BLIND CARRIER FREQUENCY OFFSET ESTIMATION FOR MULTICARRIER SYSTEMS

A Dissertation by

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BLIND CARRIER FREQUENCY OFFSET ESTIMATION FOR MULTICARRIER SYSTEMS

The following faculties have examined the final copy of this dissertation for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Doctor of Philosophy with a major in Electrical Engineering.

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To my charming wife

Heba Shatnawi
ACKNOWLEDGEMENT

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Special thanks are also due to my dearest Heba for her sincere patience, support, and for accompanying with me in the long, busy days that I needed to complete this work. I like to express my deepest gratitude to my parents, sisters, brothers, and friends for their unwavering encouragement throughout my education.
A Multicarrier Communication (MCM) system such as an Orthogonal Frequency Division Multiplexing OFDM or Discrete Multi Tone (DMT) system has been shown to be an effective technique to combat multipath fading in wireless communications. OFDM is a modulation scheme that allows digital data to be efficiently and reliably transmitted over a radio channel, even in multipath environments. OFDM transmits data by using a large number of narrow bandwidth carriers. These carriers are regularly spaced in frequency, forming a block of spectrum. The frequency spacing and time synchronization of the carriers is chosen in such a way that the carriers are orthogonal, meaning that they do not cause interference to each other. In spite of the success and effectiveness of the OFDM systems, it suffers from two well known draw backs: large Peak to Average Power Ratio (PAPR) and high sensitivity to Carrier Frequency Offset (CFO). The presence of the CFO in the received carrier will lose orthogonality among the carriers and because the CFO causes a reduction of desired signal amplitude in the output decision variable and introduces Inter Carrier Interference (ICI). It then brings up an increase of Bit Error Rate (BER). This makes the problem of estimating the CFO an attractive and necessary research problem. In this dissertation blind estimation techniques will be proposed to estimate the offset parameter.
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<td>AWGN</td>
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<td>Discrete Fourier Transform</td>
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<td>DMB-T/H</td>
<td>Digital Multimedia Broadcast-Terrestrial/Handheld</td>
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<td>DMT</td>
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<td>DOA</td>
<td>Direction of Arrival</td>
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<td>DS</td>
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<td>GSM</td>
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<td>HDTV</td>
<td>High-Definition Television</td>
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<td>IBI</td>
<td>Inter Block Interference</td>
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<td>ICI</td>
<td>Inter Carrier Interference</td>
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<td>IFI</td>
<td>Inter Frame Interference</td>
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<td>ISI</td>
<td>Inter Symbol Interference</td>
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<td>LANs</td>
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<td>LOS</td>
<td>Line Of Sight</td>
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<td>LTE</td>
<td>Long Term Evolution</td>
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<td>MCM</td>
<td>Multi Carrier Modulation</td>
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<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<td>ML</td>
<td>Maximum Likelihood</td>
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<td>MSE</td>
<td>Mean Square Error</td>
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<td>MUSIC</td>
<td>MUltiple SIgnal Classification</td>
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<td>NDFT</td>
<td>Normalized Discrete Fourier Transform</td>
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<td>NIDFT</td>
<td>Normalized Inverse Discrete Fourier Transform</td>
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<td>Non Line Of Sight</td>
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<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
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<td>OPM</td>
<td>Orthonormal PM</td>
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<td>OQAM</td>
<td>Orthogonal Quadrature Amplitude Modulation</td>
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<tr>
<td>P/S</td>
<td>Parallel /Serial</td>
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<td>PAPR</td>
<td>Peak to Average Power Ratio</td>
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<td>Power Delay Profiles</td>
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<td>PM</td>
<td>Propagator Method</td>
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<td>Phase Shift Keying</td>
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<td>Rank Revealing QR</td>
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<td>SC-FDMA</td>
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<td>SDARS</td>
<td>Satellite Digital Audio Radio Services</td>
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<td>SNR</td>
<td>Signal-to-noise ratio</td>
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<td>STBC</td>
<td>Space Time Block Coding</td>
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<td>SVD</td>
<td>Singular Value Decomposition</td>
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<td>SWEDF</td>
<td>Subspace Methods Without Eigen Decomposition</td>
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<td>TDMA</td>
<td>Time Division Multiple Access</td>
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<td>VC</td>
<td>Virtual Carrier</td>
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<td>VDSL</td>
<td>Very high data rate DSL</td>
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<td>VLSI</td>
<td>Very Large Scale Integration</td>
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CHAPTER 1

Overview

1.1 Introduction

Technology and system requirements in the telecommunications field are changing very fast. Over the previous years, since the transition from analog to digital communications, and from wired to wireless, different standards and solutions have been adopted, developed, implemented and modified, often to deal with new and different business requirements. Today, more and more, telecommunication network operators struggle to provide new advanced services in an attractive and functional way.

Wireless communications [1]-[4] is a rapidly growing piece of the communications manufacturing, with the potential to provide high-speed high-quality information exchange between the portable devices located anywhere in the world. Potential applications enabled by this technology include multimedia Internet-enabled, Global System for Mobile (GSM), smart homes, automated highway systems, video teleconferencing and distance learning, and autonomous sensor networks, just to name a few. However, supporting these applications using wireless techniques creates a significant technical challenge.

The motion in space of a wireless receiver operating in a multipath channel results in a communications link that experiences small-scale fading. The rapid fluctuations of the received power level due to small sub-wavelength changes in receiver position are described as small-scale fading [4]. Basically, mobile radio communication channels are time varying, multipath fading channels [3], [4]. In a radio communication system, there are many paths for a signal to pass through from a transmitter to a receiver. Sometimes there is a direct path where the signal travels without being obstructed, which is known as a Line Of Sight (LOS) path. In most cases,
components of the signal are refracted by different atmospheric layers or reflected by the ground and objects between the transmitter and the receiver such as vehicles, buildings, and hills, which is known as Non Line Of Sight (NLOS) paths. These components travel in different paths of different length and combine at the receiver. Thus, signals on each path suffer different transmission delays and attenuation due to the finite propagation velocity. The combination of these signals at the receiver results in a destructive or constructive interference, depending on the relative delays involved. In fact, the environment changes with time which leads to signal variation. This is called time variant environment. Also, the motion of the object influences signals. A short distance movement can cause an obvious change in the propagation paths and vary the strength of the received signals.

Orthogonal Frequency Division Multiplexing (OFDM) [5]-[10] has been shown to be an effective technique to combat multipath fading in wireless communications. OFDM is a modulation scheme that allows digital data to be efficiently and reliably transmitted over a radio channel, even in multipath environments. OFDM transmits data by using a large number of narrow bandwidth carriers. These carriers are regularly spaced in frequency, forming a block of spectrum. The frequency spacing and time synchronization of the carriers is chosen in such a way that the carriers are orthogonal, meaning that they do not cause interference to each other.

OFDM has been adopted by standardization bodies and major manufacturers for a wide range of applications. In Europe, OFDM was first standardized for Digital Audio Broadcasting (DAB) in 1995 [11], [21] and terrestrial Digital Video Broadcasting (DVB) in 1997 [22]. OFDM has already been used in a variety of applications, High-Definition Television (HDTV) broadcasting [12], [13], high bit rate Digital Subscriber Line (DSL), Asymmetric DSL (ADSL) [14], Very high data rate DSL (VDSL) [15], IEEE 802.11, multimedia mobile access
communications wireless Local Area Networks (LANs) [19], [20], Fourth-Generation (4G) wireless mobile communications and Satellite Digital Audio Radio Services (SDARS). It has recently been proposed for use in radio-over fiber based links [16] and in free-space optical communications [17], [18]. In 2007, the first complete Long Term Evolution (LTE) air interface implementation was demonstrated, including Multiple-Input Multiple-Output OFDM (OFDM-MIMO), Single Carrier Frequency-Division Multiple Access (SC-FDMA) and multi-user MIMO uplink.

OFDM can be combined with many protocols and algorithms. For example, OFDM can be combined with multiple access schemes, such as Time Division Multiple Access (TDMA), to achieve efficient bandwidth utilization in presence of multiple users. According to the standard IEEE 802.16, for example, both OFDM-TDMA and Orthogonal Frequency Division Multiple Access (OFDMA) have been adopted at 2–11 GHz band [23]. OFDM can be combined with Space Time Block Coding (STBC) and Bit Interleaved Coded Modulation (BICM) to form BICM-STBC-OFDM that achieves the maximum diversity [24].

In spite of the success and effectiveness of the OFDM systems, it suffers from two well known draw backs: large Peak to Average Power Ratio (PAPR) and high sensitivity to Carrier Frequency Offset (CFO). The presence of the CFO in the received carrier will lose orthogonality among the carriers and because the CFO causes a reduction of desired signal amplitude in the output decision variable and introduces Inter Carrier Interference (ICI). It then brings up an increase of Bit Error Rate (BER) [30]- [35]. The effect caused by CFO for an OFDM/QAM system was analyzed in [32], and it was indicated that CFO should be less than 2% of the bandwidth of subchannel to guarantee the signal to interference ratio be higher than 30 dB.
1.2 Dissertation Contributions

The estimation of the CFO is a classical problem, and it can be estimated via data added or non-blind algorithms [36]-[44], semi blind algorithms [45], and non data added or blind [46]-[65] algorithms. In this dissertation, the CFO blind estimation algorithms for OFDM systems have been studied. This dissertation focuses on the blind subspace CFO estimator performance. Two novel blind subspace CFO estimators [64], [65] have been proposed. The proposed algorithms were tested with different applications like joint time delay and frequency estimation problem [26], [27], and in channel estimation for the frequency hopping systems [28], [29].

1.3 Dissertation Outline

Chapter One introduces the dissertation and the contributions of the dissertation. Chapter Two provides the concept of OFDM, history of OFDM, mathematical system model, OFDM signal generation, multipath model, and channel classification. Chapter Three introduces one of the major drawbacks of the OFDM system which is the carrier synchronization or what known as carrier frequency offset. A brief summary of the different synchronization algorithms are also presented. Chapter Four proposes a blind CFO estimator where no training pilots or reference symbols are used. The development is proposed through the propagator method, where the classical matrix decomposition is avoided. It is well known that subspace estimation techniques are relying on the eigenvalue decomposition of the covariance matrix to extract the noise and/or the signal subspace. As a result, a significant reduction in the calculation is achieved by applying the propagator method. Chapter Five presents another blind CFO estimator based on the rank revealing QR factorization. The RRQR is a special QR factorization that is guaranteed to reveal the numerical rank of the matrix under consideration. This makes the RRQR factorization a
useful tool in the numerical treatment of many rank-deficient problems in numerical linear algebra. It is well known that the computational load of the RRQR method is insignificant compared with Eigen Value Decomposition (EVD) or Singular Value Decomposition (SVD) of the cross-spectral matrix of received signals. Finally, conclusions and future work are presented in Chapter Six.
CHAPTER 2
Orthogonal Frequency Division Multiplexing (OFDM)

2.1 Introduction

Frequency Division Multiplexing (FDM) is a scheme in which several signals are combined for transmission on a single communications line or channel. Each signal travels within its own unique carrier frequency range, which is modulated by the data [8] - [10]. In this case, the carrier signals are referred to as subcarriers, for example, a television channel is divided into subcarrier frequencies for video, color, and audio on the same conductors. Another example, DSL use different frequencies for voice and for upstream and downstream data transmission [14]. There are always some idle frequency spaces between channels, known as guardband. FDM is famously known as Multi Carrier Modulation (MCM). FDM was the first multiplexing system to enjoy wide scale network deployment, and such systems are still in use today [9].

OFDM spread spectrum technique distributes the data over a large number of carriers that are spaced apart at particular frequencies. This spacing offers the orthogonality in this technique which avoids the demodulators from seeing frequencies other than their own [10]. The OFDM transmission scheme is the optimum version of the multicarrier transmission scheme. The benefits of OFDM are high spectral efficiency, resistance to RF interference, and lower multipath distortion [6]. This is helpful because in a classic terrestrial broadcasting scenario there are multipath-channels. Since multiple versions of the signal interfere with each other, Inter Symbol Interference (ISI) appears. In order to afford robustness to multipath fading, OFDM is compatible with utilizing frequency diversity. OFDM split the wideband signal into several narrowband signals, each of which is practiced to flat fading channel instead of frequency selective fading channel.
2.2 OFDM History

Researchers first published the concept of using parallel data transmission using FDM in 1966. Early in 1970 a United States patent was issued. The idea of the patent was to employ parallel data streams and frequency division multiplexing with overlapping subchannels to avoid the use of high speed equalization and to handle the impulsive noise, and to prevent the multipath distortion as well as to fully use the existing bandwidth. The early applications were in military communications. In the telecommunications area, the terminologies of Discrete Multi Tone (DMT) and Multi Channel Modulation (MCM) are broadly used, and frequently they are exchangeable with OFDM [5], [10].

For a large number of subchannels, the arrays of sinusoidal generators and coherent demodulators requested in a parallel system became unreasonably expensive and complicated. The receiver needs an accurate phasing of the demodulating carriers and sampling times in order to keep crosstalk between subchannels at an acceptable level. The modulation and demodulation process were implemented by applying the Discrete Fourier Transform (DFT) to parallel data. In addition to eliminating the banks of subcarrier oscillators and coherent demodulators required by FDM, a complete digital implementation could be built around special purpose hardware performing the Fast Fourier Transform (FFT). Recent development in Very Large Scale Integration (VLSI) technology allows making high speed chips that can achieve large size FFT at affordable prices.

In the 1980s, OFDM was considered for digital mobile communications, high density recording, and high speed modems. In the 1990s, OFDM was exploited for wideband data communications over mobile radio Frequency Modulation (FM) channels, Asymmetric Digital Subscriber Lines (ADSL, 1.5 Mb/s) [14], High bit rate Digital Subscriber Lines (HDSL, 1.6
Mb/s), Very High-Speed Digital Subscriber Lines (VHDSL, 100 Mb/s) [15]. OFDM has been successfully used in the European DAB [11], DVB systems, HDTV terrestrial broadcasting [12], and IEEE 802.11a-1999 or 802.11a [25].

In the 2000s, OFDM was implemented in the Digital Terrestrial Multimedia Broadcast (DTMB) [13]. The standard was formerly named as Digital Multimedia Broadcast-Terrestrial/Handheld (DMB-T/H). This standard was applied in China, and covers both fixed and mobile terminals. It will eventually serve more than half of the television viewers there, especially those in suburban and rural areas. OFDM was implemented in the DVB - Handheld (DVB-H). In addition, the first complete Long Term Evolution (LTE) air interface implementation was demonstrated, including OFDM-MIMO, Single Carrier FDMA (SC-FDMA), and multi user MIMO uplink.

2.3 OFDM Advantages and Drawbacks

The OFDM is a promising transmission scheme, which has been considered extensively, as it has the following key advantages [5]-[10]:

- OFDM makes efficient use of the spectrum.
- OFDM becomes more resistant to frequency selective fading than single carrier systems by converting the frequency selective fading channel into narrowband flat fading subchannels.
- OFDM eliminates Inter Symbol Interference (ISI) and Inter Frame Interference (IFI) through use of a Cyclic Prefix (CP).
- OFDM recovers the symbols lost due to the frequency selectivity of the channel by using adequate channel coding and interleaving.
- OFDM makes channel equalization simpler than single carrier systems by using adaptive equalization techniques.
- OFDM seems to be less sensitive to sample timing offsets in comparison with single carrier systems.
- OFDM provides good protection against co channel interference and impulsive parasitic noise.
- OFDM makes it possible to use Maximum Likelihood (ML) decoding with reasonable complexity. OFDM is computationally efficient with FFT techniques.

The several advantages of the OFDM systems could only appear if the main three drawbacks were treated carefully. OFDM has the following negative aspect:
- OFDM signal has a noise like amplitude with a very large dynamic range; therefore, it requires RF power amplifiers with a high peak to average power ratio, which may require a large amplifier power back off and a large number of bits in the Analog to Digital (A/D) and Digital to Analog (D/A) designs.
- OFDM is very sensitive to Carrier Frequency Offset (CFO) caused by Doppler effect. Hence, CFO should be estimated and cancelled completely.
- OFDM receiver suffers from the difficulty to make a decision about the starting time of the FFT symbol.
2.4 Discrete Fourier Transform

The Normalized Discrete Fourier Transform (NDFT) sinusoids are given by:
\[
\phi_k(n) = \frac{1}{\sqrt{N}} e^{j2\pi kn/N}
\]  
(2.1)

The NDFT sinusoids are forming an orthonormal sinusoidal basis signals satisfy:
\[
\langle \phi_i(n), \phi_k(n) \rangle = \sum_{n=0}^{N-1} \phi_i(n) \phi_k^*(n) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}
\]

The NDFT of a time domain signal \(x[n]\) is given by:
\[
X[k] \triangleq NDFT\{x[n]\} \triangleq \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0,1,2, ..., N - 1
\]  
(2.2)

The operation of the NDFT can be inverted to recover a time-domain signal from its frequency representation. This is done with the Normalized Inverse Discrete Fourier Transform (NIDFT). The NIDFT is given by:
\[
x[n] \triangleq NIDFT\{X[k]\} \triangleq \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}
\]  
\[
n = 0,1,2, ..., N - 1
\]  
(2.3)

2.5 Matrix Formulation of the NDFT and the NIDFT

The NDFT and the NIDFT can be formulated as a complex matrix multiply. Based on (2.2) and (2.3) we may write the following:

\[
\begin{bmatrix}
X(0) \\
X(1) \\
X(2) \\
\vdots \\
X(N-1)
\end{bmatrix} = \frac{1}{\sqrt{N}} \begin{bmatrix}
W^{-0.0} & W^{-1.0} & W^{-2.0} & \cdots & W^{-(N-1).0} \\
W^{-0.1} & W^{-1.1} & W^{-2.1} & \cdots & W^{-(N-1).1} \\
W^{-0.2} & W^{-1.2} & W^{-2.2} & \cdots & W^{-(N-1).2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
W^{-0.(N-1)} & W^{-1.(N-1)} & W^{-2.(N-1)} & \cdots & W^{-(N-1).(N-1)}
\end{bmatrix} \begin{bmatrix}
x(0) \\
x(1) \\
x(2) \\
\vdots \\
x(N-1)
\end{bmatrix}
\]
\[
\begin{bmatrix}
  x(0) \\
x(1) \\
x(2) \\
\vdots \\
x(N-1)
\end{bmatrix} = \frac{1}{\sqrt{N}} \begin{bmatrix}
  W^{0.0} & W^{1.0} & W^{2.0} & \cdots & W^{(N-1).0} \\
  W^{0.1} & W^{1.1} & W^{2.1} & \cdots & W^{(N-1).1} \\
  W^{0.2} & W^{1.2} & W^{2.2} & \cdots & W^{(N-1).2} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  W^{0.(N-1)} & W^{1.(N-1)} & W^{2.(N-1)} & \cdots & W^{(N-1).(N-1)} 
\end{bmatrix} \begin{bmatrix}
  X(0) \\
  X(1) \\
  X(2) \\
  \vdots \\
  X(N-1)
\end{bmatrix}
\]

Define the NIDFT matrix \( W \) as:

\[
W = \frac{1}{\sqrt{N}} \begin{bmatrix}
  W^{0.0} & W^{1.0} & W^{2.0} & \cdots & W^{(N-1).0} \\
  W^{0.1} & W^{1.1} & W^{2.1} & \cdots & W^{(N-1).1} \\
  W^{0.2} & W^{1.2} & W^{2.2} & \cdots & W^{(N-1).2} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  W^{0.(N-1)} & W^{1.(N-1)} & W^{2.(N-1)} & \cdots & W^{(N-1).(N-1)} 
\end{bmatrix}
\] (2.4)

where \( W = e^{j2\pi/N} \), the matrix \( W \) is a unitary matrix of size \( L \times L \), and \( W^H W = I \). We may write the NDFT and the NIDFT as:

\[
X = W^H x \tag{2.5}
\]

\[
x = W X \tag{2.6}
\]

where the time domain data vector is \( x \) and the frequency domain data vector is \( X \). Equations (2.5) and (2.6) represent the matrix version of the equations given by (2.2) and (2.3).

2.6 Orthogonality Principle

OFDM is considered as a block transmission technique. In the baseband, complex-valued data symbols modulate a large number of closely collected carrier waveforms. The transmitted OFDM signal multiplexes a number of low rate data streams, and each data stream is associated with a given subcarrier. The main benefit of this concept in a radio environment is that each of the data streams deals with an almost flat fading channel. In slowly fading channels, the ISI and Inter Carrier Interference (ICI) within an OFDM symbol can be avoided with a small loss of
transmission energy using the concept of the CP. An OFDM signal consists of $N$ orthogonal subcarriers modulated by $N$ parallel data streams. Each baseband subcarrier $\phi_k(t)$ is of the form

$$
\phi_k(t) = e^{j2\pi f_k t}, \quad k = 1, 2, 3, ..., N
$$

where $f_k$ is the frequency of the $k^{th}$ subcarrier. The subcarrier frequencies $f_k$ are equally spaced as:

$$
f_k = \frac{k}{NT}
$$

which makes the subcarriers $\phi_k(t)$ on $0 \leq t \leq NT$ orthogonal on each other

$$
\langle \phi_i(t), \phi_k(t) \rangle = \begin{cases} 
NT, & i = k \\
0, & i \neq k
\end{cases}
$$

Figure 2.1 also shows the orthogonality among five subcarriers.
2.7 OFDM Transmitter

Figure 2.2 shows the block diagram of a simple OFDM transmitter. The OFDM signal is obtained by using the orthogonal filters

\[ \psi_n(t) = u(t) e^{-j2\pi nt/PT}, \quad n = 0, 1, 2, ..., P - 1 \]

where \( u(t) \) is a rectangular time window, \( P \) is the number of the input symbols, and \( T \) is the symbol transmission time. The time window is given by:

\[ u(t) = \begin{cases} \frac{1}{\sqrt{PT}} & 0 \leq t \leq PT \\ 0 & \text{else} \end{cases} \]

The \( k^{th} \) transmitted signal (corresponding to the \( k^{th} \) frame) can be formulated as:

\[ x^k(t) = \sum_{n=0}^{P-1} s_n^k \psi_n(t - kPT) \]

where \( s_n^k \) is the \( n^{th} \) symbol to be transmitted in the \( k^{th} \) frame. The stream of data belonging to a Phase Shift Keying (PSK) or QAM is Serial to Parallel (S/P) converted. Let \( s_P(k) \) is the \( k^{th} \) block of size \( P \) to be transmitted.

\[ s_P(k) = [s_0(k) \ s_1(k) \ ... \ s_{P-1}(k)]^T \]

Figure 2.2 Simple OFDM Transmitter.
To avoid aliasing problems at the receiver, $N-P$ subcarriers at the edge of the spectrum are not used. Thus, the vector $s_p$ of size $P$ will be extended to the vector $s$ of size $N$, by padding $N-P$ zeros, which is known as Virtual Carrier (VC) or unused carriers.

$$s(k) = \begin{bmatrix} s_0(k) & s_1(k) & \cdots & s_{P-1}(k), & 0,0,\ldots,0 \end{bmatrix}^T$$

(2.7)

The vector $s(k)$ is fed to the $N$-point IDFT unit. The output vector (of size $N$) of the IDFT unit for each block, which is called “time domain” block vector, is given by:

$$x(k) = W^H s(k)$$

(2.8)

$$x(k) = [x_0(k) \ x_1(k) \ \cdots \ x_{N-1}(k)]^T$$

We may write (2.8) as:

$$x(k) = W_p s_p(k)$$

(2.9)

Where $W_p$ is the first $P$ column of the matrix $W$ and given by:

$$W_p = \frac{1}{\sqrt{N}} \begin{bmatrix} W^{0.0} & W^{1.0} & \cdots & W^{(P-1).0} \\
W^{0.1} & W^{1.1} & \cdots & W^{(P-1).1} \\
W^{0.2} & W^{1.2} & \cdots & W^{(P-1).2} \\
\vdots & \vdots & \ddots & \vdots \\
W^{0.(N-1)} & W^{1.(N-1)} & \cdots & W^{(P-1).(N-1)} \end{bmatrix}$$

(2.10)

$$W_p = [w_p^0 \ w_p^1 \ w_p^2 \ \cdots \ w_p^{P-1}]$$

where $w_p^k$ is the $k^{th}$ column of $W_p$. 

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2.8 Cyclic Prefix

Two difficulties take place when the OFDM signal is transmitted over a dispersive channel. One difficulty is that channel dispersion destroys the orthogonality between subcarriers and causes ICI. In addition, a system may transmit multiple OFDM symbols in a series so that a dispersive channel causes ISI between successive OFDM symbols. The insertion of a guard period between successive OFDM symbols would avoid ISI in a dispersive environment, but it does not avoid the loss of the subcarrier orthogonality. The cyclic prefix (CP) both preserves the orthogonality of the subcarriers and prevents ISI between successive OFDM symbols. Consequently, equalization at the receiver is very simple. This frequently encourages the use of OFDM in wireless systems. To eliminate ISI almost totally, a guard time is established for each OFDM symbol. To avoid multipath components from one symbol to interfere with the next symbol (ISI), the guard time that is selected should be larger than the expected delay spread. The CP is a copy of the last samples from the IFFT, which are placed at the beginning of the OFDM frame. More precisely, there are two reasons to insert a CP:

1. The convolution between the channel impulse response and the data will operate like a circular convolution instead of a linear one. Circular convolution makes equalization easier.

2. Interference from the previous symbol will only have an effect on the CP, which is not needed in the receiver.

Both reasons assume that the CP is longer than the channels impulse response. If the CP is shorter than the impulse response, the convolution will not be circular and ISI will arise. However, if the number of samples in the CP is large, the data transmission rate will decrease significantly since the CP does not carry any useful data.
At the output of the IFFT, a guard symbols of length $N_G$ is inserted at the beginning of each block. In other words, the vector in (2.6) is extended by $N_G$ symbols to form either Cyclic Prefix Orthogonal Frequency-Division Multiplexing (CP-OFDM) or Zero Padding Orthogonal Frequency Division Multiplexing (ZP-OFDM).

$$x_{cp} = \begin{pmatrix} x_{N-N_G}, \ldots, x_{N-1} \cr x_0, x_1, \ldots, x_{N-N_G}, \ldots x_{N-1} \end{pmatrix}_{N_G/N}$$ \hspace{1cm} (2.11.a)$$

$$x_{zp} = \begin{pmatrix} x_0, x \ldots, x_{N-1}, 0,0, \ldots, 0 \end{pmatrix}_{N/N_G}$$ \hspace{1cm} (2.11.b)$$

Due to the cyclic extension or zero padding, the efficiency of OFDM transmissions reduces by a factor of $\frac{N}{N+N_G}$ after a redundant $N_G$ symbols are inserted between each $x(k)$. The resulting blocks $x_{cp}$ or $x_{zp}$ are finally sent sequentially through the channel. The total number of time-domain samples per transmitted block is $N + N_G$. Thus, it is important to choose the minimum possible CP to maximize the system efficiency. The guard symbols also eliminate the need for a pulse-shaping filter, and it reduces the sensitivity to time synchronization problems.

The CP/ZP can be added by extending the NIDFT matrix to $W_p^{CP}$ / $W_p^{ZP}$ of size $(N + N_G) \times P$

$$W_p^{CP} = \frac{1}{\sqrt{N}} \begin{bmatrix} W^{0.0} & W^{1.(N-N_G)} & W^{1.(N-N_G+1)} & \ldots & W^{(P-1).(N-N_G)} \\
W^{0.0} & W^{1.0} & W^{1.(N-N_G+1)} & \ldots & W^{(P-1).(N-N_G+1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
W^{0.2} & W^{1.2} & \ldots & \vdots & W^{(P-1).2} \\
W^{0.(N-1)} & W^{1.(N-1)} & W^{1.(N-1)} & \ldots & W^{(P-1).(N-1)} \end{bmatrix}$$ \hspace{1cm} (2.12.a)$$

$$W_p^{ZP} = \frac{1}{\sqrt{N}} \begin{bmatrix} \text{zeros}(N_G \times P) \end{bmatrix}$$ \hspace{1cm} (2.12.b)$$

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where $w_p^{CP_k}$ is the $k^{th}$ column of $W_p^{CP}$. In this thesis the CP-OFDM is considered. We may write (2.8) as:

$$x_{cp}(k) = W_p^{CP} \cdot s_p(k)$$

(2.13)

### 2.9 Multipath and Fading Channel

Multipath is defined as the propagation phenomenon that results in radio signals reaching the receiver by several paths. As it appears in Figure 2.3, reasons of multipath consist of ionospheric reflection, atmospheric ducting, refraction, and reflection from terrestrial objects such as mountains, buildings, and trees. The effects of multipath include constructive, destructive interference in addition to phase shifting of the signal. In literature, the standard statistical model of fading is a Rayleigh distribution. In digital radio communications, multipath fading can cause severe errors and affect the quality of communications badly. The type of fading experienced by a signal propagating through a channel can be determined by the nature of the transmitted signal with respect to the characteristics of that channel. Factors influencing fading are multipath
propagation environment, the speed of the mobile, the speed of the surrounding objects, and the bandwidth of the transmitted signal.

2.9.1 Mathematical Modeling

A simple mathematical model can be presented by assuming that the transmitted signal is an ideal pulse at time zero. At the receiver, due to the presence of the multiple electromagnetic paths, several pulses will be received; thus, each one of them will arrive at different times. In fact, since the electromagnetic signals travel at the speed of light, and since every path has a length probably different from that of the others, there are different traveling times. Thus, the received signal will be expressed by:

\[ y(t) = x(t) * h(t) \]

\[ = \sum_{n=0}^{N-1} \rho_n \cdot e^{j\phi_n} \cdot \delta(t - \tau_n) \]

where \( N \) is the number of received pulses which is exactly the number of possible paths, and may be very large number, \( \tau_n \) is the time delay of the \( n^{th} \) pulse, and \( \rho_n e^{j\phi_n} \) represents the complex amplitude (magnitude and phase) of the received pulse. By definition, \( y(t) \) represents the impulse response function \( h(t) \) of the equivalent multipath model. In general, the consideration of time variation of the geometrical reflection conditions yields an impulse response that varies with time.

2.9.2 Power Delay Profiles

Power Delay Profiles (PDP) are generally modeled as plots of relative received power as a function of excess delay with respect to a fixed time delay reference. The maximum delay time spread parameter is used to denote the severity of multipath surroundings. It is also called the
multipath time $T_M$, and it is defined as the time delay existing between the first and the last received impulses. The delay spread is the square root of the second central moment of the power delay profile and given by:

$$\sigma = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2}$$

where

$$\bar{\tau}^2 = \frac{\sum_k P(\tau_k) \cdot \tau_k^2}{\sum_k P(\tau_k)}$$

$$\bar{\tau} = \frac{\sum_k P(\tau_k) \cdot \tau_k}{\sum_k P(\tau_k)}$$

$\bar{\tau}$ is the first moment of the PDP known as the mean excess delay. Typical values of delay spread are on the order of microseconds in outdoor mobile radio channels and on the order of nanoseconds in indoor radio channels. The multipath can be characterized by either the channel transfer function $H(f)$, or the impulse response $h(t)$, where they are related to each other by the continuous time Fourier transform

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt$$

$$= \sum_{n=0}^{N-1} \rho_n e^{j\phi_n} e^{-j2\pi f \tau_n}$$

The obtained multipath channel transfer function characteristic has a typical appearance of a sequence of peaks (maxima) and notches (minima); it can be shown that, on average, the distance in Hertz between two consecutive peaks is roughly inversely proportional to the multipath time. The coherence bandwidth is defined as:

$$B_c = \frac{k}{T_M}$$
The coherence bandwidth is defined to be the range of frequencies over which two frequencies have possibility for amplitude correlation [2]. If two sinusoids with a frequency separation of greater than $B_c$ are propagating in the same channel, they are affected quite differently by the channel. Delay spread and Coherence bandwidth describe perfectly the time dispersive nature of the channel. They do not offer any information about the time varying nature of the channel caused by the relative motion of the transmitter and/or the receiver.

Doppler spread (DS) is a measure of spectral expansion caused by motion, the time rate of change of the mobile radio channel, and is defined as the range of frequencies over which the received DS is essentially non-zero. It offers information about the fading rate of the channel. Knowing DS in mobile communication systems can improve detection and help to optimize transmission at the physical layer, as well as higher levels of the protocol stack. If the baseband signal bandwidth is much less than the DS, then effect of the DS is negligible at the receiver. Doppler spread $D_s$, is defined as the maximum Doppler shift and given by:

$$D_s = f_m = \frac{v}{\lambda}$$

Coherence time is inversely related to DS. Coherence time is defined to be the time duration over which the channel impulse response is mainly invariant with time [1]. If the symbol time of the baseband signal is larger than the coherence time, then the signal will distort, since the channel will change during the transmission of the single symbol time. Doppler Spread and coherence time are parameters which express the time varying nature of the channel in a small-scale region. By definition, Coherence time implies that two signals arriving with a time separation greater than $T_c$ are affected differently by the channel.
2.9.3 Frequency Selective and Flat Fading

By comparing the channel coherence bandwidth and the signal bandwidth, the channel can be classified into frequency selective fading or flat fading. In frequency selective fading, the coherence bandwidth of the channel is smaller than the bandwidth of the signal [3]. Different frequency components of the signal therefore experience uncorrelated fading. In a frequency-selective fading channel, because different frequency components of the signal are affected separately, it is highly doubtful that all parts of the signal will be simultaneously affected by a deep fade. Some modulation techniques such as OFDM and Code Division Multiple Access (CDMA) are compatible with employing frequency diversity to offer robustness to selective fading. OFDM divides the wideband signal into several narrowband subcarriers, each of which are practiced to flat fading instead of frequency selective fading. The length of the CP is chosen for the maximum anticipated multipath spread; for the IEEE 802.11a standard, this is 25% of OFDM symbol duration, indicating a significant loss in utilization. In flat fading, the coherence bandwidth of the channel is larger than the bandwidth of the signal, and the symbol period of the transmitted signal is much larger than the Delay Spread of the channel. Therefore, all frequency components of the signal will be subjected to the same magnitude of fading.

2.9.4 Fast and Slow Fading

Slow fading scenario arises when the channel coherence time is larger than the channel delay constraint. In this scenario, the channel can be considered roughly constant over the period of use, the amplitude and phase change are small enough to be neglected. Slow fading can be founded by events such as shadowing, where a large obstruction such as a hill or large building obscures the main signal path between the transmitter and the receiver [4]. Fast fading occurs when the coherence time of the channel is small relative to the delay constraint of the channel
In this system, the amplitude and phase change imposed by the channel varies considerably over the period of use. In a fast fading channel, the transmitter may take advantage of the variations in the channel conditions by using time diversity to help increase the robustness of the communication system.

### 2.9.5 OFDM Channel Model

In OFDM systems, the CP length needs to be larger than the maximum excess delay of the channel. If this information is not available, the worst case channel condition is used for system design, which makes CP a significant portion of the transmitted data, thus reducing spectral efficiency. One way to increase spectral efficiency is to adapt the length of the cyclic prefix depending on the radio environment. This adaptation requires an estimation of the maximum excess delay of the radio channel, which is also related to the frequency selectivity of the channel. Other OFDM parameters that could be changed adaptively using the knowledge of the dispersion are OFDM symbol duration and OFDM subcarrier bandwidth. The CP length is assumed to be known or pre-estimated. The channel can be represented by an equivalent discrete time model and its effects can be represented by a linear Finite Impulse Response (FIR) filter with the Channel Impulse Response (CIR):

\[
c_N = [c_0, c_1, c_2, ..., c_{L-1}, 0, 0, 0, ... 0]
\]

The channel transfer function \( H(z) \) is given by:

\[
H(n) = \sum_{k=0}^{L-1} c_k e^{j2\pi \frac{n}{N}}, \quad n = 0, 1, 2, ..., N - 1
\]

\[
= c_0 + c_1 e^{j2\pi /N} + c_2 e^{j4\pi /N} ... + c_{L-1} e^{j2\pi (L-1)/N}
\] (2.14)

Using the matrix notation, let

\[
H = W^H \cdot c_N
\]
\[ = [H(0), H(1), H(2), ..., H(N)]^T \]
\[ = [h_1, h_2, h_3, ..., h_N]^T \] \hspace{1cm} (2.15)

2.10 OFDM Receiver

The CP is added between each OFDM block in order to transform the linear convolution into a circular convolution. After Parallel to Serial (P/S) and Analog to Digital (A/D), the signal is sent through a frequency selective channel. The block diagram of the OFDM receiver is shown in Figure 2.4. For noiseless received signals and given channel state information, reversed steps to the transmitter operation are applied. The discrete time received signal with CP guard interval, denoted as \( r^{CP} \), is given by the following expression:

\[
\begin{bmatrix}
  r_1^{CP}(k) \\
  r_2^{CP}(k) \\
  \vdots \\
  r_{N+N_G}^{CP}(k)
\end{bmatrix}
= \begin{bmatrix}
  x_{N-N_G}(k) \\
  \vdots \\
  x_0(k) \\
  x_{N-1}(k)
\end{bmatrix} + \begin{bmatrix}
  x_{N-N_G}(k-1) \\
  \vdots \\
  x_0(k-1) \\
  x_{N-1}(k-1)
\end{bmatrix}
\]

\[= H_{ISI} \cdot x_{cp}(k) + H_{IBI} \cdot x_{cp}(k - 1) \] \hspace{1cm} (2.16)

where \( H_{ISI} \) represents inter-symbol interference generated by the frequency selective behavior of

![Figure 2.4 Simple OFDM receiver](image URL)
the channel inside an OFDM block at time $k$. $H_{ISI}$ is a square matrix of size $(N + N_G) \times (N + N_G)$ and is given by:

$$
H_{ISI} = \begin{bmatrix}
  c_0 & 0 & \cdots & \cdots & \cdots & 0 \\
  \vdots & \ddots & \ddots & \ddots & & \vdots \\
  c_{L-1} & \ddots & \ddots & \ddots & & \vdots \\
  0 & \ddots & \ddots & \ddots & & 0 \\
  \vdots & \ddots & \ddots & \ddots & & \vdots \\
  0 & \cdots & 0 & c_{L-1} & \cdots & c_0
\end{bmatrix}
$$

$H_{IBI}$ corresponds to Inter Block Interference (IBI) between two consecutive block transmissions at $k$ and $k + 1$ and given by:

$$
H_{IBI} = \begin{bmatrix}
  0 & \cdots & 0 & c_{L-1} & \cdots & c_1 \\
  \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
  \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\
  \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
  0 & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}
$$

At the receiver end, in order to cancel the IBI, the first $N_G$ samples of the frame are discarded.

$$
\begin{bmatrix}
  r_{1+N_G}^{CP}(k) \\
  r_{2+N_G}^{CP}(k) \\
  \vdots \\
  r_{N+N_G}^{CP}(k)
\end{bmatrix}
= \begin{bmatrix}
  r_1(k) \\
  r_2(k) \\
  \vdots \\
  r_N(k)
\end{bmatrix}
= \begin{bmatrix}
  c_{L-1} & \cdots & c_0 & 0 \\
  \vdots & \ddots & \ddots & \ddots \\
  \vdots & \ddots & \ddots & \ddots \\
  0 & \cdots & c_{L-1} & c_0
\end{bmatrix} \cdot x(k)
$$

This can be rewritten as:
The use of cyclic redundancy has enabled us to convert the linear convolution to a circular convolution. Since any circulant matrix is diagonal in the Fourier basis [16], it is very easy to diagonalize the channel effect by FFT processing at the receiver as shown below:

\[
\begin{bmatrix}
 c_0 & 0 & \cdots & c_{L-1} & c_1 \\
 \vdots & \ddots & \ddots & \vdots & \vdots \\
 c_{L-1} & \ddots & \ddots & c_{L-1} & \vdots \\
 0 & \ddots & \ddots & \ddots & \vdots \\
 0 & \cdots & 0 & c_{L-1} & c_0
\end{bmatrix} W^H \begin{bmatrix}
 s_0(k) \\
 \vdots \\
 s_{p-1}(k) \\
 0
\end{bmatrix}
\]

(2.19)

\[
\begin{bmatrix}
 y_0(k) \\
 y_1(k) \\
 \vdots \\
 y_{N-1}(k) \\
 y_N(k)
\end{bmatrix} = W \begin{bmatrix}
 r_1(k) \\
 \vdots \\
 r_{N}(k)
\end{bmatrix} = W \begin{bmatrix}
 c_0 & 0 & \cdots & c_{L-1} & c_1 \\
 \vdots & \ddots & \ddots & \vdots & \vdots \\
 c_{L-1} & \ddots & \ddots & c_{L-1} & \vdots \\
 0 & \ddots & \ddots & \ddots & \vdots \\
 0 & \cdots & 0 & c_{L-1} & c_0
\end{bmatrix} \times \begin{bmatrix}
 s_0(k) \\
 \vdots \\
 s_{p-1}(k) \\
 0
\end{bmatrix}
\]

\[
= \text{diag}[h_1, h_2, \ldots, h_N] \cdot s
\]

\[
= \text{diag}[h_1, h_2, h_3, \ldots, h_p] \cdot s_p
\]

(2.20)

\[
y_i(k) = h_i s_i(k)
\]

From the simple properties, convolution in one domain is equivalent to multiplication in the other domain. Convolution here yields a multiplication in the frequency domain. The signal \(s\) is transmitted over \(N\) parallel flat fading channels. Each channel is subjected to complex frequency attenuation. As shown in Figure 2.5. In the case of noisy transmission, the time Gaussian added noise vector is multiplied at the receiver by the FFT demodulator, the statistics of a Gaussian vector does not change by orthogonal transformation.
Figure 2.5 Parallel channels via OFDM.
CHAPTER 3

Carrier Frequency Offset Estimation

3.1 Introduction

OFDM is a great technique to handle impairments of wireless communication channels such as multipath propagation. Hence, OFDM is a practical candidate for future 4G wireless communications techniques [1] - [4]. On the other hand, one of the major drawbacks of the OFDM communication system is the drift in reference carrier. The offset present in received carrier will lose orthogonality among the carriers as shown in Figure 3.1. Hence, the CFO causes a reduction of desired signal amplitude in the output decision variable and introduces ICI. Then it brings up an increase of BER. The effect caused by CFO for OFDM system was analyzed in [30]-[35]. In [30] BER upper bound of OFDM system is analyzed without ICI self cancellation [31] and BER of OFDM system is analyzed using self cancellation, but this method is less accurate. In [33], it is indicated that CFO should be less than 2% of the bandwidth of the subchannel to guarantee the signal to interference ratio to be higher than 30 dB. A critically sampled OFDM/OQAM system is also not robust to CFO [33], even when optimal pulses are used as shaping filters [34]. Thus, carrier frequency offset greatly degrades system performance. Therefore, practical OFDM systems need the CFO to be compensated with sufficient accuracy, and this has led to a whole lot of literature on CFO estimation algorithms. In [35], a formula for the BER analysis of OFDM system with the conjugate cancellation scheme has been derived.

Most of the existing CFO estimators for OFDM are based on periodically transmitted pilot symbols [36] - [41]. Yet, the pilot symbols transmission loses a significant bandwidth, especially in the case of continuous transmissions. Therefore, pilot-based schemes are mainly suited for packet oriented applications.
Semi blind approaches proposed in the literature are the first step to improve the bandwidth efficiency [45]. Those usually depend on various assumptions such as the usage of a single pilot symbol, two identical consecutive OFDM data blocks, or some specific structure within the OFDM symbol.

Recently, blind, or non data aided methods have received extensive attention, as the bandwidth will be totally kept for real data. Among different classes of blind methods, subspace based methods [36] - [45] are the famous category which were lately shown to be equivalent to the ML estimator [40]. Those methods depend on the low rank signal model induced by either some unmodulated carriers or virtual carriers (VC) at the edges of the OFDM block, which aim at minimizing the interference caused to adjacent OFDM systems. While OFDM systems are suited by design to multipath transmission, many existing CFO estimators deal only with
frequency flat channels. Extension of ML methods to multipath Rayleigh fading channels may be found in [59]. More recently, non-circularity introduced by real-valued modulations was exploited in [55]. In [66] a blind CFO estimation algorithm has been derived by exploiting the conjugate second-order cyclostationarity of the received OFDM signal in the case of noncircular transmissions. In [67] this method, designed for standard OFDM systems, has been extended and analyzed in the context of OFDM/OQAM transmissions. On the other hand, the derived estimator assures adequate performance only when a large number of OFDM symbols is considered. In [68] a blind joint CFO and symbol timing estimator based on the unconjugate cyclostationarity property of the OFDM/OQAM signal has been derived. Constant Modulus (CM) constellations allow highly accurate CFO estimation [57]. Most of the CFO estimation algorithms in the literature exploit second order cyclostationarity [61].

In the Blind CFO Estimator the used subchannels will be totally used to transmit real data and the CP will not be extended by any extra guard intervals. The blind estimators are considered as a band width efficient ones. The blind estimators of the CFO in the OFDM system can be built basically based on the structure of the OFDM frame or its components: Blind CFO estimators based on the used carriers [7], VC based blind CFO estimators [49], and the CP based blind CFO estimators. In the following subsection different blind estimators based on used carriers are introduced.

### 3.2 Blind CFO Estimators Based on the Used Carriers

An OFDM system is implemented by IDFT and DFT each of size $N$ for modulation and demodulation, respectively. As introduced in Chapter Two, the $N$ samples of the IDFT output are given by:
\[ x(k) \equiv Ws(k) \] (3.1)

where \( W \) is the NIDFT matrix, given by (2.4) and \( s(k) \) is the \( k^\text{th} \) block of size \( N \) (including VC) to be transmitted given by (2.5)

\[ s(k) = [s_0(k) \ s_1(k) \ldots s_{N-1}(k)]^T \] (3.2)

In practical OFDM system the number of used subcarriers \( P \) is generally less than the DFT block size \( N \). The remaining unused subchannels \((N-P)\) is known as virtual carriers, which are padded by zeros. The QPSK or QAM data symbol to be transmitted through the \( k^\text{th} \) block is given by:

\[ s_p(k) = [s_0(k) \ s_1(k) \ldots s_{P-1}(k)]^T \] (3.3)

The removal of the guard samples at the receiver end makes the received sequence the circular convolution of the transmitted sequence with the Channel Impulse Response (CIR) \( h(l) \), \( l = 0,1,...,L_c - 1 \), where \( L_c \) is the channel length. Inside the \( k^\text{th} \) block only the guard portion of the signal will be distorted since the channel length \( L_c < G \). The receiver input based on used subcarriers for the \( k^\text{th} \) block is given by:

\[
y(k) \equiv [y_0(k) \ y_1(k) \ldots y_{N-1}(k)]^T \\
= W_pHs(k) + z(k) \] (3.4)

where

\[
H = \text{diag}[H(0), H(1), \ldots, H(P-1)] \] (3.5)

\[
H(i) = \sum_{l=0}^{L_c-1} h(l)w^{-il} 
\]

In the existence of the CFO, the receiver input for the \( k^\text{th} \) block given by:

\[
y(k) = EW_pHs(k)e^{j(k-1)\varphi(N+G)} + z(k) \] (3.6)

where

\[
E = \text{diag}(1, e^{j\varphi}, \ldots, e^{j(N-1)\varphi})
\]
and $\varphi$ is the carrier offset. Comparing (3.5) with (2.5), a new term $E$ due to CFO has come into sight. Applying DFT to (3.6), will not lead to the subcarrier recovery as the orthogonality is destroyed by $E$.

$$W_p^H E W_p \neq I$$

To maintain the orthogonality among the subchannel carriers and to avoid ICI, the matrix $E$ must be estimated and compensated before applying the DFT to (3.6)

$$W_p^H E^{-1} E W_p \approx I$$

The task now is to estimate $E$ blindly, which is a function of $\varphi$ only, assuming that the $K$ received noisy data blocks (each of length $P$) are the only measurements available. No training data will be used; the used subchannels will be totally used to transmit real data.

### 3.3 Blind CFO Estimation using ESPRIT Algorithm

The standard ESPRIT algorithm exploits the shift invariant structure available in the signal subspace, and estimates the parameters of interest through subspace decomposition and generalized eigenvalue calculation [7]. Given the $k^{th}$ block of the received signal (10), one can form $N - M$ block of $M + 1 \geq P + 1$ consecutive samples in both the forward and backward directions as follows:

$$y^F_i := [y_{i-1}(k) \ y_i(k) \ ... \ y_{i+M-1}(k)]^T$$

$$y^B_i := [y_{N-i}(k) \ y_{N-i-1}(k) \ ... \ y_{N-i-M}(k)]^T, \quad i = 1,2,\ldots,N-M \quad (3.7)$$

From (3.6) it can be easily verified that

$$y^F_i(k) = E_{M+1} W_{M+1} \Delta^i \hat{s}(k) \quad (3.8)$$

where $W_{M+1}$ is the first $M + 1$ rows of the $W_p$, and the diagonal matrix $E_{M+1}$ is given by:
\[ E_{M+1} = \text{diag}(1, e^{j\varphi}, \ldots, e^{jM\varphi}) \]

The diagonal matrix \( \Delta \) with the carrier frequency offset information is given by:

\[ \Delta = \text{diag}(e^{j\varphi}, e^{j(\omega+\varphi)}, \ldots, e^{j(\omega(P-1)+\varphi)}) \] (3.9)

and \( \omega = 2\pi / N \). Similarly backward vector is given by \( y^i_B \) by

\[
y^i_B(k) = E_{M+1}W_{M+1}\Delta^i e^{-j\varphi(N-1)} \begin{bmatrix} 1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & e^{j(P-1)(N-1)\omega} \end{bmatrix} \bar{s}^*(k)
= E_{M+1}W_{M+1}\Delta^i r(k) \tag{3.10}
\]

where \([\mathbf{\bullet}]^*\) denotes complex conjugate. The sample covariance matrix can be expressed as:

\[
R_{M+1} = \frac{1}{K(N-M)} \sum_{k=1}^{K} \sum_{i=1}^{N-M} \left[ y^i_F(k)(y^i_F(k))^H + y^i_B(k)(y^i_B(k))^H \right]
= A_{M+1}.E[(\bar{s}(k) + r(k))(\bar{s}(k) + r(k))^H].A_{M+1}^H \tag{3.11}
\]

where

\[
A_{M+1} := E_{M+1}W_{M+1}
\]

The \( P \) eigenvectors of the sample covariance that span the signal subspace can be obtained by using SVD. The shift invariant structure of the covariance matrix is exploited to find the eigenvalues of the diagonal matrix \( \Delta \).

\[
\Delta = (A_{M+1}(1:M, 1:P))^\dagger (A_{M+1}(2:M + 1,1:P)) \tag{3.12}
\]

The CFO can be calculated as:
\[ \varphi = \text{Angle}\left[ \frac{\text{trace}(\Delta)}{\sum_{k=0}^{P-1} e^{j k \omega}} \right] \]  

(3.13)

3.4 Blind CFO Estimation using MUSIC

The MUSIC search technique [48] provides a high accuracy carrier estimate by taking advantage of the inherent orthogonality among OFDM subchannels. Indeed, even when the OFDM signal is distorted by an unknown carrier offset, the received signal possesses an algebraic structure, which is a direct function of the carrier offset. It shows in the following that this property permits the formulation of a cost function, which yields a closed-form estimate of the carrier offset. Since the \( W_p \) is the \( P \) columns of the \( N \times N \) IDFT matrix \( W \), its orthogonal complement is given by:

\[ \mathbf{W}^\perp = [w_{p+1} \ w_{p+2} \ldots \ w_N] \]

and is known as a prior information. Hence, in absence of carrier offset, means \( \varphi = 0 \), we have:

\[ w_{P+i}^H y(k) = w_{P+i}^H W_p s(k) = 0, \quad i = 1, 2, \ldots, N - P \]  

(3.14)

The above result is deviated from zero when \( \varphi \neq 0 \). However, if we consider the matrix \( Z \) as:

\[ Z = \text{diag}(1, z, z^2, \ldots, z^{N-1}) \]

it can be easily understood that

\[ w_{P+i}^H Z^{-1} y(k) = w_{P+i}^H QZ^{-1} EW_p s(k) = 0, \quad i = 1, 2, \ldots, N - P \]

This observation leads to development of cost function with bounded data vectors as:

\[ P(z) = \sum_{i=1}^{L} \sum_{k=1}^{K} \| w_{P+i}^H Z^{-1} y(k) \|^2 \]

\[ = \sum_{i=1}^{L} \sum_{k=1}^{K} w_{P+i}^H Z^{-1} y(k) y^H(k) Z W_{P+i} \]  

(3.14)
where \( L \leq N - P \). In a system with many virtual carriers, we may choose \( L \ll N - P \) to reduce computational complexity without loss of performance. Clearly, \( P(z) \) is zero when \( z = e^{j\varphi} \).

Therefore, one can find the carrier offset by evaluating \( P(z) \) along the unit circle, as in the well-known MUSIC algorithm in array signal processing. On the other hand, it is noted that \( P(z) \) forms a polynomial of with order \( 2(N - 1) \). Such allows a closed-form estimate of \( \varphi \) through polynomial rooting. In particular, \( e^{j\varphi} \) can be identified as the root of \( P(z) \).

### 3.5 Blind CFO Estimation Using Propagator Method

The propagator method could be used to estimate the CFO. By using this method [78] matrix decomposition would be avoided, and the null space could be extracted directly from the observation matrix. Chapter Four introduces the idea of this estimator.

### 3.6 Blind CFO Estimation Using Rank Revealing QR Factorization

Rank Revealing QR Factorization [79] is an efficient method to estimate the CFO. Chapter Five introduces this estimator in details.
CHAPTER 4

The Propagator Method

4.1 Introduction

In the estimation problems such as Direction of Arrival (DOA) estimation and frequency estimation, the identification of the signal and the noise subspaces plays a fundamental role. This identification process was generally obtained by the EVD or SVD of the spatial correlation matrix of the observations. Specifically, let us express the SVD of the matrix \( A \) as

\[
A_{n \times p} = U_{n \times n} \cdot S_{n \times p} \cdot V_{p \times p}^T
\]  

(4.1)

where \( U \) is a unitary square matrix, the columns of \( U \) are the left singular vectors, \( S \) has singular values and is diagonal, \( V \) is a unitary square matrix and \( V^T \) has rows that are the right singular vectors. The SVD represents an expansion of the original data in a coordinate system where the covariance matrix is diagonal. Calculating the SVD [70] consists of finding the eigenvalues and eigenvectors of \( A^T A \) and \( A A^T \). The eigenvectors of \( A^T A \) make up the columns of \( V \) and the eigenvectors of \( A A^T \) make up the columns of \( U \). Also, the singular values in \( S \) are square roots of eigenvalues from \( A A^T \) or \( A^T A \). The singular values are the diagonal entries of the \( S \) matrix and are arranged in descending order. The singular values are always real numbers. The eigenvalues can be separated into two distinct groups: the signal eigenvalues and the noise eigenvalues. Accordingly, the eigenvectors can be separated into the signal and noise eigenvectors. The columns of signal eigenvectors span signal subspace, whereas those of noise eigenvectors span its orthogonal complement, which is the noise subspace. Unfortunately, the EVD is computationally intensive and time consuming [71] especially when the number of sensors or the assumed order of the signal model is large. Consequently, to decrease the computational load of EVD, many efficient techniques have been developed from different perspectives, such as computing only a
few eigenvectors or a subspace basis, approximating the eigenvectors or basis, and recursively updating the eigenvectors or basis. Recently, simple computational subspace based methods have been proposed for estimating the directions of narrow band signals efficiently [78], [80], [82] where the need for computation of EVD/SVD is avoided. The representative methods are the Bearing Estimation Without Eigen decomposition (BEWE) [80], PM and Orthonormal PM (OPM) [83] and Subspace Methods Without Eigen Decomposition (SWEDE) [81], in which the exact signal/noise subspace is easily obtained from the array data based on a partition of the array response matrix. In the chapter, the PM will be introduced. Subspace based methods have been widely used for the parameter estimation in the array signal processing problems because of their high resolution and computational simplicity [84]. The propagator method [78] belongs to a subspace based methods for DOA which requires only linear operations but does not involve any EVD or SVD as it is popular in the subspace techniques. In other words, the propagator is a linear operator which only depends on steering vectors and which can be easily extracted from the direct data set or the covariance matrix. It is known that computational loads and the processing time of the PM can be significantly smaller, e.g., one or two order, than MUSIC and ESPRIT [88].

The PM, in particular, has been well studied in various aspects in the recent decade [84]. The PM has been used to estimate the frequencies of multiple real sinusoids. The PM achieved a fast algorithm and a high resolution estimates. An extensive use of the PM is found in the DOA problems [85]-[87]. Recently, the PM was utilized in the joint time delay and frequency estimation of sinusoidal signals, received at two separated sensors problem [26], [27]. The PM estimator gave a wonderful performance in comparison to the conventional methods [84]. In the next section the propagator formulation will be introduced.
4.2 Propagator Formulation

Given a matrix $A$ of size $M \times N$, we may partition $A$ into two sub-matrices $A_1$ and $A_2$ of size $P \times N$ and $(M - P) \times N$ respectively.

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

We defined a propagator matrix $P$ of size $(M - P) \times P$ satisfying:

$$P . A_1 = A_2$$

(4.2)

we may rewrite (4.2) as

$$P . A_1 - A_2 = 0$$

$$[P \quad -I] \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0$$

$$E . A = 0$$

(4.3)

Matrix $E$ of size $(M - P) \times M$ is representing an orthogonal space of matrix $A$; then each column of $A$ is orthogonal to each of the rows of $E$. In other words, $A$ is the null space of $E$ that is the orthogonal complement of the row space of $E$. Or we may partition $A$ into two sub-matrices $A_1$ and $A_2$ of size $M \times P$ and $M \times (N - P)$ respectively.

$$A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}$$

We defined a propagator matrix $P$ of size $P \times (N - P)$ satisfying:

$$A_1 . P = A_2$$

(4.4)

$$A_1 . P - A_2 = 0$$

$$[A_1 \quad A_2] . \begin{bmatrix} P \\ I \end{bmatrix} = 0$$

$$A . E = 0$$

(4.5)
Matrix $E$ of size $N \times (N - P)$ is representing an orthogonal space of matrix $A$; then each row of $A$ is orthogonal to each of the column of $E$.

### 4.3 CFO Problem Formulation

We will use the PM in conjunction with the multiple signal classification (MUSIC) algorithm \[84\] for estimating the carrier offset in the received signal. We will use the existing structure of OFDM system to form a propagator to explore the presence of carrier offset. The receiver input based on used subcarriers for the $k^{th}$ block is given by (3.6), and can be written in vector notation as

$$ y(k) \overset{\text{def}}{=} [y_0(k) \; y_1(k) \; ... \; y_{N-1}(k)]^T $$

where $[\;]^T$ denotes transpose. The $K$ blocks of the received data are collected in matrix $Y$ of size $(N \times K)$

$$ Y = [y(1) \; y(2) \; ... \; y(K)] + Z $$  \hspace{1cm} (4.6)

where $Z$ is the corresponding additive white gaussian noise matrix. Constructing $(N - M + 1)$ sub-matrices from $Y$, each of size $M \times K$ such as $M \geq P$, the first three matrices are given by:

$$ Y^1 = \begin{bmatrix} y_0(1) & y_0(2) & ... & y_0(K) \\ y_1(1) & y_1(2) & ... & y_1(K) \\ \vdots & \vdots & \ddots & \vdots \\ y_{M-1}(1) & y_{M-1}(2) & ... & y_{M-1}(K) \end{bmatrix} $$  \hspace{1cm} (4.7)

$$ Y^2 = \begin{bmatrix} y_1(1) & y_1(2) & ... & y_1(K) \\ y_2(1) & y_2(2) & ... & y_2(K) \\ \vdots & \vdots & \ddots & \vdots \\ y_M(1) & y_M(2) & ... & y_M(K) \end{bmatrix} $$
In general, the $i^{th}$ sub-matrix is given by:

$$Y^i = \begin{bmatrix}
y_{i-1}(1) & y_{i-1}(2) & \ldots & y_{i-1}(K) \\
y_{i}(1) & y_{i}(2) & \ldots & y_{i}(K) \\
\vdots & \vdots & \ddots & \vdots \\
y_{i+M-1}(1) & y_{i+M-1}(2) & \ldots & y_{i+M-1}(K)
\end{bmatrix}$$

(4.8)

we may rewrite (4.8) as:

$$Y^i = [y^i(1) \ y^i(2) \ y^i(3) \ldots \ y^i(K)] + Z^i$$

(4.9)

where

$$y^i(k) = [y_{i-1}(k), y_{i}(k), \ldots, y_{i+M-1}(k)]^T + z^i(k)$$

$$i = 1, 2, \ldots, N - M, \ k = 1, 2, \ldots, K$$

(4.10)

Collecting $(N - M + 1)$ sub matrices calculated in (4.9) each of size $M \times K$ to form a matrix $X$ of size $M \times K(N - M + 1)$

$$X = [Y^1 \ Y^2 \ldots \ Y^{N-M+1}]$$

(4.11)

it can be easily shown that (4.11) is equivalent to:

$$X = [A\Phi^0 S \ A\Phi^1 S \ldots \ A\Phi^{N-M} S]$$

$$= A \begin{bmatrix} \Phi^0 & \Phi^1 & \ldots & \Phi^{N-M} \end{bmatrix} S$$

(4.12)

where:

$$A := E_M W_M$$

$W_M$ consists of the first $M$ rows of $W_p$

$$E_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & e^{j\phi} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & e^{j(M-1)\phi} \end{bmatrix}$$
\[ \Phi = diag(e^{j\varphi}, e^{j(\omega+\varphi)}, \ldots, e^{j(\omega(P-1)+\varphi)}) \], the unknown \( P \times P \) diagonal matrix including the information of carrier offset, with \( \omega = 2\pi/N \).

The structure in (4.12) is similar to ESPRIT structure in DOA problems [86], and the frequency estimation problem [26], hence shift invariance property can be applied. Partitioning \( A \) into two sub-matrices \( A_1 \) and \( A_2 \) of size \( P \times P \) and \( (M-P) \times P \) respectively. We defined a propagator matrix \( P \) of size \( P \times (M-P) \) satisfying:

\[ P^H A_1 = A_2 \] (4.13)

Multiplying (4.13) by \([\Phi_0^T \Phi_1^T \ldots \Phi_{N-M}^T].S\) we obtain

\[ P^H A_1[\Phi_0^T \Phi_1^T \ldots \Phi_{N-M}^T].S = A_2[\Phi_0^T \Phi_1^T \ldots \Phi_{N-M}^T].S \]

\[ P^H X_1 = X_2 \] (4.14)

So we can partition the received data matrix \( X \) into two sub-matrices \( X_1 \) and \( X_2 \) with the dimensions \( P \times K(N-M+1) \) and \( (M-P) \times K(N-M+1) \) respectively and given by

\[ X_1 = A_1[\Phi_0^T \Phi_1^T \ldots \Phi_{N-M}^T].S \]

\[ X_2 = A_2[\Phi_0^T \Phi_1^T \ldots \Phi_{N-M}^T].S \]

The Propagator matrix \( \hat{P} \) can be estimated by

\[ \hat{P} = \arg \min \|X_2 - P^H X_1\|^2 \]

\[ = (X_1 X_1^H)^{-1} X_1 X_2^H \] (4.15)

where \( \| \cdot \|^2 \) denotes Euclidean norm. Matrix \( E \) can be defined as

\[ E = [P \ -I] \] (4.16)

where \( I \) is the identity matrix of size \( (M-P) \times (M-P) \). Clearly here,

\[ E^H X = P^H X_1 - X_2 = 0 \] (4.17)

In the noisy channel the basis of the matrix \( E \) is not orthonormal. Therefore, with an introduction of the orthogonal projection matrix \( Q \) which represents the noise sub-space, we may write:
where the orthonormal projection matrix is given by:

\[ Q = E(E^H E)^{-1}E^H \]  

(4.18)

MUSIC [8] like search algorithm is applied to estimate the frequency offset using following function:

\[ \hat{\varphi}_{MU} = \frac{1}{A(\varphi)^H QA(\varphi)} \]  

(4.19)

Instead of searching for the peaks in (4.19), an alternative is to use a root-MUSIC. The frequency estimates may be taken to be the angles of the roots of the polynomial \( D(z) \) that are closest to the unit circle.

\[ D(z) = \sum_{i=0}^{M-1} V_i(z) V_i^*(1/z^*) \]  

(4.20)

where \( V_i(z) \) is the z-transform of the \( i^{th} \) column of the projection matrix \( Q \) [9].

### 4.4 CFO Simulations Results

Extensive computer simulations are done to validate our proposed method. We considered OFDM system with \( N=64 \) carriers, of which \( P=40 \) are the used subcarriers while the remaining \( N-P=24 \) are the virtual carriers. Transmitted symbols are drawn from equiprobable QPSK constellation. The CP length was selected to be eleven symbols and the frequency offset \( \varphi \) is assumed to be 0.1\( \omega \). The experiment was run under AWGN environment. The performance of the proposed method is compare with the ESPRIT [7]. The estimator performance was evaluated using the normalized mean square error (MSE) and is given by

\[ MSE_{dB} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \frac{\varphi - \hat{\varphi}}{\omega} \right)^2 \]  

(4.21)
Figure 4.1 plots the normalized CFO MSE of the propagator method as a function of the structure parameter M at fixed 10 dB SNR with $N_t = 1000$ independent Monte Carlo realizations. Different numbers of blocks were assumed: 8, 10, 20, 30, and 40. It was found that the optimal value is 41, which is $P+1$. We will use the optimal M to modify (4.21) by constructing $(N-P)$ sub matrices each of size $(P+1) \times K$ to form a matrix $X$ of size $(P+1) \times K(N-P)$. It is worth mentioning that selecting the optimal value of the parameter M is significant not only to minimize the MSE but to reduce the computational load. In other words, selecting an improper M will lead to a high MSE even if the block number is very large. To guarantee fair comparison with our reference, the ESPRT method [], we will test all the possible values for the structure parameter M. In all of our comparisons we will pick the optimal

![Figure 4.1 Normalized MSE for propagator method versus structure parameter M using $K=2, 4, 6, 8, 10, 20, 30, 40, MC=200$ and fixed 10 dB SNR.](image-url)
parameter for each method. Figure 4.2 and Figure 4.3 are showing the MSE as a function of parameter M for the ESPRT method [7] with the same experiment setup. This step is important to guarantee a fair comparison between the two methods. The optimal value of the structure parameter is not fixed as the propagator method. The optimal value is a function of the number of frames K. Similar to the propagator method, selecting the improper value of M will lead to a high MSE. For example, selecting M=45 would give the same error even the number of blocks changed from 8 to 40. Table 2 summarizes the optimal values of the structure parameter M for each number of used blocks, K.

![Normalized MSE for ESPRT method versus structure parameter M using K=8, 10, 20, 30, 40, MC=200 and fixed 10 dB SNR](image)

Figure 4.2 Normalized MSE for ESPRT method versus structure parameter M using K=8, 10, 20, 30, 40, MC=200 and fixed 10 dB SNR
Figure 4.3 Normalized MSE for ESPRT method versus structure parameter M using K=2, 4, 6 MC=200 and fixed 10 dB SNR

<table>
<thead>
<tr>
<th>K</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>43</td>
<td>47</td>
<td>50</td>
<td>53</td>
<td>53</td>
<td>57</td>
<td>59</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 4.1: The optimal value of the structure parameter for each block size.

Figure 4.4 plots the MSE as a function of signal to noise ratio (SNR) for both estimators with different numbers of used blocks (2, 4, 6, 8, 10, 20, 30, 40) and with the optimal value of M. The frequency offset is assumed to be .1ω. PM shows a fantastic result compare with the ESPRIT [7]. It is obvious that PM with block only can almost achieve the same results that the ESPRIT achieves with 20 blocks, hence PM can achieve the same performance with one tenth the number of blocks. The experiment was run under AWGN environment with \( N_t = 300 \) independent Monte-Carlo realizations. Figure 4.5 plots the MSE as a function of the number of
Figure 4.4 PM and ESPRIT Estimators performance versus SNR using $K=2, 4, 6, 8, 10, 20, 30$ with optimal structure parameter.

Figure 4.5 PM and ESPRIT estimators performance versus the number of blocks $K$. 

45
blocks $K$ for both estimators with different SNR (5 dB, 15dB, 20dB) and with the optimal value of $M$. The frequency offset is assumed to be $\omega_0$. PM shows a fantastic result compare with the ESPRIT [7]. The experiment was run under AWGN environment with $N_t = 300$ independent Monte-Carlo realizations. Figure 4.6 shows the performance of both estimators as a function of frequency offset at different signal to noise ratios. Again the superiority of PM over the ESPRIT is evident in the different situations.

4.5 Conclusions

A novel propagator based method in conjunction with the MUSIC based search algorithm or root-MUSIC based algorithm for estimating CFO for OFDM systems is projected. For the same experiment set up, almost 15 dB is achieved in SNR for the proposed PM based method over the ESPRIT one. The proposed method is showing equivalent performance in
comparison with the well known ESPRIT type estimator at one tenth of the block acquisitions. We considered blocked data of length 10 and the structure parameter M considered 60 for ESPRIT algorithm, while it is 41 for the proposed PM based algorithm.
CHAPTER 5

Rank Revealing QR Method

5.1 Introduction

The RRQR [79] is a good alternative to conventional subspace decomposition techniques [70] like SVD and EVD, because it has a lower computational cost. Moreover, it is quite supportive in rank deficient least square problems.

5.2 RRQR Method

Collecting \((N - M + 1)\) sub matrices calculated in (4.9) each of size \(M \times K\) to form a \(LM \times K(N - M - L + 2)\) matrix \(X\)

\[
Y = XS = \begin{bmatrix}
Y^1 & Y^2 & \cdots & Y^{N-M-L+2} \\
Y^2 & Y^3 & \cdots & Y^{N-M-L+3} \\
\vdots & \vdots & \ddots & \vdots \\
Y^L & Y^{L+1} & \cdots & Y^{N-M+1}
\end{bmatrix}
\]

(5.1)

Also, it can be easily shown that

\[
X = \begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_L
\end{bmatrix} = \begin{bmatrix}
A\Phi^0 & A\Phi^1 & \cdots & A\Phi^{N-M-L+1} \\
A\Phi^1 & A\Phi^2 & \cdots & A\Phi^{N-M-L+2} \\
\vdots & \vdots & \ddots & \vdots \\
A\Phi^{L-1} & A\Phi^L & \cdots & A\Phi^{N-M}
\end{bmatrix}
\]

(5.2)

The structure in (5.2) is similar to the well known structure in DOA problems and hence shift invariance property can be applied. The matrix \(X\) can be partitioned into two subgroups of same size \(X^e\) and \(X^o\) (assuming \(L\) is even), where group matrices \(X^e\) and \(X^o\) are given by even and odd submatrices of matrix \(X\). It can be noticed that the matrices \(X^o\) and \(X^e\) are related by

\[
X^e = X^o\Phi
\]

(5.3)
\[ X^o = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{L-1} \\ X_L \end{bmatrix}, \quad X^e = \begin{bmatrix} X_2 \\ X_4 \\ \vdots \\ X_L \end{bmatrix} \] (5.4)

Applying RRQR factorization to above matrix here, we have

\[ X = QR = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \\ \vdots & \vdots \\ Q_{L1} & Q_{L2} \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \] (5.5)

where the L sub-matrices \( Q_{11}, Q_{21} \ldots Q_{L1} \) are of dimensions \( M \times P \) and collectively forming signal sub-space in matrix \( Q \). The submatrix \( R_{11} \) is upper triangular square full rank matrix while \( R_{12} \) is holding remaining important information with dimensions \( P \times K(N - M - L + 2) \). Because of rank revealing QR factorization, it is interesting to note here that the submatrix \( R_{22} \) is approximately equal to the null matrix. Therefore, it hardly contributes in construction of either signal space or null space of the matrix; hence (5.5) can be approximated as

\[ \bar{X} = \begin{bmatrix} Q_{11} \\ Q_{21} \\ \vdots \\ Q_{L1} \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \end{bmatrix} \] (5.6)

Then we can rewrite (5.4) in following form

\[ X^o \simeq Q^o [R_{11} \quad R_{12}] \] (5.7)

\[ X^e \simeq Q^e [R_{11} \quad R_{12}] \] (5.8)

where group matrices \( Q^e \) and \( Q^o \) are given by even and odd submatrices of signal subspace.

\[ Q^o = \begin{bmatrix} Q_{11} \\ Q_{13} \\ \vdots \\ Q_{1,L-1} \end{bmatrix}, \quad Q^e = \begin{bmatrix} Q_{12} \\ Q_{14} \\ \vdots \\ Q_{1L} \end{bmatrix} \] (5.9)

from (5.7), we get

\[ [R_{11} \quad R_{12}] = Q^o X^o \] (5.10)
where [\(\mathbf{M}\)]\(^\dagger\) is the pseudo inverse of the matrix and the matrix \(\mathbf{Q}^{o\dagger} = (\mathbf{Q}^{oH}\mathbf{Q}^{o})^{-1}\mathbf{Q}^{oH}\).

Substituting the above equation into (5.8)

\[
\mathbf{X}^e \cong \mathbf{Q}^e \mathbf{Q}^{o\dagger} \mathbf{X}^o
\]

(5.11)

Using (5.3) we may write (5.11) as

\[
\mathbf{X}^o\Phi = \mathbf{Q}_0 \mathbf{X}^o
\]

(5.12)

where the matrix \(\mathbf{Q}_0 = \mathbf{Q}^e \mathbf{Q}^{o\dagger}\). Equation (4.12) can be reformulated as

\[
\Phi_{ii}\mathbf{X}^o_i = \mathbf{Q}_0 \mathbf{X}^o_i, \quad i = 1, 2, \ldots, P
\]

(5.13)

Equation (5.13) is a classical eigenvalue problem with the eigenvector \(\mathbf{X}^o_i\) and the eigenvalue \(\Phi_{ii}\). The eigenvector \(\mathbf{X}^o_i\) is the \(i\)th column of the matrix \(\mathbf{X}^o\) and the \(\Phi_{ii}\) is the \(i\)th diagonal element of the diagonal matrix \(\Phi\). Clearly, the \(P\) eigenvalues of the matrix \(\mathbf{Q}_0\) correspond to the \(P\) diagonal elements of the diagonal matrix \(\Phi\). Therefore, \(\text{trace}(\mathbf{Q}_0) = \text{trace}(\Phi)\), then the CFO can be estimated as

\[
\exp(j\varphi) = \frac{\text{trace}(\mathbf{Q}_0)}{\sum_{k=0}^{P-1} e^{jk\omega}}
\]

(5.14)

5.3 CFO Simulations Results

Extensive computer simulations are done to validate our proposed method. In the first experiment, we considered OFDM system with \(N=64\) carriers, of which \(P=40\) are used carriers. Transmitted symbols are drawn from equiprobable QPSK constellation. The cyclic prefix (CP) length is eleven symbols, the matrix structure parameter \(L\) is assumed to be two and the frequency offset is assumed to be \(0.1\omega\). The experiment is verified under AWGN environment with \(N_t = 1000\) independent monte-carlo realizations. The estimation performance is evaluated by mean square error (MSE) and given by
\[ MSE_{dB} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \frac{\varphi - \hat{\varphi}}{\omega} \right)^2 \] (5.15)

The normalized MSE is compared with two different number of blocks (K=2 and K=4) acquisition. Even for a small number of block acquisition our algorithm performs much better than the classical Esprit [7] type algorithm. For example, to achieve the same MSE performance with just K=4, the reference algorithm requires an approximately 20 dB of additional SNR.

The second figure is comparing the performance of the proposed and the reference algorithm under a varying number of blocks. It is evident that our proposed algorithm is showing better performance compared with the reference algorithm [7] especially at lower block acquisitions. Significant achievement can be seen below K=10 block acquisition by the RRQR based closed form algorithm.

Figure 5.1 Normalized MSE versus SNR at \( \varphi = 0.1\omega \)
Figure 5.2 Normalized MSE versus block acquisition.

Figure 5.3 Normalized Processing time versus block acquisition.
The third figure is focusing on processing time involving in each of the methods. We compared each method using normalized processing time with respect to the different block realization. More than double the calculation is required by reference algorithm [7] in contrast with the proposed method. The RRQR based method is more efficient.

5.4 Conclusion

New blind OFDM CFO Estimation algorithm was presented in this Chapter. The main advantages with the proposed algorithm are that it does not use any training symbols, and it is equipped with closed-form formula. The proposed algorithm is equipped with lower complexity and computationally efficient with respect to its peer ones. Moreover, EVD or SVD based complex spectral decomposition is avoided. Through simulation we achieved significant performance compared with the reference methods.
CHAPTER 6

Conclusion

OFDM is a great technique to handle impairments of the frequency selective channel. Hence, OFDM is a practical candidate for future 4G wireless communications techniques. On the other hand, one of the major drawbacks of the OFDM communication system is the drift in reference carrier. The offset present in received carrier will lose orthogonality among the carriers, and hence, the CFO causes a reduction of desired signal amplitude in the output decision variable and introduces ICI, then brings up an increase of BER. This leads to the necessity to estimate the CFO in order to cancel it in next stage. This dissertation proposes two novel estimators one based on the propagator based method and the other passed on the RRQR. The main advantages with the proposed algorithms are that they do not use any training symbols and it is equipped with closed-form formula. The proposed algorithm is equipped with lower complexity and computationally efficient with respect to its peer ones. Moreover, EVD or SVD based complex spectral decomposition is avoided.

A novel propagator based method in conjunction with the MUSIC based search algorithm or root-MUSIC based algorithm for estimating CFO for OFDM systems is presented in Chapter Four. Almost 15 dB is achieved in SNR for the proposed PM based method over the ESPRIT one. The proposed method is showing equivalent performance in comparison with the well known ESPRIT type estimator at one tenth of the block acquisitions. By introducing RRQR estimator we achieved a significant performance compared with the reference methods especially when the number of the available block is small, which makes this estimator a very good candidate for the fast fading channel as shown in chapter six.
In terms of future work, it is worth to mention that these blind methods may be applied in the OFDMA case and MIMO-OFDM. Many developments can be achieved by improving the estimation function or by obtaining an accurate noise and signal subspaces.
REFERENCES
LIST OF REFERENCES


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