\pi f_n \Delta V}{g} \quad (3.1)$$
The natural frequency $f_n$ represents the inverse of the time frame for which the shock pulse will be released. Based on the static weight of the system to be protected and the required frequency $f_n$ to reduce the shock pulse, a shock mitigation device can be selected using

$$
(2\pi f_n)^2 = \frac{K}{m}
$$

The output response $G_{out}$ from the shock mitigation device should not exceed the fragility limit of the component or system being protected or human tolerable limits. For mechanical systems, fragility is defined as the highest acceleration level the system will tolerate and still function.

In vehicles and aircraft, mechanical shock or impulse can cause severe injury to occupants if appropriate protection is not provided in their systems. Foam materials, airbag cushions, rubber, and other elastomers are used as energy absorption materials in the attenuation of impact shocks of low intensity, while for higher acceleration shock pulses, the design of a proper energy-absorbing system is necessary. These types of materials lower the natural frequency of the protected system providing this spreading-out effect by delaying the response of the protected system to the shock input. Due to the importance of comfort in vehicles and aircraft, foam materials are the most commonly used material to protect occupants. Beyond the concern of comfort, mechanical energy-absorber systems are used to keep the impulse applied to the occupant within human tolerable limits. For crashworthy design requirements, an energy-absorbing system to absorb the impact energy rather than to store it is preferred. Hence, systems with high-damping or energy-absorbing capabilities are preferred. The rotorcraft or helicopter seat energy absorber is an example of an acceleration pulse attenuation system using structural plastic deformation.

For elastic impact design, to size the component, the strain energy capacity is set equal to the impact energy. For an axial element, the strain energy in the element is given by
where the maximum stress in the axial element is limited to \(S_y\) during an elastic impact. Substituting \(P = S_y A\), the strain energy capacity of the axial component is then evaluated as

\[
U_{axial} = \int_0^L \frac{P^2}{2AE} \, dx = \frac{S_y A L}{2E}
\]  

In a similar method, for a pure bending element, the strain energy in the element is given by

\[
U_{bending} = \frac{M^2 L}{2EI}
\]  

For elastic conditions, the maximum stress in the bending element is limited to \(S_y\) during the impact, thus the moment \(M\) is bound by

\[
M = \frac{S_y J}{c}
\]  

where \(c\) is the extreme fiber distance from the cross-sectional neutral axis. The strain energy capacity for a pure bending element is then given by

\[
U_{bending, max} = \frac{S_y^2 IL}{2Ec^2}
\]  

Therefore the elastic energy absorption for all types of loading is controlled by material properties and component geometry. The “modulus of resilience” is defined as \(\frac{S_y^2}{2E}\). The impact energy that a component can elastically absorb is then a function of its modulus of resilience times the components volume. The yield strength influences the energy absorption quadratically and the Young’s modulus by being inversely proportional to energy capacity. A good crashworthy design is the use of materials with high yield strength and low Young’s modulus. The impact energy capacity can also be improved by increased material volume. Comparing the modulus of resilience of mild steel to rubber, rubber is 20 times better at elastic-absorbing impact energy than steel. That is why rubber is a good shock attenuator for low-impulse shocks.
As the intensity of an impact increases, the resultant stresses and strains in the contact components increase. Hence, for low- and medium-impact impulses, it is important to determine whether the yielding of the material occurs due to contact stresses. For structural impacts and vehicular accidents, due to high-impact impulse during any crash event, the elastic portion of energy absorption is negligible compared to the energy absorbed due to plastic deformation of the system.

To understand how yield occurs in contacts, an appropriate expression for the force indentation relation is needed to predict any damage due to plastic deformation. Beyond the elastic loading, two stages are considered: “elastic-plastic” and “fully plastic.” In the elastic-plastic stage, plastic deformation is small enough to be accommodated by an expansion of the surrounding area. As the load increases, the plastic zone grows, and the displaced material flows to the sides of the indenter. For this analysis, the rigid-perfectly plastic material model is commonly used. It assumes that the elastic deformation is small enough to be negligible, and the material flows plastically at a constant stress \( S_y \). For a sphere-sphere contact, Johnson [5] shows that, under those assumptions, yield will initiate when the mean contact pressure \( P_m \) is \( 1.1S_y \), and the flow will become fully plastic at about \( P_m = 3S_y \). Stronge [6] takes the same approach to derive an expression for the restitution coefficient that reflects the dissipation due to plastic work under different conditions of friction. Based on the rigid-perfectly plastic model and Hertz theory of impact, Johnson [5] calculates the impact velocity \( V_y \) necessary to initiate yield. For a sphere striking the plane surface of a massive body, this study shows that

\[
\rho \frac{V_y^2}{S_y} = 26\left(\frac{S_y}{E}\right)^4
\]

where \( \rho \) is the sphere material density, and \( E \) is an equivalent elastic modulus. For example, for a medium hard steel, \( S_y = 1000 \text{ Mpa} \) and \( V_y = 0.14 \text{ m/sec} \).
Johnson [7] suggested the non-dimensional parameter \( \frac{\rho V^2}{S_{yd}} \) as a way to characterize impact between two metallic bodies, in which \( S_{yd} \) is the “dynamic yield strength.” Table 3.1 provides a preliminary guide to distinguish the impact regime in terms of \( \frac{\rho V^2}{S_{yd}} \).

**TABLE 3.1. Impact regimes as function of \( \frac{\rho V^2}{S_{yd}} \) [7]**

<table>
<thead>
<tr>
<th>Regime</th>
<th>( \frac{\rho V^2}{S_{yd}} )</th>
<th>Approximate Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>(&lt;10^{-6})</td>
<td>(&lt;0.1)</td>
</tr>
<tr>
<td>Fully Plastic</td>
<td>(\sim10^{-3})</td>
<td>(\sim5)</td>
</tr>
<tr>
<td>Shallow Indentation</td>
<td>(\sim10^{-1})</td>
<td>(\sim100)</td>
</tr>
<tr>
<td>Large Plastic Flow, Hydrodynamic Behavior</td>
<td>(\sim10)</td>
<td>(\sim1000)</td>
</tr>
<tr>
<td>Hypervelocity (e.g., Laser Beams, Meteorites)</td>
<td>(\sim10^{3})</td>
<td>(\sim10000)</td>
</tr>
</tbody>
</table>

3.3 **Impact Attenuation Using Structural Plastic Deformation**

For many years, the basis for structural design has been the “allowable stress” concept. Allowable stress was usually taken to be the yield stress of the material, and the design stress was then taken to be some fraction of the allowable stress, depending on the factor of safety used. There is really no reason for assuming that the stress in the structure should never exceed the elastic limit. Therefore, the practice is becoming more widespread to design structures into the plastic range. In these design procedures, no attempt is made to determine the stress and strains in the structure, but rather what is sought is the load-carrying capacity or limiting load at the point in which the structure will collapse. This type of analysis is called “limit design” or “plastic design,” and the load at collapse is called the “plastic collapse load.” When a structure collapses, it adds some kinematic degrees of freedom to it. In another words, the portion of the structure becomes a mechanism. One of the objectives of limit analysis is determining when a
mechanism will be formed and, in the case when more than one mechanism is possible for a system, to determine which one is the system-collapse mechanism. Limit analysis is concerned with finding the critical load sufficient to cause plastic collapse with the physical requirements of static equilibrium, yield conditions for the materials, and consistent geometry considerations. The principles of limit analysis have been well developed by a number of authors [8-10].

In structural engineering beam theory, the term “plastic hinge” is used to describe the deformation of a section of a beam where plastic bending occurs. In limit analysis of structural members subjected to bending, it is assumed that an abrupt transition from elastic to ideally plastic behavior occurs at a certain level of bending moment, known as plastic moment $M_p$. Member behavior between $M_{yp}$ and $M_p$ is considered to be elastic, where $M_{yp}$ is the onset of the yielding of the member at the extreme fiber from the neutral axis. When $M_p$ is reached, a plastic hinge is formed in the member. In contrast to a frictionless hinge that permits free rotation, it is postulated that the plastic hinge allows large rotations to occur at constant plastic moment $M_p$.

Plastic hinges extend along short lengths of beams. Actual values of these lengths depend on cross sections and load distributions. But detailed analyses have shown that it is sufficiently accurate to consider beams rigid-plastic, with plasticity confined to plastic hinges at points. While this assumption is sufficient for limit analysis, finite element formulations are available to account for the spread of plasticity along plastic hinge lengths.

In the design of machines and components, a plastic hinge is a type of energy-dampening device allowing plastic rotation of an otherwise rigid column connection. This device is composed of a weakened portion of the column that is prevented from rotating by relatively small steel members. These small bars are designed to yield and allow rotation before the capacity of the column is reached, thus acting as mechanical fuses protecting the column from
fatigue. By inserting a plastic hinge at a plastic load limit into a statically determinate beam, a kinematic mechanism permitting an unbounded displacement of the system can be formed. This is known as the collapse mechanism. For each degree of static indeterminacy of the beam, an additional plastic hinge must be added to form a collapse mechanism.

Energy is dissipated through the plastic deformation of specific zones—plastic hinges—without collapsing the rest of the structure. In order for this hinge to be effectively used as an energy-dampening device, the moment capacity of the hinge, determined by the position, axial and bending stiffness, and yielding strength of the fuse bars, must be lower than that of the column.

Sizing the components and members for plastic impacts are much more difficult compared to that of elastic design. If the geometry has any complexity at all, an experimental or virtual analysis, or FEA, is required. As a first-cut approximation in sizing a component for plastic impact, the sum of the plastic moments $M_p$ times their plastic rotation $\theta_p$ can be equated to the impact energy.

$$\frac{1}{2} m_2 V_2^2 (1 - e^2) = \sum_{i=1}^{n} M_p \theta_p$$  \hspace{1cm} (3.9)

where $m_2$ is the mass of the impactor with velocity $V_2$ and the coefficient of impact of $e$. The evaluation method for the structural deformation ($\theta$) under impact loading was presented in detail in Chapter Two. As the mass $m_2$ exceeds the mass of the target, this relation becomes a better approximation to the energy absorbed by the target. Application of the first-cut approximation for plastic impact to a cantilever beam, shown in Figure 3.2, yields

$$\frac{1}{2} m_2 V_2^2 (1 - e^2) = M_p \theta_p$$  \hspace{1cm} (3.10)
For a beam simply supported at the ends, as shown in Figure 3.3, application of the impact energy balance yields

\[ \frac{1}{2} m_2 v_2^2 (1 - e^2) = M_p (2 \theta_p) \]  
(3.11)

3.4 Impact Damping

Figure 3.4 illustrates a linear mass-spring system with an applied force. Although the spring is linear, the presence of impact velocity discontinuities, when the mass strikes the barrier, makes the response of the system non-linear. Systems such as these are often referred to as “impact dampers” or impact absorbers, since energy is lost, or dissipated, due to structural damping or local deformations. Although they can be noisy, impact dampers have advantages over other types of dampers, particularly at very high temperatures when ordinary damping materials are not applicable [4]. Another interesting application of the one-sided barrier model is a ship tethered adjacent to a dock in the presence of wave motion.
Depending on the available impact parameters and methodology utilized, impact energy can be evaluated. Using the stereomechanics approach, the dissipated energy due to an impact can be calculated as

$$T_L = -(V_n P_n + V_t P_t)$$

where $V$ is the average velocity of the impactor during the impact, and $P$ is the impulse. The normal and tangential directions of the impact are represented by $n$ and $t$. Equation (3.12) can be rewritten in terms of impact parameters as

$$T_L = \frac{1}{2} m (1 + e)(V_n^{closing})^2 [(1 - e) + 2\mu r - (1 + e)\mu^2]$$

where $V^{closing}$ is the closing velocity in the normal or tangential direction, $m$ is the impactor mass, $e$ is the coefficient of restitution, $\mu$ is the coefficient of tangential restitution, and $r = \frac{V_t^{closing}}{V_n^{closing}}$. For a normal impact, equation (3.13) can be summarized as

$$T_L = \frac{1}{2} m (1 - e^2)(V^{closing})^2$$

From a contact mechanics viewpoint, it is shown that for impact between bodies at low impact velocities, hysteresis damping is the primary factor for energy dissipation [11]. At fairly moderate or high velocities of collision, especially in the case of metallic solids, permanent indentations are left behind on the colliding surfaces [12 and 13]. Hence, local plasticity of the
surfaces in contact becomes the dominant source of energy dissipation during impact. Figure 3.5 shows the impact force due to local deformation and the energy loss due to impact.

Figure 3.5. Low-Velocity Impact: (a) Hysteresis Damping Force Model, (b) Local Plastic Contact Force Model

For both cases, the energy loss can be evaluated as

$$T_L = \int_0^{\delta_m} F_{\text{com}} d\delta + \int_{\delta_p}^{\delta_m} F_{\text{rest}} d\delta$$

where $F_{\text{com}}$ is the contact force during the “compression phase,” $F_{\text{rest}}$ is the contact force during the “restitution phase,” $\delta_m$ is the maximum penetration due to impact, and $\delta_p$ is the permanent local deformation, which is equal to zero at the hysteresis contact force method. The energy dissipation relationship for each method was presented in detail in the Chapter 2.

Particle damping is a derivative of impact damping where multiple auxiliary masses of small size are placed in a cavity attached to the vibrating structure. Particle damping can perform at elevated temperatures where most other forms of passive damping cannot. Studies in recent years have demonstrated the effectiveness and potential application of particle dampers and have shown that they are highly nonlinear dampers with energy dissipation, or damping, derived from a combination of loss mechanisms. The relative effectiveness of these mechanisms changes based on various system parameters.
An impact damper is one of the dynamic vibration absorbers in which motion of auxiliary mass is limited by the motion-limiting stops placed inside a container. The subject of impact dampers is of great importance for several engineering applications; however, it is not in the scope of this research, and interested readers are referred to studies carried out by Brach [4], Moon and Shaw [14], Shaw and Holmes [15], Papalou and Masri [16], Masri [17], Masri and Ibrahim [18], Davies [19], Soong and Dargush [20].

In this chapter, the basics of impact dynamics of mechanical systems have been examined. Different methodologies for impact analyses of mechanical systems or structures have been investigated and compared. The nature of energy dissipation in impact has also been examined for low- and high-velocity impacts. In Chapters Four to Seven, several test cases of impact design are presented, and the investigation of each test case is conducted in terms of crash energy management, impulse mitigation, and/or impact injury biomechanics.

3.5 References


CHAPTER FOUR

CASE STUDY ONE: APPLICATION TO CRASH ENERGY MANAGEMENT—
INFLUENCE OF SIDE UNDERRIDE GUARD HEIGHT OF TRUCK ON CABIN
INTRUSION AND OCCUPANT INJURY POTENTIAL OF A SMALL CAR IN
CAR/LARGE-TRUCK SIDE CRASHES

4.1 Abstract

In car/large truck collisions, the size, weight, and stiffness mismatch results in a much
larger structural deformation of the car compared to that of the truck, and larger potential for car
occupants injuries. This is further aggravated when the passenger vehicle underrides the rear or
side of the taller truck. In the U.S., underride guards are required for only the rear of large trucks,
although a review of fatal truck-car crashes in the National Crash Severity Study/
Crashworthiness Data System (NCSS/CDS) accident databases indicates that fatal underride
crashes involving the sides of large trucks are almost as common as rear underride crashes. A
methodology using validated finite element models of a passenger car and a truck is utilized in
this study to quantify the influence of a side guard attached to a large truck in reducing the
intrusion of the car and thus reducing the injury sustained by the occupants of a car in a side
crash. A formulation is proposed based on a normalized passenger compartment intrusion and its
relation to the occupant injury potential. The transmitted deceleration severity and the occupant
compartment intrusion are evaluated for the car impacting a rigid underride guard. A NHTSA
underride test is simulated first for model validation. Once model validation is conducted for a
specific guard design, further simulations are conducted at the speed range of 30 to 50 mph and

1This entire chapter has been published in the following source:

intrusion and occupant injury potential of a small car in car/large-truck side crashes,” International Journal
for 90- and 45-degree impact angles. The results from a parametric study are used to identify the critical guard height resulting in severe cabin deceleration and compartment intrusion of the small car. This study quantifies the vulnerability of car occupants in truck side underride crashes and the effectiveness of a side guard at different ground heights in reducing the injury potential to car occupants.

**Acronyms and Nomenclature**

- $h_g$: Side Guard Height (mm)
- $h_f$: Side Main Frame Height (mm)
- $\Theta$: Angle of Impact (Degree)
- $PCI$: Passenger Compartment Intrusion (mm)
- $NPCI_w$: Weighted Normalized Passenger Compartment Intrusion
- $\delta_{s_b}$: Steering Wheel Backward Displacement (mm)
- $\delta_{s_u}$: Steering Wheel Upward Displacement (mm)
- $\delta_{a_b}$: Closing Distance between A- and B-Pillars (mm)
- $\delta_{i_b}$: Instrument Panel Backward Displacement (mm)
- $\delta_f$: Footwell Backward Resultant Displacement (mm)
- $\alpha$: Weight Factor for Steering Wheel Longitudinal Displacement in $NPCI_w$
- $\beta$: Weight Factor for Steering Wheel Vertical Displacement in $NPCI_w$
- $\eta$: Weight Factor for A-Pillar Longitudinal Displacement in $NPCI_w$
- $\gamma$: Weight Factor for Instrument Panel Longitudinal Displacement in $NPCI_w$
- $\lambda$: Weight Factor for the Footwell Longitudinal Displacement in $NPCI_w$
- $NPCI$: Normalized Passenger Compartment Intrusion
- $A_p$: Passenger Transmitted Acceleration (G)
4.2 Introduction and Background

According to the fatality analysis reporting system (FARS) database, nearly 70,000 large trucks were involved in fatal injury crashes in the United States. In 2008, accidents initiated at the sides of large trucks accounted for 17% of fatal crashes, compared to 19% of accidents initiated at the rear of large trucks [1]. Due to a relatively large mismatch of the size and mass of the large trucks with other on-road vehicles, most cars impacting the side or rear of large trucks would be associated with the so-called underride accidents [2].

When a large truck or trailer makes a lane change on a highway, an adjacent car may not be readily perceived by the truck driver, and the car can be trapped in the long open side of the trailer and crushed by the trailer’s rear wheels. In addition, the trailer’s side structure can crush into the car’s roof structure. Underride accidents have also occurred when a tractor trailer makes a turn at an intersection or pulls out onto the highway in front of oncoming traffic [3]. Figure 4.1 shows a typical side-underride crash of a small car into a large truck or trailer, for which significant intrusion of the passenger car due to underride can be observed.
Figure 4.1. Angular Side Underride Impact of Passenger Car and Large Truck  
(Photo Courtesy of NHTSA)

Truck underride protection was the longest-standing unresolved regulatory issue for the National Highway Traffic Safety Administration (NHTSA). In the U.S., underride guards are required only for the rear of large trucks, although the review of fatal truck-car crashes in NCSS/CDS accident databases indicates that fatal underride crashes involving the sides of large trucks are almost as common as fatal rear underride crashes [4]. To reduce the impact severity of rear underride crashes with passenger vehicles, the NHTSA requires that trailers manufactured after January 1998, with a gross vehicle weight rating of 10,000 pounds and above, have rear-impact guards [5, 6]. This safety standard prevents the underriding of smaller vehicles during rear-end collisions with large trucks. The post-1998 guard specifications have done much to deal with the intrusion issues, but some concern still exists, especially for passenger vehicles in the subcompact class [3, 7, 8, and 9]. The U.K. and Japan mandate side under-ride protective devices designed to prevent pedestrians and motorcyclists from going underneath the large trucks, but a stronger design will be needed to prevent automobiles from underriding the sides of large trucks [8, 10]. On the other hand, the trucking industry has argued against the lower height of the side guards, which might cause a problem of maneuverability of the trucks during the off-road conditions. The industry is asking for extra ground clearance.
The prevention of passenger compartment intrusion is clearly the primary purpose of having an underride guard. However, the acceleration transmitted to the driver and passenger of a small car due to impact of a lower side guard of a truck with a car’s main frame is also important. The crash response of a car impacting the lower side guard of a truck tends to be similar to the crash response of a car impacting a rigid barrier.

In a collision involving a large truck and a smaller vehicle, the latter is likely to move underneath the truck. Direct contact to the A-pillars, windshield, and roof of the car, referred to as a “greenhouse impact,” is typical of passenger vehicles contacting the side of a truck or semi-trailer, or rear of a semi-trailer with inadequate rear protection or significant offset [3]. In the case of underride, the stiff truck bed may directly hit the occupants of small passenger cars at the head and chest level by breaking the windshield. In these cases, passive safety devices such as airbags and seat belts, and the energy absorption capability of the car may not work properly because parts of both the car and the truck often intrude into the smaller vehicle passenger compartment [3, 11]. Common injuries that occur due to underride will be severe brain trauma, extensive facial fractures, loss of eyes, neck and chest injuries, and even decapitation of the driver or occupants [12, 13]. It would appear that equipping such heavy vehicles with rear and side underride guards would result in a reduction of the number of fatalities and the severity of injuries.

Trego et al. performed a series of underride impacts of passenger cars with the side of a semi-trailer [14]. The primary utility of this study was the development of a relation for the roof structure damage to impact velocity, referred to as the Damage Index Matrix (DIM). The passenger compartment deformation and the driver-transmitted acceleration are significant factors affecting occupant injuries and consequently the appropriate triage. It would be
particularly useful to know the type of car and height of the underride guard of the truck involved in the crash, thus allowing the hospital to organize the appropriate rescue teams, plan triage in advance, and provide emergency medical personnel with all the necessary information before their arrival at the scene of the accident [15].

Currently there is no generally accepted methodology to document and scientifically reconstruct a side underride accident to demonstrate the effect of the different geometries of the side guard on the injury potential of impacted car occupants.

4.3 Methodology

The primary objective of this study is to obtain a relation between the geometric characteristics of the side guard of a large truck with the passenger compartment intrusion and the transmitted acceleration to a small vehicle during side underride impact crashes.

A validated finite element model of a small-size car (Geo Metro) and a mid-size car (Ford Taurus) are used as passenger cars impacting a rigid-side guard of a large truck (Ford F800) [16]. It is assumed that the crash behavior of these cars represents the underride behavior of the same class of cars. To show the effect of installation of the side guard, a rigid guard structure is attached to the truck side/rear at different ground heights. Figure 4.2 depicts a typical side guard attached to a truck or trailer at a height of hg. The impact of a small car to the truck side occurs at the angle $\theta$, as shown in Figure 4.2.
In any frontal crash event, the front part of the impacting vehicle comes to a halt, while the remainder of the vehicle continues to undergo high deceleration. Substantial compression contact forces are generated between the front and rear regions of the vehicle at this time. For commercial vehicles and four-wheel drives, the proportion of mass in the rear is usually greater than that of passenger cars, particularly when laden. This, combined with a stiffer front structure, can place a quite severe demand on the structure of the passenger compartment [17]. The passenger compartment intrusion (PCI) and the driver transmitted acceleration are possible factors affecting the occupant/driver sustained injury level in all impact scenarios, practically in side underride impacts. A large PCI will cause contact injuries to the face, head, neck, and chest, or even decapitation of the driver of the light vehicle, and the acceleration input will cause inertial injuries to the brain, head, or neck injuries [3, 12, and 13].

The passenger compartment should keep its shape in the event of a crash. According to the Insurance Institute for Highway Safety (IIHS), the PCI is measured at different parts which are crucial in determining the injury to vehicle occupants [18, 19]. Figure 4.3 shows those crucial points at which intrusion needs to be measured. The steering column, instrument panel, roof, roof pillars, pedals, and floor panels should not be deformed excessively inwards, where they are more likely to injure the occupants. Measurements used by the IIHS represent the residual
movement (pre-crash/post-crash difference) of interior structures in front of the driver dummy. The movement of seven points on the vehicle interior plus the closing of the distance between the A- and B-pillars are the foundations of the institute’s structural ratings. Due to large diversity and level of importance of these variables in frontal and offset impacts, five different parameters are used to demonstrate the PCI of the impacting cars in this study: steering wheel rearward and backward movement, A-pillar rearward movement, instrument panel rearward movement, and footwell intrusion, which includes the maximum backward resultant movement of the footrest, brake pedal, and left and right toe pan areas [19].

A weighted normalized passenger compartment intrusion \( NPCI_w \) is defined as

\[
NPCI_w = \alpha \frac{\delta_{sx}}{\delta_{sx0}} + \beta \frac{\delta_{sz}}{\delta_{sz0}} + \eta \frac{\delta_{Ax}}{\delta_{Ax0}} + \gamma \frac{\delta_{Ix}}{\delta_{Ix0}} + \lambda \frac{\delta_{Fx}}{\delta_{Fx0}} \tag{4.1}
\]

where \( \alpha, \beta, \eta, \gamma, \) and \( \lambda \) are the weight factors of each parameter affecting the \( NPCI \), so that \( \alpha + \beta + \eta + \gamma + \lambda = 1 \); \( \delta_{sx} \) and \( \delta_{sz} \) are the steering wheel backward and upward displacements respectively; \( \delta_{Ax} \) is the closing distance between the A- and B-pillars; \( \delta_{Ix} \) is the instrument panel backward displacement; and \( \delta_{Fx} \) is the footwell backward resultant displacement. The pre-crash and post-crash locations of these points are measured with respect to a coordinate system originating on the passenger-side B-pillar. The measured movement of the interior seven points is adjusted to reflect movement toward the driver seat, which is represented by the locations of its attachment to the vehicle floor. Thus, movement of the driver seat with respect to the reference coordinate system is not reflected in evaluations of vehicle structure. Also the forward movement of the driver due to the impact deceleration of the car is not considered in this study. It can be assumed that the driver is utilizing the appropriate seat belt system to prevent any forward movement. The denominator values are defined as basic values of each parameter according to IIHS and the European Economic Community (EEC) \([19, 20]\). Figure 4.4 shows the
IIHS recommended ranges for these measurements and associated structural ratings. The upper bound of acceptable ranging is considered in this study as the allowable values. According to the European regulations, the allowable values for the steering wheel displacements are 127 mm upward and backward displacement of the steering wheel.

Figure 4.3. Passenger Compartment Intrusion Measured at Different Locations Inside Car [Photo Courtesy of NHTSA]

Figure 4.4. IIHS Guidelines for Rating Occupant Compartment Intrusion (cm) [19]

The effect of passenger compartment intrusion on the injury of the driver and occupant has been addressed in the literature [13, 21], but no study has identified the importance of each PCI variable in a frontal crash scenario. Stefanopoulos et al. [15] conducted research on the relationship between the intrusions of the passenger compartment with the injury severity of the
occupants. However, the different areas of intrusion in the crash were not discretized, and the amount of the intrusion was not specified. Evans et al. [21] developed a relationship between the PCI and injury level: abbreviated injury scale (AIS) and injury severity score (ISS); however, neither the type of crash nor the car regions contributing to the PCI was identified. In that study, the PCI was also reported as a number not showing the intrusion of the specific part of the car. Since the weight factors for each PCI parameter on occupant/driver injury level have not been studied, all parameters are assumed here to have the same level of importance. Equation (1) could then be written as

$$NPCI = 0.2\left( \frac{\delta_{sx}}{\delta_{sx0}} + \frac{\delta_{sz}}{\delta_{sz0}} + \frac{\delta_{Ax}}{\delta_{Ax0}} + \frac{\delta_{Ix}}{\delta_{Ix0}} + \frac{\delta_{Fx}}{\delta_{Fx0}} \right)$$  \hspace{1cm} (4.2)

In equation (4.2), when $NPCI$ is equal to zero, no intrusion at any specified location occurs, and when $NPCI$ is equal to unity, the normalized passenger compartment intrusion values are in the boundary of the acceptable-marginal rating. The value for $NPCI$ can be any real number, depending on the displacements of the specified locations.

The acceleration level sustained by the driver can be shown as proportional to the total acceleration applied to the vehicle center of gravity (CG) for the same scenario:

$$A_p = \kappa A_{VCG}$$  \hspace{1cm} (4.3)

where $A_p$ is the magnitude of the passenger-transmitted acceleration, $A_{VCG}$ is the magnitude of the vehicle center of gravity acceleration, and $\kappa$ is the proportionality coefficient. For consistency in all impact scenarios, the frontal impact of a small-size car with a rigid wall at the speed of 30 mph is defined as a reference test, according to the EEC for steering wheel protection in a frontal crash [20]. By dividing $A_p$ into the acceleration of the same point at the reference test, the normalized acceleration of car occupant can be obtained as
In equation (4.4), when the acceleration level is equal to that of the reference test, the normalized acceleration $NA_p$ would be unity.

As mentioned previously, the injury potential of the driver/occupant of the underriding car depends on both the $NP CI$ and the acceleration applied to the driver/occupant:

$$IP_r = \mu \cdot NP CI + \rho \cdot NA_p$$

(4.5)

where $\mu$ and $\rho$ are the weight factors for each parameter, and $IP_r$ is the relative injury potential level. The term “relative” is used here because the injury potential is compared to the reference case, and then $IP_r$ for the reference case is considered to be 1. Here, it is assumed that the normalized acceleration has the same weight factor as the intrusion parameters, and hence, the injury potential can be rewritten as

$$IP_r = 1/6(NP CI + NA_p)$$

(4.6)

In equation (4.6), when $IP_r$ is equal to 1, the relative injury level from the normalized acceleration and intrusion is equal to the mentioned allowable values.

Accident data and case evaluations indicate that the vast majority of truck underrides occur in the impact speed range of 30 to 50 mph [3]. Side-crash simulations are conducted for different side guard heights from the ground at different impact speeds of 30 and 50 mph at direct frontal and oblique impacts. Due to direct contact and interaction of the side guard with the small-size car, the main objective of this study is to evaluate the effect of side guard height on the occupant injury potential of the a small-size car. However, for comparison purposes, the simulations are repeated for the same impact scenarios for a mid-size passenger car as well. A truck with guard heights of 18 in (457.2 mm), 20 in (508 mm), 22 in (558.8 mm), 24 in (609.6 mm), 26 in (660.4 mm), 32 in (812.8 mm), and 39 in (1000 mm), and also a truck without a side
guard are investigated in this study. Impact angles of 90 degrees (12 o’clock) and 45 degrees are both considered. Finally, the $NPCI$ and $IP_r$ are evaluated for each case, and the best design for the truck side guard height is recommended.

4.4 Simulation Results and Discussion

4.4.1 Model Validation

A validated finite element model of a small-size car (Geo Metro) and a mid-size car (Ford Taurus) are used as passenger cars impacting the rigid side guard of a large truck (Ford F800) [16]. To validate the guard-impact scenario, the simulation of a mid-size car impacting a rigid guard is validated against the NHTSA Ford Taurus underride test [22]. LS-DYNA- 971 has been utilized in this study for constructing all small-car/large-truck side-impact scenarios. Figure 4.5 shows a comparison of the overall deformation of the mid-size car front area from this study and the NHTSA test at an impact speed of 10 mph.

![Simulation from this study](image1) ![NHTSA test [20]](image2)

Figure 4.5. Mid-Size Vehicle After-Impact Deformation from This Study (left) and Mid-Size Vehicle Impact with Rigid Guard at Impact Speed of 10 mph from NHTSA Report [20] (right)

Figure 4.6 (a) shows the vehicle center of gravity transmitted longitudinal acceleration generated by the car model employed in this study compared with that of the NHTSA report at an impact speed of 10 mph. A fairly acceptable agreement can be seen in gross pattern and peak values. Simulations of impacts at higher speeds were also conducted. The impact tests of NHTSA at high speeds, however, were cumulative and not comparable with an intact vehicle
structure. Nonetheless, Figure 4.6 (b) shows a reasonable profile correlation between the resultant transmitted CG acceleration of the car at impact speed of 34.7 mph from this study and from the NHTSA test.

![Graph showing acceleration profiles](image)

Figure 4.6. Mid-Size Vehicle Center of Gravity Transmitted X-Axis Acceleration (Left) and Resultant Acceleration (Right) in G at the Point of Impact with a Rigid Guard at Speed of 10 mph and 34.7 mph, Respectively, from This Study and NHTSA Test [20]

### 4.4.2 Parametric Study

Validated finite element models of the small car and truck are utilized in this study from the National Crash Analysis Center (NCAC) [16]. The passenger car impact simulations with a large truck side augmented with a rigid side guard at various heights from the ground are conducted using LS-DYNA. Two different impact configurations, the 90-degrees impact, or 12 o’clock impact, and the 45-degrees impact, are conducted in this study. The underriding of the small car depended on the geometric characteristics of both the front of the small car and the side guard of the truck. Figures 4.7 and 4.8 depict the after-impact configurations of the small-size car impacting at a 90-degree impact angle with the truck side guard at different heights from the ground at 50 mph and 30 mph, respectively. It can be seen that the $IP_r$ is not linearly proportional to the impact speed. Figure 4.9 shows the after-impact configurations and related $NPCI$, $NA_p$, and $IP_r$ for the small-size passenger car impacting the truck-side guard at different heights from the ground at a low-impact speed and an impact angle of 45 degrees. The sliding of
the angular impacted car along the truck’s side guard can be seen in Figure 4.9 (a, b) as well as rotation of the car during the angular impact in Figure 4.9 (c).

Figure 4.7. Small Car Impacted with Truck with and without Side Guard at Impact Speed of 50 mph and Impact Angle of 90 Degrees

Figure 4.8. Small Car Impacted with Truck with and without Side Guard at Impact Speed of 30 mph and Impact Angle of 90 Degrees

Figure 4.9. Small Car Impacted with Truck with and without Side Guard at Impact Speed of 30 mph and Impact Angle of 45 Degrees
For each impact scenario, the passenger compartment intrusions at the specified locations are measured, and the normalized PCI are derived according to equation (4.2). Figures 4.10 and 4.11 show the PCI of a small-size car impact with a large-truck side guard at different ground heights for a 90-degree impact at 50 and 30 mph, respectively. Figure 4.12 shows the $NPCI$ for low- and high-speed car impacts at different impact angles for all side guard heights.

**Figure 4.10.** Small-Size Car Passenger Compartment Intrusions at Impact Speed of 50 mph and Impact Angle of 90 Degrees

**Figure 4.11.** Small-Size Car Passenger Compartment Intrusions at Impact Speed of 30 mph and Impact Angle of 90 Degrees
Figure 4.12. Small-Size Car NPCI for Low- and High-Speed Impacts

Since the stiffness of the vehicle structure is not identical in different directions, the occupant transmitted acceleration is not the same for different impact angles. The energy absorption capability of cars is, in general, higher for frontal impacts compared to other directional impacts, because the volume available for impact energy dissipation is larger for frontal impacts. Typically, the bending stiffness of the transverse links and structures are lower in frontal impacts than in directional impacts. Due to higher stiffness of an angled crash of a small car, the acceleration level for the angular impact is relatively higher than that of the frontal impact. Figure 4.13 shows the transmitted acceleration of the small car center of gravity for frontal impact at high- and low-speed impacts. However, in all impacts with the same speed, the CG acceleration decreases with increasing guard height until the front part of the small car makes contact with the underbelly rigid parts of the large truck or trailer, which causes a slight increase in acceleration level. The impact energy during a low-speed impact is absorbed by the car’s front deformable parts, such as the bumper, radiator, or hood. Finally, the $IP_r$ from equation (4.6) is shown in Figure 4.14 for all small-car/heavy-truck side impacts for straight and directional impacts at both low- and high-speed impacts for different heights of truck side guards.
To illustrate the efficiency of the side guard for all sizes of cars, Figure 4.15 depicts the high-speed impact of a mid-size passenger car with the side of a large truck having side guards 22 in (558.8 mm) and 32 in (812.8 mm) from the ground as well as having no side guard. The values of the related $NPCI$ and $NA_p$ and $IP_r$ are identified at the small car reference system. Since the front area of the mid-size car is larger in size and stiffer than that of the small-size car, the intrusion experienced by the mid-size car is smaller, but the transmitted acceleration to the occupant is larger than that of a small-size car, contributing to the same impact scenario. Figures 4.16 and 4.17 show the $NPCI$ and occupant relative injury potential of a mid-size car impacting a large truck at both low and high speeds, respectively. Note that the small-size car reference test data is considered as a reference for the mid-size car as well.
The impact angle between the underriding vehicle and the truck or trailer affects the pattern of the damage and the way energy is dissipated during a collision. For impacts other than 90 degrees, the vehicle will be in contact with the trailer over a larger distance and slide along the side of the trailer, as shown in Figure 4.9 (a, b). This means that the underriding vehicle will have a greater opportunity of contacting the underbelly structures of the truck, such as the...
dollies, spare tires and/or hangers, toolboxes, and tractor or trailer tires. Also a significant amount of energy may be dissipated as the vehicle moves and slides along the side guard. Depending on the angle of impact, there may be a greater likelihood that the underriding vehicle will rotate as shown in Figure 4.9 (c).

The transmitted acceleration of the small-car center of gravity depends on the directional stiffness of the impact and the mass of both the small car and truck or trailer as well. It is observed that for identical mass and dimensions of the car and truck, the angular impacts result in relatively larger acceleration to the small car than the 90-degree impact. However, the sliding of the small car along the truck side guard slightly attenuates this acceleration. For impacts other than 12 o’clock, the stiffness properties and energy-absorption capabilities of the underriding car parts differ from that of the frontal impact for which the car mainly has been designed. By increasing the guard height from the ground, the acceleration applied to the small car is mitigated due to less probability of impact of the rigid guard with the underriding car’s stiff components such as the subframe, suspension, and engine. For a side guard height larger than 30 inches from the ground, although the rigid guard makes an impact with less stiff parts of the small car, mostly the hood and A-pillars, the impact of the car’s frontal parts occurs with the underbelly rigid parts of the large truck or trailer. Hence, the acceleration level slightly increases for the high guard cases but is still less than that of low height guard. As the size of the impacting car increases, the transmitted acceleration increases, and the level of PCI decreases quite rapidly. As a result, the relative occupant injury potential decreases, although non-linearly.

Sliding of the small car along the side guard of the truck for angular impacts decreases the PCI, compared with the 12 o’clock impact. As the height of the guard increases and underriding of the small car takes place, the PCI increases quickly due to more underriding
probability compared with the 90-degree impact. For directional and 12 o’clock impacts involving a side guard height of about 18 in (457.2 mm), no small-car underride occurs. Starting from a side guard height of 22 in (558.8 mm), a slight side underriding of the small car can be observed. The significant underriding of the small car starts at a side guard height of 32 in (812.8 mm), in which the full front hood area of the car penetrates underneath of the large truck. As a result, the relative injury potential of the occupants increases up to about 250 percent. For the 90-degree impact of a small car and truck with no guard or a large guard height, the impact of the small-car parts with various parts of the truck at the same time and at different locations causes the low acceleration and PCI in the small car. However, based on Figures 4.7, 4.8, 4.9, and 4.15, the underriding of the small car can take place. This might lead to a disaster in the case where the truck has a relative longitudinal impact velocity. Hence, these cases are not desired in the design of the truck and trailer side guard. This is shown in Figures 4.10 to 4.14, and Figures 4.16 and 4.17, by the fading red color starting at the side guard height of 30 in (762 mm).

For low-velocity impacts, the impact of the truck’s rigid guard occurs with the outer body parts, and due to less impact energy, the deformation of the outer body will absorb all the impact energy. This will avoid the impact of the relatively rigid chassis, suspension, or engine components of the underriding car with the rigid guard.

Considering the side guard height of above 30 in (762 mm) as underriding cases for the small car, it is observed that the side guard design region will be limited to the maximum guard height of 30 in (762 mm). On the other hand, due to truck manufacturers’ and truck users’ objections related to less maneuverability of the truck for lower side guards, the design region of the side guard is suggested to be between the heights of 20 and 23 in (508 and 584 mm) from the
ground. This will prevent the underriding of the small car and keep the occupant injury potential to a minimum.

4.5 Conclusions

A methodology using finite element modeling of a small car (bullet vehicle) and a large truck (target vehicle) was utilized in this study to quantify the influence of the side guard attached to the truck in reducing the intrusion of the car and thus reducing any injuries sustained by the occupants of the car in side-impact crashes. The car occupant cabin deceleration severity and the occupant compartment intrusion were compared for a small car when impacting a rigid underride guard to prevent the underride. A formulation based on passenger compartment intrusion and its relation to occupant injury potential was proposed in this paper. The formulation was expanded to predict the injury potential of a driver/occupant of the underriding car based on both the normalized passenger compartment intrusion and the CG acceleration of the car. Using this paper’s methodology, the aggressiveness of the open side of trucks and trailers has been demonstrated and quantified in a scientific manner. It was shown that severe injury would be sustained by the driver or occupants of a small car impacting the side of a large truck or trailer. In the process, an NHTSA underride test was simulated for model validation. The IIHS guidelines for rating the occupant compartment intrusions were used in this study.

One observation from this study was that the size of the impacting car poses a significant effect on passenger compartment intrusion as well as injury potential to the occupants. The smaller the size of the impacting car, the lower the required height of the side guard to keep the impact in the safe region. For a small-size car, the addition of the side guard reduces the probability of severe injury of occupants by about 250 percent compared to the no-side guard configuration. Furthermore, it prevents the underriding probability of the small car, which
increases the injury potential catastrophically in the case of a moving truck or trailer. As long as the maneuverability of the large truck and trailer is in the desirable range, any design between the heights of 20 and 23 in (508 and 584 mm) seems to correspond to the lowest injury potential to the small-car occupants.

Further research needs to be conducted on the weighting factors for the accelerations sustained by the occupants and the passenger compartment intrusion in a frontal-impact scenario. It is also recommended that a study be conducted on the correlation between the IPr and the related injury severity score or abbreviation injury scale. It would also be beneficial to investigate the allowable limits for each intrusion value in order to standardize the limits and unify the method of approach for these types of studies, so that the base values for the injury thresholds for specific AIS values can be determined.

4.6 References


CHAPTER FIVE

CASE STUDY TWO: APPLICATION TO CRASH ENERGY MANAGEMENT AND IMPULSE MITIGATION—LUMBAR LOAD ATTENUATION FOR ROTORCRAFT OCCUPANTS USING A DESIGN METHODOLOGY FOR SEAT IMPACT ENERGY-ABSORBING SYSTEM

5.1 Abstract

Aircraft occupant crash-safety considerations require a minimum cushion thickness to limit the relative vertical motion of the seat-pelvis during high vertical impact loadings in crash landings or accidents. In military aircraft and helicopter seat design, due to the potential for high vertical accelerations in crash scenarios, the seat system must be provided with an energy absorber to attenuate the acceleration level sustained by the occupants. Because of the limited stroke available for the seat structure, the design of the energy absorber becomes a trade-off problem between minimizing the stroke and maximizing the energy absorption. The available stroke must be used to prevent bottoming out of the seat as well as to absorb maximum impact energy to protect the occupant. In this study, the energy-absorbing system in a rotorcraft seat design is investigated using a mathematical model of the occupant/seat system. Impact theories between interconnected bodies in multibody mechanical systems are utilized to study the impact between the seat pan and the occupant. Experimental responses of the seat system and the occupant are utilized to validate the results from this study for civil and military helicopters according to FAR 23 and 25 and MIL-S-58095 requirements. A model for the load limiter is

1 This entire chapter has been submitted for publication to the following source:

proposed to minimize the lumbar load for the occupant by minimizing the relative velocity between the seat pan and the occupant’s pelvis. The modified energy absorber/load limiter is then implemented for the seat structure so that it absorbs the energy of impact in an effective manner and below the tolerable limit for the occupant in a minimum stroke. Results show that for a designed stroke, the level of occupant lumbar spine injury would be significantly attenuated using this modified energy-absorber system.

5.2 Introduction

The safety of occupants in aircraft and rotorcraft during any survivable crash has been a primary concern of the aviation industry for many years [1-3]. The need for improved crashworthy seats in aircraft was initially established by the Aviation Crash Injury Research (AvCIR) Division of the Flight Safety Foundation during the late 1950s [4]. A limiting load imparted from the seat was necessary to improve the chance of survival and to minimize spinal fracture and risk of paraplegia. The major mode of injury in aircraft occupants as a result of vertical loading is in their lumbar or pelvic region [5]. Chandler [6] and Shanahan and Shanahan [7] developed a 6675 N (1500 lbf) lumbar load injury criterion. This criterion is now incorporated into FAR Parts 23, 25, and 27 for rotary- and fixed-wing civil aircraft [8, 9, 10]. The military specification originally defined a spinal injury criterion in the form of seat pan acceleration, called a “load factor.” Later, it became apparent that the seat pan acceleration criterion was potentially flawed as a reliable predictor of compressive spinal injury risk. Military seating specifications continue to use the measurement of seat pan acceleration instead of lumbar spine loading criteria [10], although it is generally accepted that tolerable lumbar loads for military aircraft applications are in the range of 8000–9790 N (1,800–2,200 lbf).
It is clear that a crashworthy seat shall be utilized only if it is comfortable, which has made the task of designing seats more difficult [5]. In the past, this has been accomplished by utilizing nets or extremely thick, soft cushions. It was found that low spring rates of soft cushions allow large relative velocities to build up between the occupant and the seat pan during the imposition of impulsive loads, thus increasing the lumbar/pelvis load to the occupant. According to MIL-S-58095, the total thickness of the compressed cushion at the buttocks reference point should be minimized to 1/2–3/4 in (13–19 mm) at 1 G load [11, 12]. Payne [13] and Stech and Payne [14] demonstrated the increase in potential injury associated with the amplification effect of seat cushions on the dynamic response index (DRI), using a general model employing the single mass-spring-damper system to simulate the biomechanical response of the human body. Therefore, thoughtful integration of several mechanical components is required to produce an effective crashworthy energy-absorbing seat design.

In general, characteristics of the contact between an occupant and a seat determine whether there is significant dynamic overshoot within the body. While a slow rate of onset allows the body to come to the equilibrium-deformed configuration gradually, a rapid rate of onset effectively results in an impact between the seat and the body, creating high transient stresses and severe injuries. The soft cushion creates a plateau region in the load-deflection curve, which causes a multibody system to behave as if this region is a clearance between the seat pan and occupant. It is known that the existence of clearance in joints leads to load amplification, where the magnitude rises in proportion to the amount of clearance [15]. A dynamic compression index (DCI) was defined and utilized [16, 17] as an inexpensive and time-conserving methodology for replacing seat cushions for aircraft seats. The DCI is the cushion compression distance available to the occupant before the cushion bottoms out during impact. A
further study indicated that an increase in the DCI increased the lumbar load in a vertical impact crash scenario [18].

The energy-absorption capability of a seat structure must be considered in evaluating the dynamic strength of the seat. For high-peak impulses, to reduce the impact between the seat pan and occupant, the seat structure must possess sufficient energy-absorption capacity to reduce the occupant’s relative velocity before it bottoms out. This seat structure capability is referred to as “load limit.” The seat structure begins plastic deformation when the acceleration or load experienced by the occupant and seat reaches a level corresponding to the critical load limit, which corresponds to the human tolerable limit to provide the intended protective function. Attempts have been made to design seats that absorb impact energy for occupant protection without the use of an external energy absorber attached to the seat [19]. One such crew seat, shown in Figure 5.1 (a), uses S-shaped tubular steel front legs designed to form plastic hinges to limit the load and provide energy absorption [20]. Although cost efficient, these seats are inefficient energy absorbers, and their performance is dependent upon the direction of impact. Such a concept has not been used in the design of successful military crew seats; it has only been used in general aviation and transport aircraft seats.

Figure 5.1. (a) Crew Seat with Energy-Absorbing Legs, (b) Cockpit Seat with Monolithic Bucket

[4]
As seat technology was developing, many energy-absorbing mechanisms or concepts were proposed and adopted in operational systems. A desirable energy absorber should be as light and as small as possible with high specific energy absorption. The system should stroke at a constant force, resist loads in the opposite direction to the stroking (rebound), and be rate insensitive [12].

Today, most military aviation aircraft and rotorcraft are equipped with crashworthy seat systems, such as the one shown in Figure 5.1 (b). Most of these seats use an energy-absorbing device, referred to as a fixed-load energy absorber (FLEA), that applies a constant load to decelerate the occupant. A manually adjustable or variable-load energy absorber (VLEA) was developed to produce the same level of protection for all sizes of occupants [4]. Later on, the variable-profile energy absorber (VPEA) seat structure was developed to take advantage of the dynamic response of the occupant. The ultimate design is an automatic energy absorber (AEA), which provides the maximum amount of protection to a larger range of occupants [4].

It was determined in the early 1970s that the limit load or total dynamic stroking load [22, 23] of an energy-absorbing system had to be set to a lower value (18 G) than originally thought. This was based on the observation of seat performance relative to tolerance data assembled by Eiband [24], in order to account for dynamic overshoot and to keep the load-duration environment within the human tolerable range [4]. Following extensive tests, it was recommended that the load factor should be retained at 14.5 G for U.S. army aircraft seats, and between 11 G and 13 G for commercial and light aircraft, by using the energy-absorber system to protect the spine [21-23, 25]. The 14.5 G design criterion considers the dynamic response of the seat and occupant. During testing, the standard cushion used in a UH-60A crew-seat bucket was utilized. The load factor of 14.5 G was defined based on the maximum load sustained by the occupant, which was influenced by the cushion’s characteristics. If a rigid seat had been used in
the tests, the load factor could have been higher, due to elimination of the cushion compliance, and the occupant pelvic compliance was the only contributing factor to the load limit.

Reducing the energy-absorber load limit to a low value to prevent decelerative spinal injuries may actually increase the rate of spinal injury, because in a greater number of accidents, the seat bottoms out, increasing the likelihood of spinal injury [4]. The desirability of certified seats with the shortest possible stroke is a concern, and encourages seat suppliers to be innovative and develop seats with energy-absorbing systems that require short strokes [4].

In a study by Carr and Phillips, it was concluded that an energy absorber could be designed to take advantage of the dynamic response of the human body [26]. A notched load-stroke profile was suggested in order to compress the springs in the human body more quickly by imposing a high initial load spike. Then it would lower the load rapidly to minimize the occupant’s spine load overshoot as the body springs loaded up and bottomed out. The energy-absorbing load would then be increased again slowly to reach a plateau, which could be sustained with the body springs loaded and compressed for the rest of the stroke. The characteristics of this variable-profile energy absorber (VPEA) are shown in Figure 5.2.

![Figure 5.2. Stroke Characteristics of Variable-Profile Energy Absorber (VPEA) [4]](image)

If the energy-absorbing system is to provide only one load setting, then that load should be sized for the effective weight of 50th-percentile occupants to ensure a tolerable stroke for the
majority of occupants while not exceeding the stroke limitations of the seats [4]. Then 50th-percentile occupants would be the only occupants decelerated at the load limit selected as the tolerable limit. Lighter occupants would be decelerated at a higher rate, and heavier occupants would be decelerated a lower rate than would the 50th-percentile occupants.

Crew seats should be designed to stroke a minimum distance of 30 cm (12 in) when they are in the lowest position of the adjustment range [4], in order to absorb the residual energy associated with the vertical design pulse. Even with a 30 cm stroke, heavier occupants in more severe impact crashes will exhaust the available stroke distance and bottom out.

As observed, a number of studies have addressed the design of the energy-absorbing system for rotorcraft seats. The objective of the present study is to build on this knowledge and expand the development of different energy-absorbing systems for rotorcraft seats based on the examination of load-limit curves and corresponding strokes. This is done using a combination of simplified mathematical modeling of the seat energy-absorbing system and occupant, detailed biodynamic modeling of the seat cushion and occupant, experimental testing and model validation, and a parametric study to evaluate the different designs.

5.3 Methodology

In this study, based on the kinematics of a seat and its occupant during a vertical impact, first the seat stroke and stroking stop time are evaluated in terms of the energy-absorber load factor. Due to the existence of several variables in the system and the efficiency of the load limiter, the evaluated seat stroke might not correlate entirely with experimental tests but could still be a good factor in seat-structure design. Then, a computational model of an aircraft seat and occupant is developed in MADYMO, and a set of experiments are utilized to validate the model. An industry seat cushion is tested to obtain the load-deformation curve under a dynamic loading
condition. An FAA Hybrid-III 50th-percentile anthropomorphic test device (ATD) model is used as the occupant of a seat-system model. The cushion load-deformation results are implemented in the model, and the occupant/seat model is subjected to a vertical impact test condition to validate the computational model. The seat energy absorber is then augmented to the occupant/seat model and validated with experimental results under different test conditions [18, 27]. Different energy-absorber load-limiting characteristics are evaluated to identify the most suitable design concepts for the energy absorber. The energy-absorber design iterations are examined to maintain the occupant lumbar load below the tolerable limit and to maximize the seat stroke. Such a design would be a great benefit for military rotorcraft, which have limited room beneath the seats, in order to utilize the maximum energy-absorbing capability of the seat system. In this study, the design of the seat energy absorber is investigated based on a mid-size male aviator. However, the idea could be extended to all occupant sizes, and an appropriate energy absorber could be designed. Figure 5.3 outlines the methodology utilized to design the seat energy absorber in this study. The steps in the design methodology are described in detail in the next section.

5.3.1 Mathematical Evaluation of Load Limits

Impact loading is different from static loading, where a state of equilibrium is reached. Both objects involved in an impact can be subjected to elastic and/or plastic deformations [28]. During impact, some energy is stored through elastic strain, stress wave propagation, plastic deformation, material dampening, or other phenomenon such as sound or heat [29]. The impact energy that a component can elastically absorb is a function of its modulus of resilience times the components’ volume. The superior modulus of resilience is the reason that rubber is used as a shock-mitigation material.
In the design of a shock-mitigation system, the goal is to spread out the input impulse over time, thereby reducing its magnitude to acceptable levels for the system being protected. In other words, the input pulse is absorbed by the shock-arresting device, and the shock energy is released over a broader time, lowering the magnitude of the pulse to the protected system, as shown in Figure 5.4.
Figure 5.4. Typical Input/Output in Shock Mitigation System

From the magnitude and time duration of the input pulse, the change in velocity \( \Delta V \) of the impactor can be determined. The output response \( G_{\text{out}} \) from the shock-mitigation device should not exceed the fragility or tolerable limit of the component or system being protected. Then, the natural frequency \( f_n \), representing the inverse of the time frame for which the shock pulse will be released, can be determined as

\[
G_{\text{out}} = \frac{2\pi f_n \Delta V}{g} \tag{5.1}
\]

Based on the mass of the system to be protected \( m \) and the required frequency \( f_n \), a shock-mitigation material stiffness \( K \) can be selected as

\[
(2\pi f_n)^2 = \frac{K}{m} \tag{5.2}
\]

to reduce the shock pulse. The most efficient process for limiting loads is the one that absorbs or dissipates energy rather than the one that stores it [4]. Plastic deformation of material, primarily metals, results in a reasonably efficient energy-absorbing process. However, sizing components for plastic impacts are much more difficult. As a first-cut approximation in sizing a component for plastic impact, the sum of the crush force \( \bar{F} \) or moment \( \bar{M} \) over its plastic displacement \( X \) or \( \theta \) can be equated to the impact energy as

\[
\frac{1}{2} m \Delta V^2 = \frac{1}{2} m V^2 (1 - e^2) = \bar{F} X \text{ or } \bar{M} \theta \tag{5.3}
\]
where \( m \) is the mass of the impactor, and \( e \) is a coefficient of restitution representing the energy absorbed in the impact (\( 0 \leq e \leq 1 \)). As observed from equation (5.3), for the same absorbed energy, the larger the crush distance, the lower the average load on the mass.

Each seat system for military crew and troops as well as civil rotorcraft occupancy must successfully complete dynamic tests, or be demonstrated by rational analysis supported by dynamic tests, in accordance with specific conditions defined in MIL-S-58095(AV), MIL-S-85510(AS), and SAE AS8049, respectively [11, 30, 31]. Federal performance requirements for civil helicopters are established in the U.S. Code of Federal Regulations (CFR), Title 14, Parts 27 and 29 [9,10]. These tests must be conducted using an occupant simulated by an ATD, defined in CFR 49, Part 572, Subpart B, commonly known as a Hybrid-II 50th-percentile dummy, or an FAA-approved equivalent, such as the FAA Hybrid-III dummy, with a nominal weight of 77 kg (170 pounds) and seated in the normal upright position [33]. The seating configurations and deceleration-time pulse developed for the design and test of the seat system are shown in Figure 5.5, where \( t_r \) is the acceleration rise time, and \( G_p \) is the peak acceleration.

![Figure 5.5. Military and FAR Seat Dynamic Test Configurations and Acceleration Input](image)

The seat energy-absorbing stroke simply lengthens the stopping distance of the occupant. The seat may continue to stroke until the kinetic energy of the seat occupant has been exhausted.

By defining a time-step function at time \( t_r \) as
The input acceleration $\ddot{S}(t)$ and the corresponding velocity and displacement $\dot{S}(t)$ and $S(t)$ in the vertical (lumbar axis) direction can be calculated by direct integration of the acceleration function, respectively, as

\[
\ddot{S}(t) = -\left(\frac{G_p}{t_r} + 2\frac{G_p}{t_r} \frac{t - t_r}{t_r} + \frac{G_p}{2t_r} \frac{(t - t_r)^2}{t_r} \right)
\]

\[
\dot{S}(t) = V_0 - \frac{G_pl^2}{2t_r} + \frac{G_p}{t_r} \frac{t - t_r}{t_r} + \frac{G_p}{2t_r} \frac{(t - t_r)^2}{t_r}
\]

\[
S(t) = V_0t - \frac{G_pl^3}{6t_r} + \frac{G_p}{3t_r} \frac{(t - t_r)^3}{t_r} + \frac{G_p}{6t_r} \frac{(t - t_r)^2}{t_r}
\]

where $V_0$ is the system initial velocity. The equilibrium condition at time $2t_r$ yields

\[
V_0 = G_p t_r
\]

Then,

\[
S(2t_r) = G_p t_r^2
\]

For the seat/occupant system, due to the load-limiter effect at time $t_l$, the seat vertical acceleration $\ddot{S}_s(t)$ would be constant until the energy absorber stops stroking at time $t_f$. Using the same method as above, the seat acceleration, velocity, and displacement functions can be written, respectively, as

\[
\ddot{S}_s(t) = -\left(\frac{G_p}{t_r} - \frac{G_p}{t_r} \frac{t - t_l}{t_r} \right)
\]

\[
\dot{S}_s(t) = V_0 - \frac{G_pl^2}{2t_r} + \frac{G_p}{2t_r} \frac{t - t_l}{t_r} + G_l \frac{(t - t_f)}{t_r} + \frac{\dot{S}(t)}{t - t_f}
\]

\[
S_s(t) = V_0t - \frac{G_pl^3}{6t_r} + \frac{G_p}{3t_r} \frac{(t - t_l)^3}{t_r} + \frac{G_p}{6t_r} \frac{(t - t_l)^2}{t_r} + S(t) \frac{(t - t_f)}{2}
\]

For a seat/occupant effective mass of $M_{\text{eff}}$, the force applied to the seat, or the load-limiter load $F_s(t)$ or $F_{LL}(t)$, can be calculated as
\[ F_s(t) = F_{LL}(t) = M_{\text{eff}} \beta_s(t) \quad (5.13) \]

Figure 5.6 shows the acceleration, velocity, and displacement of both the floor and the occupant/seat pan in the presence of a load-limiting energy absorber for two cases: low and high load-limit factors. The area between the velocity-time graphs for the floor and seat pan for each case equals the energy-absorber stroke, which can be evaluated from the above equations for the cases \( t_f > 2t_r \) and \( t_f < 2t_r \), respectively.

Let \( \frac{G_l}{G_p} = \frac{t_f}{t_r} = K \). Then the determination of whether the stroking stop-time is greater or less than time \( 2t_r \) can be made by equating velocities for the floor and the seat pan at time \( t_f \) as

\[ t_f = 2.414 \sqrt{2 - K} t_r \quad (5.14) \]

Then, for \( K = 0.586 \), we obtain \( t_f = 2t_r \). For \( K \leq 0.586 \) or \( t_f > 2t_r \),

\[
\begin{align*}
S(t_f) &= G_p t_r^2 \\
S_s(t_f) &= V_0 t_f - \frac{G_p t_f^3}{6t_r} + \frac{G_p < t_f - t_t >^3}{6t_r} 
\end{align*}
\]

Subtracting distances for each case yields

\[ \Delta S = G_p t_r^2 \left( \frac{1}{2K} - \frac{K^3}{24} - 1 \right) \quad (5.15) \]

For \( K \geq 0.586 \) or \( t_f < 2t_r \),

\[
\begin{align*}
S(t_f) &= V_0 t_f - \frac{G_p t_f^3}{6t_r} + \frac{G_p < t_f - t_t >^3}{6t_r} \\
S_s(t_f) &= V_0 t_f - \frac{G_p t_f^3}{6t_r} + \frac{G_p < t_f - t_t >^3}{6t_r} 
\end{align*}
\]

Then,

\[ \Delta S = G_p t_r^2 (5.093 K^3 - 12.36 K^2 + 9.449 K - 2.178) \quad (5.16) \]

This has been illustrated in Figure 5.6. As shown, the occupant’s velocity is reduced at a lower rate than that of the seat frame, and the occupant has stroked at a larger distance than the floor.
Figure 5.6. (a) Deceleration, (b) Velocity, and (c) Displacement Versus Time for Airframe and Seat Pan for \( K < 0.586 \) and \( K > 0.586 \)

As the result of ignoring the importance of impact direction on the level of seat acceleration, the concept of the load-limit factor has created some confusion in stroke calculations. The load-limit factor is not necessarily the seat-acceleration level, unless the direction of the dynamic acceleration is horizontal. Examples of the vertical acceleration test and non-vertical acceleration test are provided here to show the differences in the calculation for \( G_L \). As an example, for the model subjected to acceleration according to the MIL-S-58095 vertical test \( (t_f = 0.027 \text{ sec}, \quad V_0 = 12.8 \text{ m/sec}(42 \text{ ft/sec}), \quad G_p = 48G) \) with a static load-limit factor of 14.5 G, the seat-base acceleration level and corresponding stroke can be evaluated, respectively, as

\[
F_{LL} - mG = mG_1; \quad \text{then} \quad G_1 = 13.5G
\]

\[
K = 0.281, \quad t_f = 2.73t_r, \quad \Delta S = 31.5 \text{ cm (12.4 in)}
\]

Calculation of the stroke for the seat acceleration limit of 14.5 G yields erroneous results as

\[
K = 0.302, \quad t_f = 2.685t_r, \quad \Delta S = 27.7 \text{ cm (10.9 in)}
\]

The preceding calculations are based on mathematics and could not be reached in a practical test. Usually, results from the above equations should be considered the minimum stroke distance required, and allowance for additional stroke should be provided. In a very rough calculation, using a simple linear dynamic model of the 50th-percentile occupant, application of the load
limit of 14.5 G results in a lumbar load of \( F_{\text{Lumb}} = M_2(G_1) = 4830 \text{ N} \), where \( M_1 = 43 \text{ kg} \) for the lower body part, and \( M_2 = 34 \text{ kg} \) for the upper body part [32].

Based on the principles of rigid-body dynamics, in order to protect a mid-size occupant spine for the axial load of less than 6675 N (1500 lbf), a static load-limit factor of 20 G can be applied to the system, which is still below the Eiband limit for tolerable acceleration. The maximum lumbar load at a load-limit factor of 14.5 G is far beyond the results of rigid-body dynamics. The compliance of the pelvis area along with that of the seat cushion initiates a relative velocity between the occupant and the seat pan. This relative velocity provides an impact between the seat bucket and occupant, and amplifies the inertial load applied from the accelerated floor (seat pan). Obviously, reducing the cushion compliance mitigates the overshoot, at the expense of reducing the comfort level. Elimination of the spine load overshoot is not possible due to a certain compliance interaction of the pelvis with the rigid seat. However, if the relative velocity is decreased before the cushion/pelvis bottoms out, then the overshoot would be reduced. Thus, the load limit could be increased to a higher level, still keeping the lumbar load at the required level. This could be achieved using a load limiter with a certain constant load level for a short period of time and then increasing it gradually to a higher level. The benefit of this load limiter would be to minimize the stroke required for a certain load-limit factor. In other words, with the same stroke designed for a seat, the occupant’s lumbar spine load level would be significantly attenuated by reducing the initial load-limit factor and increasing it to a higher level.

For purposes of the load-limit calculation, the effective weight in the vertical direction of a seated occupant is assumed to be approximately 80 percent of the occupant’s total weight, because the lower extremities are partially supported by the floor [29]. Assuming a 27 kg (60 lb)
movable seat mass (and portions of a crew’s clothing, helmet, and boots), the total effective masses that the load limit system must be designed for are 75.5 kg (166.7 lb), 88.8 kg (196 lb), and 104 kg (229.4 lb), for 5th-percentile, 50th-percentile, and 95th-percentile aviators, respectively. The load limit FLL for a 14.5 G load-limit factor is calculated as

\[ F_{LL} = G_L W_{eff} \]

A load limit of \( F_{LL} = 12642 \) N (2,842 lb) is calculated for a 50th-percentile male aviator. The load factors for the 95th-percentile and 5th-percentile aviators are then calculated as

\[ G_{L-95\text{th}} = \frac{F_{LL}}{W_{eff-95\text{th}}} = 12.4 \]

\[ G_{L-5\text{th}} = \frac{F_{LL}}{W_{eff-5\text{th}}} = 17 \]

This means that with the same load limit factor of 14.5 G, the heavier occupant experiences a lower lumbar load, and a smaller-size occupant sustains a higher lumbar load, provided that the required stroke is available for the heavier occupant.

5.3.2 Full ATD/Seat/Cushion Model Development and Validation

A MADYMO [34] model was developed and employed in this study to evaluate the lumbar load for the crash test. Planes and ellipsoids were used to create models of the seat pan, seat back, sled, feet steps, and seat cushion. The Hybrid-III 50th-percentile FAA ATD was utilized as the occupant. A five-point harness configuration, which is primarily utilized in the aviation industry for crew seats, was utilized to restrain the occupant on the seat. Figure 5.7 depicts the entire MADYMO model including the seat system, sled, and dummy created for this crash simulation. Effort was taken to create the model as comparable as possible to real-world sled tests. The required deceleration pulses were applied to the system using a translational joint motion in the vertical direction. The belt was allowed to slip on the dummy by considering a typical friction coefficient of 0.2 between each segment and the body. A typical friction
coefficient of 0.2 between the occupant and the seat back and seat cushion, and a value of 0.3 between the feet and the base was utilized in the model. For validation purposes, a 95th-percentile aviator male model was also developed using MADYScale [34].

![Figure 5.7. MADYMO Models of Seat, Cushion, ATD, and Seat Belt for Vertical Test Configuration for: (a) 50th Percentile Dummy and (b) 90th Percentile Dummy](image)

The compliance of the cushion was implemented by using the data obtained from an experimental test for a 5-in industry cushion, as shown in Figure 5.8, and also from the literature (Table 5.1) for a 1.56-in-thick layer of typical polyethylene slow-rebound foam [29]. Seating geometry was applied according to MIL-STD-1333A for 50th-and 95th-percentile male aviators [35]. The energy-absorber system was modeled using a translational joint with a suggested restraint between the floor and bucket back as the system load limiter. To reach the equilibrium condition, the model was run for a short period of time before application of the vertical impulse.

The model was validated for a rigid seat and a seat with an industry cushion of 5-in thickness under Part 25 and Part 23 dynamic Test-I conditions, respectively. The experimental test results conducted by the NIAR [18] are shown in Figure 5.9. The lumbar load responses of an occupant on a rigid metal seat and the seat covered by a low rate-sensitive industry cushion are shown in Figure 5.10.
Figure 5.8. Experimental Setup to Generate Load-Deflection Curve of an Industry Seat Cushion

TABLE 5.1 Typical Cushion Properties for Crew Seat Utilized in This Study [29]

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Deformation (in)</th>
<th>Unloading Rate (lb/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>89</td>
<td>0.1</td>
<td>800</td>
</tr>
<tr>
<td>625</td>
<td>0.7</td>
<td>800</td>
</tr>
<tr>
<td>2280</td>
<td>1.10</td>
<td>1000</td>
</tr>
<tr>
<td>4400</td>
<td>1.25</td>
<td>2000</td>
</tr>
</tbody>
</table>

Figure 5.9. Full-Scale Dynamic Sled Test for Certification of Seat Cushions
Comparing the lumbar load from the rigid-seat sled test and simulation from this study shows quite reasonable agreement within the range of 5% at the peak value and rise time. The peak lumbar load also correlates with the linear analysis calculation for a rigid seat and dummy, for which the lumbar load is 4830 N (1087 lbf). Due to the difference in the belt system from this simulation, which is a five-point belt system equipped with retractor, compared to a simple two-point lap belt utilized in the sled test, the rebound of the occupant causes a small tension in the lumbar load. Also, the occupant’s upper body in the test results shows more rotation compared to the shoulder-belted occupant in the simulation results, which in turn attenuate the lumbar load in the sled test. Differences between the peak lumbar load and rise time could also be related to different dummy models utilized in the test and in this study.

To validate the energy absorber system further, the system of a 95th-percentile male aviator on a seat with an energy absorber was tested under a vertical acceleration of 51.6 G, rise time of 0.027 sec, velocity change of 14.3 m/sec (47 ft/sec), and load-limiting force of 25310 N (5696 lbf). Results were then compared with the experimental test [29], as shown in Figure 5.11. To correlate the results from this study with the published data, the mass of the seat system was set to be 68.9 kg (152 lb); hence, the total movable mass of 144 kg (318 lb) was considered.
Results from the simulation show a seat stroke of 32.7 cm (12.9 in), which is comparable to 32.8 cm (12.9 in) from the test. This test was conducted in a low environmental temperature, which simulated the cushion and dummy stiffness characteristics as almost rigid characteristics. This reduced the effect of pelvic and cushion compliances and, hence resulted in lower pelvic deceleration peak compared to the simulation results. On the other hand, the energy absorber in the test was not as ideal as in the simulation, which in turn caused some differences in results between the test and the simulation. Overall, the results were within ±10% of each other, and hence, good confidence was obtained for the model validity.

5.3.3 Results from Design Iterations (Parametric Study)

The seat energy absorber was subjected to a parametric study to design an efficient energy-absorbing system to minimize the occupant injury level while minimizing the seat stroke.
A 50th percentile FAA ATD model on a seat with a moveable mass of 27.2 kg (60 lb), shown in Figure 5.8, was subjected to the different seat energy-absorber systems, as shown in Figure 5.12. This figure identifies four different load-limit cases: simple EA, two-phase EA, two-phase EA with initial spike, and two-phase EA with ramp.

![Graph showing load-liming curves for energy absorber](image)

**Figure 5.12. Load-Limiting Curves for Energy Absorber**

Figure 5.13(a) shows lumbar load history from the MADYMO model for a seat implemented with a one-stage load limiter of 14.5 G (12640 N). The relative velocity built up between the pelvis and seat bucket is also depicted in Figure 5.13(b). Results show that after the first spike in lumbar load as the result of impact between the pelvis and seat pan, the velocity is reduced to a relatively small value due to the small rebound. Then, the average lumbar load takes approximately the linear dynamic load value (4850 N) for a period of time and is gradually reduced to zero. The seat base and pelvis vertical accelerations along with airframe acceleration are plotted in Figure 5.13(c). Figure 5.13(d) shows the seat stroke for the case of the simple load-limiter energy absorber.
Simulation results for a two-stage load limiter are depicted in Figure 5.14. The load limiter is designed to give a relative velocity of zero at the end of the first step and onset of the second step of the load limiter. The value of the second peak load is set to have a lumbar load below that of the first overshoot value, which is in the body tolerable range. Since the area under the load-deflection of the energy absorber is equal to the energy absorbed by the system, it is obvious that more energy is absorbed in the two-stage load limiter compared to that of the one-stage load limiter. Hence, the stroke in the two-stage load limiter would be less than that of the one-stage load limiter, which is shown in Figure 5.15. After a few iterations, to find the force level at the second stage of the energy-absorbing system, the load level was set to 15300 N (17.5 G).
Figure 5.14. Occupant Lumbar Load, Seat-Base Acceleration, Seat/Occupant Relative Velocity, and Seat Stroking Curve for Two-Stage Load Limiter

Figure 5.15. Occupant Lumbar Load, Seat-Base Acceleration, Seat/Occupant Relative Velocity, and Seat Stroking Curve for Two-Stage Load Limiter with Initial Spike

Figure 5.15 shows results for the initially spiked load limiter as mentioned by Carr and Phillips [26]. For this purpose, the same two-stage load limiter as shown in Figure 5.12 was used,
with an initial spike of about 10% during the first stage. Results for the two-stage energy absorber with ramp between stages are shown in Figure 5.16. A comparison of the lumbar loads for energy absorbers with different load limiter curves is depicted in Figure 5.17.

Figure 5.16. Occupant Lumbar Load, Seat-Base Acceleration, Seat/Occupant Relative Velocity, and Seat Stroking Curve for Two-Stage Load Limiter with Ramp

Figure 5.17. Comparison of Occupant Lumbar Load for Different Load Limiters
5.4 Discussion

A mathematical model was utilized to predict the sustained lumbar load when subjected to the vertical dynamic test conditions according to MIL-S-58095 for civil rotorcraft. The relative displacement and velocity of occupants with respect to the seat pan were also monitored. The model was validated with experimental dynamic sled tests as well as published data.

The two-stage VLEA profile, recommended in this study, uses a constant load displacement between stage one and stage two. The magnitude of the load-displacement is subject to change, based on different-size occupants and seat weights. For the first stage, the suggested starting-point stroking load of 12650 N, based on a 50\textsuperscript{th}-percentile occupant weight and a movable seat mass of 60 lb, is equal to a load factor of 14.5 G. For the second stage of the aforementioned weight conditions, the suggested starting-point stroking load of 17200 N, equal to a load factor of 20 G, was set to keep the maximum lumbar load in the human tolerable limits. The simulation indicates the second stage should “kick in” after one in (2.5 cm) of stroke, which strongly depends on the seat cushion properties and thickness. Utilizing the modified two-stage energy absorber, the stroke needed for the same level of occupant injury would be reduced. For the recommended two-stage energy absorber, the stroke is reduced by more than 3 in (7.5 cm), as shown in Figures 5.13 to 5.16. In another word, with the same stroke designed for a specific seat, the occupant lumbar spine injury level would be significantly attenuated.

At least three variables were used to examine each specific seat energy absorber. The first was the stage-one load limit value, which is the load limit based on load factor and the effective movable mass of the seat occupant. The second was the stage-two load limit value, which is the load level that does not create lumbar load more than that of the first peak. The third was the starting point for the stage-two load limit value, which depends on the relative velocity built up
at stage one and the contact characteristics between the seat and the occupant’s pelvis. The design of the energy absorber was such that it kept the lumbar load within the tolerable limit while using the maximum energy-absorbing capability of the seat system and minimizing the seat stroke.

The second stage could be a step-load input (simplest design) or could be ramped if a significant performance benefit is expected. The simulation indicates a negligible improvement for an energy absorber with a ramped load limit. The ramp function for the load limiter indicates that the two humps of lumbar load can merge together at its maximum level.

The recommended two-stage load-displacement profile allows the body to be decelerated at a higher average load. Since the maximum load in the spine typically results from dynamic overshoot, minimizing the overshoot allowed the average load to be raised while keeping the spine load within human tolerance limits.

As an inevitable fact, any contact of the occupant’s arm with other body parts or seat system at the onset of the impulse changes the lumbar load peak significantly, as observed from simulations. The arm support acts like a short cut for the load to be transferred to the upper body and subtracts the portion of the inertial force of the arm from the lumbar load. This indicates a need for more detail in describing the test requirement for occupant configuration as well as seat and cushion system.

It was also observed that unlike what has been suggested in some previous studies, any initial spike at load-displacement characteristics of the energy absorber does not help in reducing the lumbar load or the absorbed energy by the system. In contrast, since the load path starts with impact with an occupant’s pelvis, any initial increase in the load-limiting level increases the lumbar load while not significantly helping the stroke of the seat.
5.5 Conclusions

In this study, a mathematical model was first developed based on the kinematics of a rotorcraft seat and occupant for a one-stage load limiter at different load factors. The computational model was then further developed, and a two-stage energy-absorber system for a military and civil rotorcraft crew seat was implemented to minimize the seat stroke when subjected to vertical dynamic test conditions according to MIL-S-58095. The relative displacement and velocity of the occupant with respect to the seat pan were monitored for different energy-absorber system designs. The models were validated with experimental dynamic sled tests as well as published data.

Results from this study indicate that by utilizing the proposed two-stage energy absorber, while keeping the occupant lumbar load below the human tolerable limit, the seat stroke requirement could be significantly attenuated. It was observed that the seat cushion thickness significantly influenced the lumbar load overshoot at certain load-limit factors. This study also concludes that depending on the cushion thickness, the load limit must be set at a load factor considerably below the tolerable level in order to limit the occupant response to a tolerable level, particularly for seats with high movable mass. The larger the movable seat mass, the higher the impact impulse to the occupant lumbar, and the lower the required load-limit factor. The load factor of 14.5 G for a 50th-percentile male aviator and a given seat mass leads to a higher lumbar load and smaller seat stroke for a lighter effective mass of seat and occupant (i.e., 5th-percentile aviator) and a lower lumbar load and high seat stroke for a heavier effective mass of seat and occupant (i.e., 95th-percentile aviator). For any occupant and seat mass, the recommended two-stage load-limiter design demonstrated an improvement in seat stroke. This improvement was shown to be about 25% for a 50th-percentile aviator with a light moveable seat mass.
The methodology and mathematical models developed in this study can be utilized in the design of energy-absorbing systems for seats. Results, however, indicate the need for further experimental testing to define the accurate load-limit factor, independent of environmental and system variables and based on human tolerable limits.

5.6 References


6.1 Abstract

A review of accident data reveals that in most pedestrian accidents, the head and lower extremity injuries are the predominant areas of injury to the pedestrian. The front geometry profile and stiffness of the vehicle as well as impact speed are important factors governing pedestrian kinematics. Accident data shows that the fatality rate for pedestrian/utility vehicle impact is greater than that for pedestrian/passenger car impact. The addition of a front guard on light trucks and sport utility vehicles to mitigate damage during off-road activity or to provide mounting points for extra lights, makes the pedestrian more vulnerable to the impact. In this study, a computational technique is utilized to quantify the influence of the added front guard on the impacted pedestrian. A CAD model of a typical commercial frontal guard is developed, and the finite element (FE) analysis along with impact test, are conducted to obtain the stiffness properties of the guard. Different sizes of pedestrian models in the MADYMO code are utilized, and the validated facet-surface model of a pickup truck is used to generate a vehicle front surface. The entire model is validated by comparing the pedestrian kinematics and injury parameters with the published data. This study demonstrates that for all sizes of pedestrians, the mid-body region is more vulnerable when a guard is added to the vehicle. The results from this study have been published in the following source:

study can be utilized in the design of front guards, the frontal crash zone of utility vehicles, and installation of these aftermarket guards in order to protect vulnerable road users.

6.2 Introduction

Statistical data have indicated the vulnerability of pedestrians and cyclists when impacting a motor vehicle in urban areas. In 2009, pedestrians accounted for 12% of all highway fatalities in the United States [1]. In 2005, 8.7% of vehicle-pedestrian impacts in the U.S. were fatal, whereas the corresponding fatality rate for occupants in vehicle crashes was 1.3% [2]. The Fatality Analysis Reporting System (FARS) database on pedestrian fatalities shows that light trucks and vans (LTVs) are the major source of pedestrian death [3]. This difference can be attributed to the different sizes, shapes, and stiffness of LTVs compared to passenger cars. The analysis of real-world crash data in the U.S. indicates that 11.5% of pedestrians struck by larger sport utility vehicles are killed, compared to 4.5% of pedestrians struck by passenger cars [4].

In the last two decades, the on-road fleet in the U.S. has experienced a nearly 200% increase in the number of LTVs [5-7]. In general, these vehicles impact more aggressively than passenger cars [7], which have led to greater concerns about pedestrian safety when they are involved in impacts. Data indicates that the size of the pedestrian and the vehicle frontal area are the dominant factors in injury outcomes. Hence, design innovations to the front of the vehicle are sought to mitigate impact loads on the pedestrian [3, 8]. It has been demonstrated that the shape of a sport utility vehicle (SUV)’s frontal structure is a factor resulting in higher pedestrian injuries to the mid-body region as compared to the shape of passenger cars. In pedestrian-SUV impacts, the pedestrian is struck more centrally with respect to the body’s center of gravity, increasing the momentum transfer in the primary impact [9]. This clearly indicates the higher aggressiveness of SUVs than passenger cars toward pedestrians [4, 10-12]. Car manufacturers
address the mitigation of injuries to pedestrians by distinguishing the frontal crush area of the vehicle based on the probability of impacting a pedestrian in the frontal region [13, 14]. Protection of pedestrians has recently attracted increased attention because of regulations implemented or proposed in Europe, Korea, and Japan [15, 16]. Pedestrian regulations, such as Global Technical Regulations (GTRs), have also been developed since 1987 as a part of efforts to increase pedestrian safety [17-20].

Many consumers of pick-up trucks and SUVs attach frontal guards to their vehicles after purchasing them. The purpose of the guard is mainly to deflect brush and other vegetation from fragile vehicle parts in order to mitigate damage during off-road activity. However, use of these guards is alarmingly high in urban areas, where they are typically purchased for cosmetic purposes and styling of LTVs. The presence of a guard significantly changes the vehicle’s front-end dynamic crash-energy dissipation characteristics from the original design [6]. The most important consideration of an LTV retrofitted guard is in the alteration of the crash response and crash-energy dissipating area as well as change of the original equipment manufacturer (OEM)-designed and tested crash-zone specification for pedestrians struck by these vehicles.

Pedestrian kinematics is highly influenced by vehicle front-end geometry and pedestrian anthropometry. Furthermore, a stiff guard on the front of a vehicle significantly affects pedestrian kinematics at any given speed [21]. Vehicle front-end geometry and stiffness characteristics along with pedestrian size and pre-impact configuration are important factors determining the injury level [22-30]. Moradi et al. found that light trucks and vans with frontal guards were associated with more than two times higher risk of severe pelvis injuries in comparison with the original-designed LTVs [21].
6.3 Methodology

In order to investigate the effect of a vehicle front guard on the kinematics of different pedestrian sizes, a computational model was utilized in this study. MADYMO pedestrian models, namely the six-year-old child, the 50% (mid-size) male, and 95% (large-size) male, were impacted with a vehicle front-facet model. The CAD model of a typical front guard, consisting mainly of two vertical cantilever I-beams and four horizontal tubular beams, was created, and the finite element (FE) analysis along with the impact test were conducted to obtain the guard’s stiffness properties. The facet rigid model of a guard was then fixed rigidly to the front of the vehicle at the appropriate location. The overall MADYMO computational model setup consisted of three systems—the road, the vehicle, and the pedestrian. Simulations were carried out for vehicle impact speeds of 25 km/hr, 36 km/hr, 40 km/hr (for validation purposes), and 54 km/hr. All simulations were executed with and without the front guard. In 80% to 90% of cases, a pedestrian crossing the street in urban areas is struck laterally by the frontal area of a vehicle at a medium speed of about 40 km/hr [27, 31-33]; therefore, the model is developed based on the pedestrian/LTV lateral-impact scenario. To examine the differences between pedestrian models versus a standing dummy model, the results were compared with those of Moradi et al. [21]. Pedestrian-ground, pedestrian-vehicle, and ground-vehicle friction coefficients of 0.6, 0.3, and 0.5, respectively, were chosen, based on data from the literature [9, 25, 34].

6.3.1 LTV Modeling

The validated Chevy Silverado FE model from the National Crash Analysis Center (NCAC) was utilized in this study. The front region of this model was used to create a facet model, which included the bumper, hood, bonnet, fenders, and windshield. Load-deflection characteristics for the facet model were defined from published results and assigned to various
portions of the front end. Figure 6.1(a) shows the typical front-end pedestrian rating from the European New Car Assessment Programme (Euro NCAP) [35]. Figure 6.1(b) shows the MADYMO facet model of the front-end zones. The load-deflection characteristics of each zone are shown in Figure 6.2 [6, 9, 35].

Figure 6.1. (a) Euro NCAP Typical Pedestrian Rating for Front-End Zone [34], (b) MADYMO Facet Model for LTV Front-End Zone

Figure 6.2. Load-Deflection Characteristics for Vehicle Front End [6, 9, 35]

6.3.2 Front-Guard Modeling

A CAD model of a typical frontal guard used for an LTV was developed and is shown in Figure 6.3. The mass of this guard is 36 kg, and the design mainly consists of two vertical cantilever I-beams and four horizontal tubular beams. The two vertical beams are separated from the centerline by 700 mm. The horizontal tube was fabricated from a 38.1 mm outer diameter using A-513 grade steel with wall thickness of 2.1 mm. The frontal guard characteristics were
derived from FE analysis followed by an experimental test for its validation. Figure 6.4 (a, b, and c) shows the process of FE analysis and the drop test conducted on a front guard and the instrumentation to extract the force-displacement data at different impact velocities. Figure 6.4 (d) shows the load-deflection characteristic extracted and used for the guard [6]. As observed, the guard has significantly higher stiffness compared to the bumper.

Figure 6.3. Guard Model Used for Simulation

Figure 6.4. (a) FE Analysis, (b) Instrumentation, (c) Drop Test of Guard from Temporary Drop Tower, and (d) Extraction of Load-Deflection Characteristics Used for Guard in This Study
6.3.3 Human Models

Validated MADYMO pedestrian models of a six-year-old child, and mid- and large-size males were used in this study, as shown in Figure 6.5(a) [36]. The initial standing configuration of the pedestrian models was selected to match the test conducted by Nierderer and Schumpf [27] and Simms and Wood [9]. In pedestrian impacts, leg bending and resulting bone fracture are commonly observed. To account for this in the pedestrian model, the bending and fracture properties were implemented at several locations in the femur and tibia using bending/fracture joints, as shown in Figure 6.5 (b) [36]. The large spots show the hip, knee, and ankle joints, while the small spots show the locations of the bending and/or fracture joints. All fracture joints are spherical joints that are initially locked until a pre-defined fracture trigger signal exceeds the fracture tolerance level.

![Pedestrian Family](image)

Figure 6.5. (a) Pedestrian Family, from Left to Right: Three-Year-Old Child, Six-Year-Old Child, Small Female, Mid-Size Male, and Large-Size Male; (b) Bending and Fracture Joints in Pedestrian Leg [36].

6.4 Model Validation

Validation of the entire LTV/guard/pedestrian model was conducted by comparing the mid-size pedestrian kinematics with the published results for similar impact conditions. The results from Simms and Wood [9], Moradi et al. [21], Kerrigan et al. [22], and Niederer and
Schumpf [27] were used for the validation of the model in the current study. The comparison between the computational model and the published results shows a reasonably good correlation, as can be seen in Figure 6.6 for the walking mid-size pedestrian at 40 km/h. Figure 6.7 shows the validation for the kinematics of body parts. Figure 6.7 (a, b) shows the head and pelvis center of gravity (CG) trajectories of a walking mid-size pedestrian when struck by an LTV at 40 km/hr, which shows a fair correlation with the cadaver or post-mortem human surrogate (PMHS) test results from the work of Kerrigan et al. [22], as shown in Figure 6.7 (c, d). Figure 6.8 depicts the kinematic results from this study, which indicates reasonably good correlation with results from the simulation of a standing dummy laterally impacted by an LTV [21] and experimental results from PMHS tests [27]. One exception is in the lower extremities, which have a greater flexibility in the MADYMO human pedestrian models but no flexibility in the MADYMO standing dummy models. The greater flexibility of the lower leg in the human models is due to the fracture joints that are used in them, compared with only physical joints used in the dummy or dummy models.

Figure 6.6. Walking Pedestrian and LTV Impact at 40 km/hr

\[ t = 0 \quad t = 20 \quad t = 40 \quad t = 60 \quad t = 80 \quad t = 100 \quad t = 200 \quad t = 300 \]
Figure 6.7. Head and Pelvis CG Trajectories of Walking Pedestrian in Vehicle Reference System at Impact Speed of 40 km/hr: (a) and (b) Results from This Study; (c) and (d) Results from Kerrigan et al. Study [22].

Figure 6.8. Walking Mid-Size Pedestrian/ LTV Impact at 25 km/hr
6.5 Results

6.5.1 Pedestrian Kinematics Results

Figure 6.9 shows the impact of a child pedestrian with an LTV with and without a guard at an impact speed of 36 km/h. The impact kinematics for the six-year-old pedestrian is entirely different from those of the mid- and large-size pedestrians, since the translational movement is dominant in this impact. The bumper of the original LTV impacts with the upper leg and hip of the child pedestrian, followed by the hood edge impacting the head and neck. The pedestrian is thrown away (forward projection) from the car front after a slight rotation of upper-body parts (wrapping behavior). For the LTV with the guard, the pedestrian impact occurs directly with the rigid guard, and the forward projection of the pedestrian occurs without any wrapping around the car hood. In this case, due to the direct impact of all body parts with the stiff guard, the rise time of post-impact velocity of body parts decreases compared with adult pedestrian impacts and the LTV without guard impacts.

![Image of pedestrian impact](image.png)

(a) Child pedestrian/LTV impact without guard

(b) Child pedestrian/LTV impact with guard

$t = 0$  $t = 50$  $t = 100$  $t = 200$  $t = 300$

Figure 6.9. Walking Six-Year-Old Pedestrian/LTV Impact at 36 km/hr with and without Guard

In the original LTV/mid-size pedestrian side impact, the bumper impacts the pedestrian’s thigh, and after a small rotation of the pedestrian, the hood edge comes in contact with the lower torso, elbow, and upper arm of the model. The body of the pedestrian gradually rotates and attains a horizontal position. The kinematics of the mid-size pedestrian impacting the
car involves the entire body translational movements and upper body rotational movements. The primary head impact occurs with the middle area of the hood for the 25 km/h impact speed, and then the head slides down the hood. For higher impact velocities of 36 and 54 km/h, the body is thrown ahead of the vehicle, and a significant rotational movement of the pedestrian is observed, as shown in Figure 6.10. The secondary impact of the pedestrian’s head with the ground surface in medium- and high-speed impacts is more severe than the primary impact with the car. This scenario changes slightly for the LTV with the guard, due to the change in rotational point and the first contact of the upper leg and pelvis region with the stiff guard. Hence, the head impact occurs at the frontal part of the hood, which is stiffer than the middle part.

![Walking Mid-Size Pedestrian/LTV Impact at 36 km/hr with and without Guard](image)

Figure 6.10. Walking Mid-Size Pedestrian/LTV Impact at 36 km/hr with and without Guard

The impact scenario for a large-size pedestrian differs slightly from that of the mid-size pedestrian. The bumper impacts the pedestrian’s knee and then the hip, and the hood edge comes in contact with the lower arm. As shown in Figure 6.11, as the body rotates and leans against the hood, the head impacts with the rear part of the car hood, compared to the middle-hood location of the mid-size pedestrian head impact. This results in severe head impact at high speeds for a vehicle without the guard compared to a vehicle that has a guard, whereby the guard causes the body to rotate at a higher point, and the head impacts the middle part of the vehicle hood.
Figure 6.11. Walking Large-Size Pedestrian/LTV Impact at 36 km/hr *with and without Guard*

Figure 6.12 shows the trajectory of the head, upper torso, and pelvis of different pedestrian sizes impacting the LTV with and without the guard. Each trajectory is plotted from time $t = 0$ to a short time after impact. The generic rotational behavior of the upper body parts of different pedestrian sizes is depicted in Figure 6.13, in which the pedestrian is assumed to be a multibody system articulated at the point of impact.

Figure 6.12. Head, Upper torso, and Pelvis CG Trajectories of Different Sizes of Walking Pedestrians/LTV Trajectories at 25 km/hr without Guard (a, b, and c, respectively), and Pedestrian/LTV Impact *with Guard* (d, e, and f, respectively) in Vehicle Reference System at Low-Speed Impact of 25 km/hr
Figure 6.13. After-Impact Configurations of Small- and Large-Size Pedestrians Impacting with LTV with and without Front Guard

### 6.5.2 Pedestrian Injury Results

Injury criteria are used to relate physical parameters, such as accelerations and forces, to the injuries on human body parts. The tolerances and injury criteria are the basis for legislation and evaluation injury prevention measures. A value of 1000 with a maximum time window size of 36 milliseconds is specified for the head injury criteria (HIC), denoted as HIC36 as the concussion tolerance level, which has been associated with a 50% risk of skull fracture [37]. The U.S. Federal Motor Vehicle Safety Standard (FMVSS) 208 includes a limit of 700 for the HIC, with a maximum window size of 15 milliseconds to be a better predictor of head injuries. This limit is denoted as HIC15. The U.S. FMVSS 214 stipulates a peak acceleration tolerance of 130 G for the pelvis in a side impact. A commonly stated human tolerance level for severe chest injury (AIS ≥ 4) is a maximum linear acceleration in the center of gravity of the upper thorax of 60 G, sustained for 3 milliseconds or longer based on FMVSS 208. Thus, the criterion is not based on a single maximum value but rather on a sustainable level of linear acceleration. From this study, the predicted injuries to a pedestrian’s head, thorax, and pelvis for all impact scenarios are depicted in Figures 6.14 and 6.15.
Figure 6.14. (a) HIC\textsubscript{15}; (b) Upper Torso Cumulative 3-ms Acceleration in G; (c) Pelvis Acceleration in G, at Different LTV Impact Speeds for Different-Size Pedestrians and LTV without Front Guard

Figure 6.15. (a) HIC\textsubscript{15}; (b) Upper Torso Cumulative 3-ms Acceleration in G; (c) Pelvis Acceleration in G, at Different LTV Impact Speeds and for Different-Size Pedestrians and LTV with Front Guard
6.6 Discussion

The aim of this study was to evaluate the influence of an LTV rigid frontal guard on the kinematics of, and injury potential to, different sizes of pedestrians. The addition of a rigid guard on the front of a vehicle amplifies the stiffness characteristics of the vehicle’s frontal structure and changes its front geometry profile. These are the most important factors relative to kinematics and injury potential to a pedestrian impacted by a vehicle. This change is quantified by comparing pedestrians’ kinematics and potential injury parameters with and without the guard after impact.

The complete kinematics sequence of the entire impact scenario consists of the pedestrian’s primary impact (with the vehicle) and secondary impact (with the ground). The severity of these impacts is influenced by the vehicle impact speed as well as size of the pedestrian. At the same vehicle impact speed for a pedestrian, the linear momentum transfer for a six-year-old pedestrian is much more than that for a mid-size or large-size pedestrian.

The interaction of the LTV and the pedestrian is quite different from that corresponding to the interaction of a car and pedestrian. In the latter scenario, the pedestrian tends to rotate and fall on the hood of the car (wrap). The LTV’s higher bonnet height results in direct contact with the mid-region of the pedestrian body, which leads to the body’s forward projection. This may result in the pelvis being a more vulnerable body region when impacted by the LTV. This situation is further aggravated due to the presence of the rigid guard in the front of the vehicle. The presence of the guard alters the lead angle of the vehicle front profile and results in a significant change in the impact kinematics since the probability of forward projection is increased. It was observed that with the guard, pelvis acceleration increased by more than two-
fold for all pedestrian sizes, as compared to the case with no guard, which makes the LTV with a guard more aggressive, even at low-speed impacts.

At an impact speed of 25 km/hr or less, the primary head contact of the mid-size pedestrian with the vehicle is absent with and without the guard, and the large-size pedestrian experiences a very slow touch. In the same situation, the six-year-old pedestrian sustains relatively severe head impact with the LTV’s front area or rigid guard, far beyond the acceptable injury level. Addition of the guard increases the height of the center of rotation of the pedestrian’s body on the vehicle’s front area, and as a result, head impact occurs at the front and middle area of the hood for a mid- and large-size pedestrian, respectively. This results in no change or slightly lower HIC value for a large-size pedestrian, and a slight increase in that of a mid-size pedestrian at a high-speed impact compared to the same impact scenario without the guard.

Torso acceleration is in the safe region for adult pedestrians for a wide range of impact speeds, but at the same time, this is beyond the critical level for a six-year-old child, for LTVs both with and without the guard. It was observed that the thorax injury level for the pedestrian is not significantly influenced by the presence of the guard but mainly depends proportionally on the impact speed.

When the LTV with or without the front guard impacts the mid- and large-size pedestrian laterally at low- and medium-impact speeds, the secondary impact of the pedestrian with the road surface is more life threatening than the primary impact with the car in terms of head injury. At low- and medium-impact speeds, pelvis injury increases with the addition of the frontal guard. At impact speeds greater than 40 km/hr, the secondary impact is less important since the primary impact with the car front is the first life-threatening impact. On the other hand, due to the high
momentum transferred to the pedestrian at impact speeds of greater than 40 km/hr, the addition of a guard is not the dominant life-threatening feature of the high-speed impact. This is entirely different for a small-size pedestrian, who is dominantly influenced by the front guard even at low-speed impacts. For a six-year-old pedestrian, the body and head have less rotation before the impact, compared with the other two pedestrians. As a result, the head acceleration in the small-size pedestrian is much higher than that for the mid- and large-size pedestrians. This is more significant in the presence of a guard, since the rise time for head velocity is less than that involving the vehicle without a guard.

Evaluating the after-impact movement of all pedestrian-LTV impacts, Figure 6.13 depicts the general configurations of pedestrian impacts on a vehicle with different front-end geometries. Despite some local deformation for both vehicle and pedestrian, it can be observed that the small-size pedestrian is likely to have forward projection when impacted by the LTV with a front guard. When impacted by the LTV without a front guard, the small-size pedestrian behaves as an articulated multibody having angular momentum in the upper body after the first impact with the bumper. The head impacts with the car front at a slightly higher velocity compared to the vehicle with the guard. The articulated multibody impact behavior is observed more in the adult pedestrian than the child pedestrian, for both front-vehicle configurations. Because the angular momentum depends on the body dimensions and moments of inertia as well as angular velocities, the pedestrian’s upper-body rotation velocity after impact with the LTV with a guard would be less than after impact with the LTV without a guard. This results in the HIC value being slightly lower for the adult pedestrian impacted with the LTV with a guard, compared to that of the original LTV without a guard. Hence, in addition to the energy-absorption
consideration, minimizing the angular momentum transfer could be utilized in the design of the frontal guard.

6.7 Conclusions

A computational model was utilized in this study to investigate the influence of the addition of a vehicle front guard on pedestrian kinematics as a result of impact with the guard. This model was constructed from the FE and multibody models of the LTV, the guard, and the pedestrian. The model was then validated using the experimental test and published results by demonstrating the correlation between the pedestrian kinematics. The following conclusions can be made from this study.

- The stiff frontal guard alters the leading angle of the vehicle front profile, resulting in forward projection of the pedestrian instead of rotation around the hood.
- The influence of the addition of the guard in pedestrian injury depends on the size of the pedestrian.
- In general, the existence of the front guard increases the pelvis acceleration by more than 200% for all sizes of pedestrians and has a low effect on upper-torso acceleration.
- For the small-size pedestrian, the head acceleration increases catastrophically (about ten-fold) with the addition of the front guard, but no significant change in the head acceleration was observed for the mid- and large- size pedestrians.

This study clearly demonstrates that LTVs with the addition of a stiff front guard show poor performance in preventing severe injury to pedestrians. The presence of a rigid guard on the vehicle front changes the pedestrian kinematics and, due to the high stiffness characteristics of the guard, makes the pedestrian more vulnerable to injuries, as demonstrated. On the other hand, because the angular momentum of the pedestrian upper body is reduced with the addition of a
front guard, the relative impact velocity of the upper body parts is in turn reduced, and the injury levels in those areas, specifically head injury, may be reduced. Overall, this study documents that special attention must be paid to the design and installation of such frontal guards in terms of regulations to attenuate potential injury to pedestrians. Also, compliance tests for the manufacturers of these front guards are recommended. Furthermore, consumers must be informed of the potential dangers of utilizing these guards.

6.8 References


CHAPTER SEVEN

CASE STUDY FOUR: APPLICATION TO IMPULSE MITIGATION AND IMPACT INJURY BIOMECHANICS—MULTIBODY MODELING AND DESIGN OF EXPERIMENT INVESTIGATION OF MOTORCYCLIST IMPACT ON ROADSIDE BARRIERS AT UPRIGHT AND SLIDING CONFIGURATIONS

7.1 Abstract

Roadside guard systems such as concrete and wire barriers and steel guard rails are mainly developed to protect occupants of errant cars or trucks. Yet motorcycle riders are vulnerable to these barriers and guard systems, and impact on these barriers may result in major injuries. The objective of this study is to examine the major factors causing injuries in motorcycle-barrier accidents. For this purpose, a mathematical multibody motorcycle model with a motorcycle anthropometric test device, MATD, is developed in the MADYMO. The model of the motorcycle and the model of the motorcycle and rider are validated using full-scale crash test data available in the literature. Simulations results are found to be in a reasonable agreement with the experimental data. A parametric study is then conducted using the design of experiments (DOE) to investigate the nature of crash injuries for various impact speeds, impact angles, and bike and rider positions to assess rider kinematics and potential injuries. The results from this study can help in designing road barriers and guard systems in order to protect motorcycle riders.

7.2 Introduction

According to the National Highway Traffic Safety Administration (NHTSA) in 2008, 4,462 motorcyclists were killed in the United States. In 2009, the number of motorcyclists

1 This entire chapter has been published in the following source:

injured was 90,000 [1]. Per vehicle mile traveled in 2008, motorcyclists were about 39 times more likely than passenger car occupants to die in motor vehicle traffic crashes and 9 times more likely to be injured. In 2008, motorcyclists accounted for 14% of total traffic fatalities, 17% of all occupant fatalities, and 4% of all occupants injured. Motorcycles are more likely to be involved in fatal collisions with a fixed object rather than with other vehicles. In 2008, 25% of motorcycles involved in fatal crashes collided with fixed objects, compared to 19% for passenger cars, 14% for light trucks, and 4% for large trucks [1].

Motorcycle accidents typically result in driver injury and/or fatality due to the potential free contact of the rider with the environment. Because of the increased degrees of freedom and statically unbalance behavior of the motorcycle as well as multiple contacts of the motorcycle and the rider with the environment during impact, the motorcycle/rider multibody system undergoes complex dynamic behavior in the event of an accident. A motorcycle has no passenger cubicle, and the rider stays on the motorcycle only by his grip on the handlebars. As a result, the rider is free to move independently during the impact, and consequently gets separated from the motorcycle and exposed to the collision environment in most accidents. The motorcycle also loses balance due to the improper seating position, brakes that lock up, speeding, and weather conditions, which all occur frequently just before a collision. During an accident, the motorcycle is subjected to either yaw, or lean, or both at the same time. Motorcycles are usually involved in front-end collisions in the upright position, but when motorcycles are involved in collisions as the result of sliding, any of its parts, such as the front end, wheel, and tank, can collide with the obstacle. Before impact, the rider may be partially or completely separated from the motorcycle. Hence, the impact threat to a motorcyclist is omni-directional. Due to the
relatively small size of a motorcycle, the motorcyclist is always in danger when the cycle collides with any hindrance [2].

Over the past two decades, road safety barriers have been developed to reduce the rate of accidents by preventing errant vehicles from moving off the road or from moving in opposite directions [3-5]. They are ideally used in situations where road side hazards are located immediately behind the barrier. A conventional barrier system has proven to perform well in protecting the occupants of passenger cars, yet it might pose a significant danger to motorcyclists. Motorcycle crashes into crash barriers represent a small portion of all motorcycle accidents but a disproportionate number of motorcycle fatalities [1, 6]. The fatality risk in motorcycle-guardrail collisions is 12% and in motorcycle-concrete barrier collisions is 8%. For motorcycle-car collisions, this value is 4.8%—approximately one-third the risk of a motorcycle/guardrail collision [7]. Ouellet [8] suggested that the least injury-producing barriers for motorcyclists are smooth concrete barriers, which present no protruding surfaces to the motorcyclist in a crash event.

Approximately 60% of fatal motorcycle collisions with crash barriers involve the rider sliding with or without the cycle into the barrier, and in the other 40% of fatal collisions, the rider remains upright on the motorcycle [9]. Hell and Lobb found that the most likely areas of motorcyclists’ bodies to be injured as the result of collisions are (in order) their legs, head, and thorax [10].

7.3 Analysis Background

Generally, information on motorcycle-barrier crashes is inadequate and requires more crash testing for different crash configurations. Berg et al. conducted a series of crash tests and simulations on motorcycle impacts with roadside barriers [11]. The primary objective of the
study was to compare the different roadside barrier systems in terms of sustained rider injuries. They used the multibody system of motorcycle, dummy, and barrier to simulate the crash scenario, and compared the results with real-world crash data. A study by Nieboer et al. [12] showed the complexity of a motorcycle crash environment and a quite strict limit of utility of any simulation model. A similar strategy and simulation techniques are used in this study to simulate the motorcycle-barrier crash. In another study, Adamson et al. [13] conducted a series of real-world motorcycle crashes with barriers and cars, primarily to observe the crush profile of motorcycle. The effect of roadside guardrails on motorcycle kinematics and rider injury was evaluated by Ibitoye et al. [14]. It was found that the road condition did not change the kinematics of the rider after impact, although it changed the kinematics of the motorcycle after impact with the barrier [15].

In previous studies, the complexity of motorcycle kinematics was highlighted. In this study, the effect of the different variables on the rider kinematics and the injury level are investigated.

7.4 Methodology

Multibody techniques are used in this study since the kinematics of the system along with riders’ injury levels are of main interest. The multibody approach provides a quick and reliable method for extracting detailed information on the kinematic and dynamic behavior of the system. Contact mechanics characteristics with user-defined force-deflection model from published data are utilized to define the contact/impact characteristics between the different bodies of the multibody system.

In a broad sense, the different methods to solve the impact problem in multibody mechanical systems are the continuous and discontinuous analysis approaches [16]. The
continuous contact force method is utilized in this study to define the contact/impact properties between bodies in the system. It represents the force arising from the collisions and assumes that the forces and deformations vary in a continuous manner. This force is typically applied as a spring-damper element, which can be linear, e.g., the Kelvin-Voigt model [17], or nonlinear, e.g., the Hunt and Crossley model [18]. The later, with characteristics from the published data [8, 9], is utilized in this study.

This study involves modeling and analysis of a motorcycle crash scenario using the multibody code MADYMO 7.2. The model consists of a motorcycle with a rider and a barrier, and an inertial space (a road) over which the motorcycle, barrier, and rider are operated. The Motorcycle Anthropometric Test Device (MATD) is utilized as the rider. The MATD, specified in part 3 of the ISO 13232 document, is based on the Hybrid-III frontal impact dummy. The most important features of the dummy are a modified head; a newly designed neck; the Hybrid-III sit-stand pelvis; dummy hands that allow wrapping around the handlebars; frangible upper legs, lower legs, knees, and abdomen; and an on-board dummy data-acquisition system located in a modified spine box [19]. Figure 7.1 shows the MATD model used in this study.

Figure 7.1. MATD Model and MATD Left-Hand Showing Articulated Fingers
A Kawasaki ER 5 Twister motorcycle is used as a case study for model validation. The required dimensions and masses, such as overall length, width, height, seating height, wheel base, minimum ground clearance, and part weights, are obtained from the Kawasaki manufacture’s handbook [20].

The multibody model of the motorcycle is developed using ellipsoids consisting of seven bodies representing the main frame, seat, front and rear wheels, front and rear suspensions, and fuel tank, with a system total of 12 degrees of freedom. The main frame is considered a parent body over which the other bodies are connected through appropriate kinematic joints, as shown in Figure 7.2.

![Figure 7.2. Three-Dimensional Motorcycle Multibody Model with Kinematic Joints](image)

Revolute joints are used to connect the wheels and front upper fork. A bracket joint is used to connect the seat to the frame. Kelvin restraints are introduced for the front and rear suspensions. Since the front portion of the motorcycle impacts on the barrier, special attention is given to describe the front wheel and front suspension and also front-fork characteristics. The stiffness data for the front wheel, front suspensions, seat, and inertial properties are obtained from Nieboer et al. [12] and Ibitoye et al. [14]. The front-fork bending stiffness of the motorcycle is modeled in the revolute joint, with joint restraint according to the Nieboer et al. [12]. Due to the lack of detailed specifications for rear suspension, seat cushion, and steering, assumptions are made to match the overall kinematics of the motorcycle and the rider with the published data.
The concrete barrier is modeled as a single ellipsoid using parameters such as height of 800 mm, width of 200 mm. The material density used is 2500 kg/m³ [11].

To assess the rider kinematics and potential injuries, a parametric study is conducted for various impact speeds: 30, 45, 60, 80 km/h; impact angles: 6, 12, 18, 25, 30, 45, 60, 90 degrees; and two different rider-motorcycle impact configurations: upright and sliding.

7.5 Model Validation

Due to the complexity of motorcycle modeling related to its kinematics behavior, it is important to first validate the motorcycle model. The combined system of the motorcycle and the rider model also needs to be validated for impact with concrete barrier for each configuration.

7.5.1 Validation of Motorcycle Model

The motorcycle model is simulated and validated for the barrier test condition of 32.2 km/h velocity and 90-degree angle. The overall kinematics results are compared with results from Nieboer et al. [12], as shown in the Figure 7.3. The comparison between the simulation results of motorcycle from this study and the published data shows quite a reasonable correlation.

Figure 7.3. Simulated and Experimental Kinematics of Motorcycle for 90-Degree/32.2 km/h Barrier Test Condition
Figure 7.4 shows the acceleration and contact force results from this study compared to the experimental data and the simulation previously conducted for the same impact scenario. Some small differences in the assumptions in this study and the published data make little difference in the overall kinematics of the motorcycle and the peak values of barrier contact force as well as the separation time of the contact. However, the peak value and the rise time for the motorcycle main frame center of gravity acceleration show quite good correlation between those from this study and the published data [12]. As observed from Figure 7.4, the acceleration peak value and rise time from this study are within 2 percent of the experimental data, whereas the simulation results previously conducted for the same impact scenario were within about 30 percent of the experimental data.

Figure 7.4. Simulated and Experimental Results of Motorcycle-Barrier Impact for 90-Degree/32.2 km/h Upright Configuration

7.5.2 Validation of the Motorcycle with Rider Model

The motorcycle and rider multibody model is used in two different impact configurations, the upright and the sliding configurations. As mentioned by previous researchers [11, 12], the
model needs to be validated in both configurations. The impact on a concrete barrier at an angle of 12 degrees at a speed of 60 km/hr at upright position of the rider and motorcycle was used to validate the upright configuration of the impact. Also, the sliding impact of the model at an angle of 25 degrees and speed of 46 km/hr is used for the sliding impact configuration validation. The performance of the model is evaluated by correlating the results obtained with those from the full-scale crash test and also from the data of a computer simulation study conducted by Berg et al. [11], as shown in Figures 7.5 and 7.6.

![Figure 7.5](image1.png)

Figure 7.5. Motorcyclist’s Head Trajectory after Impact with Barrier for 12-Degree/60 km/h Upright Impact Configuration

![Figure 7.6](image2.png)

Figure 7.6. Motorcyclist’s Head Trajectory after Impact with Barrier for 25-Degree/46 km/h Sliding Impact Configuration
A comparison of the results shows relatively reasonable correlation between the trajectories of the rider from this simulation and the experimental results from Berg et al. Kinematics of the simulated and full-scale crash test were fine-tuned in order to achieve good confidence in the model and to validate the model with test results. Head injury and femur load during collision were also examined and validated with the test results. As shown in Table 7.1, the load acting on the femur, pelvis acceleration, and thoracic acceleration, and the HIC obtained from the simulation results of this study are close to the full-scale crash test done by Berg et al. [11] for most of the cases. Therefore, the motorcycle with a rider was considered as a validated model. As mentioned earlier, any change in the initial configuration of the system changes the contact point of the rider with the environment. Hence, unlike the cyclist’s overall kinematics, the local kinematics and the sustained injury level by rider are quite sensitive to the initial conditions and cannot be a good reference for validation purposes.

**TABLE 7.1. Injury Levels for Different Impact Scenarios from This Study and Experimental and Published Data**

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7.6 Results

Once good confidence was developed in the model based on the kinematics of the motorcycle and the rider, a parametric study was conducted to investigate the nature of crash injuries for various impact speeds and different impact angles to assess rider kinematics and potential injuries for the motorcycle/rider upright and sliding configurations.
Three factors—namely the motorcycle initial speed, angle of impact, and initial system configuration—were analyzed, and the contribution of these factors was evaluated. For this purpose, four speeds of 30, 45, 60, and 80 km/hr; eight different angles of collisions of 6, 12, 18, 25, 30, 45, 60, and 90 degrees; and two system configurations, upright and sliding, were considered.

To avoid repetitive graphs, the kinematics of the motorcycle with rider are shown in Figures 7.7 and 7.8 for two different scenarios only—the 45°/45 km/h impact in an upright configuration and the 90°/80 km/h impact in a sliding configuration. To show the occupant injuries for all different scenarios efficiently, the the head injury (HIC36), thoracic resultant acceleration (T3MS), pelvis resultant acceleration, and femur force criterion (FFC) representing the axial load applied to the occupant femur are shown in Figures 7.9 and 7.10 for different impact angles and speeds and system configurations.
Figure 7.9. Rider’s Sustained Injury for Different Impact Speeds and Angles for Upright Impact Configuration

Figure 7.10. Rider’s Sustained Injury for Different Impact Speeds and Angles for Sliding Impact Configuration
The value of 1000 has been specified for HIC36 as concussion threshold, which has been associated with a 50% risk of severe head injury [21]. The U.S. regulations, FMVSS 208, specify that resultant chest acceleration should not exceed 60 G, except for intervals whose cumulative duration is not more than 3 ms. The peak acceleration tolerance of 130 G for the pelvis in a side impact stipulated by the Federal Motor Vehicle Safety Standards (FMVSS) 214 is used in this study as the pelvis injury tolerance. Finally, the value of 10 kN for femur axial load (FFC), based on FMVSS 208, is specified as the femur severe injury threshold for a 50% male.

Since the motorcycle is impacted at acute angles on the right side of its rider, the left femur of the occupant does not primarily impact the concrete barrier in the upright configuration, and injury sustained by the left femur is due to a secondary impact with the road surface or motorcycle. The same scenario occurs for the right femur when the system impacts the barrier in the sliding configuration. The results for HIC values at small impact angles are quite sensitive to the initial condition of the motorcycle and the rider, and for some cases, the values correspond to the secondary impact with surrounding objects. However, results for the HIC at high speeds, and medium- and high-impact angles are primarily due to secondary impacts, and the values follow the logical pattern.

**7.7 Application of Design of Experiments**

The effect of each variable (motorcycle impact speed, the angle of impact, and motorcycle/rider configuration before impact) on rider injuries was evaluated using the design of experiments (DOE). Table 7.2 lists the factors and variables studied.

<table>
<thead>
<tr>
<th>TABLE 7.2. DOE Factors and Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact Speed, $V$ (km/h)</td>
</tr>
<tr>
<td>Impact Angle, $\Theta$ (degree)</td>
</tr>
<tr>
<td>Motorcycle/Rider Configuration</td>
</tr>
</tbody>
</table>
Results from the DOE show that the local kinematics of the body part strongly depends on the initial position of the rider; hence, any small change in the rider’s initial configuration influences the local kinematic results and then injury outcomes. This is particularly more visible in the HIC values for which the fitted DOE model for the sustained injury has no acceptable R-squared percentage, which means that the HIC values are not well predicted by the model for other impact scenarios. The T3MS, pelvic, and femur injuries are well predicted using DOE models.

Figures 7.11 and 7.12 depict the DOE-fitted models for the pelvic injury prediction at two rider configurations, upright and sliding, respectively. These figures show the safe region of impact angle and speed based on the pelvis injury parameter. Similar charts can be drawn for other injury parameters based on the best-fit polynomial equations listed in Table 7.3 as extracted from the DOE results.

![Figure 7.11. DOE Fitted Model for Rider Pelvic Injury at Upright Impacts](image1)

![Figure 7.12. DOE Fitted Model for Rider’s Pelvic Injury at Sliding Impacts](image2)

The third-order polynomial models were extracted from the DOE results with three variables in which the coefficients of the polynomial were determined based on the best spline fit for the data points. As an example, Figure 7.11 shows that for impact speeds lower than 46 km/h and for all impact angles, the pelvis injury level at the upright configuration of impact is below
the limit value of 130 G. Similar interpretations can be made for the impact angles of smaller than 20 degrees and the entire range of impact speeds in the upright impact configuration. These predictions are in reasonably good correlation with the test results and graphs of Figures 7.9 and 7.10. In all predicting equations, the impact speed \( V \) is in km/h and varies from 30 to 80 km/h. Also, the impact angle \( \Theta \) is in degrees and varies from 6 to 90 degrees. Using these polynomial models, the sustained injury potential for the rider in any other impact conditions in the mentioned range can be predicted quite well.

Table 7.3. DOE Fitted Model for Rider Injury Level

<table>
<thead>
<tr>
<th></th>
<th>Upright</th>
<th>Sliding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HIC(_{36})</strong></td>
<td>(-254 + 5.3V + 1.6\Theta + 0.08V\Theta - 0.02\Theta^2)</td>
<td>(1815 - 33V - 145\Theta + 2.8V\Theta + 0.6\Theta^2)</td>
</tr>
<tr>
<td><strong>T3MS</strong></td>
<td>(1.1 + 0.7V - 0.1\Theta - 0.03V\Theta - 0.007V^2 + 0.005\Theta^2 + 0.0005V^2\Theta)</td>
<td>(-16.6 + 0.7V + 1.4\Theta - 0.03V\Theta - 0.007V^2 - 0.007\Theta^2 + 0.0005V^2\Theta)</td>
</tr>
<tr>
<td><strong>Pelvic</strong></td>
<td>(-63.3 + V + 3\Theta + 0.04V\Theta - 0.04\Theta^2)</td>
<td>(1.4 + 0.1V + 1.2\Theta + 0.04V\Theta - 0.02\Theta^2)</td>
</tr>
<tr>
<td><strong>Femur</strong></td>
<td>(-7 + 0.03V + 0.4\Theta + 0.01V\Theta - 0.01\Theta^2 - 1.3E - 4V\Theta^2 + 8E - 5\Theta^3)</td>
<td>(9.7 - 0.2V - 0.2\Theta + 0.01V\Theta - 0.005\Theta^2 - 1.3E - 4V\Theta^2 + 8E - 5\Theta^3)</td>
</tr>
</tbody>
</table>

7.8 Discussion

The motorcycle/concrete barrier impact at different impact speeds and impact angles with two different initial configurations was simulated to evaluate the injury sustained by the motorcyclist, either primarily by the concrete barrier or secondarily by the road or motorcycle.

For the sliding configuration of motorcycle and rider, the primary impact of the rider with the road surface at impact angles of less than 18 degrees was the dominant impact causing the severe injury. However, for medium-impact angles, the impact with front parts of the motorcycle
and for 90-degree impact angles, impact with the barrier surface was the dominant impact causing severe injury. The primary impact of the rider’s leg with the barrier was dominant for low- and medium-angle impacts for the upright configuration. But for the secondary impact, the head injury of the motorcyclists for all impact speeds and impact angles was dominant.

Under the upright configuration of the motorcycle and rider, for impact angles less than 18 degrees, the rider primarily stayed at the same traffic side after impact with the barrier, and slid a bit on top of the barrier. For these cases, the primary impacts of the rider’s right knee, thigh, and leg with the barrier took place at the impact side of the rider. After sliding to the barrier, the motorcycle separated from the rider, and the secondary impact of the rider with the road surface occurred. For impact angles larger than 18 degrees, for all impact speeds, the rider vaulted to the other side of the barrier, and the secondary impact took place with the road surface. In these cases, the other side traffic accidents might be more life threatening than the impact with the road surface. Broadly speaking, as the angle of impact increases, the sliding of the motorcycle and the rider is replaced by the direct impact to the barrier. From the impact dynamics point of view, it can be stated that the compression time decreases and the restitution coefficient increases; hence, the impact force increases with the angle of impact. As a result, the motorcycle impacts harshly with the barrier, and the separation of the rider and the rebound of the motorcycle occur faster. In terms of injury levels, the upright configuration results in severe pelvis and femur injuries due to primary impact with the barrier. The sliding cyclist sustains higher head and chest injuries as a result of primary impact with motorcycle frontal parts or barrier surface, even in low-speed impacts.

The HIC values for almost all impact scenarios were less than the critical value for the upright configuration of the cyclist. For high velocity impacts, this could be associated with the
kinematics of the rider vaulting the barrier and coming down with the hands on the ground, thus preventing any direct head impact with the stiff road surface. The trend for sustained head injury to the rider shows a high dependency of HIC values to impact speed and cyclist impact configuration rather than impact angle. However, for the impact angle of 90 degrees, severe head injury occurs when the sliding cyclist impacts with a very stiff barrier.

The trend for the thoracic injury shows a constant positive slope with impact angle and impact speed for both cyclist configurations. Broadly speaking, the upper-body injuries are dominant in the sliding impacts, and lower-body injuries are dominant in the upright impact configuration.

Results from the DOE show that the HIC36 values have a scattered nature, which does not fit well with the polynomial curve. This is due to the high sensitivity of the head kinematics to the initial configuration of the rider, complexity of the exact simulation of the rider, and cycle model. Results for the thoracic and pelvic acceleration and the femur axial load show quite a good fitted polynomial model representing the rider’s injury potential for any impact speeds in the range of 30 to 80 km/h and impact angles in the range of 6 to 90 degrees.

7.9 Conclusions

The objective of this paper was to study the kinematics and potential injury to a motorcycle rider for impacts with a rigid concrete barrier, utilized as a median in highways, or a road side barrier. For this purpose, a multibody motorcycle model and rider were developed using the MADYMO multibody code and validated for 12 degree/60 km/h, 90 degree/32.2 km/h, and 25 degree/46 km/h impacts. The response from the model, including the rider kinematics during impact and acceleration of the center of gravity, and contact force for a specific test configuration, were compared with experimental data obtained from literature. The validated
model was then used for a parametric study using the DOE to investigate the nature of crash injuries for different test speeds at different impact angles and for both upright and sliding configurations.

The results from the design of experiments indicate the complexity of the multibody modeling of the rider and cycle in any impact scenario and its high dependence on the initial conditions of the system. This is more observed for the head-impact injury level due to the high sensitivity of this criterion to the impact direction.

Overall, at low-impact angles, the primary impact of the rider and barrier caused no serious injury due to the rider sliding to the barrier or on the ground and absorbing the major portion of the impact energy even for the high-velocity impacts. It was observed that for more perpendicular impact angles, the direct impact of the cyclist’s head to the barrier for sliding impact resulted in higher head and chest injuries. On the other hand, the absence of primary impact of the rider and barrier resulted in femur and pelvis injuries below the threshold level for all sliding impact scenarios. The low energy-absorbing capability in low- and medium-impact angles caused high pelvis and femur injury levels as a result of the primary impact with the barrier for upright configuration impacts.

### References


Quellet, J., “Environmental Hazards in Motorcycle Accidents,” 26th Annual Proceedings American Association for Automotive Medicine, Canada, 1982.


CHAPTER EIGHT
CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

The main goal of this dissertation was to investigate the fundamental methodologies of impact dynamics and energy dissipations in multibody systems and structures, and to contribute to the different mechanical system applications of modeling contact/impact and energy dissipation. Different approaches to the impact phenomena including stereomechanics, contact mechanics, stress wave propagation, plastic deformation, finite element methods, and the energy method were individually investigated. Advantages and disadvantages of each method were examined, and the suitability of each method and hence the area of application and degree of accuracy of each method were presented.

Studies of crashworthiness, structural impact analysis, and post-crash dynamic behavior of vehicle and aircraft occupants have drawn significant attention from investigators in both industry and academia. Occupant safety has been and continued to be a major concern in the automobile and aerospace industries. Therefore, a crashworthy design must be able to dissipate the kinetic energy of impact in a controlled manner. Proper modeling of contact forces and energy dissipation forms the basis for the design of crashworthy structures. The application of the impact dynamics methodology in different vehicular collision studies and energy management of the crash dynamics to protect the occupants were then presented in this dissertation with different applicational case studies. The case studies represented the different industrial applications of impact dynamics analysis and evaluation of impact energy absorption in different impact scenarios. It was observed that not one impact dynamics methodology of approach would lead to accurate and reasonable results for different crash applications; rather,
different methodologies of impact modeling are required for different crash-event applications based on the design function and requirements. As a general rule, for a crashworthy design, a high stiffness of the occupant compartment area and a high energy-absorbing structural design in other regions far from the occupant compartment is required. The impact impulse mitigation requirement requires the absorption of the large energy impulse and transferring the small contact force to the impactor or impacted body (target), or both. A small contact force requires a large crush distance for one or both impacting bodies, depending on the design goal of the impacting structure.

Different impact design functions for vehicular crash applications were then defined and studied for four test cases. The crash energy management, impulse mitigation, and impact injury biomechanics were investigated for the different test cases. Finite element analysis and multibody dynamics analysis were carried out to investigate the different approaches to vehicle impact design, and for each case, results were obtained and conclusions were drawn.

In the first case study, the influence of a side underride guard height of a truck on the cabin intrusion and occupant injury potential of a small car in car/large-truck side crashes were investigated. This was a test case for which the main design function was the dissipation or management of the crash energy. One observation from this study was that the size of the impacting car posed a significant effect on the passenger compartment intrusion as well as the injury potential to the occupants. It was found that for a small-size car, the addition of the side guard reduced the probability of severe injury to the occupants of the car by about 250 percent compared to the case with no side-guard configuration. Furthermore, it prevented the probability of underriding of the small car, which would increase the injury potential catastrophically in the case of impact with a moving truck or trailer. One main conclusion from this study was that as
long as the maneuverability of the large truck and trailer was in the desirable range, any design in the height range of 20-23 in (508-584 mm) seemed to correspond to the lowest injury potential to the small-car occupants. Hence, this study highlighted the importance of truck side guards in reducing the severity of impacts with colliding passenger cars and thus reducing the potential injuries to the car occupants. The study also provided valuable design guidelines for the manufacturers of these trucks.

In the second case study, the lumbar load attenuation for rotorcraft occupants using a design methodology for a seat-impact energy-absorbing system was investigated. This was a test case for which the main design functions were crash energy management and impulse mitigation. This study provided a modified two-stage energy absorber for a rotorcraft seat system. It was observed that utilizing the modified two-stage energy absorber, with the available stroke designed for a specific seat, the occupant lumbar spine injury level would be significantly attenuated. For any given occupant and seat mass, the recommended two-stage load-limiter design demonstrated an improvement in seat stroke. This improvement was shown to be about 25% for a 50th-percentile aviator with a light moveable seat mass. One main conclusion from this study was that the seat cushion thickness had significant influence on the lumbar load overshoot at certain load limit factors.

The third test case investigated the kinematics and injury potential to different-sized pedestrians impacted by a utility vehicle with a frontal guard. This was a test case for which the main design functions were impulse mitigation and impact-injury reduction.

The main conclusion from this study was that the effect of the addition of a vehicle front guard on the pedestrian injury level depended on the size of the pedestrian. Overall though, for all sizes of pedestrians, the addition of the front guard increased the aggressivity of the light truck
or van in pedestrian/LTV impacts significantly, especially in terms of pedestrian mid-body part injuries. The existence of the guard caused the mid-size and large-size pedestrian to rotate about a higher point on the mid-body region. This led to a head impact of the large-size pedestrian occurring at the middle area of the hood instead of the rear, stiff part of the hood for the case of a car without a guard. Hence, the addition of the guard slightly mitigated the primary head injury for the large-size pedestrian, but the secondary impact with the guard is still significant. It was also found that the addition of the guard increased the head injury and thoracic injury of a 6-year-old child due to direct impact of the head with the stiff guard instead of the body rotating and impacting with the front edge of the car hood.

In the fourth test case, a multibody modeling and design of experiments investigation of a motorcyclist impact on roadside barriers at upright and sliding configurations were investigated. This was a test case for which the main design functions were impulse mitigation and impact-injury reduction. The main conclusion from this study was that overall, at low-impact angles, the primary impacts of the rider and barrier resulted in no serious injury due to the rider sliding to the barrier or on the ground and absorbing the major portion of the impact energy. It was also observed that for more perpendicular impact angles, the direct impact of the cyclist’s head to the barrier for a sliding impact resulted in higher head and chest injuries. The absence of the primary impact of the rider and barrier resulted in the femur and pelvis injury levels below the threshold for all sliding impact scenarios. The low energy-absorbing capability of the motorcycle in low- and medium-impact angles caused high pelvis and femur injury levels as a result of primary impact with the barrier for upright configuration impacts.

As observed from the research on fundamental methodologies for impact analysis and the different crash applications or test cases provided, the analysis of impact in mechanical systems
and structures is quite complex and requires different methods of analysis or approach for investigation. Overall, it can be concluded that the application of multibody mechanical system analysis along with finite element analysis utilize the benefits of the low computational time of multibody analysis and the accurate geometry modeling and detailed structural analysis of finite element analysis. For occupant safety studies, the coupling of two methodologies can be used to obtain accurate results in a reasonable computational time. Structural deformation in medium-impact speeds plays a major role in impact-energy dissipation and occupant protection. Due to the effect of structure efficiency and energy-absorber system design, a design methodology needs to be developed to utilize the higher efficiency of the energy-absorber device. Material modeling and contact detection are major factors in the accuracy of impact modeling and analysis. Due to a wide variety of materials in use or in practice and their different rate dependencies, it is important to have an accurate mathematical modeling of the material behavior under the high rate or impact loading. On the other hand, contact detection and contact force calculation are also important in both finite element and multibody system analysis. Improper modeling of the contact may lead to erroneous results and considerably high computational time, which makes the modeling process ineffective.

8.2 Recommendations

Significant contributions and milestones were made in this dissertation. Some of the areas of research that could be conducted to extend the present work are summarized below.

- The stereomechanics approach could be further expanded to constrained or jointed and spatial multibody mechanical systems. As yet this has not been fully accomplished.
Different methods for the evaluation of the contact forces as well as frictional forces need to be investigated. The applicability of the different contact and frictional force models for impact analysis of mechanical systems and structures need to be further examined.

It is also recommended that the application of the stress wave propagation method in complex impact events be examined utilizing the finite element method. One potential application of such method would be the split Hopkinson pressure bar system, which utilizes the propagation of stress waves in quantifying material properties at high loading rates.

The impact dynamics methodologies discussed in this research were applicable for the impact of solid materials. The impact of solid and fluid materials also needs to be addressed. One such application would be the impact of aircraft structures on water or soil surfaces.

Occupant protection in vehicular accident still needs additional attention and research to address the proper modeling of impact and the proper mechanisms of injuries for occupants of different sizes and ages. Occupant injury mechanisms need to be investigated independent of the test parameters and the direction of impact. Further research needs to be conducted on “active human models” to be able to accurately model the response of live human beings in different crash events.

Special attention must be paid to the design and installation of frontal guards to LTVs and SUVs in terms of regulations to attenuate the potential injury to pedestrians. The addition of compliance tests for the manufacturers of these front guards is also recommended.
• Further research needs to be conducted on the weighting factors for accelerations sustained by occupants and the passenger compartment intrusion in frontal impact scenario. It would also be beneficial to investigate the allowable limits for each intrusion value in order to standardize the limits and to unify the method of approach for these types of studies, so that the base values for the injury thresholds for specific AIS values can be determined.

• Results from the rotorcraft seat occupant study indicated the need for further experimental testing to define the accurate limit load factor independent of environmental and system variables and based on human tolerable limits.