COMPARISON STUDY FOR MULTI-USER MIMO CHANNEL ESTIMATION TECHNIQUES

A Thesis by

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The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Electrical Engineering.

M. E. Sawan, Committee Chair

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DEDICATION

To my Mother and the soul of my father

ACKNOWLEDGEMENTS

My mother is the one whom I refer to all my success and happiness in this life. I am really grateful for her encouragement, patience, and her high expectations for her kids. I can't mention any accomplishments without my mother standing next to me in all of them. Not only her rewarding words and humbleness, but her experience in life that made my road towards the Graduate degree a story of success.

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ABSTRACT

Channel estimation problem is a crucial state of the art research topic that has been addressed in the literature for the past decade. The importance of such research area emerged from the fact that telecommunication scholars are searching for a wireless telecommunication system that provides a minimum error performance yet can utilize the used spectrum efficiently. Channel estimation is a common practice in most of the wireless communication systems to give the receiver an indication about the channel conditions and hence will affect his decision in detecting the transmitted information

This thesis will Compare Four methods for Channel estimation problems based in semi-blind approach for MU-MIMO systems; the Rank Revealing QR factorization RRQR, Least square method LS, CAPON method and the Eigenvector method. Performance comparison is based on the Minimum mean square error (MMSE). The introduced method in this thesis is the Eigenvector method, and it's presented in comparison to the first three methods. Compared to the RRQR the Eigenvector is based on a searching function similar to the MUSIC function with the addition of the Eigen values of the null space in the search function.

LIST OF FIGURES

LIST OF ABBREVIATIONS

AOA	Angle of Arrival
AOD	Angle of Departure
CE	Channel Estimation
CSI	Channel State Information
EV	Eigenvector
LS	Least Square
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
M-QAM	Multi-Level Quadrature Amplitude Modulation
MU-MIMO	Multi-User MIMO
MUSIC	MUltiple Signal Classification
OSTBC	Orthogonal Space Time Block Code
QAM	Quadrature Amplitude Modulation
RRQR	Rank Revealing QR

LIST OF ABBREVIATIONS (continued)

SB	Semi-Blind
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SNR	Signal-to-noise Ratio
ST	Space Time
STBC	Space Time Block Code
TOA	Time of Arrival
TS	Training Sequence

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

Chapter 1

INTRODUCTION

1.1 Telecommunication History

"*Necessity is the mother of invention*", this is how humans developed their civilization. Despite the fact that the world has had its ups and downs, and despite all the obstacles that humans faced throughout the ages, humans were able to come up with unique and extraordinary inventions that have been inherited from one generation to another. Telecommunication technology is one of these outstanding and creative innovations that will never be forgotten as long as we live on this earth. Telecommunication technology is based on the need of humans to communicate with each other. Since the beginning of humanity, people tried different methods to communicate; messengers used to transfer messages by walking or using a transportation mean. We can define this technology as a method to eliminate distance between countries, continents or persons without the need of a human to transfer the message [19]. In 1794 the mechanical telegraph has replaced the messenger, and in 1837 the copper wire was used. In 1896 the humans were able to use the electromagnetic waves in their communications, and in 1973 the optical fiber was introduced.

The word *communication* is derived from the Latin word *communicatio,* which means the social process of information exchange and the word *telecommunication* was introduced in 1904 by Edouard Estaaunie who defined it as "information exchange by means of electrical signals", since that time telecommunication refers to all means of communication that is based on electromagnetic signals [19].

The evolution of telecommunication technology can be subdivided to five eras. The first era is the one prior to 1750, during which Romans, Greeks and Persians used smoke and fire signals to transmit predefined information. This era ends with the optical telegraph developed by Claude Chappe "*father of telecommunications*" during the French revolution in which the real definition of telecommunications is fulfilled. The next era extends from 1750 to 1800 and starts with the French revolution, during which Claude proved that the optical telegraph is an effective instrument to communicate with other French cities (Marseille, Lyon and Toulon), his proposed instrument reduced the message delivery time from weeks or months by messengers to minutes. There were three main types of these optical telegraphs; the *Arms types* (it depends on a movable arms positions to represent the messages), and the *Board type* (boards moves up or down to generate signals) and the *Moved-to-fixed type* (it used spheres, boards or partitions to form signals).

The next period is from 1800 to 1850, this period can be called the era of electrical telegraph. Many European scientists showed their various approached to generate a signal by placing two different metals in an acidic liquid. Among them and the first to discover that was Alessandro Volta (1745-1827) building upon the work of Luigi Galvani (1737-1798). The work of Georg Simon Ohm (1789-1854) and Michael Faraday (1791-1867) was published during this period which leads to the invention of the electrical telegraphy.

From 1850 to 1900, the world has experienced major and various technological advances; from the invention of the telephone by Alexander G. Bell to the relativity theory by Albert Einstein the world has changed dramatically. The major telecommunication milestone in this period was the laying of the first intercontinental submarine cable between the United States and Europe.

The fourth period was during the twentieth century (1900-1950), during which the diode and the transistor were invented [19]. Both of these electronic devices have pushed the development of the telecommunication technology to new phase; the ability to multiplex telephone and telegraph channels, and radiotelephony with continuous-wave transmission.

The last and current phase in the telecommunication technology development is the new millennium; where high data rates can be achieved by the IP (internet based) technology or data communications has evolved along with the mobile radio services.

Among all the tremendous advances on this millennium, we will explore the mobile radio technology latest advances; smart Antennas or multiple antennas systems. These systems were first introduced by a Bell labs scholar (Alamouti) and evolved from that point.

In chapter two we will discuss the latest advance in the mobile radio technology (wireless telecommunication) the Multiple Input Multiple output systems (MIMO). In this chapter we will explain the reason MIMO was needed or developed. Also, we will show the various models of multiple antenna systems and compare them to the regular Single input Single Output (SISO) system.

Chapter three will explain the evolution of Space Time Block codes and their importance in the MIMO wireless communication systems. The value of these types of codes comes into hands based on their ability to provide temporal diversity that increases the capacity of the wireless system.

Chapter four is the core of this thesis; in this chapter we will explain the system model and the adopted methods for the Multi-user MIMO semi-blind estimation problem. We will be comparing four methods (the least square method, the Capon method, the RRQR and the eignvalue method). These methods will be compared in terms of the Minimum mean square error (MMSE) with the appropriate justifications.

In this thesis, a small regular letter refers to a scalar, bold small letters refers to vectors, and a bold capital letters indicates Matrices.

CHAPTER 2

MIMO SYSTEMS

2.1 Introduction

Multiple Input Multiple Output Communication Systems (MIMO) has been an active research area in the past decade. The need for such a creative idea is based on the fact that users need for higher data rates in Wireless communications Systems is becoming more demanding in a band limited echo-system. Not only high data rates can be achieved by using MIMO wireless communication systems, but a fading immune wireless channel can be implemented by constructing an antenna array at the transmitter or the receiver side.

Multiple antenna paths provide spatial diversity to the system which in turns reduces the effect of multipath fading on the MIMO communications systems. From MIMO channel modeling to MIMO channel estimation and MIMO equalizers design, the literature is still undergoing an extensive research on new methods, codes, and algorithms has been suggested/ implemented and a unique result is already in our hands.

This thesis will address the Topic of MIMO channels estimation based on a semi-blind approach for Multi-user (MU) MIMO Communication system. To build a clear approach and justify our analysis this chapter is organized as follows; section two will address the Single Input Single Output (SISO) systems. In section three we will address Multiple Input Single Output Systems (MISO), section Four will address the Single Input Multiple-Output (SIMO) System. In section five we will address the Multiple Input Multiple Output (MIMO) systems, and we will conclude this chapter with Multi-user MIMO systems.

2.2 Single Input Single Output (SISO) Systems

Single input single output Systems (SISO) is the dominant Wireless communication system since Shannon (father of information theory) addressed his famous capacity formula for such systems. Figure [2.1] is a typical block diagram of such system.

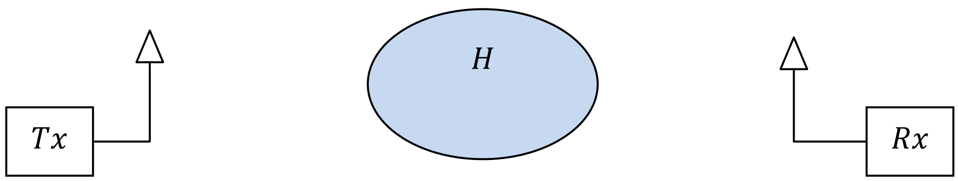

Figure 2.1 SISO Block Diagram

Where T_x is the transmitter and R_x is the receiver and H is the impulse response of the wireless channel. Shannon has proved that the Capacity of such system is [12]:

$$C_{SISO} = \log_2(1 + P/(N_o B))\ bps/Hz \tag{2.1}$$

Where P is the signal transmitted power, and B is the channel bandwidth and N_o is single side noise spectrum. Formula (2.1) represents a figure of merit that we will be referring to throughout this chapter and it's called ***Spectral efficiency***, and is defined as the total number of bit per second per hertz transmitted from one antenna\array to another. This Formula is based on the assumption that that the channel is a White Gaussian channel (fading and interference is not considered). Capacity limits can be enhanced by using channel or source coding for Digital wireless communication systems with the needed bit/error rate performance.

2.3 Multiple Input Single Output (MISO) Systems

Multiple Input single Output (MISO) wireless Communication system is development from the basic SISO model (and special case from the Multiple Input Multiple Output scheme). In this system there are multiple Antennas at the transmitter side and single Antenna at the receiver side. Figure [2.2] shows a typical block diagram for MISO system. Multiple copies of the same symbol are transmitted from the transmitter and are received by a single antenna at the receiver side. Maximum ratio combing is used with such transmission scheme to ensure that the receiver can combine the copies of each symbol correctly before it can detect which symbol has been transmitted.

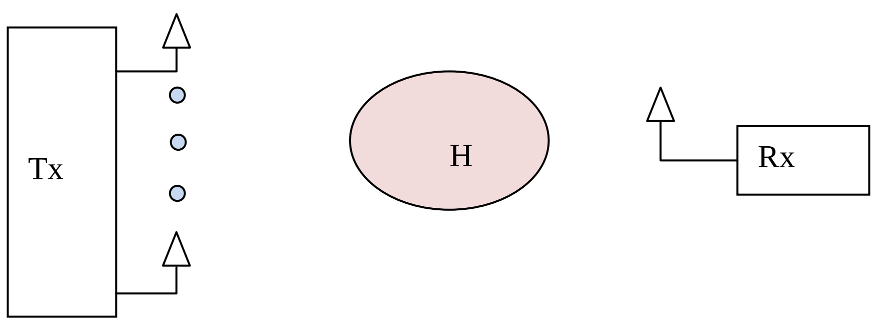

Figure 2.2 MISO Typical Block Diagram

The spectral efficiency is given below in (2.2), where n_t represents the number of antennas at the transmitter side. And P is the power allocated to the channel and ε_1 is the power gain [12].

$$C_{MISO} = \log_2\left(1 + \frac{P}{n_t}.\varepsilon_1^2\right)\ bps/Hz \tag{2.2}$$

The formula above is used with no Channel state information (CSI) at the transmitter side. There is a different approach for the spectral efficiency when CSI is known at the transmitter.

2.4 Single Input Multiple Output (SIMO)

Single Input Multiple Output (SIMO) is another special scheme of the MIMO systems. In this Scheme there is single Antenna at the transmitter side and an array of Antennas at the receiver side. Figure [2.3] shows a typical scheme for a SIMO system.

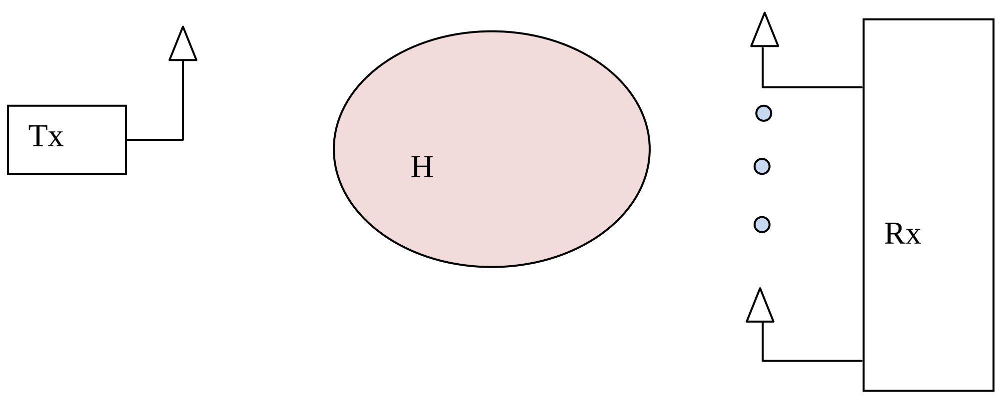

Figure 2.3 SIMO Typical block Diagram

In SIMO system the CSI doesn't affect the spectral efficiency [12] and it's given in (2.3).

$$C_{SIMO} = \log_2(1 + P.\varepsilon_1^2)\ bps/Hz \tag{2.3}$$

The reason that the CSI doesn't affect the spectral efficiency in the SIMO case is based on the Water-pouring or Water-filling algorithm. Water-pouring algorithm is used when CSI is known at the transmitter side by allocating more power to the channel that has the best conditions to maximize the channel capacity and leaving no power or minimal power to the channels with bad conditions, but in SIMO case since we have one transmitter with one antenna; all power will be allocated to that channel despite its conditions [13], so the transmitter doesn't really need to know the CSI for the power allocation mechanism.

2.5 Multiple Input Multiple Output (MIMO)

Multiple Input Multiple Output (MIMO) Communication systems are the latest advances in the wireless communication system. As mentioned earlier, such systems can provide higher data rate and better performance without any cost on the radio spectrum side [10]. Figure [2.4] shows a typical block diagram for a MIMO communication system.

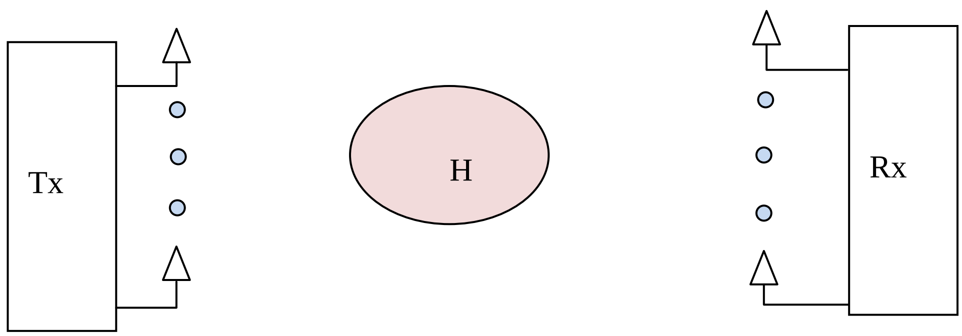

Figure 2.4 MIMO Typical Block Diagram

In this Model Multiple Copies of Symbols are transmitted by the transmitter through an Antenna array and are received by multiple antennas at the receiver side. This transmission scheme makes use of the rich scattering environment (fading baths) over the wireless medium (channel). We can think about the channel Matrix of the MIMO system as a N SISO sub channels where N is the $\min(N_t, N_r)$, where N_t is The number of transmitter antennas and N_r is the number of receiver antennas. The spectral efficiency formula for such model is given in (2.4) for no CSI at the transmitter side. So power is divided equally among all channels [12].

$$C_{MIMO} = \sum_{k=1}^{N} \log_2 \left(1 + \frac{P}{N_t} \varepsilon_k^2\right) \; bps/Hz \qquad (2.4)$$

Where P is the total power allocated by the transmitter, and ε_k^2 is the power gain.

Another important quantity that should be explained in the context of this thesis is the **Ergodic Capacity** of the MIMO channel; which is defined as the ensemble average of the information rate over the distribution of the elements of the channel matrix $\boldsymbol{H}$ [12]. So it's the capacity of the channel when every channel matrix $\boldsymbol{H}$ is an independent realization (has no relationship with to the previous matrix but typically representative of it class) [13]. Below are the formulas of the ergodic capacity for both CSI known at the transmitter and unknown CSI [12].

No CSI formula

$$\bar{C} = E\left\{\log_2 \langle det \langle I + \frac{P}{N_t}\boldsymbol{H}\boldsymbol{H}^H\rangle\rangle\right\} \tag{2.5}$$

CSI formula

$$\bar{C} = E\left\{\sum_{k=1}^{n} \log_2 \langle \boldsymbol{I} + \frac{P}{N_t}\gamma_k \varepsilon_k^2\rangle\right\} \tag{2.6}$$

Where γ_k in (2.6) is the amount of power assigned to the Kth sub-channel. There are two basic methods for the Transmitter to obtain the CSI and hence used it in the power allocation mechanism for each sub-channel. The first method is based on feedback from the receiver and the second one is based on the reciprocity principle. For the first method the transmitter depends on the CSI reported by the receiver over the reverse channel (this methods works perfectly only when the forward channel is not changing fast, otherwise the receiver should keep updating the transmitter every time the channel is changing which will cause overhead on the reverse channel). The reciprocity principle assumes that the forward and reverse channels are identical given that the time, frequency and antenna locations are the same. The former method can't be used with frequency duplex schemes. Other formulas for Capacity of MIMO system can be found in [12] and [13] for different fading MIMO channels (Rayleigh and Ricean) and for different CSI assumptions at the transmitter (partial CSI knowledge).

Graph [2.5] shows the spectral efficiency versus SNR for various schemes of MIMO systems.

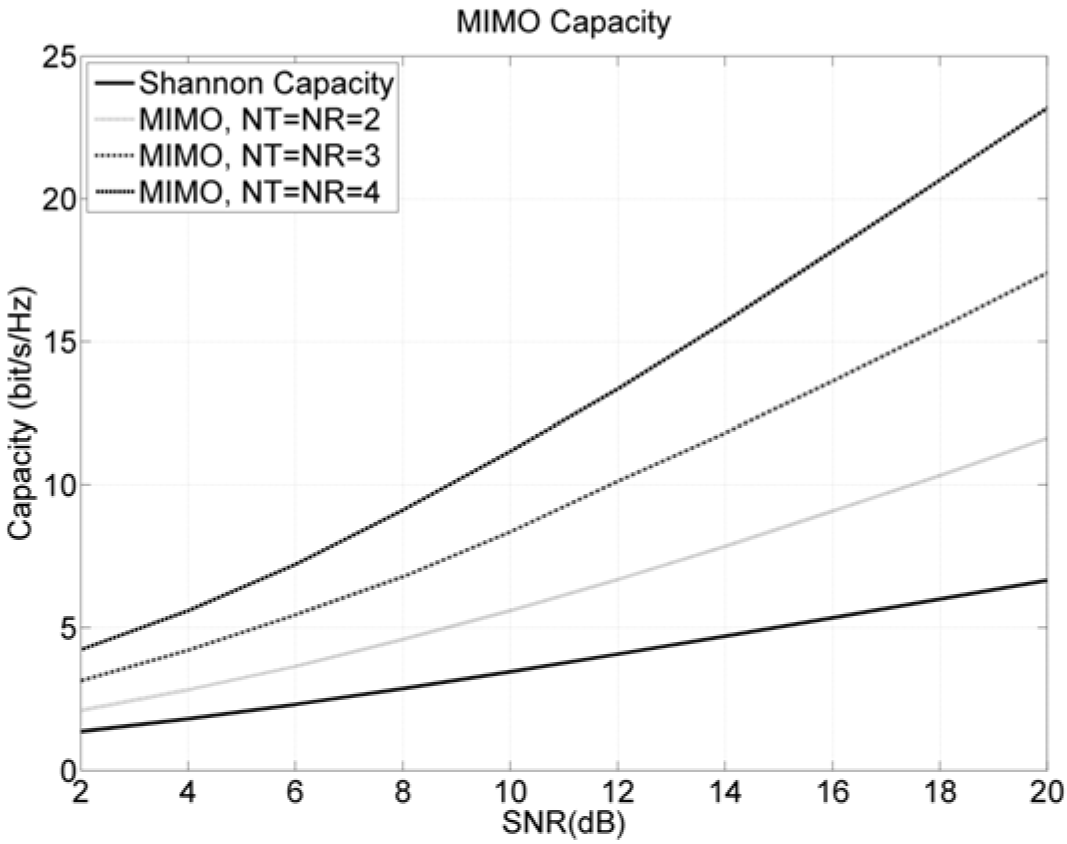

Figure 2.5 Spectral Efficiency versus SNR for Different MIMO Antenna Configuration

It's clearly seen that as the number of antennas increase at the transmitter and receiver side the spectral efficiency is improved. The Shannon line shown in the figure refers to the SISO model, interestingly speaking the spectral efficiency for MIMO systems linearly improves as we move to higher SNR ratios. In the next chapter we will address some of the available MIMO channel models.

2.5.1 MIMO channel models

There are two main distinguished approaches for MIMO channel modeling [1], a physical based approach and a non physical based model. While, the physical model describes the channel based on its crucial physical parameters; angle of arrival (AOA), angel of departure (AOD) and time of arrival (TOA), the non-physical Models are based on the channel statistical parameters. The later approach gives an accurate channel characterization [1] but have a limited description of the propagation characteristics of the MIMO channel. The second approach will be adopted in this thesis for narrow band MIMO channels (slow fading or blocking fading assumption) for its simplicity to simulate.

Before showing the statistical Model of the MIMO channel adopted in this thesis, its worth mentioning the data model used in all MIMO systems. For a MIMO communication system with *M* transmits antennas and *N* receive antennas, the baseband input-output equation can be written as below:

$$\mathrm{y(t)} = \mathbf{H}(\mathrm{t}) * \mathrm{s(t)} + \mathbf{n}(\mathrm{t}) \tag{2.7}$$

Where $s(t)$ is the transmitted signal, $y(t)$ is the received signal, $n(t)$ is the additive white Gaussian noise (AWGN), and $\mathbf{H}(t)$is *N* by *M* channel impulse response matrix. For narrowband signals (MIMO systems) formula (2.7) can be simplified to:

$$\boldsymbol{y} = \boldsymbol{Hs} + \boldsymbol{n} \tag{2.8}$$

In most research cases (eg [2]) the MIMO channel matrix elements are assumed to be independent and identically distributed (IID).

2.5.1.1 Non physical MIMO channel Models

In this subsection we will discuss the most common non-physical MIMO channel models, both of them were developed under the European Union projects for Mutli-antenna systems (IST METRA and IST SATURN).

2.5.1.1.1 Multi element transmit receive Antenna (METRA) project.

The non-line of sight (scattering environment) for channel model is proposed based on the channel power correlation matrix. For an *M* transmit antennas and *N* receiver antennas MIMO system the channel can be modeled as (without adding the noise):

$$\boldsymbol{H}(\tau) = \sum_{l=1}^{L} \boldsymbol{H}_l \delta(\tau - \tau_l) \tag{2.9}$$

Where $H(\tau)$ is the *N* by *M* matrix of channel impulse responses, $H_l$is the Matrix of channel coefficients at time delay τ_l

$$\boldsymbol{H}_l = \begin{bmatrix} H_{11}^l & \cdots & H_{1M}^l \\ \vdots & \ddots & \vdots \\ H_{N1}^l & \cdots & H_{NM}^l \end{bmatrix} \tag{2.10}$$

It's shown in [3] that the spatial correlation matrix that is used to construct the MIMO channel model can be expressed as the kronecker product of the power correlation matrices seen from the transmitter and receiver:

$$\boldsymbol{P}_H = \boldsymbol{P}_H^{Tx} \otimes \boldsymbol{P}_H^{Rx} \tag{2.11}$$

Where $\boldsymbol{P}_H$ is the power correlation matrix for the MIMO channel. Given (2.8) the MIMO channel can be simulated as:

$$Vec\ (\boldsymbol{H}_l) = \sqrt{P_l}\boldsymbol{C}\boldsymbol{a}_l \tag{2.12}$$

Where $vec\{\cdot\}$ is a vectorization operation (stacking the columns of a matrix into a vector), $\boldsymbol{a}_l$ is a column vector with IID zero mean complex Gaussian elements, **C** is a symmetric mapping matrix and P_l is the average power.

2.5.1.1.2 Smart Antenna Technology in Universal Broadband wireless Networks (SATURN) project.

In this Model the channel covariance matrix is considered rather than power correlation matrix (this will provide more information about the MIMO channel). As indicated earlier, the channel coefficients are considered to be zero mean complex Gaussian variables. In this case the first and second order moment of the channel is enough to describe it [4]. Below is the channel matrix, G is a stochastic *N* by *M* matrix with IID elements.

$$\boldsymbol{H} = (\boldsymbol{R}_H^{Rx})^{\frac{1}{2}}\boldsymbol{G}(\boldsymbol{R}_H^{Tx})^{\frac{T}{2}} \qquad (2.13)$$

2.5.2 MIMO Systems in wireless standards

The reason I have added this subsection is to give the reader a sense about the importance of MIMO communications in the wireless industry and how far MIMO systems have been implemented. The IEEE has included MIMO in its standard for Wi-Fi or (wireless Fidelity). In the IEEE802.11b the data rate supported is 54Mbps, but for IEEE802.11n the data rate supported can go up to 100Mbps because MIMO is included in this standard. MIMO communication has been included as an option in the IEEE standard 802.16 version e; this standard is for WiMAX (World Interoperability for Microwave access). The 3GPP (Third Generation partnership project) is responsible for standardizing the WCDMA technology has included MIMO in release 7 and 8 of its standard. Release 8 is considering 2×2 and 4×4 MIMO antenna configurations [11] with 64-QAM modulation. It's essential to mention that M-QAM modulation techniques is one of the most sensitive modulation to fading errors and outage, but thanks to MIMO that made the M-QAM choice possible with its immunity to fading.

Another standard that is adopting MIMO is the IEEE802.20 intended for mobile broadband wireless access; this is a packet-based air interface particularly designed for IP (Internet Protocol) based services.

It worth mentioning at this point that MIMO as a concept was first introduced in the Control theory and most of the techniques used in MIMO wireless Systems are adopted from the Control literature with the needed Modification.

2.6 Multi-User MIMO (MU-MIMO)

This thesis will consider a MU-MIMO communication system with P transmitters and one receiver and multiple access users. In addition to that we will assume that all transmitters will use the same OSTBC and flat block fading channel is used. Figure 1 shows a diagram of the assumed system.

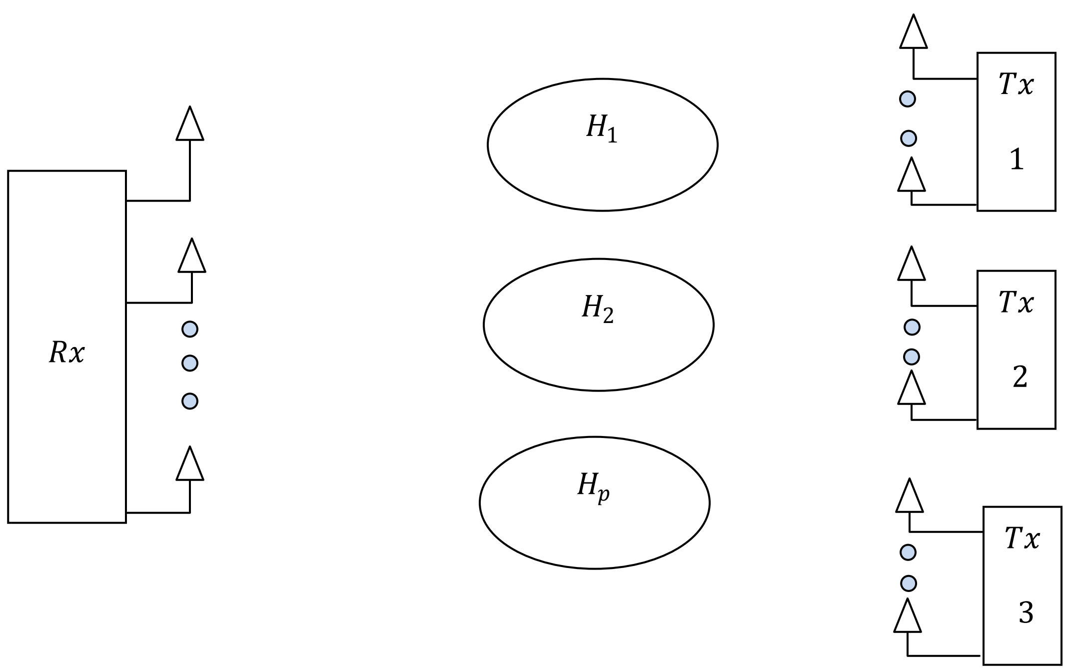

Figure 2.6 MU-MIMO Typical Block Diagram

The main difference between usual MIMO systems and MU-MIMO is the formula for the recived signal at the reciver side. Below is the formula for the recived signal for such system[15].

$$\boldsymbol{Y} = \sum_{p=1}^{P} \boldsymbol{X}(\boldsymbol{s}_p)\boldsymbol{H}_p + \boldsymbol{Z} \tag{2.14}$$

Where $\boldsymbol{Y}$ is the $(T \times M)$ received signal matrix, $\boldsymbol{X}(\boldsymbol{s}_p)$ is the $(T \times N)$ matrix of transmitted signals, $\boldsymbol{s}_p$ is the $(K \times 1)$ length information data vector. The channel matrix for the p-th transmitter and receiver is $\boldsymbol{H}_p$ of size $(N \times M)$. The matrix $\boldsymbol{Z}$ is corresponding noise matrix.

CHAPTER 3

Space-Time Block Codes (STBC)

3.1 Introduction/History

As the name implies, the data transmission in a space time (ST) system is done in two dimensions [5]. The space dimension is defined by the multiple transmit-antennas, while the time dimension is defined by the multiple time intervals on which multiple blocks are transmitted. The ST encoder maps a block of information symbols to a unique ST code matrix; the matrix below is fundamental matrix in most ST codes.

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & \cdots & x_{1N_x} \\ \vdots & \ddots & \vdots \\ x_{N_t1} & \cdots & x_{N_tN_x} \end{bmatrix} \tag{3.1}$$

Where N_t is number of transmit-antennas and N_x is number of times slots in ST system.

Code rate is defined as $\eta_{ST} = \frac{N_s}{N_x}$ symbols per channel use, where N_s is the information symbols. ST communication systems is defined as systems that deploy more than one transmit antenna. Matrix X in (3.1) is usually complex valued, and it's usually normalized to satisfy the formula in (3.2). Where $E[\cdot]$is the statistical expectation and $tr(\cdot)$is the trace

$$\frac{1}{N_x N_t} E[tr(\boldsymbol{X}\boldsymbol{X}^H)] = 1 \tag{3.2}$$

of the matrix. To relate ST transmission to channel coding, the ST functions in a similar manner to channel-coded block transmission with interleaving; without introducing any decoding delay [5]. It's essential to mention that with ST coding techniques, diversity at the receiver side is optional (which is really desirable in wireless communication systems). ST codes can be classified based on different criteria; they can be classified based on their linearity (linear Vs non-linear), or they can be classified based on the MIMO channel for which they will be used

(Flat or frequency selective MIMO channels) or (time varying MIMO channels). Another classification depends on the availability of the CSI (open-loop or closed-loop ST codes), The classification we will be interested in is the type of the ST code(Block codes Vs trellis codes) ,Among all the available ST block codes we will refer to ST block codes (STBC) for their simplicity in our problem development. In the next subsection we will explore the fundamentals of STBC.

3.2 Space-Time Block Codes (STBC)

Space-time Block codes (STBC) can be in several forms [14], but we are interested in the most widely used and preferable *linear* STBCs. The linear STBC spread the information symbols in space and time to increase/improve their diversity gain or/and spatial multiplexing rate. Formula (3.3) is a general Codeword formula for linear STBC [14].

$$\boldsymbol{C} = \sum_{1}^{Q} \boldsymbol{\Phi}_{\mathrm{q}} \Re[\mathrm{c_q}] + \boldsymbol{\Phi}_{\mathrm{q}} \Im[\mathrm{c_q}] \tag{3.3}$$

Where C is the Codeword, Φ_q are complex basis matrices, c_q stands for complex information symbol, Q is the number of complex information symbols c_q transmitted over a code word, $\Re\ and\ \Im$ stands for real and imaginary parts. The assumption is that the channel is constant over the duration of the STBC codeword T.

The first STBC was proposed by Alamouti [6] in 1998 in his attempt to suggest a transmit diversity technique for wireless communication systems. Alamouti work was extended from two transmit antennas to generic number of array by [7] and was referred to as the Orthogonal Space time block coding (OSTBC) scheme. In the following subsections we will introduce both the Alamouti and the OSTBC schemes. Although STBC provide smaller coding gain when compared to trellis codes [5] their low decoding complexity give them a flavor when receiver complexity is considered.

3.2.1 Alamouti Scheme

In 1998 Alamouti suggested a unique coding scheme as a simple transmit technique for wireless communication systems. He proved that his scheme can improve the error performance, and data rate for the wireless communication system. The coding matrix [12] for his scheme can be written as in (3.4). The decoder suggested in his work assumes that the fading channel coefficients during the two consecutive transmission periods remain constant. The transmitter transmits x_1 from the first antenna and x_2 from the second antenna in the first time slot. Then it transmits $-x_2^*$ from the first antenna and x_1^* from the second antenna in the 2nd time slot.

$$\boldsymbol{X}_1 = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \tag{3.4}$$

Figure 3.1; shows a block diagram of the Alamouti scheme for a single receiver case.

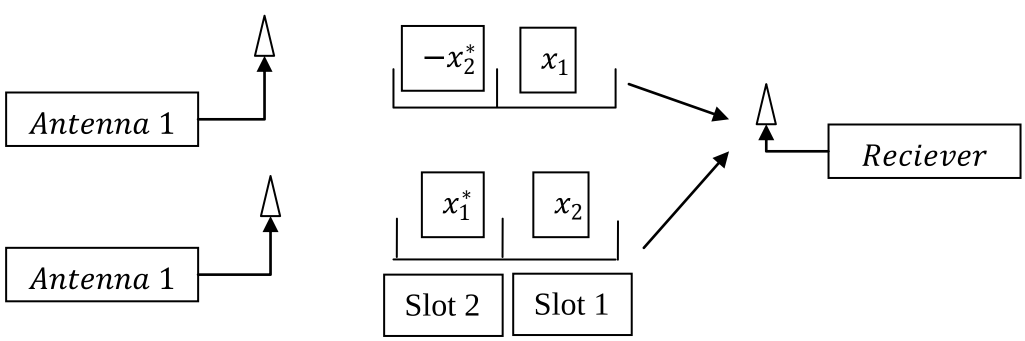

Figure 3.1 Block Diagram for Alamouti Scheme

3.2.2 Orthogonal Space Time Block Codes (OSTBC)

In 1999 Tarokh [7] introduced the concept of "Complex orthogonal Designs" to extend the STBC from 2 by 2(Alamouti Scheme) to generic number of antennas. There work evolved from "Theory of orthogonal designs" in mathematics. The objective was to design a set of $N_t \times N_t$ code matrices (for system with N_t transmit antenna) from a specific signal constellation whose columns are orthogonal to each other. The value of orthogonal columns is to ensure that a linear receiver is still optimum and the decoding complexity is still minimum [11]. The OSTBC matrix X has to satisfy the below condition:

$$\boldsymbol{X}.\boldsymbol{X}^H = p.\langle\sum_{i=1}^{n}|x_i|^2\rangle\,.\,I_{N_t} \tag{3.5}$$

Where p is a constant and n is the number of symbols x_i transmitted per transmission block in $\boldsymbol{X}$. In words, this implies that from the set of available symbols and their conjugates a non zero entries are extracted from $\{x_i, x_i^*\}$satisfying (3.5). The following code matrices are for $N_t = 3,4$.

$$\boldsymbol{X}_{3\times4} = \begin{bmatrix} x_1 & -x_2^* & -x_3^* & 0 \\ x_2 & x_1^* & 0 & -x_3^* \\ x_3 & 0 & x_1^* & x_2^* \end{bmatrix} \tag{3.3.2}$$

$$\boldsymbol{X}_{4\times4} = \begin{bmatrix} x_1 & x_2^* & -x_3^* & 0 \\ x_2 & x_1^* & 0 & -x_3^* \\ x_3 & 0 & x_1^* & x_2^* \\ 0 & x_3 & -x_2 & x_1 \end{bmatrix} \tag{3.3.3}$$

They have proved that orthogonal design of size *m* exists only for *m*=2 or 4, but the proof and lemmas used are beyond the scope of this thesis.

CHAPTER 4

Multiuser MIMO estimation problem

4.1 Introduction

This chapter will discuss in details the core of this thesis. In this chapter we will explore four techniques for estimating the MU-MIMO channel based on a semi-blind approach. The Methods that will be discussed are Least Square method (LS), Rank Revealing QR factorization, Capon method, and the Eign values method. All these methods (algorithms) estimates the subspace spanned by the user channels to extract the user channel state information (CSI) using what so called training Symbols or training blocks.

It essential in the context of this thesis to address the different methodologies addressed in the MIMO channel estimation techniques literature to give the reader a flavor of the importance of work done in this thesis. The conceptual techniques that are used for MIMO channel estimation evolved from methods used for Single input single output (SISO) estimation with the needed modifications and assumptions.

In general, three types of assumptions are made during channel estimation procedure, either a perfect channel state information (CSI) assumption, or semi-blind or fully blind assumptions. In a practical wireless communication system, channel knowledge is never known a *priori*, so blind and semi-blind assumptions are used or a pilot symbols are used that is known for the receiver. However, in a spectral efficient wireless communication system, it’s always desirable to limit the number of transmitted pilot symbols. That’s why blind and semi-blind techniques are more favorable and desired and have been studied extensively in the literature. In the semi-blind (SB) channel estimation (CE) terminology, *Training sequence* (TS) or training symbols or training blocks; refers for a limited consecutive sequence of known symbols, where as a pilot signal is a continuous stream of known symbols superimposed on the data signal [8]. The Diagram below shows at which stage channel estimations occurs for a typical wireless communication system block diagram.

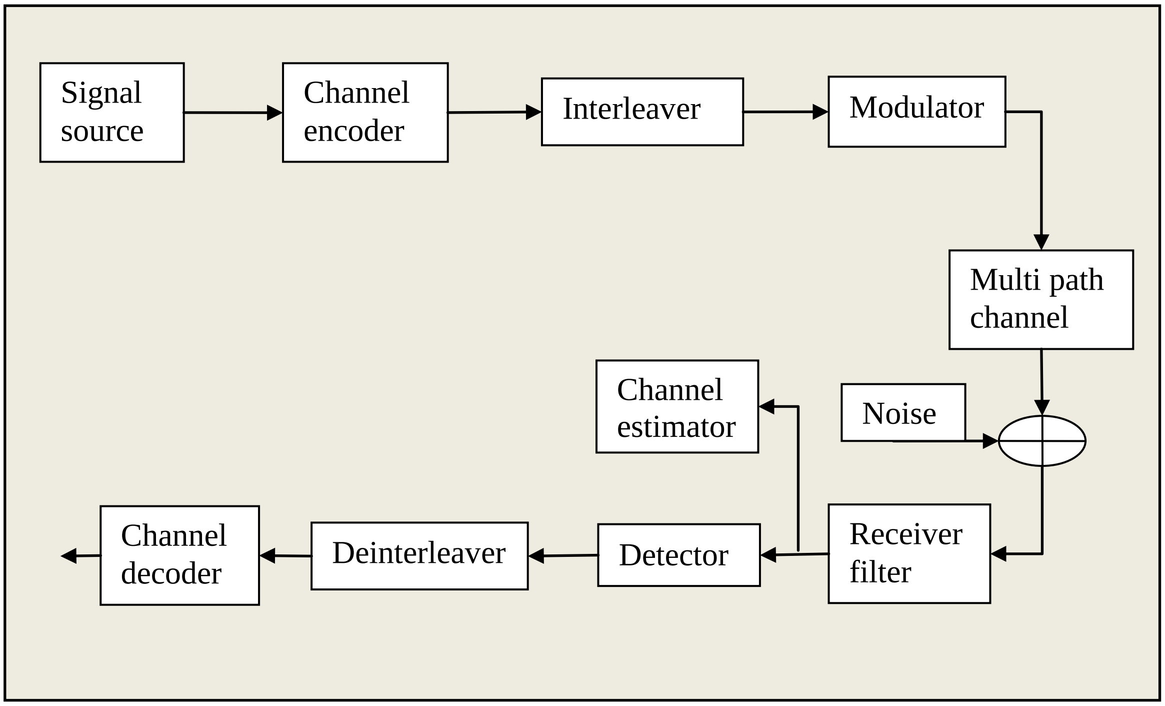

Figure 4.1 Typical Block Diagram for Telecommunication System

The remaining of this chapter is organized as follows; section two will briefly explains two main estimation techniques used with most estimation problems. Section three will address the system model that will be adopted in our simulations and comparison between the various methods. Sections four to seven explains the math of the mentioned methods (LS, RRQR, Capon and Eign values methods). Section eight will show the simulation results for the explained methods based on the system model.

4.2 Estimation techniques

Estimation theory is used in nearly all disciplines of engineering and science. Karl Fridederich Guass is considered the progenitor [9] of what is so called the "Estimation Theory" during his attempts to predict the motions of planets, when he developed the least square methods. The foundation of Karl Guass gave the chance to other scholars (Rudolph E. Kalman, and Norbert Wiener) to develop new methods and algorithms that are used in modern signal processing aspects.

The following subsections we will address different linear estimators used to estimate channel parameters for training based approach for a flat block fading channels. The basic idea in linear estimation is to optimize linear estimator W for the channel matrix in Hgiven the realizations of Y as follows [21]:

$$\widehat{\boldsymbol{H}} = \boldsymbol{WY} \tag{4.1}$$

We will address the Minimum Mean square error Estimator (MMSE), Maximum likelihood Estimator (ML).

4.2.1 Minimum Mean Square Estimator (MMSE)

In MMSE we search a function F [21] where $F$and $\widehat{H}$ is related as in (4.2) such that $\widehat{\boldsymbol{H}}$ is on average is close to the true channel realization $\boldsymbol{H}$as possible.

$$\widehat{\boldsymbol{H}} = F(\boldsymbol{Y}) \tag{4.2}$$

In other words, we try to minimize the mean square error of $\mathbf{H}$ for a given realization of $\mathbf{Y}$. the mean square error is given in (4.3) and the argument to minimize it is given in (4.4).

$$E_{H,Y}\left[\left\|\boldsymbol{H} - \widehat{\boldsymbol{H}}\right\|^2\right] \tag{4.3}$$

$$\min_{\widehat{\boldsymbol{H}}} E_{H|Y}\left[\left\|\boldsymbol{H}-\widehat{\boldsymbol{H}}\right\|\right] \tag{4.4}$$

The main advantage of the MMSE is that it tries to find the best tradeoff between the contribution of the mean squared norm of bias and the variance of the estimator based on the fact that it exploits the knowledge of about the channel and noise covariance matrices.

4.2.2 Maximum likelihood Estimator (ML)

The Maximum likelihood (ML) estimator treats **H** as deterministic parameter and no a Priori information about it. In ML finding the parameters that best fit the observation Y in (4.5) and maximize the likelihood.

$$Y \sim N_c(\boldsymbol{H}s, \boldsymbol{C}_n) \tag{4.5}$$

Where $\boldsymbol{C}_n$ is the noise covariance matrix. For a general class of noise covariance matrices, the ML problem is equivalent to weighted least-square problem. In This case $\boldsymbol{W}$can be solved as below:

$$W_{ML} = (\boldsymbol{S}^H \boldsymbol{C}_n^{-1} \boldsymbol{S})^{-1} \boldsymbol{S}^H \boldsymbol{C}_n^{-1} \tag{4.6}$$

ML estimator is address in this section for the sake of comparison with other estimation methods and will not be used in the problem development. It's crucial to mention that the performance and the quality of each of the mentioned estimators have been studied extensively and it's beyond the scope of this thesis.

The most common and basic estimation method is the training based least square (LS) estimator. For a regular MIMO channel, the LS estimator use the below equation to estimate the channel parameters:

$$\widehat{\boldsymbol{H}} = arg_H \; min\|\boldsymbol{y} - \boldsymbol{H}s\|^2 \tag{4.7}$$

The following chapters we will clearly explain how channel Matrix $\boldsymbol{H}$ is modified to account for the Multi-user MIMO channel along with the proposed estimation techniques.

4.3 System model

This section will explain in details the system adopted for our simulation purposes. Formula (2.14) shows the received signal for MU-MIMO with P transmitters (users) and one receiver. We will use formula (2.14) as our reference and we will assume that the length of the OSTBC code used as *L=1*. In the simulations we will use OSTBC with $^3/_4$ rate; the code matrix is given below.

$$\boldsymbol{X} = \begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ -s_3^* & 0 & s_1^* & -s_2 \\ 0 & -s_3^* & s_2^* & s_1 \end{bmatrix} \tag{4.8}$$

The Scenario considered here is MU-MIMO with two users $p = 2$ each with four antennas $N = 4$ and a single receiver with four antennas $M = 4$. Also in our simulations we consider one of the transmitters is stronger than the other one. This can be done by considering noise power associated in channel matrix $\boldsymbol{H}_p$ in (2.14)**.** In simulation this can be done by considering signal to noise ratio of p-th user as σ_p^2/σ^2 where σ_p^2 is the variance of channel matrix $\boldsymbol{H}_p$ while σ^2 is the noise power. The SNR of one user is assumed to be stronger than the other by 2.5dB. We will Compare the performance of each of the methods (explained in the following sections) using the root mean square error (RMSE) for *p*th user defined as below, where MC is the number of channel realizations:

$$RMSE = \frac{1}{MN_t} \sum_{n=1}^{MC} \left(\sqrt{\sum_{i=1}^{2MN} (\boldsymbol{h}_p(i) - \widehat{\boldsymbol{h}}_p(i))^2} \right) \tag{4.9}$$

In the first four graphs of the simulations results we will compare the normalized RMSE versus the SNR values for each of the users, and in the second four graphs we will compare the normalized RMSE versus the number of training blocks used to extract the CSI using the proposed method. The matrix of transmitted signals $\boldsymbol{X}(\boldsymbol{s}_p)$ is considered to be the OSTBC matrix for our system. It can be written as a summation of the real and imaginary (complex conjugates) based on (3.3) formula for linear STBC codes.

$$\boldsymbol{X}(\boldsymbol{s}_p) = \sum_{k=1}^{K} (\boldsymbol{C}_k Re\{s_k\} + \boldsymbol{D}_k Im\{s_k\}) \tag{4.10}$$

Where $\boldsymbol{C}_k \coloneqq \boldsymbol{X}(\boldsymbol{e}_k)$ and $\boldsymbol{D}_k \coloneqq \boldsymbol{X}(i\boldsymbol{e}_k)$ are the basis matrices. The $\boldsymbol{e}_k$ is a $K \times 1$ vector having one in its Kth position and zeros elsewhere [15], and i is the imaginary unit. Using (4.10) we can write (2.14) as below:

$$\overline{\boldsymbol{Y}} = \sum_{p=1}^{P} \boldsymbol{A}(\boldsymbol{h}_p)\overline{\boldsymbol{s}_p} + \overline{\boldsymbol{Z}} \tag{4.11}$$

Where $\overline{W}$ for any matrix is defined as below:

$$\overline{\boldsymbol{W}} \coloneqq \begin{bmatrix} vec\{Re(\boldsymbol{W})\} \\ vec\{Im(\boldsymbol{W})\} \end{bmatrix} \tag{4.12}$$

And $vec\{\cdot\}$ is a vectorization operator. The vector form of $\boldsymbol{A}(\boldsymbol{h}_p)$ can be written as below:

$$\boldsymbol{A}(\boldsymbol{h}_p) \coloneqq [\overline{\mathbf{C}_1\mathbf{H}_p}, \dots, \overline{\mathbf{C}_K\mathbf{H}_p}, \quad \overline{\mathbf{D}_1\mathbf{H}_p}, \dots, \overline{\mathbf{D}_K\mathbf{H}_p}] \tag{4.13}$$

Or as :

$$\boldsymbol{A}(\boldsymbol{h}_p) := [\boldsymbol{a}_1(\boldsymbol{h}_p) \ \ \boldsymbol{a}_2(\boldsymbol{h}_p) \dots \dots \boldsymbol{a}_{2K}(\boldsymbol{h}_p)] \tag{4.14}$$

Since $\boldsymbol{A}(\boldsymbol{h}_p)$ is linear in $\boldsymbol{h}_p$there are $2K$ real matrices [15] $\boldsymbol{\Phi}_{\mathrm{k}}$, $for\ k = 1,2, \dots, 2K$ with dimension $(2MT \times 2MN)$ such that:

$$\boldsymbol{a}_k(\boldsymbol{h}_p) \ = \boldsymbol{\Phi}_{\mathrm{k}}\boldsymbol{h}_p \ for\ k = 1,2, \dots, 2K \tag{4.15}$$

In this case $\boldsymbol{\Phi}_{\mathrm{k}}$ is a known specific OSTBC matrix. Using (4.15) we can rewrite $\boldsymbol{A}(\boldsymbol{h}_p)$ as:

$$\boldsymbol{A}(\boldsymbol{h}_p) \coloneqq [\boldsymbol{\Phi}_1\boldsymbol{h}_p \ \ \boldsymbol{\Phi}_2\boldsymbol{h}_p \dots \dots \boldsymbol{\Phi}_{2\mathrm{K}}\boldsymbol{h}_p] \tag{4.16}$$

And the vector form of $\boldsymbol{A}(\boldsymbol{h}_p)$ can be written as (4.17) where $\boldsymbol{\Phi} \coloneqq [\boldsymbol{\Phi}_1^{\mathrm{T}} \ \ \boldsymbol{\Phi}_2^{\mathrm{T}} \ \dots \dots \boldsymbol{\Phi}_{2\mathrm{K}}^{\mathrm{T}}]^{\mathrm{T}}$.

$$vec\{\boldsymbol{A}(\boldsymbol{h}_p)\} = \boldsymbol{\Phi}\boldsymbol{h}_p \tag{4.17}$$

To be able to show difference of the different methods explained in the next sections, we need to establish a common foundation for all of them. This foundation is the subspaces defined by the channel and noise. We will refer to the MUSIC spectrum [15] and MUSIC search method. Lets define the noise subspace $\boldsymbol{E}$ that is orthogonal to the signal subspace of a multi-user channel spanned by $\{\boldsymbol{A}(\boldsymbol{h}_p)\}$, $p = 1,2, \dots P$. Where $\boldsymbol{E}$ is $2MT \times (2MT - 2KP)$ matrix and their multiplication results in the null space $\boldsymbol{A}^{\boldsymbol{T}}(\boldsymbol{h}_p)\boldsymbol{E} = \boldsymbol{0} \quad p = 1,2, \dots \dots P$. The generalized MUSIC spectrum is defined below for such estimation problem.

$$P_{MUSIC} = \frac{1}{\left(\left\|\boldsymbol{A}^{\boldsymbol{T}}(\boldsymbol{h}_p)\boldsymbol{E}\right\|^2\right)} \tag{4.18}$$

The noise subspace $\boldsymbol{E}$ can be extracted by singular value decomposition (SVD) of the covariance matrix $\boldsymbol{R_{cov}}$ defined in (4.19). $\boldsymbol{R_{cov}}$ is a $2MT \times 2MT$ matrix formed form the received data.

$$\boldsymbol{R_{cov}} := \boldsymbol{E}\{\overline{\boldsymbol{Y}}\overline{\boldsymbol{Y}}^{\boldsymbol{T}}\} \tag{4.19}$$

Adopting the procedure in [15] we can write the MUSIC spectrum as below:

$$P_{MUSIC} = \frac{1}{tr\{\boldsymbol{A}^T(\boldsymbol{h}_p)\boldsymbol{E}\boldsymbol{E}^T\boldsymbol{A}(\boldsymbol{h}_p)\}}$$

$$= \frac{1}{vec\{\mathbf{A}(\boldsymbol{h}_p)\}^{\mathrm{T}}(\mathbf{I}_{2\mathrm{K}}\otimes\boldsymbol{E}\boldsymbol{E}^T)vec\{\mathbf{A}(\boldsymbol{h}_p)\}}$$

$$= \frac{1}{\mathbf{h}_{\mathrm{p}}^{\mathrm{T}}\boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{I}_{2\mathrm{K}}\otimes\boldsymbol{E}\boldsymbol{E}^T)\boldsymbol{\Phi}\boldsymbol{h}_p}$$

The above equations indicates that the channel vectors that belongs to the LP minor eign vectors of the matrix $\boldsymbol{\Omega} = \boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{I}_{2\mathrm{K}}\otimes\boldsymbol{E}\boldsymbol{E}^T)\boldsymbol{\Phi}$**.** So $\widehat{\boldsymbol{h}}_p$ can be estimated as below:

$$\widehat{\boldsymbol{h}}_p = \sum_{k=1}^{LP} \beta_{pk}\widehat{\boldsymbol{v}}_k \tag{4.20}$$

Where L is a OSTBC parameter, mostly one for diverse codes, and $\widehat{\boldsymbol{v}}_k, k = 1,2, \ldots LP$ are the eigenvectors of matrix $\boldsymbol{\Omega}$. Least square based method can be used to extract the parameter β [15] as explained below.

Let $\beta_k \coloneqq [\beta_{1,k}\ \ \beta_{1,k}\ \ \ldots \ldots \beta_{LP,k}\ ]$ and $\boldsymbol{\beta} = [\boldsymbol{\beta}_1^T\ \ \boldsymbol{\beta}_2^T \ldots \ldots \boldsymbol{\beta}_P^T]^T$, then$\boldsymbol{\beta}$ can be estimated as :

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{\Lambda}^{\mathrm{H}}\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}^{\mathrm{H}}\mathbf{z} \tag{4.21}$$

Where

$$\boldsymbol{\Lambda} \coloneqq [\mathbf{F}^{\mathbf{T}}(1)\ \mathbf{F}^{\mathbf{T}}(2) \ldots \mathbf{F}^{\mathbf{T}}(\mathrm{Jt})]^{\mathbf{T}} \text{ and}$$

$$\mathbf{F}(\mathbf{n}) \coloneqq [\mathbf{F_1}(\mathrm{n}) \ldots \mathbf{F}_{\mathrm{LP}}(\mathrm{n})] \quad \textit{and}$$

$$\mathbf{F}_{\mathrm{K}}(\mathrm{n}) \coloneqq [\mathbf{A}(\widehat{\boldsymbol{v}}_k)\overline{\mathbf{s}_1(\mathrm{n})}\ \ \mathbf{A}(\widehat{\boldsymbol{v}}_k)\overline{\mathbf{s}_2(\mathrm{n})}\ \ \ldots\ \ \mathbf{A}(\widehat{\boldsymbol{v}}_k)\overline{\mathbf{s}_{\mathrm{P}}(\mathrm{n})}].$$

In the following subsections we will explain the use of the estimation methods mentioned earlier based on the assumptions and simplification made on this section.

4.4 Rank Revealing QR factorization

Rank Revealing QR factorization is a special form of QR factorization, and it a valuable tool in numerical linear algebra because it can provide accurate information about the rank and null space [17]. Its main use is in the solution of rank-deficient least-square problems. RRQR has been used in [16] and it should a great performance. The following lines will explain how RRQR functions.

Using QR factorization to the covariance matrix shown in (4.19) results in the following:

$$\boldsymbol{R}_{cov} = \boldsymbol{Q}\boldsymbol{R} = \boldsymbol{Q}\begin{bmatrix}\boldsymbol{R}_{11} & \boldsymbol{R}_{12}\\ \mathbf{0} & \boldsymbol{R}_{22}\end{bmatrix} \tag{4.22}$$

In words, $\boldsymbol{R}_{cov}$ is expressed as a product of a unitary matrix and ran-revealing upper triangular matrix. $\boldsymbol{R}_{11}$is a $(2KP \times 2KP)$ upper triangular full rank matrix. Since $\boldsymbol{R}_{22}$ has small norm we can easily extract the basis of the noise space form matrix $\widehat{\boldsymbol{R}} = [\boldsymbol{R}_{11} \quad \boldsymbol{R}_{12}]$, where $\widehat{\boldsymbol{R}}$

is defined as the null space of $\boldsymbol{R}_{cov}$. It is shown in [17] that any vector that belongs to the null space should satisfy the following:

$$\widehat{\boldsymbol{R}}\boldsymbol{G} = [\boldsymbol{R}_{11} \quad \boldsymbol{R}_{12}]\begin{bmatrix}\boldsymbol{g}_1 \\ \boldsymbol{g}_2\end{bmatrix} = \boldsymbol{0} \tag{4.23}$$

Such that $\boldsymbol{R}_{11}\boldsymbol{g}_1 = -\boldsymbol{R}_{12}\boldsymbol{g}_2$, and Since $\boldsymbol{R}_{11}$ is an invertible matrix, $\boldsymbol{g}_1$ can be written in terms of $\boldsymbol{g}_2$ as $\boldsymbol{g}_1 = -\boldsymbol{R}_{11}^{-1}\boldsymbol{R}_{12}\boldsymbol{g}_2$. based on that $\boldsymbol{G}$ can be written as:

$$\boldsymbol{G} = \begin{bmatrix}\boldsymbol{g}_1 \\ \boldsymbol{g}_2\end{bmatrix} = \begin{bmatrix}-\boldsymbol{R}_{11}^{-1}\boldsymbol{R}_{12} \\ \boldsymbol{I}_{(L-P)}\end{bmatrix}\boldsymbol{g}_2 = \boldsymbol{E}\boldsymbol{g}_2 \tag{4.24}$$

In otherwords $\widehat{\boldsymbol{R}}\boldsymbol{E} = \boldsymbol{0}$. Since the columns of the basis of the null space $\boldsymbol{E}$ are not orthonormal. And to satisfy orthonormality we use orthogonal projection onto this subspace in order to improve the performance by making the basis of null space of $\boldsymbol{E}$ orthonormal.

$$\boldsymbol{E}_o = \boldsymbol{E}(\boldsymbol{E}^H\boldsymbol{E})^{-1}\boldsymbol{E}^H \tag{4.25}$$

Using (4.25) in the MUSIC spectrum defined in (4.18) and using the procedure defined in [15] the CSI can be extracted easily.

4.5 Least square (LS)

As shown in[15] the LS square method can be used to estimate the channel vectors $\{\boldsymbol{h}_p\}_{p=1}^{P}$. Using (4.11) with J_t representing the number of training symbols (training blocks). With $n = 1,2, \dots J_t$ formula (4.11) can be written as:

$$\overline{\boldsymbol{y}(n)} = \sum_{p=1}^{P} \bar{\bar{A}}\left(\overline{\boldsymbol{s}_p(n)}\right)\boldsymbol{h}_p + \overline{\boldsymbol{zz}}\,(n) \tag{4.26}$$

Where $\overline{\boldsymbol{s}_p(n)}$ and $\overline{\boldsymbol{zz}}(n)$ is a vectorized signal and noise components respectively [16]. Matrix $\bar{\bar{\boldsymbol{A}}}(\overline{\boldsymbol{s}_p})$ is a $(2MT \times 2MN)$ size with the k-th column given by:

$$\bar{\bar{\boldsymbol{A}}}\left(\overline{\boldsymbol{s}_p}\right) = \boldsymbol{A}(e_k)\overline{\boldsymbol{s}_p} \tag{4.27}$$

Where e_k is a $2MN \times 1$unit vector defined as below:

$$e_i = \begin{cases} 1, & i = k \\ 0, & i = else \end{cases}$$

(4.26) can be rewritten as :

$$\boldsymbol{y}(n) = \boldsymbol{\Gamma}(n)\boldsymbol{v} + \boldsymbol{zz}(n) \tag{4.28}$$

Where

$$\boldsymbol{v} \coloneqq \left[\mathbf{h}_1^{\mathrm{T}} \quad \mathbf{h}_2^{\mathrm{T}} \quad \ldots \ldots \ \mathbf{h}_{\mathrm{p}}^{\mathrm{T}}\right]^{\mathrm{T}}$$

and

$$\boldsymbol{\Gamma}(n) \coloneqq \left[\bar{\bar{\mathbf{A}}}\left(\overline{\boldsymbol{s}_1(\mathrm{n})}\right) \ \bar{\bar{\mathbf{A}}}\left(\overline{\boldsymbol{s}_2(\mathrm{n})}\right) \ldots \ldots \bar{\bar{\mathbf{A}}}\left(\overline{\boldsymbol{s}_{\mathrm{p}}(\mathrm{n})}\right)\right]$$

We can rearrange $\boldsymbol{y}(n)$ and $\boldsymbol{zz}(n)$ to long vectors of size $2MTJ_t \times 1$ as below:

$$\boldsymbol{\chi} \coloneqq [\mathbf{y}^{\mathrm{T}}(1) \ \ \mathbf{y}^{\mathrm{T}}(2) \ldots \ldots \ \mathbf{y}^{\mathrm{T}}(\mathrm{J_t})]^{\mathrm{T}}$$

And

$$\boldsymbol{\eta} \coloneqq [\mathbf{zz}^{\mathrm{T}}(1) \quad \mathbf{zz}^{\mathrm{T}}(2) \ldots \ldots \ \mathbf{zz}^{\mathrm{T}}(\mathrm{J_t})]^{\mathrm{T}}$$

We can define matrix $\boldsymbol{\Gamma}(n)$ of size $2MTJ_t \times 2PMN$ as below,such that $\boldsymbol{\chi}$ can be written as (4.29)

$$\bar{\bar{\boldsymbol{\Gamma}}} = [\boldsymbol{\Gamma}^T(1) \quad \boldsymbol{\Gamma}^T(1) \ldots \ldots \boldsymbol{\Gamma}^T(\mathrm{J_t})]$$

$$\boldsymbol{\chi} = \bar{\bar{\boldsymbol{\Gamma}}}\,\boldsymbol{v} + \boldsymbol{\eta} \tag{4.29}$$

Then LS estimation is given by (based on 4.7) :

$$\hat{\boldsymbol{v}} = \left(\bar{\bar{\boldsymbol{\Gamma}}}^H \bar{\bar{\boldsymbol{\Gamma}}}\right)^{-1} \bar{\bar{\boldsymbol{\Gamma}}}\,\boldsymbol{\chi} \tag{4.30}$$

Matrix $\bar{\bar{\boldsymbol{\Gamma}}}$ can be made full rank by choosing the appropriate size of $\mathrm{J_t}$.

4.6 CAPON

CAPON method was suggested by [15] for channel estimation for MU-MIMO system. When applied to MU-MIMO channel estimation problems the Capon linear receiver can be considered as a sort of spatio–temporal filter [15]. The Capon "spectrum" is defined in (4.31) and it defined as the output of the corresponding Capon receiver:

$$P_C^K(\boldsymbol{h}) = \frac{1}{a_K^T(\boldsymbol{h})\boldsymbol{R}^{-1}a_K(\boldsymbol{h})} \tag{4.31}$$

Where $\boldsymbol{R}$ is the data covariance matrix and $a_K(\cdot)$ is a linear operator in the normalized channel vector $\boldsymbol{h} = {\boldsymbol{h}_p}/{\|\boldsymbol{h}_p\|}$ for the $K = 1, \ldots, 2K$ the number of recivers to be used in parallel to estimate the symbols vector. In [15] they proposed to estimate the normalized channel vector as the P values that minimize $Q_C(\boldsymbol{h})$ given in (4.32) based on the fact that the channel vectors $\{\boldsymbol{h}_p\}_{p=1}^{P}$ are linearly independent.

$$Q_C(\boldsymbol{h}) = \boldsymbol{h}^T \left(\sum_{k=1}^{2K} \boldsymbol{\Phi}_k^T \, \boldsymbol{R}^{-1} \boldsymbol{\Phi}_k \right) \boldsymbol{h} \tag{4.32}$$

Based on (4.32) the channel vectors $\{\boldsymbol{h}_p\}_{p=1}^{P}$ are expected to belong to the subspace spanned by the minor eignvectors of the matrix given in (4.33) below, or more accurately on the eignvectors crossponding to the P smallest eignvalues of this matrix.

$$\boldsymbol{\Psi} \coloneqq \sum_{k=1}^{2K} \boldsymbol{\Phi}_k^T \, \boldsymbol{R}^{-1} \boldsymbol{\Phi}_k \tag{4.33}$$

It has been shown in [18] and [15] that $\boldsymbol{h}_p$ can be defined as below bearing in mind mind that each of the P smallest eignvalues of matrix $\boldsymbol{\Psi}$ has a multiplicity order L (that depends on the OSTBC used and the number of antennas. For our system model, and OSTBC used the value of L is one. Denoting the eignvectors corresponding to the smallest eignvalues of $\boldsymbol{\Psi}$ as $\boldsymbol{u}_k$ for $k = 1, \ldots, LP$ we can redefine $\boldsymbol{h}_p$ as below:

$$\boldsymbol{h}_p = \sum_{k=1}^{LP} \alpha_{pk}\, \boldsymbol{u}_k \tag{4.34}$$

To find $\{\boldsymbol{u}_k\}_{k=1}^{LP}$ we need only the knowledge of the data covariance matrix $\boldsymbol{R}$ which can be estimated without any training data, but training data is needed to find the real coefficients α_{pk}. These Coefficients are estimated using LS approach and the detailed proof of how these coefficients can be estimated are shown in [15].

4.6 Eigenvector Method

The Eigenvector method (EV) is a common used method for spectral estimation problems. It's closely related to the MUSIC algorithm [20]. Its estimates the Noise subspace from the peaks of the Eigen spectrum (MUSIC Spectrum) shown in (4.18), the only difference here is the definition or the method used to obtain the null space. For the RRQR case we developed the problem based on the MUSIC spectrum and obtained the Null space by multiplying the smallest Eigen vectors (that corresponds to the smallest Eigen values of the covariance Matrix) and their conjugates. In the EV case we multiple by the Eigen values that has been already obtained during the extraction of the Eigen vectors from the covariance matrix. We can define the Eigen values for the noise-subspace as $\boldsymbol{w}$ with size $1 \times (2MT - 2KP)$ and the EV space can be written as (4.35).

$$P_{EV} = \frac{1}{tr\{\boldsymbol{A}^T(\boldsymbol{h}_p)\boldsymbol{E}\boldsymbol{w}\boldsymbol{E}^T\boldsymbol{A}(\boldsymbol{h}_p)\}} \tag{3.35}$$

The same procedure applies in the derivation of matrix $\boldsymbol{\Omega}$ and then to estimate the channel vector $\widehat{\boldsymbol{h}}_p$.

4.7 Simulation Resutls

In The following pages; we will show the Simulations results for the proposed methods compared to the existing ones. Our reference methods are the LS, and the Capon suggested by [15].The First user SNR is better by 2.5 dB in each of our figures. The Figures compares the MMSE as per our mentioned formula in (4.9). The results has been generated for three different numbers of training sequences (Jt=3, 4, 5). Figures [4.1] and [4.2] compare the performance for three training symbols, while Figures [4.3] and [4.4] compares the performance for four training symbols, and finally Figures [4.5] and [4.6] compares the performance for five training symbols. In all figures the proposed method gives a better performance compared to the LS method. It's essential to mention that the number of the training symbols used in the simulation is very small, yet the results are very good for fair SNR values (10 to 15 dB).As expected the performance is better as the training Symbols number increase. In all cases the Eigen Value method and the Capon method are showing a comparable performance, and both of them perform better when compared to the RRQR method. At high SNR values the RRQR, Eigen Value, and the Capon almost perform the same, and this is expected since subspace estimation methods.

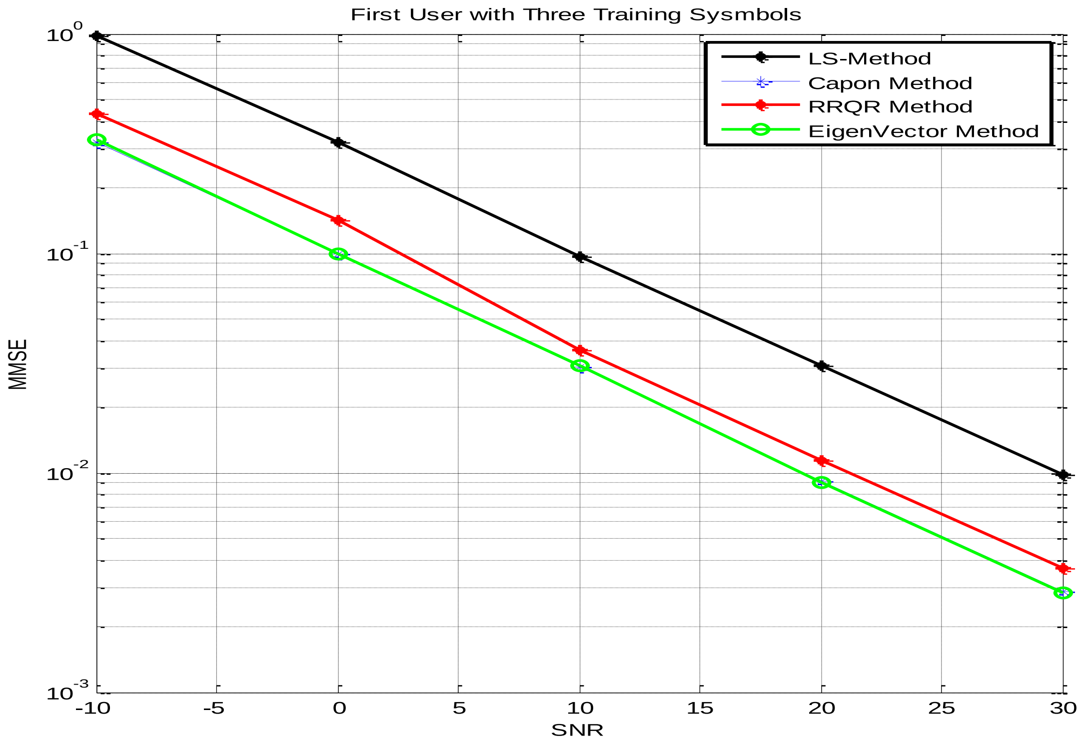

Figure 4.1 MMSE versus SNR for 1st User with 3 Training Symbols

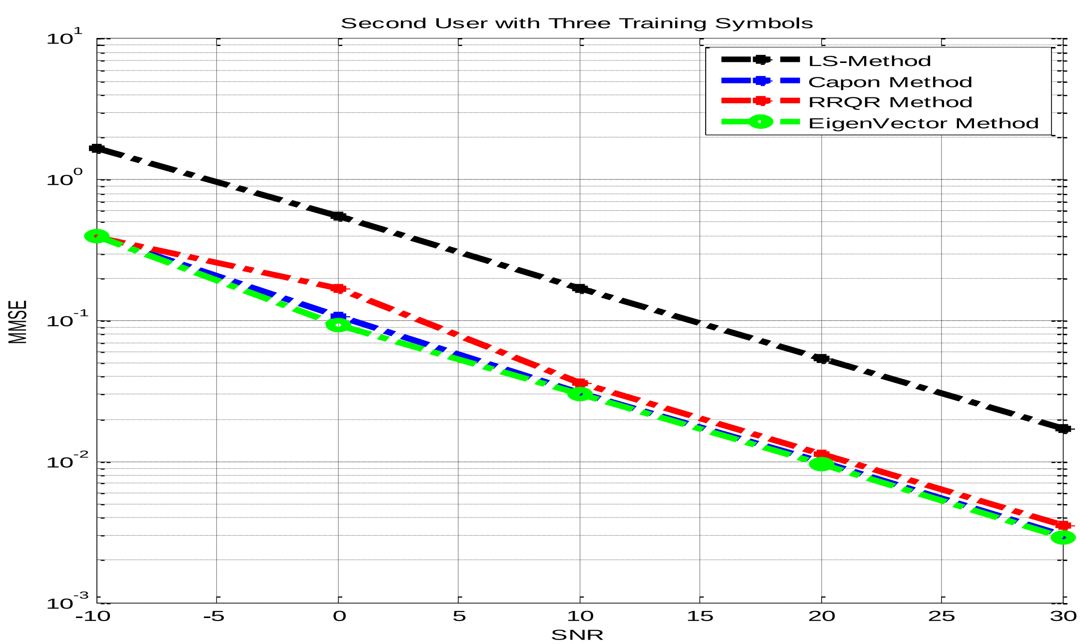

Figure 4.2 MMSE versus SNR for 2nd User with 3 Training Symbols

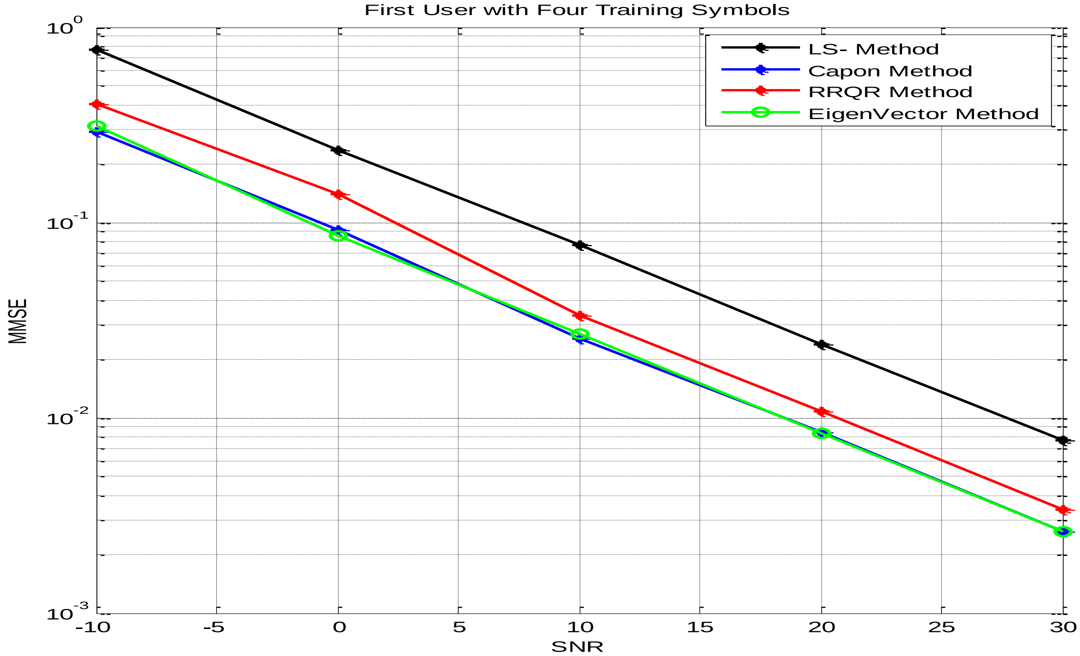

Figure 4.3 MMSE versus SNR for 1st User with 4 Training Symbols

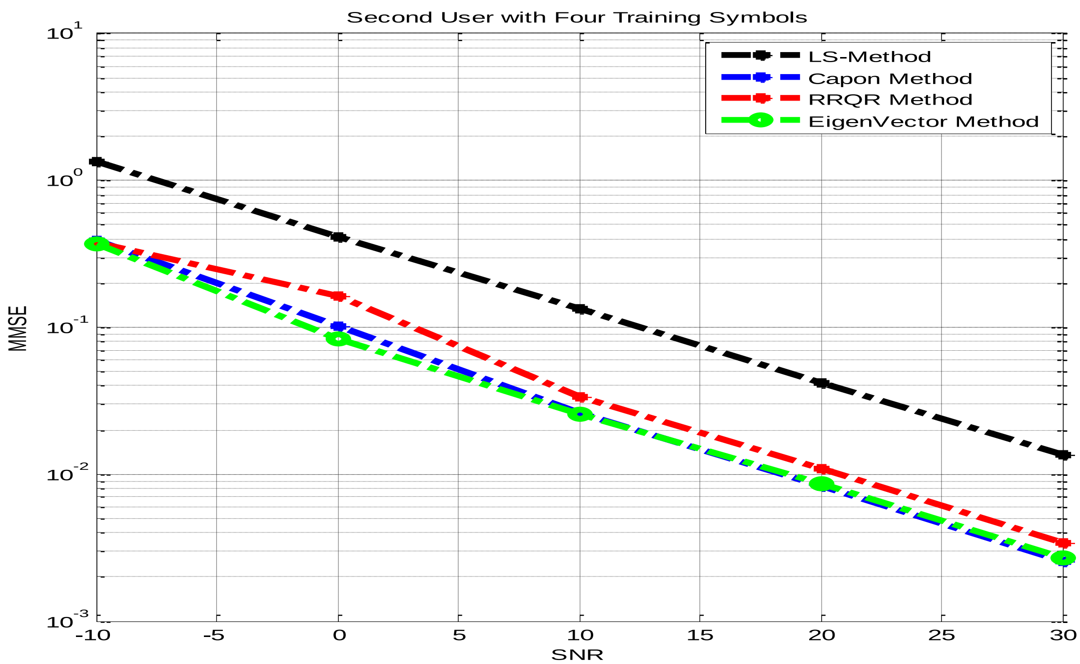

Figure 4.4 MMSE versus SNR for 2nd User with 4 Training Symbols

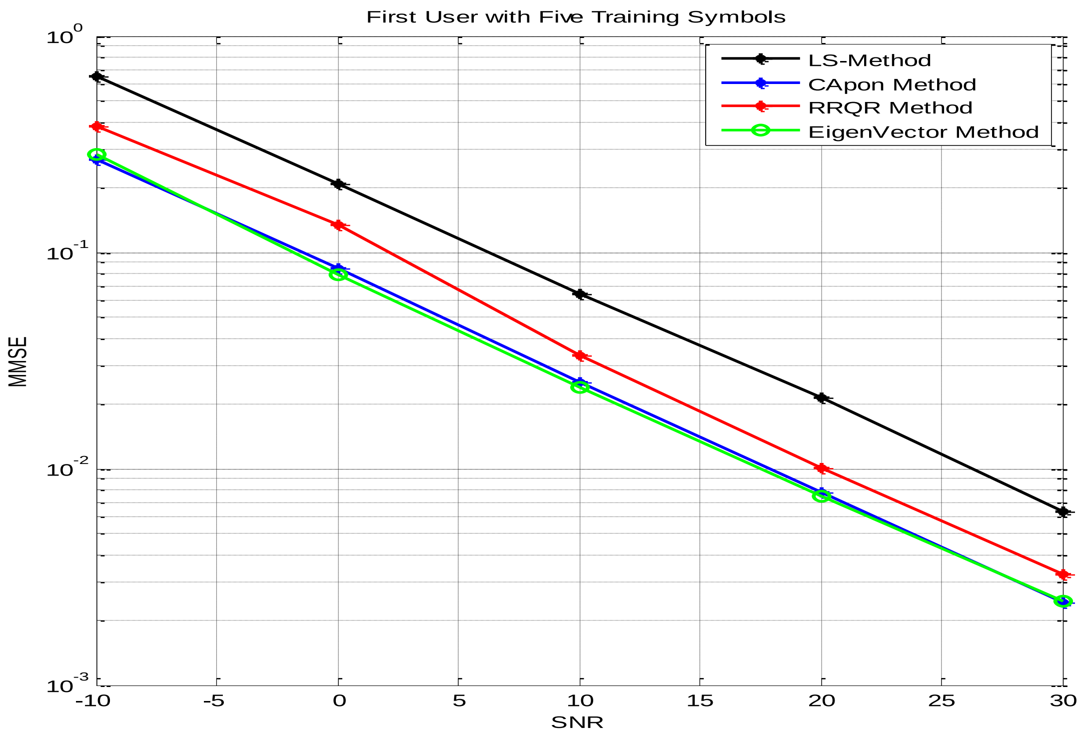

Figure 4.5 MMSE versus SNR for 1st User with 5 Training Symbols

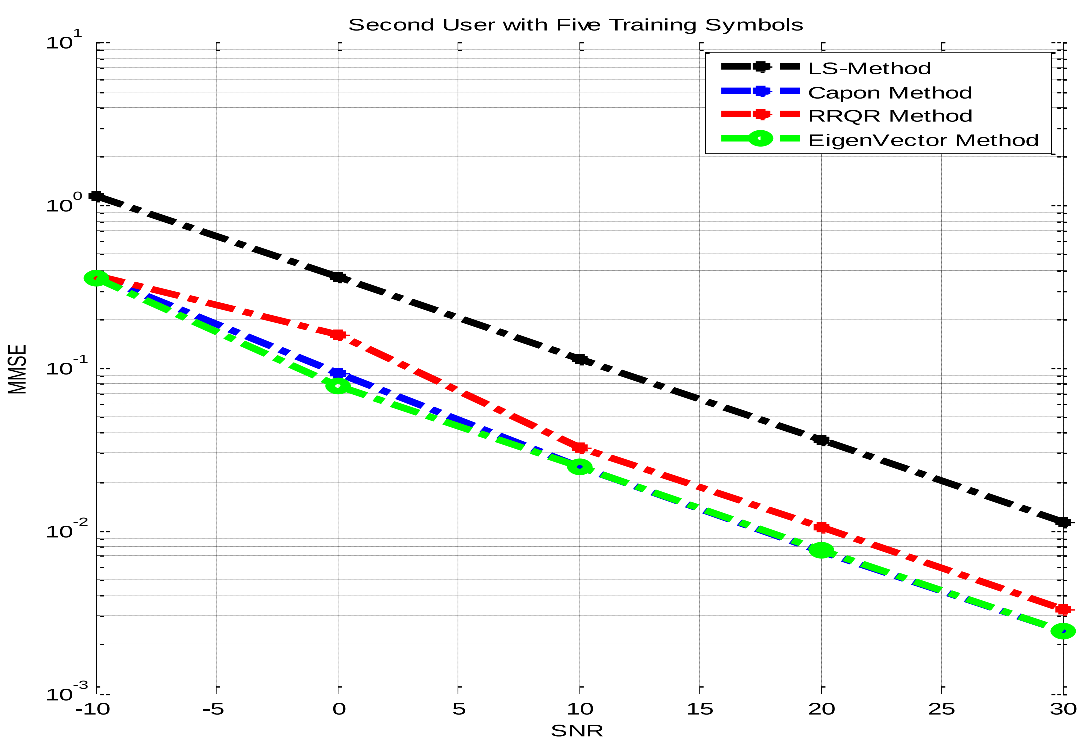

Figure 4.6 MMSE versus SNR for 2nd User with 5 Training Symbols

In the following figures, the simulation shows MMSE Vs number of training blocks(from 2 to 7 blocks) for a fixed SNR values of (-10, 0, and 10dB) for both users with the same condition (2.5dB difference) between the two users. As expected, as the number of training blocks increase the error decrease. The Capon and Eigen Value method are showing a superior performance when compared to LS-method and RRQR method.

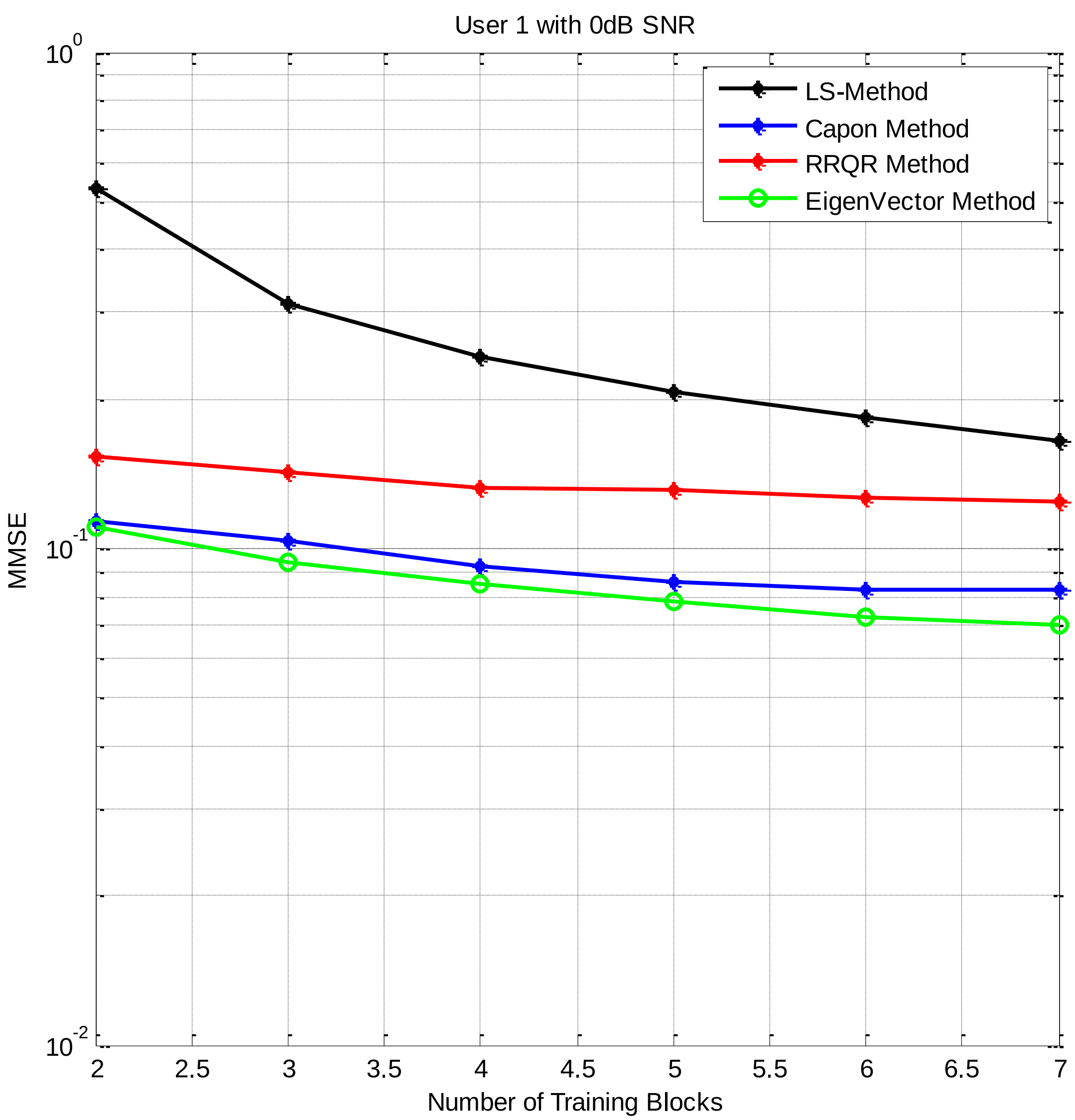

Figure 4.7 MMSE versus Training Symbols for 1st User with 0dB SNR

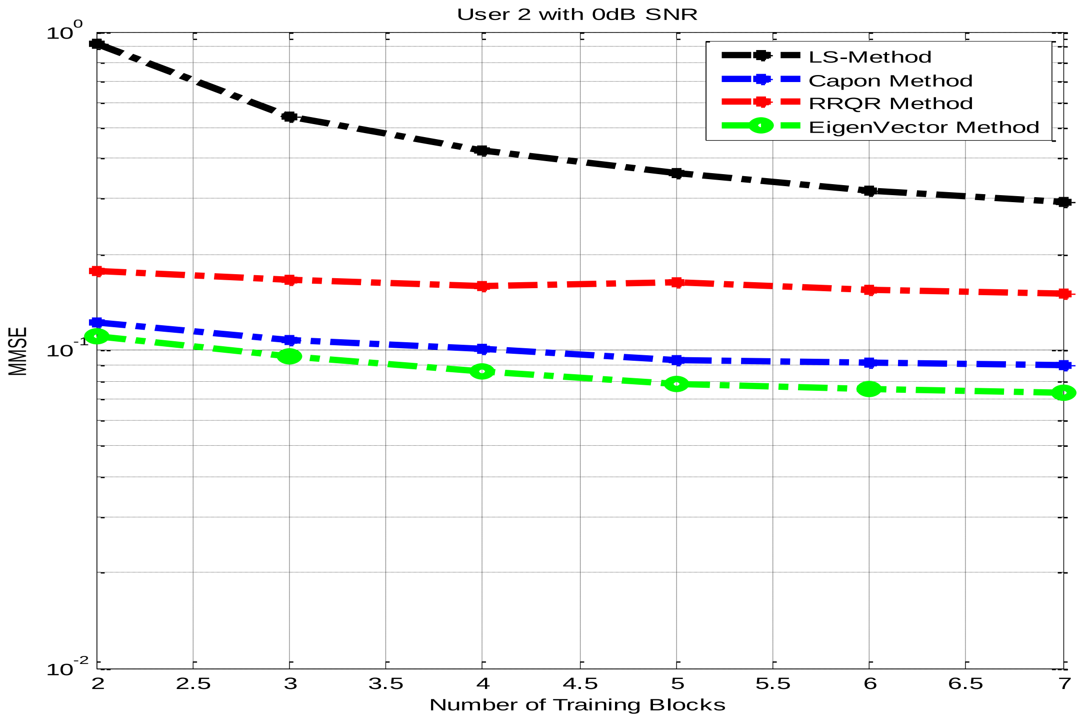

Figure 4.8 MMSE versus Training Symbols for 2nd User with 0dB SNR

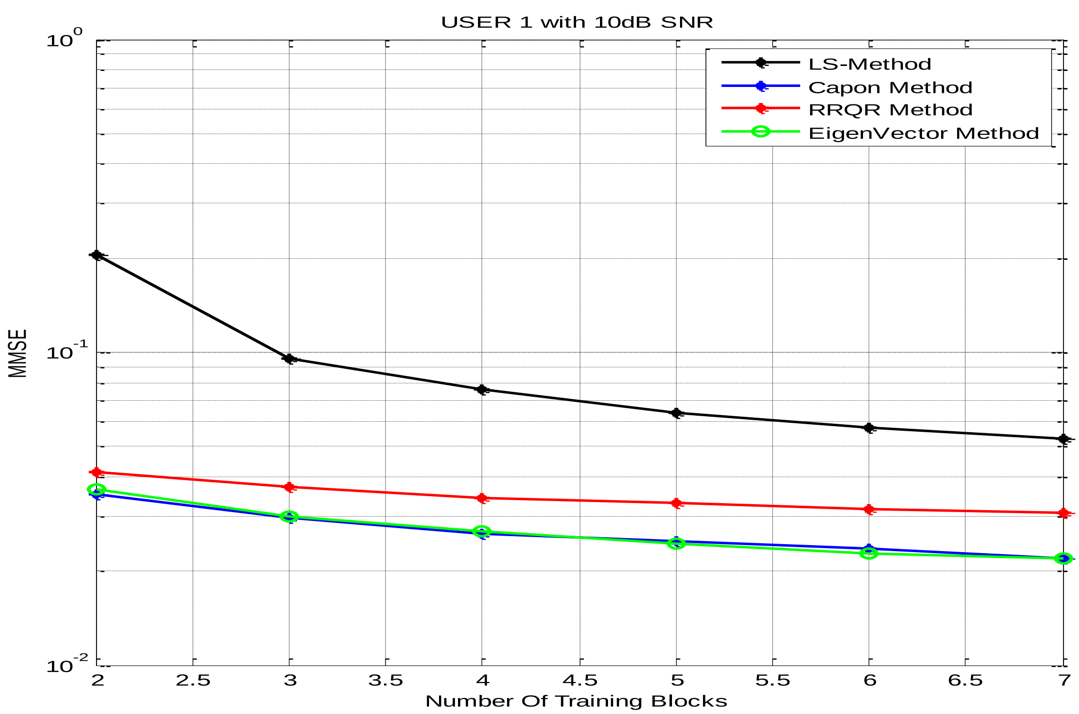

Figure 4.9 MMSE versus Training Symbols for 1st User with 10dB SNR

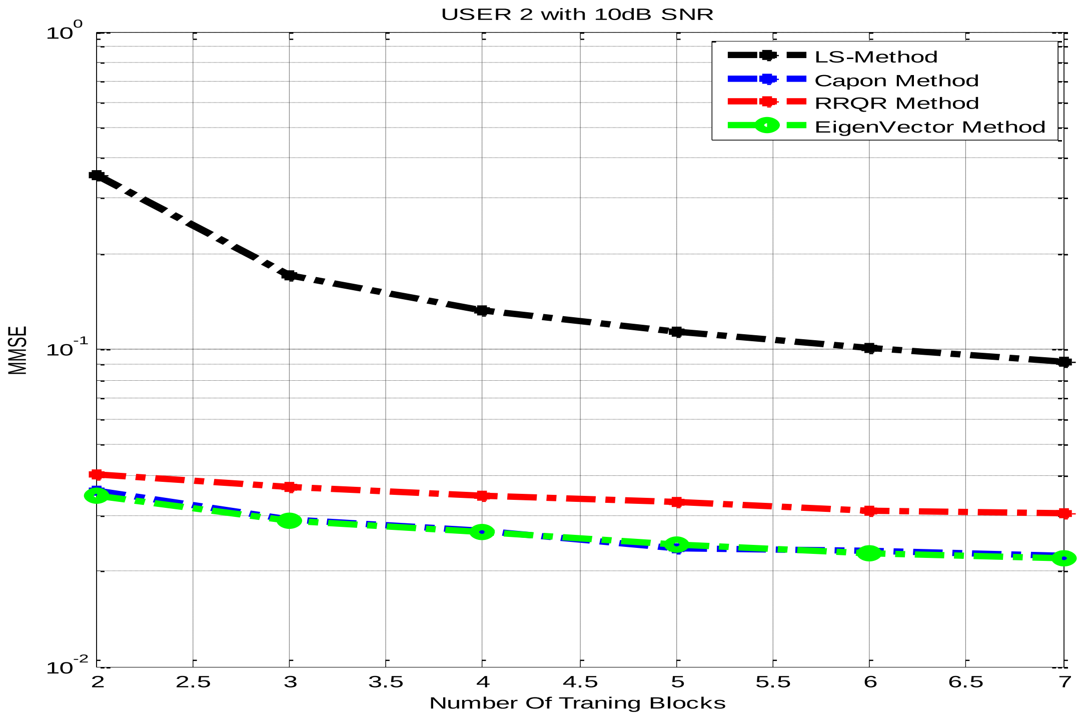

Figure 4.10 MMSE versus Training Symbols for 2nd User with 10dB SNR

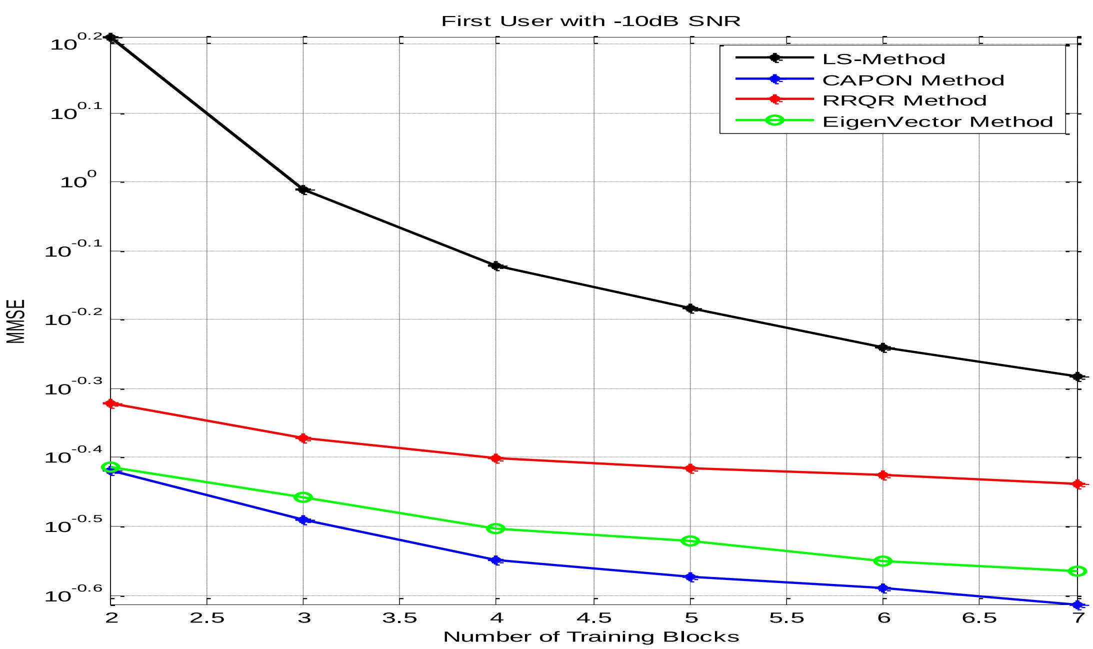

Figure 4.11 MMSE versus Training Symbols for 1st User with -10dB SNR

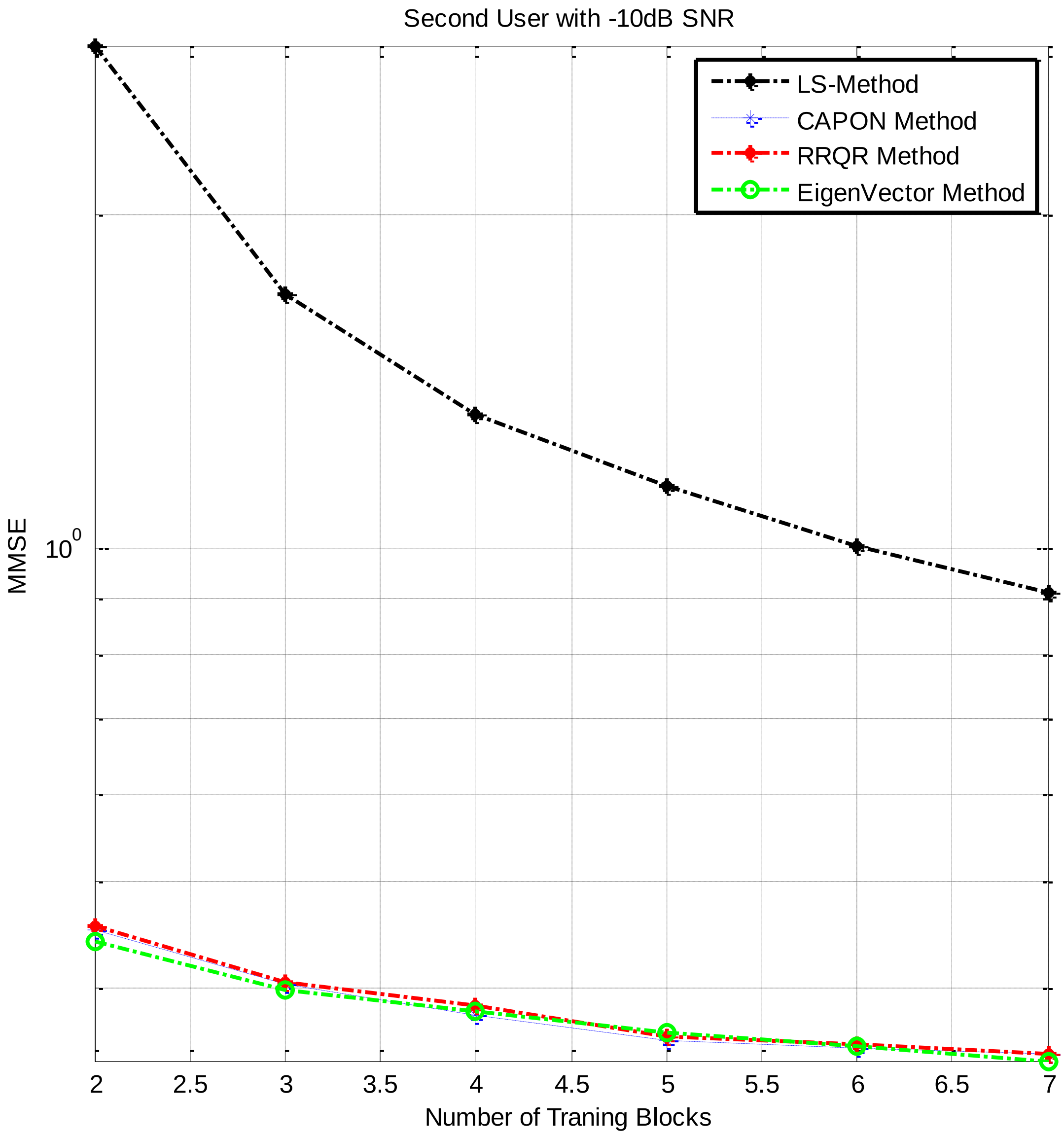

Figure 4.12 MMSE versus Training Symbols for 2nd User with -10dB SNR

It's interesting to notice the performance of the subspace methods compared to the LS- Method at low SNR values.

Chapter 5

CONCLUSIONS AND FUTURE WORK

Channel estimation problems are of vital importance to the telecommunication literature; it gives the receiver side a full knowledge of the channel state and hence helps in detecting the transmitted information precisely. Among the available approaches, we have explored three methods for Semi-blind approach, and we have introduced a common method used in Spectral estimation; the Eigenvector (EV) method to the channel estimation problem for MU-MIMO systems.

This thesis presents a comparison between three common methods for MU-MIMO semi-blind channel estimation and the EV method. The EV method showed a comparable performance in terms of the MMSE with the CAPON method and an improved performance when compared to the least Square (LS) method and the Rank Revealing QR method. The Eigenvector method depends on the extraction of the Noise-subspace that corresponds to the Null space extracted from the received data covariance matrix. When compared to the MUSIC search algorithms the EV includes the use of the Eigen values corresponding to the smallest Eigen vectors of the data covariance matrix in the extraction of the null space.

This thesis has explored techniques used for semi-blind channel estimation for Flat-block fading, a future work can be done to explore and suggest techniques to the Fast Selective channels case. Another work can be done on the signal sub-space side rather than the noise-subspace side that has been explored in this thesis. A common methods used with the signal subspace in the spectral estimation can be used in channel estimation problems like Blackmann-Tukey, Minimum Variance and Autoregressive methods.

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