

THE IMPACT ON COMPUTATIONAL FLUENCY THROUGH INSTRUCTION IN  
NUMBER SENSE

A Thesis by

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NUMBER SENSE

I have examined the final copy of this Thesis for form and content and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Education with a major in Curriculum and Instruction.

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## ABSTRACT

In a second grade classroom in an urban school district in the Midwest, there was a lack of number sense in the students' mathematics skills. District common assessments were given quarterly and each quarter the standard of number sense was low. Two points of view about teaching children mathematics were found in the research to solve the problem: (1) to teach math the way it has always been taught, or (2) a need for number sense instruction. This research project focused on the implementation of teaching number sense thirty minutes a day, four days a week for eight weeks. Madelyn Hunter's model of direct instruction was used to teach number sense strategies. Results indicated that the post test gains in computational fluency can be predicted by the post test scores in number sense.

## TABLE OF CONTENTS

Chapter		Page Number	
I.	INTRODUCTION	1	
II	REVIEW OF THE LITERATURE	3	
III	DESIGN OF THE STUDY	13	
IV	RESULTS	17	
V	DISCUSSION AND CONCLUSIONS	20	
	REFERENCES	23	
	APPENDICES	25	
	Appendix A	Pre and Post Test	26
	Appendix B	Day-by-Day Interventions	28
	Appendix C	Tens Frames	31
	Appendix D	Checklist	32

## LIST OF TABLES

Table	Page Number
1 Descriptive Statistics	17
2 Predicting Computation from Number Sense	18

## CHAPTER ONE

### INTRODUCTION

In a second grade classroom in an urban school district in the Midwest there were 136 math standards and indicators that were required to be taught by teachers and learned by students in one school year. Each year, teachers in third through fifth grades seemed to think that the previous teachers had missed teaching a particular standard. Were they? If not, what was causing the students to forget the content of the standards addressed in the curriculum? Was it the material; was it developmentally inappropriate, or did the students just lack a sense of numbers? Over the years there have been discrepancies in the understanding by teachers about what the National Council of Teachers of Mathematics (NCTM) desires for teaching computational fluency. Changes were made in the ideas of what mathematics foci should be (NCTM, 2000). Guidelines were established in the hopes that a competence in both computational fluency and mathematical reasoning would be established. “Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On the one hand, computational methods that over practice without understanding are often forgotten or remembered incorrectly... on the other hand, understanding without fluency can inhibit the problem solving process” (p 35). Extant literature was scant in the examination of the relationship between mathematical fluency and conceptual understanding in young children.

#### The Problem

Seven and eight year olds in the second grade in an urban district in the Midwest struggled continually with computational facts, the concepts of even/odd, greater than/less than, and skip counting by twos, threes, fives, and tens. On district quarterly common assessments,

one area of weakness was in the standard of number sense. What was causing that area of weakness? A review of the literature was needed to help identify what types of interventions were needed, either in computation or in number sense. This review is followed by a section reviewing the methodology.

## CHAPTER TWO

### LITERATURE REVIEW

This review is organized into three categories: (1) definitions of the terms (number sense, computational fluency, peer modeling, and direct instruction), (2) research that has been reviewed in all of these areas, and (3) a presentation of the research in the designated areas. Extant literature was reviewed for relevant and applicable studies. Current research indicates that there seems to be a correlation between a foundation in number sense and their ability to compute math problems. A review of the current research has been made.

**Definitions.** Two terms dominated this research project: The terms number sense and computational fluency were examined. A review of the literature for each follows.

**Number sense.** Berch (2005) reviewed many different ideas of number sense, which was first defined in 1954. Since then there has been much debate as to what this term actually means. “No two researchers define number sense in exactly the same way... even more problematic is that cognitive scientists and math editors define the concept of number sense in very different ways” (p. 333). After a thorough review of the literature, Berch formulated a table containing thirty items including awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, and a mental number line. After introducing the table, the idea of instructing these critical points was presented

Some controversy over the idea of “teaching” number sense exists. “Some consider number sense to be a part of our genetic endowment, whereas others regard it as an acquired skill set that develops with experience” (Berch, 2005, p. 334). One researcher viewed number sense as “a skill or kind of knowledge rather than an intrinsic process, it should be teachable...” (p. 336). On the other hand, another researcher believes, “ the emergence of rudimentary components of

number sense in young children is thought to occur ‘spontaneously without much explicit instruction’ (p. 336). In sum, the definition of number sense and its “teachability” is one that was widely discussed, debated, analyzed, and researched. The skills were introduced and loosely defined.

Carboni (2001) stated “number sense is an intuitive feel for numbers and their relationships” (¶. 1). She believed that number sense instruction was an important part of elementary mathematics, and in her opinion, there was great importance for children to work in groups to develop their mathematical thinking, as well as, their flexibility with numbers. “This intuitive feel for numbers and their relationships can be called number sense” (¶. 4). Number sense also “describes a cluster of ideas, such as the meaning of numbers, ways of representing numbers, relationships among numbers, the relative magnitude of numbers, and skill working with them.” She asserted that number sense was not a skill that could be taught in one unit of a class; number sense should be integrated throughout daily mathematics instruction (¶¶. 4-5). Several examples of effective strategies were given; i.e., using tens and ones, counting on or back from a given number, using “nice” numbers, translating to a new problem, hundreds boards, calendars, and games. (¶. 5; ¶. 10). “By developing these strategies themselves, using them to construct meaning in problems, and listening to other children describe their use of these strategies, children’s facility with numbers and confidence about their ability to do mathematics increases” (¶. 6).

For this research, the term number sense was defined as: “This intuitive feel for numbers and their relationships.” (Carboni, 2001. ¶. 4)

Computational fluency. Griffin (2003) states that “computational fluency seems to entail a well-practiced and efficient use of procedures to compute how many items were in a set or how

many there were if sets were joined or separated” (p. 306). She also asserted that NCTM may have unintentionally pushed people to believe that number sense and computational fluency do not go hand in hand. She also believes that teachers tend to look at both computational fluency and number sense as two separate standards. In 2003, the Kansas Department of Education did indeed put number sense and computation into one standard called “number and computation.” For this research, the term computational fluency was defined as the automatic retrieval of basic math facts.

Number Sense Research. Cotter (1998) discussed the difference between American taught mathematics and mathematics taught in Asian countries. “International studies, such as the TIMSS studies, show Asian students do better than American counterparts in mathematics” (¶. 1). Many contributing factors exist, such as “cultural characteristics, homogeneous population, longer school year, public value and support of education, and a philosophy of learning that hard work and good instruction, not talent, determine a student’s success” (¶. 1). She asserted that although some of the differences seen in Asian set classrooms cannot be implemented in American classes, some of the ideas can. The way that numbers were named, visualizing rather than counting and a choice of manipulatives can all help American children (¶. 2). The way numbers are named in Asian cultures is one that is very different than the way they are named in our American culture. In the Asian culture, every time a number is spoken, the value is given, i.e., twenty-one, would be stated two tens and one. Therefore, students from an Asian speaking background have “built in” number sense (¶. 3). “Asian Americans who spoke only English scored in the 54<sup>th</sup> percentile, while students who were also fluent in Chinese or Japanese scored in the 99<sup>th</sup> and 97<sup>th</sup> percentile, this contrasts with bilingual Spanish-speaking third graders who scored in the 16<sup>th</sup> percentile” (¶. 4).

Major differences exist in the ways numbers are taught from early elementary school. “In the U.S., counting was considered the basis of arithmetic; ... counting all, counting on, and counting back. Conversely, Japanese children are discouraged from counting; they are taught to recognize and visualize quantities in groups of five and ten” (¶. 5). In America, there is a push for all children to use manipulatives, until they are ready to rely on other forms of mathematical thinking. “Japanese primary classrooms have very few manipulatives, all of which the children must be able to visualize” (¶. 13). Cotter (1998) proposed a plan that implemented strategies taken directly from an Asian curriculum: visualizing, value naming of numbers, an abacus, overlapping place-value cards, part-part partitioning, and early introduction to multi-digit addition and subtraction. The experimental class used the strategies and scored between 63-94% on place value, while the control class who did traditional workbook work scored between 13-33%.

The instructional differences found in Asian countries and America are substantial. The dependency on manipulatives by those in the American culture seems to be impairing the student’s ability to produce meaningful connections in mathematics. Results of this study depicted the need for less time spent on a concrete level of mathematics and a move more quickly to the pictorial or representational stage on the developmental scale of mathematics (concrete, pictorial, abstract). (As researched by: The Access Center: Improving Outcomes for All Students K-8.)

Benjamin (2006) examined the differences between attitudes of students and parents, organization of the school day, the social environment, the teachers, and the performance in mathematics in three different countries: China, Taiwan, and the United States. Three-hundred seventy-two children in the fifth grade from rural areas in each country were assessed (pp. 1-4).

In the Chinese classrooms, there were as many as 50 students, in the Taiwanese classrooms 30, and in the American classrooms fewer than 20. Overall, both the Chinese students and the Taiwanese students out performed the American students, which nullified the researcher's hypothesis that students in rural parts of countries would perform equally. Benjamin defined several possible contributing factors. Between 10-27% of the Asian students went to a math school at night five or more nights per week. The American students were focused more on social aspects of school (friends, sports, social events, etc.), and the parental expectations (pp. 7-11).

In sum, bilingual students of Asian descent were more likely to perform better on mathematical assessments due to their attitudes and feelings towards school. Students with a "built-in" sense of numbers, due to language, and less use of manipulatives perform and achieve in mathematics. Students who speak a language that does not have number sense "built-in" and use more concrete manipulatives longer have a harder time in mathematics.

In a study conducted in Sydney, Australia, Howell and Kemp (2005) discussed the need for assessing number sense in the first year of school. The primary research question was, "If the prerequisite skills for early mathematics success could be identified, tested and taught in the early years of schooling, would students be in a better position to benefit from the learning experiences in the mathematics classroom" (p. 556). These researchers discussed the diverse issues facing many students today and asserted that children of a low socio-economic background not only have fewer words to tell what was happening around them, but also a weaker number sense (p. 556). To begin their research, they provided a questionnaire to 13 "academics" and asked their opinion of the need for students to understand the principles of "one to one correspondence, matching numeracy, rote counting to five, cardinal value within a

counting range, more than, rote counting to ten, addition of 1 or 2 to any number and its effect, subtraction by three, combining visual images, additive composition, and order irrelevance” (p. 561). Between 50-83.3% of the participants agreed that these skills were essential upon entry into kindergarten. All in all, these researchers agreed on a great need for the “skills” required for good number sense to be adequately defined and shared with professionals. If there was no consistent definition of skills, it was hard to adequately teach what needs to be taught (p. 568).

O’Nan’s (2003) research focused on increasing students’ number sense by using daily number talks in her classroom. Activities included visualization and mental computational strategies. The purpose was to insure that the students would have a larger pool of background strategies available to them when needed (pp. 10-11). The students were given a problem and then asked to solve it. For ten minutes each day, the 4<sup>th</sup> grade students were asked to verbally share their strategies (p. 11). Statistically significant gains were made over a six-week intervention, statistically significant at 0.05 (p. 30). Number talks increased her student’s abilities and strategies to solve a given problem.

Computational Fluency. O’Loughlin (2007) indicates that using an open number line produced a strategy for her students to solve computation problems, as well as word problems (p. 136). From using a number line, a number sense strategy, the students began to have discussions about the need for skips and jumps of tens and the correlations in solving their problems. The students also used a hundreds board to solve computation problems, a number sense strategy that helps the students create a mental model for solving problems. O’Loughlin attempted to replicate Fosnot and Dolk’s 2001 research in her classroom and found it to be successful. She states, “...the majority of my students are jumping back ten...” (p. 137) Through teaching only

the jumping forward strategy, students modeled to each other and investigated how they could make the same assumption for subtraction.

Flowers, Kline, and Rubenstein (2003) looked for reasons why students and teachers struggle with subtraction. They began by having the teachers write down their explanation of how to solve a subtraction problem (pp. 330-331). Then teachers decided that the best way to easily solve a problem was to chunk numbers into smaller more manageable ones. The strategy of breaking apart numbers was a number sense strategy. The teachers were then asked to use a list of strategies found effective by the researchers who were professors at the University of Michigan (p. 333). The list of number sense strategies included, making nicer numbers, using 10, 100, 1000 as anchor numbers, decomposing numbers, place value, etc. (p. 333).

Hauser (2005) used the invented strategies technique to teach his second graders how to solve two-digit addition and subtraction models, deciding that students need to use a model that makes sense to them. They needed an understanding, not just a procedure (p. 405). Carpenter et al. (1998) and Kamii and Dominick (1997; 1998) indicate “Indeed, understanding of number and operations has proved to be a strength among children who invent their own procedures” (as cited in Hauser, 2005, p. 406) Hauser used a real world “problem” to engage student interest and gave a mini-lesson for the addition and subtraction strategies needed. Each day, students were encouraged to reflect about what they did and learned. The results of this study indicated several interesting points:

1. “...straightforward computation, all students could correctly solve most problems”
2. “The degree of their comprehension was surprising.” (p. 410).
3. “Invented strategies for subtraction were less common, only half of the second graders used them” (p. 410).

4. “Three-quarters of the students were using invented strategies for addition” (p. 411). Some of the strategies that were invented to solve the problems were to use place value, to break numbers into smaller parts, to draw pictures, and to write out the procedures. All of the strategies used dealt primarily with number sense.

Baroody (2006) compared conventional wisdom and the number sense view of teaching computational facts. The comparison examined how teaching mathematics had always been done and how it ought to be done to improve student understanding. Conventional wisdom states “Mastery grows out of memorizing individual facts by rote through repeated practice and reinforcement” (p. 24). The number sense view states “Mastery that underlies computational fluency grows out of discovering the numerous patterns and relationships that interconnect the basic combinations” (p. 24). Baroody further indicated, that teaching basic facts using a “hybrid” technique, i.e., using conventional wisdom with a mixture of number sense strategies, can help students master basic facts, as well as, providing them with the ability to use their background knowledge of numbers effectively (p. 30).

### Teaching Model

Direct instruction. Direct instruction is a model of teaching that has been shown to be effective. Initially defined by Madeline Hunter as a seven step procedure for designing and implanting lessons, the model includes learning objectives, anticipatory set, stating the objectives for the students, input (teaching the lesson directly), check for understanding, guided practice and independent practice as defined in her book *Enhancing Teaching* (1994).

Saphier and Gower (1997) assert that direct instruction is best used in a skill teaching syntax. The teacher should model, check understanding, and monitor student practice. Then students should practice alone. The student is then assessed and the teacher decides whether or

not it is time to move on, "...we know that good direct instruction in groups with high time on task produces mastery of skills" (p. 284).

Klahr and Nigam (2004) indicated that direct instruction increased student learning at a quicker and deeper level than discovery learning (p. 1). They found through using the direct instruction model of teaching that their students' learning increased between 40-60% (p. 2). They also agree that the student's ability to conserve the information long term was by far better when direct instruction was used than when not taught using direct instruction (p. 2).

Peer modeling. The Center for Positive Practices has combined sets of research studies in order to depict the influences in the learning of students when peer modeling has been used within a classroom setting. Schunk (1987) found "multiple peer models increase the likelihood that children will identify with at least one of the models" presented. (p. 6). He also asserted that "the greatest impact of peer modeling appears where observers see themselves as similar in ability and other characteristics" (p. 10).

Creating mental models. In *Classroom Instruction that Works*, Marzano (2001) states, "The most direct way to generate non-linguistic representations is to simply construct a mental picture of knowledge being learned" (p. 81). A mental picture is being able to make abstract content into a physical and mental representation. His research reports the findings of creating mental models and physical representations. He refers to both of these tools as "nonlinguistic representations." Several research studies were examined. The meta-analysis indicated that the effects of using mental models showed an increase in students' scores between 19 and 40 percent (p. 74), the sample size ranging from 3 to 64.

#### The Question

After reviewing the literature regarding number sense, math fluency, and instructional practices, key research questions emerged: (1) What is the relationship between math fluency and number sense?, (2) What is the impact of direct instruction of number visualization, and visual/vocal numeracy strategies on students' number sense and math fluency?, and (3) Is there a significance difference between low and high readers achievement in number sense and computation?

It was hypothesized that number sense and math fluency are positively correlated, indicating a positive relationship between the two skills. It was also hypothesized that direct instruction in vocal numeracy positively impacts students' math fluency and number sense, and that there would be a difference between low and high readers in number sense and computation achievement.

The purpose of this study was to discover if teaching number sense strategies exclusively for eight weeks would increase, not only student's number sense, but their computational fluency, as well. Data was examined to see whether or not number sense and computational fluency go hand in hand, or are independent from each other. Within this study, students with lower reading scores data were examined to see whether or not this study helped or hindered them.

## CHAPTER THREE

### DESIGN OF THE STUDY

In the methodology section, a description of the participants is followed by procedures. The procedures include daily and weekly steps to take to implement the intervention strategies. Within the procedures section, there is also a definition of the assessments and tools used within the intervention.

**Participants.** The class of 21 students in this study was in an urban school district in the Midwest. In this room there were forty-five percent boys and fifty-five percent girls. One boy received special education services; two others were identified as “at-risk” through reading and math assessments. The demographic breakdown of the research group was 23% multi-ethnic, 50% Caucasian, 27% Hispanic, and 9% African American. One student has been retained in this group. The class was a diverse mix of very high achieving students, and students who were well below grade level with few the average achieving range. The one thing they all have in common is that each student’s lowest score on the district common assessments was in the area of number sense.

**Procedures.** Each student completed a researcher-generated number sense and math fluency pretest (see Appendix A). A checklist was created by the researcher to evaluate and record the student’s ability to use number sense to solve problems. This checklist included number sense strategies from the readings and research, from the KSDE standards for second grade math (2003), and the researcher’s experience working with second graders. The initial checklist included the following: pictures, money, tallies, hundreds board, number line, tens frames, combinations, separations, time, tens and ones, shapes, odd/even, greater and less than, and an “other” category (see Appendix B). The checklist was used on the initial pretest, on the

daily work of the students, on the weekly formative assessments, and the post test. As new strategies were identified by the students, they were added in an “other” column by the type of strategy used.

After reviewing the pretest, the teacher implemented a direct instruction approach to teaching the new concepts of solving problems using only number sense strategies. In addition, time was spent reviewing the concepts that they knew and showing them how to integrate them into a problem-solving technique. For example, students had used ten frames all year. But, when asked to show how to make a number, they were not using this strategy. So, within the instruction time, the teacher would review ten frames, and show the students how to make their own. The intervention took place over an eight-week period. During these eight weeks, the researcher conducted whole group lessons three-four days a week, depending on school calendar. Assessment occurred on the last day of the scheduled week; the number and algorithm changed each week (see Appendix A). During individual practice, the researcher monitored the room to see who was struggling and had the student tell her what they were trying to do. The Para-educator worked specifically with the students identified with special needs and/or at risk. The students were involved in both direct and indirect instruction from the teacher, as well as, their peers (see Appendix C).

On day one, the teacher explained the new routine to the students. Fifteen to twenty minutes per day were spent on “Number Talks.” Students were told they would learn new strategies to help them solve problems, and to understand numbers better.

On day one each week, the teacher re-introduced a number sense strategy, the first week being the ten frame (see Appendix B). The students had used ten frames all year long as a manipulative, but were now encouraged to move to the pictorial stage of developmental

mathematics to draw their own tens frame. After the teacher showed the presented concept, during a daily individual practice session, the students were asked to show what they knew about a given number on individual white boards at their desks. The first number was the number ten. Each student would show a different way to represent the number ten. At the same time they were expected to vocalize what skill they were using. While the students were completing the task on the white boards, the teacher assessed their procedures on a checklist, similar to the checklist used to assess the pretest (see Appendix D).

Day two each week consisted of five to eight minutes of direct instruction using the same procedures, this time using the concept of drawing a number line, essential to being competent in higher level mathematics (Berch, 2005). Students were shown how to create a number line that did not start at one, but rather how to start at the desired number and move forward. A verbal review of previous skills was included, but not demonstrated. The students were then asked to create a number line starting from a given number, and to show their methods. They were not asked in their individual daily practice to show the new skill. Student's responses were again recorded on a skill checklist.

Day three of each week began in the same way: a review of the skills taught in days one and two and an introduction of a new skill for day three. In addition, a verbal review of previous strategies cumulative to that time occurred. The skill for day three of the first week was showing combinations of numbers using only basic facts. Students were asked to show the given number for the day in any way they wanted to show it. Again, while the students were working on individual white boards during individual practice, the researcher continued to monitor progress on the checklist. Through the daily use of the same checklist daily, using different colors of pens for each day, the teacher saw the skills that were being used and could check for new

understanding. Each day the students were asked to share their strategies in their table group. At the end of the session, they reported their strategies to the whole class. These strategies were additionally recorded on the checklist.

Day four was an assessment day. The teacher reviewed the skills that were directly taught for the week. After the quick review, students were given an assessment page (see Appendix A) and asked to show as many ways as they knew to make the given number in a 15-minute period and combination that required regrouping. The assessment brought an end to the day and week.

After I reviewed the assessments, direction formed for the focus of the direct instruction for the next week. When formulating an opinion about what topics were deficit, the experience of teaching this grade for four years gave cause to believe that place value, greater than and less than, the remembering of odd and even numbers, the use of money, geometric shapes, tallies, time, and various other strategies would need to be re-introduced. However, skills were not limited to those alone. The needed skills evolved from the weekly formative assessment. For the remainder of the next seven weeks, the process and procedures remained the same for the number talks, but the content changed, based on the ongoing assessment of the students.

## CHAPTER FOUR

### RESULTS

Preliminary Analyses. Descriptive statistics were computed and are reported in Table 1. Additionally, initial data were examined for violations of normality. The Shapiro Wilks test of normality was conducted. Three of the four variables did not violate assumptions of normality (i.e., number sense pretest, number sense post test, and computation pretest,  $p$ s ranging from .11 to .62). Computation post test data did violate assumptions of normality; however, the analyses used are robust to some violations (Tabachnick & Fidell, 2001). Therefore, parametric statistics were used.

Table 1: Descriptive Statistics

Variable	M
Number sense pre	0.28
Number sense post	0.31
Computation pre	0.75
Computation post	0.95

Note. Scores represent percentage correct on measures (i.e., .28 =28% correct).

Primary Analyses. Bivariate correlations were computed to address the first research question that looked at the relation between math fluency and number sense. This statistically illustrates the relations between students' scores on the math measures. Correlations were computed for both the pre and the post test measures to investigate whether the pattern of the relations change after intervention. For pretest data, the correlation coefficient between number sense and computation was not statistically significant ( $r=.06$ ). The correlation coefficient between number sense and computation at post test was not statistically significant either ( $r=-.01$ ). Given the anecdotal evidence of a relation between increased number sense leading to increase

computation, data were submitted to a simultaneous regression model, with computation being the dependent variable and number sense (both pre and post) being the predictor variables. Interestingly, the overall model was significant,  $F(1, 19) = 3.55, p < .05$ , and accounted for 28% of the unique variance in computation. Number sense at pre test accounted for 19.4% of the unique variance in computation and number sense at post test accounted for 28.1% of the unique variance in computation. These results indicate that number sense and computation are related (see Table 2).

Table 2: Predicting Computation from Number Sense

Variable	R	$\Delta R^2$	$\beta$	F ratio
Model	.53	.28		3.52*
Number sense pre		.19	.82	2.12*
Number Sense post		.28	-1.03	-2.63*

Pre and post test scores were statistically compared using the  $t$  test to address the second research question: What is the impact of direction instruction of number visualization and visual/vocal numeracy strategies on students' number sense and math fluency? Data from the entire sample was utilized for these analyses. The analysis determined if the intervention had a statistically significant effect on students post test scores, that is, it determined if the differences between pre and post test scores were statistically significant. Results indicated that there were statistically significant differences between pre and post test scores for the entire sample on both number sense,  $t(20) = 10.38, p < .000$  as well as on the computation test,  $t(20) = 54.82, p < .000$ , indicating that statistically, differences between pre and post test scores were significant.

To address the final research question: Does the impact of the intervention vary as a function of overall reading achievement, the students were divided into two groups; those high in overall reading achievement—as defined by district quarterly assessments, computational

fluency assessments, and overall classroom performance (n=16) and those lower in overall reading achievement (n=5). Differences in performance on post test measures, based on group membership, were statistically examined using analysis of variance (ANOVA) which statistically compares group means. Gain scores for all participants were calculated (post test minus pre test scores). Results indicated that there were no statistically significant differences in gains made in either number sense,  $F(1, 19) = .228, p < .64$  or in computation,  $F(1, 19) = .067, p < .79$  based on overall reading achievement.

## CHAPTER FIVE

### DISCUSSION AND CONCLUSION

Discussion. The first research question addressed the relationship between the variables of number sense and computational fluency. When the variables were examined statistically there was no significance, but there was no instruction given in computation the duration of the study. There was a statistical significance between the pre and post test data in computation, so I looked at the regression model of statistics. When looking at the data from this standpoint, there was overall significance. The post test scores in number sense predicted gain scores in computation. The fact that there was no correlation between the two variables may be due to the small sample size. It is interesting that, although, there was no instruction in computation in the quarter the study was conducted that there was a significant gain in computation. In my opinion, it would have to be due to the students' better understanding of numbers and their relationships.

The second research question was intended to depict the necessity of teaching number sense. As previously stated in the review of literature, some math educators believe there is no sense in teaching number sense: it has been thought to be only intuitive and unable to be taught. According to this study, there is a very high statistical significance in the gain scores of pre and post tests in number sense. These scores, with a very small sample size, show unerringly that number sense can and should be taught to young children.

The final research question dealt with the impact of the study on high and low readers. The students were divided into two groups: high readers and low readers as defined by district quarterly tests, DIBELS testing, and classroom performance. Maybe the most interesting data within the study was that there was no significant difference in the achievement of low readers and high readers within this study. Therefore, one can state that whether a child can read on

grade level or not, this plan of intervention in mathematics is one that will and can work for all levels of readers with positive results.

Within this study the research-based strategies of peer modeling and creating mental models were used daily. The students were asked to create a pictorial representation of a given number or algorithm. The students began to use math vocabulary with each other. These students would say to their peers, "I showed the algorithm using expanding notation," or "I used money to find the answer to the algorithm," or "I used a clock. I knew that fifteen and fifteen was thirty, so I knew if I added fifteen more I would get forty-five." They would then show the drawing that they had created and use their new found understanding of numbers to tell each other, and anybody who happened to walk by, how they solved their problem. Watching these seven and eight year olds feel confident in their abilities to speak about mathematics and solve any problem multiply ways, seems to have been an unexpected benefit of this research.

Conclusion. It is my opinion that these strategies would best be implemented at the beginning of the school year in order that students benefit from their new found knowledge of numbers. The students in this study were asked to think about numbers and their relations, not about how best to memorize strategies to help them solve computation problems. If this intervention would take place at the beginning of each year, students would be able to use their new found knowledge of numbers and their relationships throughout the school year. The classes' average on computation during the eight weeks of this intervention increased nearly thirty percent. After the study was completed the students' scores did not decrease, they actually increased, again. The class average on addition and subtraction facts to twenty increased 8% to 99% in addition and 7% to 97% in subtraction. Seeing this data, made me feel the interventions

were successful! If the students had this knowledge throughout the entire school year, one can only imagine what their computation scores could look like.

Another suggestion would be to make the intensive intervention no longer than four days a week for six weeks. The students seemed to disengage from wanting to create new ways to make numbers and combinations, daily, at the end of that week. Continuing the skills and strategies used within this intervention are crucial for continually developing students' number sense strength. After the six weeks of intensive intervention, the suggestion for maintenance would be to engage the students in a lesson similar to those in the procedures section of this paper at least one time per week.

This investigated students' understanding of numbers, as well as to see, hear, and understand their peers' ability and understanding of numbers. Students made their own best teachers, with the teacher as guide. Their ideas and number concepts far surpassed my expectations. The ideas that were gathered and created to compile a list of number sense strategies were only fourteen strong. Within weeks of starting the intervention, students in the research group had already added new skills and strategies to the list, (i.e. the use of cards, dominos, and dice). Children have a unique way of looking at numbers. We, as adults, often take for granted what numbers are used for in the world around us. Sometimes it just takes a child to remind us of the things we know and take for granted.

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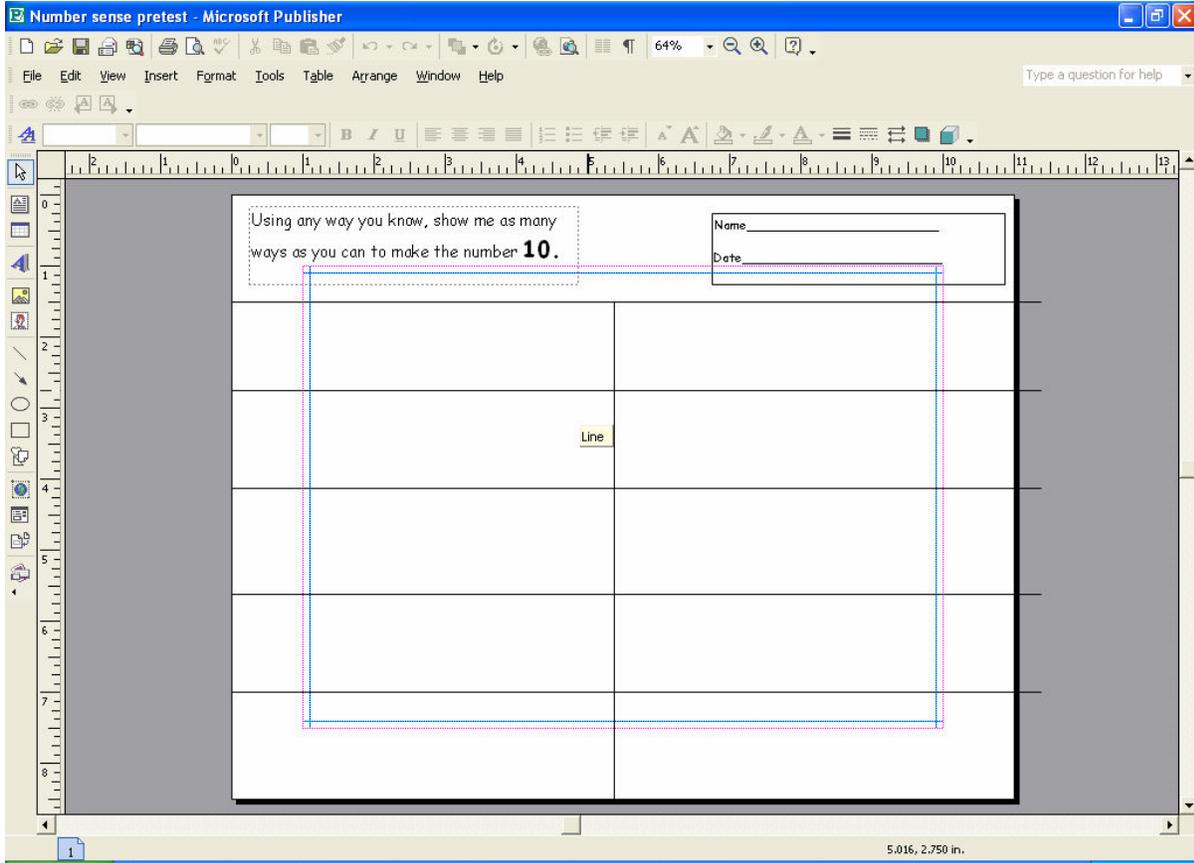
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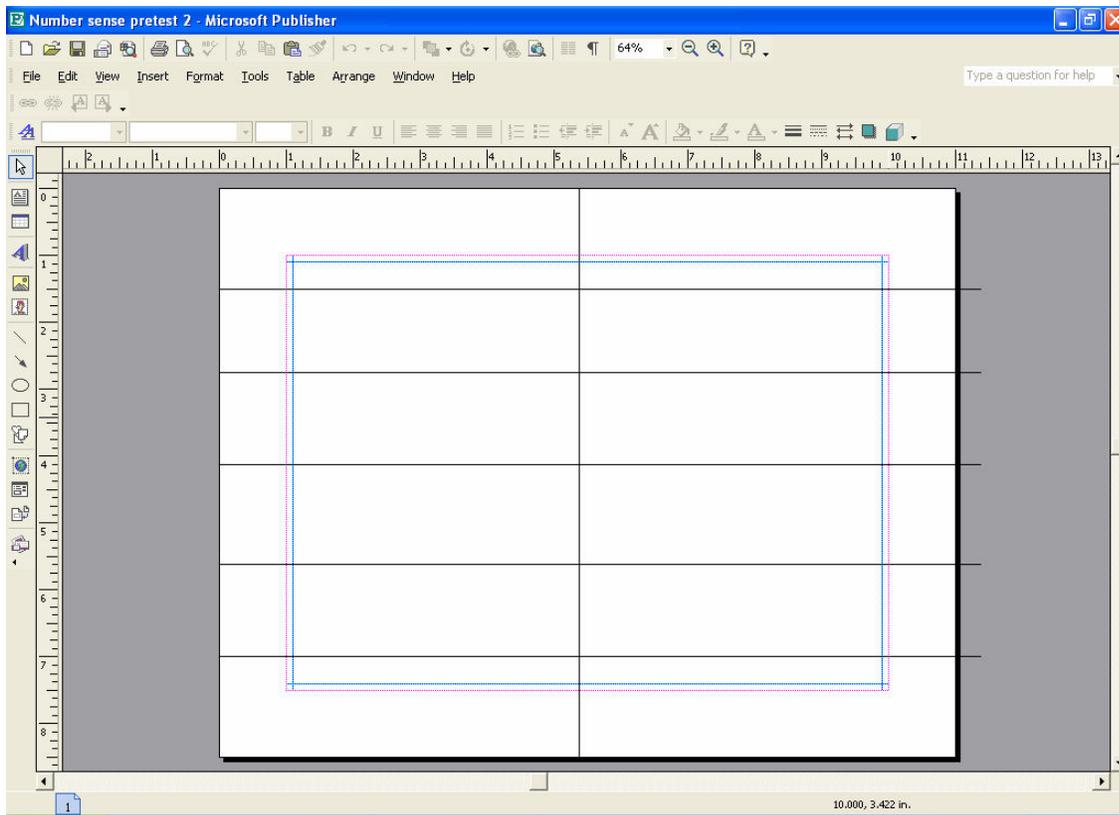
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## Appendices

# Appendix A

## Pre and Post Test Front





## The Back to All Tests

## Appendix B

### Day-by-Day Interventions

#### Number Talk Day One

Time: 20 Minutes

Day one the new routine is put into place.

#### Procedures:

Today I had all of the students come to the carpet and we reviewed ten frames (we learned them at the beginning of the year and used them a lot, but they have not been used in their personal use lately, so the decision to reintroduce them was made). I reminded them of the pretest they took and talked with them about what a ten frame was and what it could show us. I drew one ten frame, with three dots in it. (See ten frames attached). We discussed what it showed us. Some of the answers included:  $10-3=7$ ,  $10-7=3$ ,  $3+7=10$ ,  $7+3=10$ , Odd because each dot did not have a partner. We then discussed how that little picture showed us a lot of different math concepts.

I then drew a ten frame with 5 dots in it. I then pulled 5 sticks individually, (they had one child's name on each one) the kids came up and wrote down what they knew by looking at the ten frame. I repeated this procedure with the number six, seven, and eight until each child in the class had a turn.

#### Assessment Tools:

Today I used a checklist with each child's name on it, if they got the concept with no help, I put a "+" and the date" if they needed help I put "with help and the date."

#### Number Talk Day Two

Time: 30 Minutes

Today we reviewed ten frames, then we discussed how to show what you were thinking with a number line and how to draw that picture. The teacher modeled how to think aloud when showing her work, as well.

#### Procedures:

After reviewing the ten frames, a discussion of how to use and draw a number line preceded. The teacher modeled how to use it to count up and then how to draw it as a tool to solve problems when one was not available.

After the review of ten frames and re-introduction of number lines, a stick was drawn for each child to come up and model their understanding. Today, they were able to show any way they now knew how to show the number twelve. Each child got a turn and had to tell what they chose to show and why they chose the method that they did.

Assessment Tools:

Checklist with different methods and children's names (Today they were told that they next day a different method had to be chosen.)

### Number Talk Day Three

Time: 15-20 Minutes

Today a review of ten frames and number lines will take place. After the review a re-introduction of combinations were made, with a think aloud of why the teacher would chose the certain combination and how she knows that it makes the number fourteen.

Procedures:

After reviewing the ten frames and number lines, the re-introduction of combinations were made. The teacher will model her thinking of combinations and will discuss groupings and chunking of numbers.

After the review of ten frames, re-introduction of number lines and the re-introduction of combination verbalization, a stick was drawn for each child to come up and model their understanding. Today, they will show any way they now knew how to show the number fourteen. Each child will get a turn and have to tell what they chose to show and why they choose the method that they do.

Assessment Tools:

Checklist with different methods and children's names (Today they were told that they next day a different method had to be chosen.)

### Number Talk Day Four

Time: 15-20 Minutes

Today there was a review of the three ways that have been taught this week. We will review heavily ten frames, number lines, and vocalization of combinations. We will spend 15 minutes reviewing and creating different ways to make the number 16.

Procedures:

After reviewing for 15 minutes, the students were given a sheet that asks them to show any way they can to make the number 12. They were given 15 minutes to show any way they know, to create that number.

Assessment Tools:

The tests were looked at for the number of ways they can show, the different ways they made the number and the three ways reviewed this week.

## Weeks 2-8

The same routine and procedures were used, but the strategies changed, due to the needs of the students defined in the weekly formative assessments.

## Appendix C

### Ten Frames

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0	0	0	0	

