DEVELOPMENT OF FRICTION COMPENSATION CONTROL
AND APPLICATION TO FUEL VALVE CONTROL SYSTEMS

A Dissertation by

Majid Feiz

M.S., Iran University of Science and Technology, 1999
B.S., Isfahan University of Technology, 1997

Submitted to the Department of Mechanical Engineering
and the faculty of the Graduate School of
Wichita State University
in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

July 2005
© Copyright 2005 by Majid Feiz

All Rights Reserved
I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy with a major in Mechanical Engineering.

__________________________
B. Bahr, Committee Chair

We have read this dissertation and recommend its acceptance:

__________________________
J. Chaudhuri, Committee Member

__________________________
H. Lankarani, Committee Member

__________________________
E. Sawan, Committee Member

__________________________
J. Steck, Committee Member

Accepted for the College of Engineering

__________________________
Walter J. Horn, Dean

Accepted for the Graduate School

__________________________
Susan Kovar, Dean
DEDICATION

To my parents, my wife, my sister, and my brother for their support, love, and enthusiasm
ACKNOWLEDGEMENTS

I would like to thank my advisor, Professor Behnam Bahr, for his many years of thoughtful, patient guidance and support. I would like also to extend my gratitude to the members of my committee, Professor J. Chaudhuri, Professor H. Lankarani, Professor. E. Sawan, and Professor J. Steck, for their helpful comments and suggestions on all stages of this project.

I would like to thank the members of the South Carolina Institute for Energy Studies (SCIES), Dr. William H. Day and Dr. Richard Wenglarz, who provided this opportunity to be involved in their valuable University Turbine System Research (UTSR) program, and Leah Hucks, for her help during this program. I also want to thank members of Woodward Industrial Controls Company — Doug Salter, Kelly Benson, and William J. Schad — and members of the General Electric Company — Eric Butterfield, Dr. Dean Minto, and Jonathan Thatcher — for their friendship and selfless role modeling, which have contributed to my professional development.

I would like to thank my nice wife, Rana Khazaie, who was very patient during my Ph.D program; my brother, Homayoon Feiz, my uncles, Mohammad J. Feiz and Dr. Reza Feiz, who always supported me with encouragement; my families in Wichita, Mr. and Mrs. Amirani, Mr. and Mrs. Eftekhari, Mr. and Mrs. Farahdel, Mr. and Mrs. Farahnakian, Mr. and Mrs. Manani, and Mr. Joe Salim, who made me feel at home; and my friends, Amir Adibi, Farshad Barazandeh, Hamid Beheshtian, Sajad Shams and Zahir Banihashemi.

I thank God for giving me a great family, instructors, and friends.
This research involved the design of a position control system, based on a new method of observer-adaptive friction compensation, for a fuel valve that controls the flow of gas in large turbines. This fuel valve is operated by a Permanent Magnet Synchronous (PMS) motor, which is attached to the gear box and coupling joint. These unsatisfactory position response problems that occur at start-up and during operation when the pressure difference ($\Delta p$) across the valve, and consequently the friction, is the greatest includes the following: high overshoots, high steady-state error, and limit-cycle exhibition during steady-state conditions. The deficiency of the linear-based control system of the PMS motor to compensate for the nonlinearity of the system, such as friction effect and flexibility of the coupling joint, is the major source of problems with this fuel valve.

This research developed the fundamental concept of a new observer-based adaptive friction compensation for multibody actuation systems (n degrees of freedom) for both deterministic and stochastic systems, and presents the mathematical equations for molding the nonlinear system, including modeling of the PMS motor, nonlinear actuator, and gear box with friction and flexible coupling joints. Two new methods of compensating for friction, including friction compensation gain and observer-based adaptive friction compensation for a multibody system, are proposed, and the application of the latter is used for improving the position control system of the fuel valve. The initial results of using the new position control system based on observer-adaptive friction compensation have shown improvement to the response of the fuel valve system, both in steady-state and transient operations.
PREFACE

This research was sponsored by University Turbine Systems Research Program South Carolina Institute for Energy Studies Clemson University and National Energy Technology Laboratory U.S. Department of Energy.

--------------------------------------------------

This research was supported by Woodward Industrials Control and Wichita State University
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>1.2</td>
<td>Permanent Magnet Motor</td>
</tr>
<tr>
<td>1.3</td>
<td>Literature Survey of the PMS Motor</td>
</tr>
<tr>
<td>1.3.1</td>
<td>Line-Starting PMS Motor</td>
</tr>
<tr>
<td>1.4</td>
<td>The Inverter-Driven PMS Motor System</td>
</tr>
<tr>
<td>1.4.1</td>
<td>PMS Motor Modeling, Analysis, and Performance</td>
</tr>
<tr>
<td>1.4.2</td>
<td>PMS Motor Applications</td>
</tr>
<tr>
<td>1.5</td>
<td>Introduction to Chapters</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction: Gas Fuel Valve with Actuation System</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Application</td>
</tr>
<tr>
<td>2.1.2</td>
<td>Features</td>
</tr>
<tr>
<td>2.1.3</td>
<td>24 Volt Digital Driver</td>
</tr>
<tr>
<td>2.2</td>
<td>Problem Definition</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Fuel Valve Control Problems</td>
</tr>
<tr>
<td>2.3</td>
<td>Project procedure</td>
</tr>
<tr>
<td>2.4</td>
<td>Mathematical Modeling of the System</td>
</tr>
<tr>
<td>2.5</td>
<td>Permanent Magnet Synchronous (PMS) Motor</td>
</tr>
<tr>
<td>2.6</td>
<td>Phasor Diagram</td>
</tr>
<tr>
<td>2.7</td>
<td>Dynamic Model of the PMS Motor</td>
</tr>
<tr>
<td>2.8</td>
<td>Actuation Systems Model</td>
</tr>
<tr>
<td>2.9</td>
<td>Friction Characteristics</td>
</tr>
<tr>
<td>2.9.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>2.9.2</td>
<td>Friction model</td>
</tr>
<tr>
<td>2.10</td>
<td>Nonlinear Mathematical Model of Actuation System</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>3.2</td>
<td>Mathematical Modeling a Two Degrees of Freedom System with Friction</td>
</tr>
<tr>
<td>3.3</td>
<td>Friction Observer</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Conclusion</td>
</tr>
<tr>
<td>3.4</td>
<td>Obtaining the Best Valve for the Coefficient of Friction Compensation</td>
</tr>
<tr>
<td>3.5</td>
<td>Generalized Friction Compensation</td>
</tr>
<tr>
<td>3.6</td>
<td>Performance Analysis of Friction Compensation</td>
</tr>
<tr>
<td>3.7</td>
<td>Observer-Based Friction Compensation</td>
</tr>
<tr>
<td>3.7.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>3.7.2</td>
<td>Observer-Based Pole Placement Friction Compensation</td>
</tr>
<tr>
<td>3.7.3</td>
<td>Observer-Based Friction Compensation feedback Control System</td>
</tr>
</tbody>
</table>
3.8 Friction Compensation in Stochastic System ................................................. 115
3.8.1 Friction Compensation Based on Kalman Filter................................. 117

4 ANALYSIS OF THE CONTROL PROBLEM OF THE FUEL VALVE AND
THE APPLICATION OF OBSERVER-FRICTION COMPASATION IN THIS
SYSTEM.........................................................................................................................128

4.1 Introduction..................................................................................................... 128
4.2 Theoretical Background of the Control System Design ................................. 129
  4.2.1 Open Loop Control ................................................................................. 129
4.3 Simulation of the Motor Model ...................................................................... 129
  4.3.1 Mathematical Modeling of the PMS Motor ............................................ 129
  4.3.2 Mathematical Model of the Actuation System ....................................... 130
4.4 Simulation of the Closed Loop ....................................................................... 136
  4.4.1 High-Performance Closed-Loop Control................................................ 137
  4.4.2 Power Circuit .......................................................................................... 138
  4.4.3 Model for the Current Controller............................................................ 141
  4.4.4 Position Control Based on Observer-Based Friction Compensation...... 142
  4.4.5 System Response and Discussion ........................................................... 145
4.5 Conclusion ...................................................................................................... 159

LIST OF REFERENCES...............................................................................................160

APPENDICES...........................................................................................................167

A. Friction Compensation Gain Method and its Application in Control
  System of DC Motor..............................................................................................168
### TABLE OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1-1: Classification of permanent magnet motors and drives [1].</td>
<td>3</td>
</tr>
<tr>
<td>Figure 2-1: Fuel-valve response during start-up using fuel-metering valve (California site) [39].</td>
<td>14</td>
</tr>
<tr>
<td>Figure 2-2: Fuel-valve response during start-up using fuel-metering valve (Rhodia site) [39].</td>
<td>15</td>
</tr>
<tr>
<td>Figure 2-3: Steady-state response of fuel-metering valve (Black Hills II site in Wyoming) [39].</td>
<td>16</td>
</tr>
<tr>
<td>Figure 2-4: Fuel-valve response during start-up using fuel-metering valve (Black Hills I site in Wyoming) [39].</td>
<td>17</td>
</tr>
<tr>
<td>Figure 2-5: Start-up to core idle under normal operation with a 25 Hertz Dither added to the metering valve demand signal.</td>
<td>18</td>
</tr>
<tr>
<td>Figure 2-6: Metering valve demand signal with 25 Hertz Dither signal in steady-state [39].</td>
<td>18</td>
</tr>
<tr>
<td>Figure 2-7: PMS motor with two pairs of poles.</td>
<td>22</td>
</tr>
<tr>
<td>Figure 2-8: Location of the armature current $i_a$ in d-q coordinate system [40].</td>
<td>24</td>
</tr>
<tr>
<td>Figure 2-9: Actuation system of the fuel valve.</td>
<td>29</td>
</tr>
<tr>
<td>Figure 2-10: Schematic of the actuation system.</td>
<td>29</td>
</tr>
<tr>
<td>Figure 2-11: Equivalent model of the actuation system.</td>
<td>31</td>
</tr>
<tr>
<td>Figure 2-12: Examples of friction models: (a) Coulomb friction, (b) Coulomb plus viscous friction, (c) stiction plus Coulomb and viscous friction, (d) friction force decreasing continuously from the static friction level [41].</td>
<td>33</td>
</tr>
<tr>
<td>Figure 3-1: Two degrees of freedom with friction system.</td>
<td>39</td>
</tr>
<tr>
<td>Figure 3-2: Response of $M_1$ due to change of $M_1$ position.</td>
<td>44</td>
</tr>
<tr>
<td>Figure 3-3: Response of $M_2$ due to the change of $M_1$ position in a non-friction case and in a friction case.</td>
<td>45</td>
</tr>
<tr>
<td>Figure 3-4: Phase plane diagram for a non-friction system and a friction system.</td>
<td>46</td>
</tr>
<tr>
<td>Figure 3-5: Phase plane diagram for a non-friction system and a friction system on a larger scale.</td>
<td>47</td>
</tr>
<tr>
<td>Figure 3-6: Response of $M_1$ due to change of $M_2$ position.</td>
<td>48</td>
</tr>
<tr>
<td>Figure 3-7: Response of $M_2$ due to change of $M_2$ position.</td>
<td>49</td>
</tr>
<tr>
<td>Figure 3-8: Phase plane diagram for two cases of a non-friction system and a friction system.</td>
<td>50</td>
</tr>
<tr>
<td>Figure 3-9: Phase plane diagram for two cases of a non-friction system and a friction system on a larger scale.</td>
<td>51</td>
</tr>
<tr>
<td>Figure 3-10: $M_1$ response due to applied force on $M_1$.</td>
<td>52</td>
</tr>
<tr>
<td>Figure 3-11: $M_2$ response due to applied force on $M_1$.</td>
<td>53</td>
</tr>
<tr>
<td>Figure 3-12: Phase plane diagram for two cases of a non-friction system and a friction system.</td>
<td>54</td>
</tr>
<tr>
<td>Figure 3-13: $M_1$ response of the system due to applied force on $M_1$ and $M_2$.</td>
<td>55</td>
</tr>
<tr>
<td>Figure 3-14: $M_2$ response of the system due to applied force on $M_1$ and $M_2$.</td>
<td>56</td>
</tr>
<tr>
<td>Figure 3-15: Phase plane diagram showing response of the system due to applied force on $M_1$ and $M_2$.</td>
<td>57</td>
</tr>
<tr>
<td>Figure 3-16: $M_1$ response of the system due to sinusoid force on $M_1$.</td>
<td>58</td>
</tr>
</tbody>
</table>
Figure 3-17: M2 response of the system due to sinusoid force on M1. ........................................59
Figure 3-18: Phase plane diagram showing response of the system to sinusoid force on M1. ............................................................................................................................. 60
Figure 3-19: Comparing friction with friction estimation for M1, the error and M1 displacement for $\alpha_1 = \alpha_2 = 1.2, \beta_1 = \beta_2 = 100$ ................................................................. 66
Figure 3-20: Comparing friction with friction estimation for M2, the error, and M2 displacement for $\alpha_1 = \alpha_2 = 1.2, \beta_1 = \beta_2 = 100$. ................................................................. 67
Figure 3-21: Larger-scale response of friction estimation the error for both M1 and M2 for $\alpha_1 = \alpha_2 = 1.2, \beta_1 = \beta_2 = 100$. .............................................................................. 68
Figure 3-22: Response of friction estimation and error for both M1 and M2 for $\alpha_1 = \alpha_2 = 1.5, \beta_1 = \beta_2 = 1$. ............................................................................................ 69
Figure 3-23: Larger-scale response of friction estimation and error for both M1 and M2 for $\alpha_1 = \alpha_2 = 1.5, \beta_1 = \beta_2 = 1$. ........................................................................................... 70
Figure 3-24: Response of friction estimation and error for both M1 and M2 for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 1$. ......................................................................................... 71
Figure 3-25: Larger-scale response of friction estimation and error for both M1 and M2 for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 1$. ......................................................................................... 72
Figure 3-26: Response of friction estimation and error for both M1 and M2 for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10$. ......................................................................................... 73
Figure 3-27: Response of friction estimation and error for both M1 and M2 for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10$. ......................................................................................... 74
Figure 3-28: Comparing friction with friction estimation for M1, the error, and the M1 displacement for $\alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 100$ due to the change of applied force for M1 and M2 and initial conditions. ................................................................. 75
Figure 3-29: Comparing friction with friction estimation for M2, the error, and the M2 displacement for $\alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 100$, due to the change of applied force for M1 and M2 and initial conditions. ................................................................. 76
Figure 3-30: Comparing friction with friction estimation for M1, the error, and the M1 displacement for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10$ due to the change of sinusoid applied force for M1 and M2 and initial conditions. ................................................................. 77
Figure 3-31: Comparing friction with friction estimation for M2, the error and the M2 displacement for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10$ due to the change of sinusoid applied force for M1 and M2 and initial conditions. ................................................................. 78
Figure 3-32: n-degree freedom of mass-spring-dashpot ...................................................... 84
Figure 3-33: Response of position and velocity M1 of the friction system with friction compensation and friction system without friction compensation. .......................... 89
Figure 3-34: Response of position and velocity M2 of the friction system with friction compensation and friction system without friction compensation. .......................... 90
Figure 3-35: Phase diagram for the friction system with friction compensation, friction system without friction compensation, and non-friction. .......................... 91
Figure 3-36: Comparison of the response of the friction system with friction compensation, friction system without friction compensation, and non-friction system due to the change of initial conditions. .......................... 92
Figure 3-37: Comparison of the response of the friction system with friction compensation, friction system without friction compensation, and non-friction system due to the change of applied force ................................................................. 93
Figure 3-38: Phase diagram showing response of the friction system with friction compensation, friction system without friction compensation, and non-friction .... 94
Figure 3-39: Observer-based friction compensation ......................................................... 96
Figure 3-40: Estimation of position and velocity based on the M2 position response compared to the real state. .................................................................................. 97
Figure 3-41: Response of the non-friction, friction system, and friction system with observer-based friction compensation due to a change in mass M1 ....... 98
Figure 3-42: Phase diagram showing the non-friction system, friction system with observer-based friction compensation due to change of mass M1 ....... 99
Figure 3-43: Response of the non-friction system, friction system, friction system with observer-based friction compensation due to change of mass M1 after a period of time. .............................................................. 100
Figure 3-44: Phase diagram showing response of the non-friction system, friction system, and friction system with observer-based friction compensation due to change of mass M1 after a period of time. .............................................................. 101
Figure 3-45: Larger-scale phase diagram showing response of the non-friction system, friction system, and friction system with observer-based friction compensation due to change of mass M1 after a period of time. .............................................................. 102
Figure 3-46: Observer-based friction compensation feedback control system............... 103
Figure 3-47: Response of M2 due to the change of M1 displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation ......................................................... 108
Figure 3-48: Response of position and velocity of masses M1 and M2 due to the change of M1 displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation......................................................... 109
Figure 3-49: Phase plane curve of masses M1 and M2 due to the change of M1 displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation......................................................... 110
Figure 3-50: Larger-scale curve of masses M1 and M2 due to the change of M1 displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation......................................................... 111
Figure 3-51: M2 response to M1 command signal in a non-friction system, a friction system without friction compensation, and a friction system with friction compensation ......................................................... 112
Figure 3-52: Response of position and velocity of masses M1 and M2 to the command signal in a non-friction system, a friction system without friction compensation, and a friction system with friction compensation......................................................... 113
Figure 3-53: Phase plane response of masses M1 and M2 to the command signal in a non-friction system, a friction system without friction compensation, and a friction system with friction compensation ......................................................... 114
Figure 3-54: A larger scale phase plane response of masses M1 and M2 to the command signal in a non-friction system, friction system without friction compensation, and a friction system with friction compensation ......................................................... 115
Figure 3-55: LQG control diagram ................................................................. 117
Figure 3-56: Kalman filter structure ............................................................ 119
Figure 3-57: Kalman-based friction compensation ........................................ 119
Figure 3-58: Kalman filter estimation of position and velocity based on the M1 position response compared to the real state ........................................... 122
Figure 3-59: Larger-scale Kalman filter estimation of position and velocity based on the M1 position response compared to the real state .......................... 123
Figure 3-60: Response of a non-friction system, friction system, and friction system with Kalman-based friction compensation due to change of mass M1 .................. 124
Figure 3-61: Phase plane of the non-friction system, friction system, and friction system with Kalman-based friction compensation due to change of mass M1 ............. 125
Figure 3-62: Larger-scale phase plane of the non-friction system, friction system, and friction system with Kalman-based friction compensation due to change of mass M1 ................................................................. 125
Figure 3-63: Kalman filter estimation of the position and velocity due to the change of sinusoid applied force on mass M1 and constant force on mass M2 ................ 126
Figure 3-64: Response of the non-friction system, friction system, and friction system with Kalman-based friction compensation due to change of mass M1 and constant force on mass M2 ................................................................. 127
Figure 4-1: Simplified block diagram of an open loop voltage-to-frequency control of an interior or buried PMS motor ................................................................. 129
Figure 4-2: Actuation system of the fuel valve ............................................. 131
Figure 4-3: Schematic actuation system of the fuel valve ............................ 131
Figure 4-4: Simplified actuation system of the fuel valve ............................. 131
Figure 4-5: Model of three-phase PMS motor with actuator ......................... 132
Figure 4-6: Three-phase voltage \( V_{a}, V_{b}, V_{c} \) and current \( I_{a}, I_{b}, I_{c} \) ............. 132
Figure 4-7: Comparison of direct and quadrature currents in the case of friction and non-friction shafts ................................................................. 134
Figure 4-8: Velocity and position responses in high-speed and low-speed shafts in friction and non-friction cases ................................................................. 135
Figure 4-9: High-speed and low-speed torque in friction and non-friction cases . 135
Figure 4-10: Block diagram of high-performance torque control scheme for sinusoidal PMS motor using vector control concept [40] .................................................. 138
Figure 4-11: Basic circuit topology of pulse-width modulated inverter drive [40] ..... 139
Figure 4-12: Control principle of naturally sampled PWM showing one of three phase legs [40] ................................................................. 140
Figure 4-13: (a) Naturally sampled PWM and (b) symmetrically sampled 140
Figure 4-14: Regular asymmetrically sampled pulse width modulation [40] .......... 141
Figure 4-15: Basic current regulation schemes: (a) three-phase sine_triangle comparison and (b) three-phase hysteresis control [40] ................................................................. 142
Figure 4-16: Servo system with state observer and parallel friction compensation for the nonlinear PMS motor with friction actuation system ........................................ 147
Figure 4-17: Control system and model of motor with high-speed shaft and low-speed shaft resolver ................................................................. 147
Figure 4-18: Structure of nonlinear digital control system observer friction and friction model-based compensation ................................................................. 148
Figure 4-19: Comparison of the friction torque from the high-speed shaft and the friction observer ................................................................................................................... 149
Figure 4-20: Comparison of the non-friction torque from the high-speed shaft and the friction observer in a short ended time of 0.3 second. ......................................................... 149
Figure 4-21: Comparison of the response of the current to the step input in a new control system and the old system ................................................................................................................... 150
Figure 4-22: $I_d$ and $I_q$ in the new and old control systems; the old control system shows a high response during the transient operation. ............................................................... 151
Figure 4-23: $I_d$ and $I_q$ in the new and old control systems; the old control system has an oscillation around the steady-state value of the currents. ................................................. 151
Figure 4-24: Velocity and position response to the step command input and comparison between the old control system and the new system. ................................................................. 152
Figure 4-25: Steady-state behavior of the velocity and position in a two-control system. ................................................................................................................................. 152
Figure 4-26: Position response to the command signal and comparison between the old and new systems. ........................................................................................................ 153
Figure 4-27: Position response during steady-state operation; old one shows high frequency and higher amplitude compared to the new one. .............................................. 153
Figure 4-28: Position response during steady-state operation on a larger scale. ................. 154
Figure 4-29: Position response under load condition. ........................................................ 154
Figure 4-30: Position response under load condition on a larger scale. .............................. 155
Figure 4-31: Current response under load condition. .......................................................... 155
Figure 4-32: Transient current response under load condition on a larger scale. .............. 156
Figure 4-33: Steady-state current response under load condition on a larger scale. ............. 156
Figure 4-34: Position response under the worst case of load and a flexible shaft. ............. 157
Figure 4-35: Position response under the worst case of load and a flexible shaft on a larger scale. ....................................................................................................................... 158
Figure 4-36: Comparison of the steady-state current response in the two-control system. ................................................................................................................................. 158
CHAPTER 1

LITERATURE SURVEY OF PERMANENT MAGNET SYNCHRONOUS MOTORS AND DEVICES

1.1 Introduction

The objective of this research is to design a new position control system, based on a new method of observer-adaptive friction compensation, for a fuel valve that controls the flow of gas in large turbines. This fuel valve is operated by a Permanent Magnet Synchronous (PMS) motor, which is attached to the gear box and coupling joint. In order to improve the response of the system, the reconfiguration of the control system is important. Since the system here is consisted of two major subsystem such as electrical part (PMS) motor, and the mechanical part, the research and the literature review are divided in to two major part including the PMS motor, and the friction of the mechanical components. In this chapter the focus is on the literature review for the PMS motor.

In this research, the gas fuel valve is a rotary sleeve and shoe-type throttling valve. The metering port area is determined by the input shaft position from the actuator. Valve position feedback to the actuator driver is accomplished using a high-accuracy resolver. The actuator is designed for use with a 24 V digital drivers. The motor is a permanent magnet synchronous (PMS) motor with a clutch and gear head assembly. The motor uses samarium cobalt permanent magnets that are bonded and sleeved to the rotor element. The rotor position sensing is performed with a resolver.

PMS motors and drives are increasingly being used in a wide range of applications, including machine tools, robotics, aerospace generators, actuators, and electric vehicles.
This has been made possible with the advent of high-performance permanent magnets having high coercivity and residual magnetism, which make it possible for the PMS motor to have superior power density, torque-to-inertia ratio, and efficiency, compared to induction or conventional DC motors.

This chapter surveys the literature of the PMS motor and drives, and includes line-start as well as an inverter-fed application and design. The inverter-fed literature is divided into two areas: those that deal with drives using discrete feedback every 60° (electrical) for a three-phase machine, and those that deal with drives using continuous feedback. In the former, the ideal machine back electromotive force (emf) is trapezoidal, and in the latter it is sinusoidal. [1]

1.2 Permanent Magnet Motor

Permanent magnet (PM) motor drives have come of age. The invention of high-performance magnets, like Samarium Cobalt (SmCo) and Neodymium Boron Iron (NdBFe), has made it possible to achieve motor performances that can surpass the conventional DC or induction motors. PM drives can be superior with regard to power density, torque-to-inertia ratio, and efficiency. Hence, depending on the application, these motor drives are preferable in many distances.

PM machines may be classified into line-start (which uses a cage to provide starting torque) and inverter-fed, as shown in Figure 1-1. Inverter fed machines may have a cage, although frequently they do not. If a cage is used, then it is possible to operate the machine on open loop, like the line-start machine. If the machine is cageless, then normally some form of rotor position detection is necessary. This may be done with hall sensors, absolute optical encoders, resolvers or some form of sensorless position
detection. Note that in the latter, the position loop is closed, even though there is no position sensor.

![Classification of permanent magnet motors and drives](image)

It is possible to classify cageless motor drives into two types. The first uses continuous rotor position feedback to force currents into the rotor. The ideal motor back emf is sinusoidal, so that when sinusoidal currents are input, a constant torque is produced with very low ripple. It is also possible to feed this machine with PWM voltages, without any direct attempt to control the waveform of the current. This is called a PMS motor drive, since the motor was originally produced by replacing the field coil and slip rings of a wound rotor synchronous motor with a permanent magnet. It should be noted that this definition is by no means standard and that other nomenclature used include brushless AC, brushless DC, and permanent magnet AC.

The second category of cageless motor drives is based on position feedback that is not continuous but rather obtained at fixed points, for example, every 60° (electrical). This is called a brushless DC motor drive, where typically the current is held constant for at least 120° and hence is approximately rectangular in shape for a current-fed machine.
Alternately, the voltage may be fed in blocks of 120°, with a current limit to ensure that the motor current is held within the motor’s capabilities. Again, the above definition is not standard, and some of the nomenclature mentioned above with respect to the PMS motor drive is also used to describe the brushless DC motor drive. Definitions are developed with respect to the drive and not the machines. The generic term used to describe both drives is permanent magnet AC motor drives.

Position sensorless drives typically are based on brushless DC motor drives, since position information is required only every 60 electrical degrees, instead of continuously, as in the PMS motor drive. This makes sensorless implementation easier.

A large amount of research work has been done on machine design and modeling, as well as the drive design and analysis. This chapter aims to review the work done in the area of PMS motors and drives. The review categorizes the literature into a number of different areas such as machine design, analysis, and drive simulation.

Literature reviews are taken from the following sources [1]:

- IEEE Transactions on Industry Applications
- IEEE Conference Records of the Industry Applications Society Annual Meetings
- IEEE Transactions on Energy Conversion and the power Apparatus and Systems
- IEEE Transactions on Industrial Electronics
- IEEE Transactions on Magnetic
- Proceedings of the IEEE
- Proceedings of the Motor Conferences
- IEEE Conference Publications
1.3 Literature Survey of the PMS Motor

The majority of work that has been done on the PMS motor has been concerned with its operation from a fixed-frequency supply. Most of the earlier work was concerned with starting these machines using a cage on the rotor to provide induction motor torque to run the machine up to almost synchronous speed. The magnet exerts a braking torque during this initial period. The machine then pulls into synchronism with a combination of the synchronous motor torque provided by the magnet and a reluctance torque provided by unequal inductance values on the periphery of the rotor. Therefore, the first part of this literature survey deals with the so called “self-starting” PMS motor [1].

1.3.1 Line-Starting PMS Motor

The mathematical modeling and performance analysis of a PMS motor obtained by fitting a permanent magnet inside the cage rotor of an induction machine was done by Cahill and Adkins [2]. The magnets in these machines are frequently buried in the rotor to provide space for the rotor cage lying close to the surface of the rotor. Cahill and Adkins point out that the presence of the magnets provides a reluctance torque that is opposite in sign to that of a wound rotor synchronous machine. This occurs because the permeability of the magnet on the direct axis is close to that of air, while the iron on the quadrature axis has a much higher permeability, provided it is not saturated. Hence, maximum torque is obtained at a load angle greater than 90°.

Binns et al. [3] review the development of a line-start PMS motor and consider the effects of parameters variations on its performance, whereas Neumann and Tompkins [4] look at the characteristics of various NdBFe magnets and view the possibilities for use in permanent magnet line-start motors.
Several investigations have analyzed a PMS motor (with a cage rotor) operating from a fixed 60 Hertz power supply [5-8]. A detailed steady-state analysis was performed by Rahman [5], and studies were conducted on the starting performance of a PMS motor with damper windings using d-q-axis equations [6-8], and also the design of a PMS motor [9].

1.4 The Inverter-Driven PMS Motor System

1.4.1 PMS Motor Modeling, Analysis, and Performance

An inverter-driven PMS motor may have a cage, in which case its construction can be similar to that described in the previous section. More common is the motor without a cage which is advantageous. First, the cage, which is in close proximity to the magnet, is a direct source of heat on the rotor. The performance of the magnets degrades considerably with a rise in temperature. Removal of the cage reduces the major source of heat production that occurs in the stator, and more room is available with respect to the location of the magnet on the rotor.

In the absence of a cage, the PMS motor can have different rotor configurations. Rahman [10] has shown the historical development of this motor. Magnets may be mounted on the surface of the rotor (surface-mounted), inset into the rotor (inset-mounted), or buried within the rotor core (buried configuration). These magnet placements give rise to different motor characteristics, thus, affecting the behavior of the overall drive as well. The PMS motor drive needs a resolver for position/velocity feedback and appropriate current controllers to force sinusoidal currents into the machine. The literature on inverter-fed machines is divided into papers that are concerned
primarily with machine design, and those mainly concerned with the entire drive analysis or design.

Classification of an inverter-driven PMS motor was carried out by Colby [11] in terms of motor design, power circuit configuration, commutation control signals, current regulation, and principal motor control methods.

The dynamic model of a PMS motor with no damper windings was derived by Enjeti et al. [12, 13] using linear d-q axis equivalent circuit theory for surface-mounted permanent magnet motors. Modeling of both the PMS motor and the brushless DC motor (BDCM) was performed by Pillay and Krishnan [14]. In the case of the PMS motor, the d-q axis transformation was used.

A detailed field analysis of PMS motors was done by Boules [15]. DC and AC motors have been modeled [16, 17], and AC motors using both lumped and distributed parameters, and combining both field and circuit equations have been analyzed [2].

An analysis based on DC, the motor’s electromagnetic equation, was done by Davat et al. [19]. Using the ideas developed by Slemon and Gumaste [20], a steady-state analysis of the PMS motor when fed from a voltage source [21] and as a current source inverter [22] was performed. A steady-state model representing the excitation of a PMS motor as an equivalent current source was developed by Sebastian et al. [23], and a model for efficiency calculations was developed by Colby and Novotny in [24].

Pillay and Krishnan [25], offered a complete model, including the simulation and analysis of an entire drive system, for a vector-controlled PMS motor system. State space models of the motor and speed controller, as well as real-time models of the inverter switches and vector controller are used. Using both a phasor diagram and an admittance
method, Honsinger [26], predicted PMS machines’ performance. More specifically, Krause et al. [27] analyzed a PMS motor powered by an 180° phase-controlled inverter, in terms of the torque as a function of the phase between the stator voltages and the rotor angle.

Rahman et al. [28] designed a microprocessor-based speed controller. Digital modeling using the Z-transform was developed by Pillay and Krishman [29] to consider the case where the current controller was implemented using analog circuitry and a digital speed controller, and then also the case where both were digitally implemented, assuming different sampling rates.

1.4.2 PMS Motor Applications

The application characteristics of Brushless DC motors (BDC) and PMS motors were compared by Colby [11] on the basis of power density, torque per unit of current, speed range, feedback devices, inverter rating, and ripple torque. Consideration was also given to the braking capability, losses, and thermal matters.

It was shown that the axial field of a PMS motor can offer improved performance over the radial field. The possibility of replacing the standard induction motor used in industry with the PMS motor, because of its higher efficiency and power factor, was investigated [30]. In considering the PMS motor for electric vehicles, Binns et al. [31] identified several constraints in using a vector-controlled machine. The feasibility of developing PMS motors with up to 125 Hp has also been analyzed, and new applications of the PMS motor for electric vehicles [32] and the positioning of space instrumentation [33] have been researched.
The design and construction of a PMS motor as a direct drive servo motor capable of supplying 400 Nm torque at 270 rpm was carried out by Viarouge et al. [34]. Bose [35] described a complete control system for a 70 Hp interior PMS motor machine operating in the constant torque mode using vector control, and also operating in the constant power mode utilizing square wave currents. Pillay and Krishnan [36] used a pseudo differential speed controller to simulate a complete system to study various control strategies.

Zeid et al. [37] presented the systematic design procedures for both speed and current controllers. The synchronous reference frame and PI regulator was used for the speed controller as an outer loop. Moreover, two synchronous frame PI regulators were employed as inner loops to control the direct and quadrature axis current components of the motor. The complete drive was implemented in real-time using a digital signal processor (DSP) board DS-1102. Lim et al. [38] proposed a fast-position observer using current measurements and a slow generic velocity estimator for a PMS motor drive. This method uses an iterative algorithm, assuming the velocity is approximately constant over the transient period of the position observer, thus obtaining a literalized state model, which is used to estimate the position.

1.5 Introduction to Chapters

The following items are covered in this research:

- Mathematical modeling of the nonlinear PMS motor with gear box and actuator with friction using a flexible shaft.
- The friction-compensating method including the following:
  - Friction compensation gain and its application in a DC motor.
– Observer-based adaptive friction compensation for a multibody system of mass-spring-dashpot

– Kalman filter-based friction compensation.

– Position control system based on observer-adaptive friction compensation for a two-degree system.

  ▪ Position control system based on observer-adaptive friction compensation for a PMS motor with gear box and friction flexible shaft.

Chapter 1 introduces a fuel valve system based on a PMS motor driver. Problems and issues of the conventional control system are discussed in this chapter. In Chapter 2, all the mathematical equations for a PMS motor and the flexible friction actuation system is derived. Two new methods of the friction compensation method are discussed not only in Chapter 3, but also in Appendix A. In order to avoid disconnection in any chapters, the new method of friction compensation gain and its application on DC motor presented in Appendix A. In Chapter 3, the fundamental concept of the theory of observer-adaptive friction compensation proven by a two-degree system of mass-spring-dashpot and performance of the system is analyzed. The performance of using Kalman filter-based friction compensation is discussed also in this chapter. Chapter 4 presents the application of observer-adaptive friction compensation, which is used for controlling the position of a fuel valve in a PMS motor with a friction-flexible actuation system. Improvements to the new design are also discussed in this chapter.
CHAPTER 2

MATHEMATICAL MODELING OF SYSTEM AND THE PROBLEM DESCRIPTIONS

2.1 Introduction: Gas Fuel Valve with Actuation System

2.1.1 Application

The gas fuel valve is a stainless steel valve capable of metering gas flow between 23 to 18144 kg/hr (50 to 40,000 lb/hr). It is designed to be corrosion-resistant and self-cleaning, allowing it to operate in sour gas environments (high sulfur content) that can cause problems for other valves.

When used in conjunction with an actuator system and a 24 V digital driver, the fuel valve delivers gas with the demanding accuracy that is needed for precise applications. The actuator driver is entirely electric, so hydraulic contamination and maintenance problems are eliminated and cost is reduced. In addition, the actuator is designed for a long life, although it may be replaced in the field, if necessary [39].

2.1.2 Features

The motor assembly is housed in a cast aluminum, explosion-proof housing. A thermal potting compound is used to transfer heat generated by the motor to the cast housing and out to the ambient environment. The motor output shaft is directly coupled to the valve input shaft through the use of a stainless steel tensional coupling. Other advantages include the following:

• Highly accurate
• Explosion-proof design
• Entirely electric
• Low maintenance
• Designed for long life
• Integral part of the total turbine control system [39]

2.1.3 24 Volt Digital Driver

The 24 V digital driver contains an analog position controller that receives a demand signal via a 4–20 mA input. The feedback signal is generated by a brushless resolver that is mounted on the fuel-metering valve. The driver contains fault detection circuitry, which provides the status of the 4–20 mA interface, position controller, driver, and feedback to the shutdown logic. A fault condition or an external shutdown command will disable the output (removing power to the motor), which in turn causes the valve return spring to close the valve. This driver, designed for use with a Woodward NetCon® or MicroNet™ control system, consists of a real-time single-input single-output (SIO) and a driver. The real-time SIO and the digital driver exchange information over a serial communication line. The digital form of the data preserves the 16-bit feedback resolution necessary to meet the system’s accuracy requirements. The real-time SIO is configured during system initialization, with data selected off-line by GAP™ (Graphical Application Program) software. Tables 2-1, 2-2, and 2-3 show some of the valve’s specifications [39].
### Table 2-1: Gas valve specification supply characteristics [39]

<table>
<thead>
<tr>
<th>Contaminants</th>
<th>Solid particles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt;10-μm diameter 13.6 kg/hr (30 lb/hr) by volume maximum</td>
</tr>
<tr>
<td></td>
<td>&gt;10-μm diameter 0.14 kg/hr (0.3 lb/hr) by volume maximum</td>
</tr>
<tr>
<td>Metered Fuel Types</td>
<td>Natural, propane, methane service</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>0.5 to 1.05</td>
</tr>
<tr>
<td>Temperature</td>
<td>–40 to +149 °C (–40 to +300 °F)</td>
</tr>
</tbody>
</table>

### Table 2-2: Gas valve requirements [39]

<table>
<thead>
<tr>
<th>Inlet Pressure</th>
<th>6,206 kPa (900 Psia) maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Differential</td>
<td>172 to 6,206 kPa (25 to 900 psi)</td>
</tr>
<tr>
<td>Gas Flow Range</td>
<td>23 to 18 144 kg/hr (50 to 40,000 lb/hr) (0.6 spgr) maximum flow capacity is</td>
</tr>
<tr>
<td></td>
<td>dependent upon available gas conditions</td>
</tr>
</tbody>
</table>

### Table 2-3: Gas valve actuator [39]

| Mechanical            | Output shaft rotation 60° (rotation limited by valve stops)                     |
|                       | Continuous output torque +24.86 N·m (+220 lb-in) maximum                        |
|                       | Peak output torque +62.38 N·m (+552 lb-in) minimum                               |
| Electrical            | Power input 28 volt DC nominal                                                  |
| CURRENT               | 18–32 volt DC operating                                                        |
| Current               | 37 A, maximum for 20 ms                                                         |
|                       | 15 A, maximum continuous                                                        |
| Performance           | 125 ms to open the valve and 120 ms to close the valve (internal spring adding) at a nominal line voltage of 28 volt DC |
| Bandwidth             | >5 Hertz                                                                        |
| Position Accuracy     | 0.50° for analog, 0.10° for digital                                              |
| Valve                 | Pressure equipment directive (97/23/EC) compliant as Category II                |
2.2 Problem Definition

2.2.1 Fuel Valve Control Problems

Figures 2-1, 2-2, 2-3, and 2-4 show the fuel valve response during start-up at four different units from around the world. All units use same driver part number. All units show the controllability problems during start-up and operation at core idle, which occurs when the $\Delta p$ across the valve is the highest.

Figure 2-1 shows a unit from a California site during start-up using the fuel-metering valve. The black line shows demand to the valve, and the red line shows feedback from the valve.

Figure 2-2 shows the same valve in another unit during start-up using the fuel-metering valve from a Rhodia site in Europe. The black line shows demand to the valve, and the red line shows feedback from the valve.
Figure 2-2: Fuel valve response during start-up using fuel-metering valve_black is demand, red is feedback_Rhodia site [39].

Figure 2-3 shows the fuel-valve response in steady-state using the fuel-metering valve from a Black Hills site in Wyoming. The black line shows demand to the valve, and the red line shows feedback from the valve.
Figure 2-3: Steady-state response of fuel-metering valve_black is demand, red is feedback_(Black Hills II site in Wyoming) [39].

Figure 2-4 shows a unit from a Black Hills I site in Wyoming during start-up using the fuel-metering valve. The black line shows demand to the valve, and the red line shows feedback from the valve.
Figure 2-4: Fuel-valve response during start-up using fuel-metering valve_black is demand, red is feedback_ (Black Hills I site in Wyoming) [39].

Figure 2-5 shows a start-up to core idle under normal operation. Figure 2-6 shows the metering valve demand signal with a 25 Hertz Dither signal in a steady-state condition. The amplitude of the Dither signal is +/−1 percent. As shown in both of these figures, adding the 25 Hertz Dither signal improves the system response and causes the system to track better.
Figure 2-5: Start-up to core idle under normal operation with a 25 Hertz Dither added to the metering valve demand signal. Amplitude of the dither is +/-1 percent [39].

Based on this observation, injecting the Dither signal could be one way to reduce fluctuation and error amplitude, and improve system response. However, as shown, all command problems of the valve have a high amplitude error (limit cycle) during the steady-state operation. Also, in the transient response (start-up), the position tracks to the
command with fluctuations, and these fluctuations affect the performance of the valve. There are many reasons for the source of these fluctuations. These sources arise from the nonlinear nature of the system and the deficiency of the controller to meet the needs and specifications of constraint applied to the system.

The objective of this thesis is to find a reasonable solution to the control problem of the metering valve as well as to generally categorize and define the source of the fluctuations. Therefore, the fundamental expression of the qualitative behavior of the dynamic response and the limit cycle during valve operation are discussed, and a practical solution for reducing fluctuations is proposed. Various control designs and algorithms for improving the dynamic response of the actuator are presented, and the deficiency of each approach is discussed. Designing a new control algorithm more efficient under the high-friction area providing the ability of fast response and low overshoot are other considerations of this thesis. In the following section, the procedures for the dissertation will be discussed in more detail.

2.3 Project Procedure

In order to analytically investigate the control problem of the actuator, the following procedures are proposed:

1) Mathematical modeling of the system (Chapter 2), including the following:
   a) Nonlinear mathematical modeling of the system of the motor.
   b) Mathematical modeling of the system dynamic of the actuator.

2) Modeling the friction effects.

4) Developing a theory of observer-adaptive friction compensation and its performance analysis for a two-degree freedom system of spring-mass with friction (Chapter 3). This section is extended to the following investigations:
   a) Kalman filter-based friction compensation and its performance analysis for a two-degree freedom.
   b) Extent of observer-adaptive friction compensation for n-degree freedom system of a spring-mass dashpot.
   c) Position control base on friction compensation for the system of two-degree freedom with friction.
5) Implementing adaptive friction compensation for controlling the position of a PMS motor driver fuel valve with a friction flexible actuation system (Chapter 4).

2.4 Mathematical Modeling of the System

Mathematical modeling of the system is comprised of two major sections: the electrical motor and the mechanical actuator. The electrical motor is basically a three-phase PMS motor with sinusoidal flux distribution. The rotating shaft of the motor is connected to a gear box for reducing the rpms. The output of the gear box is connected to the valve through a flexible coupling joint. This actuator system makes control of the valve position very difficult, due to the nonlinearity of the flexible coupling joint. Nonlinearity of the system actuator results from the flexibility of the coupling and the backlash. Other nonlinearity arises from the valve friction. As a result of the design and the application of the valve, this kind of valve is exposed to a very large amount of friction during operation.
In order to determine the dynamic behavior of the system, the complete model of the three-phase (PMS) motor and the nonlinear mathematical model of the actuation system are essential. By combining these models, the overall mathematical model of the system dynamics is attained.

2.5 Permanent Magnet Synchronous (PMS) Motor

The PMS motor is equivalent to an induction motor where the air gap magnetic field is produced by a permanent magnet. The use of a permanent magnet to generate a substantial air gap magnetic flux makes it possible to design highly efficient PMS motors.

A PMS motor is driven by sine wave voltage coupled with the rotor position. The generated stator flux together with the rotor flux, which is generated by the rotor magnet, defines the torque, or speed, of the motor. The sinusoidal wave voltage output must be applied to the three-phase winding system so that the angle between the stator flux and the rotor flux is close to 90° in order to generate the maximum torque. To meet this criterion, the motor requires electronic control for proper operation. A common three-phase PMS motor uses a standard three-phase power stage, which utilizes six power transistors with independent switching. These power transistors are switched in the complementary mode. The sinusoidal wave output is generated using a pulse width modulated (PWM) technique.

The fuel valve is driven by a PMS motor with an interior magnet rotor and two pairs of poles, as shown in Figure 2-7.
In the steady-state range, the rotor speed is given by the input frequency-to-number of pole pairs ratio, i.e.,

\[ n_s = \frac{f}{p} \]  

(2-1)

and is equal to the synchronous speed of the rotating magnetic field produced by the stator.

As shown in Figure 2-7, the direct or \( d \)-axis is the center axis of the magnetic pole, while the quadrature or \( q \)-axis is the axis perpendicular (90° electrical) to the \( d \)-axis. The no-load rms voltage induced in one phase of the stator winding (EMF) by the DC magnetic excitation flux \( \Phi_f \) of the rotor is

\[ E_f = \pi \sqrt{2} f N_1 k_{w1} \Phi_f \]  

(2-2)

where \( N_1 \) is the number of stator turns per phase, \( k_{w1} \) is the stator winding coefficient, and \( \Phi_f \) is the magnetic flux density.

Similarly, the voltage \( E_d \) induced by the \( d \)-axis armature reaction flux \( \Phi_d \) and the voltage \( E_q \) induced by the \( q \)-axis flux \( \Phi_q \) are, respectively,
\[ E_d = \pi \sqrt{2} f N_1 k_w \Phi_d \]  \hspace{1cm} (2-3)

\[ E_q = \pi \sqrt{2} f N_1 k_w \Phi_a \]  \hspace{1cm} (2-4)

The EMFs \( E_f, E_d, E_q \), and magnetic fluxes \( \Phi_f, \Phi_d, \) and \( \Phi_q \) are used in construction of phasor diagrams and equivalent circuits.

### 2.6 Phasor Diagram

When drawing a phasor diagram of synchronous machines, two arrow systems are used: (a) Generator arrow system, i.e,

\[
E_f = V_1 + I_a R_1 + jI_d X_{sd} + jI_q X_{sq} \Rightarrow E_f = V_1 + L_d (R_1 + jX_{sd}) + I_q (R_1 + jX_{sq})
\]  \hspace{1cm} (2-5)

(b) Consumer (motor) arrow system, i.e,

\[
V_1 = E_f + I_a R_1 + jI_d X_{sd} + jI_q X_{sq} \Rightarrow V_1 = E_f + I_d (R_1 + jX_{sd}) + I_q (R_1 + jX_{sq})
\]  \hspace{1cm} (2-6)

where

\[ I_a = I_d + I_q \]  \hspace{1cm} (2-7)

and

\[
I_d = I_a \sin \psi \\
I_q = I_a \cos \psi
\]  \hspace{1cm} (2-8)

When the current arrows are in the opposite direction, the phasors \( I_a, I_d, \) and \( I_q \) are reversed by 180°. The same applies to voltage drops. The location of the armature current \( I_a \) with respect to the d-axis and q-axis for the generator and motor mode is shown in Figure 2-8.
Phasor diagrams for synchronous generators are constructed using the generator arrow system. This same system can be used for motors; however, the consumer arrow system is more convenient. In underexcited motors, the inductive current and corresponding reactive power from the line is drawn. The phasor diagrams that show the same consumer arrow system for a load current $I_a$ leading the vector $V_1$ by the angle $\Phi$ could be drawn. At this angel, the motor is, conversely, overexcited and induces, with respect to the input voltage $V_1$, a capacitive current component $I_a \sin(\psi)$.

In a simple phasor diagram, the stator core losses have been neglected. This assumption is justified only for power frequency synchronous motors with unsaturated armature cores.

2.7 Dynamic Model of the PMS Motor

Control algorithms of sinusoidal excited synchronous motors frequently use the d-q linear model of electrical machines. The d-q dynamic model is expressed in a rotating reference frame that moves at synchronous speed $\omega$. The time-varying parameters are
eliminated, and all variables are expressed in orthogonal or mutually decoupled d and q axes.

A synchronous machine is described by the following set of general equations:

\[ v_d = R_1 i_d + \frac{d\psi_d}{dt} - \omega\psi_q \] (2-9)

\[ v_q = R_1 i_q + \frac{d\psi_q}{dt} + \omega\psi_d \] (2-10)

\[ v_f = R_f I_f + \frac{d\psi_f}{dt} \] (2-11)

\[ R_d i_D + \frac{d\psi_D}{dt} = 0 \] (2-12)

\[ R_q i_Q + \frac{d\psi_Q}{dt} = 0 \] (2-13)

The linkage flux in the above equations are defined as

\[ \psi_d = (L_{ad} + L_1)i_d + L_d i_D + \psi_f = L_{sd} i_d + L_d i_D + \psi_f \] (2-14)

\[ \psi_q = (L_q + L_1)i_q + L_q i_Q = L_{sq} i_q + L_q i_Q \] (2-15)

\[ \psi_f = L_{fd} I_f \] (2-16)

\[ \psi_D = L_d i_d + (L_d + L_D) i_D + \psi_f \] (2-17)

\[ \psi_Q = L_q i_q + (L_q + L_Q) i_Q \] (2-18)

Where \( v_d \) and \( v_q \) are d- and q-axis components of the terminal voltage, \( \psi_f \) is the maximum flux linkage per phase produced by the excitation system, \( R_1 \) is the armature winding resistance, \( L_d \) and \( L_q \) are d- and q-axis components of the armature self-
inductance, \( \omega = 2\pi f \) is the angular frequency of the armature current, \( i_d \) and \( i_q \) are \( d \)- and \( q \)- components of the armature current, and \( i_D \) and \( i_Q \) are \( d \)- and \( q \)-axes components of the damper current. The field winding resistance, which exists only in the case of electromagnetic excitation, is \( R_f \), the field excitation current is \( I_f \), and the excitation linkage flux is \( \psi_f \). The damper resistance and inductance in the \( d \) axis are \( R_D \) and \( L_D \), respectively. The damper resistance and inductance in the \( q \) axis are \( R_Q \) and \( L_Q \), respectively. The resultant armature inductances are

\[
L_{sd} = L_d + L_1 \tag{2-19}
\]

\[
L_{sq} = L_q + L_1 \tag{2-20}
\]

where \( L_d \) and \( L_q \) are self-inductances in the \( d \)- and \( q \)-axis, respectively, and \( L_1 \) is the leakage inductance of the armature winding per phase. In a three-phase machine,

\[
L_d = (3/2)L'_d \quad \text{and} \quad L_q = (3/2)L'_q ,
\]

where \( L'_q \) and \( L'_d \) are self-inductances of the single-phase machine.

The excitation linkage flux is \( \psi_f = L_{fd}I_f \), where \( L_{fd} \) is the maximum value of the mutual inductance between the armature and field winding. In the case of a PM excitation, the fictitious current is \( I_f = H_hh_M \).

For machines with no damper winding, \( i_D = i_Q = 0 \), and the voltage equation in the \( d \)- and \( q \)-axis are
\[ v_d = R_i i_d + \frac{d\psi_d}{dt} - \omega \psi_q = (R_i + \frac{dL_{sd}}{dt})i_d - \omega L_{sq} i_q \]  
(2-21)

\[ v_{iq} = R_i i_q + \frac{d\psi_q}{dt} + \omega \psi_d = (R_i + \frac{dL_{sq}}{dt})i_q + \omega L_{sd} i_d + \omega \psi_f \]  
(2-22)

The matrix form of voltage equation in term of inductances \( L_{sd} \) and \( L_{sq} \) is

\[
\begin{bmatrix}
  v_d \\
  v_q
\end{bmatrix} =
\begin{bmatrix}
  R_i + \frac{dL_{sd}}{dt} & -\omega L_{sq} \\
  \omega L_{sd} & R_i + \frac{dL_{sq}}{dt}
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  \omega \psi_f
\end{bmatrix} \tag{2-23}
\]

\[
\frac{d}{dt} i_d = \frac{1}{L_{sd}} v_d - \frac{R}{L_{sd}} i_d + \frac{L_{sq}}{L_{sd}} p\omega i_q
\]

\[
\frac{d}{dt} i_q = \frac{1}{L_{sq}} v_q - \frac{R}{L_{sq}} i_q - \frac{L_{sd}}{L_{sq}} p\omega i_d - \omega \psi_f \frac{p}{L_{sq}}
\]

\( v_d, v_q \) could be defined in state space system by

\[
\begin{bmatrix}
  v_d \\
  v_q
\end{bmatrix} =
\begin{bmatrix}
  R_i + pL_q & \omega L_d \\
  -\omega L_q & R_i + pL_d
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} +
\begin{bmatrix}
  \omega \psi_f \\
  0
\end{bmatrix} \tag{2-24}
\]

For the steady-state operation

\[
(d / dt) L_{sd} i_d = (d / dt) L_{sq} i_q = 0, I_a = I_d + jI_q, V = V_d + jV_q, i_d = \sqrt{2}I_d, i_q = \sqrt{2}I_q
\]

\[ v_d = \sqrt{2}V_d, \quad v_q = \sqrt{2}V_q, \quad E_f = \omega L_{sq} I_f / \sqrt{2} = \omega \psi_f / \sqrt{2} \]

The quantities \( \omega L_{sd} \) and \( \omega L_{sq} \) are known as the d- and q-axis synchronous reactance, respectively.

The instantaneous power input to the three-phase armature is

\[ p_m = v_A i_A + v_B i_B + v_C i_C = \frac{3}{2} \left( v_d i_d + v_q i_q \right) \]  
(2-25)

The power balance equation is obtained from Eqns (2-22) and (2-23), i.e,
\[
v_d i_d + v_q i_q = R_d i_d^2 + \frac{d\varphi_d}{dt} i_d + R_q i_q^2 + \frac{d\varphi_q}{dt} i_q + \omega (\varphi_d i_q - \varphi_q i_d) \quad (2-26)
\]

The last term \(\omega (\varphi_d i_q - \varphi_q i_d)\) accounts for the electromagnetic power of a single phase, two-pole synchronous machine. For a three-phase machine

\[
P_{\text{elm}} = \frac{3}{2} \omega (\varphi_d i_q - \varphi_q i_d) = \frac{3}{2} \omega [ (L_d i_d + \varphi_f) i_q - L_d i_d i_q ]
\]

\[
= \frac{3}{2} \omega [ \varphi_f + (L_d - L_q) i_d ] i_q \quad (2-27)
\]

The electromagnetic torque of a three-phase motor with \(p\) pole pairs is

\[
T_d = p \cdot \frac{P_{\text{elm}}}{\omega} = \frac{3}{2} \cdot p [ \varphi_f + (L_d - L_q) i_d ] i_q \quad \text{Newton} \quad (2-28)
\]

The reformation from the direct- to quadrature-axis, and vice versa, is \((dq0\) transformation\)

\[
\begin{bmatrix}
  i_d \\
  i_q \\
  0
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
  \cos(wt) & \cos(wt - \frac{2\pi}{3}) & \cos(wt + \frac{2\pi}{3}) \\
  -\sin(wt) & -\sin(wt - \frac{2\pi}{3}) & -\sin(wt + \frac{2\pi}{3}) \\
  \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \cdot \begin{bmatrix}
  i_A \\
  i_B \\
  i_C
\end{bmatrix} \quad (2-29)
\]

The reverse relations, obtained by the simultaneous solution of Eqn (2-29) in conjunction with \(i_A + i_B + i_C = 0\), are

\[
\begin{bmatrix}
  i_A \\
  i_B \\
  i_C
\end{bmatrix} = \begin{bmatrix}
  \cos(wt) & -\sin(wt) & 1 \\
  \cos(wt - \frac{2\pi}{3}) & -\sin(wt - \frac{2\pi}{3}) & 1 \\
  \cos(wt + \frac{2\pi}{3}) & -\sin(wt + \frac{2\pi}{3}) & 1
\end{bmatrix} \cdot \begin{bmatrix}
  i_d \\
  i_q \\
  0
\end{bmatrix} \quad (2-30)
\]
2.8 Actuation Systems Model

Figures 2-9 and 2-10 show the actuation system for the fuel valve system. This system is composed of a shaft that is connected to the gear box and from the gear box to another shaft. A non-rigid coupling between two mechanical components in a control system often causes torsional resonances that can be transmitted to all parts of the system.

![Figure 2-9: Actuation system of the fuel valve.](image)

For the gear box part, the work done by one gear is equal to that of the other since there are assumed to be no losses. Thus
The number of teeth on the surface of the gears is proportional to the radii \( r_1 \) and \( r_2 \) of the gears, that is,

\[ r_m N_2 = r_2 N_1 \]  

(2-32)

If angular velocities of the two gears \( \omega_m \) and \( \omega_2 \) are introduced, then the above equations lead to

\[ \frac{T_1}{T_2} = \frac{\Theta_2}{\Theta_m} = \frac{N_1}{N_2} \frac{\omega_2}{\omega_m} \]  

(2-33)

Using the Newton equation of motion

\[ T_2(t) = J_2 \ddot{\Theta}_2 + B_2 \dot{\Theta}_2 + T_{f2} + K_i (\Theta_2 - \Theta_L) + B \left( \ddot{\Theta}_2 - \dot{\Theta}_L \right) \]  

(2-34)

\[ T_k = K_i (\Theta_2 - \Theta_L) + B (\ddot{\Theta}_2 - \dot{\Theta}_L) \]  

(2-35)

The torque Eqn on the side of gear \( m \) is

\[ T_m(t) = J_m \ddot{\Theta}_m + B_m \dot{\Theta}_m + T_{f1} + T_1 \]  

(2-36)

Using Eqn (2-33), Eqn (2-34) is converted to

\[ T_1(t) = \frac{N_1}{N_2} T_2(t) = \left( \frac{N_1}{N_2} \right)^2 J_2 \ddot{\Theta}_m + \left( \frac{N_1}{N_2} \right)^2 B_2 \dot{\Theta}_m + \frac{N_1}{N_2} T_{f2} \]

\[ + \frac{N_1}{N_2} K_i \left( \frac{N_1}{N_2} \Theta_m - \Theta_L \right) + \frac{N_1}{N_2} B \left( \frac{N_1}{N_2} \dot{\Theta}_m - \dot{\Theta}_L \right) \]  

(2-37)

Now by substituting Eqn (2-36) for Eqn (2-35)

\[ T_m(t) = J_m \ddot{\Theta}_m + B_m \dot{\Theta}_m + T_f + \frac{N_1}{N_2} K_i \left( \frac{N_1}{N_2} \Theta_m - \Theta_L \right) \]  

(2-38)

where
\[ J_{\text{meq}} = J_m + \left( \frac{N_1}{N_2} \right)^2 J_2 \] (2-39)

\[ B_{\text{meq}} = B_1 + \left( \frac{N_1}{N_2} \right)^2 B_2 \] (2-40)

\[ T_F = T_{f1} + \frac{N_1}{N_2} T_{f2} \] (2-41)

\[ T_L + K_r (\Theta_2 - \Theta_L) + B \left( \ddot{\Theta}_2 - \ddot{\Theta}_L \right) = J_L \ddot{\Theta}_L + B_L \dot{\Theta}_L + T_{fl} \] (2-42)

For these equations, the following apply:

- \( T_m \): Shaft torque from the motor, Nm
- \( T_L \): Fixed torque of the load, Nm
- \( J_m, J_2, J_L \): Inertia of motor shaft, low speed rotor, and load rotor
- \( B_m, B_2, B_L \): Viscous friction of motor rotor, gear shaft, and load
- \( K_r \): Spring constant (tensional coefficient) of motor, Nm/rad
- \( \Theta_m, \Theta_1, \Theta_L \): Motor Rotor, low speed shaft, and load shaft angular position
- \( T_{f1}, T_{f2}, T_{fl} \): Coulomb friction and static friction for motor rotor, low-speed shaft, and load shaft.
- \( J_{\text{meq}} \): Combined gear and motor shaft inertia
- \( B_{\text{meq}} \): Combined viscous friction for motor and gear section
- \( T_F \): Combined coulomb friction and static friction

Considering the converted Eqn (2-37), a schematic of the system could be simplified to Figure 2-11

![Figure 2-11: Equivalent model of the actuation system.](image-url)
The friction term includes coulomb friction and static friction, and plays an important role in this model. In the next section, friction will be discussed in more detail.

2.9 Friction Characteristics

2.9.1 Introduction

Friction occurs in all mechanical systems, e.g., bearings, transmissions, hydraulic and pneumatic cylinders, valves, brakes, and wheels. Friction appears at the physical interface of two surfaces in contact. Lubricants such as grease or oil are often used, but there may also be a dry contact between the surfaces. Friction is strongly influenced by contamination. Friction is carved by a wide range of physical phenomena, including elastic and plastic deformations, fluid mechanics, and the actuation system. Friction was studied extensively in classical mechanical engineering, and lately there has been a strong resurgence, which, partly from intellectual curiosity, is driven by strong engineering needs in a wide range of industries from disk drives to cars.

Friction is also very important for the control engineer who designs drive systems, high-precision servo mechanisms, robots, pneumatic and hydraulic systems, and anti-lock brakes for cars. Friction is highly nonlinear and may result in steady-state errors, limit cycles, and poor performance. Therefore, it is important for control engineers to understand and know how to deal with friction phenomena. With the computational power available today, it is possible to deal effectively with friction to improve quality, economy, and safety of a system.

Friction should be considered early in a system’s design by reducing it as much as possible with good hardware. However, cost constraints may be prohibitive. The dither method, a simple way to reduce static friction, has been used for many years. It can be
introduced electronically or mechanically by a vibrator, as was done in early auto pilots. Recent advances in computer control have also shown the possibility of reducing the effects of friction by estimation and control. It is useful for the control engineer to understand friction in order to recognize its effects on a closed loop and to design control laws that reduce it.

2.9.2 Friction model

Figure 2-12 provided examples of friction models. Friction opposes motion, and its magnitude is independent of velocity and contact area. It can therefore be described as

\[ F = F_c \cdot \text{sig}(v) \]  \hspace{1cm} (2-43)

where the friction force \( F_c \) is proportional to the normal load, i.e., \( F_c = \mu F_n \). This description of friction is termed Coulomb friction, as shown in Figure 2-12a. This Figure shows that the friction force is given by a static function, except possibly for zero velocity. Figure 2-12b shows Coulomb plus viscous friction. Stiction plus Coulomb and
viscous friction is shown in Figure 2-12c, and Figure 2-12d shows how the friction force may decrease continuously from the static friction level. The Coulomb friction model does not specify the friction force for zero velocity. It may be zero, or it can take on any value in the interval between \(-F_C\) and \(F_C\), depending on how the sign function is defined. Because of its simplicity the Coulomb friction model often has been used for friction compensation. In the nineteenth century, the theory of hydrodynamics was developed, which led to expressions for the friction force caused by the viscosity of lubricants. The term viscous friction is used for this force component, which is normally described as

\[
F = F_{v, v}
\]  

(2-44)

Stiction is an abbreviation for static friction, as opposed to dynamic friction. It describes the friction force at rest. Friction force at rest is higher than the Coulomb friction level. Stiction counteracts external forces below a certain level and thus keeps an object from moving.

Therefore, friction at rest cannot be described as a function of velocity only. Instead, it must to be modeled using the external force \(F_e\) as

\[
F = \begin{cases} 
F_e & \text{if } v = 0 \text{ and } |F_e| < F_S \\
F_S \text{ sgn}(F_e) & \text{if } v = 0 \text{ and } |F_e| \geq F_S
\end{cases}
\]  

(2-45)

The combination of stiction plus Coulomb and viscous friction is used to model the entire friction in the system [41].
2.10 Nonlinear Mathematical Model of Actuation System

Considering the friction model from section 2.9, all the mathematical models from the actuation system are rewritten to obtain the general nonlinear equation of the system as

\[
T_m(t) = \left[ J_m + \left( \frac{N_1}{N_2} \right)^2 J_2 \right] \ddot{\Theta}_m + \left[ B_1 + \left( \frac{N_1}{N_2} \right)^2 B_2 \right] \dot{\Theta}_m + \frac{N_1}{N_2} K_t (\frac{N_1}{N_2} \Theta_m - \Theta_L) + \frac{N_1}{N_2} B (\frac{N_1}{N_2} \ddot{\Theta}_m - \ddot{\Theta}_L) + T_F
\]

(2-46)

\[
T_F = T_{f1} + \frac{N_1}{N_2} T_{f2}
\]

(2-47)

\[
T_{f1} = T_{c1} \text{sgn}(\omega_1) + \begin{cases} T_m & \text{if } \omega_m = 0, \text{and } |T_m| < T_{s1} \\ T_{s1} \text{sgn}(T_m) & \text{if } \omega_m = 0, \text{and } |T_m| \geq T_{s1} \end{cases}
\]

(2-48)

\[
T_{f2} = T_{c2} \text{sgn}(\omega_2)
\]

(2-49)

\[
T_F = T_{c1} \text{sgn}(\omega_1) + \frac{N_1}{N_2} \left[ T_{c2} \text{sgn}(\omega_2) \right] + \begin{cases} T_m & \text{if } \omega_m = 0, \text{and } |T_m| < T_{s1} \\ T_{s1} \text{sgn}(T_m) & \text{if } \omega_m = 0, \text{and } |T_m| \geq T_{s1} \end{cases}
\]

(2-50)

\[
T_{FL} = T_{c1} \text{sgn}(\omega_L) + \frac{N_1}{N_2} \left[ T_{c2} \text{sgn}(\omega_2) \right]
\]

(2-51)

\[
T_{FL} = T_{c1} \text{sgn}(\omega_L) + \frac{N_1}{N_2} \left[ T_{c2} \text{sgn}(\omega_2) \right] + \begin{cases} T_m & \text{if } \omega_m = 0, \text{and } |T_m + T_L + T_K| < T_{s2} \\ T_{s2} \text{sgn}(T_m + T_L + T_K) & \text{if } \omega_m = 0, \text{and } |T_m + T_L + T_K| \geq T_{s2} \end{cases}
\]

(2-52)
\[ T_L + T_K = J_L \ddot{\Theta}_L + \]
\[ T_{cl} \text{sgn}(\omega_L) + \begin{cases} 
T_L + T_K & \text{if } \omega_m = 0, \text{and } |T_L + T_K| < T_{s2} \\
T_{s2} \text{sgn}(T_L + T_K) & \text{if } \omega_m = 0, \text{and } |T_L + T_K| \geq T_{s2} 
\end{cases} \] (2-53)

\[ T_K = K_i(\Theta_2 - \Theta_L) + B(\dot{\Theta}_2 - \dot{\Theta}_L) \] (2-54)

Eqn (2-46) to (2-54) represent the general equation for the nonlinear system of the actuation system with viscous, static, and Coulomb friction. Friction compensation is used in order to reduce the effect of friction. A new method of friction compensation is presented in this dissertation. Chapter 3 covers two methods of compensating for friction: friction compensation gain and a new method of using two parallel friction compensations. Also, the application of friction compensation gain for nonlinear DC motors will be presented in Appendix A. Chapter 3 also will provide a mathematical proof for the theory of friction compensation and its performance in a two degrees of freedom system. In addition, the generalization of observer-based friction compensation will be discussed. Chapter 4 will present the application of this new method for implementing in a fuel valve system.
CHAPTER 3

FUNDAMENTALS OF ADAPTIVE OBSERVER-BASED FRICTION COMPENSATION FOR A MULTIBODY SYSTEM

3.1 Introduction

Mathematical equations for modeling PMS motors with flexible friction actuation systems of the fuel valve were presented in Chapter 2, which also discussed the inability of the linear control system to compensate for friction. Chapter 3 presents the theory of a new method of friction compensation, based on observer adaptive-friction compensation for multibody systems. This theory is proven for a two degrees of freedom system and is generalized for an n degrees of freedom system. This section derives the mathematical equations for modeling a two-order system with friction and shows the simulation results of the position control of two degrees of freedom of mass-spring-dashpot based on friction composition.

By analyzing inner torque, outer motion of the control system, and static and kinetic friction of the high-speed and low-speed shaft, a system friction observer is designed for a two degrees of freedom system. As shown, the nonlinear mechanical friction causes a stick-slip response in the system. In high-precision positioning applications, friction compensation plays an important role in tracking accuracy requirements. Many methods have been developed to compensate for friction torques and forces acting on positioning devices. One of the goals of this thesis is to present a new method of friction compensation using a two-parallel friction compensation and also a generalized method for an n-order system.
The observer structure is based on the assumption that the velocity upon which friction depends can be measured directly, which is often, but not always, the case. It is interesting to examine how the system would perform if an estimate \( \hat{v} \) of the velocity, produced by another observer, was used in place of the true velocity. Finally, the compensator allows the control system to generate a force exactly opposite to the estimated friction. Any dynamics in the actuator would introduce a phase shift that would prevent exact cancellation. If the phase shift were appreciable, it might be necessary to include a model of the actuator in the observer [42].

The concept of an observer for a dynamic system was introduced by Friedland [43] and Shinners [44]. An observer–based application for friction estimation and compensation were also reported by Friedland and Park [45], Tafazoli et al. [46], Canudas et al. [47], and Brandenburg et al. [48]. In this section, Friedland and Park’s work is extended to a multibody system of mass, moving under the influence of some external forces, including friction. This chapter includes the following:

- Mathematical modeling of the two degrees of freedom system with friction.
- Observer-based friction compensation for the two degrees of freedom system.
- Generalized friction compensation model for n’th degrees of freedom.
- Kalman filter observer-based friction compensation for two degrees of freedom.
- Performance analysis and simulation results of friction compensation for two degrees of freedom system with friction.
- Position control system based on observer-friction compensation.
3.2 Mathematical Modeling a Two Degrees of Freedom System with Friction

The dynamic analysis of most real structures is based on multiple degrees of freedom (MDOF) models. Friction is also always present in servo-mechanisms and is responsible for tracking lags, steady-state errors, limit cycles, and undesired stick-slip motion. In this section, Lagrange’s equation is used to derive the equation of motion of two degrees of freedom system with friction.

Figure 3-1 shows the two degrees of freedom for spring, mass, and damper.

Using Lagrange’s equation, the equation of motion for a neoconservative system is shown as

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \quad i = 1,2
\]  

(3-1)

This equation is valid for a nonlinear as well as linear system. When \( q_1 \to X1 \) and \( q_2 \to X2 \), then the kinetic energy is given by
\[ T = \frac{1}{2} M_1 \dot{X}_1^2 + \frac{1}{2} M_2 \dot{X}_2^2 \]  
\hspace{1cm} \text{(3-2)}

\[ V = \frac{1}{2} K_1 X_1^2 + \frac{1}{2} C_1 \dot{X}_1^2 + \frac{1}{2} K_2 (X_2 - X_1)^2 + \frac{1}{2} C_2 (\dot{X}_2 - \dot{X}_1)^2 \]  
\hspace{1cm} \text{(3-3)}

Since this study focuses on friction (stick-slip and Coulomb friction), the damper has been removed from the system by putting \( C_1 = C_2 = 0 \).

Applying Lagrange’s equation 4-1
\[
\frac{\partial T}{\partial \dot{X}_1} = M_1 \ddot{X}_1 \]  
\hspace{1cm} \text{(3-4)}

\[
\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{X}_1} \right) = M_1 \dddot{X}_1 \]  
\hspace{1cm} \text{(3-5)}

\[
\frac{\partial T}{\partial \dot{X}_2} = M_2 \ddot{X}_2 \]  
\hspace{1cm} \text{(3-6)}

\[
\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{X}_2} \right) = M_2 \dddot{X}_2 \]  
\hspace{1cm} \text{(3-7)}

\[
\frac{\partial T}{\partial X_1} = \frac{\partial T}{\partial X_2} = 0 \]  
\hspace{1cm} \text{(3-8)}

\[
\frac{\partial V}{\partial X_1} = K_1 X_1 - K_2 (X_2 - X_1) = (K_1 + K_2) X_1 - K_2 X_2 \]  
\hspace{1cm} \text{(3-9)}

\[
\frac{\partial V}{\partial X_2} = K_2 (X_2 - X_1) = -K_2 X_1 + K_2 X_2 \]  
\hspace{1cm} \text{(3-10)}
\[ Q_1 = F_1 - F_{f1} \]  \hspace{1cm} (3-11)

\[
F_{f1} = F_{c1} \text{ sgn}(x_1) + \begin{cases} F_1 \big| \dot{x}_1 = 0 & |F_1| < F_{s1} \\ F_{s1} \text{ sgn}(F_1) & |F_1| \geq F_{s1} \end{cases}
\]  \hspace{1cm} (3-12)

\[ Q_2 = F_2 - F_{f2} \]  \hspace{1cm} (3-13)

\[
F_{f2} = F_{c2} \text{ sgn}(x_2) + \begin{cases} F_2 \big| \dot{x}_2 = 0 & |F_2| < F_{s2} \\ F_{s2} \text{ sgn}(F_2) & |F_2| \geq F_{s2} \end{cases}
\]  \hspace{1cm} (3-14)

Therefore,

\[
M_1 \ddot{X}_1 + (K_1 + K_2)X_1 - K_2X_2 = F_1 - F_{c1} \text{ sgn}(x_1)
\]
\[ + \begin{cases} F_1 \big| \dot{x}_1 = 0 & |F_1| < F_{s1} \\ F_{s1} \text{ sgn}(F_1) & |F_1| \geq F_{s1} \end{cases}
\]  \hspace{1cm} (3-15)

\[
M_2 \ddot{X}_2 - K_2X_1 + K_2X_2 = F_2 - F_{c2} \text{ sgn}(x_2)
\]
\[ + \begin{cases} F_2 \big| \dot{x}_2 = 0 & |F_2| < F_{s2} \\ F_{s2} \text{ sgn}(F_2) & |F_2| \geq F_{s2} \end{cases}
\]  \hspace{1cm} (3-16)

Defining the state for \( Z_1, Z_2, Z_3, \) and \( Z_4 \) as

\[ Z_1 = X_1 \]

\[ . \]

\[ Z_2 = X_1 \]

\[ Z_3 = X_2 \]

\[ . \]

\[ Z_4 = X_2 \]

Then the state equation of the nonlinear system will be derived as
Eqns 3-18 and 3-19 define the total mathematical model of the system of nonlinear two-degree freedom with Coulomb friction and stick-slip friction. There equations will be reconfigured to the following nonlinear state space equation:
Eqns (3-20) to (3-23) are fundamental equations for a nonlinear system with friction.

Using MATLAB, the equation is solved for the following value:

\[ K_1 = K_2 = 1 \quad N / m \]

\[ M_1 = M_2 = 1 \quad Kg \]

\[ F_1 = 1 \quad N \]

\[ F_2 = 0 \]

\[ F_{c1} = F_{Stick_2} = 0.5 \]

\[ F_{c2} = F_{Stick_2} = 0.4 \]
Figures 3-2 and 3-3 show the response of the system due to the change in $M_1$. The initial conditions for this case is $Z_0 = [10 \ 0 \ 0 \ 0]$ and $F_1 = F_2 = 0$. As shown, the steady-state condition for the system with friction is zero, 35 seconds after excitation. The non-friction system tends to move infinitely. Figures 3-4 and 3-5 show phase plane diagrams for a non-friction system and a friction system.

![Open Loop Response (Two Degrees of Freedom)](image)

Figure 3-2: Response of $M_1$ due to change of $M_1$ position.
Figure 3-3: Response of M₂ due to the change of M₁ position in a non-friction case and in a friction case.
Figure 3-4: Phase plane diagram for a non-friction system and a friction system.
Figures 3-6 and 3-7 show the response of the system due to a change in $M_2$. The initial conditions for this case is $Z_0 = [0, 0, 10, 0]$ and $F_1 = F_2 = 0$. As shown, the steady-state condition for the system with friction is zero, 25 seconds after excitation. The non-friction system tends to move infinitely. Figures 3-8 and 3-9 show phase plane diagrams for a non-friction system and a friction system.
Figure 3-6: Response of $M_1$ due to change of $M_2$ position.
Figure 3-7: Response of $M_2$ due to change of $M_2$ position.
Figure 3-8: Phase plane diagram for two cases of a non-friction system and a friction system.
Figure 3-9: Phase plane diagram for two cases of a non-friction system and a friction system on a larger scale.

Figures 3-10 and 3-11 show the response of the system due to applied force on mass M₁. In this case, the force is $F_i = 1$ N, as shown, and the amplitude of the position and the velocity in the steady-state condition are limited 25 seconds after excitation. The non-friction system tends to move infinitely. Figure 3-12 shows the phase plane diagram for a non-friction system and a friction system.
Figure 3-10: M₁ response due to applied force on M₁.
Figure 3-11: $M_2$ response due to applied force on $M_1$. 
Figures 3-13 and 3-14 show the response of the system due to applied force on mass $M_1$ and $M_2$. In this case, the forces are $F_1 = F_2 = 1$ N. As shown, the amplitude of the position and the velocity in the steady-state condition are limited 10 seconds after excitation. The non-friction system tends to move infinitely. Figure 3-15 shows the phase plane diagram for a non-friction system and a friction system.
Figure 3-13: M₁ response of the system due to applied force on M₁ and M₂.
Figure 3-14: M₂ response of the system due to applied force on M₁ and M₂.
Figures 3-15: Phase plane diagram showing response of the system due to applied force on M₁ and M₂.

Figures 3-16 and 3-17 show the response of the system due to applied sinusoid force on mass M₁. The force is applied on M₁ with an amplitude of 1 N and a frequency of 0.5 Hertz. In this case, \( F_1 = \sin(\pi t), F_2 = 0 \) N. As shown, the amplitude of the position and the velocity in steady-state condition are limited, compared to the non-friction case. Figure 3-18 shows the phase plane diagram for a non-friction system and a friction system.
Figure 3-16: M₁ response of the system due to sinusoid force on M₁.
Figure 3-17: M₂ response of the system due to sinusoid force on M₁.
3.3 Friction Observer

Friction is inevitable in mechanical systems; most systems depend upon it for proper operation. Nevertheless, friction is a nuisance to the control system designer, confounding the application of most design methods and often limiting the performance of the system. Its presence is often responsible for the inability of the system to achieve low values of steady-state error and may limit the closed-loop bandwidth to avoid limit cycling [45].

If the force (or torque) due to friction is significantly smaller than the available control force, one way of dealing with friction would be to use some of the control force...
to cancel the friction. Unfortunately, however, the physical nature of friction is such that the friction can be crudely estimated but rarely determined a priori with the accuracy required for its cancellation. Moreover, the amount of friction present in a process often changes with time and may depend on unknown environmental factors, such as temperature, lubricant condition, and similar factors that cannot be readily measured or controlled. Hence, a practical method of canceling friction in a control system would involve an on-line estimation of friction that is present in the system. The use of recursive least squares for estimating parameters in a nonlinear friction model was considered by Canudas et al. [47], who demonstrated the benefits of the technique experimentally. Brandenburg et al. [48], considered the use of a linear observer for improving the performance of a system with Coulomb friction. Friedland and Park [45] use another approach to estimate the friction by employing a “reduced-order” observer, the dynamics of which are designed to ensure asymptotic convergence of the estimation error to zero when the actual friction conforms to the classical model. If the actual model of the friction differs from the classical model, convergence of the error to zero cannot be easily shown [45].

The Friedland-Park observer [45] represents friction as a constant time the sign of velocity, which is actually the symmetric Coulomb friction model. It estimates the constant using measurements of velocity and external forces other than friction acting on the body subjected to friction. The equations of the nonlinear reduced-order observer are postulated as
\[ \hat{F} = \hat{a} \cdot \text{sgn}(v) \]  
(3-24) 

\[ \hat{a} = z - k|v|^\mu \]  
(3-25) 

\[ \dot{z} = k \cdot \mu |v|^{\mu - 1} \cdot (w - \hat{a}) \cdot \text{sgn}(v) \]  
(3-26) 

where \( \hat{F} \) is the estimated friction, \( \hat{a} \) is the parameter to be estimated, \( v \) is the measured velocity, \( w \) is the acceleration due to forces other than friction, and \( z \) is the observer state. Selection of two design parameters, gain \( k > 0 \) and exponent \( \mu > 0 \), is based on the condition that the estimation error converges asymptotically to zero, and \( v \) is bounded away from zero.

In many instances, however, positions instead of velocity measurements are available; therefore, a velocity observer was added to replace velocity measurements in the above equations. The new observer is known as the Friedland-Mentzelopoulos friction observer.

In this research, the Friedland friction observer is extended to the n-degree freedom system with stick-slip and Coulomb friction. The implementation of the method is investigated on a two-degree freedom system under the noise. A Kalman filter with the friction observer is used to estimate the state and friction. The classical concept of friction in terms of Coulomb friction for a two-degree freedom system is a force as a nonlinear function of velocity. For a two-degree freedom system, this function is defined as
\[
\begin{pmatrix}
F_{f1}^\wedge \\
F_{f2}^\wedge
\end{pmatrix} = 
\begin{pmatrix}
\wedge a_1 & 0 \\
0 & \wedge a_2
\end{pmatrix}
\begin{pmatrix}
\text{Sgn}(v_1) \\
\text{Sgn}(v_2)
\end{pmatrix} (3-27)
\]

Considering two states \(\xi_1\) and \(\xi_2\) for the friction areas of each mass, the estimation of parameters \(a_1\) and \(a_2\) in the friction model will be estimated from two sets of nonlinear equations, namely
\[
\begin{pmatrix}
\wedge a_1 \\
\wedge a_2
\end{pmatrix} = 
\begin{pmatrix}
\xi_1 \\
\xi_2
\end{pmatrix} - 
\begin{pmatrix}
\alpha_1 & 0 \\
0 & \alpha_2
\end{pmatrix}
\begin{pmatrix}
v_1^{\beta_1} \\
v_2^{\beta_2}
\end{pmatrix} (3-28)
\]

where \(\alpha_1, \alpha_2, \beta_1, \) and \(\beta_2\) are design parameters, and the variables \(\xi_1\) and \(\xi_2\) are given by
\[
\begin{pmatrix}
\xi_1 \\
\xi_2
\end{pmatrix} = 
\begin{pmatrix}
\alpha_1 \beta_1 |v_1|^{\beta_1-1} & 0 \\
0 & \alpha_2 \beta_2 |v_2|^{\beta_2-1}
\end{pmatrix}
\]
\[
\begin{pmatrix}
w_1 \\
w_2
\end{pmatrix} - 
\begin{pmatrix}
F_{f1}(v_1, a_1) \\
F_{f2}(v_2, a_2)
\end{pmatrix} - 
\begin{pmatrix}
\text{Sgn}(v_1) & 0 \\
0 & \text{Sgn}(v_2)
\end{pmatrix} (3-29)
\]

where \(w_1\) and \(w_2\) are all forces other than friction.

To study the performance of the observer, consider the error between the actual parameter \(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\) and its estimate \(\begin{pmatrix} a_1^\wedge \\ a_2^\wedge \end{pmatrix}\). The error \(e\) is defined as the difference between

the actual value and the estimated value, assuming that the true parameter vector \(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\) is constant, and
\[
\begin{pmatrix}
e_1 \\
e_2
\end{pmatrix} = 
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix} - 
\begin{pmatrix}
a_1^\wedge \\
a_2^\wedge
\end{pmatrix} (3-30)
\]

Therefore,
\[
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix} = \begin{bmatrix}
\dot{v}_1 \\
\dot{v}_2
\end{bmatrix} = \begin{bmatrix}
\alpha_1 & 0 \\
0 & \alpha_2
\end{bmatrix} \begin{bmatrix}
\beta_1 |v_1|^{\beta_1-1} v_1 \cdot \text{sgn}(v_1) \\
\beta_2 |v_2|^{\beta_2-1} v_2 \cdot \text{sgn}(v_2)
\end{bmatrix}
\]

(3-31)

\[
\begin{bmatrix}
\alpha_1 \beta_1 |v_1|^{\beta_1-1} \text{sgn}(v_1) & 0 \\
0 & \alpha_2 \beta_2 |v_2|^{\beta_2-1} \text{sgn}(v_2)
\end{bmatrix} \begin{bmatrix}
\dot{v}_1 - w_1 + f(v_1, \hat{a}_1) \\
\dot{v}_2 - w_2 + f(v_2, \hat{a}_2)
\end{bmatrix}
\]

(3-32)

\[
\begin{bmatrix}
\alpha_1 \beta_1 |v_1|^{\beta_1-1} \text{sgn}(v_1) & 0 \\
0 & \alpha_2 \beta_2 |v_2|^{\beta_2-1} \text{sgn}(v_2)
\end{bmatrix} \begin{bmatrix}
f(v_1, a_1) - f(v_1, \hat{a}_1) \\
f(v_2, a_2) - f(v_2, \hat{a}_2)
\end{bmatrix}
\]

(3-33)

\[
\begin{bmatrix}
\alpha_1 \beta_1 |v_1|^{\beta_1-1} \text{sgn}(v_1) & 0 \\
0 & \alpha_2 \beta_2 |v_2|^{\beta_2-1} \text{sgn}(v_2)
\end{bmatrix} \begin{bmatrix}
(a_1 - \hat{a}_1) \text{sgn}(v_1) \\
(a_2 - \hat{a}_2) \text{sgn}(v_2)
\end{bmatrix}
\]

(3-34)

\[
\begin{bmatrix}
\alpha_1 \beta_1 |v_1|^{\beta_1-1} & 0 \\
0 & \alpha_2 \beta_2 |v_2|^{\beta_2-1}
\end{bmatrix} \begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix}
\]

(3-35)

(3-36)

Which, for \(\alpha_1, \alpha_2, \beta_1\) and \(\beta_2 > 0\), converges asymptotically to zero if and only if \(v_1\) and \(v_2\) is bounded away from zero.

The above theory suffers from one deficiency: convergence of the estimated error to zero is dependent on whether the actual dynamics friction conforms to the classical model. If the actual model of the friction differs from the classical model, then convergence of the error to zero can not be easily shown.

For the two degrees of freedom system shown in Figure 3-1, \(w_1\) and \(w_2\) are defined as
\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2
\end{bmatrix} = \begin{bmatrix}
\frac{K_1 + K_2}{M_1} & \frac{K_2}{M_1} \\
\frac{K_2}{M_2} & -\frac{K_2}{M_2}
\end{bmatrix} \begin{bmatrix}
Z_1 \\
Z_2
\end{bmatrix} + \begin{bmatrix}
\frac{1}{M_1} & 0 \\
0 & \frac{1}{M_2}
\end{bmatrix} \begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\]

(3-37)

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \beta_1 |Z_2|^\beta_1^{-1} & 0 \\
0 & \alpha_2 \beta_2 |Z_4|^\beta_2^{-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2
\end{bmatrix} - \begin{bmatrix}
F_{f_1}(Z_2, \hat{a}_1) \\
F_{f_2}(Z_4, \hat{a}_2)
\end{bmatrix} + \begin{bmatrix}
Sgn(Z_2) & 0 \\
0 & Sgn(Z_4)
\end{bmatrix}
\]

(3-38)

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \beta_1 |Z_2|^\beta_1^{-1} & 0 \\
0 & \alpha_2 \beta_2 |Z_4|^\beta_2^{-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
-k_1 + k_2 \\
-k_2
\end{bmatrix} \begin{bmatrix}
Z_1 \\
Z_2
\end{bmatrix} + \begin{bmatrix}
\frac{1}{M_1} & 0 \\
0 & \frac{1}{M_2}
\end{bmatrix} \begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} - \begin{bmatrix}
F_{f_1}(Z_2, \hat{a}_1) \\
F_{f_2}(Z_4, \hat{a}_2)
\end{bmatrix} + \begin{bmatrix}
Sgn(Z_2) & 0 \\
0 & Sgn(Z_4)
\end{bmatrix}
\]

(3-39)

\[
\begin{bmatrix}
\hat{a}_1 \\
\hat{a}_2
\end{bmatrix} = \begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} - \begin{bmatrix}
\alpha_1 & 0 \\
0 & \alpha_2
\end{bmatrix} \begin{bmatrix}
|Z_2|^{\beta_1} \\
|Z_4|^{\beta_2}
\end{bmatrix}
\]

(3-40)

\[
\begin{bmatrix}
\hat{F}_{f_1} \\
\hat{F}_{f_2}
\end{bmatrix} = \begin{bmatrix}
\hat{a}_1 & 0 \\
0 & \hat{a}_2
\end{bmatrix} \begin{bmatrix}
Sgn(Z_2) \\
Sgn(Z_4)
\end{bmatrix}
\]

(3-41)

Figure 3-19 shows how the friction compensation estimates the friction of the system for the design parameters of \(\alpha_1 = \alpha_2 = 1.2, \beta_1 = \beta_2 = 100\). The response of the system results from the change of the displacement of the mass \(M_1\) and \(M_2\), namely \(Z_0 = [10 \ 0 \ 10 \ 0]\). As shown in Figure 3-19, between the times 30 to 40 in which the transient response is converted from transient to steady-state, the friction estimator failed to give a good estimation of the friction. The reason for this is because the friction estimator here is only capable of estimating the Coulomb friction, and during the conversion from transient to steady-state, the stick-slip friction is more dominant.
Figure 3-20 shows the same response for the mass M₂. Figure 3-21 shows the response of the friction estimation and the error for both M₁ and M₂ on a larger scale.

Figure 3-19: Comparing friction with friction estimation for M₁, the error and M₁ displacement for $\alpha_1 = \alpha_2 = 1.2$, $\beta_1 = \beta_2 = 100$. 

Figure 3-20: Comparing friction with friction estimation for M₂, the error, and M₂ displacement for \( \alpha_1 = \alpha_2 = 1.2, \beta_1 = \beta_2 = 100. \)
Figure 3-21: Larger-scale response of friction estimation the error for both M1 and M2 for 
\[ \alpha_1 = \alpha_2 = 1.2, \beta_1 = \beta_2 = 100. \]

Figure 3-22 shows the response of the friction estimation and the error for both M1 and M2 for the design parameters of \( \alpha_1 = \alpha_2 = 1.5, \beta_1 = \beta_2 = 1 \) due to the change of the initial condition, namely \( Z_0 = [10 \ 0 \ 10 \ 0] \). Figure 3-23 shows the same figure but in a larger scale. Figure 3-24 shows the response of friction estimation and error for both M1 and M2 for \( \alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 1 \) design parameter. Figure 3-25 shows the response of friction estimation and the error for both M1 and M2 for \( \alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 1 \) on a larger scale. Figure 3-26 shows the response of friction estimation and the error for both
M₁ and M₂ for \( \alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10 \). Figure 3-27 shows the response of friction estimation and the error for both M₁ and M₂ for \( \alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10 \).

Figure 3-22: Response of friction estimation and error for both M₁ and M₂ for \( \alpha_1 = \alpha_2 = 1.5, \beta_1 = \beta_2 = 1 \).
Figure 3-23: Larger-scale response of friction estimation and error for both M1 and M2 for
\[ \alpha_1 = \alpha_2 = 1.5, \beta_1 = \beta_2 = 1. \]
Figure 3-24: Response of friction estimation and error for both $M_1$ and $M_2$ for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 1$. 
Figure 3-25: Larger-scale response of friction estimation and error for both $M_1$ and $M_2$ for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 1$. 
Figure 3-26: Response of friction estimation and error for both M₁ and M₂ for 
\( \alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10. \)
Figure 3-27: Response of friction estimation and error for both M1 and M2 for 
\[
\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10.
\]

Figure 3-28 shows the comparison of friction with friction estimation for M1, the error, and the M1 displacement for \( \alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 100 \), due to the change of applied force for M1 and M2 and initial conditions, namely \( F1=1, F2=0.8, Z_0=[5 0 10 0] \).

Figure 3-29 shows the comparison of friction with friction estimation for M2, the error, and the M2 displacement for \( \alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 100 \), due to the change of applied force for M1 and M2 and initial conditions, namely \( F1 = \sin(\pi), F2 = 1, X = [10 0 0 0] \).

Figures 3-30 and 3-31 show the comparison of friction with friction estimation for M1 and M2, the error, and the M1 displacement for \( \alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10 \), due to the
change of sinusoid applied force for $M_1$ and $M_2$ and initial conditions, namely

$$F_1 = \sin(\pi t), F_2 = 1, X = [10 \quad 0 \quad 0 \quad 0].$$

Figure 3-28: Comparing friction with friction estimation for $M_1$, the error, and the $M_1$ displacement for $\alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 100$ due to the change of applied force for $M_1$ and $M_2$ and initial conditions.
Figure 3-29: Comparing friction with friction estimation for M₂, the error, and the M₂ displacement for $\alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 100$, due to the change of applied force for M₁ and M₂ and initial conditions.
Figure 3-30: Comparing friction with friction estimation for M₁, the error, and the M₁ displacement for $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 10$ due to the change of sinusoid applied force for M₁ and M₂ and initial conditions.
3.3.1 Conclusion

Figures 3-19 through 3-27 show the response of the friction estimator for different values of design parameters $\alpha_1, \alpha_2, \beta_1,$ and $\beta_2$. As shown in the figures for valves with design parameters of $\alpha_1 = \alpha_2 = 1.2$ and $\beta_1 = \beta_2 = 100$, the error of friction estimation to the Coulomb coefficient of $a_1$ and $a_2$ has the minimum value. Consequently, $\alpha_1 = \alpha_2 = 1.2, \beta_1 = \beta_2 = 100$ have been chosen for the friction compensation function. Figures 3-28 through 3-31 show the performance of the response of the friction estimator under different conditions and input. From there figures, it can be concluded that the friction estimation function provides a good estimation of the Coulomb coefficient.
3.4 Obtaining the Best Valve for the Coefficient of Fiction Compensation

In this section, the program is run for the larger range of the coefficients of \( \alpha_1, \beta_1, \alpha_2, \) and \( \beta_2 \) to determine the best value of the coefficient and to keep the response of error low. Figure 3-32a shows the changing of error versus the changing of \( \alpha_1 \). Figure 3-32b shows the changing of error versus the changing of \( \beta_1 \). Figure 3-32c shows the changing of error versus the changing of \( \alpha_2 \). Figure 3-32d shows the changing of error versus the changing of \( \beta_2 \). Figure 3-32e shows the changing of error versus the changing of \( \alpha_1 \) and \( \beta_1 \). Figure 3-32f shows the changing of error versus the changing of \( \alpha_1 \) and \( \beta_1 \) on a larger scale. Figure 3-32g shows the changing of error versus the changing of \( \alpha_2, \beta_2 \). Figure 3-32h shows the changing of error versus the changing of \( \alpha_2, \beta_2 \) on a larger scale. The best values of \( \alpha_1, \beta_1, \alpha_2, \) and \( \beta_2 \) are when the error is minimum and can be obtained from Figures 3-32f and 3-32h. In this case, the valves have been chosen as follows: \( \alpha_1 = 5, \beta_1 = 1.5, \alpha_2 = 2.6, \) and \( \beta_2 = 3. \)
Figure 3-32a: Changing of error versus changing of $\alpha_1$.

Figure 3-32b: Changing of error versus changing of $\beta_1$. 

Figure 3-32c: Changing of error versus changing of $\alpha_2$.

Figure 3-32d: Changing of error versus changing of $\beta_2$. 
Figure 3-32e: Changing of error versus changing of $\alpha_1, \beta_1$.

Figure 3-32f: Changing of error versus changing of $\alpha_1, \beta_1$ on a larger scale.
Figure 3-32g: Changing of error versus changing of $\alpha_2, \beta_2$. 
3.5 Generalized Friction Compensation

In this section, the friction compensation for a two-degree system has been extended for an n-degree system. Figure 3-32 shows n degrees of freedom of a spring-mass-dashpot. The equation in section 3 for friction compensation on a two-degree freedom system could be extended to the n’th order system as

\[
\begin{align*}
\begin{array}{c}
F_1 & F_2 \\
X_1 & X_2 \\
F_{f1} & F_{f2} \\
C_1 & K_1 K_2 \\
M_1 & M_2 \\
\vdots & \vdots \\
F_{fn} & X_n \\
C_n & M_n \\
\end{array}
\end{align*}
\]

Figure 3-32h: Changing of error versus changing of $\alpha_2, \beta_2$ on a larger scale.

Figure 3-32: n-degree freedom of mass-spring-dashpot.
\[
\left( F_{f_1} \right) \left( F_{f_2} \right) \left( F_{f_n} \right) = \begin{pmatrix}
\hat{a}_1 & 0 & 0 & 0 & 0 \\
0 & \hat{a}_2 & 0 & 0 & 0 \\
0 & 0 & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \ddots & \ddots \\
0 & 0 & 0 & 0 & \hat{a}_n
\end{pmatrix}
\begin{pmatrix}
\text{Sgn}(Z_2) \\
\text{Sgn}(Z_4) \\
\text{Sgn}(Z_n)
\end{pmatrix}
\]

(3-42)

\[
\begin{pmatrix}
\hat{a}_1 \\
\hat{a}_2 \\
\vdots \\
\hat{a}_n
\end{pmatrix} = \begin{pmatrix}
\hat{\xi}_1 \\
\hat{\xi}_2 \\
\vdots \\
\hat{\xi}_n
\end{pmatrix} \begin{pmatrix}
\alpha_1 & 0 & 0 & 0 & 0 \\
0 & \alpha_2 & 0 & 0 & 0 \\
0 & 0 & \alpha_3 & 0 & 0 \\
0 & 0 & 0 & \alpha_n & 0
\end{pmatrix}
\begin{pmatrix}
\frac{Z_2}{\beta_1} \\
\frac{Z_4}{\beta_2} \\
\frac{Z_6}{\beta_3} \\
\frac{Z_n}{\beta_n}
\end{pmatrix}
\]

(3-43)

\[
\begin{pmatrix}
\hat{\xi}_1 \\
\hat{\xi}_2 \\
\vdots \\
\hat{\xi}_n
\end{pmatrix} = \begin{pmatrix}
\alpha_1 \beta_1 Z_2^{\beta_1-1} & 0 & 0 & 0 & 0 \\
0 & \alpha_2 \beta_2 Z_4^{\beta_2-1} & 0 & 0 & 0 \\
0 & 0 & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \ddots & \ddots \\
0 & 0 & 0 & 0 & \alpha_n \beta_n Z_n^{\beta_n-1}
\end{pmatrix}
\]

(3-44)

\[
\begin{pmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_n
\end{pmatrix} \begin{pmatrix}
F_{f_1}(Z_2, \hat{a}_1) \\
F_{f_2}(Z_4, \hat{a}_2) \\
\vdots \\
F_{f_n}(Z_n, \hat{a}_n)
\end{pmatrix} = \begin{pmatrix}
\text{Sgn}(Z_2) & 0 & 0 & 0 & 0 \\
0 & \text{Sgn}(Z_4) & 0 & 0 & 0 \\
0 & 0 & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \ddots & \ddots \\
0 & 0 & 0 & 0 & \text{Sgn}(Z_n)
\end{pmatrix}
\]
Eqns (3-45) to (3-48) represent friction compensation for the n’th order system of a spring-mass-dashpot.

3.6 Performance Analysis of Friction Compensation

In order to observe the performance of friction compensation, the amount of Coulomb friction estimated by the friction observer is fed back to the system, and the response of the system is measured by changing the initial state.
\[ F_{\text{new}1} = F_1 + \hat{F}_{f_1} \]  
(3-46)

\[ F_{\text{new}2} = F_2 + \hat{F}_{f_2} \]  
(3-47)

\[
\begin{pmatrix}
F_{\text{new}1} \\
F_{\text{new}2}
\end{pmatrix} =
\begin{pmatrix}
F_1 \\
F_2
\end{pmatrix} +
\begin{pmatrix}
\xi_1 \\
\xi_2
\end{pmatrix} -
\begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix}
\begin{pmatrix}
|Z_2|^{\beta_1} \\
|Z_4|^{\beta_2}
\end{pmatrix}
\cdot
\begin{pmatrix}
\text{Sgn}(Z_2) \\
\text{Sgn}(Z_4)
\end{pmatrix}
\]  
(3-48)

\[
\begin{pmatrix}
\dot{Z}_1 \\
\dot{Z}_2 \\
\dot{Z}_3 \\
\dot{Z}_4
\end{pmatrix} =
\begin{pmatrix}
0 & 1 & 0 & 0 \\
-\frac{1}{M_1}(K_1 + K_2) & 0 & \frac{K_2}{M_2} & 0 \\
0 & 0 & 1 & \frac{K_2}{M_2} \\
\frac{1}{M_2} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4
\end{pmatrix}
\]  
(3-49)

\[
\begin{pmatrix}
\dot{Z}_1 \\
\dot{Z}_2 \\
\dot{Z}_3 \\
\dot{Z}_4
\end{pmatrix} =
\begin{pmatrix}
0 & 1 & 0 & 0 \\
\frac{1}{M_1}(K_1 + K_2) & 0 & -\frac{K_2}{M_2} & 0 \\
0 & 0 & 1 & \frac{K_2}{M_2} \\
0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4
\end{pmatrix}
\]  
(3-50)
Figures 3-33 and 3-34 show the position and velocity response of the system with friction to compensation friction, which is compared to the same system without using any compensation system to overcome the friction. This system is excited due to a change in the initial conditions of the mass $M_1$, namely $F_1 = F_2 = 0, X = [10 \ 0 \ 0 \ 0]$. As shown, the amplitude of the position and velocity is limited due to the friction of the system. But the amplitude of the position and velocity in which the friction compensation is used has a tendency not to be limited by the friction of the system. Figure 3-35 shows these phenomena more clearly. As can be seen, the limit cycle for the system with friction is almost the same as for the non-friction system. Figure 3-36 compares the response of the three systems: non-friction, system with friction with friction compensation, and system with friction without friction compensation. As shown, the response of the system with friction compensation is almost identical to the response of the non-friction case.

\[
y = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4
\end{pmatrix}
\] (3-51)
Figure 3-33: Response of position and velocity $M_1$ of the friction system with friction compensation and friction system without friction compensation.
Figure 3-34: Response of position and velocity $M_2$ of the friction system with friction compensation and friction system without friction compensation.
Figure 3-35: Phase diagram for the friction system with friction compensation, friction system without friction compensation, and non-friction.
Figure 3-36: Comparison of the response of the friction system with friction compensation, friction system without friction compensation, and non-friction system due to the change of initial conditions.

Figure 3-37 shows the position and velocity response of a two degrees of freedom system with friction with friction compensation, compared to the same system without using any compensation system to overcome friction. This system is excited due to a change in applied force, namely \( F_1 = \sin(\pi t), F_2 = 1 \). As shown, in the case of the system with friction without using friction compensation, the amplitude of the position and velocity is limited due to the friction of the system. But the amplitude of the position and velocity in which the friction compensation is used has the tendency not to be limited by the friction of the system. Figure 3-38 more clearly shows these phenomena using a phase diagram. As shown, the limit cycle for the system with friction is almost the same as the non-friction system.
Figure 3-37: Comparison of the response of the friction system with friction compensation, friction system without friction compensation, and non-friction system due to the change of applied force.
Figure 3-38: Phase diagram showing response of the friction system with friction compensation, friction system without friction compensation, and non-friction.

3.7 Observer-Based Friction Compensation

3.7.1 Introduction

The observer structure is based on the assumption that the velocity upon which the friction depends can be measured directly, which is often, but not always, the case. It would be of interest to examine how the system would perform if an estimate  \( \hat{v} \) of the velocity, produced by another observer, were used in place of the true velocity. This section, discusses the performance of two classes of observers, namely observer based on pole placement and Kalman filter.
3.7.2 Observer-Based Pole Placement Friction Compensation

By considering the equation which is defined in section 3.2, the observer-based pole placement friction compensation is designed. The estimated state then will feed to the friction compensation to estimate the friction.

\[
\begin{align*}
\dot{Z} &= A \cdot Z + B \cdot u + E \\
y &= C \cdot Z
\end{align*}
\]

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\frac{1}{M_1} (K_1 + K_2) & 0 & K_2 & 0 \\
0 & 0 & 0 & 1 \\
\frac{K_2}{M_2} & 0 & -\frac{K_2}{M_2} & 0
\end{bmatrix},
B = \begin{bmatrix}
0 \\
1 \\
0 \\
0 \end{bmatrix}
\]

\[
E = \begin{bmatrix}
- F_{c1} \text{sgn}(x_1) + \begin{cases}
F_1 |_{x_1=0} & |F_1| < F_{s1} \\
F_1 \text{sgn}(F_1) & |F_1| \geq F_{s1}
\end{cases} \\
0 \\
- F_{c2} \text{sgn}(x_2) + \begin{cases}
F_2 |_{x_2=0} & |F_2| < F_{s2} \\
F_2 \text{sgn}(F_2) & |F_2| \geq F_{s2}
\end{cases}
\end{bmatrix},
C = [0 \ 0 \ 1 \ 0]
\]

\[
C' = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\begin{bmatrix}
\hat{a}_1 \\
\frac{\xi}{\xi_2} \\
\hat{a}_2 \\
\frac{\xi}{\xi_2}
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \\
0 \\
\alpha_2 \\
0
\end{bmatrix}
\begin{bmatrix}
|Z_2|^{\beta_1} \\
|Z_2|^{\beta_2}
\end{bmatrix}
\]

Assume that the state \( \hat{Z} \) of the dynamic model
This represents the state observer for the system. \( K_e \) is the state observer gain matrix, which can be determined so that matrix \( A-C*K_e \) will yield a set of desired eigenvalues. The method of choosing \( K_e \) is discussed in Palm’s textbook [49].

Figure 3-39 shows the block diagram of the nonlinear system and the observer. The velocity of masses \( M_1 \) and \( M_2 \) is estimated by the dynamics of the system and \( M_2 \) displacement. The friction of both systems is estimated through the friction compensation system.

Figure 3-40 compares the real state and the estimated state by an observer. As shown, the observer provides a good approximation of the real state. Figure 3-41 compares the responses of three systems due to the change in the initial condition, namely \( Z_0 = [10 \ 0 \ 0 \ 0] \): non-friction system, friction system, and friction system with observer-based friction compensation. As shown, the amplitude of the
position and velocity is limited due to the friction of the system in response to the friction system. But the amplitude of the position and velocity in which the friction compensation is used has a tendency not to be limited by the friction of the system with friction compensation.

Figure 3-40: Estimation of position and velocity based on the M₂ position response compared to the real state.

Figure 3-41 here shows that the limit cycle for the system with friction using friction compensation is almost the same as for the non-friction system. Figure 3-42 compares the three responses of three systems due to the change of applied force on M₁: non-friction system, friction system, and friction system with observer-based friction compensation. As shown, the amplitude of the position and velocity is limited due to friction of the
system in response to the friction system. But the amplitude of the position and velocity in which the friction compensation is used has a tendency not to be limited by the friction of the system.

Figure 3-41: Response of the non-friction, friction system, and friction system with observer-based friction compensation due to a change in mass $M_1$. 
Figure 3-42: Phase diagram showing the non-friction system, and friction system, friction system with observer-based friction compensation due to change of mass $M_1$.

Figure 3-43 shows the same response but for a longer period of time. As can be seen, the amplitude of the position and velocity is limited due to friction of the system in response to the friction system. But the amplitude of the position and velocity in which the friction compensation is used has the tendency not to be limited by the friction of the system. However, after an appreciable amount of time, the amplitude of the velocity and position will decrease in the system with friction compensation. The reason for this is that the observed state has already been affected by the friction of the system. Mathematically speaking, the matrix $E$ will interfere in observing the state. As a result of this
interference, the state will tend to decrease even in the presence of friction compensation. However, this amplitude reduction will be in effect long time after the excitation.

![Graphs showing responses of different systems](image)

Figure 3-43: Response of the non-friction system, friction system, and friction system with observer-based friction compensation due to change of mass M₁ after a period of time.

Figures 3-43 and 3-45 show these phenomena more clearly. As can be seen, the limit cycle for the friction system is almost the same as the non-friction system. Figure 3-45 compares the response of three systems: non-friction system, friction system with friction compensation, and friction system without friction compensation. As shown, the response of the system with friction compensation is almost identical to the response of the non-friction system.
Figure 3-44: Phase diagram showing response of the non-friction system, friction system, and friction system with observer-based friction compensation due to change of mass $M_1$ after a period of time.
Figure 3-45: Larger-scale phase diagram showing response of the non-friction system, friction system, and friction system with observer-based friction compensation due to change of mass $M_1$ after a period of time.
3.7.3 Observer-Based Friction Compensation feedback Control System

Figure 3-46 shows the observer-based friction compensation feedback control system with the integral action in the feed forward path.

From this diagram, the system of equations is obtained as

\[
\begin{align*}
\dot{Z} &= A \cdot Z + B \cdot u + E, \\
y &= C \cdot Z
\end{align*}
\]

where

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\frac{1}{M_1} (K_1 + K_2) & 0 & \frac{K_2}{M_1} & 0 \\
0 & 0 & 0 & 1 \\
\frac{K_2}{M_2} & 0 & -\frac{K_2}{M_2} & 0
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0 \\
\frac{1}{M_1} & 0 \\
0 & 0 \\
0 & \frac{1}{M_2}
\end{bmatrix},
\]

\[
E = \begin{bmatrix}
0 \\
- F_{c_1} \text{sgn}(x_1) + \begin{cases} 
F_1 & x_1 < 0 \\
F_1 \text{sgn}(F_1) & x_1 \geq 0
\end{cases} \\
0 \\
- F_{c_2} \text{sgn}(x_2) + \begin{cases} 
F_2 & x_2 < 0 \\
F_2 \text{sgn}(F_2) & x_2 \geq 0
\end{cases}
\end{bmatrix}
\]

where \( F_1 \) and \( F_2 \) are the friction forces.
\[ C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad C' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[
\begin{pmatrix}
\dot{Z}_1 \\
\dot{Z}_2 \\
\dot{Z}_3 \\
\dot{Z}_4
\end{pmatrix}
= \begin{pmatrix}
0 & 0 & 0 \\
\frac{1}{M_1} (K_1 + K_2) & 0 & 0 \\
0 & 0 & 0 \\
\frac{K_2}{M_2} & 0 & -\frac{K_3}{M_2}
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 \\
0 \\
0 \\
\frac{1}{M_2}
\end{pmatrix}
\begin{pmatrix}
F_1 \\
F_2
\end{pmatrix}
+ \begin{pmatrix}
0 \\
0 \\
0 \\
\frac{1}{M_2}
\end{pmatrix}
\begin{pmatrix}
\dot{Z}_1 \\
\dot{Z}_2 \\
\dot{Z}_3 \\
\dot{Z}_4
\end{pmatrix}
\]

\[
\begin{pmatrix}
\dot{\xi}_1 \\
\dot{\xi}_2
\end{pmatrix}
= \begin{pmatrix}
\alpha \beta Z_1^{p-1} \\
\alpha \beta Z_1^{p-1}
\end{pmatrix}
\begin{pmatrix}
\frac{K_1 + K_2}{M_1} \\
\frac{K_2}{M_2} - \frac{K_3}{M_2}
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
Z_3
\end{pmatrix}
+ \begin{pmatrix}
1 \\
0
\end{pmatrix}
\begin{pmatrix}
F_1 \\
F_2
\end{pmatrix}
\begin{pmatrix}
\dot{Z}_1 \\
\dot{Z}_2 \\
\dot{Z}_3 \\
\dot{Z}_4
\end{pmatrix}
\]

\[
\begin{pmatrix}
\dot{a}_1 \\
\dot{a}_2
\end{pmatrix}
= \begin{pmatrix}
\dot{\xi}_1 \\
\dot{\xi}_2
\end{pmatrix}
- \begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix}
\begin{pmatrix}
Z_2^{p_1} \\
Z_4^{p_2}
\end{pmatrix}
\begin{pmatrix}
\dot{F}_{f_1} \\
\dot{F}_{f_2}
\end{pmatrix}
\]

\[
\hat{Z} = A \hat{Z} + Bu + K_c (y - C' \hat{Z})
\]

where \( u \) is the control signal and is written as
\[ u = -KZ + K_1\zeta + \hat{F}_1 + \hat{F}_2 \quad (3-53) \]

\[ \zeta = r - y = r - CZ \quad (3-54) \]

where \( y \) is the output signal, \( \zeta \) is the output of the integrator, and \( r \) is the reference input signal.

Having added one more state to the system, the new state equation will be as

\[
\begin{bmatrix}
\dot{Z} \\
\dot{\zeta}
\end{bmatrix}
= \begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
Z \\
\zeta
\end{bmatrix}
+ \begin{bmatrix}
B \\
0
\end{bmatrix}
\cdot u(t) + E(z) + \begin{bmatrix}
0 \\
1
\end{bmatrix}
\cdot r(t) \quad (3-55)
\]

It is possible to design an asymptotically stable system such that \( Z(\infty), \zeta(\infty) \), and \( u(\infty) \) approach a constant value, respectively. Then, at steady-state \( \dot{\zeta}(t) = 0 \), the following is obtained: \( y(\infty) = r \).

At steady-state,

\[
\begin{bmatrix}
\dot{Z}(\infty) \\
\dot{\zeta}(\infty)
\end{bmatrix}
= \begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
Z(\infty) \\
\zeta(\infty)
\end{bmatrix}
+ \begin{bmatrix}
B \\
0
\end{bmatrix}
\cdot u(\infty) + \begin{bmatrix}
0 \\
1
\end{bmatrix}
\cdot r(\infty) + E(z) \quad (3-56)
\]

Noting that \( r(t) \) is a step input, then \( r(\infty) = r(t) = r \) for \( t > 0 \). By subtracting equations

\[
\begin{bmatrix}
\dot{Z}(t) - \dot{Z}(\infty) \\
\dot{\zeta}(t) - \dot{\zeta}(\infty)
\end{bmatrix}
= \begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
Z(t) - Z(\infty) \\
\zeta(t) - \zeta(\infty)
\end{bmatrix}
+ \begin{bmatrix}
B \\
0
\end{bmatrix}
\cdot [u(t) - u(\infty)] + E(z) \quad (3-57)
\]

by defining:

\[ z(t) - z(\infty) = z_e(t) \]

\[ \zeta(t) - \zeta(\infty) = \zeta_e(t) \]

\[ u(t) - u(\infty) = u_e(t) \]

then Eqn 3-56 can be written as
where

\[ u_e(t) = -KZ_e(t) + K_I \zeta_e(t) + \hat{F}_1 + \hat{F}_2 \]  

(3-58)

defining a new (n+1) order error vector \( e(t) \) by

\[ e(t) = \begin{bmatrix} z_e(t) \\ \zeta_e(t) \end{bmatrix} \]

(3-59)

where

\[ A' = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad B' = \begin{bmatrix} B \\ 0 \end{bmatrix} \]

and Eqn (3-58) becomes

\[ u_e = -K'e + \hat{F}_1 + \hat{F}_2 \]  

(3-60)

where \( K' = [K \mid -K_I] \)

Eqns (3-62) and (3-63) describe the dynamics of the (n+1)th-order regulator system. If the system defined by Eqn (3-62) is completely state controllable, then, by specifying the desired characteristic equation for the system, matrix \( K' \) can be determined by the pole-placement technique.

The state-error equation can be obtained by substituting Eqn (3-62) into Eqn (3-59)
\[ e = (A' - B'K')e + B'(\hat{F}_1 + \hat{F}_2) + E_e(z_e) \]  

(3-61)

If the desired eigenvalues of matrix \( A' - B'K' \) (that is, the desired closed-loop poles) are specified, then the state-feedback gain matrix \( K \) and the integral gain constant \( K_I \) can be determined.

Figure 3-47 shows the \( M_2 \) response to the command signal for \( M_1 \) due to the change of initial condition of \( M_1 \), namely \( x_i, d = 0, F_i = 0, Z_o = [1 \ 0 \ 0 \ 0] \) in a non-friction system, a friction system without using friction compensation, and a friction system with friction compensation. As shown the nonlinear system without friction converges to zero without exhibiting any limit-cycle behavior. However, the same control system responds totally different in the system affected by friction. As shown, in this figure, the system exhibits the limit-cycle behavior. As a result, the steady-state error fluctuates over the zero state. The position control-based friction compensation shows lower amplitude of error in steady-state for the same input. Therefore, the position control-based friction compensation provides a better response. Figure 3-48 shows the response of position and velocity of masses \( M_1 \) and \( M_2 \) for the same input. Figures 3-49 and 3-50 show the phase plane response of masses \( M_1 \) and \( M_2 \) for the same response. Figure 3-50 is a larger scale of Figure 3-49. As shown, the limit-cycle diameter is much lower for the control system based on the friction compensation method.
Figure 3-47: Response of $M_2$ due to the change of $M_1$ displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation.
Figure 3-48: Response of position and velocity of masses $M_1$ and $M_2$ due to the change of $M_1$ displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation.
Figure 3-49: Phase plane curve of masses $M_1$ and $M_2$ due to the change of $M_1$ displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation.
Figure 3-50: Larger-scale curve of masses $M_1$ and $M_2$ due to the change of $M_1$ displacement in a non-friction system, a friction system with friction compensation, and a friction system without friction compensation.

Figure 3-51 shows the $M_2$ response to the $M_1$ command signal in zero initial condition for the $x_{d1} = 1, F_2 = 0, Z_0 = [0 \ 0 \ 0 \ 0]$ in a non-friction system, a friction system without using friction compensation, and a friction system with friction compensation. Figure 3-52 shows the response of position and velocity of masses $M_1$ and $M_2$ for the same system shown in Figure 3-51. Figures 3-53 and 3-54 show the phase plane of the same response on a larger scale. As shown in Figure 3-53, the response of the control system that is designed based on linear values provides a rather high transient
response, compared to the control system based on friction compensation. The time delay of the response and the amplitude of the fluctuation response also decrease.

Figure 3-51: $M_2$ response to $M_1$ command signal in a non-friction system, a friction system without friction compensation, and a friction system with friction compensation.
Figure 3-52: Response of position and velocity of masses $M_1$ and $M_2$ to the command signal in a non-friction system, a friction system without friction compensation, and a friction system with friction compensation.
Figure 3-53: Phase plane response of masses $M_1$ and $M_2$ to the command signal in a non-friction system, a friction system without friction compensation, and a friction system with friction compensation.
3.8 Friction Compensation in Stochastic System

Consider the set of equations in section 3.5 in state-space format

\[
\dot{x}(t) = Ax(t) + Bu(t) + E(x), \quad x(0) \text{ given} \tag{3-62}
\]

Design a feedback control law that minimizes the cost function

\[
J(t) = \frac{1}{2} x^T(T)Sx(T) + \frac{1}{2} \int_0^T [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \tag{3-63}
\]

S is a weighting matrix on the final state, Q is the weighting matrix on the states, R is the weighting matrix on the inputs, and J(t) is the scalar value.
The weighing functions \( \frac{1}{2} x^T(t)Qx(t) \) are defined as
\[
x = [x_1 \cdots x_n]^T \quad \text{and} \quad Q = \begin{bmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{n1} & \cdots & q_{nn} \end{bmatrix}
\] (3-64)

Complete the square yields with off-diagonal terms zero, as in
\[
\frac{1}{2} x^T(t)Qx(t) = \frac{1}{2} \left[ q_{11}x_1^2 + q_{22}x_2^2 + \cdots + q_{nn}x_n^2 \right]
\] (3-65)

A scalar function has a local minimum at \( x^* \) iff
\[
J(x^* + \delta x) \geq J(x^*) \quad \text{for} \ \delta x \ \text{sufficiently small, which can be re-written as}
\]
\[
\Delta J(x^*, \delta x) = J(x^* + \delta x) - J(x^*)
\]
\[
= \frac{dJ(x^*)}{dx} \delta x + \frac{1}{2} \frac{d^2J(x^*)}{dx^2} \delta x^2 + \text{H.O.T.} \geq 0
\] (3-66)

A stationary point occurs when \( \frac{dJ(x^*)}{dx} = 0 \), which is a minimum when \( \frac{d^2J(x^*)}{dx^2} > 0 \).

The last condition is ensured via the quadratic cost function. Using the constraint \( \dot{x}(t) - Ax(t) - Bu(t) = 0 \) (state equation), and a Lagrange multiplier \( \lambda \) to append the constraint to \( J(t) \), we have
\[
J(t) = \frac{1}{2} x^T(T)Sx(T) + \frac{1}{2} \int_0^T \left[ x^T(t)Qx(t) + u^T(t)Ru(t) + \lambda^T(\dot{x} - Ax - Bu) \right] dt
\] (3-67)

Using \( J_a(t) \) and from equation 3-5 we have
\[
\Delta J(x, u, \lambda, \delta x, \delta u, \delta \lambda) = J(x + \delta x, u + \delta u, \lambda + \delta \lambda) - J(x, u, \lambda) = 0
\] (3-68)

By computing Eqn 3-67 and setting the coefficients \( \delta x, \delta u, \text{and} \delta \lambda \) to zero, the optimal control solution can be calculated. The solution to the LQR problem is given by
\[ u^*(t) = -R^{-1}PB^Tx(t) = -K(t)x(t) \quad (3-69) \]

where

\[
\dot{P}(t) = P(t)A + A^TP(t) - P(t)B^TR^{-1}BP(t) + Q
\]

\[ P(T) = S \quad (3-70) \]

In infinite time solution, \( \dot{P}(t) \) is zero, so in this case we have

\[
J(t) = \frac{1}{2} \int_0^T \left[ x^T(t)Qx(t) + u^T(t)Ru(t) \right] dt
\]

\[ 0 = PA + A^TP - PB^TR^{-1}BP + Q \quad (3-71) \]

Therefore the optimal control gain is constant as Figure 3-55.

The objective of this step of the project is to design an observer based on the Kalman filter method to see the response of the system. In this step, the observer, which was designed by the pole placement method, is replaced by a Kalman filter in order to make the system more reliable under the disturbance input [50].

3.8.1 Friction Compensation Based on Kalman Filter

The Kalman filter is an optimal estimator of the state, where optimal is defined in terms of minimizing the mean square estimation error. The Kalman filter has been utilized in an extremely wide range of applications, both as a signal processing tool and as an integral component in the linear quadratic Gaussian controller [44].
The Kalman filter estimates the state of a plant given a set of known inputs and a set of measurements. The plant is described by the state model of section 3.2 and the plant noise

\[
\dot{z}(t) = A z(t) + B_u u(t) + E(z) + B_w w(t) \quad (3-72)
\]

\[
m(t) = C_m z(t) + v(t) \quad (3-73)
\]

which is driven by both a known, deterministic input \(u(t)\) and an unknown random input \(w(t)\) called plant noise. The measurement from the plant \(m(t)\) is corrupted by a random measurement noise \(v(t)\). The plant and measurement noises are assumed to be white noise vectors with spectral densities \(S_w\) and \(S_v\), respectively:

\[
E[w(t)w^T(t + \tau)] = S_w \delta(\tau) \quad (3-74)
\]

\[
E[v(t)v^T(t + \tau)] = S_v \delta(\tau) \quad (3-75)
\]

In this case, the plant noise and the measurement noise are considered to be white noise. They are assumed to be uncorrelated since they are typically generated by differing phenomena.

The Kalman filter generates the linear estimate of the plant state that minimizes the mean square estimation error. The estimated state is then fed to the friction observer in order to estimate the friction as shown on Figure 3-56 and Figure 3-57.
\[ J = E\left[ \left( z(t) - \hat{z}(t) \right)^T \left( z(t) - \hat{z}(t) \right) \right] = \sum E\left[ \left( z_i(t) - \hat{z}_i(t) \right)^2 \right] \quad (3-76) \]

Figure 3-56: Kalman filter structure.

Figure 3-57: Kalman-based friction compensation

Using the state Eqns (3-71) and (3-72), the equation of the Kalman filter is
\[ z(t) = A \hat{z}(t) + B_w u(t) + G(t)[m(t) - C_m \hat{z}(t)] \quad (3-77) \]

\[ G(t) = \sum_{e} C_m^T S_v^{-1} \quad (3-78) \]

\[ \hat{\Sigma}_e(t) = \sum_{e}(t) A^T + A \sum_{e}(t) + B_w S_w B_w^T - \sum_{e}(t) C_m^T S_v^{-1} C_m \sum_{e}(t) \quad (3-79) \]

\[
A = \begin{pmatrix}
0 & 1 & 0 & 0 \\
-\frac{1}{M_1}(K_1 + K_2) & 0 & K_2 & 0 \\
0 & 0 & 1 & 0 \\
K_2 & M_2 & -\frac{K_2}{M_2} & 0 \\
\end{pmatrix},
B = \begin{pmatrix}
0 \\
\frac{1}{M_1} \\
0 \\
0 \\
1 \\
\end{pmatrix}
\]

\[
E = \begin{pmatrix}
0 \\
-F_{c_1} \text{sgn}(x_1) + \begin{cases}
F_1 |_{x_1 = 0} & |F_1| < F_{s_1} \\
F_{s_1} \text{sgn}(F_1) & |F_1| \geq F_{s_1}
\end{cases} \\
-F_{c_2} \text{sgn}(x_2) + \begin{cases}
F_2 |_{x_2 = 0} & |F_2| < F_{s_2} \\
F_{s_2} \text{sgn}(F_2) & |F_2| \geq F_{s_2}
\end{cases}
\end{pmatrix},
C_m = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}
\]

\[
C' = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\begin{pmatrix}
\xi_1 \\
\xi_2 \\
\end{pmatrix} = \begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix}
\]

\[
\begin{pmatrix}
F_{f_1} \\
F_{f_2}
\end{pmatrix} = \begin{pmatrix} 0 & \alpha_2 \end{pmatrix} \left[ \begin{pmatrix} S_{g_2} \\
S_{g_4} \end{pmatrix} \right]
\]

The system is simulated based on the following values:

\[ S_w = 10 \]

\[ S_v = 1 \]

\[ G = [\begin{pmatrix} -0.0992 & -3.5804 & 1.8922 & 1.7902 \end{pmatrix}] \]
Figure 3-58 shows the estimation of the position and velocity state-based Kalman filter, which is compared to the real state due to the change of the \( M_1 \) position response, namely \( F_1 = F_2 = 0, Z_0 = [10 \ 0 \ 0 \ 0] \). Figure 3-58 is a larger scale of Figure 3-57. As shown, the Kalman filter provides a good estimation of states of the system under the input noise.

Figure 3-59 shows the response of a non-friction system, friction system, and friction system with Kalman-based friction compensation due to a change of mass \( M_1 \) displacement. Figure 3-61 shows the phase plane response of Figure 3-60. Figure 3-62 shows the system phase plane response on a larger scale. As can be seen, the limit cycle for the system with friction is almost the same as the non-friction system.

\[
\Sigma_e = \begin{bmatrix}
8.58 & -4.9951 & -0.0992 & 3.3926 \\
-4.995 & 17.6319 & -3.5804 & -8.8656 \\
-0.0992 & -3.5804 & 1.8922 & 1.7902 \\
3.3926 & -8.8656 & 1.7902 & 5.3789
\end{bmatrix}
\]
Figure 3-58: Kalman filter estimation of position and velocity based on the M₁ position response compared to the real state.
Observer Performance

- Displacement (m)
- Times (s)

- V1 (m/s)
- Times (s)

- Displacement2 (m)
- Times (s)

- V2 (m/s)
- Times (s)

Figure 3-59: Larger-scale Kalman filter estimation of position and velocity based on the M1 position response compared to the real state.
Figure 3-60: Response of a non-friction system, friction system, and friction system with Kalman-based friction compensation due to change of mass $M_1$. 
Figure 3-61: Phase plane of the non-friction system, friction system, and friction system with Kalman-based friction compensation due to change of mass $M_1$.

Figure 3-62: Larger-scale phase plane of the non-friction system, friction system, and friction system with Kalman-based friction compensation due to change of mass $M_1$.

The system inputs are as follows: $F_1 = \sin(\pi t)$, $F_2 = 1$, $Z_0 = [0 \quad 0 \quad 0 \quad 0]$. 

125
Figure 3-63 shows the Kalman filter estimation of the position and velocity due to the above input. Figure 3-64 shows the response of the non-friction system, friction system, and friction system with Kalman-based friction compensation due to a change of sinusoid applied force on mass $M_1$ and constant force on mass $M_2$.

![Figure 3-63: Kalman filter estimation of the position and velocity due to the change of sinusoid applied force on mass $M_1$ and constant force on mass $M_2$.](image-url)
As shown in Figures 3-63 and 3-64, the amplitude of the position and velocity is limited due to the friction of the system. But the amplitude of the position and velocity in which the friction compensation is used has a tendency not to be limited by the friction of the system with friction compensation and is more likely to follow the linear response.
4.1 Introduction

This chapter proposes an analysis of the control problem of the fuel valve with the PMS motor drive and its actuation system. Chapter 2 presented all the mathematical equations for modeling the nonlinear actuation system. In Appendix A, the friction model of the actuation system was applied to the DC motor to observe its functioning. In addition to that, a new method of compensating friction based on friction compensation gain is presented in Appendix A. Chapter 3 presented the theory of observer-based friction compensation for a multibody system, and the implementation of a control position based on friction compensation in a two-degree system was shown. After covering the necessary background for the control system of the PMS motor, this chapter will present a simulation of the dynamic system of a PMS motor and the actuation system under friction. Also, this chapter proposes the parallel observer-based friction compensation for improving the position control of the fuel valve with the PMS motor driver.
4.2 Theoretical Background of the Control System Design

4.2.1 Open Loop Control

Figure 4-1 shows a simple constant voltage-to-frequency control using a pre-programmed sinusoidal voltage PWM algorithm that can provide speed control for applications such as pumps and fans, which do not require a fast dynamic response.

![Figure 4-1: Simplified block diagram of an open loop voltage-to-frequency control of an interior or buried PMS motor.](image)

Thus, PMS motors can replace induction motors in some variable-speed drive applications to improve the drive efficiency with minimal changes to the control electronics.

4.3 Simulation of the Motor Model

Using equations from Chapter 2, the system is simulated in Matlab software. As discussed previously, this model is based on the separation of the mechanical and electrical components.

4.3.1 Mathematical Modeling of the PMS Motor

As explained in Chapter 2, state Eqns (2-23) and (2-29) for transforming the direct to quadrature-axis (dq0 transformation) and Eqn (2-28) for finding the electromagnetic torque are used to model the dynamic of the PMS motor. Namely,

\[
\frac{d}{dt}i_d = \frac{1}{L_{sd}}v_d - \frac{R}{L_{sd}}i_d + \frac{L_{sq}}{L_{sd}} p\omega_i_{sq}
\]
\[
\frac{d}{dt} i_q = \frac{1}{L_{sq}} v_q - \frac{R}{L_{sq}} i_q - \frac{L_{sd}}{L_{sq}} p \omega i_{sd} - \frac{\omega \psi_f \cdot p}{L_{sq}}
\]

\[
T_d = p \cdot \frac{P_{elm}}{\omega} = \frac{3}{2} \cdot p \left[ \psi_f + (L_{sd} - L_{sq}) i_d \right] \cdot i_q \quad \text{Newton}
\]

\[
\begin{bmatrix}
i_d \\
i_q \\
0
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos(wt) & \cos(wt - \frac{2\pi}{3}) & \cos(wt + \frac{2\pi}{3}) \\
-\sin(wt) & -\sin(wt - \frac{2\pi}{3}) & -\sin(wt + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \cdot \begin{bmatrix}
i_{aA} \\
i_{aB} \\
i_{aC}
\end{bmatrix}
\]

\[
\begin{bmatrix}
i_{aA} \\
i_{aB} \\
i_{aC}
\end{bmatrix} = \begin{bmatrix}
\cos(wt) & -\sin(wt) & 1 \\
\cos(wt - \frac{2\pi}{3}) & -\sin(wt - \frac{2\pi}{3}) & 1 \\
\cos(wt + \frac{2\pi}{3}) & -\sin(wt + \frac{2\pi}{3}) & 1
\end{bmatrix} \cdot \begin{bmatrix}
i_d \\
i_q \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
v_d \\
v_q \\
0
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos(wt) & \cos(wt - \frac{2\pi}{3}) & \cos(wt + \frac{2\pi}{3}) \\
-\sin(wt) & -\sin(wt - \frac{2\pi}{3}) & -\sin(wt + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \cdot \begin{bmatrix}
v_{aA} \\
v_{aB} \\
v_{aC}
\end{bmatrix}
\]

### 4.3.2 Mathematical Model of the Actuation System

Figures 4-2 and 4-3 show the actuation system from motor to fuel valve. This system is refiured to Figure 4-4 for modeling purposes. In Figure 4-5 the simulation of the model of a three-phase PMS motor with actuator is completed, and the model is simulated by a three-phase sinusoidal voltage with a 60° degree phase difference, as shown in Figure 4-6.
Figure 4-2: Actuation system of the fuel valve.

Figure 4-3: Schematic actuation system of the fuel valve.

Figure 4-4: Simplified actuation system of the fuel valve.
Figure 4-5: Model of three-phase PMS motor with actuator.

Figure 4-6: Three-phase voltage \((V_a, V_b, V_c)\) and current \((I_a, I_b, I_c)\).
The following equations are obtained for general nonlinear equations for the system in Chapter 2.

\[ T_m - T_1 = J_m \ddot{\Theta}_m + B_m \dot{\Theta}_m + T_{fc1} \operatorname{sgn}(\omega_1) \]
\[ + \begin{cases} T_m - T_1 & \text{if } \omega_m = 0, \text{and } |T_m - T_1| < T_{fc1} \\ T_{fc1} \operatorname{sgn}(T_m - T_1) & \text{if } \omega_m = 0, \text{and } |T_m - T_1| \geq T_{fc1} \end{cases} \tag{4-1} \]

\[ T_1 = \frac{N_1}{N_2} T_2 \]

\[ T_2 = K_r (\Theta_2 - \Theta_L) \]

\[ T_2 - T_{fcL} \operatorname{sgn}(\omega_L) + \begin{cases} T_2 & \text{if } \omega_m = 0, \text{and } |T_2| < T_{fcL} \\ T_{fcL} \operatorname{sgn}(T_2) & \text{if } \omega_m = 0, \text{and } |T_2| \geq T_{fcL} \end{cases} = J_L \ddot{\Theta}_L \]

\[ J_m \approx J_{Motor} + J_{Gear} \]

Using the following values, the results are modeled in Figures 4-6 to Figure 4-9.

\[ \frac{N_1}{N_2} = .0073 \]

\[ K_r = 48019 , \ B = 48.019 \quad B_1 = 2.6 \times 10^{-5} , \ J_m = 6.93 \times 10^{-6} \]

\[ T_{fc1} = 0.05 , \ T_{cl} = 0.05 , \ T_{fcL} = 0.4 , \ T_{cl} = 0.4 \]

Figure 4-7 shows a comparison between direct and quadrature currents in friction and non-friction-shafts. Figures 4-8 and 4-9 compare the velocity, position, and torque response in high-speed and low-speed shafts in friction and non-friction cases. In the non-friction case, the entire plot has more oscillation compared to the non-friction case. In order to simulate the system, a mathematical equation of the system is needed. In the
following procedure, all the mathematical equations are driven to obtain the non-linear model of the PMS motor with friction and gear box with flexible shaft.

Figure 4-7: Comparison of direct and quadrature currents in the case of friction and non-friction shafts.
Figure 4-8: Velocity and position responses in high-speed and low-speed shafts in friction and non-friction cases.

Figure 4-9: High-speed and low-speed torque in friction and non-friction cases.
4.4 Simulation of the Closed Loop

Knowing the dynamic behavior of the system as explained from Chapter 2, the following procedures are proposed in this research in order to analysis the control problem of the actuator:

1) Simulate the dynamic response of the PMS motor by MATLAB.
2) Discuss the theory of the control system.
3) Design a new servomechanism based on friction compensation theory to meet the needs of desired specifications of the current and position specifications.
4) Simulate the whole process including the controller (current control and position control) and the nonlinear model for dynamic response.
5) Investigate the problem of the limit cycle and the effective control factor of the limit cycle in this type of valve.
6) Investigate the effect of flexibility of the shaft in the limit cycle in the new design.
7) Design new friction compensation in order to estimate the friction and cancel its effect.

The objective of the controller is to do the following:
1) Reduce the effects of parameter variations (such as flexibility of the shaft in the coupling joint, backlash phenomena, saturation, and friction) and produce a reasonable behavior when the fiction is high.
2) Reduce the effects of disturbance inputs.
3) Improve transient response characteristics.
4) Reduce steady-state errors.
To compensate for the effect of friction, a unique model was used. This model consisted of two parallel nonlinear model-based friction compensations (based on information in Chapter 3) to predict friction and then compensate for it. Such a design is not only capable of predicting friction but is good enough for predicting the friction in coupling joints and gearboxes. The idea of using a two-parallel friction compensation is based on the nature of the model, which consisted of two shafts having two separated friction areas. A state observer (for z) was designed, and friction compensation was performed on the basis of this observation. Sections 4.4.1 to 4.4.4 cover the basics of the power circuit, current controller, and position controller of the PMS motor, and section 4.4.5 presents the response of the closed-loop system based on the friction compensation design. Section 4.5 is conclusion sections, and shows the improvement of the system over the conventional design method.

### 4.4.1 High-Performance Closed-Loop Control

To achieve high-performance motion control with a sinusoidal PMS motor, a rotor position sensor is typically required. Depending on the specific sinusoidal PMS motor drive performance, an absolute encoder or resolver providing an equivalent digital resolution of 6 bits per electrical cycle (5.6° elec.) or higher is usually required. Another condition for achieving high-performance motion control is high-quality phase-current control [40].

One of the possible approaches is through vector control, as shown in Figure 4-10 [40]. The incoming torque command $T^*_{d}$ is mapped to commands for $i^{*}_{ad}$, and $i^{*}_{aq}$ current components according to Eqn (2-27), where $\psi_{f} = N_i \Phi_f$ is the PMS flux linkage.
amplitude, and $L_{sd}$ and $L_{sq}$ are synchronous inductances under conditions of alignment with the rotor d- and q-axes, respectively. The current commands in the rotor d-q reference frame (DC quantities for a constant torque command) are then transformed into the instantaneous sinusoidal current commands for the individual stator phases $i_{ad}^*$, $i_{ab}^*$, and $i_{ac}^*$ using the rotor angle feedback and the basic inverse vector rotation equation. Current regulators for each of the three stator current phases then operate to excite the phase windings with the desired current amplitudes.

The most common means of mapping the torque command $T_d^*$ into values for $i_{ad}^*$ and $i_{aq}^*$ is to set a constraint of maximum torque-to-current operation, which is nearly equivalent to maximizing operating efficiency [40].

4.4.2 Power Circuit

The power circuit in a PMS motor drive can be classified according to the switching device, the duration of gating, the circuit topology, and the DC link control methods. The choice of the power-switching device is largely a matter of the kVA rating of the drive. The most common power circuit is the three-phase transistor bridge voltage source.
inverter, which is appropriate for the kVA rating of many PMS motor drives. It can be used in PWM mode with a current feedback loop to regulate motor currents in high-performance drives, and it can make a smooth transition to a six-step mode to operate in the voltage-limited high-speed regime [40].

Figure 4-11 illustrates the basic power circuit topology of the voltage source inverter. Only the main power handling devices are shown. The modern strategy for controlling the AC output of such power electronic converters is the technique known as pulse-width modulation (PWM), which varies the duty cycle (or mark-space ratio) of the converter switch(es) at a high switching frequency to achieve a target average low-frequency output voltage or current. In principle, all modulation schemes aim to create trains of switched pulses, which have the same fundamental volt_second average (i.e., the integral of the waveform over time) as a target reference waveform at any instant.

![Figure 4-11: Basic circuit topology of pulse-width modulated inverter drive [40].](image)

The major difficulty with these trains of switched pulses is that they also contain unwanted harmonic components that should be minimized. Three main techniques for PWM are as follows:

1. Switching at the intersection of a target reference waveform and a high-frequency triangular carrier (double-edged naturally sampled sine-triangle PWM).
Figure 4-12: Control principle of naturally sampled PWM showing one of three phase legs [40].

2. Switching at the intersection between a regularly sampled reference waveform and a high frequency triangular carrier (double-edged regular sampled sine-triangle PWM).

Figure 4-13: (a) Naturally sampled PWM and (b) symmetrically sampled PWM.

3. Switching so that the amplitude and phase of the target reference expressed as a vector is the same as the integrated area of the converter switched output over the carrier interval (space vector PWM).
4.4.3 Model for the Current Controller

It has gradually been recognized that field-oriented control allows speed-loop bandwidths far exceeding that of the DC motor (100 Hertz or more), thus making induction motor servos the device favored for demanding applications. However, such a bandwidth can only be achieved with careful tuning of the current regulator, which serves to overcome the stator transient time constant. Current regulators remain a rich area of research. However, the present methods can be categorized generally as follows:

- Sine-Triangle Current Regulation
- Hysteresis Current Regulation

The two types of regulators are illustrated in Figure 4-15 [40].
4.4.4 Position Control Based on Observer-Based Friction Compensation

In high precision positioning applications, friction compensation plays an important role in tracking accuracy requirements. One of the issues of this thesis is to present a new method of compensation using two parallel observer-based friction compensation inside an integral control system.

As shown in Chapter 3, friction compensation would be a good idea applied to systems dealing with high fluctuation in steady-state and transient response as a result of high friction in the fuel valve system. In this research, application of the new observer-based friction compensation derived in Chapter 3 is proposed for the control position of the actuator.

By analyzing, the inner electrical torque, outer motion of the control system, and static and kinetic friction of the high-speed and low-speed shafts, a friction observer of the system is designed. As shown, the nonlinear mechanical friction causes a stick-slip response in the system.
Defining the state variables \( Z_1, Z_2, Z_3, \) and \( Z_4 \) and the friction compensation state variable \( \xi_1 \) and \( \xi_2 \), an equation for the closed loop using integral action with pole placement is developed. The observer-based friction compensation used in Chapter 3 (for the system of two-degree freedom mass) will be applied to this system, namely

\[
Z_1 = \Theta_m, Z_2 = \omega_m, Z_3 = \Theta_L, Z_4 = \omega_L
\]

The above equation is reconfigured to the following matrix:
Figure 4-16 shows the servo system that is designed based on the parallel friction compensation method. Using the previous method for designing a position control based on friction compensation, the following control gain is obtained to meet the specification of the fuel valve system:

\[
K = \begin{bmatrix}
0.044197 & 0.94512 & 0.13094 & 107.32 & -2768.4
\end{bmatrix}
\]

\[
\alpha_1 = \alpha_2 = 1.1
\]

\[
\beta_1 = \beta_2 = 100
\]
4.4.5 System Response and Discussion

Figure 4-16 shows the servo system and model of motor with a high-speed shaft and low-speed shaft and resolver model. In this figure all the systems, such as the current controller and position controller, are simulated using MATLAB software. Figure 4-17 shows the servo system with the state observer and the parallel friction compensation for the nonlinear PMS motor with friction actuation system. Figure 4-18 shows the structure of a nonlinear digital control system with observer and observer friction and friction model. Figure 4-19 shows the comparison of the non-fiction torque from the low-speed shaft and observer from the same shaft. Figure 4-20 shows the comparison of the non-friction torque from the low-speed shaft and observer in a short period of time of 0.3 second. Figure 4-21 shows the comparison of the response of the current to the step input in the new control system and the old one. Figures 4-22 and 4-23 show the $I_d$ and $I_q$ in the new and old control systems. The old control system shows a high response during the transient operation. This high value of response is one of the big disadvantages of the old control systems, which is removed in the new control design. The other disadvantage of the old control system, which is shown in Figure 4-23, is oscillation around the steady-state value of the currents, which no longer exists in the new control system approach. Figure 4-24 shows the velocity and position response to the step command input, and the differences between the old control system and the new system. Figure 4-25 shows the steady-state behavior of the velocity and position in a two-control system. The old one shows a limit cycle behavior with a large amplitude and high frequency, and the new one predicts low amplitude with low-frequency oscillation. Figure 4-26 shows position response to the command signal, and comparing between the old and new system to the
command signal. The old control system has overshoot and a steady-state limit cycle that is two milliseconds faster than the new one. The new one shows a critically damped behavior. Figure 4-27 shows position response during steady-state operation. The old one has a higher frequency and higher amplitude compared to the new one. Figure 4-28 shows a large scale of the position response during steady-state operation. Figure 4-29 shows the position response under load conditions. Figure 4-30 shows a larger scale of the position response under load conditions. As can be seen from this figure, the old system has more oscillation than the new system, which means a smooth response. Figure 4-31 shows current response under the load condition. Figure 4-32 shows a larger scale of the current response under the load condition. The new control system predicts 24 maximum amperes for the currents $I_d$ and $I_q$, instead of 150 amperes predicted by the old system. Figure 4-33 shows a larger scale of the steady-state current response under the load condition. Figure 4-34 shows the position response under the worst case load and a flexible shaft. The new control system predicts a lower amplitude and frequency compared to the old version. Figure 4-35 shows a larger scale of the position response under the worst case of load and a flexible shaft. Figure 4-36 shows a comparison of the steady-state current response in the two control systems.
Figure 4-16: Servo system with state observer and parallel friction compensation for the nonlinear PMS motor with friction actuation system.

Figure 4-17: Control system and model of motor with high-speed shaft and low-speed shaft resolver.
Figure 4-18: Structure of nonlinear digital control system observer friction and friction model-based compensation.
Figure 4-19: Comparison of the friction torque from the high-speed shaft and the friction observer.

Figure 4-20: Comparison of the non-fiction torque from the high-speed shaft and the friction observer in a short ended time of 0.3 second.
Figure 4-21: Comparison of the response of the current to the step input in a new control system and the old system.
Figure 4-22: $I_d$ and $I_q$ in the new and old control systems; the old control system shows a high response during the transient operation.

Figure 4-23: $I_d$ and $I_q$ in the new and old control systems; the old control system has an oscillation around the steady-state value of the currents.
Figure 4-24: Velocity and position response to the step command input and comparison between the old control system and the new system.

Figure 4-25: Steady-state behavior of the velocity and position in a two-control system.
Figure 4-26: Position response to the command signal and comparison between the old and new systems.

Figure 4-27: Position response during steady-state operation; old one shows high frequency and higher amplitude compared to the new one.
Figure 4-28: Position response during steady-state operation on a larger scale.

Figure 4-29: Position response under load condition.
Figure 4-30: Position response under load condition on a larger scale.

Figure 4-31: Current response under load condition.
Figure 4-32: Transient current response under load condition on a larger scale.

Figure 4-33: Steady-state current response under load condition on a larger scale.
Figure 4-34: Position response under the worst case of load and a flexible shaft.
Figure 4-35: Position response under the worst case of load and a flexible shaft on a larger scale.

Figure 4-36: Comparison of the steady-state current response in the two-control system.
4.5 Conclusion

The designed control system is based on a nonlinear state feedback with a new time varying observer, and an observer friction. Simulation results show that the control system meets the specifications and predicts lower error amplitude (0.001 rad.) and frequency (0.2 Hertz) of the limit cycle during the steady-state operation, as compared to the old system, which predicted .01 rad for the amplitude and 1.25 Hertz for the frequency of limit cycle. The new control system is capable of controlling the actuators by a flexible coupling joint. Results show that the control system is stable when the shaft has higher flexibility range. However, when the shaft is highly flexible, the control system remains stable but shows limit cycle behavior during steady-state conditions. In addition, the new control system predicts 24 maximum amperes for the currents $I_d$ and $I_q$ instead of 150 amperes, which was predicted by the old system. Overall, the control system based on friction compensation shows an improvement to the actuator of the control fuel valve.
REFERENCES


APPENDICES
APPENDIX A

FRICION COMPENSATION GAIN METHOD AND ITS
APPLICATION IN CONTROL SYSTEM IF DC MOTOR

A.1. Introduction

In this research, the application of the friction model in a linear DC motor is presented. Also, a new control algorithm employing the feedback state estimation is designed for stick-slip friction compensation for controlling the tracking of a position of a DC motor system. It is well know that the major components of friction are Coulomb force and viscous force.

Stick-slip friction exists in virtually all mechanical systems. Owing to its discontinuity at zero velocity and the downward bend of the friction torque at low velocity, stick-slip friction is often responsible for positioning inaccuracy and motion intermittence in servo mechanisms. Conventional feedback control methods cannot guarantee satisfactory results in the presence of stick-slip friction. In the regulator problem, a traditional PD controller will not achieve satisfactory performance because it results in steady-state error. This error may be reduced by increasing the proportional gain, but high-gain controllers may make the system unstable.

Here, a relatively new control algorithm is proposed. This control system is based on the state observer and control gain feedback to ensure a zero steady-state error under the influence of friction. To ensure a better track and reduce the friction effectively, compensation gain is added to the control parameter. This gain breaks the friction and causes a better tracking position. In this case, the frequency response of the model has been improved and the satisfactory results have been observed.
This section is divided into the following areas:

• Linear DC motor modeling.
• Friction modeling.
• Designing the observer-based control system for the model.
• Designing the discrete control system.
• Brief review of the old version of control system and comparing of the two control systems.
• Friction compensation gain to the algorithm.
• Conclusions.

A.2 Linear DC Motor Modeling

Figure A-1 shows the linear model of the DC motor, which has three states.

![Figure A-1: Linear model of the DC motor.](image)

This model shows both the electrical and mechanical components of the system. Figures A-2, A-3, and A-4 show the step responses of the DC motor, and Figure A-5 shows the sinusoidal response of the motor.
Figure A-2: Current response of the linear DC motor.

Figure A-3: Position-time step response of the linear DC motor.
Figure A-4: Velocity response of the linear DC motor.
A.3 Friction Modeling

To incorporate the non-linear friction model into a position control system, the system is constructed by SIMULINK.

Figure A-6 shows the effect of friction when a step input is applied to the system. As shown, the actuator does not move because the input torque is less than the friction torque. Figure A-7 shows the step response of the DC motor (for 4 volts). Figure A-8 shows the sinusoidal response of the DC motor. Figure A-9 compares the two torques — one in a real model in the friction case and the other one when there is no friction.
Figure A-6: Step response of the DC motor under fiction.
Figure A-7: Step response of the DC motor.
Figure A-8: Sinusoidal response of the DC motor with friction.

Figure A-9: Torque produced by DC motor under the friction effect with sinusoidal input.
A.4 Designing the Observer-Based Control System for the Model

If the state of the plant can be estimated, then feedback compensation can be implemented using the estimated state. A full-state observer utilizes only input to the plant, $u(t)$, and output, $y(t)$ (shown in Figure A-10) to provide an estimate, $\hat{x}$, of all of the state variables. A major advantage for using an observer is the ability to implement feedback compensation with a reduction in the number of measured variables. The measurement of plant variables can be difficult, and the availability and cost of appropriate sensors may be significant factors in the selection of this design option [42].

To provide the desired operation, an observer must constitute a dynamic real-time simulation that is capable of providing an acceptably precise estimation of the state variables. The basics of the control system, proposed here, are based on the state observer by measuring a position by a revolver and calculating the observer poles by using the pole placement method. Figure A-10 shows the control system and how it is added to the nonlinear model of the system.
Figure A-10: Adding the control system to the non-linear DC motor.

Figure A-11 shows the step response of the system. As can be seen, the control system meets the specifications very well. The specifications are a rise time of 0.1s for transient response and a zero steady-state response. This figure also shows, that in this case, a zero overshoot, which is good, and the capability of a faster response in an allowable overshoot range. The other advantage of this control system is its smooth current response. In other words, the current shows little fluctuation to compensate for the friction. This smooth behavioral response helps the system to perform well in the case when the actuator slides from its position and prevents blips in the position response.
Figure A-11: Response to step input.

Figure A-12 shows the difference in torque generated by the motor when there is friction and when there is no friction. Figure A-13 shows the step response of the nonlinear model, a non-zero initial condition (actuator starts exactly in its position) compared to the linear model with no friction.
Figure A-12: Comparison of torque with and without friction.

Figure A-13: Position response with friction and without friction.
Figure A-16 shows the sinusoidal response of the linear model when the control system has been applied. As can be seen, the position tracks very well when the friction is ignored (there is no limit cycle). Figure A-17 shows the sinusoidal response of the non-linear model under the influence of friction. As can be seen, the position response tends to track the sinusoidal command as much as it can, except in some areas where static friction is very high.

Figure A-14: Sinusoidal response without friction.
High friction limits the frequency bandwidth up to 6 Hertz. So the conclusion is that this control system is good enough if the specification for frequency bandwidth is less than 6 Hertz. In this case, the performance is good and is suitable to apply to the system as a successful control system. Figure A-17 shows the position response to the sinusoidal wave command and compares the nonlinear and linear friction models. In this case, the control algorithm must be reconfigured to track better and have a wider frequency bandwidth. Figures A-18 and A-19 show the frequency response to the nonlinear model. As can be seen, the frequency bandwidth is 6 Hertz.
Figure A-16: Sinusoidal response to the controlled motor.
Figure A-17: Frequency response to motor.
A. 5 Designing the Discrete Control System

As shown, the control algorithm meets the specifications for the time rise, steady-state error, and frequency bandwidth of 6 Hertz. In this case, the control algorithm must be discredited in order to apply it to a real digital system. The next objective is to design a discrete control system using the transformation method from S domain to Z domain. The conversion of an analog controller transfer function to a digital controller algorithm is generally done by applying the approximations to the Z-transform. In this design, a matched pole-zero method is used. This method relies on the property of the true z transform that an analog controller pole of zero at \( s = -a \) converts to a pole of zero of the digital equivalent at \( z = e^{-aTs} \) [44].
Figure A-19 shows the response of the system to a step input. As can be seen, the response is identical to the analog version of the control system. Figures A-20 and A-21 show the frequency responses of the system and compare them to the discreet control system.

Figure A-19: Response of the motor with discrete control system.
Figure A-20: Frequency response of the motor in s domain and z domain.
Figure A-21: Frequency response of the system in s and z domain comparing to have time lag of 15ms.

A. 6 Review Brief of the Old Version of Control System and Comparing of the Two Control Systems:

Here, in Figures A-22 and A-23, the step response of the old version of control system is shown. Figure A-24 shows a wider frequency bandwidth compared to what is here. In the next section, a compensation friction gain is used to obtain a wider bandwidth. As can be seen, the frequency bandwidth in the old version of the control system is almost 10 Hertz.
Figure A-22: System response of the old control system.
Figure A-23: Response to the sinusoidal input in old control system.
Figure A-24: Comparing the frequency response in old and new control systems.
Figure A-25: Comparing the frequency response in old and new control systems in a large scale.

**A. 7 Friction Compensation Gain**

In this section, a new idea for reducing the effect of friction is presented. In this method, the friction compensation system is introduced to have a better tracking position with wider frequency bandwidth compared to the old controls system. Figures A-26 and A-27 show how the friction compensation gain is added to the control system. As can be seen from Figure A-28, the angular position tracks the command position. The fiction area tends to track better now. Figures A-29 and A-30 show the frequency response after adding the friction compensation. As shown, the frequency bandwidth is increased from 6 Hertz to 8 Hertz. By changing the friction compensation gain, the frequency bandwidth is increased to more than 10 Hertz. However, increasing the friction compensation gain will
affect the transient response having overshot. Consequently, when the friction compensation gain is changed, the control gain should be changed as well. In this case, an optimal way of choosing the control gain and the friction gain must be applied. In the next step, using an optimal control design is recommended to find the best location of the control pole.

Figure A-26: Friction compensation system.
Figure A-27: Control system with friction compensation.
Figure A-28: Comparing the track of the demand signal with friction compensation and without friction compensation.
Figure A-29: Frequency response of the system with friction compensation and without using it compare to the old control system.
A.8 Conclusions

The following conclusions refer to this section:

1. The control system based on the proposed observer feedback with friction compensation gain successfully meets specifications.

2. The control system meets the specifications for time rise of 0.1s, steady-state error less than .0001, and no overshoot. The frequency bandwidth is less than 7 Hertz.

3. Adding the friction compensation gain improves the frequency bandwidth up to 10 Hertz.

4. For obtaining higher bandwidth and faster response, an effective optimal control method is recommended, but the details of this approach still need to be considered.