Generalized Physical and SRB Measures for Hyperbolic Surface Diffeomorphisms

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Abstract We develop new methods to identify the *typical nature* of a Dynamical System within a big class of systems. In particular, we consider Hyperbolic Surface Diffeomorphisms and introduce the notion of "Generalized Physical Measures" for these systems. We then study the existence and uniqueness of these measures. To accomplish, we apply several tools from smooth ergodic theory including the thermodynamic formalism.

1. Introduction

Recently, there has been a considerable interest in many fields of science and engineering to understand the long-term behavior of systems that change under evolution in time. These types of systems arise naturally in Thermodynamics, Astronomy, Information Theory, Biology etc.. Dynamical Systems is the area of mathematics that deals with the analytical study of the behavior of such systems. The set of points of full measure with respect to an invariant probability measure gives the typical nature of a system. In practice, the only observable measures are those for which the set of points, whose orbit distribution converges to the measure, has a positive volume. These measures are called Physical Measures in the Literature. We consider the class of Hyperbolic Surface Diffeomorphisms and introduce the notion of Generalized Physical Measures. The main result of this paper is the proof of the existence and uniqueness of such a measure. We also study other important statistical properties of this measure.

2. Description

Let f be a C^2 diffeomorphism on a smooth n-dimensional Riemannian manifold. Let M be the space of f-invariant probability measures supported on a compact f-invariant set Λ. Given μ in M, we define the basin $B^+(\mu)$ of μ to be the set of points in the manifold whose forward orbit distribution converges to the measure μ. Analogously we define the basin $B^-(\mu)$ of μ for f^{-1}. A measure μ is called a Physical Measure if $B^+(\mu)$ has positive Lebesgue measure. Moreover, μ is called an SRB-measure if its corresponding conditional measures on the unstable manifolds are absolutely continuous with respect to the Lebesgue measure. The existence of physical and SRB-measures is well understood in the case of uniformly hyperbolic systems due to the classical work of Bowen and Ruelle. We say that an ergodic measure μ is a generalized physical measure if the Hausdorff dimension of its basin is equal to the Hausdorff dimension of the stable set of Λ. This notion was studied by Wolf in [1] in the case of certain Henon maps. In this paper we extend the work of Wolf to hyperbolic surface diffeomorphisms. We establish the existence and uniqueness of generalized physical and SRB measures for these systems.

3. Results

We now present our main results:

Theorem 1. Let f be a hyperbolic surface diffeomorphism and let Λ be a basic set of f. Then f admits a unique generalized physical measure μ^+ with the following properties:

(i) $dim_H\, B^+(\mu^+)\cap\Lambda = dim_H\, \Lambda$;

(ii) $dim_H\, B^+(\mu^+) = dim_H\, W^u_\varepsilon(x)\cap\Lambda + 1$.

Corollary 2. Let f be a hyperbolic surface diffeomorphism and let Λ be a basic set of f. Then f^{-1} admits a generalized physical measure μ^-. Moreover, generically we have: $\mu^+ \neq \mu^-$.

4. Acknowledgements

This work extends previous research of C. Wolf [1] to the class of hyperbolic surface diffeomorphisms. We would also like to thank the Department of Mathematics & Statistics, its faculty and graduate students for their inestimable help in our research.

5. References

[1] C. Wolf, Generalized Physical and SRB measures in the Henon family, to appear.