EVALUATING SPATIAL ORIENTATION AND POSITION OF AN ATD HEAD USING ACCELEROMETERS AND ANGULAR RATE SENSORS IN DYNAMIC IMPACT TESTING

A Thesis by

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The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Mechanical Engineering.

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ABSTRACT

Using three linear accelerometers and three angular rate sensors arranged to measure local acceleration in the X, Y, and Z directions and angular velocity about those axes, it is possible to calculate spatial orientation and position in a global coordinate system. The intent of this thesis is to use this calculation to provide the head trajectory of an Anthropomorphic Test Device (ATD) to supplement or replace photometric analysis. This thesis examines the various parameters of the calculations of the spatial orientation and position to determine the most accurate and efficient method. Using the local angular velocity as an input, this method determines spatial orientation as a function of a unit quaternion by numerically solving a system of ordinary differential equation.

The parameters of the numerical integration examined are the numerical integration methods, time step, and order of rotation. These functions are examined through simulation data generated by various MADYMO models. The MADYMO three-dimensional multi-body simulations output the linear accelerations and angular velocity of selected bodies in simulations similar to the data provided from accelerometers and angular rate sensors during dynamic impact testing. Simulation data is useful in the examination and validation of the different parameters used in the method due to the lack of noise and gravitational effects incurred during physical dynamic impact testing.

The method is evaluated for dynamic impact testing through a comparison between the calculated spatial orientation and position using the algorithm and photometric analysis as well as physical limitations in the test setup, i.e. rigid bulkhead. The method is demonstrated to be successfully implemented into the NIAR Crash Dynamics Laboratory at Wichita State University.
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LIST OF SYMBOLS

$\omega$  Angular Velocity

$A$  Global Acceleration

$D$  Global Displacement

$G$  Global Gravity

$P$  Global Position

$V$  Global Velocity

$c$  Initial Gravity Reading Correction

$k$  Intermediate Integration Equation

$a$  Local Acceleration

$g$  Local Gravity

$\varphi$  Phi (rotation about X)

$\psi$  Psi (rotation about Z)

$q$  Quaternion

$R$  Rotation (Transformation) Matrix

$\theta$  Theta (rotation about Y)

$h$  Time Step
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<td>AS</td>
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<td>ATD</td>
<td>Anthropomorphic Test Device</td>
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<td>CFC</td>
<td>Channel Frequency Class</td>
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<td>CFR</td>
<td>Code of Federal Regulations</td>
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<td>CG</td>
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<td>HIC</td>
<td>Head Impact Criteria</td>
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<td>Hybrid II 50th Percentile Male</td>
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<td>SAE</td>
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CHAPTER 1
INTRODUCTION

Dynamic impact testing is used in research, industry, and academia to assess occupant safety and survivability. These tests can range from full vehicle impact behavior to child restraint systems and follow different federal regulations. In this paper, the focus of testing will be aircraft seats and interiors. It is necessary to evaluate the performance of seats as well as, and in combination with, aircraft interiors, to wholly determine the possible injuries that can be sustained by the occupant.

1.1 Background

The study of biomechanics can be dated back to the 16th century to Galilei (1564-1642) and his methods of measurement techniques in biology and Harvey’s (1578-1658) discovery of blood circulation [1]. Biomechanics, closely related to physiology, is the science of applying mechanical principles to biological systems. The purpose of biomechanical research is to increase knowledge of the properties of the human body and to essentially achieve advancements in bioengineering. This research supports a wide variety of fields such as: medical diagnosis and treatment, rehabilitation, sports, safety and health, medical instruments and prosthetic devices, and comfort and ergonomics [1].

This thesis concentrates on the field of injury biomechanics, the behavior of the human body due to mechanical loading. The objectives and research methods of injury biomechanics are described by Viano as:

“The broad goal of injury biomechanics research is to understand the injury process and to develop ways to reduce or eliminate the structural and functional damage that can
occur in an impact environment. To achieve this goal, researchers must identify and define the mechanisms of impact injury, quantify the responses of body tissues and systems to a range of impact conditions, determine the level of response at which the tissues or systems will fail to recover, develop protective materials and structures that reduce the level of impact energy and force delivered to the body, and develop test devices and computer models that respond to impact in a human like manner, so that protective systems can be accurately evaluated [2].”

The end result of this thesis shall provide a method to better assess ‘protective materials and structures’ of aircraft, or vehicle, interiors to ‘reduce the level of impact energy and force delivered to the body,’ specifically the head.

Currently in dynamic impact testing, anthropomorphic test devices (ATDs), more commonly known as crash test dummies, are the occupants of choice. Due to obvious reasons, human volunteers, and the complexity of using human cadavers, as in the infancy of injury biomechanics, are not optimal, or even practical, subjects for research. ATDs are specifically designed to mimic the human body and its movement characteristics; they are designed for simplicity, anthropometry, bio-fidelity, repeatability, reproducibility, sensitivity, durability, and cost [1]. ATDs can be implemented with various instrumentation (sensors) such as accelerometers to measure acceleration, angular rate sensors to measure angular velocity, load cells to measure force, and potentiometers to measure deflection. This thesis will focus on the acceleration and angular velocity of the ATD head.

The output of the accelerometers in the head of the ATD measure acceleration in a local frame. This local acceleration, when integrated, does not provide a meaningful velocity or displacement when the local frame rotates relative to the global frame. However, with the
angular velocity known, and applied to the algorithm evaluated in this thesis, the local acceleration can be transformed into the global frame and, further, integrated to obtain velocity and displacement in the global frame.

Presently, the sole method to determine global velocity and displacement of a dynamic impact test is by photometric analysis, a cumbersome evaluation. Photometric analysis is completed by ‘tracking’ targets on the ATD by high-speed digital video of the event and is influenced by a range of parameters including camera/sensor type, lens size, focal length, pixel size, video resolution. To accommodate these factors, a lens calibration needs to be performed for each camera-lens-focus setting combination. The calibration provides lens correction; this is required because the convex shape of the lens distorts the image and therefore, the distance between two points on that image. Lens correction is the process of mathematically displacing the measured position of targets from their original location in the image plane to the location where they would have been if no lens distortion was present in the system [3].

Photometric analysis can be done two different ways, two dimensionally in the X-Z plane or three dimensionally. Two dimensional photometric analysis is the more common and traditionally used method. One camera is placed so that its optical axis is parallel to the plane of motion. Three dimensional photometric analysis is relatively new and looks to be a promising improvement upon two dimensional analysis; however, many more reference targets are needed to successfully accomplish the analysis.

Software developments in the programs utilized for analyzing the digital video have significantly improved upon previous imperfections such as parallax, perspective, depth correction, and scale factor, but some flaws still exits. According to Andy Mackey, a Research Technician in the Crash Dynamics Laboratory at the National Institute for Aviation Research
who has completed photometric analysis’ for the past five years, these flaws include target visibility, pattern recognition, and target location. The pattern recognition algorithm of the photometric analysis software may not work or may need to be tweaked if the targets are turned too much or have too much glare. The software can also have difficulties finding and mistaking targets, where it will then jump to and track any point nearby that may meet its criteria [4].

The major fault of photometric analysis is target visibility. Photometric analysis requires multiple targets, the target of interest and at least two reference targets, to be visible in the field of view for the frames of interest. As evident from many dynamic tests, the target of interest can become obscured or dislodged by the flailing arms and/or legs of an ATD, seat parts, cables, or even the ATD clothing. Actions have been taken to prevent this from occurring, such as mounting auxiliary targets to the head (also known as ‘mohawk’ targets) that can be tracked while the main target is obscured from view. The distance and angle between two auxiliary targets can be used to calculate the position of the main target. However, in some cases both the main and auxiliary targets become obscured resulting in either no valid data for that portion of the test, or an interpolation of the curve during that portion of the test.

Using the data provided from the accelerometers and angular rate sensors in the head of an ATD can provide the displacement and velocity where traditionally photometric analysis cannot due to a loss of visibility of targets. The algorithm, and its various methods, evaluated in this thesis will be shown to provide data that is accurate and can be obtained more efficiently than photometric analysis.

1.2 Literature Review

The use of accelerometer combinations to calculate spatial orientation and position have been available since 1975 [5]. At least nine linear accelerometers would be placed in a very
specific orientation and finite distance apart in order to determine linear acceleration and angular acceleration of a rigid-body. These various orientations include 3-2-2-2 [5, 6], 3-3-3 [7], and 6-6-6 [8]. The accuracy of these methods depend on the distance of the accelerometers from each other; therefore, a conflict between accuracy and compactness of the sensor array arises [9]. In situations where space is limited, it becomes impractical to use these large accelerometer orientations. Another limitation of these methods is that it requires double integration, which is prone to error, for both angular and linear displacements [9].

The combination of angular rate sensors and accelerometers has been shown to be more accurate than the linear acceleration methods [10]. With continual technology advancements, angular rate sensors have become ‘ultra small and lightweight’ and can be easily placed in objects of interest with limited space [11].

The use of a triaxial accelerometer, or three orthogonal linear accelerometers, in combination with three angular rate sensors can be used to successfully determine spatial orientation and position. From both Wu et al. [9] and Hu et al. [12], the angular velocity provided from the angular rate sensors, can be calculated into a unit quaternion based transformation matrix through numerical integration in order to determine global acceleration, velocity and displacement from the local sensor outputs. Wu et al. provides the equations necessary for an creating an algorithm [9], while Hu et al. provides a method, Runge-Kutta fourth-order numerical integration, for solving the equation relating angular velocity to the rate of change of the unit quaternions [12]. Documentation describing can be found from Schwab and Meijaard [13] as well as Diebel [14].
1.3 Objective

The objective of this thesis is to determine the most accurate and efficient method to calculate spatial orientation and position of the head of an ATD. This calculation will be accomplished from data provided by linear accelerometers and angular rate sensors located at the center of mass of the ATD head, input into an algorithm resulting in the global acceleration, velocity, and displacement of the head.
CHAPTER 2

METHODOLOGY

2.1 Algorithm

This thesis will evaluate various methods of numerical integration to solve the algorithm, as well as determining the effects of different variables to the final solution. Most equations of the initial algorithm of this paper were obtained from “Using Triaxial Angular Rate Sensor and Accelerometer to Determine Spatial Orientation and Position in Impact Tests,” by Wu [9], and will be referenced as so in the respective chapters.

The algorithm will be initially composed with Microsoft Excel and used as such when processing data for the simulation and dynamic tests in this thesis. When implemented into the NIAR Crash Dynamics Laboratory, the algorithm will be used in DIAdem, a data analysis and report generation program, and written in Visual Basic Script.

The algorithm will be evaluated using three different numerical integration methods: Euler’s Method, Runge-Kutta fourth order, and Runge-Kutta fifth order. Each method will be written and saved as a Microsoft Excel file such that inputs include local acceleration, angular velocity, time step, and initial angles of the subject of interest; while outputs include global acceleration, global velocity, and global displacement (or a relative position if applicable). The algorithm shall also be adjusted when determining what effect the method of rotation (i.e. Byant angles, Euler angles) has on the global outputs.

Once the three algorithms, one for each numerical integration method, have been composed, they will first process a series of verification simulations to determine any discrepancies that may have occurred during composition, such as linking incorrect cells or any
typos. These simulations will be of a simple design to eliminate as many variables as possible. The simulations will be of a pendulum, a ball swinging about a revolute joint, with the only force acting on the system being gravity. This simulations will first be examined in a two dimensional planar system, next a three dimensional system, and lastly a three dimensional system with a bulkhead. The two dimensional system will determine any discrepancies incurred while an object is rotating about a single axis. The data from one simulation can be manipulated and input into the algorithm so that it can be applied to the remaining two axes. The three dimensional system will determine any discrepancies incurred while an object is rotating about multiple axes. Again, the data can be input into the algorithm such that all combinations of rotational axes can be verified. The three dimensional system with a bulkhead will verify the algorithm can successfully process accelerations applied due to contact from a foreign object.

After all verification steps have been completed, the algorithms will then be evaluated on the effects of time step to the output of global accelerations. The ball-pendulum simulations will once again be utilized for this intention. The original time step of the ball-pendulum simulations will be maintained as the standard and compared to a lower value time step, a decrease, and a higher value time step, an increase.

Next, the algorithms will be applied to three test cases, described in section 2.2.2, and evaluated. The algorithm outputs from the test cases will first be evaluated from data acquired through simulations and then evaluated from data acquired through dynamic testing. The output of the algorithm from the simulations will again be examined for any differences due to a change in time step.

All simulations, ball-pendulum and test cases, will use output functions of the simulation itself to verify the accuracy of the algorithm. The dynamic tests of the test cases will utilize
multiple methods to verify the accuracy of the algorithm. These methods include high-speed digital video with known distances marked and identified in said video, a rigidly mounted bulkhead a known distance from a reference point, as well as two dimensional and three dimensional photometric analyses.

2.2 Simulation and Test Requirements

2.2.1 Federal Regulations and Procedural Recommendations

All simulations and tests conducted throughout this thesis will follow components of the federal regulations for the emergency landing dynamic conditions for transport category airplanes, formally known as, 14 CFR 25.562, Title 14 – Aeronautics and Space of the Code of Federal Regulations, Chapter 1 – Federal Aviation Administration, Department of Transportation, Part 25 – Airworthiness standards: Transport category airplanes, Section 562 – Emergency landing dynamic conditions [15]. It is important to note that not all sections of 14 CFR 25.562 will be adhered to during computer simulations and dynamic testing. SAE Aerospace Standard (AS) 8049 Revision B and FAA Advisory Circular (AC) 25.562-1B expand further upon requirements set forth by 14 CFR 25.562 and will also be referenced.

The sections of the federal regulation that will not be adhered to are not relevant to meet the objective of this thesis. The purpose of this thesis is to evaluate the kinematics of an ATD under various loading conditions; these sections denote requirements that must be met for the structural integrity of an aircraft seat, and are not applicable for this thesis due to the use of a rigid seat.

The scope of AS8049 is to provide ‘guidance for test procedures, measurements, equipment, and interpretation of results’ and ‘is also presented to promote uniform techniques and to achieve acceptable data’ [16]. The purpose of AC25.562 is to ‘provide information and
guidance regarding acceptable means of compliance with the requirements of 14 CFR part 25 applicable to dynamic testing of seats.’ As well as, providing ‘background and discussion of the reasoning behind the test procedures’ and describing ‘the test facilities and equipment necessary to conduct the tests’ [17]. There are two tests required by 14 CFR 25.562 identified as Test 1 and Test 2. The parameters of Test 1 are:

“A change in downward vertical velocity (Δv) of not less than 35 feet per second, with the airplane's longitudinal axis canted downward 30 degrees with respect to the horizontal plane and with the wings level. Peak floor deceleration must occur in not more than 0.08 seconds after impact and must reach a minimum of 14g [15].”

The purpose of Test 1 as described in section 5.3.1.1 of AS8049B is:

“Test 1, as a single row test, determines the performance of the system in a test condition where the predominant impact force component is along the spinal column of the occupant, in combination with a forward impact force component. This test evaluates the structural adequacy of the seat, critical pelvis/lumbar column forces, and permanent deformation of the structure under downward and forward combined impact loading and may yield data on Anthropomorphic Test Dummy (ATD) head displacement, velocity, and acceleration time histories [16].”

Figure 2.1. 14 CFR 25.562 Test 1
The use of Test 1 in this thesis is to provide a unique test in which the initial position of the seat and ATD are pitch rearward at 60 degrees. The parameters of Test 2 are:

“A change in forward longitudinal velocity (\( \Delta v \)) of not less than 44 feet per second, with the airplane’s longitudinal axis horizontal and yawed 10 degrees either right or left, whichever would cause the greatest likelihood of the upper torso restraint system (where installed) moving off the occupant’s shoulder, and with the wings level. Peak floor deceleration must occur in not more than 0.09 seconds after impact and must reach a minimum of 16g. Where floor rails or floor fittings are used to attach the seating devices to the test fixture, the rails or fittings must be misaligned with respect to the adjacent set of rails or fittings by at least 10 degrees vertically (i.e., out of Parallel) with one rolled 10 degrees [15].”

Figure 2.2. 14 CFR 25.562 Test 2

The purpose of Test 2 as described in section 5.3.1.2 of AS8049B is:

“Test 2, as a single row seat test, determines the performance of a system in a test condition where the predominant impact force component is along the aircraft longitudinal axis and is combined with a lateral impact force component. This test evaluates the structural adequacy of the seat, permanent deformation of the structure, the pelvic restraint and upper torso restraint (if applicable) behavior and loads, and may yield
data on ATD head displacement, velocity, and acceleration time histories and the seat leg loads imposed on the seat tracks or attachment fittings [16].”

The use of Test 2 in this thesis is to provide a representative test that will provide useful results. The simulations and tests in this thesis can also be described by section 5.3.1.4 of AS8049B:

“For seats place in repetitive rows, an additional test condition, using seats in tandem place at representative fore and aft distance between the seats (seat pitch), similar to Test 2 with or without the floor deformation directly evaluates head and femur injury criteria (the floor deformation is required if the test also demonstrates structural performance). These injury criteria are dependent of seat pitch, occupant location, and the effect of hard structures within the path of head excursions in the -10° to +10° yaw attitude range of the Test 2 conditions. The test procedure using the appropriate data obtained from Test 2 as described in 5.3.6.6 may be an alternative to multiple row testing [16].”

As indicated, it can be further described in section 5.3.6.6 as:

“In some cases, it may not be possible to measure data for head impact injury during the basic test of the seat and restraint system. The design of the surrounding interior may not be known to the designer of the seat system, or the system may be used in several applications with different interior configurations. In such cases, the head strike path and the head velocity along the path shall be documented. This will require careful placement of photo instrumentation cameras and location of targets on the ATD representing the ATD’s head center of mass so that the necessary data can be obtained. These data can be used by the interior designer to ensure either that head impact with the interior will not take place or that, should any unavoidable head impacts occur, they can be evaluated using HIC measurements in subsequent subsystem tests [16].”
This thesis will evaluate three Test Cases based on the requirements or combination of requirements set forth by 14 CFR 25.562 and specified by SAE AS8049B.

2.2.2 Test Cases

Test Case 1 is a 0 degree yaw, 16 g, 44 ft/s impact test. A rigid seat shall be placed on the test apparatus with a 0 degree yaw, within the -10° to +10° yaw attitude range indicated in AS8049B 5.3.1.4. It shall be subjected to an impact acceleration reaching 16 g in no more than 0.09 seconds and a velocity of at least 44 feet per second as indicated in 14 CFR 25.562.

Test Case 2 is a 0 degree yaw, 16 g, 44 ft/s impact test with a rigid bulkhead. This test will share the parameters of Test Case 1, but will include a bulkhead rigidly mounted forward of the seat to simulate an aircraft interior mentioned in section 5.3.6.6 of AS8049B.

Test Case 3 is a 60 degree pitch, 14 g, 35 ft/s impact test. This test shall be subjected to an impact acceleration reaching 14 g in no more than 0.08 seconds and a velocity of at least 35 feet per second while the aircraft’s longitudinal direction, and subsequently the aircraft seat, is pitched rearward 60 degrees above the horizontal.

2.2.3 Test Set Up and ATD Requirements

All simulations and tests in this thesis will adhere to the following sections of AS8049B noting that any sections excluded below are not pertinent to the objective of this thesis:

5.3.8 Procedure to Set Up the Test: Preparation for the tests will involve positioning and securing the ATD, the ATD restraint system, the seat, and the instrumentation. This will be done for the specific critical condition being tested. Preparations that pertain to the normal operation of the test facility, such as safety provisions and the actual procedures for accomplishment of the tests, are specific to the test facility and will not be addressed in this document.
5.3.8.1 The test fixture shall be oriented as required for the given test conditions.

5.3.8.3 Each ATD shall be placed in the seat in a uniform manner to enhance reproducible results. The following suggested procedures have been found to be adequate by previous experience.

a. The friction in a limb joint shall be set so that it barely restrains the weight of the limb when extended horizontally.

b. The ATD should be placed in the center of the seat, in as nearly a symmetrical position as possible.

c. The ATD’s back should be against the seat back or the shim described in paragraph h of the section without clearance. This condition can be achieved if the ATD legs are lifted as it is lowered into the seat. Then, the ATD is pushed back into the seat back as it is lowered the last few inches into the seat pan. Once all lifting devices have been removed from the ATD, it should be rocked slightly to settle it in the seat.

d. The ATD’s knees should be separated approximately 100 mm (4 in).

e. The ATD’s hands should be placed on top of its upper legs, just behind the knees. If tests on crew seats are conducted in a mockup that has aircraft controls, the ATD’s hands should be lightly tied to the controls.

f. The feet shall be in the appropriate position for the type and usage of a seat being tested (flat on the floor, on control pedals or on a 45° footrest for flight crew systems). The feet shall be placed so that the centerlines of the lower legs are approximately parallel, unless the need for placing the feet on aircraft controls dictates otherwise.
5.3.8.4 For tests where the ATD’s head is expected to impact a fixture or another seat back, the head and face of the ATD may be treated with a suitable material to mark head contact areas. The material used must not reduce the resulting HIC values.

5.3.8.5 The restraint system adjustment shall be made as follows. The restraint system shall not be tightened beyond the level that could reasonably be expected in use and the emergency locking device (inertia reel) shall not be locked prior to the impact…If manual adjustment of the restraint system is required, slack shall be removed, and the restraint system should be snug, but not excessively tight, about the ATD. For Test 2, this can normally be determined when two fingers fit snugly between the belt and the pelvis of the ATD.

5.3.8.7 A floor is not required for Test 2 structural tests, but if a floor is installed, it should not influence the behavior of the seat, or unduly restrict the movement of the ATD’s feet. This is a concern especially when floor distortion is applied. A floor should be used for tests conducted to gather head path data [16].

The ATD shall meet the requirements of 49 CFR 572 Subpart B and be of the Hybrid II design.

The ATD instrumentation and coordinate systems, as well as the laboratory coordinate system, are referenced from SAE J211-1 Revision DEC2003 whose purpose is to provide guidelines and recommendations for the techniques of measurement used in impact tests. The frequency response classes for ATD Head accelerations (linear and angular) is Channel Frequency Class (CFC) 1000 and Sled acceleration is CFC 60, unless used to integrate for velocity where it should be CFC 180 [J211]. The coordinate system and sign conventions used are described in Section 7 – Sign Convention of SAE J211 [J211]:

7.1 Right-Handed Coordinate System

To assure consistent vector directions of moments and angular velocities and accelerations produced by vector multiplications all coordinated systems used in vehicle testing will be “right-handed”. Right-handed coordinate system consists of an ordered set of three mutually perpendicular axes (x, y, z) which have a common origin and whose positive directions point in the same directions as the ordered set of the thumb, forefinger and middle finger of the right hand when positioned as shown in [Figure 2.1]. Note that one can choose the positive x-axis to point in the direction of either the thumb, forefinger or middle finger as shown in the orientations 1, 2, 3 of [Figure 2.1]. However, once this decision is made then the positive directions of the y- and z-axes must be as indicated by the corresponding orientation shown in [Figure 2.2].

![Right-hand coordinate system diagram](image)

Figure 2.3. SAE J211 Figure 3 – Right hand coordinate system [18].
7.2 Vehicle and Laboratory Coordinate Systems

For vehicle and laboratory coordinate systems, positive z-axis will be directed downward, positive x-axis will be directed forward relative to the vehicle and positive y-axis will be directed away from the vehicle’s left to its right (see SAE J670 – Vehicle Dynamics Terminology). For structures within the vehicle that have a principle axis of motion such as the steering wheel and column, the vehicle coordinate system may be rotated about the y-axis such that the positive x-axis is directed along the column axis.

7.3 Dummy Coordinate Systems

Coordinate systems can be affixed to any point on the dummy. To determine the orientation of the coordinate axes, the dummy will always be considered to be standing erect. For this posture, the positive y-axis will be directed from the dummy’s left to its right side, the positive z-axis will be directed downward from head to toe, and the positive x-axis will be directed forward. In anatomical terminology, the positive x-axis is directed from the posterior to the anterior (P-A), the positive y-axis is directed form (sic) the left to right (L-R), and the positive z-
axis is directed from superior to inferior (S-I). [Figure 2.3] shows examples of this standardized orientation for coordinate systems attached to a few body points. Note that as the dummy is articulated to sit in a vehicle or if the dummy is articulated for a test, the coordinate systems rotate with their respective dummy parts.

![Figure 2.5. SAE J211 Figure 5 – Orientations of standardized dummy coordinate systems for standing and seated postures [18].](image)

### 7.4 Polarities of Acceleration, Velocity, and Displacement

Positive recorded outputs for these transducers are to be consistent with the positive axes of the coordinate system defined for the specific dummy or vehicle point being measured. For example, a blow to the back of the dummy’s head produces an acceleration in the forward direction (+x) which shall be recorded as a positive acceleration. A blow to the top of the head produces a +z acceleration. A blow to the left side of the head produces a +y acceleration [18].
Any coordinate system in a local frame shall be designated by lower case letters \((x, y, z)\). Any coordinate system in a global frame shall be designated by upper case letters \((X, Y, Z)\). It is important to note the equations provided by Wu utilize a coordinate system where the positive \(x\)-axis is directed forward, the positive \(y\)-axis is directed left, and the positive \(z\)-axis is directed up. Therefore, any data recorded from the ATD must be transferred to the coordinate systems of the algorithm.

2.3 Simulations

All simulations, also referred to as computer models, utilized in this thesis were created by the author specifically for this project. The simulations were composed using the MADYMO software package. The MADYMO, or Mathematical Dynamic Modeling, software package simulates the dynamic behavior of physical systems. Although it can be used for a wide variety of applications, MADYMO was originally developed for studying occupant behavior during car crashes. One of the attributes of MADYMO is the ability to combine, in one simulation program, the capabilities of multi-body systems and finite element systems. [19]

MADYMO, and its 50th percentile Hybrid II occupant model, is a supported code of the FAA Advisory Circular 20-146 Methodology for Dynamic Seat Certification by Analysis for Use in Parts 23, 25, 27, and 29 Airplanes and Rotocraft [20]. Although the models created of the test cases are representative of the dynamic tests conducted, it is not the purpose of this thesis to validate the computer models, solely to use them as a step towards achieving the objective without incurring additional costs related to dynamic testing. With that said, the computer models were compared to high-speed video and data from previous dynamic tests of similar requirements to create the most equivalent model possible.
2.4 Dynamic Impact Tests

The dynamic impact tests completed for this thesis will be conducted in the Crash Dynamics Laboratory in the National Institute for Aviation Research at Wichita State University. The Crash Dynamics Laboratory uses an MTS Model 888.20 Crash Simulator, an accelerator sled. All tests will be set up and conducted as stated in section 2.2 of this thesis, and shall be done so under the direct supervision of the author.

All dynamic tests will use three linear accelerometers and three angular rate sensors to record data during the impact event, which shall be from 0 (T0) to 0.3 seconds. High-speed digital video and any necessary equipment will be used to record each event and provided two dimensional and three dimensional photometric analyses. All photometric analyses shall follow the recommendations provided in the SAE Aerospace Recommended Practice (ARP) 5482, including:

3.2.2 ATD Target Placement: As a minimum requirement, targets shall be placed on the ATD head center of gravity (CG) (as defined in the applicable ATD drawings) and the center of the knee pivot. The head target can be located at the head center of gravity by inserting a pin through the center of the target and through the head flesh into a guide hole in the skull that is at the exact location. The knee shall be fixed to the ATD and not the ATD clothing.

If rotation of the ATD head is of interest, or if the ATD head target might be obscured by ATD motion during the test, a pair of targets may be placed on the side of the head or rigidly mounted not more than 15 cm (6 inches) from the head. The relationship between these additional targets and the CG of the ATD must be
measured and documented. The target mounts should be light in weight to limit the affect on the ATD’s initial position and dynamic performance.

3.2.3 Reference Target Placement: Two Sled Reference Targets shall be mounted rigidly on the sled. These targets shall be separated by at least one third the FOV (Field of View) width, and remain visible in all frames of interest. They shall lie in a plane that is parallel to the Sled XZ plane and on a line that is at a known angle to the Sled XY plane.

3.3 Measurements: The following measurements shall be made after final placement of the camera(s), placement of the ATDs, and adjustment of the restraint system. Any target locations that are affected by floor deformation must be measured after deformation is imposed. The measurements shall have an accuracy of +/- 3.0 mm (0.1 inch) except as noted.

3.3.1 Photometric Target Measurements: The XYZ locations of all targets shall be measure in the Sled Coordinate System with respect to a common origin. However, the X and Z measurements of the ATD targets (usually the head center of gravity and knee targets) are optional. The angle between the line on which the Sled Reference Targets lie and the Sled XY plane shall be measured or calculated. If measured this measurement shall have an accuracy of +/- 0.5 degrees. Additional points measured in order to calculate this angle do not need to be targeted.

3.3.2 Seat and Seat Track Location Measurements: The purpose of measuring the location of a point on the seat is to provide the location of a rigid, known point,
relative to the Sled Reference Targets, to which the derived target paths can be referenced.

For conventional track mounted seats, the XYZ location of the intersection of the centerline of the front stud and the plane of the top of the seat track is the preferred reference point [3].

All data generated during the dynamic impact tests shall be provided in an official test report from the Crash Dynamics Laboratory as well as in a Microsoft Excel file.
CHAPTER 3

ALGORITHM DEVELOPMENT

To determine spatial orientation and position of a rigid body two reference frames, or coordinate systems, are needed. The global frame, also known as the inertial frame, is a fixed coordinated system used to describe the overall spatial orientation and position of the rigid body. The local frame, also known as the body fixed frame, is fixed to the rigid body and therefore moves relative to the global frame. The angular rate sensors and linear accelerometers are fixed to the rigid body (i.e. ATD Head) and compose the local frame [9]. The algorithm used to determine spatial orientation and position in this thesis is based on the unit quaternion, a method used to describe the rotation of an object about a vector in three-dimensional space, and rotates by a reverse Bryant method.

The equation relates local angular velocity to the rate of change of the unit quaternion and is shown in equation (3.1) [9].

\[
\dot{q} = \frac{1}{2} Q(q) \begin{bmatrix} 0 \\ \omega_b \end{bmatrix}
\]  

(3.1)

where

\[
Q(q) = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix}
\]

(3.2)

and when expanded, is shown as

\[
\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}
\]

(3.3)
The equation is a set of ordinary differential equations and, because the angular velocity \( \omega_p \) is known, can be solved by numerical integration.

### 3.1 Numerical Integration

There are various methods of numerical integration with varying complexity, error, and time required to calculate. This thesis will examine the use of the Euler, Runge-Kutta fourth order, and Runge-Kutta fifth order methods of numerical integration to solve the algorithm and any difference in the output. The initial values used in all methods are determined from the initial head angle \((\phi, \theta, \psi)\) and transformed into quaternion values \((q_0, q_1, q_2, q_3)\) by equation (3.4)-(3.7).

\[
q_0 = \left( \cos \left( \frac{\omega}{2} \right) \cos \left( \frac{\theta}{2} \right) \cos \left( \frac{\psi}{2} \right) - \sin \left( \frac{\omega}{2} \right) \sin \left( \frac{\theta}{2} \right) \sin \left( \frac{\psi}{2} \right) \right) \\
q_1 = \left( \cos \left( \frac{\omega}{2} \right) \cos \left( \frac{\theta}{2} \right) \sin \left( \frac{\psi}{2} \right) + \sin \left( \frac{\omega}{2} \right) \sin \left( \frac{\theta}{2} \right) \cos \left( \frac{\psi}{2} \right) \right) \\
q_2 = \left( \cos \left( \frac{\omega}{2} \right) \sin \left( \frac{\theta}{2} \right) \cos \left( \frac{\psi}{2} \right) - \sin \left( \frac{\omega}{2} \right) \cos \left( \frac{\theta}{2} \right) \sin \left( \frac{\psi}{2} \right) \right) \\
q_3 = \left( \cos \left( \frac{\omega}{2} \right) \sin \left( \frac{\theta}{2} \right) \sin \left( \frac{\psi}{2} \right) + \sin \left( \frac{\omega}{2} \right) \cos \left( \frac{\theta}{2} \right) \cos \left( \frac{\psi}{2} \right) \right)
\]

#### 3.1.1 Euler’s Method

Euler’s method is the least complex method and one of the most utilized due to its ease of use; however, its lack of complexity results in it being the least accurate method.

\[
y_j = y_{j-1} + h * f \left( t_{j-1}, y_{j-1} \right) \tag{3.8}
\]

Applying this method to equation (3.3) yields

\[
q_0(t) = q_0(t-1) + h * \frac{1}{2} \left( -q_1(t-1) \omega_x(t-1) - q_2(t-1) \omega_y(t-1) - q_3(t-1) \omega_z(t-1) \right) \tag{3.9}
\]

\[
q_1(t) = q_1(t-1) + h * \frac{1}{2} \left( q_0(t-1) \omega_x(t-1) - q_3(t-1) \omega_y(t-1) + q_2(t-1) \omega_z(t-1) \right) \tag{3.10}
\]
\[ q_2(t) = q_2(t-1) + h \cdot \frac{1}{2} \left( q_3(t-1) \omega_x(t-1) + q_0(t-1) \omega_y(t-1) - q_1(t-1) \omega_x(t-1) \right) \] (3.11)

\[ q_3(t) = q_3(t-1) + h \cdot \frac{1}{2} \left( -q_2(t-1) \omega_x(t-1) + q_1(t-1) \omega_y(t-1) + q_0(t-1) \omega_x(t-1) \right) \] (3.12)

### 3.1.2 Runge-Kutta Fourth Order

The second method, Runge-Kutta fourth order, is the most popular and widely used method due to its mild complexity and yields accurate results.

\[ y_j = y_{j-1} + h \left( \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \right) \] (3.13)

where

\[ k_1 = f(t_{j-1}, y_{j-1}) \] (3.14)

\[ k_2 = f(t_{j-1} + \frac{h}{2}, y_{j-1} + \frac{h}{2} k_1) \] (3.15)

\[ k_3 = f(t_{j-1} + \frac{h}{2}, y_{j-1} + \frac{h}{2} k_2) \] (3.16)

\[ k_4 = f(t_{j-1} + h, y_{j-1} + h k_3) \] (3.17)

Applying this method to equation (3.3) yields

\[ q_0(t) = q_0(t-1) + h \left( \frac{k_{1,0}}{6} + \frac{k_{2,0}}{3} + \frac{k_{3,0}}{3} + \frac{k_{4,0}}{6} \right) \] (3.18)

\[ q_1(t) = q_1(t-1) + h \left( \frac{k_{1,1}}{6} + \frac{k_{2,1}}{3} + \frac{k_{3,1}}{3} + \frac{k_{4,1}}{6} \right) \] (3.19)

\[ q_2(t) = q_2(t-1) + h \left( \frac{k_{1,2}}{6} + \frac{k_{2,2}}{3} + \frac{k_{3,2}}{3} + \frac{k_{4,2}}{6} \right) \] (3.20)

\[ q_3(t) = q_3(t-1) + h \left( \frac{k_{1,3}}{6} + \frac{k_{2,3}}{3} + \frac{k_{3,3}}{3} + \frac{k_{4,3}}{6} \right) \] (3.21)

Equations \( k_{1-4,0-3} \) can be found in Appendix A.

### 3.1.3 Runge-Kutta Fifth Order

The third and final method of numerical integration is the Runge-Kutta fifth order. It is the most accurate method used in this paper; however, it is the most complex and in most cases requires additional unnecessary steps.
\[ y_j = y_{j-1} + \frac{1}{90} h(7k_0 + 32k_2 + 12k_3 + 32k_4 + 7k_5) \]  

(3.22)

where

\[ k_0 = f(t_{j-1}, y_{j-1}) \]  

(3.23)

\[ k_1 = f\left(t_{j-1} + \frac{1}{4} h, y_{j-1} + \frac{1}{4} k_0 h\right) \]  

(3.24)

\[ k_2 = f\left(t_{j-1} + \frac{1}{4} h, y_{j-1} + \left(\frac{1}{8} k_0 + \frac{1}{8} k_1\right) h\right) \]  

(3.25)

\[ k_3 = f\left(t_{j-1} + \frac{1}{2} h, y_{j-1} + \left(-\frac{1}{2} k_1 + k_2\right) h\right) \]  

(3.26)

\[ k_4 = f\left(t_{j-1} + \frac{3}{4} h, y_{j-1} + \left(\frac{3}{16} k_0 + \frac{9}{16} k_3\right) h\right) \]  

(3.27)

\[ k_5 = f\left(t_{j-1} + h, y_{j-1} + \left(-\frac{3}{7} k_0 + \frac{2}{7} k_1 + \frac{12}{7} k_2 - \frac{12}{7} k_3 + \frac{8}{7} k_4\right) h\right) \]  

(3.28)

Applying this method to equation (3.3) yields

\[
q_0(t) = q_{0(t-1)} + \frac{1}{90} h(7k_{00} + 32k_{20} + 12k_{30} + 32k_{40} + 7k_{50})
\]  

(3.29)

\[
q_1(t) = q_{1(t-1)} + \frac{1}{90} h(7k_{01} + 32k_{21} + 12k_{31} + 32k_{41} + 7k_{51})
\]  

(3.30)

\[
q_2(t) = q_{2(t-1)} + \frac{1}{90} h(7k_{02} + 32k_{22} + 12k_{32} + 32k_{42} + 7k_{52})
\]  

(3.31)

\[
q_3(t) = q_{3(t-1)} + \frac{1}{90} h(7k_{03} + 32k_{23} + 12k_{33} + 32k_{43} + 7k_{53})
\]  

(3.32)

Equations \(k_{0-5,0-3}\) can be found in Appendix A.

### 3.2 Transformation and Algorithm Corrections

After the input of the angular velocity into the respective methods the output yields the unit quaternion values describing the orientation at each moment in time. These values are then used in a transformation matrix [9].

\[
\mathbf{R} = \begin{bmatrix}
q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\
2(q_2q_1 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\
2(q_3q_1 - q_0q_2) & 2(q_3q_2 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2
\end{bmatrix}
\]  

(3.33)
Multiplying the transformation matrix by the local accelerometer values provides the global acceleration. Throughout this paper various test cases from both computer simulations and dynamic testing will be examined using this algorithm; however, in dynamic impact tests, the effect of gravity on the accelerometer reading needs to be compensated [9].

The final output of an accelerometer reading \( r \) has three components, the actual acceleration \( a \), the influence due to gravity \( g \), and an initial gravity reading correction \( c \).

\[
r = a + g - c
\]  \hspace{1cm} (3.34)

These gravitation influences are corrected in the calculation of global acceleration. The influence due to gravity in an accelerometer is equivalent to the orientation of the local frame multiplied by gravity in the global frame \( G \). The initial gravity reading correction can be calculated by applying the initial orientation of the local components in the global frame.

\[
g = R^T G
\]  \hspace{1cm} (3.35)

\[
c = R_0^T G
\]  \hspace{1cm} (3.36)

Applying these corrections to acceleration in the global frame results in the following equation for global acceleration [9].

\[
A = R(r + c) - RR^T G = R(r + R_0^T G) - G
\]  \hspace{1cm} (3.37)

The acceleration corrections should only be applied when using data from accelerometers; these corrections need not be used when simulation data is applied. In this case global acceleration is equal to the matrix multiplication of the transformation matrix and the local accelerations.

\[
A = R(a)
\]  \hspace{1cm} (3.38)

Once the global accelerations have been calculated, global velocity and global displacement can be acquired through integration of the global acceleration and double integration of the global acceleration, respectively. The trapezoidal method of integration is used
in the algorithm to attain global velocity and global displacement. Note that if a relative position from a certain point in the global frame is needed, the initial distance values from that point to the initial point of local acceleration and angular rate can be applied to offset the displacement and determine position.

3.3 Verification

The first step, once all the algorithms were generated, is to verify that each functions properly. This can be accomplished by creating a simple ball on a pendulum simulation using the MADYMO software package. The data gathered from the simulation is extremely useful because MADYMO can output both the local linear accelerations as well as the global accelerations of the ball. Therefore, the local acceleration and angular velocity data can be input into the algorithms and the calculated global accelerations can be compared to the simulation global accelerations. Any discrepancies in the algorithms become apparent and any errors (i.e. incorrect values, sign conventions) can be corrected.

The simulation is composed of a ball and a revolute joint. The joint is placed along the Y-axis allowing the ball to swing in the X-Z plane due to acceleration cause by gravity. The data can then be rotated for use in the X-Y, and Y-Z planes while the ball rotates about the Z axis, and X axis, respectively. As shown in multiple Figures in Appendix B, once any inconsistencies in the algorithms are eliminated, the calculated global acceleration and the simulated global acceleration are identical.

Figure 3.1. 2D Ball-Pendulum
Going one step further, another simulation is generated where the ball swings in three dimensions by rotating the revolute joint by 45 degrees so it is between the X and Y axes. The purpose of this is to double check the algorithms and can verify that they work while the ball is rotating about two of its axes and translating in three dimensions. The simulation data can once again be manipulated to check all aspects of the algorithms by changing axes to rotate with X and Y, Y and Z, as well as Z and X.

![Figure 3.2. 3D Ball-Pendulum](image)

The final verification simulation is used to prove that all algorithms still function correctly if the rigid body comes in contact with another object. To show this, a bulkhead is added to the previous 45 degree simulation.

![Figure 3.3. 3D Ball-Pendulum with Bulkhead](image)
CHAPTER 4
SIMULATIONS

Using MADYMO, four simulations are used to evaluate the effects of the different numerical integration methods and different time step values. The ball and pendulum will be utilized again and the first examined in this chapter. The three other simulations will represent three test cases to be evaluated as both a simulation and a dynamic test. All test cases are in accordance, when applicable, with the Code of Federal Regulations, specifically 14 CFR 25.562. These regulations are under Title 14 – Aeronautics and Space, Chapter 1 – Federal Aviation Administration, Department of Transportation, Subchapter C – Aircraft. Part 25 of Title 14 is named ‘Airworthiness Standards: Transport Category Airplanes’ [15]. Test Cases 1 and 2 follow ‘Test 2’ under 14 CFR 25.562, and Test Case 3 follows ‘Test 1’. These Test Cases do not meet all requirements of the previously mentioned regulations; the focus of this paper is on the kinematics of the dummy throughout a simulated or dynamic impact test, not on the structural integrity of an aircraft seat.

4.1 Pendulum

The ball and pendulum simulation, with motion in a single plane, is used to determine any difference between the numerical integration methods with varying time step values. The initial location of the ball is on the positive X-axis. With the revolute joint parallel to the Y-axis, the ball will rotate in the X-Z plane. Three simulations are run with time step values of 0.005 s, 0.001 s, and 0.0005 s. The local linear acceleration and angular velocity of the ball is input into the different algorithms and the calculated global acceleration, velocity, and displacement outputs are compared to the corresponding simulation outputs.
Evaluating the Euler, RK4, and RK5 pendulum simulations for time steps of 0.005 s, 0.001 s, and 0.0005 s, it was determined the difference in the time step for all methods is negligible. This conclusion was made from the extremely minimal difference between the calculated global accelerations, velocities, and displacements and the same outputs provided by the simulation. Although no difference was shown between the time steps, all three methods and their respective three time steps compared identically to the output given from the simulation. A graphical representation is shown in Figure 4.1 with additional graphs in Appendix B.

![Graphical representation](image)

Figure 4.1. 3D pendulum, h=0.001 s, Euler, acceleration.

4.2 Test Case 1 – 0 Degree Yaw, 16 g, 44 ft/s

As discussed in the methodology, Test Case 1 follows 14 CFR 25.562 Subpart B Test 2. As previously mentioned, not all aspects of this regulation are to be followed because the focus of this paper is the kinematics of the ATD, not the structural integrity of an aircraft seat.
Therefore, the parameters of 14 CFR 25.562 Subpart B Test 2, used in Test Case 1 is for the impact pulse, or deceleration of the impact.

### TABLE 4.1

TEST CASE 1 PULSE PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Deceleration (g)</td>
<td>16</td>
</tr>
<tr>
<td>Test Velocity (ft/s)</td>
<td>44</td>
</tr>
<tr>
<td>Time to Peak (s)</td>
<td>0.09</td>
</tr>
</tbody>
</table>

### 4.2.1 MADYMO Model

The MADYMO simulation utilizes a rigid seat, floor, 50th percentile Hybrid II ATD, and a lap seat belt. With the emphasis of the Test Case 1 on the ATD, a rigid seat is used for simplicity as well as uniformity between tests. The seat pan of the rigid seat is composed of a surface plane with a length of 40cm, a width of 50cm, and an angle of 5 degrees off of the X-axis. The seat back surface plane has a height of 66cm, a width of 50cm, and an angle of 13 degrees from the Z-axis. The intersection of the seat pan and seat back is located 40cm above the floor. A surface cylinder is located along the forward most edge of the seat pan. Its purpose is to provide an additional contact to help with the intrusion of the femurs on the seat pan.
Figure 4.2. 0 deg, 16g Model.

A finite element seat belt is used across the Hybrid II ATD along with two segmented belts, one on either side, connecting to the anchor points at the intersection of the seat back and seat pan on each respective side. The FE belt uses a hysteresis slope of 6.0e9, an elastic limit of 0.01 and a material density of 800. The simple segmented belts use a hysteresis slop of 3.0e5 and an elastic limit of 1.5. The contact between the dummy and the seat uses a friction coefficient of 0.30 and a damping coefficient of 1.0. The contact between the dummy and the FE belt uses a friction coefficient of 0.30.
4.2.2 Algorithm Comparison

The results of the three numerical methods were very similar to each other, but maintained a small difference from the outputs of the simulation. The error is evident in the global accelerations calculated from the algorithm. In an attempt to eliminate this error, parameters of the algorithm were modified. These parameters include, again, time step and the transformation method, changing from a reverse Bryant method to Euler parameters.

After evaluating the three numerical methods and various parameter changes to a time of 0.17 seconds, it was concluded that none had any significant impact on the output of the algorithm. The correlation and error between the various displacements are shown in Tables 4.2 and 4.3.
### TABLE 4.2
TEST CASE 1 SIMULATION CORRELATIONS FOR 0.0001 SECOND TIME STEP

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>E.P. Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
<th>Simulation Dx</th>
</tr>
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<td>Euler Dx</td>
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<td>E.P. Dx</td>
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<th>RK5 Dy</th>
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### TABLE 4.3
TEST CASE 1 SIMULATION ERROR FOR 0.0001 SECOND TIME STEP

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<th>Euler Dx</th>
<th>E.P. Dx</th>
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<th>RK5 Dx</th>
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<tr>
<td>Maximum X Displacement (m)</td>
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<td>0.7585</td>
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<tr>
<td>Time at Max X (s)</td>
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<td>0.1695</td>
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<td><strong>2.8042</strong></td>
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<td>Difference from Simulation Y (m)</td>
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TABLE 4.4

TEST CASE 1 SIMULATION CORRELATION FOR 0.000025 SECOND TIME STEP

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<td><strong>Maximum X Displacement (m)</strong></td>
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<td><strong>Time at Max X (s)</strong></td>
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</tr>
<tr>
<td><strong>Difference from Simulation X (m)</strong></td>
<td>0.0206</td>
<td>0.0206</td>
<td>0.0206</td>
<td></td>
</tr>
<tr>
<td><strong>X % Error at Max X</strong></td>
<td>2.7199</td>
<td>2.7108</td>
<td>2.7108</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y Displacement at Max X (m)</strong></td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>-0.0001</td>
</tr>
<tr>
<td><strong>Difference from Simulation Y (m)</strong></td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Y % Error at Max X</strong></td>
<td>0.0061</td>
<td>0.0061</td>
<td>0.0061</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Z Displacement at Max X (m)</strong></td>
<td>-0.5328</td>
<td>-0.5328</td>
<td>-0.5328</td>
</tr>
<tr>
<td><strong>Difference from Simulation Z (m)</strong></td>
<td>0.0363</td>
<td>0.0362</td>
<td>0.0362</td>
</tr>
<tr>
<td><strong>Z % Error at Max X</strong></td>
<td>4.7858</td>
<td>4.7695</td>
<td>4.7695</td>
</tr>
</tbody>
</table>

It is important to note that displacement in the Y direction is not of much interest due to the ATD moving in the X-Z plane and therefore minimal to no motion out of that plane. As seen in the previous tables, the error is not improved when the smaller time step is applied. There is also no difference between Euler’s method using reverse Byrant rotations and Euler’s method using Euler parameters. Examining the difference between the numerical methods, it is determined that the RK4 and RK5 methods result in slightly less error than Euler’s method. Graphical representations of these outputs are shown in Appendix B.
4.3 Test Case 2 – 0 Degree Yaw, 16 g, 44 ft/s, with Bulkhead

4.3.1 MADYMO Model

Test Case 2 is exactly the same as Test Case 1, except that it includes a rigid bulkhead. The bulkhead simulation uses the same model of the rigid seat. The bulkhead is a surface plane located 75.5cm forward of the seat pan and seat back intersection and extends from the floor to a height of 1.0m. The bulkhead is modeled after a rigid wall composed of two sheets of plywood. The contact between the ATD and the rigid bulkhead uses a friction coefficient of 0.85 and a damping coefficient of 1380.
Figure 4.5. 0 deg, 16g Bulkhead Model.

4.3.2 Algorithm Comparison

The bulkhead test case is used to determine a worst case scenario due to the high acceleration values achieved in an extremely short time duration. To investigate these effects the Test Case 2 simulation was extended until 0.3 seconds. The algorithm outputs are similar to Test Case 1, where a difference is identifiable in the global acceleration compared to the simulation outputs. Again this difference is carried into velocity and displacement. The most notable aspect of this Test Case is the amount of error incurred as time increases, especially after contact is made with the bulkhead. The correlations and error for Test Case 2 are in Tables 4.6 and 4.7.
### TABLE 4.6
TEST CASE 2 SIMULATION CORRELATION

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
<th>Simulation Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Dx</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Dx</td>
<td>0.999999738</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Dx</td>
<td>0.999999738</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simulation Dx</td>
<td>0.992982658</td>
<td>0.992900401</td>
<td>0.992900401</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
<th>Simulation Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Dy</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Dy</td>
<td>0.999979904</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Dy</td>
<td>0.999980839</td>
<td>0.999999979</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simulation Dy</td>
<td>0.963204704</td>
<td>0.964338</td>
<td>0.964285667</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
<th>Simulation Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Dz</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Dz</td>
<td>0.99999999</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Dz</td>
<td>0.99999999</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simulation Dz</td>
<td>0.997746531</td>
<td>0.997742213</td>
<td>0.997742213</td>
<td>1</td>
</tr>
</tbody>
</table>

### TABLE 4.7
TEST CASE 2 SIMULATION ERROR

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum X Displacement (m)</td>
<td>0.6166</td>
<td>0.6166</td>
<td>0.6166</td>
</tr>
<tr>
<td>Time at Max X (s)</td>
<td>0.1438</td>
<td>0.1438</td>
<td>0.1438</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0131</td>
<td>0.0130</td>
<td>0.0130</td>
</tr>
<tr>
<td><strong>X % Error at Max X</strong></td>
<td><strong>2.1327</strong></td>
<td><strong>2.1161</strong></td>
<td><strong>2.1161</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max X (m)</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Y % Error at Max X</strong></td>
<td><strong>0.0011</strong></td>
<td><strong>0.0011</strong></td>
<td><strong>0.0011</strong></td>
</tr>
</tbody>
</table>
TABLE 4.7 (continued)

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Displacement at Max X (m)</td>
<td>-0.2338</td>
<td>-0.2338</td>
<td>-0.2338</td>
</tr>
<tr>
<td>Difference from Simulation Z (m)</td>
<td>0.0094</td>
<td>0.0093</td>
<td>0.0093</td>
</tr>
<tr>
<td>Z % Error at Max X</td>
<td>1.5208</td>
<td>1.5028</td>
<td>1.5028</td>
</tr>
</tbody>
</table>

In this test case, the values for RK4 and RK5 in the X and Z directions are exactly the same in both X and Z, while Euler’s method is the same in the Z direction and slightly higher in the X direction.

Due to the amount of error incurred throughout this test case over the duration of the entire output, various parameters were again explored to determine the cause. The parameters modified were time step, integration method, and channel filter class of the local acceleration and angular velocity. The time step was changed from 0.0001 seconds to 0.00001 seconds, utilizing 30,000 samples, and had no significant improvement. Instead of using a trapezoidal integration to achieve the global velocity from global acceleration and the global displacement from global velocity, Simpson’s rule of integration was applied and had no significant improvement. Filtering the local acceleration and angular velocity prior to being input into the algorithm did have an effect on the difference in the global acceleration; however, this effect had no significant impact on the error present in the global displacement. Graphical results are shown in Appendix B.

Another parameter utilized was the normalization of the quaternion values. Noting that the sum of the squares of the four quaternions is equal to one, equation 4.1 can be used during each time step of the algorithm to eliminate any error incurred by this function.

\[ q = \frac{1}{\sqrt{q^\top q}} q^* \]  

(4.1)
The results of the algorithm with the normalization function included in the algorithm are shown in Table 4.8 and 4.9.

**TABLE 4.8**

**TEST CASE 2 SIMULATION CORRELATION WITH NORMALIZATION**

<table>
<thead>
<tr>
<th></th>
<th>Euler D(x)</th>
<th>Euler D(x) Normalization</th>
<th>Simulation D(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler D(x)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euler D(x) Normalization</td>
<td>0.999998764</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Simulation D(x)</td>
<td>0.992982658</td>
<td>0.993166261</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler D(y)</th>
<th>Euler D(y) Normalization</th>
<th>Simulation D(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler D(y)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euler D(y) Normalization</td>
<td>0.999998939</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Simulation D(y)</td>
<td>0.963204704</td>
<td>0.963483827</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler D(z)</th>
<th>Euler D(z) Normalization</th>
<th>Simulation D(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler D(z)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euler D(z) Normalization</td>
<td>0.999999957</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Simulation D(z)</td>
<td>0.997746531</td>
<td>0.99772994</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE 4.9**

**TEST CASE 2 SIMULATION ERROR WITH NORMALIZATION**

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 D(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum X Displacement (m)</td>
<td>0.6166</td>
<td>0.6166</td>
</tr>
<tr>
<td>Time at Max X (s)</td>
<td>0.1438</td>
<td>0.1438</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0131</td>
<td>0.0132</td>
</tr>
<tr>
<td>X % Error at Max X</td>
<td>2.1327</td>
<td>2.1374</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler D(y)</th>
<th>RK4 D(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max X (m)</td>
<td>-0.0001</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Y % Error at Max X</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
</tbody>
</table>
The effect of normalizing the quaternion values at each time step actually shows a slightly higher error in the Dx direction; however, it shows almost no improvement in the Dz directions. Overall it can be determined that the inclusion of the normalization function in the algorithm is negligible.

### TABLE 4.10

**TEST CASE 2 SIMULATION ERROR WITH CFC180 FILTER**

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum X Displacement (m)</td>
<td>0.6166</td>
<td>0.6166</td>
</tr>
<tr>
<td>Time at Max X (s)</td>
<td>0.1438</td>
<td>0.1438</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0131</td>
<td>0.0134</td>
</tr>
<tr>
<td>X % Error at Max X</td>
<td>2.1327</td>
<td>2.1675</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max X (m)</td>
<td>-0.0001</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Y % Error at Max X</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Displacement at Max X (m)</td>
<td>-0.2338</td>
<td>-0.2338</td>
</tr>
<tr>
<td>Difference from Simulation Z (m)</td>
<td>0.0098</td>
<td>0.0101</td>
</tr>
<tr>
<td>Z % Error at Max X</td>
<td>1.5899</td>
<td>1.6311</td>
</tr>
</tbody>
</table>
4.4 Test Case 3 – 60 Degree Pitch, 14 g, 35 ft/s

Test Case 3 follows 14 CFR 25.562 Subpart B Test 1, which is intended to evaluate the delethalization ability of an aircraft seat during loading of the lumbar and spine of an occupant. The requirements to be used in Test Case 3 can be summarized and are shown in Table 4.10

| TABLE 4.11 |
|-----------------------------|--------|
| TEST CASE 3 PULSE PARAMETERS |

<table>
<thead>
<tr>
<th>Peak Deceleration (g)</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Velocity (ft/s)</td>
<td>35</td>
</tr>
<tr>
<td>Time to Peak (s)</td>
<td>0.08</td>
</tr>
</tbody>
</table>

4.4.1 MADYMO Model

The Test Case 3 MADYMO simulation model once again uses a rigid seat, floor, 50th percentile Hybrid II ATD, and lap seat belt. For this model, the dimension of the rigid seat remain the same as the previous two tests; however, to meet the Test 1 requirements, the floor is
pitched up to an angle of 60 degrees above the X-axis, the seat pan is then 65 degrees above the X-axis, and the seat back is 73 degrees off of the Z-axis. The finite element and segmented belts remain the same as previous simulations. A finite element seat belt is used across the Hybrid II ATD along with two segmented belts, one on either side, connecting to the anchor points at the intersection of the seat back and seat pan on each respective side. The FE belt uses a hysteresis slope of 6.0e9, an elastic limit of 0.05 and a material density of 800. The simple segmented belts use a hysteresis slope of 3.0e5 and an elastic limit of 0.1. The contact between the dummy and the seat uses a friction coefficient of 0.30 and a damping coefficient of 1.0. The contact between the dummy and the FE belt uses a friction coefficient of 0.30.

Figure 4.7. 60 deg, 19g Model.
4.4.2 Algorithm Comparison

Test Case 3 was used to compare the algorithm against the simulation when the initial position and kinematics of the ATD are significantly varied from the previous test cases. Again, as in Test Case 2, the time of the simulation went to 0.3 seconds. The correlation and error are shown in Table 4.11 and 4.12.

TABLE 4.12
TEST CASE 3 SIMULATION CORRELATION

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
<th>Simulation Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Dx</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Dx</td>
<td>0.999999951</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Dx</td>
<td>0.999999951</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simulation Dx</td>
<td>0.999545441</td>
<td>0.999549802</td>
<td>0.999549802</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
<th>Simulation Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Dy</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Dy</td>
<td>0.999999978</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Dy</td>
<td>0.999999978</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simulation Dy</td>
<td>0.993790588</td>
<td>0.993812147</td>
<td>0.993812146</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
<th>Simulation Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Dz</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Dz</td>
<td>0.999999515</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Dz</td>
<td>0.999999515</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Simulation Dz</td>
<td>0.999934385</td>
<td>0.999941514</td>
<td>0.999941514</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Euler Dx</td>
<td>RK4 Dx</td>
<td>RK5 Dx</td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td><strong>X Displacement at Max Z (m)</strong></td>
<td>0.2934</td>
<td>0.2934</td>
<td>0.2934</td>
<td></td>
</tr>
<tr>
<td><strong>Difference from Simulation X (m)</strong></td>
<td>0.0159</td>
<td>0.0161</td>
<td>0.0161</td>
<td></td>
</tr>
<tr>
<td><strong>X % Error at Max Z</strong></td>
<td>5.3361</td>
<td>5.4183</td>
<td>5.4183</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y Displacement at Max Z (m)</strong></td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Difference from Simulation Y (m)</strong></td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Y % Error at Max Z</strong></td>
<td>0.0073</td>
<td>0.0073</td>
<td>0.0073</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Maximum Z Displacement (m)</strong></td>
<td>0.2974</td>
<td>0.2974</td>
<td>0.2974</td>
</tr>
<tr>
<td><strong>Time at Max Z (s)</strong></td>
<td>0.2230</td>
<td>0.2230</td>
<td>0.2230</td>
</tr>
<tr>
<td><strong>Difference from Simulation Z (m)</strong></td>
<td>0.0058</td>
<td>0.0062</td>
<td>0.0062</td>
</tr>
<tr>
<td><strong>Z % Error at Max Z</strong></td>
<td>1.9491</td>
<td>2.0901</td>
<td>2.0901</td>
</tr>
</tbody>
</table>

It is important to note that for Test Case 3 examples, the error was determined from the maximum Z displacement rather than the maximum X displacement as in Test Case 1 and Test Case 2. Addressing the numerical methods, the results of the error are higher in the Dx direction and similar in the Dz direction from Test Case 2; RK4 and RK5 have the same amount of error in all directions, while Euler’s method is contains less error. The graphical results are shown in Appendix B.
Figure 4.8. Test Case 3 Head Trajectory, h=0.0001s, Euler
CHAPTER 5
SLED TESTS

The sled tests, or dynamic impact tests, conducted for this thesis utilize the same Test Cases as used for the simulation models. The sled tests were conducted in the Crash Dynamics Laboratory in the National Institute for Aviation Research at Wichita State University. Two tests are run for each test case. The procedure used for each test is similar to that of the simulations; however, additional steps are required to gather essential information for comparing the data generated from the algorithm to the data acquired by photometric analysis. One of these steps includes gathering 3-dimensional locations of the ATD Head CG and any other necessary points of interest through the use of a CMM, specifically a FARO arm. Knowing the initial position of the ATD head and a relative fixed point on the sled or rigid seat, position can be determined in through both the algorithm and photometric analysis. Position, as opposed to displacement used in the simulations, will be the output used to compare the algorithm output against the photometric output. The instrumentation used for all tests is in Table 5.1. The certificate of calibration and other related calibration documents for the instrumentation can be found in Appendix F.
Dynamic tests, given the Crash Dynamics Lab test names, RH11A-01 and RH11A-02 were conducted to meet the requirements of Test Case 1 as previously mentioned. A zero degree mounting plate is attached to the sled, and then the rigid seat, also known as an iron seat, is attached to the zero degree mounting plate. Once the ATD is instrumented with accelerometers and angular rate sensors in its head and all its joints check to meet 1g requirements, the ATD is placed in the rigid seat, positioned correctly, and secured with a lap belt. The lap belt used for all sled tests is an AMSAFE model 2011-1561-2551. Next, the FARO measurements were taken. Without knowing the initial location of the head position cannot be determined; however, the displacement is still able to be calculated. The coordinate system of the FARO measurements was setup so the X-axis is parallel with the fore-aft direction of the sled, the Y-direction is left-right, and Z-direction is up-down, where forward is +X, right is +Y, and down is +Z. The origin of the coordinate system is then transferred to the fixed point on the rigid seat and the points were taken of the Head CG, Head Auxiliary Target #1, Head Auxiliary Target #2, Reference Target #1, and Reference Target #2. These locations are shown in Figure 5.1.
Figure 5.1. Target locations.

Once each test is completed the data acquired is provided by the Crash Dynamics Laboratory. All final test reports are included in Appendix D. The acceleration and angular rate data is input into the algorithm, which now includes the gravitational effect on the accelerometers shown in equation 3.33. All data is converted to Metric for comparability against simulation data. Once the global displacement is calculated, the initial position values are applied to offset the displacement resulting in the global position of the ATD Head CG. This data can be graphically compared to the photometric data in its native state; however, it is necessary to reduce the calculated position to numerically compare to the photometric data. Since the high-speed cameras record at a rate of 1000 frames/second, the photometric analysis
can only provide data with a sample rate of 1000 Hz, or a time step of 0.001 seconds. Therefore, the calculated positions need to be reduced by a factor of 10; the sample rate is 10000 Hz, time step of 0.0001 seconds. To do this, an averaging method was used to average every 10 samples of the calculated position and then every 10\textsuperscript{th} data point was extracted to meet the 1000 Hz sample rate.

5.1 Test Case 1, RH11A-01

For test RH11A-01 the previously mentioned locations and the initial head angles are shown in Table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head CG</td>
<td>-0.3420</td>
<td>-0.2758</td>
<td>-0.9489</td>
</tr>
<tr>
<td>Head Auxiliary #1</td>
<td>-0.4636</td>
<td>-0.3043</td>
<td>-1.0416</td>
</tr>
<tr>
<td>Head Auxiliary #2</td>
<td>-0.4723</td>
<td>-0.3028</td>
<td>-0.9467</td>
</tr>
<tr>
<td>Reference #1</td>
<td>0.4049</td>
<td>0.2093</td>
<td>0.3313</td>
</tr>
<tr>
<td>Reference #2</td>
<td>-0.6391</td>
<td>0.2099</td>
<td>0.3323</td>
</tr>
<tr>
<td>Head Angle</td>
<td>-0.2</td>
<td>4.7</td>
<td>0</td>
</tr>
</tbody>
</table>

The algorithmic results are comparable to the photometric results; however, due to the loss of the head targets, the photometric results are only given until a time of 0.182 seconds. The graphical results are shown in Appendix E. The correlation and error are shown in Tables 5.3 and 5.4.
TABLE 5.3
RH11A-01 CORRELATION

<table>
<thead>
<tr>
<th></th>
<th>Euler Px</th>
<th>RK4 Px</th>
<th>RK5 Px</th>
<th>Photometric Px</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Px</td>
<td>1</td>
<td>0.9999999992</td>
<td>0.99999999993</td>
<td>0.9999994981</td>
</tr>
<tr>
<td>RK4 Px</td>
<td>0.99999999992</td>
<td>1</td>
<td></td>
<td>0.999994981</td>
</tr>
<tr>
<td>RK5 Px</td>
<td>0.99999999999</td>
<td>1</td>
<td>1</td>
<td>0.999994848</td>
</tr>
<tr>
<td>Photometric Px</td>
<td>0.9999994981</td>
<td>0.9999948484</td>
<td>0.999994843</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Py</th>
<th>RK4 Py</th>
<th>RK5 Py</th>
<th>Photometric Py</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Py</td>
<td>1</td>
<td>0.999998427</td>
<td>0.999998031</td>
<td>-0.380471312</td>
</tr>
<tr>
<td>RK4 Py</td>
<td>0.999998427</td>
<td>1</td>
<td></td>
<td>-0.381965644</td>
</tr>
<tr>
<td>RK5 Py</td>
<td>0.999998031</td>
<td>0.999999964</td>
<td>1</td>
<td>-0.38219858</td>
</tr>
<tr>
<td>Photometric Py</td>
<td>-0.380471312</td>
<td>-0.381965644</td>
<td>-0.38219858</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Pz</th>
<th>RK4 Pz</th>
<th>RK5 Pz</th>
<th>Photometric Pz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Pz</td>
<td>1</td>
<td>0.999999926</td>
<td>0.999999921</td>
<td>0.999992506</td>
</tr>
<tr>
<td>RK4 Pz</td>
<td>0.999999926</td>
<td>1</td>
<td></td>
<td>0.999992857</td>
</tr>
<tr>
<td>RK5 Pz</td>
<td>0.999999921</td>
<td>1</td>
<td>1</td>
<td>0.99999287</td>
</tr>
<tr>
<td>Photometric Pz</td>
<td>0.999992506</td>
<td>0.999992857</td>
<td>0.99999287</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE 5.4
RH11A-01 ERROR

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum X Displacement (m)</td>
<td>0.4821</td>
<td>0.4821</td>
<td>0.4821</td>
</tr>
<tr>
<td>Time at Max X (s)</td>
<td>0.1720</td>
<td>0.1720</td>
<td>0.1720</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0074</td>
<td>0.0072</td>
<td>0.0072</td>
</tr>
<tr>
<td>X % Error at Max X</td>
<td>1.5376</td>
<td>1.4904</td>
<td>1.4858</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max X (m)</td>
<td>0.2828</td>
<td>0.2828</td>
<td>0.2828</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0237</td>
<td>0.0237</td>
<td>0.0237</td>
</tr>
<tr>
<td>Y % Error at Max X</td>
<td>4.9140</td>
<td>4.9162</td>
<td>4.9172</td>
</tr>
</tbody>
</table>
Evaluating the numerical methods, it is shown that the RK4 and RK5 methods provide a mild improvement over the algorithm utilizing Euler’s method; however, there is only a minimal improvement of RK5 over RK4.
5.2 Test Case 1, RH11A-02

Test RH11A-02 is the same test setup as RH11A-01. The initial conditions are shown in Table 5.5.

TABLE 5.5
INITIAL POSITIONS FOR RH11A-02

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head CG</td>
<td>-0.351</td>
<td>-0.271</td>
<td>-0.950</td>
</tr>
<tr>
<td>Head Auxiliary #1</td>
<td>-0.472</td>
<td>-0.307</td>
<td>-1.042</td>
</tr>
<tr>
<td>Head Auxiliary #2</td>
<td>-0.480</td>
<td>-0.303</td>
<td>-0.947</td>
</tr>
<tr>
<td>Reference #1</td>
<td>-0.640</td>
<td>0.210</td>
<td>0.331</td>
</tr>
<tr>
<td>Reference #2</td>
<td>0.405</td>
<td>0.209</td>
<td>0.330</td>
</tr>
<tr>
<td>φ (deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ (deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ (deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Head Angle</td>
<td>-0.3</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

As with RH11A-01, the photometric results are provided for the full length of the test; they give data up until 0.179 seconds. The graphical results are shown in Appendix E. The correlation and error for RH11A-02 are shown in Tables 5.6 and 5.7.

TABLE 5.6
RH11A-02 CORRELATION

<table>
<thead>
<tr>
<th></th>
<th>Euler Px</th>
<th>RK4 Px</th>
<th>RK5 Px</th>
<th>Photometric Px</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Px</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Px</td>
<td>0.9999999992</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Px</td>
<td>0.9999999992</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Px</td>
<td>0.999972423</td>
<td>0.999973011</td>
<td>0.999973011</td>
<td>1</td>
</tr>
</tbody>
</table>
### TABLE 5.6 (continued)

<table>
<thead>
<tr>
<th>Euler Py</th>
<th>RK4 Py</th>
<th>RK5 Py</th>
<th>Photometric Py</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Py</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Py</td>
<td>0.999998395</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>RK5 Py</td>
<td>0.999998346</td>
<td>0.999999967</td>
<td>1</td>
</tr>
<tr>
<td>Photometric Py</td>
<td>-0.753946509</td>
<td>-0.754601761</td>
<td>-0.75469753</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Euler Pz</th>
<th>RK4 Pz</th>
<th>RK5 Pz</th>
<th>Photometric Pz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Pz</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Pz</td>
<td>0.999999923</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>RK5 Pz</td>
<td>0.999999923</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Photometric Pz</td>
<td>0.999951949</td>
<td>0.999953704</td>
<td>0.999953704</td>
</tr>
</tbody>
</table>

### TABLE 5.7

**RH11A-02 ERROR**

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum X Displacement (m)</td>
<td>0.4542</td>
<td>0.4542</td>
<td>0.4542</td>
</tr>
<tr>
<td>Time at Max X (s)</td>
<td>0.1720</td>
<td>0.1720</td>
<td>0.1720</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0083</td>
<td>0.0080</td>
<td>0.0080</td>
</tr>
<tr>
<td>X % Error at Max X</td>
<td>1.8194</td>
<td>1.7698</td>
<td>1.7698</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max X (m)</td>
<td>0.2859</td>
<td>0.2859</td>
<td>0.2859</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0195</td>
<td>0.0195</td>
<td>0.0195</td>
</tr>
<tr>
<td>Y % Error at Max X</td>
<td>4.3009</td>
<td>4.3036</td>
<td>4.3039</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Displacement at Max X (m)</td>
<td>0.4724</td>
<td>0.4724</td>
<td>0.4724</td>
</tr>
<tr>
<td>Difference from Simulation Z (m)</td>
<td>0.0143</td>
<td>0.0138</td>
<td>0.0138</td>
</tr>
<tr>
<td>Z % Error at Max X</td>
<td>3.1550</td>
<td>3.0298</td>
<td>3.0298</td>
</tr>
</tbody>
</table>
For RH11A-02 in both the X and Z directions, the RK4 and RK5 both result in the same amount of error. The error of RK4 and RK5 are both slightly less than that of the algorithm using Euler’s method.

5.3 Test Case 2, RH11A-03

For RH11A-03, which follows the Test Case 2 test setup, a rigid bulkhead is placed in front of the ATD. This setup is exactly the same as Test Case 1 except for the inclusion of the bulkhead, knee bolster, and foot stop. The bulkhead used in the sled testing for this thesis was constructed to allow 3D photometric tracking of the Head CG targets throughout the entirety of the test. Normally a bulkhead is composed of two rigid stanchions and multiple sheets of plywood extending most of the width of the sled. This type of bulkhead would result in a loss of the target from the front isometric camera view which is needed in the 3D photometric analysis.
Therefore, a bulkhead with a width less than that of the ATD head is utilize to provide the front isometric camera view with full visibility of the Head CG target throughout the duration of the test. The initial conditions are in Table 5.8.

**TABLE 5.8**  
INITIAL POSITIONS FOR RH11A-03

<table>
<thead>
<tr>
<th></th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head CG</td>
<td>-0.353</td>
<td>-0.280</td>
<td>-0.951</td>
</tr>
<tr>
<td>Reference #1</td>
<td>-0.639</td>
<td>0.209</td>
<td>0.329</td>
</tr>
<tr>
<td>Reference #2</td>
<td>0.404</td>
<td>0.209</td>
<td>0.329</td>
</tr>
<tr>
<td>Bulkhead Point #1</td>
<td>0.390</td>
<td>-0.363</td>
<td>-0.985</td>
</tr>
<tr>
<td>Bulkhead Point #2</td>
<td>0.397</td>
<td>-0.361</td>
<td>-0.615</td>
</tr>
<tr>
<td>Head CG to Bulkhead</td>
<td>0.746</td>
<td></td>
<td></td>
</tr>
<tr>
<td>φ (deg)</td>
<td>0.1</td>
<td>4.4</td>
<td>0</td>
</tr>
<tr>
<td>θ (deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ (deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The photometric data was able to be given for the duration of the test, until 0.3 seconds. The graphical results are shown in Appendix E. The correlation and errors for test RH11A-03 are shown in Tables 5.9 and 5.10.

**TABLE 5.9**  
RH11A-03 CORRELATIONS

<table>
<thead>
<tr>
<th></th>
<th>Euler Px</th>
<th>RK4 Px</th>
<th>RK5 Px</th>
<th>Photometric Px</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Px</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Px</td>
<td>0.9999999194</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Px</td>
<td>0.9999999171</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Px</td>
<td>0.967770784</td>
<td>0.968088532</td>
<td>0.968093041</td>
<td>1</td>
</tr>
</tbody>
</table>
TABLE 5.9 (continued)

<table>
<thead>
<tr>
<th></th>
<th>Euler Py</th>
<th>RK4 Py</th>
<th>RK5 Py</th>
<th>Photometric Py</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Euler Py</strong></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RK4 Py</strong></td>
<td>0.999999063</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RK5 Py</strong></td>
<td>0.999999008</td>
<td>0.999999299</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>Photometric Py</strong></td>
<td>0.982229298</td>
<td>0.982099858</td>
<td>0.982214156</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Pz</th>
<th>RK4 Pz</th>
<th>RK5 Pz</th>
<th>Photometric Pz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Euler Pz</strong></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RK4 Pz</strong></td>
<td>0.999998732</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RK5 Pz</strong></td>
<td>0.999998678</td>
<td>0.999999999</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>Photometric Pz</strong></td>
<td>0.967757251</td>
<td>0.96735543</td>
<td>0.967346776</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE 5.10

RH11A-03 ERROR

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum X Displacement (m)</td>
<td>0.3123</td>
<td>0.3123</td>
<td>0.3123</td>
</tr>
<tr>
<td>Time at Max X (s)</td>
<td>0.1570</td>
<td>0.1570</td>
<td>0.1570</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0007</td>
<td>0.0008</td>
<td>0.0008</td>
</tr>
<tr>
<td><strong>X % Error at Max X</strong></td>
<td><strong>0.2263</strong></td>
<td><strong>0.2636</strong></td>
<td><strong>0.2633</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max X (m)</td>
<td>0.2801</td>
<td>0.2801</td>
<td>0.2801</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0246</td>
<td>0.0242</td>
<td>0.0246</td>
</tr>
<tr>
<td><strong>Y % Error at Max X</strong></td>
<td><strong>7.8666</strong></td>
<td><strong>7.7435</strong></td>
<td><strong>7.8776</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Displacement at Max X (m)</td>
<td>0.7018</td>
<td>0.7018</td>
<td>0.7018</td>
</tr>
<tr>
<td>Difference from Simulation Z (m)</td>
<td>0.0184</td>
<td>0.0184</td>
<td>0.0184</td>
</tr>
<tr>
<td><strong>Z % Error at Max X</strong></td>
<td><strong>5.9019</strong></td>
<td><strong>5.8966</strong></td>
<td><strong>5.8968</strong></td>
</tr>
</tbody>
</table>
The error of the displacement in the X direction is shown to be minimal; however, the error in the Z direction is higher than that in the Test Case 1 examples.

5.4 Test Case 2, RH11A-04

Test RH11A-04 uses the same test setup as RH11A-03. The initial conditions are listed in Table 5.11.
TABLE 5.11
INITIAL POSITIONS FOR RH11A-04

<table>
<thead>
<tr>
<th></th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head CG</td>
<td>-0.3514</td>
<td>-0.2906</td>
<td>-0.9529</td>
</tr>
<tr>
<td>Reference #1</td>
<td>-0.6396</td>
<td>0.21007</td>
<td>0.33146</td>
</tr>
<tr>
<td>Reference #2</td>
<td>0.40472</td>
<td>0.20925</td>
<td>0.32959</td>
</tr>
<tr>
<td>Bulkhead Point #1</td>
<td>0.3948</td>
<td>-0.3598</td>
<td>-1.0921</td>
</tr>
<tr>
<td>Bulkhead Point #2</td>
<td>0.40303</td>
<td>-0.3593</td>
<td>-0.5323</td>
</tr>
<tr>
<td>Head CG to Bulkhead</td>
<td>0.75032</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>φ (deg)</th>
<th>θ (deg)</th>
<th>ψ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head Angle</td>
<td>0.3</td>
<td>4.4</td>
<td>0</td>
</tr>
</tbody>
</table>

The photometric data for RH11A-04 is able to provide data for almost the entirety of the test, up to 0.287 seconds. The graphical results can be found in Appendix E. The correlation and error between the numerical methods and the photometric data is shown in Tables 5.12 and 5.13.

TABLE 5.12
RH11A-04 CORRELATION

<table>
<thead>
<tr>
<th></th>
<th>Euler Px</th>
<th>RK4 Px</th>
<th>RK5 Px</th>
<th>Photometric Px</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Px</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Px</td>
<td>0.999999261</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Px</td>
<td>0.999999258</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Px</td>
<td>0.988669747</td>
<td>0.98885027</td>
<td>0.988850602</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Py</th>
<th>RK4 Py</th>
<th>RK5 Py</th>
<th>Photometric Py</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Py</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Py</td>
<td>0.999986382</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Py</td>
<td>0.999972981</td>
<td>0.999997719</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Py</td>
<td>0.479723263</td>
<td>0.477472985</td>
<td>0.476561368</td>
<td>1</td>
</tr>
</tbody>
</table>
TABLE 5.12 (continued)

<table>
<thead>
<tr>
<th></th>
<th>Euler Pz</th>
<th>RK4 Pz</th>
<th>RK5 Pz</th>
<th>Photometric Pz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Pz</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Pz</td>
<td>0.999998438</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Pz</td>
<td>0.999998412</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Pz</td>
<td>0.990783999</td>
<td>0.990545443</td>
<td>0.990543395</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE 5.13

RH11A-04 ERROR

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum X Displacement (m)</td>
<td>0.3184</td>
<td>0.3184</td>
<td>0.3184</td>
</tr>
<tr>
<td>Time at Max X (s)</td>
<td>0.1550</td>
<td>0.1550</td>
<td>0.1550</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0030</td>
<td>0.0028</td>
<td>0.0028</td>
</tr>
<tr>
<td><strong>X % Error at Max X</strong></td>
<td>0.9366</td>
<td>0.8934</td>
<td>0.8939</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dy</th>
<th>RK4 Dy</th>
<th>RK5 Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max X (m)</td>
<td>0.2794</td>
<td>0.2794</td>
<td>0.2794</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0235</td>
<td>0.0231</td>
<td>0.0235</td>
</tr>
<tr>
<td><strong>Y % Error at Max X</strong></td>
<td>7.3694</td>
<td>7.2508</td>
<td>7.3788</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Displacement at Max X (m)</td>
<td>0.7106</td>
<td>0.7106</td>
<td>0.7106</td>
</tr>
<tr>
<td>Difference from Simulation Z (m)</td>
<td>0.0167</td>
<td>0.0167</td>
<td>0.0167</td>
</tr>
<tr>
<td><strong>Z % Error at Max X</strong></td>
<td>5.2576</td>
<td>5.2491</td>
<td>5.2492</td>
</tr>
</tbody>
</table>
The error of the displacement for test RH11A-04 is higher than that of RH11A-03 in the X direction and slightly improved in the Z direction.

5.5 Test Case 3, RH11A-05

RH11A-05 utilizes the parameters of Test Case 3, where the rigid seat is place on a fixture modified to pitch backwards 60 degrees to meet the Test 1 requirements. The same origin is used for test RH11A-05 and RH11A-06; however, the coordinate system is setup in respect to the sled as opposed to the rigid seat in its pitched position. All measurement points were taken after the dummy was placed into the seat, buckled, and seat pitched back. No auxiliary targets are needed on the ATD head for this type of test due to the different kinematic motion of the ATD; not much forward motion or flailing of the arms occurs. The initial conditions for test RH11A-05 are shown in Table 5.14.
The photometric data for RH11A-05 is able to provide data for almost the entirety of the test, up to 0.298 seconds. The graphical results can be found in Appendix E. The correlation and error between the numerical methods and the photometric data is shown in Tables 5.15 and 5.16.
<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X Displacement at Max Z (m)</strong></td>
<td>-0.4821</td>
<td>-0.4821</td>
<td>-0.4821</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0384</td>
<td>0.0384</td>
<td>0.0384</td>
</tr>
<tr>
<td><strong>X % Error at Max Z</strong></td>
<td>6.8376</td>
<td>6.8429</td>
<td>6.8429</td>
</tr>
<tr>
<td></td>
<td>Euler Dy</td>
<td>RK4 Dy</td>
<td>RK5 Dy</td>
</tr>
<tr>
<td><strong>Y Displacement at Max Z (m)</strong></td>
<td>0.3151</td>
<td>0.3151</td>
<td>0.3151</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0150</td>
<td>0.0150</td>
<td>0.0150</td>
</tr>
<tr>
<td><strong>Y % Error at Max Z</strong></td>
<td>2.6694</td>
<td>2.6663</td>
<td>2.6663</td>
</tr>
<tr>
<td></td>
<td>Euler Dz</td>
<td>RK4 Dz</td>
<td>RK5 Dz</td>
</tr>
<tr>
<td><strong>Maximum Z Displacement (m)</strong></td>
<td>0.5611</td>
<td>0.5611</td>
<td>0.5611</td>
</tr>
<tr>
<td>Time at Max Z (s)</td>
<td>0.1890</td>
<td>0.1890</td>
<td>0.1890</td>
</tr>
<tr>
<td>Difference from Simulation Z (m)</td>
<td>0.0513</td>
<td>0.0512</td>
<td>0.0512</td>
</tr>
<tr>
<td><strong>Z % Error at Max Z</strong></td>
<td>9.1420</td>
<td>9.1170</td>
<td>9.1170</td>
</tr>
</tbody>
</table>
5.6 Test Case 3, RH11A-06

Test RH11A-06 uses the same test setup as test RH11A-05. The initial conditions are listed in Table 5.17.

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head CG</td>
<td>-1.000</td>
<td>-0.277</td>
<td>-0.192</td>
</tr>
<tr>
<td>Reference #1</td>
<td>-0.487</td>
<td>0.249</td>
<td>1.020</td>
</tr>
<tr>
<td>Reference #2</td>
<td>0.557</td>
<td>0.247</td>
<td>1.020</td>
</tr>
<tr>
<td>Head Angle</td>
<td>0</td>
<td>-56.1</td>
<td>0</td>
</tr>
</tbody>
</table>

*TABLE 5.17*

INITIAL POSITIONS FOR RH11A-06

Figure 5.6. Test Case 3, RH11A-05 Head Trajectory, Euler
The photometric data for RH11A-06 is able to provide data for almost the entirety of the test, up to 0.294 seconds. The graphical results can be found in Appendix E. The correlation and error between the numerical methods and the photometric data is shown in the tables below.

**TABLE 5.18**

**RH11A-06 CORRELATION**

<table>
<thead>
<tr>
<th></th>
<th>Euler Px</th>
<th>RK4 Px</th>
<th>RK5 Px</th>
<th>Photometric Px</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Px</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Px</td>
<td>0.999999998</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Px</td>
<td>0.999999998</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Px</td>
<td>0.999944054</td>
<td>0.999944125</td>
<td>0.999944125</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Py</th>
<th>RK4 Py</th>
<th>RK5 Py</th>
<th>Photometric Py</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Py</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Py</td>
<td>0.999999994</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Py</td>
<td>0.999999994</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Py</td>
<td>0.979093512</td>
<td>0.979111493</td>
<td>0.97911156</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Pz</th>
<th>RK4 Pz</th>
<th>RK5 Pz</th>
<th>Photometric Pz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Pz</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK4 Pz</td>
<td>0.999999521</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RK5 Pz</td>
<td>0.999999521</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Photometric Pz</td>
<td>0.987705238</td>
<td>0.987854088</td>
<td>0.987854088</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE 5.19**

**RH11A-06 ERROR**

<table>
<thead>
<tr>
<th></th>
<th>Euler Dx</th>
<th>RK4 Dx</th>
<th>RK5 Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>X Displacement at Max Z (m)</td>
<td>-0.4563</td>
<td>-0.4563</td>
<td>-0.4563</td>
</tr>
<tr>
<td>Difference from Simulation X (m)</td>
<td>0.0212</td>
<td>0.0212</td>
<td>0.0212</td>
</tr>
<tr>
<td>X % Error at Max Z</td>
<td><strong>3.8604</strong></td>
<td><strong>3.8640</strong></td>
<td><strong>3.8640</strong></td>
</tr>
</tbody>
</table>
TABLE 5.19 (continued)

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Displacement at Max Z (m)</td>
<td>0.2750</td>
<td>0.2750</td>
<td>0.2750</td>
</tr>
<tr>
<td>Difference from Simulation Y (m)</td>
<td>0.0021</td>
<td>0.0021</td>
<td>0.0021</td>
</tr>
<tr>
<td>Y % Error at Max Z</td>
<td>0.3806</td>
<td>0.3805</td>
<td>0.3805</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Euler Dz</th>
<th>RK4 Dz</th>
<th>RK5 Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Z Displacement (m)</td>
<td>0.5490</td>
<td>0.5490</td>
<td>0.5490</td>
</tr>
<tr>
<td>Time at Max Z (s)</td>
<td>0.1890</td>
<td>0.1890</td>
<td>0.1890</td>
</tr>
<tr>
<td>Difference from Simulation Z (m)</td>
<td>0.0286</td>
<td>0.0285</td>
<td>0.0285</td>
</tr>
<tr>
<td>Z % Error at Max Z</td>
<td>5.2158</td>
<td>5.1931</td>
<td>5.1931</td>
</tr>
</tbody>
</table>

Figure 5.7. Test Case 3, RH11A-06 Head Trajectory, Euler
CHAPTER 6
IMPLEMENTATION

A need is present to implement the algorithm for use in the Crash Dynamics Laboratory in the National Institute for Aviation Research at Wichita State University. For all previously mentioned examples, a spreadsheet was utilized with the algorithm to calculate the global values from the data acquired from the local instrumentation. This method is not very efficient and requires data from multiple inputs to be exported into the Microsoft Excel format. It is not practical to utilize this method on a daily basis. Currently, the Crash Dynamics Laboratory uses National Instruments DIAdem™ software package for data analyzing and report generation for all tests. The algorithm used in this thesis can be written in Visual Basic Script and used in DIAdem™. The script described below is solely to run the algorithm of this thesis; additional measures are taken to fully implement the algorithm into the Crash Dynamics Laboratory report generation script. The full script is located in Appendix G.

The first step in the script determines if the file enabling the variables used through multiple scripts of the Crash Dynamics Laboratory is activated, and if not, will do so. This process is accomplished by using an if-else-end statement. The script written to execute the algorithm was done in a way to reduce the number of variables; however, a few variables are still needed and utilized. Next, the values of the head angular rate sensor are converted from their native unit of degree/second to radians/second. All channels used in the algorithm are copied and inserted into the Data Portal with an extension on their original channel name. This is done because of the difference in the coordinate systems used by the SAE standard and of those used
in the algorithm. Therefore, the next step in the script is to invert the head accelerations and angular velocities of the Y and Z directions.

Two time channels are created, one with the time step values and one with the time value. A time step channel is not necessarily needed, however, it was used in this script to allow and change in the future without modifying the value of the time step in all the algorithmic calculations. Now only one value needs to be changed to use a different time step in the algorithm. Since all of the time step values are the same, the channel can be created by inserting the chosen time step for the duration of the channel. Most tests conducted at the Crash Dynamics Laboratory run until 0.3 seconds, or according to DIAdem, from the channel values 1 to 3001. Therefore, the time step, 0.0001 seconds in this instance, is inserted into the ‘Time Step’ channel for all 3001 instances. The time channel is created using a do-loop statement, where variable ‘A1_’ determines both the channel row number and the time value. ‘A1_’ is set to equal 1, the do-loop statement is started and set to repeat until ‘A1_’ is equal to 3002. The desired value is input into the selected channel (‘Time’) row number (‘A1_’) with the value of ‘A1_/10000’, where the sample rate is 10000 Hz, or frequency is 0.0001 seconds. The value of ‘A1_’ is then increased by one and the statement is repeated until the ‘A1_’ value of 3002 is met.

The initial values of the head angle are needed to determine the initial quaternion values for the utilized Runge-Kutta fourth order method. A dialog box is called and appears; the dialog box request the initial head angle in the \( \phi \), \( \theta \), and \( \psi \) directions in degrees and determines what unit set the user would prefer the results, English or Metric. The initial values input into the dialog box are first saved as the variables ‘A4_’, ‘A5_’, and ‘A6_’ for the \( \phi \), \( \theta \), and \( \psi \) directions, respectively. These values are then input into the newly created channel ‘Head Initial Values’ where values 1-3 of the channel are the \( \phi \), \( \theta \), and \( \psi \) head angles in degrees, 4-6 of the channel are
the same head angles in radians, and then 7-10 are the calculated initial quaternion values $q_0(0)$,
$q_1(0)$, $q_2(0)$, $q_3(0)$.

Next, all of the channels used within the algorithm are created; the quaternion channels
(0-3), Runge-Kutta channels (1-4, 0-3), and the transformation matrix values (1-3, 1-3). The first
set of values for the Runge-Kutta channels and quaternion channels are calculated using the data
from the ‘Head Initial Values’ channel input into equations (A.1-A.16) From the second value
of the channels to the last, a do-loop statement is used. The do-loop cannot be used for the first
value due to the fact the equations require the script to extract values from the ‘Head Initial
Values’, while during the loop, those same values are extracted from the previous channel value.

The calculation of the transformation matrix follows equation (3.33) and again uses a do-
loop statement to calculate all values. After this, the ‘Global Head Ax’, ‘Global Head Ay’, and
‘Global Head Az’ channels are created and calculated using a do-loop statement for all values.
The global head acceleration values are calculated in g’s. The script then splits into two seconds
using an if-then-else statement depending on the user selection of English or Metric units. In
both cases the overall process is the same; only scale is different based on the units. The global
head accelerations are changed from a numerical channel to a waveform channel based on the
‘Time’ channel created previously. The global head acceleration channels are integrated
(DIAadem uses the trapezoidal method) to achieve the global head velocities. The global head
velocities in the Y and Z direction are converted into the waveform type channel based on the
‘Time’ channel. X, Y, and Z global head velocities are scaled (by 32.1741 to get ft/s, or 9.80665
to get m/s). The sled velocity is subtracted from the global head velocity in the X direction; this
operation results in the new ‘Global Head Vx’ which is natively in the waveform channel format
from the subtraction operation. The new ‘Global Head Vx’ and ‘Vy’ and ‘Vz’ channels are then
integrated to displacement. The displacement channels are transformed into a waveform channel and then scaled by 12 to get inches or 100 for centimeters. Lastly in the if-else statement, the units are applied to all channels.

The last part of the script determines the time of the maximum and minimum values. Using that specific command, the values are found and are then stored into the properties section of each channel, ‘MINMAX_MAXTIME’ and ‘MINMAX_MINTIME’. These values in the properties section allow these values to be recalled in the report. The maximum and minimum values are natively included in the properties, therefore, only the time at which they occur needs to be calculated.

Not included in this script is the conversion from global displacement to position. This conversion is included in a different script and is relatively simple. The script is composed of a user dialog box to input the initial positions and the command to offset the respective displacement channels by the input values. This is not included with the main algorithm script as the output of displacement or position may change by client preference.
CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The work of this thesis has provided a better understanding of the algorithm presented. Through multiple variations and parameter changes, an algorithm was not found to be accurate for the entire time duration presented; however it can successfully approximate the maximum excursion of an ATD head for 14 CFR 25.562 Test 2. As seen from the data in the previous chapters and the graphs in Appendices B and E, the algorithm provides an accurate representation of the global displacement or position until approximately 0.1 to 0.2 seconds; at that point the displacement or position begins to diverge from the data provided by either the simulation or the photometric analysis.

This algorithm and method of calculating global displacement or position may be used to substitute photometric analysis. Benefits of using this method include not having to use specialized software costing upwards of $30,000, greatly reduced time to provide the head trajectory data, and it will provide consistent results. Traditional photometric analysis can take, at the very least, one hour to four hours to calculate and, depending if there is a backlog, can take half a day to a week to reach the customer. Photometric analyses for the same test can vary depending on the software package used and on the user completing the analysis. The algorithm depends solely on the data recorded from the instrumentation and the initial head angle. This can be implemented at various locations and used with different sled systems with consistency.

Multiple variations of the algorithm were applied to eliminate error caused through the integration of the global acceleration to global velocity and furthermore to global displacement.
These methods included using a smaller time step and changing from a trapezoidal method of integration to utilizing Simpson’s rule. The results of applying these changes did not result in any significant improvements in the overall output of the global displacement.

Additional variations of the algorithm were then applied; prior to the transformation calculation. These changes were deemed necessary due to the error present between the algorithm and simulation global accelerations. In an attempt to correct this, the input local acceleration and angular velocity were filtered at multiple filter classes and in various combinations with each other. A normalization equation was also applied to the quaternion values. Neither of these processes had a significant change in the overall output of the global displacement.

7.2 Recommendations

From the information and data provided from this thesis, it is recommended that further investigation is warranted between the calculation for the transformation of the local acceleration and angular velocity into the global acceleration. As previously mentioned, the error between the algorithm calculated global acceleration and the provided simulation global acceleration is considerable. This error is the cause of the global displacement values diverging from their expected values. Other recommendations include utilizing different combinations of error-eliminating parameters such as using normalization in conjunction with filtered data. Also, adjusting the full scale value of the instrumentation during dynamic testing for a more accurate reading.
REFERENCES


APPENDICES
EQUATIONS OF NUMERICAL INTEGRATION METHODS FOR ANGULAR VELOCITY TRANSFORMATIONS

**Euler’s Method**

\[ y_j = y_{j-1} + h \times f(t_{j-1}, y_{j-1}) \]  \hspace{1cm} (3.4)

\[ q_0(t) = q_{0(t-1)} + h \times \frac{1}{2} (-q_1(t-1)\omega_x(t-1) - q_2(t-1)\omega_y(t-1) - q_3(t-1)\omega_z(t-1)) \]  \hspace{1cm} (3.5)

\[ q_1(t) = q_{1(t-1)} + h \times \frac{1}{2} (q_0(t-1)\omega_x(t-1) - q_3(t-1)\omega_y(t-1) + q_2(t-1)\omega_z(t-1)) \]  \hspace{1cm} (3.6)

\[ q_2(t) = q_{2(t-1)} + h \times \frac{1}{2} (q_3(t-1)\omega_x(t-1) + q_0(t-1)\omega_y(t-1) - q_1(t-1)\omega_z(t-1)) \]  \hspace{1cm} (3.7)

\[ q_3(t) = q_{3(t-1)} + h \times \frac{1}{2} (-q_2(t-1)\omega_x(t-1) + q_1(t-1)\omega_y(t-1) + q_0(t-1)\omega_z(t-1)) \]  \hspace{1cm} (3.8)
APPENDIX A (continued)

Runge-Kutta Fourth Order

\[ y_j = y_{j-1} + h \left( \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \right) \]  \hspace{1cm} (3.9)

where

\[ k_1 = f(t_{j-1}, y_{j-1}) \]  \hspace{1cm} (3.10)
\[ k_2 = f(t_{j-1} + \frac{h}{2}, y_{j-1} + \frac{h}{2} k_1) \]  \hspace{1cm} (3.11)
\[ k_3 = f(t_{j-1} + \frac{h}{2}, y_{j-1} + \frac{h}{2} k_2) \]  \hspace{1cm} (3.12)
\[ k_4 = f(t_{j-1} + h, y_{j-1} + h k_3) \]  \hspace{1cm} (3.13)

Therefore

\[ q_0(t) = q_{0(t-1)} + h \left( \frac{k_{1,0}}{6} + \frac{k_{2,0}}{3} + \frac{k_{3,0}}{3} + \frac{k_{4,0}}{6} \right) \]  \hspace{1cm} (3.14)
\[ q_1(t) = q_{1(t-1)} + h \left( \frac{k_{1,1}}{6} + \frac{k_{2,1}}{3} + \frac{k_{3,1}}{3} + \frac{k_{4,1}}{6} \right) \]  \hspace{1cm} (3.15)
\[ q_2(t) = q_{2(t-1)} + h \left( \frac{k_{1,2}}{6} + \frac{k_{2,2}}{3} + \frac{k_{3,2}}{3} + \frac{k_{4,2}}{6} \right) \]  \hspace{1cm} (3.16)
\[ q_3(t) = q_{3(t-1)} + h \left( \frac{k_{1,3}}{6} + \frac{k_{2,3}}{3} + \frac{k_{3,3}}{3} + \frac{k_{4,3}}{6} \right) \]  \hspace{1cm} (3.17)

where

\[ k_{1,0} = \frac{1}{2} \left( -q_1(t-1) \omega_x(t-1) - q_2(t-1) \omega_y(t-1) - q_3(t-1) \omega_z(t-1) \right) \]  \hspace{1cm} (A.1)
\[ k_{1,1} = \frac{1}{2} \left( q_0(t-1) \omega_x(t-1) - q_3(t-1) \omega_y(t-1) + q_2(t-1) \omega_z(t-1) \right) \]  \hspace{1cm} (A.2)
\[ k_{1,2} = \frac{1}{2} \left( q_3(t-1) \omega_x(t-1) + q_0(t-1) \omega_y(t-1) - q_1(t-1) \omega_z(t-1) \right) \]  \hspace{1cm} (A.3)
\[ k_{1,3} = \frac{1}{2} \left( -q_2(t-1) \omega_x(t-1) + q_1(t-1) \omega_y(t-1) + q_0(t-1) \omega_z(t-1) \right) \]  \hspace{1cm} (A.4)
\[ k_{2,0} = \frac{1}{2} \left( -\left( q_{1(t-1)} + \frac{h}{2} k_{1,1} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) - \left( q_{2(t-1)} + \frac{h}{2} k_{1,2} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) - \left( q_{3(t-1)} + \frac{h}{2} k_{1,3} \right) \left( \frac{\omega_z(t-1) + \omega_z(t)}{2} \right) \right) \] (A.5)

\[ k_{2,1} = \frac{1}{2} \left( \left( q_{0(t-1)} + \frac{h}{2} k_{1,0} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) - \left( q_{3(t-1)} + \frac{h}{2} k_{1,3} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) + \left( q_{2(t-1)} + \frac{h}{2} k_{1,2} \right) \right) \] (A.6)

\[ k_{2,2} = \frac{1}{2} \left( \left( q_{3(t-1)} + \frac{h}{2} k_{1,3} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) + \left( q_{0(t-1)} + \frac{h}{2} k_{1,0} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) - \left( q_{1(t-1)} + \frac{h}{2} k_{1,1} \right) \right) \] (A.7)

\[ k_{2,3} = \frac{1}{2} \left( -\left( q_{2(t-1)} + \frac{h}{2} k_{1,2} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) + \left( q_{1(t-1)} + \frac{h}{2} k_{1,1} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) + \left( q_{0(t-1)} + \frac{h}{2} k_{1,0} \right) \right) \] (A.8)

\[ k_{3,0} = \frac{1}{2} \left( -\left( q_{1(t-1)} + \frac{h}{2} k_{2,1} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) - \left( q_{2(t-1)} + \frac{h}{2} k_{2,2} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) - \left( q_{3(t-1)} + \frac{h}{2} k_{2,3} \right) \right) \] (A.9)

\[ k_{3,1} = \frac{1}{2} \left( \left( q_{0(t-1)} + \frac{h}{2} k_{2,0} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) - \left( q_{3(t-1)} + \frac{h}{2} k_{2,3} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) + \left( q_{2(t-1)} + \frac{h}{2} k_{2,2} \right) \right) \] (A.10)

\[ k_{3,2} = \frac{1}{2} \left( \left( q_{3(t-1)} + \frac{h}{2} k_{2,3} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) + \left( q_{0(t-1)} + \frac{h}{2} k_{2,0} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) - \left( q_{1(t-1)} + \frac{h}{2} k_{2,1} \right) \right) \] (A.11)

\[ k_{3,3} = \frac{1}{2} \left( -\left( q_{2(t-1)} + \frac{h}{2} k_{2,2} \right) \left( \frac{\omega_x(t-1) + \omega_x(t)}{2} \right) + \left( q_{1(t-1)} + \frac{h}{2} k_{2,1} \right) \left( \frac{\omega_y(t-1) + \omega_y(t)}{2} \right) + \left( q_{0(t-1)} + \frac{h}{2} k_{2,0} \right) \right) \] (A.12)
\[ k_{4,0} = \frac{1}{2} \left( -(q_{1(t-1)} + h k_{3,1}) \omega_x(t) - (q_{2(t-1)} + h k_{3,2}) \omega_y(t) - (q_{3(t-1)} + h k_{3,3}) \omega_z(t) \right) \]  
(A.13)

\[ k_{4,1} = \frac{1}{2} \left( (q_{0(t-1)} + h k_{3,0}) \omega_x(t) - (q_{3(t-1)} + h k_{3,3}) \omega_y(t) + (q_{2(t-1)} + h k_{3,2}) \omega_z(t) \right) \]  
(A.14)

\[ k_{4,2} = \frac{1}{2} \left( (q_{3(t-1)} + h k_{3,3}) \omega_x(t) + (q_{0(t-1)} + h k_{3,0}) \omega_y(t) - (q_{1(t-1)} + h k_{3,1}) \omega_z(t) \right) \]  
(A.15)

\[ k_{4,3} = \frac{1}{2} \left( -(q_{2(t-1)} + h k_{3,2}) \omega_x(t) + (q_{1(t-1)} + h k_{3,1}) \omega_y(t) + (q_{0(t-1)} + h k_{3,0}) \omega_z(t) \right) \]  
(A.16)
APPENDIX A (continued)

Runge-Kutta Fifth Order

\[ y_j = y_{j-1} + \frac{1}{90} h (7k_0 + 32k_2 + 12k_3 + 32k_4 + 7k_5) \]  

(3.18)

where

\[ k_0 = f(t_{j-1}, y_{j-1}) \]  

(3.19)

\[ k_1 = f \left( t_{j-1} + \frac{1}{4} h, y_{j-1} + \frac{1}{4} k_0 h \right) \]  

(3.20)

\[ k_2 = f \left( t_{j-1} + \frac{1}{4} h, y_{j-1} + \left( \frac{1}{8} k_0 + \frac{1}{8} k_1 \right) h \right) \]  

(3.21)

\[ k_3 = f \left( t_{j-1} + \frac{1}{2} h, y_{j-1} + \left( -\frac{1}{2} k_1 + k_2 \right) h \right) \]  

(3.22)

\[ k_4 = f \left( t_{j-1} + \frac{3}{4} h, y_{j-1} + \left( \frac{3}{16} k_0 + \frac{9}{16} k_3 \right) h \right) \]  

(3.23)

\[ k_5 = f \left( t_{j-1} + h, y_{j-1} + \left( -\frac{3}{7} k_0 + \frac{2}{7} k_1 + \frac{12}{7} k_2 - \frac{12}{7} k_3 + \frac{8}{7} k_4 \right) h \right) \]  

(3.24)

Therefore

\[ q_0(t) = q_{0(t-1)} + \frac{1}{90} h (7k_{0,0} + 32k_{2,0} + 12k_{3,0} + 32k_{4,0} + 7k_{5,0}) \]  

(A.14)

\[ q_1(t) = q_{1(t-1)} + \frac{1}{90} h (7k_{0,1} + 32k_{2,1} + 12k_{3,1} + 32k_{4,1} + 7k_{5,1}) \]  

(A.18)

\[ q_2(t) = q_{2(t-1)} + \frac{1}{90} h (7k_{0,2} + 32k_{2,2} + 12k_{3,2} + 32k_{4,2} + 7k_{5,2}) \]  

(A.19)

\[ q_3(t) = q_{0(t-1)} + \frac{1}{90} h (7k_{0,3} + 32k_{2,3} + 12k_{3,3} + 32k_{4,3} + 7k_{5,3}) \]  

(A.20)
where

\[ k_{0,0} = \frac{1}{2} \left( -q_1(t-1)\omega_x(t-1) - q_2(t-1)\omega_y(t-1) - q_3(t-1)\omega_z(t-1) \right) \]  \hspace{1cm} (A.21)\\
\[ k_{0,1} = \frac{1}{2} \left( q_0(t-1)\omega_x(t-1) - q_3(t-1)\omega_y(t-1) + q_2(t-1)\omega_z(t-1) \right) \]  \hspace{1cm} (A.22)\\
\[ k_{0,2} = \frac{1}{2} \left( q_3(t-1)\omega_x(t-1) + q_0(t-1)\omega_y(t-1) - q_1(t-1)\omega_z(t-1) \right) \]  \hspace{1cm} (A.23)\\
\[ k_{0,3} = \frac{1}{2} \left( -q_2(t-1)\omega_x(t-1) + q_1(t-1)\omega_y(t-1) + q_0(t-1)\omega_z(t-1) \right) \]  \hspace{1cm} (A.24)
\[ k_{1,0} = \]
\[
\frac{1}{2} \left( -\left( q_{1(t-1)} + \frac{1}{4} k_{0,1} h \right) \left( \frac{\left( \omega_x(t-1) + \omega_y(t) \right)}{2} + \omega_x(t-1) \right) \right) - \\
\left( q_{2(t-1)} + \frac{1}{4} k_{0,2} h \right) \left( \frac{\left( \omega_y(t-1) + \omega_y(t) \right)}{2} + \omega_y(t-1) \right) - \left( q_{3(t-1)} + \frac{1}{4} k_{0,3} h \right) \left( \frac{\left( \omega_x(t-1) + \omega_z(t) \right)}{2} + \omega_x(t-1) \right) \]  
\( (A.25) \)

\[ k_{1,1} = \]
\[
\frac{1}{2} \left( q_{0(t-1)} + \frac{1}{4} k_{0,0} h \right) \left( \frac{\left( \omega_x(t-1) + \omega_x(t) \right)}{2} + \omega_x(t-1) \right) - \left( q_{3(t-1)} + \frac{1}{4} k_{0,3} h \right) \left( \frac{\left( \omega_y(t-1) + \omega_x(t) \right)}{2} + \omega_y(t-1) \right) + \\
\left( q_{2(t-1)} + \frac{1}{4} k_{0,2} h \right) \left( \frac{\left( \omega_y(t-1) + \omega_z(t) \right)}{2} + \omega_y(t-1) \right) \]  
\( (A.26) \)

\[ k_{1,2} = \]
\[
\frac{1}{2} \left( q_{3(t-1)} + \frac{1}{4} k_{0,3} h \right) \left( \frac{\left( \omega_x(t-1) + \omega_x(t) \right)}{2} + \omega_x(t-1) \right) + \left( q_{0(t-1)} + \frac{1}{4} k_{0,0} h \right) \left( \frac{\left( \omega_y(t-1) + \omega_x(t) \right)}{2} + \omega_y(t-1) \right) - \\
\left( q_{1(t-1)} + \frac{1}{4} k_{0,1} h \right) \left( \frac{\left( \omega_x(t-1) + \omega_z(t) \right)}{2} + \omega_x(t-1) \right) \]  
\( (A.27) \)
\[
\begin{align*}
k_{1,3} &= \frac{1}{2} \left(-\left(q_{2(t-1)} + \frac{1}{4} k_{0,2} h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right) + \\
&\left(q_{1(t-1)} + \frac{1}{4} k_{0,1} h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right) + \left(q_{0(t-1)} + \frac{1}{4} k_{0,0} h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right)\right) \\
(A.28) \\
\end{align*}
\]

\[
\begin{align*}
k_{2,0} &= \frac{1}{2} \left(-\left(q_{1(t-1)} + \left(\frac{1}{8} k_{0,1} + \frac{1}{8} k_{1,1}\right) h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right) - \left(q_{2(t-1)} + \frac{1}{8} k_{0,2} + \\
&\frac{1}{8} k_{1,2} h\right) \left(\frac{\left(\omega_y(t-1) + \omega_x(t-1)\right)}{2}\right) - \left(q_{3(t-1)} + \left(\frac{1}{8} k_{0,3} + \frac{1}{8} k_{1,3}\right) h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right)\right) \\
(A.29) \\
\end{align*}
\]

\[
\begin{align*}
k_{2,1} &= \frac{1}{2} \left(\left(q_{0(t-1)} + \left(\frac{1}{8} k_{0,0} + \frac{1}{8} k_{1,0}\right) h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right) - \left(q_{3(t-1)} + \frac{1}{8} k_{0,3} + \\
&\frac{1}{8} k_{1,3} h\right) \left(\frac{\left(\omega_y(t-1) + \omega_x(t-1)\right)}{2}\right) + \left(q_{2(t-1)} + \left(\frac{1}{8} k_{0,2} + \frac{1}{8} k_{1,2}\right) h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right)\right) \\
(A.30) \\
\end{align*}
\]

\[
\begin{align*}
k_{2,2} &= \frac{1}{2} \left(\left(q_{3(t-1)} + \left(\frac{1}{8} k_{0,3} + \frac{1}{8} k_{1,3}\right) h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right) + \left(q_{0(t-1)} + \frac{1}{8} k_{0,0} + \\
&\frac{1}{8} k_{1,0} h\right) \left(\frac{\left(\omega_y(t-1) + \omega_x(t-1)\right)}{2}\right) - \left(q_{1(t-1)} + \left(\frac{1}{8} k_{0,1} + \frac{1}{8} k_{1,1}\right) h\right) \left(\frac{\left(\omega_x(t-1) + \omega_y(t-1)\right)}{2}\right)\right) \\
(A.31) \\
\end{align*}
\]
\[ k_{2,3} = \frac{1}{2} \left( -q_2(t-1) + \left( \frac{1}{8} k_{0,2} + \frac{1}{8} k_{1,2} \right) h \left( \frac{(\omega_x(t-1) + \omega_x(t))}{2} \right) + q_1(t-1) + \left( \frac{1}{8} k_{0,1} + \frac{1}{8} k_{1,1} \right) h \left( \frac{(\omega_y(t-1) + \omega_y(t))}{2} \right) \right) \]

\[ = \frac{1}{8} k_{1,1} h \left( \frac{(\omega_y(t-1) + \omega_y(t))}{2} \right) - \left( q_0(t-1) + \left( \frac{1}{8} k_{0,0} + \frac{1}{8} k_{1,0} \right) h \left( \frac{(\omega_x(t-1) + \omega_x(t))}{2} \right) \right) \]

(A.32)

\[ k_{3,0} = \]

\[ \frac{1}{2} \left( -q_1(t-1) + \left( \frac{1}{2} k_{1,1} + k_{2,1} \right) h \left( \frac{(\omega_x(t-1) + \omega_x(t))}{2} \right) - \left( q_2(t-1) + \left( \frac{1}{2} k_{1,2} + k_{2,2} \right) h \left( \frac{(\omega_y(t-1) + \omega_y(t))}{2} \right) \right) \right) \]

(A.33)

\[ k_{3,1} = \]

\[ \frac{1}{2} \left( \left( q_0(t-1) + \left( \frac{1}{2} k_{1,0} + k_{2,0} \right) h \left( \frac{(\omega_x(t-1) + \omega_x(t))}{2} \right) \right) - \left( q_3(t-1) + \left( \frac{1}{2} k_{1,3} + k_{2,3} \right) h \left( \frac{(\omega_y(t-1) + \omega_y(t))}{2} \right) \right) \right) \]

(A.34)

\[ k_{3,2} = \]

\[ \frac{1}{2} \left( \left( q_3(t-1) + \left( \frac{1}{2} k_{1,3} + k_{2,3} \right) h \left( \frac{(\omega_x(t-1) + \omega_x(t))}{2} \right) \right) + \left( q_0(t-1) + \left( \frac{1}{2} k_{1,0} + k_{2,0} \right) h \left( \frac{(\omega_y(t-1) + \omega_y(t))}{2} \right) \right) \right) - \left( q_1(t-1) + \left( \frac{1}{2} k_{1,1} + k_{2,1} \right) h \left( \frac{(\omega_x(t-1) + \omega_x(t))}{2} \right) \right) \]

(A.35)
\[ k_{3,3} = \]
\[ \frac{1}{2} \left( -\left( q_{2(t-1)} + \left( -\frac{1}{2} k_{1,2} + k_{2,2} \right) h \right) \left( \frac{\omega_{y(t-1)} + \omega_{x(t)}}{2} \right) + \right. \]
\[ \left. \left( q_{1(t-1)} + \left( -\frac{1}{2} k_{1,1} + k_{2,1} \right) h \right) \left( \frac{\omega_{y(t-1)} + \omega_{x(t)}}{2} \right) - \left( q_{0(t-1)} + \left( -\frac{1}{2} k_{1,0} + k_{2,0} \right) h \right) \left( \frac{\omega_{y(t-1)} + \omega_{x(t)}}{2} \right) \right) \]
\[ (A.36) \]
\begin{equation}
k_{4,0} = \frac{1}{2} \left( - \left( q_{1(t-1)} + \left( \frac{3}{16} k_{0,1} + \frac{9}{16} k_{3,1} \right) h \left( \frac{(\omega x(t-1) + \omega x(t)) + \omega x(t)}{2} \right) \right) - \left( q_{2(t-1)} + \left( \frac{3}{16} k_{0,2} + \frac{9}{16} k_{3,2} \right) h \left( \frac{(\omega y(t-1) + \omega y(t)) + \omega y(t)}{2} \right) \right) \right)
\end{equation}
\begin{equation}
(A.37)
\end{equation}

\begin{equation}
k_{4,1} = \frac{1}{2} \left( q_{0(t-1)} + \left( \frac{3}{16} k_{0,0} + \frac{9}{16} k_{3,0} \right) h \left( \frac{(\omega x(t-1) + \omega x(t)) + \omega x(t)}{2} \right) \right) - \left( q_{3(t-1)} + \left( \frac{3}{16} k_{0,3} + \frac{9}{16} k_{3,3} \right) h \left( \frac{(\omega y(t-1) + \omega y(t)) + \omega y(t)}{2} \right) \right)
\end{equation}
\begin{equation}
(A.38)
\end{equation}

\begin{equation}
k_{4,2} = \frac{1}{2} \left( q_{3(t-1)} + \left( \frac{3}{16} k_{0,3} + \frac{9}{16} k_{3,3} \right) h \left( \frac{(\omega x(t-1) + \omega x(t)) + \omega x(t)}{2} \right) \right) + \left( q_{0(t-1)} + \left( \frac{3}{16} k_{0,0} + \frac{9}{16} k_{3,0} \right) h \left( \frac{(\omega y(t-1) + \omega y(t)) + \omega y(t)}{2} \right) \right)
\end{equation}
\begin{equation}
(A.39)
\end{equation}

\begin{equation}
k_{4,3} = \frac{1}{2} \left( - \left( q_{2(t-1)} + \left( \frac{3}{16} k_{0,2} + \frac{9}{16} k_{3,2} \right) h \left( \frac{(\omega x(t-1) + \omega x(t)) + \omega x(t)}{2} \right) \right) + \left( q_{1(t-1)} + \left( \frac{3}{16} k_{0,1} + \frac{9}{16} k_{3,1} \right) h \left( \frac{(\omega y(t-1) + \omega y(t)) + \omega y(t)}{2} \right) \right) \right)
\end{equation}
\begin{equation}
(A.40)
\end{equation}
APPENDIX A (continued)

\[ k_{5,0} = \frac{1}{2} \left\{ \left( -q_1(t-1) + \left( -\frac{3}{7} k_{0,1} + \frac{2}{7} k_{1,1} + \frac{12}{7} k_{2,1} - \frac{12}{7} k_{3,1} + \frac{8}{7} k_{4,1} \right) h \right) (\omega_x(t)) - \left( q_2(t-1) + \left( -\frac{3}{7} k_{0,2} + \frac{2}{7} k_{1,2} + \frac{12}{7} k_{2,2} - \frac{12}{7} k_{3,2} + \frac{8}{7} k_{4,2} \right) h \right) (\omega_y(t)) - \left( q_3(t-1) + \left( -\frac{3}{7} k_{0,3} + \frac{2}{7} k_{1,3} + \frac{12}{7} k_{2,3} - \frac{12}{7} k_{3,3} + \frac{8}{7} k_{4,3} \right) h \right) (\omega_z(t)) \right\} \] (A.41)

\[ k_{5,1} = \frac{1}{2} \left\{ \left( q_0(t-1) + \left( -\frac{3}{7} k_{0,0} + \frac{2}{7} k_{1,0} + \frac{12}{7} k_{2,0} - \frac{12}{7} k_{3,0} + \frac{8}{7} k_{4,0} \right) h \right) (\omega_x(t)) - \left( q_3(t-1) + \left( -\frac{3}{7} k_{0,3} + \frac{2}{7} k_{1,3} + \frac{12}{7} k_{2,3} - \frac{12}{7} k_{3,3} + \frac{8}{7} k_{4,3} \right) h \right) (\omega_y(t)) + \left( q_2(t-1) + \left( -\frac{3}{7} k_{0,2} + \frac{2}{7} k_{1,2} + \frac{12}{7} k_{2,2} - \frac{12}{7} k_{3,2} + \frac{8}{7} k_{4,2} \right) h \right) (\omega_z(t)) \right\} \] (A.42)

\[ k_{5,2} = \frac{1}{2} \left\{ \left( q_3(t-1) + \left( -\frac{3}{7} k_{0,3} + \frac{2}{7} k_{1,3} + \frac{12}{7} k_{2,3} - \frac{12}{7} k_{3,3} + \frac{8}{7} k_{4,3} \right) h \right) (\omega_x(t)) + \left( q_0(t-1) + \left( -\frac{3}{7} k_{0,0} + \frac{2}{7} k_{1,0} + \frac{12}{7} k_{2,0} - \frac{12}{7} k_{3,0} + \frac{8}{7} k_{4,0} \right) h \right) (\omega_y(t)) - \left( q_1(t-1) + \left( -\frac{3}{7} k_{0,1} + \frac{2}{7} k_{1,1} + \frac{12}{7} k_{2,1} - \frac{12}{7} k_{3,1} + \frac{8}{7} k_{4,1} \right) h \right) (\omega_z(t)) \right\} \] (A.43)

\[ k_{5,3} = \frac{1}{2} \left\{ \left( -q_2(t-1) + \left( -\frac{3}{7} k_{0,2} + \frac{2}{7} k_{1,2} + \frac{12}{7} k_{2,2} - \frac{12}{7} k_{3,2} + \frac{8}{7} k_{4,2} \right) h \right) (\omega_x(t)) + \left( q_1(t-1) + \left( -\frac{3}{7} k_{0,1} + \frac{2}{7} k_{1,1} + \frac{12}{7} k_{2,1} - \frac{12}{7} k_{3,1} + \frac{8}{7} k_{4,1} \right) h \right) (\omega_y(t)) - \left( q_0(t-1) + \left( -\frac{3}{7} k_{0,0} + \frac{2}{7} k_{1,0} + \frac{12}{7} k_{2,0} - \frac{12}{7} k_{3,0} + \frac{8}{7} k_{4,0} \right) h \right) (\omega_z(t)) \right\} \] (A.44)
APPENDIX B

GRAPHICAL RESULTS OF SIMULATION TEST CASES

2-D Pendulum

Figure B.1. 2D pendulum, h=0.005 s, Euler, acceleration.
APPENDIX B (continued)

Figure B.2. 2D pendulum, h=0.005 s, RK4, acceleration.

Figure B.3. 2D pendulum, h=0.005 s, RK5, acceleration.
APPENDIX B (continued)

Figure B.4. 2D pendulum, h=0.001 s, Euler, acceleration.

Figure B.5. 2D pendulum, h=0.001 s, RK4, acceleration.
APPENDIX B (continued)

Figure B.6. 2D pendulum, h=0.001 s, RK5, acceleration.

Figure B.7. 2D pendulum, h=0.0005 s, Euler, acceleration.
APPENDIX B (continued)

Figure B.8. 2D pendulum, h=0.0005 s, RK4, acceleration.

Figure B.9. 2D pendulum, h=0.0005 s, RK5, acceleration.
APPENDIX B (continued)

3-D Pendulum

Figure B.10. 3D pendulum, h=0.001 s, Euler, acceleration.

Figure B.11. 3D pendulum, h=0.001 s, RK4, acceleration.
APPENDIX B (continued)

Figure B.12. 3D pendulum, h=0.001 s, RK5, acceleration.

3-D Pendulum with Bulkhead

Figure B.13. 3D pendulum with bulkhead, h=0.001 s, Euler, acceleration.
APPENDIX B (continued)

Test Case 1

Figure B.14. Test Case 1, h=0.0001 s, Euler, acceleration.

Figure B.15. Test Case 1, h=0.0001 s, Euler, velocity.
Figure B.16. Test Case 1, h=0.0001 s, Euler, displacement.

Figure B.17. Test Case 1, h=0.0001 s, RK4, acceleration.
APPENDIX B (continued)

Figure B.18. Test Case 1, h=0.0001 s, RK4, velocity.

Figure B.19. Test Case 1, h=0.0001 s, RK4, displacement.
APPENDIX B (continued)

Figure B.20. Test Case 1, h=0.0001 s, RK5, acceleration.

Figure B.21. Test Case 1, h=0.0001 s, RK5, velocity.
APPENDIX B (continued)

Figure B.22. Test Case 1, h=0.0001 s, RK5, displacement.

Figure B.23. Test Case 1, h=0.0001 s, Euler w/Euler Parameters, acceleration.
Figure B.24. Test Case 1, h=0.0001 s, Euler w/Euler Parameters, velocity.

Figure B.25. Test Case 1, h=0.0001 s, Euler w/Euler Parameters, displacement.
Figure B.26. Test Case 1, h=0.0001 s, Euler w/normalization, acceleration.

Figure B.27. Test Case 1, h=0.0001 s, Euler w/normalization, velocity.
APPENDIX B (continued)

Figure B.28. Test Case 1, h=0.0001 s, Euler w/normalization, displacement.

Figure B.29. Test Case 1, h=0.000025 s, Euler, acceleration.
APPENDIX B (continued)

Figure B.30. Test Case 1, h=0.000025 s, Euler, velocity.

Figure B.31. Test Case 1, h=0.000025 s, Euler, displacement.
Figure B.32. Test Case 1, h=0.000025 s, RK4, acceleration.

Figure B.33. Test Case 1, h=0.000025 s, RK4, velocity.
APPENDIX B (continued)

Figure B.34. Test Case 1, h=0.000025 s, RK4, displacement.

Figure B.35. Test Case 1, h=0.000025 s, RK5, acceleration.
APPENDIX B (continued)

Figure B.36. Test Case 1, h=0.000025 s, RK5, velocity.

Figure B.37. Test Case 1, h=0.000025 s, RK5, displacement.
APPENDIX B (continued)

Test Case 2

Figure B.38. Test Case 2, h=0.0001 s, Euler, acceleration.

Figure B.39. Test Case 2, h=0.0001 s, Euler, velocity.
APPENDIX B (continued)

Figure B.40. Test Case 2, h=0.0001 s, Euler, displacement.

Figure B.41. Test Case 2, h=0.0001 s, RK4, acceleration.
APPENDIX B (continued)

Figure B.42. Test Case 2, h=0.0001 s, RK4, velocity.

Figure B.43. Test Case 2, h=0.0001 s, RK4, displacement.
APPENDIX B (continued)

Figure B.44. Test Case 2, h=0.0001 s, RK5, acceleration.

Figure B.45. Test Case 2, h=0.0001 s, RK5, velocity.
APPENDIX B (continued)

Figure B.46. Test Case 2, \( h=0.0001 \) s, RK5, displacement.

Figure B.47. Test Case 2, \( h=0.0001 \) s, Euler w/normalization, acceleration.
APPENDIX B (continued)

![Graph 1: Test Case 2, h=0.0001 s, Euler w/normalization, velocity.](image1)

**Figure B.48.** Test Case 2, h=0.0001 s, Euler w/normalization, velocity.

![Graph 2: Test Case 2, h=0.0001 s, Euler w/normalization, displacement.](image2)

**Figure B.49.** Test Case 2, h=0.0001 s, Euler w/normalization, displacement.
APPENDIX B (continued)

Figure B.50. Test Case 2, h=0.00001 s, Euler, acceleration.

Figure B.51. Test Case 2, h=0.00001 s, Euler, velocity.
APPENDIX B (continued)

Figure B.52. Test Case 2, h=0.00001 s, Euler, displacement.

Test Case 3

Figure B.53. Test Case 3, h=0.0001 s, Euler, acceleration.
APPENDIX B (continued)

Figure B.54. Test Case 3, \( h=0.0001 \) s, Euler, velocity.

Figure B.55. Test Case 3, \( h=0.0001 \) s, Euler, displacement.
Figure B.56. Test Case 3, h=0.0001 s, RK4, acceleration.

Figure B.57. Test Case 3, h=0.0001 s, RK4, velocity.
Figure B.58. Test Case 3, h=0.0001 s, RK4, displacement.

Figure B.59. Test Case 3, h=0.0001 s, RK5, acceleration.
Figure B.60. Test Case 3, h=0.0001 s, RK5, velocity.

Figure B.61. Test Case 3, h=0.0001 s, RK5, displacement.
Application of Various Filters

CFC600

Figure B.62. Test Case 2, acceleration filtered to CFC600, Euler, acceleration.
Figure B.63. Test Case 2, acceleration filtered to CFC600, Euler, velocity.

Figure B.64. Test Case 2, acceleration filtered to CFC600, Euler, displacement.
Figure B.65. Test Case 2, angular velocity filtered to CFC600, Euler, acceleration.

Figure B.66. Test Case 2, angular velocity filtered to CFC600, Euler, velocity.
Figure B.67. Test Case 2, angular velocity filtered to CFC600, Euler, displacement.

Figure B.68. Test Case 2, acceleration and angular velocity filtered to CFC600, Euler, acceleration.
APPENDIX B (continued)

Figure B.69. Test Case 2, acceleration and angular velocity filtered to CFC600, Euler, velocity.

Figure B.70. Test Case 2, acceleration and angular velocity filtered to CFC600, Euler, displacement.
APPENDIX B (continued)

CFC180

Figure B.71. Test Case 2, acceleration filtered to CFC180, Euler, acceleration.

Figure B.72. Test Case 2, acceleration filtered to CFC180, Euler, velocity.
Figure B.73. Test Case 2, acceleration filtered to CFC180, Euler, displacement.

Figure B.74. Test Case 2, angular velocity filtered to CFC180, Euler, acceleration.
Figure B.75. Test Case 2, angular velocity filtered to CFC180, Euler, velocity.

Figure B.76. Test Case 2, angular velocity filtered to CFC180, Euler, displacement.
Figure B.77. Test Case 2, acceleration and angular velocity filtered to CFC180, Euler, acceleration.

Figure B.78. Test Case 2, acceleration and angular velocity filtered to CFC180, Euler, velocity.
Figure B.79. Test Case 2, acceleration and angular velocity filtered to CFC180, Euler, displacement.
CFC60

Figure B.80. Test Case 2, acceleration filtered to CFC60, Euler, acceleration.

Figure B.81. Test Case 2, acceleration filtered to CFC60, Euler, velocity.
APPENDIX B (continued)

Figure B.82. Test Case 2, acceleration filtered to CFC60, Euler, displacement.

Figure B.83. Test Case 2, angular velocity filtered to CFC60, Euler, acceleration.
Figure B.84. Test Case 2, angular velocity filtered to CFC60, Euler, velocity.

Figure B.85. Test Case 2, angular velocity filtered to CFC60, Euler, displacement.
APPENDIX B (continued)

Figure B.86. Test Case 2, acceleration and angular velocity filtered to CFC60, Euler, acceleration.

Figure B.87. Test Case 2, acceleration and angular velocity filtered to CFC60, Euler, velocity.
Figure B.88. Test Case 2, acceleration and angular velocity filtered to CFC60, Euler, displacement.
APPENDIX C
SLED TEST SETUP PHOTOS

Initial Set-up and RH11A-01

Figure C.1. Accelerometers and angular rate sensors.  
Figure C.2. Accelerometers and angular rate sensors.

Figure C.3. Accelerometers and angular rate sensors in ATD head.  
Figure C.4. NIAR Velocity Generator.
Figure C.5. 3D photometric targets.

Figure C.6. NIAR Velocity Generator.

Figure C.7. Rigid seat.

Figure C.8. Seat belt 1.

Figure C.9. Seat belt 2.

Figure C.10. FARO Arm.
APPENDIX C (continued)

Figure C.11. FARO Arm.  Figure C.12. Visual identifiers.

Figure C.13. 3D photometric known distance.  Figure C.14. 3D photometric known distance.

Figure C.15. RH11A-01 seat belt tension.  Figure C.16. RH11A-01 seat belt location.
Figure C.17. RH11A-01 set-up.

Figure C.18. RH11A-01 set-up.

Figure C.19. RH11A-01 set-up.

Figure C.20. RH11A-01 initial angle.

Figure C.21. RH11A-01 initial angle.
APPENDIX C (continued)

RH11A-02

Figure C.22. RH11A-02 set-up.

Figure C.23. RH11A-02 seat belt tension.

Figure C.24. RH11A-02 set-up.

Figure C.25. RH11A-02 initial angle.
Figure C.26. RH11A-02 initial angle.

RH11A-03

Figure C.27. RH11A-03 set-up.  
Figure C.28. RH11A-03 set-up.
Figure C.29. RH11A-03 set-up.  Figure C.30. RH11A-03 set-up.

Figure C.31. RH11A-03 bulkhead.  Figure C.32. RH11A-03 ATD head.

Figure C.33. RH11A-03 seat belt tension.  Figure C.34. RH11A-03 seat belt location.
APPENDIX C (continued)

Figure C.35. RH11A-03 initial angle.

Figure C.36. RH11A-03 initial angle.

Figure C.37. RH11A-03 bulkhead post-test.

Figure C.38. RH11A-03 ATD head post-test.

Figure C.39. RH11A-03 bulkhead post-test.

Figure C.40. RH11A-03 bulkhead post-test.
RH11A-04

Figure C.41. RH11A-04 set-up.  
Figure C.42. RH11A-04 set-up.  

Figure C.43. RH11A-04 seat belt tension.  
Figure C.44. RH11A-04 seat belt location.
Figure C.45. RH11A-04 bulkhead.

Figure C.46. RH11A-04 initial angle.

Figure C.47. RH11A-04 initial angle.

Figure C.48. RH11A-04 ATD head post-test.

Figure C.49. RH11A-04 bulkhead post-test.

Figure C.50. RH11A-04 bulkhead post-test.
APPENDIX C (continued)

RH11A-05

Figure C.51. RH11A-05 set-up.  
Figure C.52. RH11A-05 set-up.  
Figure C.53. RH11A-05 seat belt tension.  
Figure C.54. RH11A-05 seat belt location.
Figure C.55. RH11A-05 initial angle.

Figure C.56. RH11A-06 set-up.

Figure C.57. RH11A-06 set-up.
Figure C.58. RH11A-06 seat belt tension.

Figure C.59. RH11A-06 seat belt location.

Figure C.60. RH11A-06 initial angle.
APPENDIX D
NIAR CRASH DYNAMICS LABORATORY TEST REPORTS

RH11A-01

Figure D.1. RH11A-01 Report Page 1.
APPENDIX D (continued)

Figure D.2. RH11A-01 Report Page 2.
Figure D.3. RH11A-01 Report Page 3.
Figure D.4. RH11A-01 Report Page 4.
Figure D.5. RH11A-01 Report Page 5.
Figure D.6. RH11A-02 Report Page 1.
Figure D.7. RH11A-02 Report Page 2.
Figure D.8. RH11A-02 Report Page 3.
Figure D.9. RH11A-02 Report Page 4.
Figure D.10. RH11A-02 Report Page 5.
APPENDIX D (continued)

RH11A-03

Figure D.11. RH11A-03 Report Page 1.
APPENDIX D (continued)

Figure D.12. RH11A-03 Report Page 2.
Figure D.13. RH11A-03 Report Page 3.
APPENDIX D (continued)

Figure D.14. RH11A-03 Report Page 4.
Figure D.15. RH11A-03 Report Page 5.
Figure D.16. RH11A-03 Report Page 6.
Figure D.17. RH11A-04 Report Page 1.
Figure D.18. RH11A-04 Report Page 2.
Figure D.19. RH11A-04 Report Page 3.
Figure D.20. RH11A-04 Report Page 4.
Figure D.21. RH11A-04 Report Page 5.
Figure D.22. RH11A-04 Report Page 6.
Figure D.23. RH11A-05 Report Page 1.
Figure D.24. RH11A-05 Report Page 2.
Figure D.25. RH11A-05 Report Page 3.
Figure D.26. RH11A-05 Report Page 4.
Figure D.27. RH11A-05 Report Page 5.
Figure D.28. RH11A-06 Report Page 1.
Figure D.29. RH11A-06 Report Page 2.
Figure D.30. RH11A-06 Report Page 3.
Figure D.31. RH11A-06 Report Page 4.
Figure D.32.  RH11A-06 Report Page 5.
Figure E.1. RH11A-01 positions – Euler
APPENDIX E (continued)

Figure E.2. RH11A-01 positions – RK4

Figure E.3. RH11A-01 positions – RK5
Figure E.4. RH11A-02 positions – Euler

Figure E.5. RH11A-02 positions – RK4
APPENDIX E (continued)

Figure E.6. RH11A-02 positions – RK5

RH11A-03

Figure E.7. RH11A-03 positions – Euler
Figure E.8. RH11A-03 positions – RK4

Figure E.9. RH11A-03 positions – RK5
APPENDIX E (continued)

RH11A-04

Figure E.10. RH11A-04 positions – Euler

Figure E.11. RH11A-04 positions – RK4
APPENDIX E (continued)

Figure E.12. RH11A-04 positions – RK5

RH11A-05

Figure E.13. RH11A-05 positions – Euler
Figure E.14. RH11A-05 positions – RK4

Figure E.15. RH11A-05 positions – RK5
Figure E.16. RH11A-06 positions – Euler

Figure E.17. RH11A-06 positions – RK4
Figure E.18. RH11A-06 positions – RK5
Figure F.1. Sled accelerometer 29917.
Figure F.2. Sled accelerometer 29924.
Figure F.3. ATD head accelerometer P60375.
Figure F.4. ATD head accelerometer P60376.
Figure F.5. ATD head accelerometer P60377.
### Certificate of Calibration

**Model #:** Angular Rate Sensor  
**Serial #:** ARS-1148  
**Date Received:** 17 January 2011  
**Date Calibrated:** 19 January 2011  
**Next Calibration:** 19 January 2012  
**RMA Number:** 10688  
**Item Received:** Pass  
**Item Returned:** Pass  
**Temperature:** 72°F/22.2°C  
**Humidity:** 22%  
**Customer:** Wichita State University  
**Address:** 1845 N. Fairmount  
**Wichita, KS 67260**

DTI has been audited by the American Association for Laboratory Accreditation (A2LA) and found in compliance with ISO/IEC 17025:2005. Accredited calibrations performed within the DTI Scope of Accreditation are indicated by the presence of the A2LA Logo and Certificate Number on this Certificate of Calibration. DTI calibration standards are processed and calibrated in accordance with the DTI Quality Assurance System, traceable to the National Institute of Standards and Technology (NIST). All calibrations have been performed using processes having a test uncertainty ratio of four or more times greater than the unit calibrated, unless otherwise noted on the report. Uncertainties have been estimated at a 95 percent confidence level (k=2). Calibration at a 4.17°C provides reasonable confidence that the instrument is within the manufacturer's published specifications. The reported data is the raw recorded data and is not corrected for uncertainty or environmental effects. Any number of factors can cause a unit to drift out of tolerance at any time following its calibration. This report only applies to the item identified above, and shall not be reproduced except in full, without the written approval of DTI. Limitations on the use of this instrument are detailed in the manufacturer's operating instructions.

**Notes:** Operating range is 5V to 14V DC. Excitation is internally regulated and the output signal is NOT proportional to excitation. Calibration was performed with 5V excitation. Output is centered at 2.5 volts (nominal) with respect to excitation. Signal remains stationary at 2.5 volts, while Signal moves ±2 volts (with respect to 2.5 volts) in response to physical stimuli. The ARS has no provision for adjustment; the "As Received" and "As Left" data is the same.

**Remarks:** Removed connector for calibration. Unit returned with wire plug-tailed and connector in box with ARS.

#### Standards Used

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<th>Model</th>
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<td>34401A</td>
<td>Digital Multimeter, 6.5 Digit</td>
<td>21-May-2010</td>
<td>21-May-2011</td>
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<td>Agilent</td>
<td>34401A</td>
<td>Digital Multimeter, 6.5 Digit</td>
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<td>19-Feb-2011</td>
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<td>DTI</td>
<td>ARS-CA6</td>
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#### Results

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<th>Upper Limit</th>
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<th>As Returned</th>
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<td>Pass</td>
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<td>Pass</td>
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<td>-9.402919 mV</td>
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<td>Sensitivity</td>
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<td>0.18</td>
<td>Pass</td>
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<td>Pass</td>
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Calibration Site: 41204 Bridge St.  
Novi, MI 48375  
Calibrated By: Jim Platte  
Technical Support Engineer
Figure F.7. ATD head angular rate sensor 1148 page 2.
**Certificate of Calibration**

<table>
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<tr>
<td>Serial #:</td>
<td>ARS/1151</td>
</tr>
<tr>
<td>Procedure Name:</td>
<td>ARS Calibration</td>
</tr>
<tr>
<td>RMA Number:</td>
<td>10666</td>
</tr>
<tr>
<td>Customer:</td>
<td>Wichita State University</td>
</tr>
<tr>
<td>Wichita, KS 67260</td>
<td></td>
</tr>
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</table>

**Certification Details**

<table>
<thead>
<tr>
<th>Date Received</th>
<th>17 January 2011</th>
</tr>
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<tbody>
<tr>
<td>Date Calibrated</td>
<td>19 January 2011</td>
</tr>
<tr>
<td>Next Calibration</td>
<td>19 January 2012</td>
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<tr>
<td>Item Received</td>
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</tr>
<tr>
<td>Item Returned</td>
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<tr>
<td>Temperature</td>
<td>72°F/22.2°C</td>
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<tr>
<td>Humidity</td>
<td>22%</td>
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</tbody>
</table>

DTS has been audited by the American Association for Laboratory Accreditation (A2LA) and found in compliance with ISO/IEC 17025:2005. Accredited calibrations performed within the DTS Scope of Accreditation are indicated by the presence of the A2LA Logo and Certificate Number on the Certificate of Calibration. DTS reference standards are processed and calibrated in accordance with the DTS Quality Assurance System, and traceable to the National Institute of Standards and Technology (NIST). All calibrations have been performed using processes having a test uncertainty ratio of four or more times greater than the unit calibrated, unless otherwise noted on the report. Uncertainties have been estimated at a 95 percent confidence level (k=2). Calibration at a 4-1 TUR provides reasonable confidence that the instrument is within the manufacturer’s published specifications. The reported data is the new recorded data and is not corrected for uncertainty or environmental effects. Any number of factors can cause a unit to drift out of tolerance at any time following its calibration. This report only applies to the item identified above, and shall not be reproduced except in full, without the written approval of DTS. Limitations on the uses of this instrument are detailed in the manufacturer’s operating instructions.

**Notes:**
- Operating range is 5V to 14V DC. Excitation is internally regulated and the output signal is NOT proportional to excitation. Calibration was performed with 5V excitation.
- Output is centered at 2.5 volts (nominal) with respect to -Excitation. Signal remains stationary at 2.5 volts, while +Signal moves ±2 volts with respect to 2.5 volts in response to physical stimulus.
- The ARS has no provision for adjustment; the “As Received” and “As Left” data is the same.

**Standards Used**

<table>
<thead>
<tr>
<th>Serial #</th>
<th>Manufacturer</th>
<th>Model</th>
<th>Description</th>
<th>Cal Date</th>
<th>Due Date</th>
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<tbody>
<tr>
<td>MY4707610</td>
<td>Agilent</td>
<td>34401A</td>
<td>Digital Multimeter, 6.5 Digit</td>
<td>21-May-2010</td>
<td>21-May-2011</td>
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<td>MY4707673</td>
<td>Agilent</td>
<td>34401A</td>
<td>Digital Multimeter, 6.5 Digit</td>
<td>19-Feb-2010</td>
<td>19-Feb-2011</td>
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<td>ARS/1151</td>
<td>DTS</td>
<td>ARS/1151</td>
<td>Rate Table</td>
<td>24-Mar-2010</td>
<td>24-Mar-2011</td>
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**Results**

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<tr>
<th>Test Description</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>As Received</th>
<th>Pass/Fail</th>
<th>As Returned</th>
<th>Pass/Fail</th>
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<tbody>
<tr>
<td>Visual Inspection</td>
<td>5 mA</td>
<td>7 mA</td>
<td>Pass</td>
<td>Pass</td>
<td>Pass</td>
<td>Pass</td>
</tr>
<tr>
<td>Input Current</td>
<td>-100 mV</td>
<td>100 mV</td>
<td>Pass</td>
<td>Pass</td>
<td>Pass</td>
<td>Pass</td>
</tr>
<tr>
<td>Sensor Offset</td>
<td>0.11</td>
<td>0.18</td>
<td>0.1258 mV/deg/sec</td>
<td>Pass</td>
<td>0.1288 mV/deg/sec</td>
<td>Pass</td>
</tr>
</tbody>
</table>

Calibration Site: 41204 Bridge St., Novi, MI 48375

Calibrated By: Jim Platte
Technical Support Engineer

Page 1 of 2

Figure F.8. ATD head angular rate sensor 1151 page 1.
Figure F.9. ATD head angular rate sensor 1151 page 2.
Figure F.10. ATD head angular rate sensor 1153 page 1.
Figure F.11. ATD head angular rate sensor 1153 page 2.
APPENDIX G
DiaDEM ALGORITHM SCRIPT

Option Explicit  'Forces the explicit declaration of all the variables in a script.
If VSA("J:\Diadem\reportdialogvariables.vas")=True Then
Else Call UserVarCompile("J:\Diadem\reportdialogvariables.vas")
End If

Call ChnUnitConvert("[1]/Head Rx "+F11_,"/Head Rx "+F11_,"rad/s") '... Y,E,TargetUnit
Call ChnUnitConvert("[1]/Head Ry "+F11_,"/Head Ry "+F11_,"rad/s") '... Y,E,TargetUnit
Call ChnUnitConvert("[1]/Head Rz "+F11_,"/Head Rz "+F11_,"rad/s") '... Y,E,TargetUnit
Call ChnCopy("[1]/Head Ax "+F11_,"[1]/Head Ax "+F11_" (T))
Call ChnCopy("[1]/Head Ay "+F11_,"[1]/Head Ay "+F11_" (T))
Call ChnCopy("[1]/Head Az "+F11_,"[1]/Head Az "+F11_" (T))
Call ChnCopy("[1]/Head Rx "+F11_,"[1]/Head Rx "+F11_" (T))
Call ChnCopy("[1]/Head Ry "+F11_,"[1]/Head Ry "+F11_" (T))
Call ChnCopy("[1]/Head Rz "+F11_,"[1]/Head Rz "+F11_" (T))
Call ChnLinScale("[1]/Head Ay "+F11_" (T),"/Head Ay "+F11_" (T),-1,0) '...
Y,E,ChnScaleFactor,ChnScaleOffset
Call ChnLinScale("[1]/Head Az "+F11_" (T),"/Head Az "+F11_" (T),-1,0) '...
Y,E,ChnScaleFactor,ChnScaleOffset
Call ChnLinScale("[1]/Head Ry "+F11_" (T),"/Head Ry "+F11_" (T),-1,0) '...
Y,E,ChnScaleFactor,ChnScaleOffset
Call ChnLinScale("[1]/Head Rz "+F11_" (T),"/Head Rz "+F11_" (T),-1,0) '...
Y,E,ChnScaleFactor,ChnScaleOffset

Call Data.Root.ChannelGroups(1).Channels.Add("Time Step",DataTypeFloat64)
Call DataBlInsertVal("[1]/Time Step",1,3001,0.0001,0)

Call Data.Root.ChannelGroups(1).Channels.Add("Time",DataTypeFloat64)
A1_=1
Do Until A1_=3002
    Call DataBlInsertVal("[1]/Time",A1_,1,A1_/10000,0)
    A1_=A1_+1
Loop

Call SudDlgShow("Dlg1","J:\Diadem\Angular Rate")
Call Data.Root.ChannelGroups(1).Channels.Add("Head Initial Values",DataTypeFloat64)
Call DataBlInsertVal("[1]/Head Initial Values",1,1,A4_,0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
Call DataBlInsertVal("[1]/Head Initial Values",2,1,A5_,0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
APPENDIX G (continued)

Call DataBlInsertVal("[1]/Head Initial Values",3,1,A6_.,0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
Call DataBlInsertVal("[1]/Head Initial Values",4,1,A4_.*pi/180,0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
Call DataBlInsertVal("[1]/Head Initial Values",5,1,A5_.*pi/180,0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
Call DataBlInsertVal("[1]/Head Initial Values",6,1,A6_.*pi/180,0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
'A4=phi, A5=Theta, A6=psi
Call DataBlInsertVal("[1]/Head Initial Values",7,1,((cos(CHD(4,"[1]/Head Initial
Values")/2)*cos(CHD(5,"[1]/Head Initial Values")/2)*cos(CHD(6,"[1]/Head Initial
Values")/2))-(sin(CHD(4,"[1]/Head Initial Values")/2)*sin(CHD(5,"[1]/Head Initial
Values")/2)*sin(CHD(6,"[1]/Head Initial Values")/2))),0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
Call DataBlInsertVal("[1]/Head Initial Values",8,1,((cos(CHD(4,"[1]/Head Initial
Values")/2)*cos(CHD(5,"[1]/Head Initial Values")/2)*sin(CHD(6,"[1]/Head Initial
Values")/2))+(sin(CHD(4,"[1]/Head Initial Values")/2)*sin(CHD(5,"[1]/Head Initial
Values")/2)*cos(CHD(6,"[1]/Head Initial Values")/2))),0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
Call DataBlInsertVal("[1]/Head Initial Values",9,1,((cos(CHD(4,"[1]/Head Initial
Values")/2)*sin(CHD(5,"[1]/Head Initial Values")/2)*cos(CHD(6,"[1]/Head Initial
Values")/2))-(sin(CHD(4,"[1]/Head Initial Values")/2)*cos(CHD(5,"[1]/Head Initial
Values")/2)*sin(CHD(6,"[1]/Head Initial Values")/2))),0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite
Call DataBlInsertVal("[1]/Head Initial Values",10,1,((cos(CHD(4,"[1]/Head Initial
Values")/2)*sin(CHD(5,"[1]/Head Initial Values")/2)*sin(CHD(6,"[1]/Head Initial
Values")/2))+(sin(CHD(4,"[1]/Head Initial Values")/2)*cos(CHD(5,"[1]/Head Initial
Values")/2)*cos(CHD(6,"[1]/Head Initial Values")/2))),0) '...
ChnNoStr,ChnRow,ValNo,ValueInsert,ValueOverwrite

Call Data.Root.ChannelGroups(1).Channels.Add("q0",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("q1",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("q2",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("q3",DataTypeFloat64)

Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k1-0",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k1-1",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k1-2",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k1-3",DataTypeFloat64)

Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k2-0",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k2-1",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k2-2",DataTypeFloat64)
APPENDIX G (continued)

Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k2-3",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k3-0",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k3-1",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k3-2",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k3-3",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k4-0",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k4-1",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k4-2",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("RK4 k4-3",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r11",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r12",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r13",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r21",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r22",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r23",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r31",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r32",DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("r33",DataTypeFloat64)

'First k values
A1_ = 1
'k1 values
Call Data.BlInsertVal("[1]/RK4 k1-0",A1_,1,0.5*(-1*CHD(8,"[1]/Head Initial Values")*0-CHD(9,"[1]/Head Initial Values")*0-CHD(10,"[1]/Head Initial Values")*0),0)
Call Data.BlInsertVal("[1]/RK4 k1-1",A1_,1,0.5*(CHD(7,"[1]/Head Initial Values")*0-CHD(10,"[1]/Head Initial Values")*0+CHD(9,"[1]/Head Initial Values")*0),0)
Call Data.BlInsertVal("[1]/RK4 k1-2",A1_,1,0.5*(CHD(10,"[1]/Head Initial Values")*0+CHD(7,"[1]/Head Initial Values")*0+CHD(9,"[1]/Head Initial Values")*0),0)
'k2 values
Call Data.BlInsertVal("[1]/RK4 k2-0",A1_,1,0.5*(-1*(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-1")))*(0+CHD(A1_,"[1]/Head Rx "+F11_+" (T)"))/2-(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-2")))*(0+CHD(A1_,"[1]/Head Ry "+F11_+" (T)"))/2-(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-3")))*(0+CHD(A1_,"[1]/Head Rz "+F11_+" (T)"))/2),0)
APPENDIX G (continued)

Call DataBlInsertVal("[1]/RK4 k2-1",A1_,1,0.5*((CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-0"))*(0+CHD(A1_,"[1]/Head Rx "+F11_+" (T)))/2-(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-3"))*(0+CHD(A1_,"[1]/Head Ry "+F11_+" (T)))/2+(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-2"))*(0+CHD(A1_,"[1]/Head Rz "+F11_+" (T)))/2),0)
Call DataBlInsertVal("[1]/RK4 k2-2",A1_,1,0.5*((CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-3"))*(0+CHD(A1_,"[1]/Head Rx "+F11_+" (T)))/2+(CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-0"))*(0+CHD(A1_,"[1]/Head Ry "+F11_+" (T)))/2-(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-1"))*(0+CHD(A1_,"[1]/Head Rz "+F11_+" (T)))/2),0)
Call DataBlInsertVal("[1]/RK4 k2-3",A1_,1,0.5*(-1*(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-2"))*(0+CHD(A1_,"[1]/Head Rx "+F11_+" (T)))/2-(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-3"))*(0+CHD(A1_,"[1]/Head Ry "+F11_+" (T)))/2+(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k1-1"))*(0+CHD(A1_,"[1]/Head Rz "+F11_+" (T)))/2),0)
'k3 values
Call DataBlInsertVal("[1]/RK4 k3-0",A1_,1,0.5*(-1*(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-1"))*(0+CHD(A1_,"[1]/Head Rx "+F11_+" (T)))/2-(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-2"))*(0+CHD(A1_,"[1]/Head Ry "+F11_+" (T)))/2-(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-3"))*(0+CHD(A1_,"[1]/Head Rz "+F11_+" (T)))/2),0)
Call DataBlInsertVal("[1]/RK4 k3-1",A1_,1,0.5*((CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-0"))*(0+CHD(A1_,"[1]/Head Rx "+F11_+" (T)))/2-(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-3"))*(0+CHD(A1_,"[1]/Head Ry "+F11_+" (T)))/2+(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-2"))*(0+CHD(A1_,"[1]/Head Rz "+F11_+" (T)))/2),0)
Call DataBlInsertVal("[1]/RK4 k3-2",A1_,1,0.5*((CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-3"))*(0+CHD(A1_,"[1]/Head Rx "+F11_+" (T)))/2+(CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-0"))*(0+CHD(A1_,"[1]/Head Ry "+F11_+" (T)))/2-(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-1"))*(0+CHD(A1_,"[1]/Head Rz "+F11_+" (T)))/2),0)
CALL DataBlInsertVal("[1]/RK4 k3-3",A1_,1,0.5*(-1*(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-2")))*(0+CHD(A1_,"[1]/Head Rx "+F11_+("(T)")))/2+(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-1"))*(0+CHD(A1_,"[1]/Head Ry "+F11_+("(T)")))/2+(CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")/2)*CHD(A1_,"[1]/RK4 k2-0"))*(0+CHD(A1_,"[1]/Head Rz "+F11_+("(T)")))/2),0)

'k4 values

CALL DataBlInsertVal("[1]/RK4 k4-0",A1_,1,0.5*(-1*(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-1")))*CHD(A1_,"[1]/Head Rx "+F11_+("(T)"))-(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-2")))*CHD(A1_,"[1]/Head Ry "+F11_+("(T)"))-(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-3")))*CHD(A1_,"[1]/Head Rz "+F11_+("(T)")),0)

CALL DataBlInsertVal("[1]/RK4 k4-1",A1_,1,0.5*((CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-0")))*CHD(A1_,"[1]/Head Rx "+F11_+("(T)"))-(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-3")))*CHD(A1_,"[1]/Head Ry "+F11_+("(T)"))+(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-2")))*CHD(A1_,"[1]/Head Rz "+F11_+("(T)")),0)

CALL DataBlInsertVal("[1]/RK4 k4-2",A1_,1,0.5*((CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-3")))*CHD(A1_,"[1]/Head Rx "+F11_+("(T)"))+(CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-0")))*CHD(A1_,"[1]/Head Ry "+F11_+("(T)"))-(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-1")))*CHD(A1_,"[1]/Head Rz "+F11_+("(T)")),0)

CALL DataBlInsertVal("[1]/RK4 k4-3",A1_,1,0.5*((CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-2")))*CHD(A1_,"[1]/Head Rx "+F11_+("(T)"))+(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-1")))*CHD(A1_,"[1]/Head Ry "+F11_+("(T)"))+(CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k3-0")))*CHD(A1_,"[1]/Head Rz "+F11_+("(T)")),0)

'q values

CALL DataBlInsertVal("[1]/q0",A1_1,1,(CHD(7,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k1-0")/6)+(CHD(A1_,"[1]/RK4 k2-0")/3)+(CHD(A1_,"[1]/RK4 k3-0")/3)+0)),0)

CALL DataBlInsertVal("[1]/q1",A1_1,1,(CHD(8,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k1-1")/6)+(CHD(A1_,"[1]/RK4 k2-1")/3)+(CHD(A1_,"[1]/RK4 k3-1")/3)+(CHD(A1_,"[1]/RK4 k4-1")/6)),0)

CALL DataBlInsertVal("[1]/q2",A1_1,1,(CHD(9,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*CHD(A1_,"[1]/RK4 k1-2")/6)+(CHD(A1_,"[1]/RK4 k2-2")/3)+(CHD(A1_,"[1]/RK4 k3-2")/3)+(CHD(A1_,"[1]/RK4 k4-2")/6)),0)
APPENDIX G (continued)

Call DataBlInsertVal("[1]/q3",A1_,1,(CHD(10,"[1]/Head Initial Values")+(CHD(A1_,"[1]/Time Step")*((CHD(A1_,"[1]/RK4 k1-3")/6)+(CHD(A1_,"[1]/RK4 k2-3")/3)+(CHD(A1_,"[1]/RK4 k3-3")/3)+(CHD(A1_,"[1]/RK4 k4-3")/6)))),0)

'Rest of k values
A1_=2
Do Until A1_=3002
  'k1 values
  Call DataBlInsertVal("[1]/RK4 k1-0",A1_,1,0.5*(-1*(CHD(A1_-1,"[1]/q1")*CHD(A1_-1,"[1]/Head Rx "+F11_" (T)"))+(CHD(A1_-1,"[1]/q3")*CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)
  Call DataBlInsertVal("[1]/RK4 k1-1",A1_,1,0.5*(CHD(A1_-1,"[1]/q0")*CHD(A1_-1,"[1]/Head Rx "+F11_" (T)"))+(CHD(A1_-1,"[1]/q2")*CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)
  Call DataBlInsertVal("[1]/RK4 k1-2",A1_,1,0.5*((CHD(A1_-1,"[1]/q2")*CHD(A1_-1,"[1]/Head Rx "+F11_" (T)"))+(CHD(A1_-1,"[1]/q3")*CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)
  Call DataBlInsertVal("[1]/RK4 k1-3",A1_,1,0.5*(-1*(CHD(A1_-1,"[1]/q2")*CHD(A1_-1,"[1]/q3")*CHD(A1_-1,"[1]/Head Rx "+F11_" (T)"))+(CHD(A1_-1,"[1]/q1")*CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)
  'k2 values
  Call DataBlInsertVal("[1]/RK4 k2-0",A1_,1,0.5*(-1*(CHD(A1_-1,"[1]/q1")+(CHD(A1_-1,"[1]/Time Step")*CHD(A1_-1,"[1]/ RK4 k1-1")))+(CHD(A1_-1,"[1]/q3")+(CHD(A1_-1,"[1]/Head Rx "+F11_" (T)")))+(CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)
  Call DataBlInsertVal("[1]/RK4 k2-1",A1_,1,0.5*((CHD(A1_-1,"[1]/q0")+(CHD(A1_-1,"[1]/Time Step")*CHD(A1_-1,"[1]/ RK4 k1-2")))+(CHD(A1_-1,"[1]/q3")+(CHD(A1_-1,"[1]/Head Rx "+F11_" (T)")))+(CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)
  Call DataBlInsertVal("[1]/RK4 k2-2",A1_,1,0.5*((CHD(A1_-1,"[1]/q0")+(CHD(A1_-1,"[1]/Time Step")*CHD(A1_-1,"[1]/ RK4 k1-3")))+(CHD(A1_-1,"[1]/q3")+(CHD(A1_-1,"[1]/Head Rx "+F11_" (T)")))+(CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)
  Call DataBlInsertVal("[1]/RK4 k2-3",A1_,1,0.5*(-1*(CHD(A1_-1,"[1]/q2")*CHD(A1_-1,"[1]/q3")*CHD(A1_-1,"[1]/Head Rx "+F11_" (T)"))+(CHD(A1_-1,"[1]/q1")*CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))+(CHD(A1_-1,"[1]/Head Rx "+F11_" (T)"))+(CHD(A1_-1,"[1]/Head Ry "+F11_" (T)"))),0)

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APPENDIX G (continued)

Call DataBlInsertVal("[1]/RK4 k2-3",A1_,1,0.5*(-1*(CHD(A1_ - 1,"[1]/q1")+(CHD(A1_,"[1]/[Time Step]"/(2))*CHD(A1_,"[1]/RK4 k1-1")/*CHD(A1_ - 1,"[1]/Head Rx "+F11_+ (T)")-CHD(A1_,"[1]/Head Ry "+F11_+ (T)")+(CHD(A1_,"[1]/Head Rz "+F11_+ (T)")},0)

k3 values
Call DataBlInsertVal("[1]/RK4 k3-0",A1_,1,0.5*(-1*(CHD(A1_ - 1,"[1]/q1")+(CHD(A1_,"[1]/[Time Step]"/(2))*CHD(A1_,"[1]/RK4 k1-1")/*CHD(A1_ - 1,"[1]/Head Rx "+F11_+ (T)")-CHD(A1_,"[1]/Head Ry "+F11_+ (T)")+(CHD(A1_,"[1]/Head Rz "+F11_+ (T)")},0)

Call DataBlInsertVal("[1]/RK4 k3-1",A1_,1,0.5*(-1*(CHD(A1_ - 1,"[1]/q1")+(CHD(A1_,"[1]/[Time Step]"/(2))*CHD(A1_,"[1]/RK4 k1-1")/*CHD(A1_ - 1,"[1]/Head Rx "+F11_+ (T)")-CHD(A1_,"[1]/Head Ry "+F11_+ (T)")+(CHD(A1_,"[1]/Head Rz "+F11_+ (T)")},0)

k4 values
Call DataBlInsertVal("[1]/RK4 k4-0",A1_,1,0.5*(-1*(CHD(A1_ - 1,"[1]/q1")+(CHD(A1_,"[1]/[Time Step]"*(2))*CHD(A1_,"[1]/RK4 k1-1")/*CHD(A1_ - 1,"[1]/Head Rx "+F11_+ (T)")-CHD(A1_,"[1]/Head Ry "+F11_+ (T)")-CHD(A1_,"[1]/Head Rz "+F11_+ (T)")},0)
APPENDIX G (continued)

Call DataBlInsertVal("[1]/RK4 k4-1", A1_1, 1, 0.5*((CHD(A1_1,"[1]/q0")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-0")))*CHD(A1_1,"[1]/Head Rx "+F11_+" (T")-(CHD(A1_1,"[1]/q3")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-3")))*CHD(A1_1,"[1]/Head Ry "+F11_+" (T")+(CHD(A1_1,"[1]/q2")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-2")))*CHD(A1_1,"[1]/Head Rz "+F11_+" (T")",0),)

Call DataBlInsertVal("[1]/RK4 k4-2", A1_1, 1, 0.5*((CHD(A1_1,"[1]/q3")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-3")))*CHD(A1_1,"[1]/Head Rx "+F11_+" (T")+(CHD(A1_1,"[1]/q0")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-0")))*CHD(A1_1,"[1]/Head Ry "+F11_+" (T")-(CHD(A1_1,"[1]/q1")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-1")))*CHD(A1_1,"[1]/Head Rz "+F11_+" (T")",0),)

Call DataBlInsertVal("[1]/RK4 k4-3", A1_1, 1, 0.5*(-1*(CHD(A1_1,"[1]/q2")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-2")))*CHD(A1_1,"[1]/Head Rx "+F11_+" (T")+(CHD(A1_1,"[1]/q1")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-1")))*CHD(A1_1,"[1]/Head Ry "+F11_+" (T")+(CHD(A1_1,"[1]/q0")+(CHD(A1_1,"[1]/Time Step")*CHD(A1_1,"[1]/RK4 k3-0")))*CHD(A1_1,"[1]/Head Rz "+F11_+" (T")",0),)

' q values
Call DataBlInsertVal("[1]/q0", A1_1, 1, (CHD(A1_1,"[1]/q0")+(CHD(A1_1,"[1]/RK4 k1-0")/6)+(CHD(A1_1,"[1]/RK4 k2-0")/3)+(CHD(A1_1,"[1]/RK4 k3-0")/3)+(CHD(A1_1,"[1]/RK4 k4-0")/6)),0)
Call DataBlInsertVal("[1]/q1", A1_1, 1, (CHD(A1_1,"[1]/q1")+(CHD(A1_1,"[1]/RK4 k1-1")/6)+(CHD(A1_1,"[1]/RK4 k2-1")/3)+(CHD(A1_1,"[1]/RK4 k3-1")/3)+(CHD(A1_1,"[1]/RK4 k4-1")/6)),0)
Call DataBlInsertVal("[1]/q2", A1_1, 1, (CHD(A1_1,"[1]/q2")+(CHD(A1_1,"[1]/RK4 k1-2")/6)+(CHD(A1_1,"[1]/RK4 k2-2")/3)+(CHD(A1_1,"[1]/RK4 k3-2")/3)+(CHD(A1_1,"[1]/RK4 k4-2")/6)),0)
Call DataBlInsertVal("[1]/q3", A1_1, 1, (CHD(A1_1,"[1]/q3")+(CHD(A1_1,"[1]/RK4 k1-3")/6)+(CHD(A1_1,"[1]/RK4 k2-3")/3)+(CHD(A1_1,"[1]/RK4 k3-3")/3)+(CHD(A1_1,"[1]/RK4 k4-3")/6)),0)

A1_1 = A1_1 + 1
Loop

'Transformation Matrix
A1_1 = 1
Do Until A1_1 = 3002
  Call DataBlInsertVal("[1]/r11", A1_1, 1, ((CHD(A1_1,"[1]/q0")^2+(CHD(A1_1,"[1]/q1")^2-(CHD(A1_1,"[1]/q2")^2-(CHD(A1_1,"[1]/q3")^2))/2),0)
  Call DataBlInsertVal("[1]/r12", A1_1, 2, ((CHD(A1_1,"[1]/q1")*CHD(A1_1,"[1]/q2")-(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q3"))/2),0)
  Call DataBlInsertVal("[1]/r13", A1_1, 2, ((CHD(A1_1,"[1]/q1")*CHD(A1_1,"[1]/q3")+(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q2"))/2),0)
  Do Until A1_1 = 3002
    Call DataBlInsertVal("[1]/r11", A1_1, 1, ((CHD(A1_1,"[1]/q0")^2+(CHD(A1_1,"[1]/q1")^2-(CHD(A1_1,"[1]/q2")^2-(CHD(A1_1,"[1]/q3")^2))/2),0)
    Call DataBlInsertVal("[1]/r12", A1_1, 2, ((CHD(A1_1,"[1]/q1")*CHD(A1_1,"[1]/q2")-(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q3"))/2),0)
    Call DataBlInsertVal("[1]/r13", A1_1, 2, ((CHD(A1_1,"[1]/q1")*CHD(A1_1,"[1]/q3")+(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q2"))/2),0)
  Loop

End
Call DataBlInsertVal("[1]/r21",A1_1,1,2*((CHD(A1_1,"[1]/q2")*CHD(A1_1,"[1]/q1")))+(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q3"))),0)
    Call DataBlInsertVal("[1]/r22",A1_1,1,((CHD(A1_1,"[1]/q0")^2+(CHD(A1_1,"[1]/q2")^2-(CHD(A1_1,"[1]/q3")))^2),0)
    Call DataBlInsertVal("[1]/r23",A1_1,1,2*((CHD(A1_1,"[1]/q2")*CHD(A1_1,"[1]/q3"))-(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q1"))),0)
    Call DataBlInsertVal("[1]/r31",A1_1,1,2*((CHD(A1_1,"[1]/q3")*CHD(A1_1,"[1]/q1"))-(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q2"))),0)
    Call DataBlInsertVal("[1]/r32",A1_1,1,2*((CHD(A1_1,"[1]/q3")*CHD(A1_1,"[1]/q2"))+(CHD(A1_1,"[1]/q0")*CHD(A1_1,"[1]/q1"))),0)
    Call DataBlInsertVal("[1]/r33",A1_1,1,((CHD(A1_1,"[1]/q0"))-(CHD(A1_1,"[1]/q1")))^2-(CHD(A1_1,"[1]/q2"))+(CHD(A1_1,"[1]/q3")))^2),0)
    A1_1 = A1_1 + 1
Loop

Call Data.Root.ChannelGroups(1).Channels.Add("Global Head Ax "+F11_,DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("Global Head Ay "+F11_,DataTypeFloat64)
Call Data.Root.ChannelGroups(1).Channels.Add("Global Head Az "+F11_,DataTypeFloat64)
A1_1 = 1
Do Until A1_1 = 3002
    Call DataBlInsertVal("[1]/Global Head Ax
    "+F11_1,A1_1,1,((CHD(A1_1,"[1]/r11"))+(CHD(A1_1,"[1]/Head Ax "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values"))+(CHD(A1_1,"[1]/r12"))+(CHD(A1_1,"[1]/Head Ay "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values")))+(CHD(A1_1,"[1]/r13"))+(CHD(A1_1,"[1]/Head Az "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values")))
    Call DataBlInsertVal("[1]/Global Head Ay +F11_1,A1_1,1,((CHD(A1_1,"[1]/r21"))+(CHD(A1_1,"[1]/Head Ax "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values")))+
    (CHD(A1_1,"[1]/r22"))+(CHD(A1_1,"[1]/Head Ay "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values"))+(CHD(A1_1,"[1]/Head Az "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values")))
    Call DataBlInsertVal("[1]/Global Head Az +F11_1,A1_1,1,((CHD(A1_1,"[1]/r23"))+(CHD(A1_1,"[1]/Head Ax "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values")))+
    (CHD(A1_1,"[1]/r31"))+(CHD(A1_1,"[1]/Head Ay "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values"))+(CHD(A1_1,"[1]/Head Az "+F11_1, (T)"))+(CHD(A1_1,"[1]/Head Initial Values")))
    A1_1 = A1_1 + 1
Loop
APPENDIX G (continued)

If A2 = 0 Then
    Call ChnToWfChn("[1]/Time","[1]/Global Head Ax " + F11_0, 0) '... X,ChnNoStr,XChnDelete
    Call ChnToWfChn("[1]/Time","[1]/Global Head Ay " + F11_0, 0) '... X,ChnNoStr,XChnDelete
    Call ChnToWfChn("[1]/Time","[1]/Global Head Az " + F11_0, 0) '... X,ChnNoStr,XChnDelete
    Call ChnIntegrate("[1]/Time","[1]/Global Head Ax " + F11_,"/Global Head Vx " + F11_) '... XW,Y,E
    Call ChnIntegrate("[1]/Time","[1]/Global Head Ay " + F11_,"/Global Head Vy " + F11_) '... XW,Y,E
    Call ChnIntegrate("[1]/Time","[1]/Global Head Az " + F11_,"/Global Head Vz " + F11_) '... XW,Y,E
    Call ChnLinScale("[1]/Global Head Vx " + F11_,"/Global Head Vx " + F11_0,32.1741,0) '... Y,E,ChnScaleFactor,ChnScaleOffset
    Call ChnLinScale("[1]/Global Head Vy " + F11_,"/Global Head Vy " + F11_0,32.1741,0) '... Y,E,ChnScaleFactor,ChnScaleOffset
    Call ChnLinScale("[1]/Global Head Vz " + F11_,"/Global Head Vz " + F11_0,32.1741,0) '... Y,E,ChnScaleFactor,ChnScaleOffset
    Call ChnSub("[1]/Global Head Vx " + F11_0,"[1]/Sled Velocity","/Global Head Vx " + F11_) '... Y,Y1,E
    Call ChnIntegrate("[1]/Time","[1]/Global Head Vx " + F11_","/Global Head Dx " + F11_) '... XW,Y,E
    Call ChnIntegrate("[1]/Time","[1]/Global Head Vy " + F11_","/Global Head Dy " + F11_) '... XW,Y,E
    Call ChnIntegrate("[1]/Time","[1]/Global Head Vz " + F11_","/Global Head Dz " + F11_) '... XW,Y,E
    Call ChnToWfChn("[1]/Time","[1]/Global Head Dx " + F11_0, 0) '... X,ChnNoStr,XChnDelete
    Call ChnToWfChn("[1]/Time","[1]/Global Head Dy " + F11_0, 0) '... X,ChnNoStr,XChnDelete
    Call ChnToWfChn("[1]/Time","[1]/Global Head Dz " + F11_0, 0) '... X,ChnNoStr,XChnDelete
    Call ChnLinScale("[1]/Global Head Dx " + F11_0,"/Global Head Dx " + F11_0,12,0) '... Y,E,ChnScaleFactor,ChnScaleOffset
    Call ChnLinScale("[1]/Global Head Dy " + F11_0,"/Global Head Dy " + F11_0,12,0) '... Y,E,ChnScaleFactor,ChnScaleOffset
    Call ChnLinScale("[1]/Global Head Dz " + F11_0,"/Global Head Dz " + F11_0,12,0) '... Y,E,ChnScaleFactor,ChnScaleOffset
    Call Chnpropvalset("[1]/Global Head Ax " + F11_0,"unit_string","g")
    Call Chnpropvalset("[1]/Global Head Ay " + F11_0,"unit_string","g")
    Call Chnpropvalset("[1]/Global Head Az " + F11_0,"unit_string","g")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","ft/s")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","ft/s")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","ft/s")
    Call Chnpropvalset("[1]/Global Head Dx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","ft")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","ft")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","ft")
    Call Chnpropvalset("[1]/Global Head Dx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Dy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vx " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vy " + F11_0,"unit_string","in")
    Call Chnpropvalset("[1]/Global Head Vz " + F11_0,"unit_string","in")
APPENDIX G (continued)

Call Chnpropvalset("[1]/Global Head Dz "+,F11_,","unit_string",",in")
Elseif A2 = 1 Then
  Call ChnToWfChn("[1]/Time","[1]/Global Head Ax "+,F11_,",0") '... X,ChnNoStr,XChnDelete
  Call ChnToWfChn("[1]/Time","[1]/Global Head Ay "+,F11_,",0") '... X,ChnNoStr,XChnDelete
  Call ChnToWfChn("[1]/Time","[1]/Global Head Az "+,F11_,",0") '... X,ChnNoStr,XChnDelete
  Call ChnIntegrate("[1]/Time","[1]/Global Head Ax "+,F11_,",/Global Head Vx "+,F11_,") '... XW,Y,E
  Call ChnIntegrate("[1]/Time","[1]/Global Head Ay "+,F11_,",/Global Head Vy "+,F11_,") '... XW,Y,E
  Call ChnIntegrate("[1]/Time","[1]/Global Head Az "+,F11_,",/Global Head Vz "+,F11_,") '... XW,Y,E
  Call ChnToWfChn("[1]/Time","[1]/Global Head Vy "+,F11_,",/Global Head Vx "+,F11_,") '... X,Y,E
  Call ChnToWfChn("[1]/Time","[1]/Global Head Vz "+,F11_,",/Global Head Vx "+,F11_,") '... X,Y,E
  Call ChnLinScale("[1]/Global Head Vx "+,F11_,",/Global Head Vx "+,F11_,",9.80665,0") '... Y,E,ChnScaleFactor,ChnScaleOffset
  Call ChnLinScale("[1]/Global Head Vy "+,F11_,",/Global Head Vy "+,F11_,",9.80665,0") '... Y,E,ChnScaleFactor,ChnScaleOffset
  Call ChnLinScale("[1]/Global Head Vz "+,F11_,",/Global Head Vx "+,F11_,",9.80665,0") '... Y,E,ChnScaleFactor,ChnScaleOffset
  Call ChnSub("[1]/Global Head Vx "+,F11_,",[1]/Sled Velocity","/Global Head Vx "+,F11_,") '... Y,Y1,E
  Call ChnIntegrate("[1]/Time","[1]/Global Head Vx "+,F11_,",/Global Head Dx "+,F11_,") '... XW,Y,E
  Call ChnIntegrate("[1]/Time","[1]/Global Head Vx "+,F11_,",/Global Head Dy "+,F11_,") '... XW,Y,E
  Call ChnIntegrate("[1]/Time","[1]/Global Head Vx "+,F11_,",/Global Head Dz "+,F11_,") '... XW,Y,E
  Call ChnToWfChn("[1]/Time","[1]/Global Head Dx "+,F11_,",0") '... X,ChnNoStr,XChnDelete
  Call ChnToWfChn("[1]/Time","[1]/Global Head Dy "+,F11_,",0") '... X,ChnNoStr,XChnDelete
  Call ChnToWfChn("[1]/Time","[1]/Global Head Dz "+,F11_,",0") '... X,ChnNoStr,XChnDelete
  Call ChnLinScale("[1]/Global Head Dx "+,F11_,",/Global Head Dx "+,F11_,",100,0") '... Y,E,ChnScaleFactor,ChnScaleOffset
  Call ChnLinScale("[1]/Global Head Dy "+,F11_,",/Global Head Dy "+,F11_,",100,0") '... Y,E,ChnScaleFactor,ChnScaleOffset
  Call ChnLinScale("[1]/Global Head Dz "+,F11_,",/Global Head Dz "+,F11_,",100,0") '... Y,E,ChnScaleFactor,ChnScaleOffset
  Call Chnpropvalset("[1]/Global Head Ax "+,F11_,","unit_string",",g")
  Call Chnpropvalset("[1]/Global Head Ay "+,F11_,","unit_string",",g")
  Call Chnpropvalset("[1]/Global Head Az "+,F11_,","unit_string",",g")
  Call Chnpropvalset("[1]/Global Head Vx "+,F11_,","unit_string",",m/s")
  Call Chnpropvalset("[1]/Global Head Vy "+,F11_,","unit_string",",m/s")
  Call Chnpropvalset("[1]/Global Head Vz "+,F11_,","unit_string",",m/s")
  Call Chnpropvalset("[1]/Global Head Dx "+,F11_,","unit_string",",cm")
APPENDIX G (continued)

Call Chnpropvalset("[1]/Global Head Dy "+F11_,"unit_string","cm")
Call Chnpropvalset("[1]/Global Head Dz "+F11_,"unit_string","cm")
Else
  Call MsgBoxdisp("Need to select units")
End If
Call Chnpropvalset("[1]/Time","unit_string","s")

Call ChnTMinTMaxCalc("","[1]/Global Head Vx "+F11_')... XW,Y
Call ChnPropValSet("[1]/Global Head Vx "+F11_,"MINMAX_MAXTIME",Maxtime)
Call ChnPropValSet("[1]/Global Head Vx "+F11_,"MINMAX_MINTIME",Mintime)
Call ChnTMinTMaxCalc("","[1]/Global Head Vy "+F11_')... XW,Y
Call ChnPropValSet("[1]/Global Head Vy "+F11_,"MINMAX_MAXTIME",Maxtime)
Call ChnPropValSet("[1]/Global Head Vy "+F11_,"MINMAX_MINTIME",Mintime)
Call ChnTMinTMaxCalc("","[1]/Global Head Vz "+F11_')... XW,Y
Call ChnPropValSet("[1]/Global Head Vz "+F11_,"MINMAX_MAXTIME",Maxtime)
Call ChnPropValSet("[1]/Global Head Vz "+F11_,"MINMAX_MINTIME",Mintime)
Call ChnTMinTMaxCalc("","[1]/Global Head Dx "+F11_')... XW,Y
Call ChnPropValSet("[1]/Global Head Dx "+F11_,"MINMAX_MAXTIME",Maxtime)
Call ChnPropValSet("[1]/Global Head Dx "+F11_,"MINMAX_MINTIME",Mintime)
Call ChnTMinTMaxCalc("","[1]/Global Head Dy "+F11_')... XW,Y
Call ChnPropValSet("[1]/Global Head Dy "+F11_,"MINMAX_MAXTIME",Maxtime)
Call ChnPropValSet("[1]/Global Head Dy "+F11_,"MINMAX_MINTIME",Mintime)