# Study of Kinematic and Kinetic Aspects of Mechanisms Using Tools of CAD Solid Modelers

Blair J McDonald[1]
Prathivadi Ravikumar[1]
Il-Seop Shin[1]
Khaled Zbeeb[1]
Puneeth N.[2]
[1]*Western Illinois University-Quad Cities*
[2]*Vivekananda College of Engineering and Technology*
*b-mcdonald2@wiu.edu*; *pb-ravikumar@wiu.edu*; *i-shin@wiu.edu*; *k-zbeeb@wiu.edu*;
*puneethn0015@gmail.com*

**Abstract:** In engineering education, the kinematic and kinetic study of mechanisms is a subject that is as challenging to teach as it is to learn. Modern computer aided design (CAD) and solid modeling software, such as SolidWorks, enables the synthesis, analysis and evaluation of mechanisms throughout the design process and in greater detail than has ever been available. In the past, a designer carefully selected a limited number of instants in time where position, velocity, acceleration, force and torque were painstakingly determined to evaluate performance. This was a complex process that required skill and attention to detail. Now, using solid modeler tools such as Sketch, Assembly and Motion Study, these parameters can be continuously evaluated throughout the cycle of a mechanism whether it be simple or complex. As an example, a relatively simple rocker linkage is modeled. The results of motion parameters at specific stages of the cycle are determined theoretically and compared to those obtained using the solid modeler.

## 1. INTRODUCTION

The design 'stem' of mechanical engineering involves statics (Plesha, 2010), dynamics (Beer, 2019), vibrations (Thomson, 1998), design processes (Ullman, 2015), the mechanical design of machine elements (Budynas, 2014), manufacturing engineering (Elshennawy, 2015), and the focus of this paper, mechanisms (Norton, 2012). The study of mechanisms involves synthesis (Section 2), kinematics (Section 3) and kinetics (Section 4), all of which will be accomplished, in this paper, using tools available in computer aided design (CAD) modeling software, namely SolidWorks (Howard, 2018) (SolidWorks, 2020). Synthesis is the process of first creating a conceptual mechanism that can achieve some required path of motion. Formal methods of synthesis, where available, must be utilized. Synthesis using the sketch tools in SolidWorks will be illustrated. Kinematics is the study of any mechanism to gain a better understanding and control of parameters such as velocities and accelerations involved in motion. Synthesis is also kinematics with a focus on position. A method combining the kinematic equations and vector polygons using SolidWorks sketches will be used to study the kinematics at the starting position of a synthesized mechanism without loss of generality. To study kinematics over a period of motion of the mechanism, it is more efficient to use the Motion Study tool within SolidWorks. In this study, sketches alone are insufficient, solid models are required. The results of both methods, for a particular position of the mechanism or instance in time, can then be compared. When the results match, the accuracy and equivalence of the studies is demonstrated. To quantify the forces and torques that interact to produce motion parameters, such as position, velocity and acceleration, a kinetic study must follow. A theoretical kinetic solution of the starting position, using free-body-diagrams and Newton's Laws of Motion, is illustrated. As with the kinematics, to study the kinetics over a period of motion, a solid model and the

Motion Study tool is used. Results of the solid model obtained for the same position and instance in time as theoretical solution, when compared, will confirm the accuracy of the model.

## 2. SYNTHESIS OF A MECHANISM

The motion to be achieved through a mechanism is a rocker with an angular displacement that is between 60 and 65 degrees. The rocker movement needs to be in the second quadrant, be 10 inches long between pin centers and be part of a Grashof crank-rocker 4-bar mechanism. The fixed link of the 4-bar mechanism needs to be horizontal and 19 inches long between pin centers. Some kinematic conditions, such as the crank's initial position, initial angular velocity, angular velocity over time and initial angular acceleration are available and provided during the kinematic study. Loads on the mechanism are also available and provided during the kinetic study.

It is only possible to synthesize a mechanism to meet the above requirements if they are all realizable. Since there is no objective way to determine this ahead of time, there may need to be a certain amount of give and take in the requirements during the synthesis process. An efficient way to implement the graphical methods established for synthesizing a mechanism is to use the sketch tools within SolidWorks. Figures 1 through 4 illustrate the outcome of using the sketch tool following methods outlined by Robert L. Norton in *Design of Machinery* on pages 102-119 (Norton, 2012).

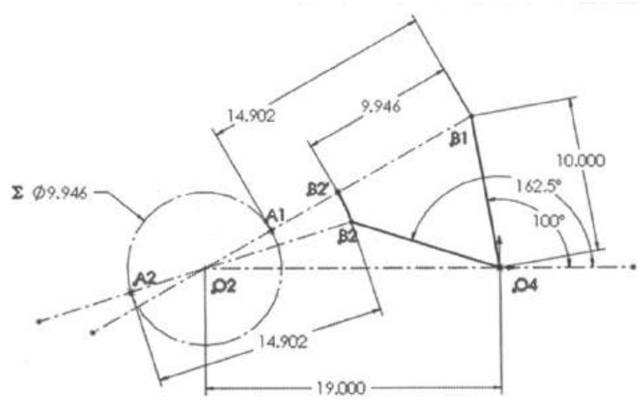


**Figure 1. Synthesis of the basic mechanism**

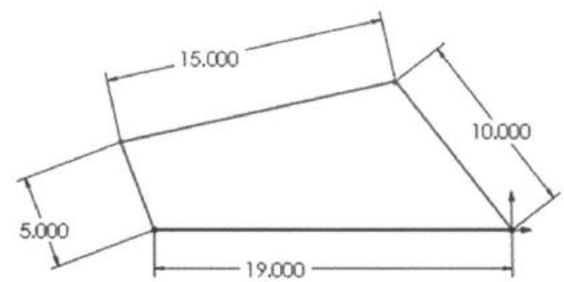


**Figure 2. The refined basic mechanism**

The basic mechanism shown in Figure 1 meets the stated requirements; the angular displacement of the rocker is 62.5 degrees, and the displacement is within the second quadrant. This mechanism is a Grashof crank-rocker because the sum of the shortest and longest links is less than the sum of the lengths of the other two links and one of the links next to the shortest link is grounded. Hence, having met the Grashof condition, the crank can completely rotate, and the rocker oscillates between the extremes shown. Since integer values are preferred for the length of links, the basic mechanism can be adjusted by rounding the crank and coupler lengths to the closest integers as shown in Figure 2. A quick check of the criteria confirms that the Grashof conditions are still met.

The sketches shown in Figures 1 and 2 were drawn in SolidWorks as separate sketch layers, one on top of the other using "Hide" and "Show" options, to facilitate faster construction and changes. The importance of synthesis and the outcome achieved thus far in this example should not be underemphasized to engineering students because real-world challenges are a lot more about synthesis than analysis even though most engineering courses focus on the analysis of a given design. Practicing engineers are often solely responsible for the synthesis stages of design work.

## 3. KINEMATICS OF THE SYNTHESIZED MECHANISM

The synthesis process resulted in a skeletal linkage that can now be used for further kinematic and kinetic studies. The mechanism is grounded using a fixed link, Link 1. Without loss of generality, a starting position for the investigation can be arbitrarily established with the Crank (Link 2) oriented at 60 degrees, as shown in Figure 3. A hybrid method using kinematic equations for velocity or acceleration in combination with graphical polygon constructions for velocity or acceleration developed in SolidWorks sketch mode can be used to analyze the starting position. Obviously, the same methodology can be applied to any other Crank position; however, to evaluate a complete cycle in this manner would be an arduous and time-consuming process. The Motion Study simulation tools within SolidWorks provide an alternative that continuously evaluates and reports parameters throughout the entire cycle. With the Crank in the starting position, parameter values obtained using both methods can be compared and assessed to evaluate the reliability of the CAD model simulation.

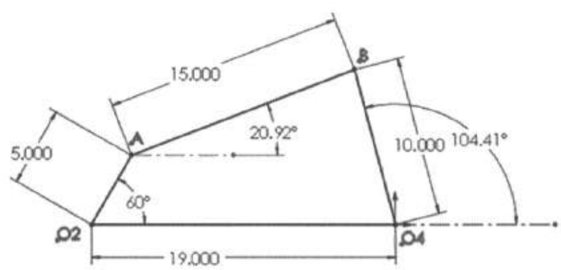


**Figure 3. Mechanism starting position**

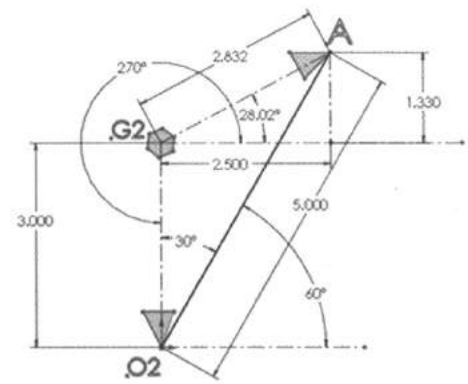


**Figure 4. Crank center of mass**

The mechanism being demonstrated is a single degree of freedom system; therefore, one coordinate is sufficient to associate position, angular velocity and angular acceleration at any instant in time. The center of mass of each link is required to calculate linear velocity and linear acceleration. Figures 4, 5 and 6 respectively show details for the center of mass of the moving links: Crank (Link 2), Coupler (Link 3) and Rocker (Link 4). Link 1 is fixed; hence, its center of mass information is not required. The ***P*** shown in Figure 5, is an external load. In these figures, the position vectors originate at the center of mass (hexagon) of each link and terminate at link connecting points. If vector methods are used, this method of locating the center of mass and connecting points is helpful when writing the equations of motion during rotation. These figures were drawn using the SolidWorks Sketch tools; their use simplifies determining the horizontal and vertical components of the position vectors.

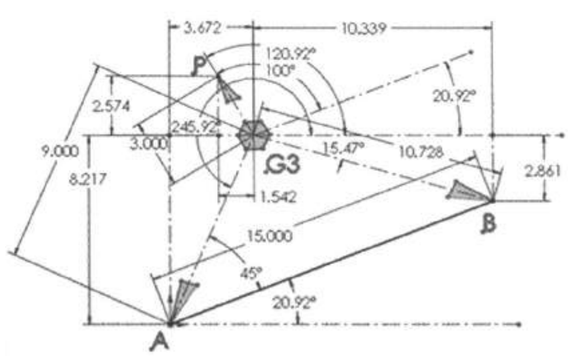


**Figure 5. Coupler center of mass**

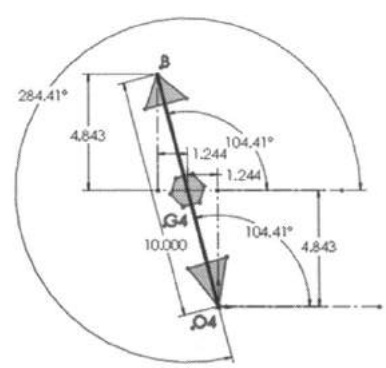


**Figure 6. Rocker center of mass**

At the initial time, with the Crank (Link 2) in the 60-degree starting position, the initial angular velocity is 1432.39 deg/s (25 rad/s) and the initial acceleration is -2291.83 deg/s$^2$ (-40 rad/s$^2$). Using these parameters, the linear velocities and accelerations of key points, such as link connecting points, and the angular velocities and accelerations of each link can be determined. The acceleration polygons and center of mass accelerations for each link in the mechanism are shown in Figures 7, 8, 9 and 10.

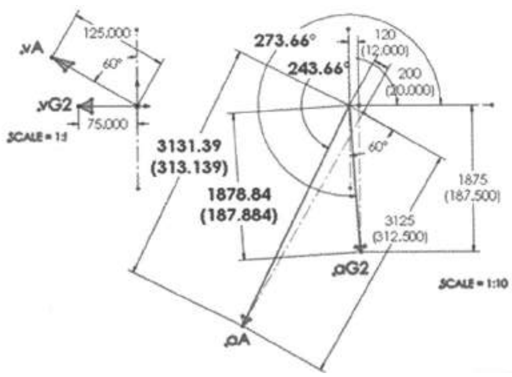


**Figure 7. Crank velocity polygon and acceleration polygon**

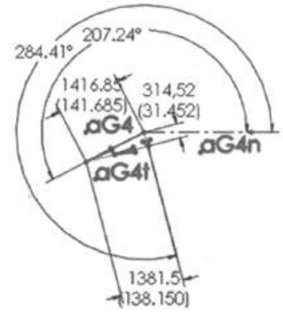


**Figure 8. Rocker acceleration polygon for acceleration of the center of mass**

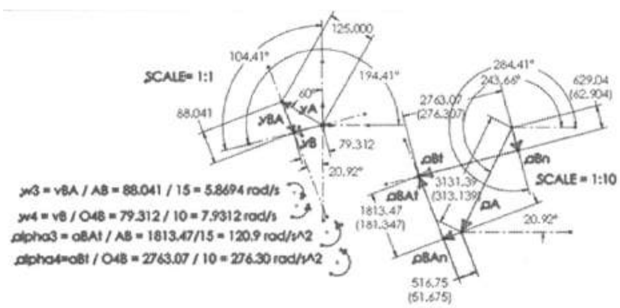


**Figure 9. Coupler velocity polygon and acceleration polygon**


**Figure 10. Coupler acceleration polygon for acceleration of the center of mass**

Table 1 summarizes the key kinematic values computed by the combined method of analysis using kinematic equations with polygons sketched in SolidWorks. These quantities along with external load information provide the data necessary to study the kinetics of the mechanism.

**Table 1. Summary of key kinematic values**

| | Angular Position (degrees) see Figure 6. | Angular Velocity (rad/s) & (deg/s) | Angular Acceleration (rad/s$^2$) & (deg/s$^2$) | Acceleration of Center of Mass (in/s$^2$ @ θ deg) |
|---|---|---|---|---|
| **Crank** | 60 | 25 (1432.40) | -40 (-2291.83) | 1878.84@273.66 |
| **Coupler** | 20.92 | -5.87 (-336.33) | 120.9 (6927.06) | 3646.6@226.51 |
| **Rocker** | 104.41 | 7.93 (454.36) | 276.30 (15830.83) | 1416.85@207.24 |
| Negative values indicate clockwise direction. | | | | |

## 4. KINETIC STUDY OF THE SYNTHESIZED MECHANISM

A kinetic study of the mechanism involves the calculation of source and internal forces and torques. These forces, along with the external loads and torques, produce the motion characterized by the kinematic study. The values presented in Table 1 characterize the motion of the mechanism at one instant, the starting position. This information, along with the external loads, mass and mass moment of inertia ($I_G$) of each moving link, at that same instant, is sufficient to find the other forces and torques necessary to satisfy the laws of motion at that instant. Table 2 provides the weight ($W$) and mass moment of inertia about the center of mass of each moving link. The external loads at the starting position are provided in Figure 11.

**Table 2. Weight and mass moment of inertia of each moving link**

| | Weight, $W$ (lb) | Mass Moment of Inertia about Center of Mass, $I_G$ (lb·in·sec$^2$) |
|---|---|---|
| **Crank** | 1.5 | 0.4 |
| **Coupler** | 7.7 | 1.5 |
| **Rocker** | 5.8 | 0.8 |

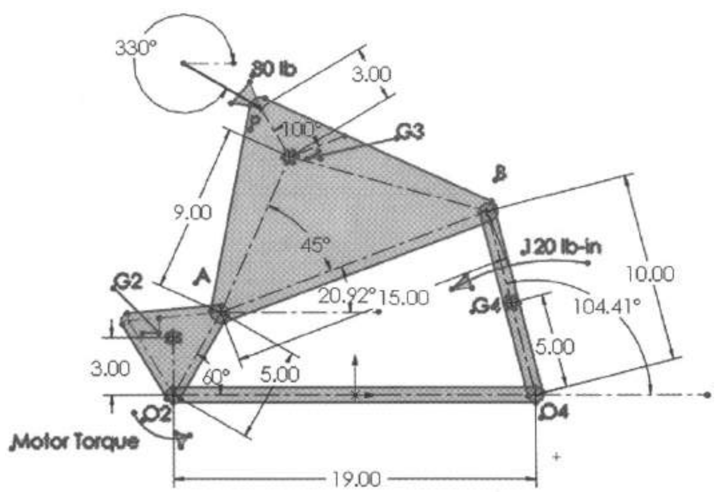


**Figure 11. External loads of 80 lb. force at *P* on the Coupler and 120 lb·in of torque on the Rocker; the source motor torque on the Crank is desired**

Newton's Laws of Motion are now applied to solve for the source motor torque applied to the Crank (Link 2) and the forces applied by each link on connecting links at the pin locations. This requires drawing free body diagrams of each moving link before applying the laws of motion. These diagrams are created in the CAD solid modeler and shown as Figures 12, 13 and 14.

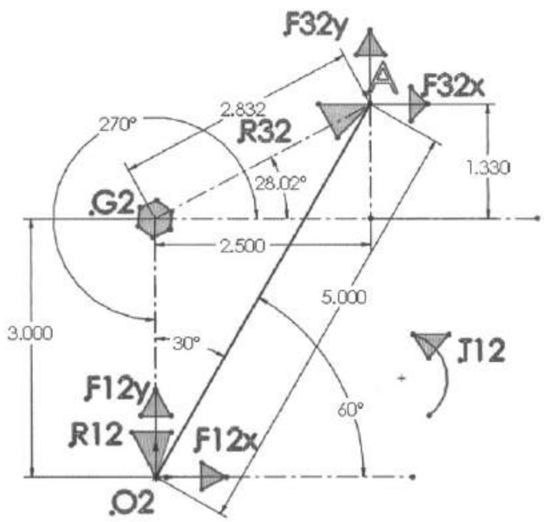


Figure 12. Free body diagram of the Crank

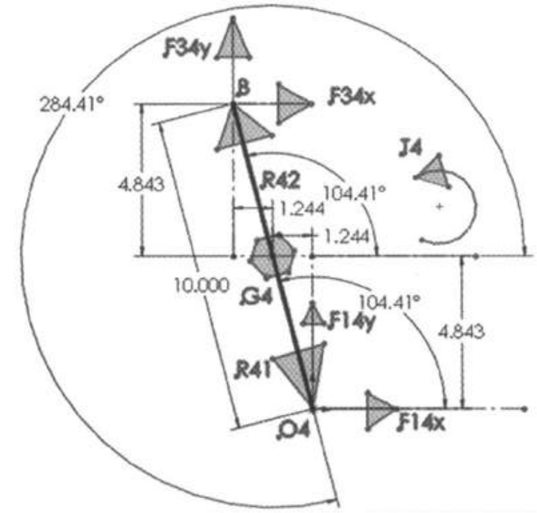


Figure 13. Free body diagram of the Rocker

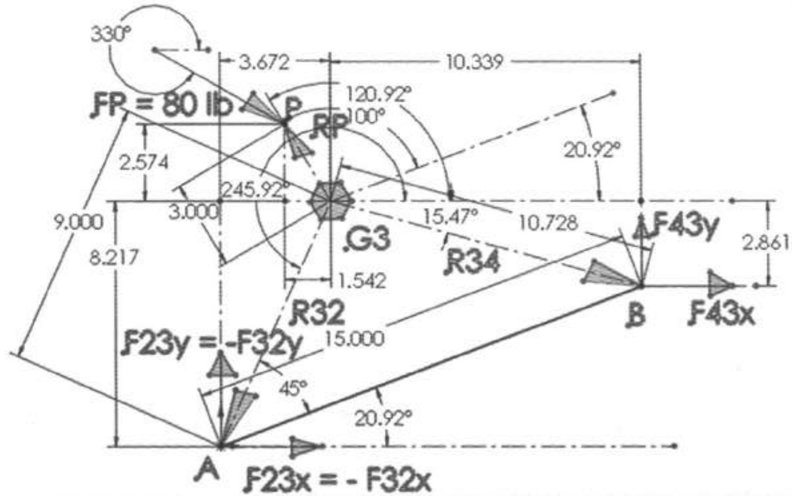


Figure 14. Free body diagram of the Coupler

There are three equations of motion for each of the three-moving links, or nine equations available to solve for the eight unknown forces $F_{12x}$, $F_{12y}$, $F_{32x}$, $F_{32y}$, $F_{43x}$, $F_{43y}$, $F_{14x}$, $F_{14y}$ *and* $T_{12}$ shown in the free body diagrams. Exercise care in setting up the equations; convert the weights given in Table 2 to masses and the mass moment of inertia units from lb·in·sec$^2$ to lb·in$^2$. The equations of motion for each moving link are as follows, for the Crank (Link 2):

$$F_{12_x} + F_{32_x} = m_2 a_{G2_x} \qquad \text{Eqn. 1}$$

$$F_{12_y} + F_{32_y} = m_2 a_{G2_y} \qquad \text{Eqn. 2}$$

$$T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{32_x} F_{32_y} - R_{32_y} F_{32_x}\right) = I_{G_2} \alpha_2 \qquad \text{Eqn. 3}$$

For the Coupler (Link 3):

$$F_{43_x} + F_{32_x} + F_{P_x} = m_{23} a_{G3_x} \qquad \text{Eqn. 4}$$

$$F_{43_y} + F_{32_y} + F_{P_y} = m_3 a_{G3_y} \qquad \text{Eqn. 5}$$

$$\left(R_{43_x}F_{43_y} - R_{43_y}F_{43_x}\right) + \left(R_{23_x}F_{32_y} - R_{23_y}F_{32_x}\right) + \left(R_{P_x}F_{P_y} - R_{P_y}F_{P_x}\right) = I_{G_3}\alpha_3 \quad \text{Eqn. 6}$$

For the Rocker (Link 4):

$$F_{14_x} - F_{43_x} = m_4 a_{G4_x} \quad \text{Eqn. 7}$$

$$F_{14_y} - F_{43_y} = m_4 a_{G4_y} \quad \text{Eqn. 8}$$

$$\left(R_{14_x}F_{14_y} - R_{14_y}F_{14_x}\right) - \left(R_{34_x}F_{43_y} - R_{34_y}F_{43_x}\right) + T_4 = I_{G_4}\alpha_4 \quad \text{Eqn. 9}$$

The resulting nine equations can be compared to those given in an example published by Norton in *Design of Machinery* on pages 592-595 (Norton, 2012). The results shown in Table 3 were obtained by solving these nine equations simultaneously using MATLab (The MathWorks, Inc., 2020); they match those published by Norton. Using SolidWorks to create the free body diagrams minimizes the risk of errors, is generally faster than preparing hand drawn figures and generates detailed diagrams from which the parameter values required to solve the equations of motion are easily obtained.

**Table 3. Results of the solution of the nine kinetic equations for the unknown forces and torque**

| $F_{12x}$ (lb) | $F_{12y}$ (lb) | $F_{32x}$ (lb) | $F_{32y}$ (lb) | $F_{43x}$ (lb) | $F_{43y}$ (lb) | $F_{14x}$ (lb) | $F_{14y}$ (lb) | $T_{12}$ (lb·in) |
|---|---|---|---|---|---|---|---|---|
| -117.55 | -107.50 | 118.02 | 100.21 | -1.32 | 87.45 | -20.25 | 77.70 | 243.05 |

## 5. USING MOTION STUDY TOOLS FOR KINEMATIC AND KINETIC SIMULATION

The Motion Study tool within SolidWorks will now be used to repeat the kinematic and kinetic study. Thus far, not much attention has been paid to the actual size, shape or materials used for the links. The link length (between pin locations), mass, center of mass and mass moment of inertia values have all been assumed. A solid model will be created using these assumed values rather than by specifying a detailed geometry and material from which the values would then be determined. This is possible in SolidWorks using an over-ride feature that allows the designer to specify values for geometric and/or material dependent parameters such as mass and mass moment of inertia. It is recognized that for a final design, these properties must by derived for the actual final shape being used, and that this shape is also required to determine assembly interference avoidance and manufacturing processes; however, for this study (and most preliminary designs), the actual shape is not critical. Figure 15 shows a trimetric view of the solid models of the four links and their assembly. The model is approximate as it is based solely upon the link lengths between pins and the proper mates between links. Link 1 is fixed, to make the assembly a proper mechanism, and the Crank (Link 2) is positioned at 60 degrees (the start position).

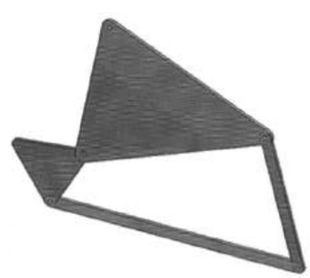

**Figure 15. Trimetric view of the assembly of solid models of the four links**

The Motion Study tool in the SolidWorks assembly file is used to conduct kinematic and kinetic simulations. The tool features three levels of simulation: Animation, Basic Motion and Motion Analysis. The Motion Analysis simulation includes all the analysis and outcomes desired for this study. To run the simulation, a rotary motor is attached to the Crank (Link 2) and the motion expressions shown below are set up to provide the Crank the starting angular velocity of 25 rad/s and angular acceleration of -40 rad/s. The horizontal and vertical components of the 80 lb. external force applied at point ***P*** on the Coupler (Link 3) and 120 lb·in external torque on the Rocker (Link 4) are set up. These external forces and torque are assumed to be constant throughout the motion in magnitude and direction; although, they could be variable and set up as expressions if needed. The animation and plotting parameters are then set up.

motor angular velocity = 565.6961*sin(6.283185307*TIME+2.271547)+1000 deg/s

motor angular acceleration = 3554.373*cos(6.283185307*TIME+2.271547 deg/$s^2$

The simulation provides visualization of the motion of the mechanism using animation and plots. For the rocker mechanism studied, continuous plots of the angular velocity and angular acceleration of the Coupler (Link 3), and the reaction force 1 and motor torque 2 on the Crank (Link 2) are shown for a duration of two seconds of time in Figures 16 through 19. The plots were created in the Results section of the Property Manager within the SolidWorks Motion Study tool. These are just a few of the plots that could be prepared since this, and similar information, can be extracted for any object, part or location in the solid model. The plots shown here confirm that the start time (t=0) values, as shown in Table 1 or Table 3, are matched by the simulation. This study demonstrates the functionality of solid modeling and applicability of using a hybrid method of analysis where information from the solid model is combined with classic analysis techniques. In the real-world, simulation is used to minimize the potential for error, maximize efficiency, and provide improved design solutions through the iterative processes that are typical of design. Simulations provide better visualization and allow more iterations.

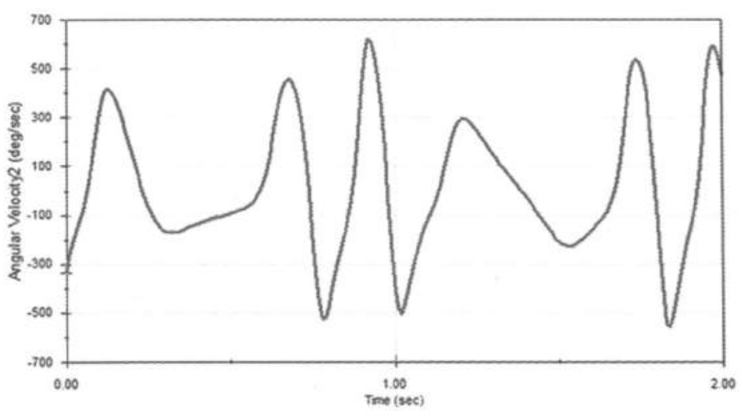


**Figure 16. Angular velocity (deg/s) of the Coupler (Link 3) vs Time (sec).**

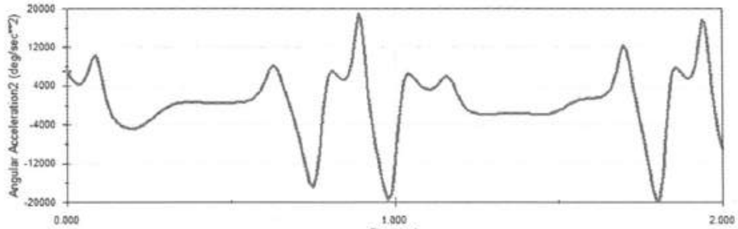


**Figure 17. Angular acceleration (deg/$s^2$) of the Coupler (Link 3) vs Time (sec).**

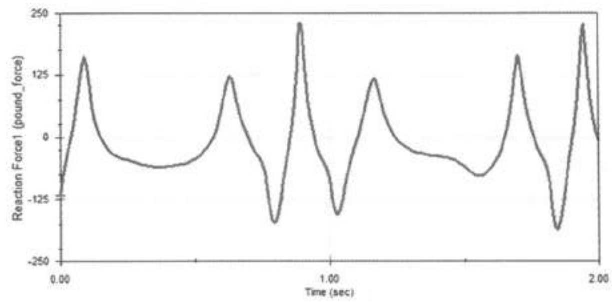


**Figure 18. Horizontal force $F_{12x}$ (lb) by Link 1 (Fixed) on the Crank (Link 2) at $O_2$ vs Time (sec).**

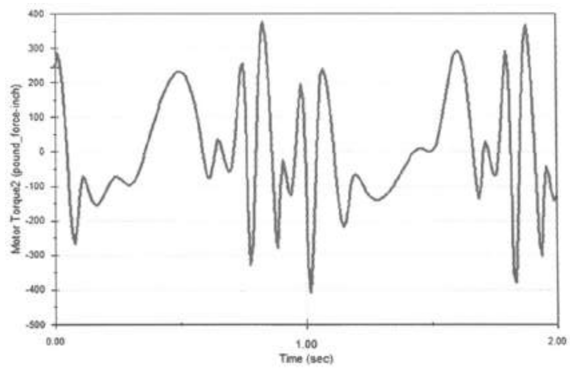


**Figure 19. Motor torque (lb·in) on the Crank (Link 2) vs Time (sec).**

## 6. CONCLUSIONS

The practice of using a solid modeler simulation that is supported by an alternative means of analysis to establish the simulation accuracy and reliability has been presented through the study of a simple rocker mechanism. The ability to quickly solve advanced synthesis and analysis problems involved with mechanism design has been demonstrated, as was the ability to merge CAD sketches with traditional methods of analysis thus creating hybrid analysis method and an alternative means of checking the validity of the simulation results. While in school, engineering students learn how to combine classical methods of analysis with modern computing tools, such as CAD solid modelers, to work efficiently and effectively in today's exciting world of design.

## 7. REFERENCES

[1] Beer, F. J. (2019). Vector Mechanics for Engineers: Statics and Dynamics, 12th ed. New York, New York: McGraw-Hill.
[2] Budynas, R. N. (2014). Shigley's Mechanical Engineerign Design, 10th ed. New York, New York: McGraw-Hill.
[3] Elshennawy, A. W. (2015). Manufacturing Processes & Materials. Dearborn, Michigan: SME.
[4] Howard, W. M. (2018). Introduction to Solid Modeling Using SolidWorks 2018. New York, New York: McGraw-Hill.
[5] Norton, R. (2012). Design of Machinery, 5th ed. New York, New York: McGraw-Hill.
[6] Plesha, M. G. (2010). Engineering Mechanics: Statics; 1st ed. New York, New York: McGraw-Hill.
[7] SolidWorks. (2020, January 7). SolidWorks Homepage. Retrieved from SolidWorks: https://www.solidworks.com
[8] The MathWorks, Inc. (2020, January 7). MATLAB. Retrieved from Mathworks.com: https://www.mathworks.com/products/matlab.html
[9] Thomson, W. (1998). Theory of Vibration with Applications, 5th ed. Boston, Massachusetts: Pearson.
[10] Ullman, D. (2015). The Mechanical Design Process, 5th ed. New York, New York: McGraw-Hill.