An Improved Frequency Estimator

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***Abstract*— A novel method of estimating the differential delay of a sinusoidal signal is considered. The new method utilizes the Propagator Method (PM) which does not require the eigen-decomposition of the cross-spectral matrix (CSM) in estimating the signal frequencies. Such frequency estimation is based on the observation and/or covariance matrices. Computer simulation is performed to validate the new procedure.**

I. Introduction

The estimation of time delay and frequencies [1] [2] has been a research topic of practical importance in many areas (Radar, Sonar, Ultrasonic, Seismology, Mobile communications etc.) by spatially separated sensors. Similarly frequency estimation [3] has been universally addressed in signal processing literature. We begin with the discrete-time sinusoidal signals $x(n)$ measurements satisfying

$$x(n) = s(n) + e_1(n),\ n = 0,1,\dots,N-1 \quad (1)$$

where

$$s(n) = \sum_{i=1}^{P} A_i \exp(j\omega_i n) \quad (2)$$

The source signal $s(n)$ is modeled by a sum of P complex sinusoids where the amplitudes (A_i) are unknown complex-valued constants, and the normalized radian frequencies (ω_i) are different. Without loss of generality, we considered $\omega_1 < \omega_2 \dots < \omega_P$.To simplify the problem we have assumed the number of sources P either known or pre-estimated [4]. The additive noise parameters $e_1(n)$ and $e_2(n)$ are uncorrelated zero-mean complex white Gaussian processes with variances σ_e^2. Also parameter N represents the number of samples collected at each channel respectively.

A subspace algorithm based on state-space realization has been proposed [5] for joint time delay and frequency estimation of sinusoidal signals received at two separated sensors. The frequency estimates are obtained directly from the eigenvalues of the state transition matrix; while the delay is determined using the observation matrix and the estimated frequencies. A generalized Yule–Walker solution is suggested in [6] literature to determine (ω_i) separately.

The Propagator Method (PM) is subspace-based method [7], [8] which does not require the eigen-decomposition of cross-spectral matrix (CSM) of received signals. It is well known that the computational load of PM based method is significant as it does not involve eigenvalue decomposition (EVD) or singular value decomposition (SVD). We compared our frequency estimator performance with [5].

II. Development of Proposed Method

Using the N received data $x(0), x(1), \dots\dots\dots x(N-1)$ given by (1), we form the $L \times (N-L+1)$ Hankel Matrix

$$\boldsymbol{X} = \begin{bmatrix} x(0) & x(1) & \dots & x(N-L) \\ x(1) & x(2) & \dots & x(N-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(L-1) & x(L) & \dots & x(N-1) \end{bmatrix}$$

$\boldsymbol{X}$ can be rewritten as

$$\boldsymbol{X} = [\boldsymbol{r(0)}\ \boldsymbol{r(1)} \dots\dots\dots \boldsymbol{r(N-L)}] \quad (3)$$

where the i^{th} column of $\boldsymbol{X}$ is given by

$$\boldsymbol{r}(i) = \boldsymbol{A_L(\omega)}(\boldsymbol{\varphi(\omega)})^i \boldsymbol{s} + \boldsymbol{n_{1i}}, \qquad i = 0,1,\dots\dots L-1$$

$$\boldsymbol{A_L(\omega)} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\omega_1} & e^{j\omega_2} & \dots & e^{j\omega_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(L-1)\omega_1} & e^{j(L-1)\omega_2} & \dots & e^{j(L-1)\omega_P} \end{bmatrix}$$

$$\boldsymbol{\varphi(\omega)} = diag(e^{j\omega_1}\ \ e^{j\omega_2} \dots\dots\ e^{j\omega_P}) \quad (4)$$

We can formulate the received data matrix as

$$\boldsymbol{X} = [\boldsymbol{A_L(\omega)s}\ \ \boldsymbol{A_L(\omega)\varphi(\omega)s} \dots\dots \boldsymbol{A_L(\omega)}(\boldsymbol{\varphi(\omega)})^{\boldsymbol{L-1}}\boldsymbol{s}] + [\boldsymbol{n_{10}}\ \boldsymbol{n_{11}} \dots\dots \boldsymbol{n_{1L-1}}]$$

Partitioning $\boldsymbol{A_L(\omega)}$ into two sub-matrices $\boldsymbol{A_{L1}(\omega)}$ and $\boldsymbol{A_{L2}(\omega)}$ with dimensions $P \times P$ and $(L-P) \times P$ respectively. We defined Propagator matrix $\boldsymbol{P}$ satisfying following condition

$$\boldsymbol{P^H A_{L1}(\omega)} = \boldsymbol{A_{L2}(\omega)} \quad (5)$$

Where $(.)^{H}$ denotes hermitian transpose and dimension of matrix $\boldsymbol{P^H}$ is $(L-P)\times P$. From (5) $\boldsymbol{P}$ is calculated as

$$\boldsymbol{P} = (\boldsymbol{A}_{L1}\boldsymbol{A}_{L1}^{H})^{-1}\boldsymbol{A}_{L1}\boldsymbol{A}_{L2}^{H}$$

Similarly, partitioning received data matrix $\boldsymbol{X}$ into two sub-matrices $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ with dimensions $P\times(N-L)$ and $(L-P)\times(N-L)$ respectively.
The propagator can be estimated as

$$\widehat{\boldsymbol{P}} = (\boldsymbol{X_1}\boldsymbol{X_1^H})^{-1}\boldsymbol{X_1}\boldsymbol{X_2^H} \tag{6}$$

Matrix $\boldsymbol{E}$ can be define as

$$\boldsymbol{E} = \begin{bmatrix}\boldsymbol{P}\\ -\boldsymbol{I}\end{bmatrix} \tag{7}$$

where $\boldsymbol{I}$ is identity $(L-P)\times(L-P)$ matrix. Clearly here

$$\boldsymbol{E^H}\boldsymbol{A_L}(\boldsymbol{\omega}) = \boldsymbol{P^H}\boldsymbol{A_{L1}}(\boldsymbol{\omega}) - \boldsymbol{A_{L2}}(\boldsymbol{\omega}) = \boldsymbol{0} \tag{8}$$

In a noisy channel, the basis of matrix $\boldsymbol{E}$ is not orthonormal. By introducing orthogonal projection matrix $\boldsymbol{Q}$ we have

$$\boldsymbol{Q}\boldsymbol{A_L}(\boldsymbol{\omega}) = \boldsymbol{0} \tag{9}$$

where $\boldsymbol{Q} = \boldsymbol{E}(\boldsymbol{E^H}\boldsymbol{E})^{-1}\boldsymbol{E^H}$ **.** Apply MUSIC [1] like search algorithm to estimate the frequencies using following function

$$f(P) = \frac{1}{\boldsymbol{A_L}(\boldsymbol{\omega})^{\boldsymbol{H}}\boldsymbol{Q}\boldsymbol{A_L}(\boldsymbol{\omega})} \tag{10}$$

III. Simulation Results

Extensive computer simulations had been done to validate our proposed method. The first experiment is comparing the performance of the PM Method with the State-Space realization. The scenario considered similar to [5]. The source signal $s(n)$ was a sinusoidal signal of the form (2) with $A_1 = A_2 = 1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s. We simulated estimator performance under AWGN environment with different SNRs and 500 independent monte-carlo (MC) realizations. The numbers of signal samples were 150 and the value of $\boldsymbol{L}$ was chosen to be 100. The mean square error defined below is employed as a performance measure of the frequencies estimator

$$MSE_{dB} = 10log_{10}\left(\frac{1}{N_t P}\sum_{i=1}^{N_t}\sum_{j=1}^{P}\left(\omega_j - \widehat{\omega}_j\right)^2\right) \tag{11}$$

where $\widehat{\omega}_j$ is the estimate of ω_j and N_t is the number of Monte Carlo (MC) trials. Fig. 1 plots root mean square error (RMSE) of the frequencies estimate and compared with state-space realization method [5]. Significant improvements in performance were achieved by the proposed method in an estimation of frequencies, especially SNR ≥ -5 dB.

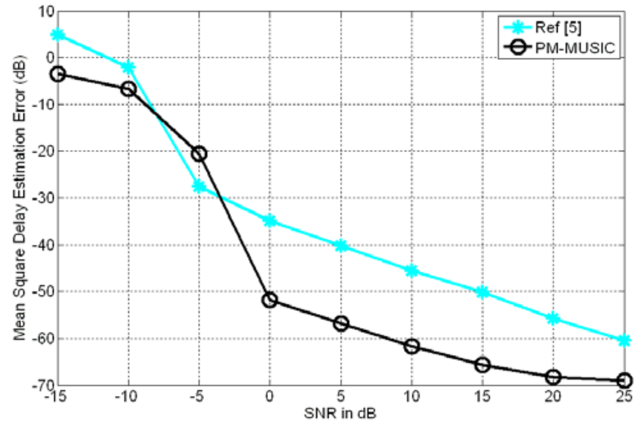

Fig.1. MSE of all frequencies Vs SNR

IV. Conclusion

We proposed a new technique for frequencies estimation of received sinusoidal signals by applying the PM based method. The frequencies estimated either by observation matrix or through covariance matrix of received data matrix. The frequency estimator is showing outstanding performance compared with state space realization illustrated in [5]. In terms of future directions, it would be an interesting to explore computation load behavior of these algorithms.

References

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