STRESS CONCENTRATIONS DUE TO COUNTERSUNK HOLES IN ADHESIVELY BONDED BI-LAYERED ALUMINUM SUBJECTED TO TENSILE LOADING

A Thesis by

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STRESS CONCENTRATIONS DUE TO COUNTERSUNK HOLES IN ADHESIVELY BONDED BI-LAYERED ALUMINUM SUBJECTED TO TENSILE LOADING

The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Aerospace Engineering.

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DEDICATION

To my teachers
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ABSTRACT

The adhesively bonded layered aluminum is used in aircraft structures to avoid knife edge situations when flush head fasteners are used with minimum gage skins. Due to the countersunk hole and adhesive bonding, stress flow becomes more complicated. Extensive knowledge of the different parameters that affect the behavior of the bonded joints with countersunk holes is essential for dependable and effective design. A 3-D finite element model was used to estimate the location and magnitude of stress concentration under remote tension for the aforementioned problem.

The influence of the various parameters on stress concentration was investigated for a counter sunk angle of 100°. Different parameters such as ratio of young’s modulus of adhesive to aluminum, position of adhesive layer, countersunk sunk depths, ratio of thickness to radius and ratio of width to radius have been addressed in this study.

The stress flow varies significantly when the plates are filled with fasteners of different pre-tension loads. Also the effects of pre-tension loading were compared for the cases of open hole and fastener filled hole without pre-tension for bonded, monolithic and straight shank hole. The results obtained from the finite element analysis for the monolithic cases have been validated against those reported in literature.
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<td>$E_{adh}$</td>
<td>Young’s modulus of adhesive</td>
</tr>
<tr>
<td>$E_{al}$</td>
<td>Young’s modulus of aluminum</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Countersunk depth</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of the hole</td>
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<tr>
<td>$C_a$</td>
<td>Location of the adhesive</td>
</tr>
<tr>
<td>$2w$</td>
<td>Width of plate</td>
</tr>
<tr>
<td>$2h$</td>
<td>Total height of plate</td>
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<tr>
<td>$2\alpha$</td>
<td>Counter sunk angle</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of specimen</td>
</tr>
<tr>
<td>$t_{adh}$</td>
<td>Thickness of adhesive</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>Applied remote tension</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Maximum Stress concentration (gross)</td>
</tr>
<tr>
<td>$K_{bk}$</td>
<td>Shear Stress Concentration</td>
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<td>$\sigma_{yy}(z)$</td>
<td>Hoop’s stress</td>
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$\sigma_{\theta z}$ Shear stress along $\theta$-$z$ direction

$N$ Number of Elements across the Thickness

$P$ Pre tension Load

$X, Y, Z$ Coordinates
CHAPTER 1

INTRODUCTION

1.1 Background

Joints are necessary in aircraft structures to transfer loads across two or more members of a structural assembly. A complex structure can be formed by using different joining processes such as fastener joints (also refereed as bolted or riveted joints), welded joints and adhesive joining process. Fastener joints are preferred over the other joining processes because of their ability to transfer high loads and also facilitate disassembly of structure for the purposes of inspection and repair. The fastener joints are widely used in critical structures such as aircraft wings, fuselage, rudder, civil structures etc. Many of these mechanical joints form vital elements in the structure which are most likely to fail. Thus, it is necessary for the designer to recognize the complex stress state arising in these joints to ensure safe operation of the structures under operational loads and to minimize the weight of the structure.

A typical fastener joint is illustrated in figure 1.1. The fastener holds the two parts (plates, sheets, substrates, etc) together between its head and nut, while the shank passes through the hole in the parts being joined.

Figure 1.1. Typical Bolted Joints [1]
Based on the type of loading, bolted joints are classified into two categories. In tensile joints, the load acts along the axis of bolt. Tension joints are also called as preloaded or pre-tension joints [1]. When the forces act perpendicular to the axis of the bolt, they are called shear joints. In shear joints, the loads are transferred between the joined parts by shearing (and bending) of the fastener. The load transfer between the two (or more) parts and the fastener occurs through bearing and in some instances friction between the two parts contributes to the load transfer process.

Depending on the specific application, different fastener designs may be used. The two primary fastener types are the “protruding head” and “flush head” fastener. With the protruding head fasteners, the fastener head protrudes out of the joint surface as illustrated in figure 1.1. Such protrusions may be undesirable in airframe structures where an aerodynamic surface is required. For such applications, “countersunk” or flush-head fasteners are used. Unlike the regular fastener holes, a countersink seats the head of the fastener, thereby producing a smooth surface. Typical countersunk/flush-head fasteners are illustrated in figure 1.2. Due to the smaller head size, these fasteners are used primarily in shear loaded joints.

![Figure 1.2. Flush head fastener [1].](image-url)
The fastener joints suffer from the stress concentrations arising (in the parts being joined) due to the presence of the hole. The stress concentrations arise in the presence of remote (bypass loads) as well as the bearing loads acting on the hole surface [2].

Unlike the joints using protruding head fasteners where the full thickness of the parts being joined is available for bearing load transfer, the joints using flush head fasteners use a fraction of the part thickness owing to the countersinking of the hole. In structural applications where the parts are thin to begin with, e.g., skins of airframes, the countersinking of the holes could lead to a situation termed as ‘knife-edge’ as illustrated in figure 1.3. Due to the limited hole surface available for bearing load transfer, the knife-edge regions are prone to yielding and reduced fatigue lives [3] due to elevated stress levels.

![Figure 1.3. Knife Edge Situation](image-url)
The knife-edge scenario is often alleviated using an adhesively bonded doubler which adds to the bearing area as shown in figure 1.3. This configuration of adhesively bonded layers is referred to as “layered” or “laminated” construction. In addition, the doubler(s) could serve as an additional load path and could contribute to altering the stress concentration at the hole.

![Layered construction of countersunk fastened Joint](image)

**Figure 1.4. Layered construction of countersunk fastened Joint**

The evaluation of stress concentration around open-holes with straight shanks and countersunk holes has received considerable amount of attention in open-literature [4, 5]. Analytical expressions [2] based on curve fitting of data from experiments and numerical models are available for predicting the stress concentration in plain sheets with countersunk holes under remote loading. However, the analysis and prediction of stress concentration in layered / laminated aluminum sheets with countersunk holes are rarely reported in open-literature. Some fatigue data [3] has been reported for layered construction, highlighting the key difference in comparison to the monolithic case.

In addition to the stress concentration arising due to remote loads, the layered construction presents added complexity in terms of the stress distribution across the layers, shear stress...
concentration in the adhesive, etc. The additional degrees of freedom associated with the choice of doubler thickness and adhesive properties, adds to the complexity of the problem.

In the present study, the stress concentrations in layered sheets with countersunk holes have been analyzed using non-linear finite element analysis. In addition to the variables associated with the countersink and the layered construction, the effects of hole filling due to the fastener and the clamp-up loads introduced during fastener installation, on the resulting stress concentration has been addressed.
CHAPTER 2
LITERATURE REVIEW

2.1 Research Survey

The earliest study on 3-D plain (straight shank) holes was conducted by Sternberg and Sadowsky [6] in which they suggested an analytical solution for cylindrical hole in an infinite plate of arbitrary thickness by using modification of Ritz method in elasticity approach. A 3-dimensional solution to the problem was considered by superimposing plane stress solution on the residual plane stress. The stress distribution in this case was found by using the principle of superposition of plane hydrostatic and plane pure shearing. The results obtained indicated that as the thickness ratio (which is defined as ratio of thickness to radius) increases the hoop stress along the thickness shows a drastic rise and asymptotically approaches plain strain value. The change in hoop stress due to the change in the thickness ratio was negligible. Hence it can be concluded that the stress in the direction of thickness has significant effect on stress concentration while rest of the stress components are negligible.

Folias and Wang [7] continued the study on 3-D plain holes of arbitrary thickness by covering different value of radius to thickness ratio. A 3-D plate of arbitrary thickness $2h$ and radius of the hole $a$ is subjected to a far field tension of $\sigma_o$. The analysis revealed that, stress concentration factor varies across thickness and was also a function of radius to thickness ratio. The analysis also conveyed that, as the Poisson’s ratio $\nu \rightarrow 0$, the stress concentration factor tends to 3. From this study they deduced that stress concentration factor decreases as it approaches the free surface. However, for a ratio of radius to thickness that is smaller than 0.2, the variation of stress concentration is negligible along the thickness and the maximum value occurs at the free surface. The results indicate that as radius to thickness ratio tends to zero, there exists a boundary
layer solution closer to the free surface. It was thus concluded that stress concentration is due to hoop stress which depends on dimensionless parameters such as ratio of thickness to radius and function of thickness. However the abnormal behavior of stress concentration factor for a very small radius to thickness ratio could not be explained.

Very few papers have been published on stress concentration in the countersunk holes [2, 8, -11]. The first experimental work was done by Wharley [8] using birefringent coating on the surface of the plates to estimate stress concentration. Two different sets of specimens one with plain holes and other with countersunk holes subjected to a fatigue loading at 60 cpm with a maximum stress of 46,700 psi, and resulting lives were compared. This evaluation assisted in finding out the maximum stress concentration factors for countersunk holes. Countersink angle of 100° and depth of half the thickness was used in preparing the specimen with countersunk holes. The specimens were coated with type-S photo stress birefringent plastic. These experiments proved that stress concentration in countersunk holes is higher than the plain holes. The results indicated that the stress concentration (which was inferred from the fatigue tests) was maximum at the countersink edge and varied along the thickness. The maximum stress concentration for countersunk holes was 13 to 23 percent higher than that of plain (cylindrical) holed specimens. The maximum stress concentration was found to be highest at the countersink edge which is 13 to 23 percent higher than the values at the free surfaces which was inferred from the fatigue test.

Stress concentration as function of the ratio of thickness to radius and thickness as mentioned by Folias and Wang [7] were ignored in this work.

Cheng [9] used a stress freezing technique to measure stress concentration along the thickness of the plate. A total of 13 different configurations, by varying countersunk angle, depth and radius of hole were investigated. A photo elastic epoxy resin was used as a model specimen
which is loaded in tension by using dead weights and four point bending by using rollers and dead weights. These specimens along with a calibration disk were passed through stress freezing cycle in a temperature controlled furnace. After the stress freezing cycle the specimens were halved and the maximum fringe order and hoops stress were determined. The results confirmed that stress concentration is highest at the countersink edge when the specimen is loaded in tension. The deviation in stress along the thickness in countersunk holes was higher than that of straight shank holes. Results indicated that the variation of small change in countersunk angle, depth to thickness ratio had no significant effect on stress concentration. From these experiments it was observed that finite width had a significant effect on stress concentration, which was 33% higher than that of the cylindrical hole. In case of bending, it was noted that the neutral plane does not remain at mid thickness of specimen. It was also noted that in case of bending hoops stress at the free surfaces were not equal. The neutral plane was shifted slightly below the countersink edge towards the straight shank portion and the maximum hoops stress occurred at the free edge of the straight shank portion.

In 1993 Young and Lee [10] conducted a parametric study of factors effecting stress concentration in countersunk holes using finite element analysis on the 3- dimensional plate subjected to remote tension and provided a design equation for the stress concentration. Various configurations were made by using different design parameters such as thickness to diameter ratio, edge distance (width of plate), countersunk depth and angle. A quarter symmetric model of geometry was used in developing a finite element model with gradient mesh which was finer at the surface of the hole. The results indicated that stress concentration was maximum at the countersink edge. When compared with cylindrical holes, the value of stress concentration at free surfaces was lower than that of plain holes but higher at the countersink edge. The value of stress
concentration increased with increase in the thickness to diameter ratio (which is also the thickness of plate as the fastener sizes are of standard diameters). Similarly, the stress concentration was also found to increase linearly with countersunk depth. For a small change in countersink angle from 90° to 100°, the authors observed that the change of stress concentration was negligible. With the increase in edge distance (width of plate) the value of stress concentration decreased, which is in agreement with the St.Venants free edge effects. Considering the above parameters and normalizing the maximum value of stress concentration with that of the plain holes, it was observed that countersunk depth was the only independent parameter affecting relative stress concentration. However solutions of Young and Lee were based on a very coarse model and the equation developed by them did not condense to a 2-D plane stress solution as mentioned in the handbooks [4-5].

Later, this work was continued by Shivakumar and Newman [11] in which they conducted 3-dimensional stress analysis on various configurations of straight shank and countersunk holes. Stress concentration along the thickness was obtained for various configurations such as thickness to diameter ratio, countersink depth and angle for a very wide plate subjected tension and bending. In case of straight shank holes along with tension and bending, wedge loading was also applied. In case of tension, the stress concentration was observed to increase linearly with the countersunk depth and thickness to radius ratio. Similar to Young and Lee’s [10] model for a small change in countersunk angle the variation of maximum stress concentration was negligible. In case of bending, the stress concentration was almost unaffected by countersunk angle and depth. Maximum stress concentration in tension for a plate with countersunk hole was about 37% higher than the 2-D solution presented in the hand books [4-5], whereas it is 30% higher for 3-D straight shank holes. In case of bending, maximum stress concentration was approximately same as that of
the straight shank plate. Based on the finite element results, a multi-parameter least square fits were performed to obtain an equation for stress concentration for tension, bending and wedge loading. By using the principle of superposition, tension and wedge loading the results for pin loading in cylindrical holes were obtained. A FORTRAN program was developed, which included an interpolation technique for determining stress concentration for various kinds of loading. The major drawback of this work was that the results were limited to very wide plates.

Shivakumar et.al [2] continued their work on countersunk holes in which a 3-D finite element analysis was performed on various configurations of countersunk and straight shank holes. Various countersunk parameters such as, width to radius ratio and thickness to radius ratio were considered in developing an equation for stress concentration factor. Similar to [10] a quarter symmetric model of the geometry was modeled by using appropriate symmetry boundary conditions. A remote tension of $\sigma_o = 1$ was applied so that the resulting stresses directly gave the value of stress concentration. A very fine mesh was created at the surface of the hole which gradually became coarser away from the hole. From the results obtained it was concluded that, for a small change in countersunk angle the deviation of maximum stress concentration can be ignored. The authors observed a monotonous increase in the value of stress concentration with countersunk depth and thickness to radius ratio. The maximum stress concentration for very thick plates was observed to be at the countersunk edge and for thin plates it was below (5% of thickness) the countersunk edge in straight shank portion. Maximum stress concentration was observed to decrease with increase in width of plate. For very wide plates the stress concentration remains constant throughout the thickness of the plate. An equation for the stress concentration as a function of plate thickness, countersunk depth and width was developed by using finite element results. The developed equation had a maximum error of 7% when compared with finite element
results. Hence the equation obtained is accurate enough for all limiting conditions including the
knife edge situation.

Lanciotti and Polese [3] studied fatigue properties of monolithic and metal laminated
aluminum open-hole (straight shank) specimens. Thin sheets of aluminum were bonded by using
an adhesive to obtain the desired thickness, which were called metal laminated materials. Four
different kinds of specimens were prepared for conducting fatigue and crack propagation tests.
They are

1. Monolithic specimens which were drilled, reamed and de-burred.

2. Monolithic specimens which were drilled, reamed by stacking ten specimens on
   milling machine and two aluminum plates pressed against the packing. Holes were not
de-burred.

3. Similar to the second case but pressing is done by placing plastic foil in between the
   plates. Holes were not de-burred.

4. Metal laminates with adhesive bonding used were drilled, reamed and de-burred.

The fatigue tests results indicated that the monolithic specimens with holes de-burred has a
have a longer life compared to that of monolithic specimen in which holes were not de-burred.
Further, the life of specimens which were drilled after placing a plastic foil were less due to the
large discontinuities caused by the plastic foil. Metal laminates had better life period when
compared to monolithic specimens under low fatigue loads, whereas both behaved similarly in
case of higher fatigue loads. For crack propagation test, a single corner flaw was induced at the
edge of hole. In metal laminated specimens cracks nucleated early but the propagation is slow. In
monolithic specimens the nucleation of crack was delayed but propagation was very fast. Fatigue
crack propagation lives of metal laminates were about 5 times higher than that of monolithic.
Ratwani [11] conducted a parametric study of fatigue crack growth behavior in adhesively bonded metallic structure. The stress intensity factors were obtained by using a finite element model and analytical predictions were compared with those of experimental values. The finite element model considered each layer as a 2-D dimensional structure under plain stress. Adhesive material was considered as a shear spring rather than elastic continuum. The simulations were carried out considering the presence elliptical de bonds with minor to major axis ratio of 0.1. The author concluded that the use of brittle adhesive would decrease the crack growth life of a structure. Also as the thickness of the adhesive layer increases, the life of the structures was observed to be decrease.

Due to adhesive bonding and countersunk hole the stress flow becomes more complicated. There hasn’t been any work done on the bonded joints with countersunk holes in open literature which is a practical design used in aircraft industry. The present study has been taken up to give an accurate idea of tensile stress concentration in adhesively bonded joints with countersunk holes and the properties of adhesive affecting the stress concentration. A comprehensive 3-D finite element analysis with very fine modeling will be conducted on bonded joints with countersunk holes. A parametric study includes ratio of thickness to radius, Young’s modulus of adhesive to aluminum, position of adhesive, countersunk depth to thickness and finite width effect. The structure is also affected by fastener filled holes with different pre-tension loading. Based on the results the stress concentration in bonded joints will be compared with that of the monolithic plates. Hence all these parameters are to be studied to design of adhesively bonded joint with countersunk hole.
3.1 Statement of problem

The objective of this work is to characterize the influence of factors effecting the stress concentration around fastener holes in adhesively bonded bi-layered aluminum. According to Lanciotti and Polese [3], the crack propagation rate in bonded aluminum is slow compared to the monolithic aluminum. In addition, the bonded aluminum is used to avoid the knife edge situations when flush head fasteners are used with minimum gage skins as illustrated in figure 3.1 [3]. Countersinking and adhesive bonds complicates the stress flow and could further increase the local stresses. Therefore it is very important characterize these local stresses to predict joint strength and fatigue life.

Figure 3.1. Crack propagation in monolithic plates and metal laminates [3]
Discontinuities introduced to the countersunk holes changes the load path. Additionally, these joints are subjected to various sets of loads such as tension, bearing and bending. An accurate study demands an investigation of stress concentration factors bonded bi-layered aluminum with countersunk holes. However, due to complicated stress flow in bonded aluminum, it will be more detrimental to the fatigue behavior. Exhaustive study on various parameters such as countersunk depth, thickness to radius ratio, width, material properties of adhesive, location of adhesive and countersunk angle effecting the stress concentration have been studied. In addition to these parameters elastic–plastic model for a particular configuration has also been studied to determine effects of residual stresses on stress concentration in open holes.

In the present study, the stress fields and specifically stress concentration factors in monolithic and bi-layered aluminum sheets has been studied using finite element modeling. Both straight shank and countersunk holes will be studied for comparison. The effects of the presence of fastener and associated clamp-up loads will also be addressed in this study.

### 3.2 Approach

A finite element analysis was performed to approximate the position and magnitude of stress concentration under tensile loading for various parameters. To study effects of these parameters on stress concentrations under tensile loading, a quarter symmetric model of the geometry was modeled with suitable boundary conditions in ABAQUS. The results of the model are validated with the results reported in references [2] and [6]. A non linear elastic-plastic analysis with load increment method has been studied to find out the effects of residual stresses on stress concentration. The parameters investigated in this study are listed in following table 3.1.
### TABLE 3.1

**SELECTION OF PARAMETERS**

<table>
<thead>
<tr>
<th>Type of plate</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open hole</td>
<td>1. Countersunk Depth</td>
</tr>
<tr>
<td></td>
<td>2. Young’s modulus of adhesive</td>
</tr>
<tr>
<td></td>
<td>3. Width to radius ratio</td>
</tr>
<tr>
<td></td>
<td>4. Thickness to radius ratio</td>
</tr>
<tr>
<td></td>
<td>5. Position of adhesive</td>
</tr>
<tr>
<td></td>
<td>6. Countersunk angle</td>
</tr>
<tr>
<td></td>
<td>7. Elastic plastic model</td>
</tr>
<tr>
<td>Fastener Filled hole</td>
<td>Straight Shank hole</td>
</tr>
<tr>
<td>With pre-tension</td>
<td>Countersunk Hole</td>
</tr>
</tbody>
</table>

A contact analysis with between the fastener and the plate is done to find out effects of filled holes on stress concentration compared to that of open holes. The important feature of the 3-D modeling is contact surface between the plate and the fastener. A comparative study between the straight shank hole and countersunk hole was also done.
CHAPTER 4
FINITE ELEMENT MODEL DEVELOPMENT

4.1 Introduction

This chapter discusses in detail the 3D finite element model developed to predict the stress concentration factors around the countersunk hole in an adhesively bonded bi-layered aluminum for both open hole and fastener filled holes. The FE model was validated for the case of monolithic plates using the results reported in [2].

4.2 Finite Element Modeling

The finite element model was assembled using ABAQUS, a commercial finite element analysis software. ABAQUS pre- and post-processor has been used to generate various finite element models and to analyze the results.

4.3 Development of Model

The model for this analysis consists of two components, plates and fastener which are shown in the figures 4.1 and 4.2 respectively.
Figure 4.1. Quarter symmetric model of bi-layered plate

Figure 4.2. Schematic of countersunk fastener
4.4 Open Hole Configuration

The configuration of a countersunk plate of height $2h$ and width $2w$, with a fastener hole is shown in figure 4.3. Let the thickness of the plate be $t$, radius of hole shank be $r$ is assumed to be 0.077 in [2] and the same radius is also used in the present analysis, the depth of straight shank be $C_s$, thickness of adhesive be $t_{adh}$ and countersunk depth be $C_r$.

A practical countersunk angle of 100° was used in a majority of the simulations, although the effect of countersink angle is discussed in the present study. The effect of countersink angle was studied by varying the countersunk angle from 60° to 110°.

The $t/r$ ratio was found to be the main parameter in evaluating stress from theoretical analysis. This parameter is varied from 0.5$r$,1$r$,2$r$ till 4$r$. Commonly used cases in the aircraft industry [3] are from 0.5$r$ to 2$r$. The countersunk depth $C_s$ is varied as 0.25$t$, 0.5$t$ and 0.75$t$. The cases such as $C_s$=0 $t$ and $C_s$ = $t$ are ignored as they are representing the cases of straight shank hole and knife edge situation. Finite width effects are studied by considering half width $w$ = 3$r$,4$r$,6$r$ and 15$r$. The cases of 3$r$ to 6$r$ are practically used in aircraft industry. In Sternberg and Sadowsky’s case, the solution was developed for a case of infinite width plate [6]. According to St.Venants principle of dissipation of localized stresses, there will be a finite width beyond which the stresses can be considered to approach that in a case of an infinite width plate.

Further, the ratio of Young’s modulus of adhesive to aluminum is also considered to be key parameter for determining the stress concentration in bonded joints. It is varied for $E_{adh}/E_{al}$ =0.02, 0.05, 0.08, and 0.1. When the ratio of young’s modulus of adhesive increases the situation tends to become a case of monolithic plate. Practical adhesives used in the industry result in a ratio of 0.1 to 0.2. The position of the adhesive is also a key parameter affecting stress
concentration. The position of adhesive is varied for $C_a = 0.2t, 0.4t, 0.5t$ and $0.7t$. Commonly a case of $0.5t$ is used in aircraft industry as other cases like $0.2t$ and $0.4t$ are practically difficult to use. The above parameters are varied to carry out different sets of simulation.
a) y-z plane

b) x-y plane

Figure 4.3. Schematic of the problem showing the geometric variables.
4.5 Boundary Conditions

The figure 4.4 shows the boundary and loading conditions of the mechanical model for bi-layered aluminum.

![Figure 4.4 Boundary conditions](image)

A quarter symmetric model of the geometry was modeled by applying suitable boundary conditions x-symmetric ($u_1=0$) and y-symmetric ($u_2=0$) in $x=0$ and $y=0$ planes respectively. The study was conducted for an average far field stress of $\sigma_0 = 1000$ psi. To obtain uniform strains in both adhesive and plate, the stress in aluminum and adhesive layer are proportional to their Young’s modulii. However, the average stress is still 1000 psi.
4.5.1 Loading Conditions for Elastic-Plastic Model

The investigation was conducted for $\sigma_o = 10, 15, 20$ and $25$ ksi and stress along the nodal line A-B-C were taken which gave the stress concentration $K_1(z)$. A constant countersink angle of $100^\circ$ was used in all simulations. The thickness of adhesive was maintained constant for all simulation as $t_{adh} = 0.005$in. A nonlinear elastic-plastic analysis was conducted by load incremental method for calculating residual stresses. For calculating the residual stresses amplitude of the load is defined as shown in the Table 4.1.

**TABLE 4.1:**

<table>
<thead>
<tr>
<th>Total Time Steps</th>
<th>Load factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
</tr>
<tr>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
</tr>
</tbody>
</table>

The configuration of the model used for the elastic-plastic analysis was $C_s/t = 0.5$, $C_a/t = 0.5$, $E_{adh}/E_{ad} = 0.05$, $t/r = 0.5$, $w/r = 15$ and $h/r = 15$. 
4.6 Material Model

For the open hole simulations, both the plate and adhesive are assumed to be linear elastic. The material is assumed is to be isotropic Hookean which can be used in combination with different kinds of elements. The stress-strain curve of the elastic model is represented by Hooke’s law as shown in equation 4.1.

\[ \sigma = E\varepsilon \]  

(4.1)

![Stress Strain diagram](image)

Figure 4.5. Stress Strain diagram

Where \( \sigma \) represents the uniaxial stress and the \( \varepsilon \) represents the strain induced due to the applied stress. The plates are assumed to have properties of Al-2024-T3 which is used in aircraft industry.

The adhesive is also assumed to be isotropic-Hookean material. To investigate the effects of young’s modulus of adhesive on stress concentration, it is assumed to be varying parameter.
4.6.1 Elastic Plastic model

The material behavior in assumed to be linearly elastic until it reaches the yield stress $\sigma_y$. When the applied stress reaches beyond the yield stress, material experiences work hardening, which in turn results in both elastic and plastic deformation. The material is assumed to have properties of Al 2024-T3 and undergo isotropic hardening. The Table 4.2 lists all the properties of the material used in this present study.

<table>
<thead>
<tr>
<th>Material</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum(Al 2024-t3) (Both Plates and fastener)</td>
<td>Elastic Modulus -10.7 msi</td>
</tr>
<tr>
<td>Adhesive</td>
<td>Poisson’s ratio=0.3</td>
</tr>
<tr>
<td></td>
<td>Yield Stress -50 ksi with 0% Elongation</td>
</tr>
<tr>
<td></td>
<td>Yield Stress -65 ksi With 15 % Elongation</td>
</tr>
<tr>
<td></td>
<td>Elastic Modulus –varying parameter</td>
</tr>
<tr>
<td></td>
<td>Poisson’s Ratio -0.3[3]</td>
</tr>
</tbody>
</table>

4.7 Element Formation

Different meshing techniques were applied to mesh the plates and adhesive used in the simulations. The figure 4.6 shows the typical finite element mesh of the components used in the analysis.

The geometry of plates has been partitioned into two halves so that the desired mesh is obtained. Gradient mesh was used because the stress at the hole face is anticipated to be highest, a finer mesh is created at the hole face (to accurately determine the local stresses) which
progressively becomes coarser away from the hole. An 8-node hexahedron element (in ABAQUS C3D8R) with reduced integration and hourglass control was used in the simulation for both the plate and adhesive material. Linear geometric order of the element was used for both plate and adhesive in all simulations.

Figure 4.6. Gradient mesh of the plate
The test of convergence was carried out on one of the models with fine tuning of mesh and number of elements across the thickness ($N$) and near hole was done to ensure if the solution will converge as shown in figure 4.7. Based on this convergence study 50 elements were used across the thickness for the simulation as the curve starts converging from that point.

Figure 4.7. Convergence of solution with different mesh size

4.8 Fastener Filled Holes

A non-linear contact analysis between the plates (with both circular, countersunk hole) and fastener, to find out the effects of pre-tension on stress concentration by varying the maximum pre-tension load. The configuration of the model used for straight shank hole used is $t/r = 0.5$, $w/r = 15$, $h/r = 15$, $t = 0.077\, \text{in}$. The configuration of the model used for the countersunk holes was $C_s/t = 0.5$, $C_o/t = 0.5$, $E_{adh}/E_{aol} = 0.05$, $t/r = 0.5$, $w/r = 15$ and $h/r = 15$. 

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4.8.1 Definition of Contact between Plate and Fastener

A general surface-to-surface contact interaction [12] is made between the plate and fastener. The contact surface connecting the plate and fastener is chosen as frictionless sliding interface. The function of this contact interaction is to avoid penetration of elements of both components. Frictionless contact is chosen to simplify the problem, on the stress concentration. Newton’s iterative technique along with adaptive stabilization [12] is used for solving this problem.

4.8.2 Loading conditions for Fastener filled holes

The pre-tension section will allow the contacting surfaces to interfere with each other, i.e., the top and bottom half of the bolt may overlap without a contact being detected. If there are no sheets between the bolt head and the nut, the two halves of the bolt simply overlap each other without any pre-tension being generated as shown in the figure 4.8 and 4.9. However, when a sheet is present between the head and nut, it will get squeezed and resist the overlapping of the two parts of the bolt and thus generate the pre-tension. This pre-tension-load may be locked by fixing the ‘control’ node displacement shown in figure 4.10.

4.8.2.1 Load Steps

The investigation was conducted for various pre-tension loads 50,100,150,200 lbs in the fastener and far field tension of $\sigma_o = 1000 \text{ psi}$ in plates as shown in the figure 4.11.

4.8.2.2 Apply Pre-tension Load

Pre-tension load is applied on the fastener in the first step of analysis which has automatic stabilization for attaining load equilibrium on an internal bolt surface. In this step the far field tension in the plates is zero.
Figure 4.8. Pre-tension surface of straight shank fastener

Figure 4.9. Pre-tension surface of countersunk shank fastener
Figure 4.10. Pre-tension Node

Figure 4.11. Loading conditions of Pre-tension fastener
4.8.2.3 Far field Loading

The step 2 starts after the step 1 is completed. In this step, the bolt deformation is fixed at the value which is obtained at the end of first step. The overlap between the head and nut is fixed. However the length of the bolt can change.

4.9 Model Validation

A detailed evaluation of present finite element model with available data was done to validate the accuracy of model. The number elements used across the thickness were 50. The finite element model developed in this study has been validated by comparing the results of finite element model results reported in [2] and [6]. A monolithic plate was modeled with same configuration as in [6] and analyzed with the same boundary conditions. The variation of stress concentration with the thickness per ref [6] was also studied in the analysis. The configuration of model used for validation was \( \frac{t}{r} = 0.5 \), \( \frac{w}{r} = 15 \) and \( \frac{h}{r} = 15 \) and the hole radius is 0.077in. The results summarized are shown in figures 4.12 to 4.15.
Figure 4.12. Comparison with [2] for $\frac{C_s}{t} = 1$

Figure 4.13. Comparison with [2] for $\frac{C_s}{t} = 0.25$
Figure 4.14. Comparison with [2] for $\frac{C_s}{t} = 0.5$

Figure 4.15. Comparison with [2] for $\frac{C_s}{t} = 0.75$
The marginal differences between the results of present simulation in comparison with [2] are due to the different mesh used. From the Figures it is also evident that the stress concentration varies along the thickness as suggested by [6].

The differences noticed among these two approaches are attributed to the numerical errors associated with finite element analysis.
CHAPTER 5

STRESS CONCENTRATION FACTORS IN PLATES WITH OPEN HOLE

5.1 Introduction

This chapter gives a detailed overview of the stress fields computed using finite element model for open countersunk holes. FE analysis has been conducted for a variety of configurations by varying the ratio of Young’s modulus of adhesive to aluminum \( \frac{E_{adh}}{E_{al}} \), countersunk angle \( 2\alpha \), position of adhesive bondline \( C_a \), ratio of \( \frac{C_s}{t} \), ratio of thickness to radius \( \frac{t}{r} \) and ratio of width to radius \( \frac{w}{r} \). In all the simulations the distance of the loading edge from the hole has been fixed such that \( \frac{h}{r} = 15 \). A constant adhesive thickness of 0.005 inches was used for all analyses.

5.2 Coordinate system and Stress Distribution around the Hole

In the countersunk hole, there are three regions where the stress gradient is high near the two free surfaces and at the countersink edge. From [7] it is observed that the stress concentration varies along the thickness for 3-D configurations. The distribution of von-Mises stress around the hole surface and along the thickness directions is shown in the figure 5.1. It indicates that the highest stress gradient is located at the countersunk edge. The countersunk holes make the problem asymmetrical about the thickness which results in a stress distribution that is asymmetrical across the thickness, resulting in out-of-plane displacement/bending as shown in figure 5.2. The distribution of \( \sigma_{yy} \) the along \( Y \)-direction and shear stress \( \tau_{xz} \) component developed in the region of the hole are shown in the figures 5.3 and 5.4.
Figure 5.1. Distribution of von-Mises stress around the hole

Figure 5.2. Out of Plane Deformation of plate
Figure 5.3. Distribution of Stress in $Y$-Direction ($\sigma_{yy}$) around the hole

Figure 5.4. Distribution of Shear Stress ($\tau_{\theta z}$) around the hole
5.3 Effect of Countersunk Angle \((2\alpha)\)

The effect of countersunk angle on the stress concentration around the hole in this work is studied by altering the half countersunk angle for quarter symmetric model. In this study, the countersunk angle is varied from 60° to 110° to determine the effects of countersunk angle on stress concentration. Variation of \(K_t\) was found to be relatively negligible with the small change in countersunk angle over the range of angles investigated. This was confirmed by varying the countersunk angle for two different configurations of \(t/r = 1\) and 0.5, while keeping the other parameters \(h/r = w/r = 15\), \(E_{adh}/E_{al} = 0.1\), \(C_s/t = 0.5\) and \(C_a/t = 0.5\) and \(r = 0.077\) in constant for both configurations.

The stress concentration factor \(K_t(z)\) increases monotonically with increase in the value of \(2\alpha\) as shown in figures 5.5a and 5.5b. The reason for monotonic increase is due to uninterrupted channeling of load path in the region of the countersink edge. Maximum value of \(K_t\) is observed at countersink edge and it decreases towards the material surfaces where the stress concentration is lower. Decrease of \(K_t\) in countersunk portion (B-C) is much faster than that of the straight shank portion (A-B). The minimum stress concentration is found at countersunk free surface (C). The change in maximum stress concentration for change in \(\alpha\) from 90° to 100° and 100° to 110° is about 1.6%, which is shown in Figure 5.6. Hence the variation of \(K_t\) is negligible for small change in countersunk angle. A constant countersunk angle of 100° is used in all following simulations because it is commonly used in aircraft industry.

The variation in the value of \(K_t\) for monolithic plates is slightly lower than that of Sivakumar and Newman [11]. The results are in agreement with the results from [11], but are
opposite to the trend obtained by Young and Lee’s model [10]. This may be due to the use of coarser mesh by Young and Lee [10].

Figure 5.5 (a). Effect of countersunk angle on $K_i(z)$

Figure 5.5 (b). Effect of countersunk angle on $K_i(z)$
5.4. Effect of Ratio of Young’s modulus of the adhesive to aluminum

The effect of Young’s modulus of adhesive on the stress concentration around the hole in this work is studied by altering the ratio of Young’s modulus of the adhesive to aluminum (properties are shown in Table 4.2). Simulations were carried out for modulus ratios of 0.02, 0.05, 0.08 and 0.1 for a fixed geometric configuration of $C_s/t = 0.5$, $C_a/t = 0.5$, $t/r = 1$ and $w/r = 15$. The thickness of the adhesive was maintained constant at 0.005 in.

The results from the analysis are presented in the figures 5.7 and 5.8. There is a difference of 2.4% in the maximum stress concentration when $E_{adh}/E_{al} = 0.02$ is compared with $E_{adh}/E_{al} = 0.1$. The difference decreases to less than 1% when $E_{adh}/E_{al} = 0.08$ is compared with $E_{adh}/E_{al} = 0.1$. The value of $K_t(z)$ is observed to be monotonically decreasing.
with increasing ratio of $\frac{E_{adh}}{E_{al}}$. As the Young’s modulus of adhesive increases, the situation tends towards monolithic case. The comparative study is shown in the figure 5.9. For a ratio of $\frac{E_{adh}}{E_{al}} = 0.07$, the value of $K_i$ is nearly the same as that of monolithic plates. For smaller ratios of $\frac{E_{adh}}{E_{al}} < 0.07$ the value of $K_i$ is higher in case of bonded plates compared to that of monolithic plates, whereas for higher ratios of $\frac{E_{adh}}{E_{al}} > 0.07$, it is less than that of the monolithic plate.

The reason for higher variation in $K_i$ is due to discontinuity in the load path because of bonding. With higher Young’s modulus, there is a reduction in stress value. One can expect that the stress around the hole to be reduced when stiffer adhesives are used. Based on these results it may be concluded that $K_i$ in adhesively bonded plates is higher than their monolithic counterparts, and is a function of the ratio of Young’s modulus of adhesive to aluminum (sheet).
Figure 5.7. Effect of $\frac{E_{adh}}{E_{al}}$ on stress concentration.

Figure 5.8. Variation Maximum Stress Concentration with Young's modulus of adhesive.
Figure 5.9. Comparison of stress concentrations bonded plates with monolithic plates for different $E_{adh}/E_{al}$ ratios.

5.5. Effect of Thickness to Radius ($t/r$) Ratio on Stress Concentration

The effect of sheet thickness to radius ratio on the stress concentration around the hole in this work is studied by varying the thickness of the plates, while holding the hole radius at 0.077 in. In this study, the thickness of the plates is varied from 0.5r to 4r to determine the effects of thickness to radius ratio on stress concentration. Values of $t/r = 0.5, 1$ and 2 represent the practical range of holes used in aircraft industry and $t/r > 2$ is used for thick plates. The variation covers from thin plates to thick plates. Simulations were carried out for different thickness ratio $t/r = 0.5, 1, 2$ and 4, for fixed values of $E_{adh}/E_{al} = 0.1$, $C_s/t = 0.5$, $C_w/t = 0.5$ and $w/r = 15$. The thickness of the adhesive bondline was maintained constant at 0.005 in.

The variation of $K_t(z)$ with the thickness to radius ratio is shown in figure 5.10. With
increase in $t/r$ ratio the value of $K_i$ increases monotonically. For smaller $t/r$ ratios ($t/r < 1$), the value of $K_i$ is observed to be at the countersink edge. At the countersunk edge the $K_i$ value appears to have a maximum value of 3.47 for the case of $t/r = 0.5$. A similar trend was observed in straight shank holes in [7]. There is difference of 16% in the maximum stress concentration between $t/r = 4$ and $t/r = 0.5$ is shown in figure 5.11.

These results are in agreement with the trend reported by Sternberg and Sadowsky [6]. A comparison between monolithic and bonded plates is shown in figure 5.12. The stress concentration in case of bonded plates is lower than monolithic plates for higher adhesive modulus. The difference between maximum stress concentrations for thinner plates ($t/r = 0.5$) is about less than 1% whereas the difference between maximum stress concentrations for thicker plates is around 4% when compared with monolithic. This implies that for thin plates stress concentration does not vary much between the bonded or monolithic plates.
Figure 5.1. Effect of $t/r$ on stress concentration

Figure 5.2. Variation Maximum Stress Concentration with $t/r$ ratio
Figure 5.3. Comparison between Bonded and Monolithic plates for maximum stress concentration for $E_{adh}/E_{al} = 0.1$ and $C_s/t = 0.5$

5.5.1 Variation of Shear Stress in the Adhesive Layer Due to $t/r$ Ratio

The shear stress $\sigma_{tk}$ increases with an increase in value of $t/r$, as shown in figure 5.13. Values of $\sigma_{tk}$ at $x = r$ along the circumference (i.e. from $\theta = 0^\circ$ to $90^\circ$) of mid thickness of the adhesive layer were obtained from finite element results. As shown in figure 5.13, the shear stress $\sigma_{tk}$ along the circumference of adhesive was found to be minimum at $x = r$ and $y = r$ and it is maximum at an angle of $45^\circ$ from $x = 0$.

The shear stress $\sigma_{tk}$ is an important factor in investigation of stress concentration factors in laminated plates. The crack initiation in the bonded plates is due to the de-bonding at the interface of adhesive and plates where the maximum stress concentration occurs [3]. The reason for earlier crack initiation in [3] is attributed to shear stress $\sigma_{tk}$ which is absent in the monolithic
plates. Although the crack initiates earlier, due to the bonding between the plates, the crack takes a longer time to propagate.

The difference between maximum shear stress for thinner plates \( \left( \frac{t}{r} = 0.5 \right) \) is about less than 90% compared to that of the thicker plates \( \left( \frac{t}{r} = 4 \right) \). This is shown in figure 5.14. This huge variation suggests that crack initiation at bond line between adhesive and aluminum is earlier in thicker plates compared to that of thinner plates.

![Figure 5.4. Variation of shear stress along circumference of adhesive](image)

\[ E_{adh}/E_{al} = 0.1, C_s/t = 0.5, C_a/t = 0.5, w/r = 15 \]
5.6 Effect of Bondline Location on Stress Concentration

The effect of adhesive location on the stress concentration around the hole has been studied by varying the location of the adhesive. In this study, the location of the adhesive is varied from 0.2t to 0.8t, to determine the effects of the adhesive location on stress concentration. Simulations were carried out for different ratios of \( \frac{C_a}{t} = 0.2, 0.5, 0.7 \) and 0.8 for a fixed configuration of \( \frac{C_t}{t} = 0.5 \), \( \frac{E_{adh}}{E_{al}} = 0.1 \), \( \frac{t}{r} = 0.5 \) and \( \frac{w}{r} = 15 \) and thickness of the adhesive was maintained constant at 0.005 in.

The variation of \( K_i(z) \) across the thickness is shown in figure 5.14. The stress concentration value is highest at the countersink edge for all \( \frac{C_a}{t} \) values. There is a difference of 4% in the maximum stress concentration between \( \frac{C_a}{t} = 0.5 \) and \( \frac{C_a}{t} = 0.2 \) is shown in figure 5.15. The difference in the maximum stress concentration for small variation of \( C_a \) from 0.8t to 0.7t is less than 1%. The stress concentration value increases in the straight shank portion and is highest at the countersink edge. The stress concentration decreases rapidly in the countersink region. When the adhesive layer is located in the countersunk section away from the countersunk edge, the value of \( K_i \) is observed to be lowest. The value of \( K_i \) is highest when the adhesive layer is positioned next to the counter sunk edge.

A comparison between monolithic and bonded plates for different position of adhesives is shown in figure 5.16. The stress concentration is highest when the adhesive is positioned next to the countersink edge in straight shank portion. The reason for maximum value of \( K_i \) at this location is due to discontinuity of load path at the countersink edge due to the adhesive.
Figure 5.5. Effect of adhesive position on stress concentration

Figure 5.6. Variation of maximum stress concentration due to $\frac{C_{ad}}{t}$
5.7 Finite Width Effects

The effect of width to radius ratio on the stress concentration around the hole in this work is studied by varying the width of the plates. In this study, the width of the plates was varied from 3 to 15 to determine the effects of width to radius ratio on stress concentration. Values of \( \frac{w}{r} = 3, 4, 6 \) and 15 represent the different width of plates covered in this study. The case of \( \frac{w}{r} = 3 \) represents the classical case of stress concentration in straight shank holes. Similarly the case of \( \frac{w}{r} = 15 \) represents the infinite width problem. Simulations were carried out for different modulus ratio \( \frac{w}{r} = 3, 4, 6 \) and 15 for different configurations \( \frac{t}{r} = 0.5 \) and 1 with parameters remaining constant \( \frac{E_{\text{adh}}}{E_{\text{al}}} = 0.1, \frac{C_s}{t} = 0.5 \) and \( \frac{C_a}{t} = 0.5 \). The thickness of the adhesive was maintained constant at 0.005 in.
The variation of $K_i(z)$ with $\frac{w}{r}$ ratio is shown in figure 5.17. The $K_i(z)$ value decreases monotonically with an increase in the width of the plate. These results were in concurrence with the experimental results of [6]. The maximum $K_i$ occurs at the central region and declines towards the free surface. The free edge phenomenon, which is a result of the non-zero Poisson’s ratio has been explained by [6]. The value of $K_i(z)$ is highest in the straight portion slightly below the countersunk edge. The rate of increase in $K_i$ decreases as $\frac{w}{r}$ ratio increases above 5. There is a difference of 17% in the maximum stress concentration between $\frac{w}{r} = 3$ and $\frac{w}{r} = 15$ is shown in figure 5.18. The difference in the maximum stress concentration for the variation of $\frac{w}{r}$ from 6 to 15 is less than 3%.

A comparison between monolithic and bonded plates is shown in figure 5.19. The stress concentration in case of bonded plates is lower than monolithic plates. The difference in the maximum stress concentration for bonded and monolithic is 5%. Discontinuity due to bonding is the reason for the difference in the value of $K_i$ between monolithic and bonded specimens.
Figure 5.8. Effect of $w/r$ on stress concentration

Figure 5.9. Variation of maximum stress concentration due to $w/r$
5.8 Effect of Countersunk Depth on Stress Concentration

The effect of countersunk depth on the stress concentration around the hole was studied by varying the countersunk depth. In this study, the countersunk depth varied from 0.25t to 1t to determine the effects of countersunk depth on stress concentration. The countersunk depth of zero is a case of straight shank hole whereas the countersunk depth of 1t represents the knife edge situation. This variation covers thin doublers to thick doublers. Simulations were carried out for different ratios of $\frac{C_s}{t} = 0.25, 0.5, 0.75$ and 1 for two different configurations of $\frac{t}{r} = 1$ and 0.5 with other parameters $\frac{E_{adh}}{E_{al}} = 0.1$, $\frac{C_a}{t} = 0.5$ and $\frac{w}{r} = 15$ held constant. The thickness of the adhesive was maintained constant at 0.005 in.

The variation of $K_t(z)$ along $\frac{z}{t}$ for varying countersunk depths is shown in figure 5.20. For thinner plates maximum stress concentration occurs at the countersunk edge and as thickness
increases, maximum stress concentration is slightly away from the countersunk edge in straight shank portion. The value of $K_r$ increases monotonically with increase in $\frac{C_s}{t}$. The knife-edge situation is also shown in figure 5.21. The trend was also noted by [2]. There is a difference of 14% in the maximum stress concentration between $\frac{C_s}{t} = 3$ and $\frac{C_s}{t} = 15$ is shown in figure 5.21. The difference in the maximum stress concentration for variation of $\frac{C_s}{t}$ from 6 to 15 is less than 3%. There is a difference of 22% in maximum stress concentration of knife edge situation and straight hole classical problem ($K_r = 3$) and 17% for problem straight shank hole with finite thickness. There is a significant difference in the value of $K_r$ between monolithic and bonded plates as shown in Figure 5.22. The stress concentration in case of bonded plates is lower than monolithic plates. The difference in the maximum stress concentration for bonded and monolithic is 3.5%
Figure 5.10. Effect of countersunk depth on stress concentration

Figure 5.11. Variation of maximum stress concentration due to countersunk depth

\[ K(t) = \begin{cases} 3.88298 & \text{if } Cs/t = 1 \\ 3.82365 & \text{if } Cs/t = 0.75 \\ 3.74459 & \text{if } Cs/t = 0.5 \\ 3.33268 & \text{if } Cs/t = 0.25 \end{cases} \]
5.9 Effect Residual Stresses on Stress Concentration

A nonlinear elastic-plastic analysis was conducted by load incremental method to study the effects of residual stresses on the stress concentration around the hole. In this study, the maximum remote stress was varied from 10 to 25 ksi to determine the effects of residual stresses on stress concentration. The loading conditions and material model are shown in Tables 4.1 and 4.2. Simulations were carried out for different remote stress levels, $\sigma_o = 10, 15, 20$ and $25$ for a particular configuration of $t/r = 0.5$, $E_{adl}/E_{al} = 0.05$, $C_{a}/t = 0.5$, $C_{t}/t = 0.5$ and $w/r = 15$.

Normalized von-Mises stress is plotted against the total time for all load cases. From the figure 5.23 it is evident that normalized von-Mises is maximum for $\sigma_{max} = 10$ksi. As the load reaches the maximum value during the analysis, von-Mises stress also reaches the maximum value then it starts decreasing the as the load is decreasing. The plates will be within the elastic region when subjected to a load of 10ksi. Therefore the residual stresses in such plates are almost zero. Whereas, the plates are subjected to loads such as 15ksi, 20ksi and 25ksi will go beyond the...
elastic region into the plastic region. In this case, the compressive residual stresses are developed. Therefore, von-Mises stress increases during the last steps of analysis.

von-Mises stress in adhesive layer increases with an increase in maximum load which is shown in figure 5.24. For the cases, 10ksi and 15 ksi von-Mises stress is almost zero at end of last step, whereas for the 20ksi and 25ksi some residual stresses were developed. This is due to the increase in the residual compressive stress developed in the plates with the increase in the load.

The normalized value of $\sigma_{yy}$ is plotted against the ratio $\frac{z}{t}$ which is shown in the figure 5.25. The value of $\sigma_{yy}$ almost remains constant in straight shank and it is maximum in countersunk region (approximately 10% of thickness away from countersunk edge). Similarly for the cases of $\sigma_{\text{max}} = 10$ ksi and 15 ksi, the plates are within the elastic region and for $\sigma_{\text{max}} = 20$ ksi and 25 ksi compressive stresses are developed.

From the results it is evident that the large plastic deformation occurs along the nodal line A-B-C. Due to the stress concentration, the stresses increase approximately 3.5 times the applied maximum load which causes a large plastic deformation in the plates. When the maximum load is increased the residual stresses developed also increases. For the maximum load of 10ksi the plates are stressed within the elastic region since the maximum stress developed is less than the yield stress.
Figure 5.13. Normalized von-Mises vs. Step Time

Figure 5.14. von-Mises Stress in Adhesive layer
Figure 5.15. Effect of Residual stresses along the nodal line
CHAPTER 6

STRESS CONCENTRATION FACTORS IN FASTENER FILLED HOLE

6.1 Introduction

In this chapter, a detailed overview of the stress fields computed using finite element model for fastener filled countersunk holes is presented. The investigation was conducted for pre-tension loads of 50, 100, 150 and 200 lbs in the fastener and far-field tension of \( \sigma_o = 1000 \) psi in plates. A non-linear contact analysis was performed in two different steps with the first step of applying pre-tension load and second step applying the far-field load as described previously in chapter 4 (see figure 4.10). A comparative study between the straight shank, monolithic countersunk and bonded countersunk plates was conducted. Also, the results were compared with the case of open hole and fastener filled hole without pre-tension.

6.2 Stress Distribution around the Fastener Filled Hole

The von-Mises stress as developed in the region of the hole is shown in the figure 6.1. The out of plane deformation of the plates with fastener is shown in figure 6.2, when both the pre-tension and far-field load are applied.

The distribution of the normal stress along Y-direction \( (\sigma_{yy}) \) component is shown in the Figure 6.3. Shear stress component developed around the hole are shown in the Figure 6.4. As shown in the figure, the stresses in the region of hole are high, which eventually lead to commencement of failure. The pre-tension load produces compressive stress in out-of-plane direction in the plate. The tangential stresses developed in the vicinity of hole are compressive in nature both after preload and far-field load. This is because of the Poisson’s effect, due to which
material around the hole (away from the clamp-up region) resists the expansion in the radial direction.

The contact pressure is developed due to pre-tension and contact between the fastener and plate. The pre-tension loads are applied in the fastener as shown in figures 4.7 and 4.8. Due to nature of the pre-tension loading, the fastener elongates in axial direction because of which the flanges of the fastener exert compressive forces on plates. Contact pressure is very high nearer to the free surfaces in straight shank holes, whereas contact pressure in the case of countersunk holes is seen in the countersunk region and not at the free surface \( z = 1 \). This is shown in figures 6.5 and 6.6.
Figure 6.1. von-Mises stress around the hole (psi)

Figure 6.2. Out of Plane deformation (in)
Figure 6.3. Distribution of Normal Stress ($\sigma_{yy}$) around the hole (psi)

Figure 6.4. Distribution of Shear Stress ($\sigma_{xy}$) around the hole (psi)
Figure 6.5. Contact Pressure Distribution In Straight Shank Holes (psi)

Figure 6.6. Contact Pressure Distribution Countersunk Holes (psi)
6.3 Effect of Pre-Tension Loading in Straight Shank Holes

The effect of fastener pre-tension on the stress concentration around the hole is studied varying pre-tension loads and by observing stress distribution in vicinity of the straight shank hole. Four different levels of fastener pre-tension loads 50 lbs, 100 lbs, 150 lbs and 200 lbs were applied on the bolt surface along the Z-direction for a particular configuration of \( \frac{h}{r} = 15, \frac{w}{r} = 15 \) and \( \frac{t}{r} = 1 \).

The distribution of the Normal stress \( \sigma_{yy} \) around the hole for different pre-tension loads is plotted against X-direction at the top free surface \( (z = 1) \) in the after the pre-tension loading and after the far field loading figures 6.7 and 6.8. It can be observed that with the increase in pre-tension load there is an increase in the stress around the hole for both the cases (after pre-tension loading and far-field loading).

The tangential stress around the hole increases with the increase of pre-tension load as shown in the figure 6.9. The increase in pre-tension load from 50 to 100 lbs causes an increase of 38% in the stress values and for the pre-tension load of 200 lbs the stress increases by 64%. There is difference of 65% in maximum stress concentration for pre-tension load of 200 lbs to that of the problem of straight shank hole with finite thickness. The difference between maximum normal stresses for case of \( P = 200 \) lbs after far field loading is around 14 % when compared to that after pre-tension loading. The highest difference of 40 % in normal stress is observed in case of pre-tension loading 50lbs after far field loading when compared with after pre-tension loading.

The variation normal stress along the nodal line A-B-C is shown in figure 6.10 and 6.11 for the cases of after pre-tension loading and after far-field loading. From the graph it is evident that the maximum stress concentration is at the \( z = 0 \) free surface. Stress along the \( \frac{z}{t} \) is
compressive due to the pre-tension load applied on the fastener. The stress concentration at both the free surfaces is almost equal. As the pre-tension load increases, compressive stress at the free boundary surfaces also increases.

Figure 6.7. Distribution of Normal stress \( \sigma_{yy} \) along the \( Y=0 \) after pre-tension loading
Figure 6.8. Distribution of Normal stress ($\sigma_{yy}$) along the $Y=0$ after far field loading

Figure 6.9. Comparison of $\sigma_{yy}$ with Different pre-tension loads
Figure 6.10. Variation of Normal Stress ($\sigma_{yy}$) along the $Z$-direction after Pre Tension loading

Figure 6.11. Variation of Normal Stress along the $Z$-direction after Far field loading
6.4 Effect of Pre Tension Loading in Countersunk Monolithic Holes

The effect of fastener pre-tension on the stress concentration around the hole is studied varying pre-tension loads and by observing stress distribution in vicinity of the countersunk monolithic holes. Four different levels of fastener pre-tension loads 50 lbs, 100 lbs, 150 lbs and 200 lbs were applied on the bolt surface along the \( Z \) direction for a particular configuration of \( \frac{h}{r} = 15, \frac{w}{r} = 15, \frac{C_s}{t} = 0.5, \frac{t}{r} = 1 \).

The distribution of the stress (\( \sigma_{yy} \)) around the hole for different pre-tension loads is plotted against \( X \)-direction at the countersunk free surface (\( z = 1 \)) in the after the pre-tension loading and after the far field loading figures 6.12 and 6.13. It can be observed that with the increase in pre-tension load there is an increase in the stress around the hole for both the cases (after pre-tension loading and farfield loading).

The Tangential stress around the hole increases with the increase of fastener pre-tension value as shown in the figure 6.14. The increase in pre-tension load from 50 to 100 lbs causes an increase of 42% in the stress values and for the pre-tension load of 200 lbs the stress increases by 66%. There is difference of 40% in maximum stress concentration of for pre-tension load of 50 lbs for problem countersunk monolithic hole with finite thickness. The difference between maximum normal stress for case of P=200 lbs after far field loading is around 11% when compared to that after pre-tension loading. The highest difference of 27% in normal stress is observed in case of pre-tension loading 50 lbs after far field loading when compared with after pre-tension loading.

The variation Tangential stress along the nodal line A-B-C is shown in figure 6.15 and 6.16 for the cases of after pre-tension loading and after far field loading. From the graph it is evident that the maximum stress concentration is at the \( z = 0 \) free surface. Stress along the
\[ \frac{z}{t} = 1 \] is compressive due to the pre-tension load applied on the fastener. The stress concentration at both the free surfaces is almost equal. As the pre-tension load increases, compressive stress at the free boundary surfaces also increases.

Figure 6.12. Distribution of Normal stress (\( \sigma_{yy} \)) along the \( X \) (Y=0) after pre-tension loading in monolithic plates
Figure 6.13. Distribution of Normal stress ($\sigma_{yy}$) along Y=0 after far field loading in monolithic plates.

Figure 6.14. Comparison of $\sigma_{yy}$ with Different pre-tension loads.
Figure 6.15. Variation of Normal Stress along the $Z$-direction after Pre Tension loading

Figure 6.16. Variation of Normal Stress along the $Z$-direction after Far Field loading
6.5 Effect of Pre Tension Loading in Countersunk Adhesively Bonded Holes

The effect of fastener pre-tension on the stress concentration around the hole is studied varying pre-tension loads and by observing stress distribution in vicinity of the hole in the bonded countersunk holes. Four different levels of fastener pre-tension loads 50 lbs, 100 lbs, 150 lbs and 200 lbs are being applied on the bolt surface along the \( Z \) direction for a particular configuration of \( h/r = 15, w/r = 15, C_s/t = 0.5, E_{adh}/E_{al} = 0.1, t/r = 1 \).

The distribution of the normal stress \( (\sigma_{yy}) \) around the hole for different pre-tension loads is plotted against \( X \)-direction at the countersunk free surface \( (z = 1) \), before and after the far-field loading as shown in figures 6.17 and 6.18. It can be observed that with the increase in pre-tension load there is an increase in the stress around the hole for both the cases (after pre-tension loading and far field loading).

The normal stress around the hole increases with the increase of fastener pre-tension value as shown in the figure 6.19. The increase in pre-tension load from 50 to 100 lbs causes an increase of 42% in the stress values and for the pre-tension load of 200 lbs the stress increases by 68%. There is difference of 42% in maximum stress concentration for pre-tension load of 50 lbs for the problem of bonded countersunk open hole with finite thickness. The difference between maximum normal stresses for case of \( P = 200 \) lbs after far field loading is around 23% when compared to that after pre-tension loading. The highest difference of 27% in normal stress is observed in case of pre-tension loading 50 lbs after far field loading when compared with after pre-tension loading. Stress along the \( z/t \) is compressive due to the pre-tension load applied on the fastener. The stress concentration at both the free surfaces is almost equal.
Figure 6.17. Distribution of Normal stress ($\sigma_{yy}$) in psi along $Y=0$ after pre-tension loading in bonded plates

Figure 6.18. Distribution of Normal stress ($\sigma_{yy}$) in psi along $Y=0$ after far-field loading in bonded plates
Comparison of Pre-Tension effects

A comparison between monolithic, bonded and straight shank plates after pre-tension step and after far-field loading step is shown in figure 6.20 and 6.21 respectively. The configurations of the model were same as the ones used in section 6.3 to 6.5. The maximum normal stress in case of bonded plates is higher than that of monolithic and straight shank plates. There is difference of 9.5% in maximum stress concentration of pre-tension load of 200 lbs between bonded and monolithic whereas it is 59% between bonded and straight shank after far field loading. There is difference of 3.5% in maximum stress concentration of pre-tension load of 200 lbs between monolithic and bonded whereas it is 54% between bonded and straight shank after pre-tension loading.
Figure 6.20. Comparison of maximum normal stress (psi) between bonded, monolithic and straight shank after pre-tension loading (lbs)

Figure 6.21. Comparison of maximum normal stress between bonded, monolithic and straight shank after Far field loading
Due to the complications and dependence on many material and geometric variables, the stress concentration factors of adhesively bonded laminated aluminum sheets with countersunk holes cannot be inferred from those of monolithic sheets. Broad knowledge of the various parameters that affect the behavior of the bonded joints with countersunk holes is essential for effective and safe design. A three dimensional finite element model has been assembled to study the effects of various parameters such as ratio of young’s modulus adhesive to aluminum, location of adhesive layer, countersunk sunk depths, ratio of thickness to radius and width on the stress distribution developed around the hole. An investigation was also conducted for various pre-tension loads in the fastener filled holes. A comparative study has been done between the straight shank, monolithic countersunk and bonded countersunk plates. The results obtained from the finite element analysis have been validated by using the results reported by Sivakumar. Etal. [2] on monolithic sheets with countersunk holes. The primary conclusions of the investigation in this thesis are summarized below:

1. Ratios of Young’s modulii of adhesive to aluminum, location of adhesive, countersunk sunk depth to plate thickness, and thickness to radius, and width to radius, are the factors which effect $K_t(z)$ in bonded layered specimens with straight shank and countersunk holes.

2. The change in maximum stress concentration for change in $\alpha$ from 90° to 100° and 100° to 110° is about 1.6%. Hence the variation of $K_t$ is negligible for small change in countersunk angle. A constant countersunk angle of 100° is used in all following simulations because it is
commonly used in aircraft industry.

3. The results established that the stress concentration value is lower for bonded specimens when compared with monolithic plates when Ratio of young’s modulus adhesive to aluminum is >0.07 for countersunk holes.

4. For thinner plates maximum stress concentration is nearly equal to monolithic scenario where as for thicker plates it is less than that of the monolithic plates. These results can be seen in the investigation of different $t/r$ ratios.

5. Due to the low shear strength of adhesive materials, the shear stress $\sigma_{\theta z}$ in the bond line/interfaces is an important factor in investigation of stress concentration factors in bonded plates because the $\sigma_{\theta z}$ is responsible for initiating cracks at the bond lines.

6. The stress concentration is highest when the adhesive is positioned next to the countersink edge in straight shank portion and nearly equal to that of monolithic case whereas the stress concentration was lower with the other locations of adhesive.

7. The stress concentration in case of bonded plates is lower than monolithic plates for both finite and infinite widths. For different countersunk depths, the stress concentration in case of bonded plates is lower than monolithic plates. It was almost equal to monolithic for a case $C_s/t=0.5$.

8. The effect of fastener pre-tension was observed to be more in bonded layered plates with countersunk holes when compared to monolithic and straight shank plates.

9. The effects of pre-tension loading were compared with the case of open hole and fastener filled hole without pre-tension for bonded, monolithic and straight shank hole. The maximum normal stress in case of bonded plates is higher than that of monolithic and straight shank plates.
7.2 Recommendation

It is important to study the complex modes of stress concentration associated with bending, wedge loading, pin loading etc. It is also necessary to study the crack propagation in bonded layered aluminum with countersunk hole for better fatigue design. The scope of the study should also include multilayered laminated plates with countersunk holes and fasteners.
REFERENCES


APPENDICES
APPENDIX 1 A

Linear open hole problem

*Heading
** Job name: tr1edh01 Model name: Model-1
** Generated by: Abaqus/CAE 6.9-EF1
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=Part-7
*End Part
**
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=Part-7-1, part=Part-7

*Node***************************************************************************/Node Numbers and their Coordinates***************************************************************************/

1, 0., 0.654500008, 0.
2, 0., 0.654500008, 0.0384999998
3, 0.654500008, 0., 0.0384999998
.
.
.

*Element, type=C3D8R***************************************************************************/Element Number and their Nodal Connectivity***************************************************************************/

1, 398, 8742, 25882, 2580, 1, 29, 1488, 134
2, 8742, 8743, 25883, 25882, 29, 30, 1489, 1488
3, 8743, 8744, 25884, 25883, 30, 31, 1490, 1489
4, 8744, 8745, 25885, 25884, 31, 32, 1491, 1490
.
.
.

*Nset, nset=_PickedSet_4_#15, internal***************************************************************************/Node Sets***************************************************************************/

1, 2, 3, 4, 5, 6, 7, 8, 29, 30, 31, 32, 33, 34, 35, 36
37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52
53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68

*Elset, elset=__PickedSurf200_S3, internal, instance=Part-7-1

102401, 102402, 102403, 102404, 102405, 102406, 102407, 102408, 102409, 102410, 102411, 102412, 102413, 102414, 102415, 102416
102417, 102418, 102419, 102420, 102421, 102422, 102423, 102424, 102425, 102426, 102427, 102428, 102429, 102430, 102431, 102432
102433, 102434, 102435, 102436, 102437, 102438, 102439, 102440, 102441, 102442, 102443, 102444, 102445, 102446, 102447, 102448

*Elset, elset=._PickedSet220, internal, instance=Part-7-1
  1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 601
  602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 1201, 1202
  1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1801, 1802, 1803
  1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 18

*Surface, type=ELEMENT, name=_PickedSurf217, internal
  _PickedSurf217_S3, S3
  _PickedSurf217_S4, S4
*Elset, elset=._PickedSurf218_S3, internal, instance=Part-7-1
  114416, 114417, 114418, 114419, 114420, 114421, 114422, 114423, 114424, 114425, 114426, 114427, 114428, 114429, 114430, 114431
  114432, 114433, 114434, 114435, 114436, 114437, 114438, 114439, 114440, 114441, 114442, 114443, 114444, 114445, 114446, 114447
  114448, 114449, 114450, 114451, 114452, 114453, 116396, 116397, 116818
*Elset, elset=._PickedSurf218_S4, internal, instance=Part-7-1, generate
  115537, 115555, 1
*Surface, type=ELEMENT, name=_PickedSurf218, internal
  _PickedSurf218_S3, S3
  _PickedSurf218_S4, S4
*End Assembly

**
** MATERIALS ************/Materials and Their Properties/****************************
**
*Material, name=Material-2
*Elastic
  1.07e+06, 0.3
*Material, name=aluminum
*Elastic
  1.07e+07, 0.3

**
** BOUNDARY CONDITIONS ************/Boundary Conditions/****************************
**
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
  _PickedSet219, YSYM
** Name: BC-2 Type: Symmetry/Antisymmetry/Encastre
*Boundary
  _PickedSet220, XSYM
**
** STEP: SECTION LOAD ************/ Time Step/****************************
**
*Step, name="SECTION LOAD"
apply surface traction as far field stress
*Static
1., 10., 0.001, 10.
**
** LOADS ********************************/Applying Loads/**********************************
**
** Name: Load-1 Type: Surface traction
*Dsload
_PickedSurf217, TRVEC, 1000., 1., 0., 0.
** Name: Load-2 Type: Surface traction
*Dsload
_PickedSurf218, TRVEC, 100., 1., 0., 0.
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U
*Element Output, directions=YES
S,
*Output, history, frequency=0
*End Step
APPENDIX 1B

Elastic Plastic model

*Heading
elastic plastic aluminum
** Job name: eadh005plas_def Model name: x05teadh005
** Generated by: Abaqus/CAE Version 6.8-1
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**

** PARTS
**
*Part, name=PART-7-1

*Node**********************/*Node Numbers and their Coordinates************

1,  1.11000001,  -1.50999999,  0.0384999998
2,  0.10899997,  -1.50999999,  0.0384999998
3,  0.10899997,  -1.50999999,       0.
4,  1.11000001,  -1.50999999,       0.
5,  1.11000001,  0.800000012,  0.0384999998
6,  -1.20000005,  0.800000012,  0.0384999998

*Element, type=C3D8R************/*Element Number and their Nodal Connectivity /*****

1,   2660,   2661,  29167,  29120,     38,     39,  1559,  1558
2,   2661,   2662,  29168,  29167,     39,     40,  1560,  1559
3,   2662,   2663,  29169,  29168,     40,     41,  1561,  1560
4,   2663,   2664,  29170,  29169,     41,     42,  1562,  1561
5,   2664,   2665,  29171,  29170,     42,     43,  1563,  1562
6,   2665,   2666,  29172,  29171,     43,     44,  1564,  1563

*Elset, elset=__PICKEDSURF18_S4_1, internal, instance=PART-7-1

7801,  7802,  7803,  7804,  7805,  7806,  7807,  7808,  7809,  7810,  7811,  7812,  7813,  7814,  7815,  7816

*Surface, type=ELEMENT, name=_PICKEDSURF18, internal
__PICKEDSURF18_S3_1, S3
__PICKEDSURF18_S4_1, S4

*End Assembly

*Amplitude, name=Amp-1, time=TOTAL TIME*****/* Load Incrementing amplitude/*****

0.,  0.,  20.,  0.5,  40.,  1.,  60.,  0.

**

** MATERIALS***********/Materials and Their Properties/********************

**
*Material, name=ADHESIVE
*Elastic
535000., 0.3
*Material, name=ALUMINUM
*Elastic
1.07e+07, 0.3
*Plastic
50000., 0.
65000., 0.15
** ____________________________________________________________
**
** STEP: SECTION LOAD *****************/ Time Step/****************************
**
*Step, name="SECTION LOAD", inc=60
apply surface traction as far field stress
*Static, direct, nlgeom=YES
1., 60.,
**
** BOUNDARYCONDITIONS*****************/Boundary Conditions/****************************
**
**
** Name: Disp-BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PICKEDSET16, XSYMM
** Name: Disp-BC-2 Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PICKEDSET17, YSYMM
**
** LOADS ***********************/Applying Loads/****************************
**
** Name: SurfaceTraction-1 Type: Surface traction
*Dsload, amplitude=Amp-1
_PICKEDSURF18, TRVEC, 20000., 1., 0., 0.
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U
**
** FIELD OUTPUT: F-Output-2
**
*Element Output, directions=YES
S.
*Output, history, frequency=0
*End Step
Appendix 1 C

Fastener Filled Holes

*Heading
** Job name: tr1precounterb50 Model name: Model-1
** Generated by: Abaqus/CAE 6.9-EF1
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=Part-7
*End Part
**
*Part, name=Part-8
*End Part
**
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=Part-8-1, part=Part-8
-1.38777878078145e-17, -6.93889390390723e-18, -0.0385
-1.38777878078145e-17, -6.93889390390723e-18, 0.577350279552042,
0.577350279552042, 0.538850279552042, 119.999999109416
*Node
***************/Node Numbers and their Coordinates**************

1, 0.0192499999, 0.0
2, 0.0192499999, 0.0769999996
3, 0.0384999998, 0.0769999996
4, 0.0384999998, 0.

*Element, type=C3D8R
***************/Element Number and their Nodal Connectivity

1, 398, 25882, 25882, 29, 1488, 1488
2, 8742, 8743, 25883, 25882, 30, 31, 1490, 1489
3, 8743, 8744, 25884, 25883, 30, 31, 1490, 1489
4, 8744, 8745, 25885, 25884, 31, 32, 1491, 1490
5, 8745, 8746, 25886, 25885, 32, 33, 1492, 1491
6, 8746, 8747, 25887, 25886, 33, 34, 1493, 1492

*Elset, elset=__PickedSurf200_S4, internal, instance=Part-7-1
107968, 107969, 107970, 107971, 107972, 107973, 107974, 107975, 107976, 107977, 107978, 107979, 107980, 107981, 107982, 107983
107984, 107985, 107986, 107987, 107988, 107989, 107990, 107991, 107992, 107993, 107994, 107995, 107996, 107997, 107998,

*Surface, type=ELEMENT, name=__PickedSurf200_S3, S3
__PickedSurf200_S4, S4
*Elset, elset=__PickedSurf201_S3, internal, instance=Part-7-1
  114416, 114417, 114418, 114419, 114420, 114421, 114422, 114423, 114424, 114425, 114426, 114427,
  114428, 114429, 114430, 114431
  114432, 114433, 114434, 114435, 114436, 114437, 114438, 114439, 114440, 114441, 114442, 114443,
  114444, 114445, 114446, 114447
  114448, 114449, 114450, 114451, 114452, 114453, 116396, 116397, 116818
*Elset, elset=__PickedSurf201_S4, internal, instance=Part-7-1, generate
  115537, 115555. 1
*Surface, type=ELEMENT, name=_PickedSurf201, internal
__PickedSurf201_S3, S3
__PickedSurf201_S4, S4
*Node
  1, 0., 0., 0.
*Nset, nset=_Load-1_blrn_, internal

1,
** Pre-Tension Section for Bolt Load: Load-1************/Pre Tension Node /*********************

1,
** Pre-Tension Section, surface= _PickedSurf199, node=_Load-1_blrn_
-2.02227e-21, 6.12323e-17, 1.
*End Assembly
**
** MATERIALS***********/Materials and Their Properties/********************
**
**
*Material, name=Material-2
*Elastic
  1.07e+06, 0.3
*Material, name=aluminum
*Elastic
  1.07e+07, 0.3
**
** INTERACTION PROPERTIES
**
*Surface Interaction, name=IntProp-1
  1.,
*Friction
  0.,
**
** BOUNDARY CONDITIONS*************/Boundary Conditions/********************
**
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
  _PickedSet202, YSYM
** Name: BC-2 Type: Symmetry/Antisymmetry/Encastre
*Boundary
  _PickedSet203, XSYM
**
** INTERACTIONS ***********/Contact Formulation/***********************
** Interaction: Int-1
*Contact Pair, interaction=IntProp-1, small sliding, type=SURFACE TO SURFACE
  Part-8-1.Surf-3, Part-7-1.Surf-3
** Interaction: Int-2
*Contact Pair, interaction=IntProp-1, small sliding, type=SURFACE TO SURFACE, adjust=0.0
  Part-8-1.Surf-1, Part-7-1.Surf-1
** Interaction: Int-3
*Contact Pair, interaction=IntProp-1, small sliding, type=SURFACE TO SURFACE, adjust=0.0
*-----------------------------------------------

** STEP: SECTION LOAD**********/Pre Tension Step/**********************
**
*Step, name="SECTION LOAD", nlgeom=YES
apply surface traction as far field stress
*Static, stabilize=0.0002, allsdtol=0.05, continue=NO
  1e-05, 1., 1e-06, 1.
**
** LOADS
**
** Name: Load-1   Type: Bolt load**********/Applying Pre Tension Load/**************

*Cload _Load-1_blrm_, 1, 50.
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
  CF, RF, U
*Element Output, directions=YES
  S.
*Contact Output
  CDISP, CFORCE, CSTRESS
*Output, history, frequency=0
*End Step
** -----------------------------------------------
**
** STEP: Step-2**********/Far Field Loading/****************************
**
**
*Step, name=Step-2, nlgeom=YES
*Static, stabilize=0.0002, allsdtol=0.05, continue=NO
  1e-05, 1., 1e-06, 1.
**
** BOUNDARY CONDITIONS
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary, op=NEW
_PickedSet202, YSYMM
** Name: BC-2 Type: Symmetry/Antisymmetry/Encastre
*Boundary, op=NEW
_PickedSet203, XSYMM
** Name: Load-1 Type: Bolt load
*Boundary, op=NEW, fixed
_Load-1_blrn_, 1, 1
**
** LOADS************************/Applying Loads/****************************
**
** Name: Load-1  Type: Bolt load
*Cload, op=NEW
** Name: Load-2  Type: Surface traction
*Dsload
_PickedSurf200, TRVEC, 1000., 1., 0., 0.
** Name: Load-3  Type: Surface traction
*Dsload
_PickedSurf201, TRVEC, 100., 1., 0., 0.
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U
*Element Output, directions=YES
S,
*Contact Output
CDISP, CFORCE, CSTRESS
*Output, history, frequency=0
*End Step