FACILITY LAYOUT DESIGN CONSIDERING RISK FOR SINGLE-PERIOD AND MULTI-PERIOD CASES

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I have examined the final copy of this dissertation for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Doctor of Philosophy with a major in Industrial Engineering.

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ABSTRACT

The most desirable characteristic of a facility layout is its ability to maintain its efficiency over time while coping with the uncertainty in product demand. A traditional facility layout design method is governed by the flow intensity between departments, which is the product flow quantity between two departments. Hence, an error in the product demand assessment can render the layout inefficient with respect to material handling costs. Most of this research integrates uncertainty in the form of probability of occurrence of different from-to charts. In an environment where the variability of each product demand is independent, the derivation of "probabilistic from-to chart" based scenarios cannot be used to address uncertainty of individual demands.

This dissertation presents a facility layout problem approach to deal with the uncertainty of each product demand in the design of facility layout. Two procedures are presented: the first procedure is utilized to assess the risk associated with the layout, while the second procedure is used to develop the layout that minimizes risk. Results from case studies have shown that the procedure results in reduction of risk by as much as 68 percent.

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CHAPTER 1

INTRODUCTION

1.1 Problem Background

Under current global manufacturing paradigms, the reduction in material handling cost is essential to maintaining a competitive edge in the market. Material handling cost is estimated to contribute up to 20 to 50 percent of the operating expenses in a manufacturing facility, and it is shown that this operating expense can be reduced by 10 to 30 percent with efficient facility layout design (Tompkins and White, 1996). In general, the facility layout problem (FLP) deals with the allocation of departments, machines, aisle space, etc., to available floor space with an objective to minimizing overall material handling costs. Traditionally, these allocations are determined based on product flow intensities between machines or departments. However, due to the current market environment where product demand and level of product mix are continuously changing, material handling cost fluctuates and is often increased. In addition, introduction of new products and machines or discontinuation of existing products and machines can also alter the material handling requirements. Material handling cost is also commonly categorized as a non-value adding cost. Therefore, firms must focus their efforts to reduce this cost to the minimum level.

In order to maintain the efficiency of the facility layout, the layout design should be evaluated periodically to ensure its optimality under a new production setting. A new production setting may involve numerous changes in production requirements. In general, five changes may cause a reduction in layout efficiency: (1) change in product demand and product mix, (2) change in process sequences, (3) change in scheduling and sequencing strategies, (4) change in resources, and (5) change in safety regulations.

Most of these factors are interrelated, and variations in any of these factors may increase the material handling cost. In most cases, when a change in product demand and product mix occurs, a change in process sequences and scheduling and sequencing strategies may be necessary. In addition, changes in product demand and product mix may consequently lead to changes in resource requirements, which in turn trigger the change in capacity level and number of material handling devices. Frequent additions and adjustment in safety constraints and regulations can also disrupt the existing facility. As a result, an effective method to design a facility that is flexible, responsive, and adaptive is a must in order to maintain the overall efficiency of the facility. Extensive research has been conducted in the area of facility layout design to address these issues, and they are summarized in the following sections.

1.2 Facility Layout Problem (FLP)

Research in the facility layout area is commonly categorized as a static or dynamic FLP problem, depending on the nature of the input requirements and the time periods under consideration. The static facility layout problem (SFLP) approach generally assumes that flow between machines, product demand, and level of product mix are constant and usually performed for a single time period. On the contrary, a dynamic facility layout problem (DFLP) approach is performed a periodic layout evaluation and modification with respect to changes. In addition, in recent years, many researchers have addressed the SFLP and DFLP in a stochastic environment where many of the factors (i.e., product demand, level of product mixed, and process sequence) considered in the FLP are subjected to variability. The stochastic FLP approach aims to incorporate the true nature of many manufacturing environments, where the uncertainties are unavoidable. Much research in the area of SFLP, DFLP, and stochastic FLP can be found in the literature. The following sections provide a brief overview of these approaches.

1.2.1 Static Facility Layout Problem

In the static facility layout problem approach, the flow relationship between departments is used as input, and data is usually in the form of a from-to chart. SFLP assumes that there is no variability in product demand. Comprehensive reviews of the SFLP can be found in Kusiak and Heragu (1987) and Meller and Gau (1996). Several optimization formulations have been developed to solve SFLP: quadratic assignment problem (Koopmans and Beckman, 1957, and Armour and Buffa, 1963), quadratic set-covering problem (Bazaraa, 1975), linear integer programming problem (Lawler, 1963, Love and Wong, 1976), mixed-integer programming problem (Kaufman and Broeckx, 1978, Bazaraa and Sherali, 1980, Burkard and Bonninger, 1983, Frieze and Yadegar, 1983, and Montreuil, 1990), or graph theoretic problem (Foulds and Robinson, 1976).

1.2.2 Dynamic Facility Layout Problem

In the situation where product demands vary from one period to next, the DFLP is facilitated. Fluctuations in product demands could be the result of changes in product mix, introduction of new products, or discontinuation of existing products. Changes in product demands render the current facility layout inefficient and can increase material handling costs (Afentakis et al., 1990). In order to maintain good facility layout, it is necessary to continuously assess the changes in product demands, the flow between departments/machines, and the existing layout to determine the need for redesigning the layout. There have been several attempts in the area of dynamic facility layout problems.

Rosenblatt (1986) proposed a procedure that considered material handling costs as well as rearrangement costs to generate an optimal layout for multiple periods. The approach develops an upper-bound solution to the minimization problem by identifying the best layout for each period and adding material handling costs and transition cost from one period to the next for rearranging the layout. A second upper-bound value is generated by identifying the minimal total costs when the same layout is used for all periods. A population of solutions is generated randomly for each period, and the best solution is identified. Research in the DFLP following this attempt can be categorized into that which that develops either a single robust layout for multiple production periods or flexible or "agile" (Kochhar and Heragu, 1999) layouts, which can be easily customized to adapt to changes in production requirements.

The robust layout approach assumes that the product demands are known in the initial facility layout design stages. This approach generates a single layout that minimizes cost over the entire period under consideration (Kouvelis and Kiran, 1991). This is typically used when the cost of transition for a layout from one period to another is high. It also can be employed to determine the most robust layout for a period under consideration with respect to multiple scenarios (Rosenblatt and Lee, 1987, Rosenblatt and Kropp, 1992).

The periodic redesign approach assumes that layouts can be changed from time to time. When there are wide changes in product mix and demands and when the redesign cost is low, the periodic redesign approach is more effective in reducing material handling costs than the single robust layout design approach. As in the SFLP, heuristic approaches have been used to generate layout design for the DFLP. Urban (1993) facilitated a steepest-descent pair-wise exchange method to obtain the layout design, which minimizes material handling costs in dynamic facility layout problems. This procedure utilized a time-window approach to consider the multiple periods under consideration. Balakrishnan et al., (2000) extended Urban's procedure and used a backward process with a single time window to improve the solution. They also combined Urban's procedure with a dynamic programming approach for each of the periods under consideration to generate multiple layouts. Conway and Venkataramanan, (1994) used a genetic algorithm (GA) to solve the DLP. Balakrishnan and Cheng (2000) proposed a nested-loop GA, which used an inner loop to replace unfit solutions and an outer loop to replace solutions at random. Balkrishnan et al. (2003) developed a hybrid genetic layout algorithm, which used dynamic programming and pairwise exchange heuristics to find solutions along with the GA. Baykasoglu and Gindy (2001) proposed a simulated annealing (SA) approach to solve the DFLP, which was shown to be computationally efficient compared to GA approaches. In most DFLP approaches, researchers assumed discrete data points at the end of the period as the production demand for each period. Krishnan et al. (2006) modeled product demands using a continuous function rather than discrete data points. They developed the concept of the dynamic from-between chart (DFBC), which tracks the changes in demand continuously and identifies the need for redesign based on demand fluctuations. DFBC was found to be effective in reducing material handling costs in the presence of time-varying production demand.

1.2.3 Stochastic Facility Layout Problem

All SFLP and DFLP research assumes that a single deterministic product demand is forecasted for an individual period. However, product demand forecasts are usually inaccurate, and thus manufacturers develop several possible demand scenarios for the period under consideration. One approach to designing layouts in such situations is to plan for the most likely scenario. However, this may result in large losses if a different scenario occurs. Another approach is to develop a compromise layout that can be used for multiple scenarios. A robust approach to a single-period plant layout problem for multiple scenarios in the same period is given by Rosenblatt and Lee (1987). Their model develops layouts to minimize the total expected material handling costs and minimize the maximum losses using a total enumeration strategy. However, as the problem size increases, the total enumeration strategy becomes computationally infeasible, and it becomes worse in terms of computational feasibility when solving multi-period multi-scenario problems. Rosenblatt and Kropp (1992) employed the methodology of "compressing" flow matrices, whereby a set of probable scenario-based deterministic flow matrices are chosen to evaluate a stochastic flexible layout for a single period.

Mehrabi et al. (2000) addressed the need for reconfigurable manufacturing systems to deal with demand scenarios that are highly unpredictable. They showed that it is vital to generate layouts that are quickly adaptable and reliable. Irani and Huang (2000) proposed a module-based technique such that different material flow patterns can be decomposed into a set of modules and these modules can be reconfigured on a timely basis to deal with uncertainties. Bruccoleri et al. (2003) concluded that mass customization is a key factor for any industry's survival. When uncertainty associated with demand is high, reconfigurable layouts are capable of addressing such environments. This paper detailed the complexities of the decision-making process in reconfigurable layouts and proposed an agent-based system to handle reconfigurable systems.

Several researchers have also used fuzzy models to cope with the uncertainty of production or demand in FLP. Dweiri and Meier (1996) employed the Analytical Hierarchy Process (AHP) to generate an activity relationship chart that yields better layouts. Enea et al. (2005) used a fuzzy model to represent the fuzzy flow patterns between departments and a genetic search approach to deal with uncertain production demand scenarios. A robust layout that can perform effectively for a wide variety of demand scenarios by maximizing the system efficiency over a long term was designed. The model adopted arithmetic operators to calculate the material handling cost based on possible flows of finite-capacity constraint between departments. However, the flows are assumed to be deterministic.

Benjaafar and Sheikhzadeh (2000) presented a demand scenario-based procedure, which assigns the layout strategically to achieve reduction in total flow. Multiple finite-demand scenarios along with their probabilities of occurrence are used to generate multiple from-to flow matrices. A heuristic approach iterates the problem separately in two steps: a facility layout problem and a flow allocation problem. Yang and Peters (1998) developed layouts to reduce the material handling cost using the expected flow density (EFD) model for multiple production scenarios. The EFD approach may not be the best option in such situations where widely varying production scenarios exist, because the layout generated is biased to the scenario with the highest probability of occurrence rather than compromising between different scenarios. Krishnan et al. (2008) developed models for the multi-scenario-single-period problem and the multi-period-multi-scenario problem. They used a GA to minimize the maximum losses for both problems.

1.3 Research Purpose

In a manufacturing environment, where product demands are highly volatile, assessing future product demand is very critical. Forecasting is one of the most widely used techniques for predicting the future product demand. However, such forecasts may not be accurate. Indeed, in many cases, the uncertainty of projected product demands should be considered during the design of the facility layout. Any facility layout design has an element of risk, and risk factors should be quantified. Usually, the larger the variances of the product demands, the higher the risk associated with the layout. To reduce the risk, the layout design process should incorporate advanced techniques that reduce the impact of errors.

1.4 Research Objective

The objectives of this dissertation were to investigate the models required for quantifying risk in facility layout design and to develop the facility layout problem approach for a single-period and multi-period case, which minimize risk of the facility layout design. Numerous tasks are required to complete, verify, and validate the developed framework:

- Risk assessment method development
    - Mathematical formulation method
    - Simulation method
- Risk based facility layout design development for single-period case
    - Mathematical formulation
    - Solution algorithm using GA
    - Case studies (9, 16, and 25 departments)
- Risk based facility layout design development for multi-period case
    - Mathematical formulation
    - Solution algorithm using GA
    - Case study (9, 16, and 25 departments)

1.5 Chapter Outline

This dissertation is organized into seven chapters. Chapter 2 presents a detailed survey of the existing FLP approaches in the literature. This survey mainly covers the most relevant literature on four topics: (1) SFLP approach, (2) DFLP approach, (3) Stochastic FLP, and (4) GA application in the FLP. Chapter 3 presents the development of a facility layout risk assessment method in detail. This includes product demand probability of occurrence assessment and methods to quantify its relative change, and the facility layout risk calculation. Chapter 4 discusses procedure to solve the single-period FLP, which minimizes risk of the layout and development of the GA algorithm. In addition, several numerical examples to demonstrate the usage of the framework for a single period are also presented in this chapter. The risk-based facility layout approach for solving the multi-period FLP is presented in Chapter 5. The mathematical formulation along with the solution procedure and several case studies are demonstrated in this chapter. Chapter 6 provides additional case studies to demonstrate use of the risk-based facility layout approach for different types of projected demand scenarios. Finally, Chapter 7 provides the conclusions of the developed framework. It also presents the importance or contribution of this research to the FLP research area, and provides possible extensions of this research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The purpose of this research was to address the facility layout problem (FLP) in a stochastic environment, both statically and dynamically. To offer a comprehensive understanding of the FLP, this chapter provides a thorough review of the existing literature in the FLP research area. It is categorized into four major sections, which address different aspects of the FLP: the static facility layout problem (SFLP), the dynamic facility layout problem (DFLP), the stochastic facility layout problem, and the heuristics for solving the FLP.

First, an overview of the SFLP along with existing solution approaches is presented. Next, an in-depth review of the DFLP, factors that are commonly considered in the DFLP and existing approaches for solving the DFLP are discussed. Furthermore, the concept of stochastic facility layout problem is introduced, and a detailed survey on the existing solution procedures is offered. Lastly, a review on applications of the heuristic approach, namely genetic algorithms (GA) to solve the SFLP and DFLP are discussed.

2.2 Static Facility Layout Problem

In the past, the SFLP approach has been widely used as a tool to generate facility layouts. To determine the effectiveness of a layout generated by such an approach, material handling cost is one of the most commonly used performance measures. Material handling cost comprises the cost of handling, distance between two locations, and number of trips required to move products from one location to another. The objective function in SFLP approach aims to minimize the total material handling cost while maximizing the closeness ratio. This type of approach assumes that the product demand and cost of handling are certain and already known for a single period.

Comprehensive reviews of the SFLP were conducted by Kusiak and Heragu (1987) and Meller and Gau (1996). The following three sections provide a brief overview of SFLP modeling techniques, solution approaches, and additional considerations and applications of the SFLP.

2.2.1 SFLP Formulation

Conventionally, the SFLP is formulated as one of the following: quadratic assignment problem (Koopmans and Beckman, 1957; and Armour and Buffa, 1963), quadratic set-covering problem (Bazaraa, 1975), linear integer programming problem (Lawler, 1963; Love and Wong, 1976), mixed integer programming problem (Kaufman and Broeckx, 1978; Burkard and Bonninger, 1983; Frieze and Yadegar, 1983; and Montreuil, 1990) or graph theoretic problem (Foulds and Robinson, 1976).

In quadratic assignment problem (QAP), the objective function is derived in the form of a second-degree function of the variable, while constraints are formulated in the form of a linear function of the variable. The principle of the model is to assign every machine/department to one position and only one machine/department to each position. The cost of allocating a machine/department at a specific position depends on the position of the interacting machine/department (Meller and Gau, 1996). However, the QAP is frequently used to solve the FLP, but its application is limited to an equal-area FLP. This is due to the input requirement of the QAP, which mandates the location of the machine or department to be known and specified ahead of time. A modified version of the QAP was later proposed to enable the traditional QAP to solve an unequal-area FLP (Kusiak and Heragu, 1987; and Liao, 1993). In the modified QAP model, the departments are broken into smaller grids of equal size, and a large artificial flow between those grids of the same department is assigned to guarantee that they are not split to solve an unequal-area problem. However, the problem size that this modified QAP can handle is very limited due to an increase in the numbers of departments that need to be considered (Meller and Gau, 1996).

The quadratic set-covering problem (QSP) is an alternative formulation method to solve the FLP. The formulation of QSP is similar to those of QAP with an additional concept of dividing the total area taken by the facilities into smaller blocks. Due to the formulation similarity and the input requirement, likewise, QSP can only be used to solve an equal area FLP.

Linear integer programming (LIP) is another modeling technique used to represent the FLP. In 1963, Lawler (1963) proposed LIP for solving an FLP and also proved that the proposed LIP is an equivalent of the QAP. Due the number of variables and constraints required to model the FLP using LIP (Lawler, 1963), an alternate simpler LIP, which significantly reduces the numbers of required variables and constraints, was proposed (Love and Wong, 1976).

FLP can also be solved using graph theoretic (GT) techniques. The objective of a GT is to maximize the weight on the adjacencies (arcs) between department pairs (nodes) (Meller and Gau, 1996). To solve the GT, the following three steps are performed: (1) develop the adjacent graph from department relationships, (2) construct the dual graph of the adjacent graph, and (3) convert the dual graph into an adjacent block (Meller and Gau, 1996). However, as in the QAP, the GT can only solve a small-sized problem. A more detailed the GT approach to solve a FLP can be found in the study of Foulds and Robinson, (1976).

Mixed integer programming (MIP) is also one of the most extensively used modeling techniques to formulate a SFLP. One of these models was proposed by Montreuil in 1990. Unlike the traditional QAP, which is a discrete framework, he uses the distance-based objective for a continuous layout representation (Montreuil, 1990). The objective function was based on the flow time rectilinear distance between department centroids. The following constraints in the formulation: each department must be within the facility, maximum and minimum length of the department, and the departments should not overlap. Later on, a special case of Montreuil's MIP model, which allows the user to specify width, length and orientation of the department priory, was developed to provide a more powerful and detail formulation (Heragu and Kusiak, 1991). Although the MIP formulation was proven to be a powerful tool in solving an SFLP, due to its associated computational intensity, it only solved a small-sized (less than six departments) problem optimally (Meller and Gau, 1996).

2.2.2 SFLP Approach

As shown in the previous section, it is evident that the conventional optimization approaches are not capable of solving a large-sized FLP. In addition, these optimization approaches have also been proven to be NP-Hard, which implies that computational complexities increase as the number of departments increase (Kusiak and Heragu, 1987). Alternatively, heuristics approaches, such as simulated annealing (SA) (Meller and Bozer, (1996); Montreuil et al., (1993)), genetic algorithms (GA) (Tam and Chan, 1998), and tabu search (TS) (Chiang and Kouvelis, 1996), have been developed to overcome this computational difficulty. Other heuristics, such as pair-wise interchange methods, computerized relative allocation of facilities technique (CRAFT), layout optimization using guillotine induced cuts (LOGIC), SHAPE, nonlinear optimization layout technique (NLT), qualitative layout analysis using automated recognition of patterns (QLAARP), MULTI-floor plant layout evaluation (MULTIPLE), and FLEXible bay structure (FLEX-BAY) (Meller and Gau, 1996), have also been used to generate an SFLP solution.

2.2.3 Additional Considerations and Applications in SFLP

One of the most significant issues of the FLP is how to consider or compare the qualitative and quantitative attributes of the FLP in the problem formulation simultaneously. An earlier attempt to address such a problem was conducted by Rosenblatt (1979). He presented a heuristic for combined quantitative and qualitative attributes, which addresses the multiple purposes of the FLP. The heuristic aims to minimize the quantitative attributes (material handling cost) while maximizing the qualitative attributes (closeness ratio). Likewise, an analytical hierarchy process (AHP) was proposed to quantify the qualitative attributes of the layout (Shang 1993). The relative weight that was generated from the AHP could be used for combining qualitative and quantitative attributes of facility layout in a multiple-objective QAP problem (Shang 1993). Similarly, the dominant index (DI) concept was also introduced to resolve the issues of unit and comparability of qualitative and quantities criterion considered in the FLP (Chen and Sha, 1999). Many other methods such as the expert system (Kumara et al., 1988; Malakooti and Tsurushima, 1989; Heragu and Kusiak, 1990) and the fuzzy set (Evan et al., 1987) also can be used to formulate the multi-criteria facility layout problem.

Another important constraint in the facility layout design is space allocation (Tam, 1992). One of the objectives is minimizing the interdepartmental flow between cells considering the geometric constraints (Tam, 1992). To formulate the space-allocation problem, both space allocation and geometric properties of the cell is defined. A slicing tree structure was used to define the space allocation, whereas the geometric properties of the cell were defined by the associated aspect ratio (Tam, 1992).

In addition, another aspect of the space-allocation problem was pointed out by Bozer et al. (1994), Randhawa and West (1995), and Kochhar and Heragu (1998). For certain industries like semiconductor manufacturing, steel production, artillery combat vehicle fabrication, etc., the facility is required to be located vertically due to the horizontal space limitation (Randhawa and West, 1995). Therefore, material handling has become a major concern, since a traditional facility layout design algorithm can only address a single-floor problem. To solve the multiple floors FLP, Bozer et al., (1994) introduced the space-filing curves concept to address both single and multiple floor problems. Similar to the multi-floor FLP, the multi-bay FLP was formulated as an MIP, which can be solved using the dynamic programming to determine optimal layouts (Meller, 1997). In addition to the two approaches mentioned, a MULTI-HOPE algorithm, which is an extension of the HOPE algorithm, can also be used to solve such a problem (Kochhar and Heragu, 1998).

As shown, the formulations and solution approaches to solve the SFLP have been well established. However, with the recent market trend where product demand and level of product mix changes continuously, the existing SFLP approaches may fall short of providing an effective solution in such an environment. Numerous researchers ventured into a new area of FLP trying to address and resolve the shortcoming of the SFLP. An overview of such research is provided in the following section.

2.3 Dynamic Facility Layout Problem

In an environment where changes in product mix, introduction of new products, and discontinuation of existing products are presented, the facility layout is obligated to be redesigned in order to maintain the efficiency of the facility. According to Afentakis et al. (1990), changes in product demand can render the current facility layout inefficient and increase material handling costs. To maintain or redesign an efficient facility layout, it is necessary to continuously evaluate the changes in product demand, the flow between departments/machines, and the existing layout. Although many models have been developed to address the FLP with multiple relative constraints, current volatile changes in the market will demand that the layout be robust and agile to adapt to any dynamic change.

2.3.1 DFLP Formulation

Similar to the SFLP, the DFLP was commonly formulated as an extension of the QAP, considering time and cost associated with alteration of the layout design. Two types of cost are introduced when the layout is altered: (1) loss in production time and (2) cost of moving departments from one location to another (Kochhar and Heragu, 1999). Most DFLP approaches use these costs to evaluate the worthiness of redesigning the layout. Hence, the objective is to find a balance between these two costs and the benefit of redesigning the layout (Kochhar and Heragu, 1999). A comprehensive review of the DFLP related literatures is given by Koren et al. (1999). As in the traditional QAP, the computational intensity required to solve a large-sized problem is a challenge. To effectively solve the DFLP, numerous heuristics were developed, and the overview of these heuristics is provided in the following section.

2.3.2 DFLP Approach

An earlier attempt to address the FLP dynamically was conducted by Rosenblatt (1986), who showed that the problem can be formulated using dynamic programming (DP). However, by considering the total number of layout combinations at each period, the problem became very large. To overcome this problem, a simplifying procedure using the concept of upper-bound and lower-bound heuristics was proposed. However, one of the major drawbacks of this approach was the inhabited nature of DP, which can not solve the problem with more than twelve departments (curse of dimensionality) (Urban, 1993). Hence, a more effective algorithm with moderate computation intensity was required. Many other researchers based their work on improving the algorithm to solve the DFLP effectively and efficiently. In 1993, Urban proposed a heuristic based on the steepest-descent pair-wise interchange procedure to resolve this issue. The procedure generated the layout from the varying time window with an additional consideration of rearrangement cost.

As an attempt to develop an effective algorithm to solve the DFLP, the modification of five existing QAP algorithms was proposed (Lacksonen and Enscore, 1993): CRAFT (Armour and Buffa, 1963), cutting planes (Burkard and Bonniger, 1983), branch-and-bound (Pardalos and Crouse, 1989), dynamic programming (Rosenblatt, 1986), and cut trees (Gomory and Hu, 1961). Among these, the cutting plane algorithm was found to be the efficient approach to solve the DFLP. In addition, an improved pair-wise exchange heuristic of Urban's (1993) heuristic was proposed to solve the DFLP (Balakrishnan et al., 2000). This heuristic was performed in two steps: (1) perform the backward pass from the solution obtained by Urban's (1992) heuristic and (2) combining the Urban's (1992) heuristic and DP, and solve for solutions. Since the Urban's (1993) heuristic is a forward pass in nature, the quality of the solution of the later state depended on the quality of the solution in the preceding state, which can be perceived as a disadvantage. Therefore, incorporating the backward pass helped to improve the quality of the generated solution (Balakrishnan et al., 2000).

To improve the solution quality while reducing the computational time, numerous researchers have used the GA to solve a DFLP. One of these earlier attempts was conducted by Conway and Venkataramanan (1994). Similarly, a new algorithm called DHOPE was developed (Kochhar and Heragu, 1999), which was the extension of the previously developed algorithm for static layouts known as MULTI-HOPE. DHOPE was designed to generate layouts for two periods. The generated layouts were evaluated based on the reduction in rearrangement costs and material handling costs. The roulette wheel selection method was proposed for the selection of the most likelihood string (parent). Experimental results indicated that by using the developed methodology, DHOPE could provide a significant reduction in cost when compared to the usual methods. Subsequently, research aimed to enhance GA procedure proposed by Conway and Venkataramanan (1994) was conducted, and the resulting improved GA procedure was developed (Balakrishnan and Cheng, 2000). This GA procedure used a new crossover operator to increase the search space with the addition of a mutation process and a generational replacement approach to diversify the solutions. This GA also used nested loop, which is comprised of an inner loop and an outer loop. The inner loop substituted the most unfit individual from each generation, while the outer loop replaced the large number of unlucky individual in a generation (Balakrishnan and Cheng, 2000). Likewise, the concept of a hybrid GA was proposed in 2003 to further improve the efficiency of the existing GA algorithms (Balkrishnan et al., 2003). This hybrid GA algorithm adopted a new crossover operator and a mutation process to improve the quality of solutions. Dynamic programming was used as the crossover operator, while the CRAFT was employed for the mutation process. In addition, Urban's (1993) pair-wise interchange method was used to generate the initial population. This proposed GA was tested against the GA algorithm proposed by Balakrishnan and Cheng (2000) and the SA procedure proposed by Baykasoglu and Gindy (2001). The results showed that the proposed hybrid GA provided better solutions when compared with other GAs and SAs in most cases.

Simulated annealing (SA) is one of the heuristic approaches, which is well known to be an efficient and effective tool to solve a combinatorial optimization problem (Baykasoglu and Gindy, 2001). In 2001, an attempt to apply SA to solve the DFLP was conducted by Baykasoglu and Gindy (2001). The proposed SA algorithm was tested and compared with the DP proposed by Rosenblatt (1986) and the GA algorithm proposed by Conway and Venkataramanan (1994). The results indicated that the proposed SA performed better than the other evaluated algorithms (Baykasoglu and Gindy, 2001). Another effective heuristic procedure that can be used to solve the DFLP is the tabu search (TS). In the study of Kaku and Mazzola (1997), a two-staged TS was employed. The first stage executed the diversified search of the solution space using a constructive approach to obtain the initial solution, while the second stage performed the intensification around the number of promising solutions identified in the first stage. This method proved to be very effective in obtaining good solutions when compared with other evaluated heuristic approaches namely the algorithm by Lacksonen and Enscore (1993) and Urban (1993).

Through literature surveys, DFLP approaches proved to be effective tools to solve an FLP, where the product demand and product mix fluctuates continuously. Numerous approaches and formulations were developed to provide an FLP solution in such environment. Nevertheless, in the environment where the product demand and product mix are varied and unpredictable rather than changing but certain, a new FLP approach is needed. The following section discusses this type of FLP approach.

2.4 Stochastic Facility Layout Problem

As shown in the two previous sections, both the SFLP and DFLP approaches proved to be efficient in solving an FLP where the production requirements are priory known and certain. However, in the manufacturing environment, where the production volume and the level of product mix are highly unpredictable, the need for a different FLP approach has emerged. Traditionally, most FLP approaches assume that the production volume and level of product mix are deterministic or priory known, based on either a forecasting technique or an historical trend.

However, the inherent uncertainties or variations of the resulting forecast are inevitable. Therefore, the layout design based on such a forecast is also subjected to uncertainty, which consequently compromises the layout effectiveness. In the literature, there are two distinct type of approaches handle the uncertain production environment in the FLP: (1) the probabilistic scenario-based approach and (2) the fuzzy approach. The following sections provide an overview of research in the stochastic SFLP and DFLP, and the FLP fuzzy approaches.

2.4.1 Stochastic FLP Method (Static)

An earlier effort to address the uncertainty in production volume and product mix was put forward by Rosenblatt and Lee (1987). As a result, a robust approach to a single-period plant layout problem under a stochastic environment was developed. In the situation where there are many possible states of demand to be considered, assigning equal probabilities to each state may not be the most appropriate option (Rosenblatt and Lee, 1987). Based on this argument, the proposed approach categorized the probability of occurrence of production scenarios into three levels (high, medium, and low), which aimed to improve the efficiency of generated layouts.

Subsequently, a more advance probabilistic method was proposed to solve an FLP in a single-period stochastic layout problem (Rosenblatt and Kropp, 1992). This method simultaneously considered all anticipated production scenarios by assigning each production scenario with its associate probability of occurrence. The problem was formulated as a QAP, which was used to generate the optimal layout for each production scenario. In addition, the following assumptions were made: (1) a finite number of possible product mixes scenarios can occur, (2) each scenario can be represented by a unique from-to matrix, (3) each scenario has a predetermined probability of occurrence, and (4) the probabilities are mutually exclusive and jointly exhaustive (Rosenblatt and Kropp, 1992). In order to simplify the problem to consider all possible production scenarios, a weight-average flow matrix was used. The purpose of this matrix was to compress numerous flow matrices into a single flow matrix, which can be calculated as

$$\bar{F} = \sum_{s=1}^{S} p_s F_s \tag{2.1}$$

where p_s represents the probability of occurrence for scenario s and F_s is the associated flow matrix for scenario s. This weight-average flow matrix is used as input to generate the best layout, which minimizes the total expected material handling cost over all scenarios.

2.4.2 Stochastic FLP Method (Dynamic)

To improve the range of applications of the existing stochastic FLP approaches, several studies were conducted in the stochastic DFLP area. One study developed an approach similar to that of Rosenblatt and Kropp (1992) with an extension of the multi-period case for an unequal-area FLP (Yang and Peters, 1998). The approach aimed to minimize the expected material handling cost and machine rearrangement cost. Two types of layout were taken into an account: robust and dynamic. For robust layouts, the entire planning horizon was considered as one planning window. On the other hand, dynamic layouts constituted several sets of planning windows. The machine rearrangement cost was used to assist in selecting the type of layout. In cases where the machine rearrangement costs were high, the robust layout was preferred. On the other hand, if the machine rearrangement costs were low, the adaptable layout was favored. The expected material handling cost was obtained based on the expected flow density (EFD) between two departments i and j, and was calculated as

$$F_{k,T}(i,j) = \sum_{t=k}^{k+T} \sum_{\pi(t) \in \prod(t)} P\left[\pi(t)\right] f\left[\pi(t)\right] \tag{2.2}$$

where $\pi(t)$ represents the possible production scenarios in period t, $f\left[\pi(t)\right]$ is the associated flow matrices of $\pi(t)$, and $P\left[\pi(t)\right]$ denotes the probability of occurrence of $\pi(t)$.

Two years later, a similar study was conducted to address the stochastic DFLP. Similar to the approach by Rosenblatt and Kropp (1992) and Yang and Peters (1998), this approach aimed to solve an FLP where multiple finite demand scenarios are considered with the additional consideration of product demand distributions (Benjaafar and Sheikhzadeh, 2000). This approach utilized the compress flow matrix concept to simplify the problem. This matrix was also used as the input to develop the best layout.

Similarly, another method to solve the stochastic DFLP or stochastic dynamic plant layout problem (SDPLP) was proposed (Palekar et al., 1992). The objective of this method was to minimize the expected material handling cost and relocation cost for each period. In their study, the Markov chain theory was used to estimate the likelihood (probability of occurrence) of each of the demand scenarios to occur and assumed that the probability of occurrence of a product demand in a period is most likely to be conditioned to the situation in the previous period. Four assumptions were made regarding the availability of the product information: (1) each flow matrix represents a unique state in the Markov chain, and there is a finite set of flow matrices, (2) the transition probabilities from one state to another are known, (3) the distance matrix is known, and (4) The material handling cost and relocation cost are known. The probability of each demand scenario to occurring in each period is calculated as

$$p_s^t = \sum_{h=1}^{S} p_h^{t-1} O_{hs} \ \ and \ \ p_s^0 = O_{\phi s} \qquad (2.3)$$

where S is the number of possible flow matrix or state, and O_{hs} represents the probability of one step transition from state s to h. These probabilities are then used to calculate the expected material handling cost.

Further, Montreuil and Laforge (1992) attempted to address the FLP problem under uncertainty using the scenario tree of probable futures. This method can help to provide a set of layouts, which offers a robust master plan to accommodate the potential futures. Figure 2.1 depicts of the typical scenario tree of probable futures.

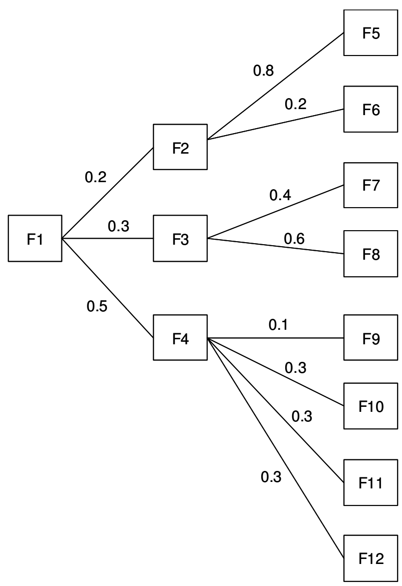


Figure 2.1. Typical scenario tree of probable futures (Montreuil and Laforge, 1992).

2.4.3 Fuzzy Approach in the FLP

In recent years, the fuzzy approach has been used to incorporate the uncertain nature of product demand into the FLP in the research of Evans et al. (1987), Grobenlyn (1987), Dweiri and Meier (1996), Aiello and Enea (2001), Deb and Bhattacharyya (2003), and Enea et al. (2005). To solve the block design layout problem, the fuzzy linguistic heuristic based on fuzzy relation and fuzzy ranking was applied to determine the location of each department in a manually generated layout (Evans et al., 1987). In addition, the fuzzy implication and fuzzy consistency method was employed to solve the stochastic FLP by defining a grade of satisfaction for the optimality criterion to identify the most suitable location for each department (Grobenlyn, 1998). In addition, the AHP and fuzzy decision-making procedure were also used collectively to generate a facility activity chart (Dweiri and Meier, 1996).

According to Aiello and Enea (2001), the previous stochastic FLP algorithm such as Rosenblatt and Lee (1987) robust approach did not consider the limitation in the capacity of the facility. Consequently, in some instances, the obtained solutions may be infeasible. To overcome this problem, a fuzzy approach that uses a constrained arithmetic operator to mimic the uncertainty nature of the product demand was developed (Aiello and Enea, 2001). This approach also claimed to provide a reduction in computational complexity when compare with the other stochastic models.

To handle the uncertain and vague data, another methodology based on fuzzy decision-making system was developed to solve the FLP (Deb and Bhattacharyya, 2003). A multi-factors fuzzy inference was used for selecting the location of the department in an open continual plane considering both quantities and qualitative factors. To generate the robust layout that is effective over a wide range of production scenarios, Enea et al. (2005) proposed the use of fuzzy number and constrained arithmetic operators to formulate the FLP with the objective of minimizing the fuzzy material handling cost.

In the previous sections, all three types of FLP approaches and formulations were discussed. As shown, most of these approaches were formulated as a QAP, which was categorized as an NP-hard problem. To reduce the computational intensity required to solve a

QAP, numerous heuristics were used, such as GA, TS, and SA. Among these, GA is one of the most widely used heuristics to solve an FLP. The next section provides a survey of GA application in the FLP research area.

2.5 Genetic Algorithm

In the early 1970s, Holland (1975) developed a heuristic inspired by Darwin's theory of evolution by natural selection known as the genetic algorithm. The main advantage of using a GA is due to the randomness induced in the algorithm, which consequently broadens the search space. A larger search space helps to increase the possibility that the algorithm will obtain a better solution. Based on the general GA procedure, the candidate solutions are represented in form of a chromosome, which is randomly generated. The GA evaluates all solution candidates in a feasible region, and selects the best solution. The best solutions are identified using a fitness function, which is developed based on the objective of the problem. Once the best solutions (parents) are selected, the GA repopulates another set of candidate solutions (offspring) by the reproduction process called crossover and mutation, which are applied to the parents. The GA repeats these processes until a specific number of iterations is met, and the best solution is selected among the last set of solution candidates in the last iteration based on the fitness function.

2.5.1 Application of GA in FLP

Due to a reduction in computational time offered by the GA and the combinatorial nature of the FLP problem, numerous researchers have employed the GA as an alternative approach to solve an SFLP. For instance, the GA was used to obtain the optimal/near optimal solution for an unequal area FLP (Tate and Smith, 1995). It has also been used to solve a cell layout problem to obtain a path skeleton design (Banerjee et al., 1997). In the following year, a GA called HOPE was developed and applied to solve both an equal and unequal-area FLP (Kochhar et al., 1998). Further, Rajasekharan et al. (1998) proposed a GA-based decomposition strategy to solve the flexible manufacturing system (FMS). This proposed methodology comprised a two-step hierarchical decomposition approach, which demonstrated an ability to generate a good result while using a minimum amount of time. In addition, Balakrishnan et al. (2003) extended the existing GA algorithm to a hybrid GA to address the DFLP.

To further improve the efficiency of the GA, Tam and Chan (1998) employed a parallel GA for solving the FLP. The layout was represented using a slicing tree approach and the chromosomes were generated such that the internal and external nodes are considered as substrings, which help to broaden the search space for the GA. Mak et al. (1998) proposed a mathematical model and new crossover operator for the GA to solve the FLP. Experimental results indicated that the effectiveness of the proposed approach was competent. In 1998, Islier proposed the use of modeling chromosomes in three parts to solve the FLP. The first part was modeled as the sequence of departments, the second part represented the area for these departments, and the third part was used for representing the concatenated cells in a string. Three factors are considered in the objective function, which are load, shape and deviation. An improved GA to facilitate the design of efficient layouts for the DFLP was proposed by Balakrishnan and Cheng (2000). A new crossover operator along with the use of a mutation and generation replacement strategy was proposed to increase population diversity and efficiency in computation time. Based on the work of Tam and Chan (1998), Hakim (2000) further modified their slicing tree representation in the GA algorithms to solve the FLP. He also pointed out the problems of using the existing crossover and mutation operators and presented a new transplanting method, which can generate efficient off-springs. A new slicing tree structure in the

GA was proposed by Shayan and Chittilappilly (2004). This method removes the repairing procedure constraint and induces the penalty function to generate feasible layout.

2.6 Research Motivation

Although, extensive research on methodologies to solve the FLP considering many different type of conditions and constraints has been conducted, research on uncertain production environment is limited. The current research on the development of facility layouts involving uncertain production environment used multiple from-to scenarios. Assigning a probability value to the from-to chart or a scenario assumes that every flow in the from-to chart is dependent on other flows within the chart. In other words, all product demands are dependent. For instance, Yang and Peters (1998) considered multiple scenarios for a single period by assigning the probability of occurrence for each scenario. Similarly, Benjaafar and Sheikzadeh (2000) identified the scenario in the same manner as Yang and Peters (1998) with additional consideration of the probability distribution. In addition, Krishnan et al. (2008) developed models for the multi-scenario-single-period problem and the multi-period-multi-scenario problem, aiming to minimize the maximum losses for both problems. However, in the environment where each product demand by itself demonstrates high variability and is independent from one another, the scenario-based method may not be suitable for addressing overall uncertainty of demands.

In the following chapters, the stochastic FLP framework called the risk-based facility layout approach is presented. This approach aims to provide a more elaborate variability or uncertainty assessment of product demand. It also includes the FLP formulation and solution approaches to the stochastic FLP in a single-period case and a multi-period case.

CHAPTER 3

FACILITY LAYOUT RISK ASSESSMENT METHOD

3.1 Introduction

In most FLP approaches, layout design or allocation of departments is directly driven by flow intensity, which is the total flow between a pair of departments for all products. A change in the demand of any products will modify the strength of the relationship between the relevant departments. When the change is large enough to cause the flow intensity between one pair of departments to exceed or fall below that of another pair of departments, the layout could be compromised. When there is uncertainty associated with product demand, it is necessary to take into consideration the associated risk. Specifically, risk is defined as "a measure of the potential loss occurring due to natural or human activities" (Modarres, 2006). In the context of facility layout design, risk can be viewed as potential loss caused by perturbations in projected product demand.

Two fundamental issues must be addressed when dealing with uncertain product demands in FLPs. The first is determining the best layout for the projected demand. The second is the assessment of risk associated with the layout with respect to variability in product demands. Trade-off between the optimal layout for the projected demand and the potential loss associated with possible variations in product demand must be analyzed. One of the most desirable characteristics of a layout design is the ability of maintaining its flexibility and efficiency against uncertainty in product demands.

3.2 Notations

The following notations are used in the facility layout risk assessment method:

N	Total number of departments in layout
f_{ij}	Flow intensity between departments i and j
f_{kl}	Flow intensity between departments k and l
x_h	Demand per period for product h, where $h = 1 \ldots H$
C	Material handling cost of moving one product per unit distance
D_{ij}	Distance between departments i and j of the current layout
$p_{f_{ij}}^{f_{kl}}$	Probability that flow intensity between departments i and j exceeds that between departments k and l
$g(x_h)$	Probability density function for the demand of product h
$\varphi(f_{ij})$	Probability density function for the flow intensity between departments i and j
$E[f_{ij}^{kl}]$	Expected relative change in f_{ij} with respect to f_{kl} when $f_{ij} > f_{kl}$
F_{ij}	Projected flow intensity between departments i and j
F_{kl}	Projected flow intensity between departments k and l
$G(f_{ij})$	Cumulative density function for flow intensity f_{ij}
$R_{f_{ij}}^{f_{kl}}$	Risk that flow intensity between departments i and j exceeds that between departments k and l

3.3 Facility Layout Risk Assessment Procedure

In this research, the influence of demand uncertainty on facility layout is assessed using the risk assessment method. Mathematically, risk can be expressed as (Modarres, 2006)

$$RISK = \sum_i u_i c_i . \quad (3.1)$$

where u_i is the probability of event i and c_i denotes the associated consequence. For the FLP, the probability of occurrence is defined as the probability that flow intensity between departments i and j exceeds that between k and l. The consequence can be viewed as the resulting increase in the material handling cost. The risk assessment of the facility layout is conducted using the following four steps:

- Identify all significant flow intensities, which are at risk.
- Assess the relative increase of the significant flow intensity.
- Assess the layout risk.
- Rank the risk

3.3.1 Identification of Significant Flows

Traditionally, product demands are determined from historical data or forecasting techniques. In this research, these demands are referred to as projected demand. Due to fluctuation in the current market demand, when the projected demand values are obtained through forecasting techniques, these demand values may or may not be the same as the expected demand, which is typically obtained based on the demand distribution. In this research, the projected demand values, which are different from the expected demand values are analyzed. Random product demands and sequence data for nine-department five-product case are shown in Table 3.1, and the associated from-between chart is presented in Table 3.2.

TABLE 3.1

SUMMARY OF PRODUCT DEMAND AND PROCESS SEQUENCE

Product	Product Demand Distribution	Projected Demand	Product Sequence
1	Uniform (100, 200)	150	1-3-5-7-9
2	Uniform (275, 320)	280	1-2-7-4-6
3	Uniform (200, 310)	240	4-5-6
4	Uniform (50, 150)	60	3-5-7-8-6
5	Uniform (40, 400)	50	1-8

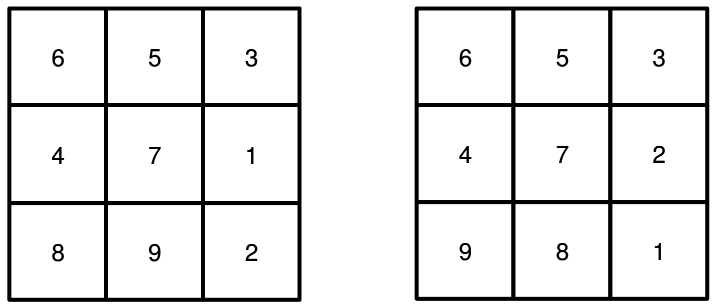
TABLE 3.2

FROM-BETWEEN CHART

Dept.	1	2	3	4	5	6	7	8	9
1		280	150					50	
2							280		
3					210				
4					240	280	280		
5						240	210		
6								60	
7								60	150
8									
9									

For the data provided, the demand for product 2 is 280 units and the one for product 5 is 50 units. Based on probability density functions (*pdf*) of the product demands, there is a possibility that the demand for product 5 is higher than that for product 2. If this scenario occurs, the layout design based on the projected demand values (Table 3.1, Column 3) may no longer be the best layout. Figure 3.1 shows two layout designs. The first is generated based on projected demand values, and the second is generated based on the demand scenario in which flow intensities consisting of product 3 are equal to those of flow intensities consisting of product 2.

This case is an extreme case and can be used to demonstrate the impact of uncertainty in product demand.



(a) (b)

Figure 3.1. Facility layout based on projected demand values (a) and the scenario where flow intensity consisting of product 3 is equal to product 2 (b).

Based on this example, the flow intensities that are exposed to risk can be identified by checking the following conditions:

1. Is the probability that the flow intensity between departments *i* and *j* will exceed the flow intensity between any other departments *k* and *l* equal to zero?
2. Are departments *i* and *j* adjacent to one another in the layout generated using projected demand values?

The first condition is used to check if there is any possibility that the flow intensity under consideration can be altered due to uncertainty, which in turn will change the relationship between its corresponding pair of departments. The second condition checks if departments *i* and *j* are adjacent. If they are adjacent, the material handling costs cannot be reduced further, even if the flow intensity between departments *i* and *j* increases. Flow intensities that do not satisfy both conditions are considered to be subject to risk.

For the data provided in Tables 3.1 and 3.2, the best layout for the projected demand is shown in Figure 3.1a. A total of thirteen flow intensities (f_{12}, f_{13}, f_{18}, f_{27}, f_{35}, f_{45}, f_{46}, f_{47}, f_{56}, f_{57}, f_{68}, f_{78}, *and* f_{79}) is considered in the from-between chart (Table 3.2). By evaluating each of these flow intensities through the risk conditions, only f_{18} and f_{45} are at risk.

3.3.2 Assessment of Relative Increase in Flow Intensity (Mathematical Formulation Method)

The previous section presented the method of identifying those flow intensities that are at risk. In this section, the assessment of the probability of occurrence for flow intensity between departments *i* and *j* exceeding that of departments *k* and *l* procedure, along with the method to quantify these relative changes are detailed.

In order to calculate the probability of occurrence of flow intensity between departments *i* and *j* exceeding that of departments *k* and *l* ($p_{f_{ij}}^{f_{kl}}$), the *pdf* of each flow intensity is needed. In general, the flow intensity between two departments may either be the result of a single-product demand or multiple-product demands. In the case of one product, the *pdf* of the flow intensity is the *pdf* of the product demand itself. When multiple products flow between the two departments, the *pdf* of the flow intensity is attributed to all of those products. To estimate the *pdf* of a combined-product demand, the cumulative distribution function *(cdf)* of combined-product demands can be calculated using convolution theory and it can be calculated as

$$\varphi(f_{ij}) = P(\ X_1 + X_2\ < x) = \int_{-\infty}^{\infty} g(x - x_1\ )g(x_1\ )dx \qquad (3.2)$$

where $g(x_1\ )$ denotes the *pdf* of the demand of the first product, and x is the variable for the total flow intensity level of the combined product demand. It is assumed that the distributions of these demands are known and independent, and they are of continuous types. Once the *pdf* of each flow is determined, $p_{f_{ij}}^{f_{kl}}$ can be calculated using

$$p_{f_{ij}}^{f_{kl}} = Pr(f_{ij} \geq f_{kl}) = \int_0^{\infty} \int_{f_{kl}}^{\infty} \varphi(f_{ij})\varphi(f_{kl})\, df_{ij}\, df_{kl} \tag{3.3}$$

To illustrate the proposed method, a uniform distribution is assumed for all product demands due to its usefulness in dealing with limited information regarding product demands. Similar derivations using other distributions can also be developed. The *pdf* of the uniform distribution is expressed as

$$\begin{gathered} g(x)=\frac{1}{b-a}, \quad a \leq x \leq b \\ \text{Otherwise},\ g(x)=0 \end{gathered} \tag{3.4}$$

where *a* and *b* are the lower and upper bounds, respectively, of the distribution. From equation (3.2), the *pdf* of a flow intensity consisting of two product types can be expressed as follows:

$$\boldsymbol{\varphi}(f_{ij}) = \begin{cases} \int_{a_{x_1}}^{x-a_{x_2}} g(x-x_1)g(x_1)\,dx = \dfrac{x-a_{x_1}-a_{x_2}}{(b_{x_1}-a_{x_1})(b_{x_2}-a_{x_2})}, \text{ where} (a_{x_1}+a_{x_2}) \leq x < (b_{x_1}+a_{x_2}) \\ \int_{x-a_{x_2}}^{x-b_{x_1}} g(x-x_1)g(x_1)\,dx = \dfrac{a_{x_2}-b_{x_1}}{(b_{x_1}-a_{x_1})(b_{x_2}-a_{x_2})}, \text{where} (b_{x_1}+a_{x_2}) \leq x < (a_{x_1}+b_{x_2}) \\ \int_{x-b_{x_1}}^{b_{x_2}} g(x-x_1)g(x_1)\,dx = \dfrac{b_{x_1}+b_{x_2}-x}{(b_{x_1}-a_{x_1})(b_{x_2}-a_{x_2})}, \text{where} (a_{x_1}+b_{x_2}) \leq x \leq (b_{x_1}+b_{x_2}) \\ 0, \text{elsewhere} \end{cases} \tag{3.5}$$

where a_{x_h} and b_{x_h} denote the lower and upper bounds, respectively, of the distribution of product h. For example, considering the data provided in Tables 3.1 and 3.2, the flow intensity between departments 3 and 5 comprises product 1 and 3, which follow uniform distribution with upper and lower bounds of 100 and 200, and 50 and 150, respectively. By using equation (3.5), the *pdf* of the flow intensity between departments 3 and 5 ($\boldsymbol{\varphi}(f_{35})$) is derived as

$$\varphi(f_{35}) = Pr(X_1 + X_4 < f_{35}) = \begin{cases} \left(\dfrac{f_{35} - a_{x_1} - a_{x_4}}{(b_{x_1} - a_{x_1})(b_{x_4} - a_{x_4})}\right), & 150 \le f_{35} < 250 \\ \left(\dfrac{b_{x_1} + b_{x_2} - f_{35}}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})}\right), & 250 \le f_{35} \le 350 \end{cases}$$

Once the *pdf* of flow intensity is determined, and then $p_{f_{ij}}^{f_{kl}}$ can be obtained from equation (3.3).

For example, based on the information provided in Tables 3.1 and 3.2, $p_{f_{18}}^{f_{35}}$ can be calculated as

If $150 \le f_{35} < 250$, then

$$p_{f_{ij}}^{f_{kl}} = \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} \varphi(f_{ij})\varphi(f_{kl})\, df_{ij}\, df_{kl} = \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{f_{kl} - a_{x_1} - a_{x_2}}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})}\right)\left(\frac{1}{(b_{f_{ij}} - a_{f_{ij}})}\right) df_{ij}\, df_{kl}$$

$$= \frac{\left(\dfrac{b_{f_{ij}} + (a_{x_1} + a_{x_2})}{2}\right)(b_{f_{kl}}^2 - a_{f_{kl}}^2) - \left(\dfrac{b_{f_{kl}}^3 - a_{f_{kl}}^3}{3}\right) - b_{f_{ij}}(a_{x_1} + a_{x_2})(b_{f_{kl}} - a_{f_{kl}})}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})}$$

$$= \frac{\left(\dfrac{400 + (50 + 100)}{2}\right)(250^2 - 150^2) - \left(\dfrac{250^3 - 150^3}{3}\right) - 400(50 + 100)(250 - 150)}{(150 - 50)(200 - 100)(400 - 40)}$$

$$= 0.254$$

If $250 \le f_{35} \le 350$, then

$$p_{f_{ij}}^{f_{kl}} = \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} \varphi(f_{ij})\varphi(f_{kl})\, df_{ij}\, df_{kl} = \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{b_{x_1} + b_{x_2} - f_{kl}}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})}\right)\left(\frac{1}{(b_{f_{ij}} - a_{f_{ij}})}\right) df_{ij}\, df_{kl}$$

$$= \frac{b_{f_{ij}}(b_{x_1} + b_{x_2})(b_{f_{kl}} - a_{f_{kl}}) - \left(\dfrac{b_{f_{ij}} + (b_{x_1} + b_{x_2})}{2}\right)(b_{f_{kl}}^2 - a_{f_{kl}}^2) + \dfrac{1}{3}(b_{f_{kl}}^3 - a_{f_{kl}}^3)}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})}$$

$$= \frac{400(150+200)(350-250)-\left(\frac{400+(150+200)}{2}\right)(350^2-250^2)+\frac{1}{3}(350^3-250^3)}{(150-50)(200-100)(400-40)}$$

$$= 0.162$$

where $a_{f_{ij}}$ and $b_{f_{ij}}$ are the lower and upper bounds, respectively, of the distribution of flow intensity f_{ij}, and $a_{f_{kl}}$ and $b_{f_{kl}}$ are the lower and upper bounds, respectively, of the distribution of flow intensity f_{kl}.

Traditionally, the material handling cost is the product of the flow intensities, distance between corresponding pairs of departments, and material handling cost per unit distance. And, as defined previously, the risk of facility layout is the potential increase in material handling cost caused by the uncertainty of product demand. Therefore, to estimate this increase in material handling cost, the expected relative changes in flow intensity between departments *i* and *j* with respect to that of departments *k* and *l* ($E[f_{ij}^{kl}]$) is needed and can be obtained using the following equations:

$$E[f_{ij}^{kl}] = \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} (f_{ij} - f_{kl} + Q)\varphi(f_{ij})\varphi(f_{kl})\, df_{ij}\, df_{kl}, \text{where } Q = F_{kl} - F_{ij} \quad (3.6)$$

For demonstration, the estimate of $E[f_{18}^{35}]$ can be calculated using equation (3.6) as

If $150 \leq f_{35} < 250$, then

$$E[f_{ij}^{kl}] = \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} (f_{ij} - f_{kl} + Q)\varphi(f_{ij})\varphi(f_{kl})\, df_{ij}\, df_{kl}, where\ Q = F_{kl} - F_{ij}$$

$$= \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{(f_{ij} - f_{kl} + Q)(f_{kl} - a_{x_1} - a_{x_2})}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})} \right) df_{ij}\, df_{kl}$$

$$= \int_{a_{f_{kl}}}^{b_{f_{kl}}} \int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{f_{ij} f_{kl} - f_{kl}^2 + Q f_{kl} - (a_{x_1} + a_{x_2})(f_{ij} - f_{kl} + Q)}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})} \right) df_{ij}\, df_{kl}$$

Let $(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{kl}} - a_{f_{kl}}) = M$, then

$$E[f_{ij}^{kl}] = \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}}\left(\frac{f_{ij}\,f_{kl} - f_{kl}^{2} + Qf_{kl}}{M}\right)df_{ij}\,df_{kl} - \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}}\left(\frac{(a_{x_1} + a_{x_2})(f_{ij} - f_{kl} + Q)}{M}\right)df_{ij}\,df_{kl}$$

Let $E[f_{ij}^{kl}]_1 = \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}}\left(\frac{f_{ij}\,f_{kl} - f_{kl}^{2} + Qf_{kl}}{M}\right)df_{ij}\,df_{kl}$

$$= \frac{(b_{f_{kl}}^{2} - a_{f_{kl}}^{2})\left(\frac{b_{f_{ij}}^{2}}{4} + \frac{Qb_{f_{ij}}}{2}\right) - (b_{f_{kl}}^{3} - a_{f_{kl}}^{3})\left(\frac{b_{f_{ij}}}{3} + \frac{Q}{3}\right) + \frac{1}{8}(b_{f_{kl}}^{4} - a_{f_{kl}}^{4})}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})}$$

$$= \frac{(250^2 - 150^2)\left(\frac{400^2}{4} + \frac{(160)(400)}{2}\right) - (250^3 - 150^3)\left(\frac{400}{3} + \frac{160}{3}\right) + \frac{1}{8}(250^4 - 150^4)}{(200 - 100)(150 - 50)(400 - 40)}$$

$= 282.87$

And let

$$E[f_{ij}^{kl}]_2 = \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}}\left(\frac{(a_{x_1} + a_{x_2})(f_{ij} - f_{kl} + Q)}{M}\right)df_{ij}\,df_{kl}$$

$$= \frac{(a_{x_1} + a_{x_2})\left((b_{f_{kl}} - a_{f_{kl}})\left(\frac{b_{ij}^{2}}{2} + b_{ij}Q\right) - (b_{f_{kl}}^{2} - a_{f_{kl}}^{2})\left(\frac{b_{f_{kl}}}{2} + \frac{Q}{2}\right) + \frac{1}{6}(b_{f_{kl}}^{3} - a_{f_{kl}}^{3})\right)}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})}$$

$$= \frac{(100 + 50)\left((250 - 150)\left(\frac{400^2}{2} + (400 * 160)\right) - (250^2 - 150^2)\left(\frac{400}{2} + \frac{160}{2}\right) + \frac{1}{6}(250^3 - 150^3)\right)}{(200 - 100)(150 - 50)(400 - 40)}$$

$= 218.40$

Therefore, $E[f_{ij}^{kl}] = E[f_{ij}^{kl}]_1 - E[f_{ij}^{kl}]_2 = E[f_{18}^{35}]_1 - E[f_{18}^{35}]_2 = 282.87 - 218.40 = 64.47.$

If $250 \le f_{35} \le 350$, then

$$E[f_{ij}^{kl}] = \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}} (f_{ij} - f_{kl} + Q)\varphi(f_{ij})\varphi(f_{kl})\, df_{ij}\, df_{kl}, where\ Q = F_{kl} - F_{ij}$$

$$= \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{(f_{ij} - f_{kl} + Q)(b_{x_1} + b_{x_2} - f_{kl})}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})} \right) df_{ij}\, df_{kl}$$

$$= \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{f_{kl}^2 - f_{ij} f_{kl} - Qf_{kl} + (b_{x_1} + b_{x_2})(f_{ij} - f_{kl} + Q)}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})} \right) df_{ij}\, df_{kl}$$

Let $(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{kl}} - a_{f_{kl}}) = M$. Then

$$E[f_{ij}^{kl}] = \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{f_{kl}^2 - f_{ij} f_{kl} - Qf_{kl}}{M} \right) df_{ij}\, df_{kl} + \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{(b_{x_1} + b_{x_2})(f_{ij} - f_{kl} + Q)}{M} \right) df_{ij}\, df_{kl}$$

Let $E[f_{ij}^{kl}]_1 = \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{f_{kl}^2 - f_{ij} f_{kl} - Qf_{kl}}{M} \right) df_{ij}\, df_{kl}$

$$= \frac{\left(\frac{b_{f_{ij}} + Q}{3} \right)(b_{f_{kl}}^3 - a_{f_{kl}}^3) - \left(\frac{Qb_{f_{ij}}}{2}(b_{f_{kl}}^2 - a_{f_{kl}}^2) \right) - \frac{b_{f_{ij}}^2}{4}(b_{f_{kl}}^2 - a_{f_{kl}}^2) - \frac{1}{8}(b_{f_{kl}}^4 - a_{f_{kl}}^4)}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})}$$

$$= \frac{\left(\frac{400+160}{3} \right)(350^3 - 250^3) - \left(\frac{(160)(400)}{2}(350^2 - 250^2) \right) - \frac{400^2}{4}(350^2 - 250^2) - \frac{1}{8}(350^4 - 250^4)}{(200-100)(150-50)(400-40)}$$

$$= -172.45$$

And let

$$E[f_{ij}^{kl}]_2 = \int_{a_{f_{kl}}}^{b_{f_{kl}}}\int_{f_{kl}}^{b_{f_{ij}}} \left(\frac{(b_{x_1} + b_{x_2})(f_{ij} - f_{kl} + Q)}{M} \right) df_{ij}\, df_{kl}$$

$$= \frac{(b_{x_1} + b_{x_2})\left((b_{f_{kl}} - a_{f_{kl}})\left(\frac{b_{ij}^2}{2} + b_{ij}Q \right) - (b_{f_{kl}}^2 - a_{f_{kl}}^2)\left(\frac{b_{f_{kl}}}{2} + \frac{Q}{2} \right) + \frac{1}{6}(b_{f_{kl}}^3 - a_{f_{kl}}^3) \right)}{(b_{x_1} - a_{x_1})(b_{x_2} - a_{x_2})(b_{f_{ij}} - a_{f_{ij}})}$$

$$= \frac{(150+200)\left((350-250)\left(\frac{400^2}{2}+(400*160)\right)-(350^2-200^2)\left(\frac{400}{2}+\frac{160}{2}\right)+\frac{1}{6}(350^3-200^3)\right)}{(200-100)(150-50)(400-40)}$$

$= 208.22$

Therefore, $E[f_{ij}^{kl}] = E[f_{ij}^{kl}]_1 + E[f_{ij}^{kl}]_2 = E[f_{18}^{35}]_1 + E[f_{18}^{35}]_2 = -172.45 + 208.21 = 35.76$.

Hence, $E[f_{18}^{35}]$ is 64.47 + 35.76 = 100.23. Table 3.3 is a summary of the $E[f_{ij}^{kl}]$ values for flow intensities that are exposed to risk.

TABLE 3.3

SUMMARY OF $E[f_{ij}^{kl}]$ VALUES (MATHEMATICAL FORMULATION METHOD)

$E[f_{ij}^{kl}]$	$E[f_{ij}^{12}]$	$E[f_{ij}^{13}]$	$E[f_{ij}^{27}]$	$E[f_{ij}^{35}]$	$E[f_{ij}^{45}]$	$E[f_{ij}^{46}]$
$E[f_{18}^{kl}]$	80.31	157.41	80.31	100.23	107.13	80.31
$E[f_{45}^{kl}]$	6.02	-	6.02	-	-	6
$E[f_{ij}^{kl}]$	$E[f_{ij}^{47}]$	$E[f_{ij}^{56}]$	$E[f_{ij}^{57}]$	$E[f_{ij}^{78}]$	$E[f_{ij}^{79}]$	$E[f_{ij}^{86}]$
$E[f_{18}^{kl}]$	80.31	107.13	100.23	151.85	157.41	151.85
$E[f_{45}^{kl}]$	6.02	-	-	-	-	-

3.3.3 Assessment of Relative Increase in Flow Intensity (Simulation Method)

In a real manufacturing setting, the distribution that the product demands follow can come in many forms. The previous section presented the mathematical formulation for the facility layout risk assessment. Although, the method is proven to be useful in quantifying layout risk, the time required to derive the necessary formulations can be very extensive when the product demands follow a more complex distribution (more variables) and when more than two products are combined in a flow intensity. To provide an alternative method of estimating $E[f_{ij}^{kl}]$, the simulation method for facility layout risk assessment model was developed. Figure 3.3 demonstrates the general steps to estimate $E[f_{ij}^{kl}]$ using the simulation method.

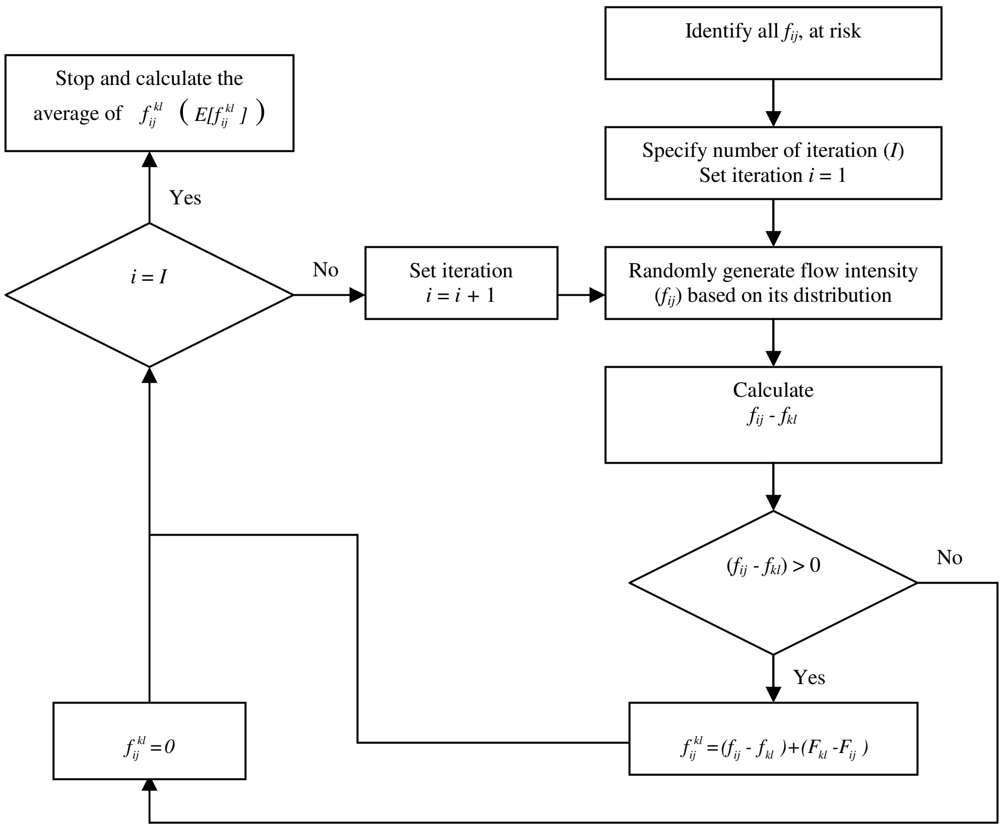


Figure 3.2. Simulation process flow chart.

To illustrate the usage of the simulation method, the estimation of $E[f_{18}^{13}]$ is simulated for 50 iterations using @Risk software. Results of the simulation are presented in Table 3.4. Based on these results, the average of f_{18}^{13} ($E[f_{18}^{13}]$) is 134.37, and its corresponding 95 percent confidence interval is (102.69, 166.05).

TABLE 3.4

RESULTS OF $E[f_{18}^{13}]$ VALUES USING SIMULATION (50 ITERATIONS)

Iteration	f_{18}^{13}	Iteration	f_{18}^{13}	Iteration	f_{18}^{13}	Iteration	f_{18}^{13}	Iteration	f_{18}^{13}
1	108.9	**11**	230.2	**21**	288.0	**31**	260.7	**41**	0.0
2	0.0	**12**	130.6	**22**	204.4	**32**	253.5	**42**	239.5
3	138.2	**13**	0.0	**23**	246.3	**33**	0.0	**43**	0.0
4	0.0	**14**	209.2	**24**	0.0	**34**	169.5	**44**	144.2
5	0.0	**15**	204.5	**25**	285.5	**35**	0.0	**45**	137.8
6	107.7	**16**	0.0	**26**	361.4	**36**	0.0	**46**	288.2
7	228.9	**17**	0.0	**27**	0.0	**37**	171.8	**47**	0.0
8	0.0	**18**	258.5	**28**	293.0	**38**	166.3	**48**	126.3
9	0.0	**19**	135.6	**29**	182.4	**39**	157.2	**49**	188.2
10	0.0	**20**	292.7	**30**	271.1	**40**	0.0	**50**	237.1

For a more accurate estimation of $E[f_{ij}^{kl}]$ values, the simulation is run for 10,000 iterations, and the resulting values are shown in Tables 3.5 and 3.6 is the summary of the calculated 95 percent confidence interval of $E[f_{ij}^{kl}]$ values.

TABLE 3.5

SUMMARY OF THE $E[f_{ij}^{kl}]$ VALUES (SIMULATION METHOD)

$E[f_{ij}^{kl}]$	$E[f_{ij}^{12}]$	$E[f_{ij}^{13}]$	$E[f_{ij}^{27}]$	$E[f_{ij}^{35}]$	$E[f_{ij}^{45}]$	$E[f_{ij}^{46}]$
$E[f_{18}^{kl}]$	81.32	157.63	80.32	99.17	108.27	80.32
$E[f_{45}^{kl}]$	6.34	-	6.34	-	-	6.34
$E[f_{ij}^{kl}]$	$E[f_{ij}^{47}]$	$E[f_{ij}^{56}]$	$E[f_{ij}^{57}]$	$E[f_{ij}^{78}]$	$E[f_{ij}^{79}]$	$E[f_{ij}^{86}]$
$E[f_{18}^{kl}]$	81.32	108.27	99.17	152.16	157.63	152.16
$E[f_{45}^{kl}]$	6.34	-	-	-	-	-

TABLE 3.6

SUMMARY OF CALCULATED 95 PERCENT CONFIDENCE INTERVAL OF $E[f_{ij}^{kl}]$ VALUES USING SIMULATION METHOD

$E[f_{ij}^{kl}]$	**95 Percent Confidence Interval**	$E[f_{ij}^{kl}]$	**95 Percent Confidence Interval**
$E[f_{18}^{12}]$	[78.79, 82.87]	$E[f_{18}^{57}]$	[96.77, 101.58]
$E[f_{18}^{13}]$	[155.24, 160.69]	$E[f_{18}^{78}]$	[150.12, 155.1]
$E[f_{18}^{27}]$	[78.79, 82.87]	$E[f_{18}^{79}]$	[155.24, 160.69]
$E[f_{18}^{35}]$	[96.77, 101.58]	$E[f_{18}^{86}]$	[150.12, 155.1]
$E[f_{18}^{45}]$	[105.63, 110.34]	$E[f_{45}^{12}]$	[6.00, 6.45]
$E[f_{18}^{46}]$	[78.79, 82.87]	$E[f_{45}^{27}]$	[6.00, 6.45]
$E[f_{18}^{47}]$	[78.79, 82.87]	$E[f_{45}^{46}]$	[6.00, 6.45]
$E[f_{18}^{56}]$	[105.63, 110.34]	$E[f_{45}^{47}]$	[6.00, 6.45]

To provide more statistical insight on these two methods, the $E[f_{ij}^{kl}]$ values acquired by both methods were examined and compared. Through observations, all of the $E[f_{ij}^{kl}]$ values obtained by mathematical formulation method (Table 3.3) fall within the range of the 95 percent confidence interval of $E[f_{ij}^{kl}]$ acquired via the simulation method (Table 3.6). Hence, it can be concluded that $E[f_{ij}^{kl}]$ values generated by both methods are not different.

3.3.4 Facility Layout Risk Calculation

In the previous two sections, the method of quantifying the impact of uncertainty in product demand was proposed in term of relative increase in flow intensity ($E[f_{ij}^{kl}]$). Once all $E[f_{ij}^{kl}]$ values are obtained, the risk associated with the facility layout can be determined. The risk value ($R_{f_{ij}}^{f_{kl}}$) is an increase in material handling cost as a result of the probable increases in demand and is calculated using equation (3.7).

$$R_{f_{ij}}^{f_{kl}} = E[f_{ij}^{kl}]\ C\ D_{ij} \qquad (3.7)$$

These calculated risk values provide an estimate of probable increases in material handling cost due to the probability that flow intensity between departments *i* and *j* surpasses that between departments *k* and *l*. A risk-flow table, comprises of $R_{f_{ij}}^{f_{kl}}$ values, is used to represent the estimated risk. Table 3.7 shows the format of a risk-flow table.

TABLE 3.7

RISK-FLOW TABLE

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{12}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{23}}$	....	$R_{f_{ij}}^{f_{kl}}$
$R_{f_{12}}^{f_{kl}}$	-	$R_{f_{12}}^{f_{13}}$	$R_{f_{12}}^{f_{23}}$		$R_{f_{12}}^{f_{kl}}$
$R_{f_{13}}^{f_{kl}}$		-	$R_{f_{13}}^{f_{23}}$		$R_{f_{13}}^{f_{kl}}$
$R_{f_{2j}}^{f_{kl}}$			-		
.				-	$R_{f_{ij}}^{f_{kl}}$
$R_{f_{ij}}^{f_{kl}}$					-

For example, equation (3.7) is used to calculate $R_{f_{18}}^{f_{35}}$, which is obtained as

$$R_{f_{18}}^{f_{35}} = E[f_{18}^{35}]\ C\ D_{18} = 87*1*100.23 = \$8,730$$

where $E[f_{18}^{35}]$ value is calculated previously in section 3.2.1, C is assumed to be \$1/unit distance, and D_{18} is the distance between departments 1 and 8 of the layout generated based on projected demand (Figure 1a). The remaining $R_{f_{ij}}^{f_{kl}}$ values are calculated and are presented in Table 3.8.

TABLE 3.8

RISK-FLOW TABLE FOR NINE-DEPARTMENT CASE STUDY

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{12}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{27}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{45}}$	$R_{f_{ij}}^{f_{46}}$
$R_{f_{18}}^{f_{kl}}$	$7,180	$14,072	$7,180	$8,730	$9,577	$7,180
$R_{f_{45}}^{f_{kl}}$	$340	-	$340	-	-	$340
$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{47}}$	$R_{f_{ij}}^{f_{56}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{f_{78}}$	$R_{f_{ij}}^{f_{79}}$	$R_{f_{ij}}^{f_{86}}$
$R_{f_{18}}^{f_{kl}}$	$7,180	$9,577	$8,730	$13,576	$14,072	$13,576
$R_{f_{45}}^{f_{kl}}$	$340	-	-	-	-	-

3.3.5 Risk-Ranking

To generate the ranking of each flow intensity, a risk-flow table is used, and the significant flow can be identified by calculating the maximum risk for the flow from departments *i* to *j* to surpass the others ($R_{f_{ij}}^{*}$). Thus, $R_{f_{ij}}^{*}$ can be expressed as

$$R_{f_{ij}}^{*} = \left[\max (R_{f_{ij}}^{f_{kl}}) \right] \forall f_{kl} \tag{3.8}$$

The $R_{f_{ij}}^{*}$ values are sorted and ranked in descending order in this step. These rankings are generated to assist in indicating which flow intensities contribute to a significant increase in risk value, when product demand is altered with respect to the others. The interpretation of these risk values can be subjective since it depends upon the risk attitude of the decision maker. Based on the example shown in the previous section, by using equation (3.8), $R_{f_{18}}^{*}$ is $14,072 and $R_{f_{45}}^{*}$ is $340, and the risk ranking is $R_{f_{18}}^{*}$ followed by $R_{f_{45}}^{*}$.

3.4 Conclusion

In this chapter, the definition of risk in the context of facility layout design is established. Risk here is defined as the potential increase in material handling cost due to uncertainty in product demand. Subsequently, the development of a facility layout risk assessment method is detailed. The proposed method aims to quantify the impact of the uncertainty in product demand to the layout in terms of an increase in material handling cost. To calculate this increase, two methods for the estimation of expected relative increase in flow intensity ($E[f_{ij}^{kl}]$) are offered: (1) a mathematical formulation method and (2) a simulation method. The estimated $E[f_{ij}^{kl}]$ value is later used to calculate the risk associated with the layout. The next chapter presents an approach for solving an FLP utilizing these risk values. The goal of this approach is to reduce the impact of the product demands uncertainty to the facility layout.

CHAPTER 4

RISK-BASED FACILITY LAYOUT APPROACH FOR SINGLE-PERIOD PROBLEM

4.1 Introduction

In Chapter 3, the facility layout risk assessment method was presented. This method aims to quantify the impact of the uncertainty in product demand to the efficiency of the facility layout in terms of risk. Once the risk of a facility layout is measured, it can be incorporated with the design of the facility layout process. In this chapter, the development of a risk-based facility layout approach for a single-period case is discussed. The objective of this approach was to minimize the risk of the layout. In other words, the facility layout was designed to minimize the maximum expected increase in material handling cost of the layout. A risk-based facility layout approach was performed using the three-step procedure:

1. Develop the best layout based on projected demand values.
2. Perform facility layout risk assessment.
3. Develop the layout to minimize the maximum risk.

The following sections describe the development of a risk-based facility layout approach, and detail the mathematical formulation and GA procedure required for layout generation.

4.2 Notations

The notions used in the risk-based facility layout approach for a single-period case are the following:

Z	Material handling cost for layout based on projected demand values
P	Total number of locations in layout; $p = 1 \ldots P$ and $q = 1 \ldots P$
N	Total number of departments in layout where i and j are indices for departments

R	Risk of facility layout
D_{pq}	Distance between any two locations *p* and *q*
X_{ip}	Binary variable, 1 if department *i* is allotted to location *p* or location *p* is allotted to department *i*, 0 otherwise without considering risk
Y	GA population size
G	GA number of generation
D^*_{pq}	Distance between any two locations *p* and *q* for risk-based layout
X^*_{ip}	Binary variable, 1 if department *i* is allotted to location *p* or location *p* is allotted to department *i*, 0 otherwise for the risk based layout
y_{gst}	y^{th} population of g^{th} generation of s^{th} risk scenario in t^{th} period
y_{gt}	y^{th} population of g^{th} generation in t^{th} period; $y = 1 \ldots Y$
g_{st}	g^{th} generation of s^{th} risk scenario in t^{th} period
g_t	g^{th} generation of t^{th} period; $g = 1 \ldots G$
Z_i	Cost of best solution in i^{th} population
Z^*	Cost of best solution in any population
R_i	Risk of best solution in i^{th} population
R^*	Risk of best solution in any population
F_{ij}	Projected low intensity between departments *i* and *j*
F^*_{ij}	Flow intensity between departments *i* and *j* considering risk

4.3 Layout Generation for Projected Demand

As mentioned earlier, the risk associated with a facility layout is defined as the potential increase in material handling cost caused by uncertainty in product demand. Hence, to estimate the additional material handling cost, the best layout based on the projected demand values was first determined. This layout problem can be formulated as

$$\text{Min } Z = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{p=1}^{P} \sum_{q=1}^{P} F_{ij} CD_{pq} X_{ip} X_{jq} \tag{4.1}$$

Subject to the following:

$$\left(\sum_{i=1}^{N} X_{ip} = 1 \right) \forall\ p \tag{4.2}$$

$$\left(\sum_{p=1}^{P} X_{ip} = 1 \right) \forall\ i \tag{4.3}$$

$$\{i=1 \ldots N\text{-}1, j=i+1 \ldots N, p=1 \ldots P, \text{and } q=1 \ldots P\}$$

where constraint equation (4.2) was used to ensure that there is only one department per location, and constraint equation (4.3) was used to guarantee that each location is allotted to only one department.

4.4 GA Procedure

Since the mixed-integer programming shown in section 4.3 is categorized as a classical quadratic assignment problem (QAP) for facility layout design, a GA approach was employed. A one-dimensional array chromosome was used to represent the order of departments to be placed in a layout. The chromosomes were represented by numerical representation (e.g., 01, 02, 03,…etc.) of a string placement scheme for the layout generation. The departments were placed in successive rows from left-to-right and then from right-to-left. The width and height of the facility were specified for placement of the departments. For example, the placement of departments for the string 11090204080605071203 0110 is shown in Figure 4.1.

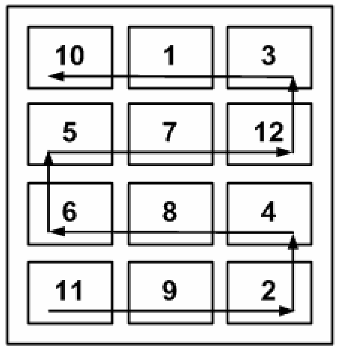


Figure 4.1. Department placement scheme.

The GA cost function is provided in equation (4.1). This cost function attempts to minimize the material handling cost for the projected demand. The fitness function is given in equation (4.4). The GA algorithm was verified using several existing cases to verify the code.

$$\text{Fitness Value} = v(i) = K(Z_i)^{\alpha\text{-}1} e^{-\left(\left(\frac{Z_i - Z^*}{Z^*}\right)\beta\right)} \tag{4.4}$$

where $\alpha = 0.5$, and β is a dynamic factor that is continuously modified as time increases. For problems in the single-period scenario, after experimentation, the following ranges of values are used for β:

$$\beta = \begin{cases} 0.002Z^*, & \text{when } 0<t<T/5 \\ 0.004Z^*, & \text{when } T/5 \le t<2T/5 \\ 0.006Z^*, & \text{when } 2T/5 \le t<3T/5 \\ 0.008Z^*, & \text{when } 3T/5 \le t<4T/5 \\ 0.01Z^*, & \text{when } 4T/5 \le t \le T \end{cases}$$

where i is the current generation, Z^* is the cost of the best solution in any population, T is the total run time, and t is the current time. The value of β used in the fitness function is dependent on time as well as minimum cost. This fitness function was designed such that as the cost function value increased, the corresponding fitness value decreased. The probability of accepting a bad solution also decreased as the time increased.

The steps used in generating the layout using the GA procedure are given below:

Step 1: Determine population size (Y) and time (T).

Step 2: Generate a random layout (string/chromosome), and set $y_{gst} = 1$. Conduct a string feasibility check. The condition for infeasibility exists when a department is represented twice in a string. In a case where the string is not feasible, eliminate the second occurrence of the same department, and replace it with a department that is not represented in the string (corrective action). Evaluate the fitness of this string. Set $y_{gst} = y_{gst} + 1$.

Step 3: If $y_{gst} + 1 < Y$, then go to Step 2; otherwise, set $g_{st} = 1$. Save the ten best-fit strings according to fitness values, and use the ten best-fit solutions for crossover and mutation.

Step 4: Perform the roulette wheel selection method for crossover in the selection of the parents based on fitness values obtained. After the crossover and mutation operations, check the new strings obtained for feasibility, if required, and perform corrective action. Add strings into a new generation $g_{st} + 1$. Set $y_{gst+1} = y_{gst+1} + 1$.

Step 5: If $y_{gst+1} + 1 < Y$, then go to Step 4; otherwise, set $g_{st} = g_{st} + 1$. Retain the ten best-fit strings based on fitness value. Perform the elitism operation by keeping the ten best-fit solutions from the combined set of layouts generated in the two runs. Continue the process until $t = T$ is satisfied.

4.4.1 Analysis of GA Procedure

Although the GA was proven to provide a good solution in a moderate amount of time, solution quality is still a function of time. Longer heuristic run times increased the chance of acquiring a better solution. This was due to the randomness induced in the GA. The different factors in the proposed GA procedure have to be evaluated to determine their effectiveness in providing a good solution. One factor has been shown to affect the solution quality of a GA is problem size. The larger the problem size, the longer the GA is required to be run to obtain a good solution. Another factor tested was the percentage of crossover operation. In this research, the percentage of crossover operation was also hypothesized to have an effect on the solution quality produced by the GA. Hence, to find the best GA setting for the case studies, the general factorial design was used with three factors, which are run time (minutes), problem size (number of departments), and percentage of crossover operation. Run times for the study were 0.5, 1, 5, and 10 minutes; problem sizes were sixteen-department and twenty five-department problems and the percentage of crossover operation were 0, 50 and 100 percent. The sample size was 5 for each setting.

The performance measure taken into consideration was the deviation from the best solution. Due to the size of the problem considered (sixteen and twenty-five departments), the optimal solution for both cases was unknown. Therefore, the best solution, obtained by running them for 24 hours, was used as the benchmark to calculate the deviation from the best solution. The design matrix and entire results can be found in the Appendix A. Table 4.1 is the resulting ANOVA table. The data was transformed using square root transformation.

Based on the resulting ANOVA table, only problem size, percentage of crossover operation and their interaction had a significant effect on the percentage deviation form best solution at a significant level of 0.05 (p-value less than 0.5). Table 4.2 shows the reduced ANOVA table.

TABLE 4.1

ANALYSIS OF VARIANCE TABLE FOR FULL MODEL

Source	SSE	Degrees of Freedom	MSE	F Value	Prob > F
Model	3.75062774	29	0.129331991	18.72964989	< 0.0001
Time (A)	0.01951727	4	0.004879318	0.706614982	0.5889
Problem Size (B)	0.90746367	1	0.907463669	131.4174217	< 0.0001
Percentage Crossover (C)	2.43115115	2	1.215575573	176.0376895	< 0.0001
Interaction between AB	0.01778156	4	0.00444539	0.643774138	0.6323
Interaction between AC	0.05949676	8	0.007437095	1.077028114	0.3838
Interaction between BC	0.25587172	2	0.127935859	18.52746427	< 0.0001
Interaction between ABC	0.05934562	8	0.007418202	1.074292006	0.3857
Pure Error	0.82862408	120	0.006905201		
Total	4.57925182	149			

TABLE 4.2

ANALYSIS OF VARIANCE TABLE FOR REDUCED MODEL

Source	SSE	Degrees of Freedom	MSE	F Value	Prob > F
Model	3.59448653	5	0.718897307	105.1227264	< 0.0001
Problem Size (B)	0.90746367	1	0.907463669	132.6963589	< 0.0001
Percentage Crossover (C)	2.43115115	2	1.215575573	177.7508654	< 0.0001
Interaction between BC	0.25587172	2	0.127935859	18.70777114	< 0.0001
Residual	0.98476529	144	0.006838648		
Lack of Fit	0.15614121	24	0.006505884	0.94217156	0.5461
Pure Error	0.82862408	120	0.006905201		
Total	4.57925182	149			

As for the model adequacy check, assumptions of normality, constant, and independent of the error were valid. The normal probability plot, residuals plots, and outlier plot can be found in Appendix B. The interaction plot between problem size and percentage of crossover operation and three-dimensional interaction plot of the same are shown in Figures 4.2 and 4.3, respectively.

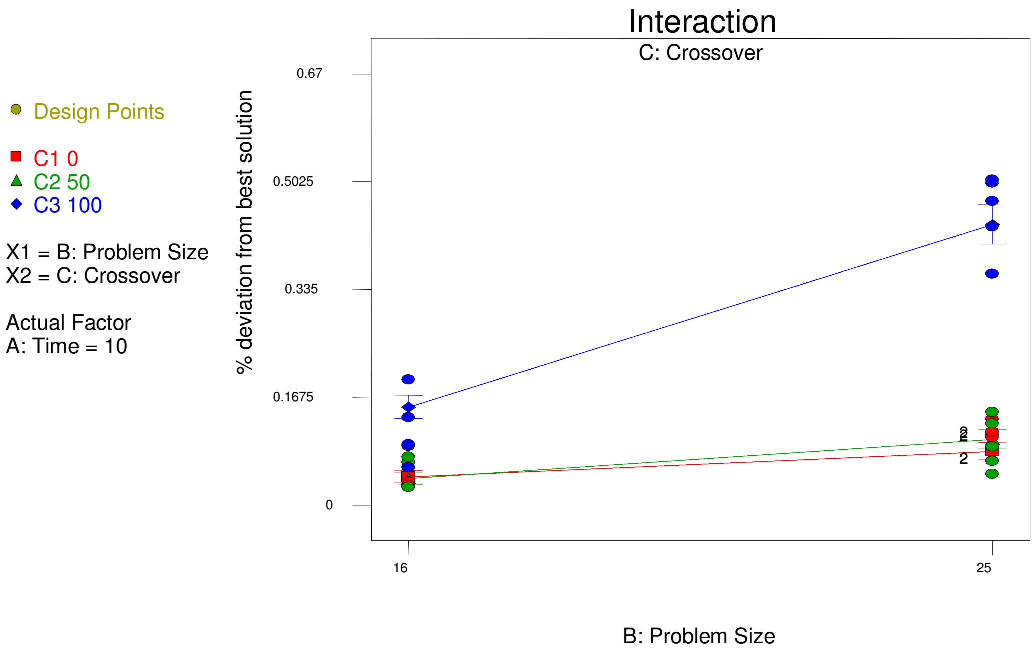


Figure 4.2. Interaction plot between problem size and percentage crossover operation.

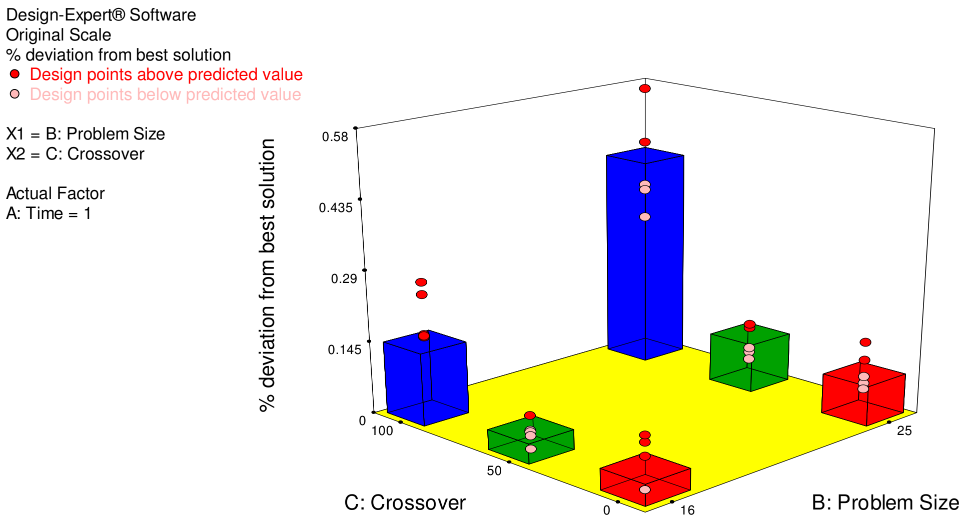


Figure 4.3. Three-dimensional interaction plot between problem size and percentage crossover operation.

The interaction plot (Figure 4.2) and three-dimensional interaction plot (Figure 4.3) indicate that the GA produces the best-quality solution in terms of percentage deviation from the best solution when crossover operation is performed at either 0 or 50 percent for both problem size. Hence, for all case studies, the crossover operation was set at 50 percent for the GA.

4.5 Risk-Based Facility Layout Design Procedure (Single-Period Case)

After generating the best layout based on the projected demand, the layout that minimizes the risk can be obtained using the following mixed-integer programming formulation. The objective function attempted to minimize the maximum increased cost associated with the risky flow intensities.

$$\text{Min } R = \left[\left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{p=1}^{P} \sum_{q=1}^{P} F_{ij}^{*} C\, D_{pq}^{*}\, X_{ip}^{*} X_{jq}^{*} \right) - Z \right] \tag{4.5}$$

Subject to the following:

$$\left(\sum_{i=1}^{N} X_{ip}^{*} = 1 \right) \forall\ p \tag{4.6}$$

$$\left(\sum_{p=1}^{P} X_{ip}^{*} = 1 \right) \forall\ i \tag{4.7}$$

$$\{ i=1 \ldots N\text{-}1, j=i+1 \ldots N, p=1 \ldots P, \text{and } q=1 \ldots P \}$$

where constraints equations (4.6) and (4.7) are similar to equations (4.2) and (4.3). To reflect impact of the uncertainty in the product demand values in the facility layout design process, the ($E[f_{ij}^{kl}]$) values were added to their corresponding projected flow intensities where the risk is presented using the following equation:

$$F_{ij}^{*} = \begin{cases} \left(\left[\max(E[f_{ij}^{kl}]) \right] \forall\, f_{kl} \right) + F_{ij}, & \text{when } p_{f_{ij}}^{f_{kl}} > 0, \text{for at least one } f_{kl} \\ F_{ij}, \text{ when } p_{f_{ij}}^{f_{kl}} = 0, \text{for all } f_{kl} \end{cases} \tag{4.8}$$

The same GA procedure for layout generation based on projected demand can be used to obtain the risk-based facility layout design (risk-based layout), with the exception of cost function and fitness function. To minimize layout risk, the modified cost function in equation (4.5) was used. The fitness function is given in equation (4.9).

$$\text{Fittness Value} = v(i) = K(R_i)^{\alpha-1} e^{-((\frac{R_i - R^*}{R^*})\beta)} \quad (4.9)$$

Similar to the fitness function used to generate layout based on projected demand, $\alpha = 0.5$ and β is a dynamic factor that is continuously modified as the time increases. The values of β were adjusted according to time.

$$\beta = \begin{cases} 0.002R^*, \text{ when } 0<t<T/5 \\ 0.004R^*, \text{ when } T/5 \le t<2T/5 \\ 0.006R^*, \text{ when } 2T/5 \le t<3T/5 \\ 0.008R^*, \text{ when } 3T/5 \le t<4T/5 \\ 0.01R^*, \text{ when } 4T/5 \le t \le T \end{cases}$$

where R^* is the risk associated with the best solution in any population.

4.6 Single-Period Case Studies

To illustrate the proposed procedure, two nine-department case studies are shown: one with no risk and another with risk. The no-risk situation occurred in two scenarios: (1) when the variances in product demand were not high and (2) when product sequences were similar, in which the flow intensities varied together. In addition, two more case studies, sixteen-department and twenty-five-department, are demonstrated using the simulation for risk assessment method. The following assumptions were made for these case studies:

- All departments are equal in size (40 × 40).
- The process sequence of each product is known.

- Probability density functions of products demand follow a known distribution, and are of a continuous type, and are independent.
- Material handling cost is $1/unit distance.
- Distances between departments are calculated based on a centroid to centroid procedure for the nine-department case and a rectilinear procedure for the sixteen- and twenty-five-department cases.

4.6.1 Nine-Department Case Study (Zero Risk)

In this case study, risk conditions were used to detect the presence or absence of risk. Consider the following nine-department problem with four products for which the projected product demand, product demand distribution, process sequence, and from-between chart are summarized in Tables 4.3 and 4.4. The optimal layout based on projected demand values is represented in Figure 4.4.

TABLE 4.3

PRODUCT DEMAND AND PROCESS SEQUENCE SUMMARY FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY WITH ZERO RISK

Product	Product Demand Distribution	Projected Demand	Product Sequence
1	Uniform (150, 210)	180	1-3-5-7-9
2	Uniform (275, 300)	280	1-2-7-4-6
3	Uniform (200, 270)	240	4-5-6-8
4	Uniform (200, 250)	225	1-3-5-7-9

TABLE 4.4

FROM-BETWEEN CHART FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY WITH ZERO RISK

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		280	405						
2							280		
3					405				
4					240	280	280		
5						240	405		
6								240	
7									405
8									
9									

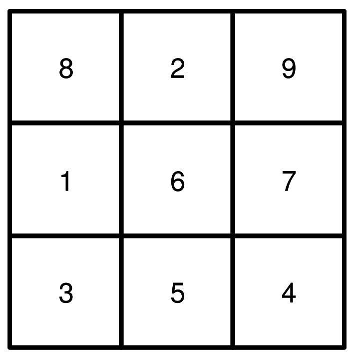


Figure 4.4. Layout based on projected demand value for nine-department single-period case study (zero risk).

A total of eleven flow intensities were considered in this case study. By evaluating all flow intensities through the risk conditions, flow intensities f_{45}, f_{56}, *and* f_{68}, which obtained a contribution only from product 3, were eliminated, since they had zero probability of surpassing any other flow intensities based on the *pdf*. Similarly, flow intensities f_{12}, f_{27}, f_{74}, *and* f_{46}, which obtained a contribution only from product 2, also were eliminated. In addition, based on the *pdf* of product 1, there was a probability that the demand of product 1 could exceed the demand of product 4. However, product 1 and product 4 had the same process sequence; hence, the flow intensities associated with this sequence were not at risk. As a result, there was no risk associated with the current layout design.

4.6.2 Nine-Department Case Study (With Risk)

Based on the data provided in Tables 4.5 and 4.6 for the nine-department case study, the best layout based on projected demand values is shown in Figure 4.5. A total of thirteen flow intensities were considered ($f_{12}, f_{13}, f_{18}, f_{27}, f_{35}, f_{45}, f_{46}, f_{47}, f_{56}, f_{57}, f_{68}, f_{78}$, *and* f_{79}) in the from-between Chart (Table 4.6).

TABLE 4.5

PRODUCT DEMAND AND PROCESS SEQUENCE FOR NINE-DEPARTMENT SINGLE- PERIOD CASE STUDY WITH RISK

Product	Product Demand Distribution	Projected Demand	Product Sequence
1	Uniform (100, 200)	150	1-3-5-7-9
2	Uniform (275, 320)	280	1-2-7-4-6
3	Uniform (200, 310)	240	4-5-6
4	Uniform (40, 120)	60	3-5-7-8-6
5	Uniform (40, 400)	50	1-8

TABLE 4.6

FROM-BETWEEN CHART FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY WITH RISK

Dept.	1	2	3	4	5	6	7	8	9
1		280	150					50	
2							280		
3					210				
4					240	280	280		
5						240	210		
6								60	
7								60	150
8									
9									

6	5	3
4	7	1
8	9	2

Figure 4.5. Best layout based on projected demand for nine-department single-period case study with risk.

In this case study, only f_{18} and f_{45} were exposed to risk. Table 4.7 shows a summary of the calculated $E[f_{ij}^{kl}]$ values. The $R_{f_{ij}}^{f_{kl}}$ values were calculated and are presented in the risk-flow table below (Table 4.8). Table 4.9 shows the from-between chart input generated using equation (4.8) for risk-based layout generation.

TABLE 4.7

SUMMARY OF $E[f_{ij}^{kl}]$ VALUES FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY WITH RISK

$E[f_{ij}^{kl}]$	**Expected Value**
$\left[\max(E[f_{18}^{kl}])\right] \forall f_{kl}$	157
$\left[\max(E[f_{45}^{kl}])\right] \forall f_{kl}$	6

Utilizing the information presented in Table 4.9, the facility layout design that minimizes risk was developed. The GA approach discussed in the previous section was used for the layout generation. The layout that minimizes risk is presented in Figure 4.6. The resulting $R_{f_{18}}^{*}$ value is now $6,296, which is a 55 percent reduction in risk. However, $R_{f_{45}}^{*}$ remains the same.

TABLE 4.8

RISK-FLOW TABLE FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY WITH RISK

$R^{f_{kl}}_{f_{ij}}$	$R^{f_{12}}_{f_{ij}}$	$R^{f_{13}}_{f_{ij}}$	$R^{f_{27}}_{f_{ij}}$	$R^{f_{35}}_{f_{ij}}$	$R^{f_{45}}_{f_{ij}}$	$R^{f_{46}}_{f_{ij}}$	
$R^{f_{kl}}_{f_{18}}$	$7,180	$14,072	$7,180	$7,730	$9,577	$7,180	
$R^{f_{kl}}_{f_{45}}$	$340	-	$340	-	-	$340	
$R^{f_{kl}}_{f_{ij}}$	$R^{f_{47}}_{f_{ij}}$	$R^{f_{56}}_{f_{ij}}$	$R^{f_{57}}_{f_{ij}}$	$R^{f_{78}}_{f_{ij}}$	$R^{f_{79}}_{f_{ij}}$	$R^{f_{86}}_{f_{ij}}$	$R^{*}_{f_{ij}}$
$R^{f_{kl}}_{f_{18}}$	$7,180	$9,577	$7,730	$13,576	$14,072	$13,576	$14,072
$R^{f_{kl}}_{f_{45}}$	$340	-	-	-	-	-	$340

TABLE 4.9

FROM-BETWEEN CHART OF RISK-BASED LAYOUT FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY WITH RISK

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		280	150					207	
2							280		
3					210				
4					246	280	280		
5						246	210		
6								60	
7								60	150
8									
9									

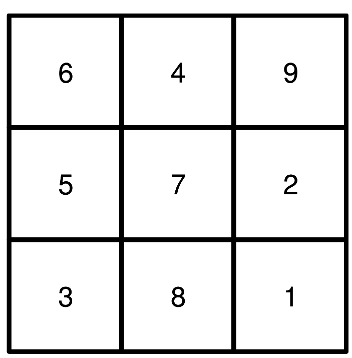


Figure 4.6. Risk-based layout for nine-department single-period case study with risk.

Although the proposed methodology was proven to be effective in reducing the risk of the layout, the effect of this compromised layout design on the material handling cost of the layout had to be analyzed. In this case study, f_{18} and f_{45} were identified as significant flows, which had the highest risk to the layout, and they correspond to $R^*_{f_{18}}$ and $R^*_{f_{45}}$, respectively. For the analysis of the performance of layout designs with respect to material handling costs, four scenarios that result in maximum risks were considered.

1. Projected demands are true
2. $x_5 \geq x_1, x_5 = 207\, units$
3. $x_3 \geq x_2, x_3 = 246\, units$
4. $x_5 \geq x_1 + x_4,\ x_5 = 207\, units$ and $x_3 \geq x_2,\ x_3 = 246\, units$

The analysis was performed with respect to material handling cost for layouts based on projected demand values and risk-based layout. Table 4.10 shows a material handling cost comparison of the layout based on projected demand values and the risk-based layout for these scenarios. Based on the results, given that the projected demand values are true, the risk-based layout incurred an additional $959 in overall material handling cost (0.8 percent increase). For scenario 3 (12.38 percent probability of occurrence), the risk-based layout also incurred an additional cost of $959. On the other hand, for both scenario 2 (69.02 percent probability of occurrence) and 4 (8.55 percent probability of occurrence), the risk-based layout resulted in a reduction of material handling cost by $6,797 (5.3 percent reduction).

TABLE 4.10

SUMMARY OF MATERIAL HANDLING COST COMPARISION FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY WITH RISK

Scenario	Likelihood	Material Handling Cost		Increase in Material Handling Cost
		Layout Based on Projected Demand	Risk-Based Layout	
1	-	$114,040	$114,999	$959
2	69.02%	$128,076	$121,279	-$6,797
3	12.38%	$114,619	$115,578	$959
4	8.55%	$128,655	$121,858	-$6,797

4.6.3 Sixteen-Department Case Study

In the sixteen-department case study, the simulation method was utilized to obtain the required $E[f_{ij}^{kl}]$ values due to the number of the products combined in flow intensity and the complexity of distribution that these product demands follow. The summary of the case study setting is presented in Tables 4.11 and 4.12, and the best layout based on projected demand values is shown in Figure 4.7.

TABLE 4.11

PRODUCT DEMAND AND PROCESS SEQUENCE FOR SIXTEEN-DEPARTMENT SINGLE-PERIOD CASE STUDY

Product	Product Demand Distribution	Projected Demand	Process Sequence
1	Normal (140, 10)	150	1-3-5-7-9-11-13-15
2	Normal (260, 20)	280	2-4-6-8-10-12-14-16
3	Uniform (200, 310)	240	7-9-11-13
4	Uniform (40, 120)	60	1-3-5-7-9
5	Uniform (40, 500)	50	1-2-3-4-5-6-10-14-15

TABLE 4.12

FROM-BETWEEN CHART FOR SIXTEEN-DEPARTMENT
SINGLE-PERIOD CASE STUDY

Depts.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		50	210													
2			50	280												
3				50	210											
4					50	280										
5						50	210									
6								280		50						
7									450							
8										280						
9											390					
10												280		50		
11													390			
12														280		
13															150	
14															50	280
15																
16																

15	13	11	9
1	3	5	7
2	4	6	8
16	14	12	10

Figure 4.7. Best layout based on projected demand values for sixteen-department single-period case study.

A total of twenty two flow intensities were considered. By evaluating these flow intensities, five flow intensities were identified as at-risk (f_{23} , f_{45} , $f_{6,10}$, $f_{10,14}$ and $f_{14,15}$). To obtain the estimation of $E[f_{ij}^{kl}]$ values, @Risk software was used to simulate the required values. The simulation was run for 100,000 iterations. The estimated $E[f_{ij}^{kl}]$ values are summarized in Table 4.13, and the resulting risk-flow table is presented in Table 4.14. The risk ranking is $R^{*}_{f_{14,15}}$, followed by any of the flow intensities because they have the same risk value.

TABLE 4.13

SUMMARY OF $E[f_{ij}^{kl}]$ VALUES FOR SIXTEEN-DEPARTMENT SINGLE-PERIOD CASE STUDY

$E[f_{ij}^{kl}]$	**Expected Values**
$\left[\max(E[f_{23}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{45}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{6,10}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{10,14}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{14,15}^{kl}])\right]\forall f_{kl}$	219

TABLE 4.14

RISK-FLOW TABLE FOR SIXTEEN-DEPARTMENT SINGLE-PERIOD CASE STUDY

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{f_{79}}$	$R_{f_{ij}}^{f_{9,11}}$	$R_{f_{ij}}^{f_{11,13}}$	$R_{f_{ij}}^{f_{13,15}}$	
$R_{f_{23}}^{f_{kl}}$	$14,606	$14,606	$14,606	$2,423	$7,295	$7,295	$17,549	
$R_{f_{45}}^{f_{kl}}$	$14,606	$14,606	$14,606	$2,423	$7,295	$7,295	$17,549	
$R_{f_{6,10}}^{f_{kl}}$	$14,606	$14,606	$14,606	$2,423	$7,295	$7,295	$17,549	
$R_{f_{10,14}}^{f_{kl}}$	$14,606	$14,606	$14,606	$2,423	$7,295	$7,295	$17,549	
$R_{f_{14,15}}^{f_{kl}}$	$29,213	$29,213	$29,213	$4,847	$14,590	$14,590	$35,098	
$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{24}}$	$R_{f_{ij}}^{f_{46}}$	$R_{f_{ij}}^{f_{68}}$	$R_{f_{ij}}^{f_{8,10}}$	$R_{f_{ij}}^{f_{10,12}}$	$R_{f_{ij}}^{f_{12,14}}$	$R_{f_{ij}}^{f_{14,16}}$	$R^{*}_{f_{ij}}$
$R_{f_{23}}^{f_{kl}}$	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$17,549
$R_{f_{45}}^{f_{kl}}$	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$17,549
$R_{f_{6,10}}^{f_{kl}}$	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$17,549
$R_{f_{10,14}}^{f_{kl}}$	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$14,672	$17,549
$R_{f_{14,15}}^{f_{kl}}$	$29,345	$29,345	$29,345	$29,345	$29,345	$29,345	$29,345	$35,098

Table 4.15 is the from-between chart generated using equation (4.8) for the risk-based layout, which is shown in Figure 4.8.

TABLE 4.15

FROM-BETWEEN CHART OF RISK-BASED LAYOUT FOR SIXTEEN-DEPARTMENT SINGLE-PERIOD CASE STUDY

Depts.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1		269	210													
2			269	280												
3				269	210											
4					269	280										
5						269	210									
6								280		269						
7									450							
8										280						
9											390					
10												280		269		
11													390			
12														280		
13															150	
14															269	280
15																
16																

7	5	3	1
9	6	4	2
11	8	10	12
13	15	14	16

Figure 4.8. Risk-based layout for sixteen-department single-period case study.

The generated risk-based layout helped to reduce $R^*_{f_{14,15}}$ from $35,098 to $8,774 (75 percent reduction) and $R^*_{f_{10,14}}$ from $17, 549 to 8,774 (50 percent reduction). However, $R^*_{f_{23}}$, $R^*_{f_{45}}$, and $R^*_{f_{6,10}}$ remained unchanged. Table 4.16 is a summary of the risk ranking and risk reduction generated using the risk-based facility layout approach.

TABLE 4.16

SUMMARY OF RISK RANKING AND RISK REDUCTION FOR SIXTEEN-DEPARTMENT SINGLE-PERIOD CASE STUDY

Flow Intensity		**Maximum Risk**		**Reduction in Material Handling Cost**	**Reduction**
		Layout Based on Projected Demand	**Risk-Based Layout**		
x_5	**2-3**	$17,549 (5)*	$17,549	$0	0%
	4-5	$17,549 (4)*	$17,549	$0	0%
	6-10	$17,549 (3)*	$17,549	$0	0%
	10-14	$17,549 (2)*	$8,774	$8,774	50%
	14-15	$35,098 (1)*	$8,774	$26,323	75%

*Number in parenthesis indicates ranking of risk.

Table 4.17 provides a comparison of material handling costs. For this specific case study, only two scenarios were considered:

1. Projected demands are true.
2. $x_5 \geq x_1 = 269$ *units.*

Based on the results, given that all projected demand values were true, the risk-based layout incurred an additional $3,200 in overall material handling cost (1.7 percent increase). On the other hand, for scenario 2 (78.54% probability of occurrence), the risk-based layout resulted in a reduction of material handling cost by $31,898 (9.9 percent reduction).

TABLE 4.17

SUMMARY OF MATERIAL HANDLING COST COMPARISON FOR SIXTEEN-DEPARTMENT SINGLE-PERIOD CASE STUDY

Scenario	Likelihood	Material Handling Cost		Increase in Material Handling Cost
		Layout Based on Projected Demand	Risk-Based Layout	
1	-	$188,800	$192,000	$3,200
2	78.54%	$320,417	$288,519	-$31,898

4.6.4 Twenty-Five-Department Case Study

Similar to the sixteen-department case study, the simulation method was used to obtain the required $E[f_{ij}^{kl}]$ values. The summary of the case study setting is presented in Table 4.18, and the best layout based on projected demand values is shown in Figure 4.9. A from-between chart for this case study can be found in Appendix C. A total of forty five flow intensities were considered. Among these flow intensities, eighteen flow intensities were at risk (Table 4.19).

TABLE 4.18

PRODUCT DEMAND AND PROCESS SEQUENCE FOR TWENTY-FIVE-DEPARTMENT SINGLE-PERIOD CASE STUDY

Product	Product Demand Distribution	Projected Demand	Process Sequence
1	Normal (140, 5)	150	1-9-3-7-5-11-19-13-17-15-21-25-23
2	Normal (275, 15)	280	2-10-4-8-6-12-20-14-18-16-24-22
3	Uniform (200, 310)	240	5-11-13-19-21
4	Uniform (40, 120)	60	6-12-14-16-18
5	Uniform (40, 500)	50	1-2-3-4-5--6-7-8-9-10-11-12-13
6	Triangular (40, 200, 500)	300	13-19-21-6-12-14
7	Triangular (100, 170, 500)	150	20-21-22-23-24-25

2	10	3	9	1
25	4	7	5	11
23	8	6	21	19
24	22	12	20	13
16	18	14	15	17

Figure 4.9. Best layout based on projected demand values for twenty-five-department single-period case study.

To obtain the estimate of $E[f_{ij}^{kl}]$ values, @Risk software was used to simulate the required values. The simulation was run for 100,000 iterations, and the estimated $E[f_{ij}^{kl}]$ values are summarized in Table 4.19. The resulting risk-flow table can be found in Appendix D and the input from-between chart for the risk-based layout can be found in Appendix E. Once the facility layout risk assessment was completed, the risk-based facility layout design procedure was performed, and the resulting layout is shown in Figure 4.10.

A summary of risk ranking and risk reduction are presented in Table 4.20. In this case study, the generated risk-based layout provided 50 percent reduction for seven flow intensities, 67 percent reduction for one of the flow intensities, 75 percent reduction for two flow intensities, and 80 percent reduction in one of the flow intensities**.** However, for six flow intensities, the associated risk remained unchanged, and the risk of one flow intensity increased by 50 percent.

TABLE 4.19

SUMMARY OF $E[f_{ij}^{kl}]$ VALUES FOR TWENTY-FIVE-DEPARTMENT SINGLE-PERIOD CASE STUDY

$E[f_{ij}^{kl}]$	**Expected Value**	$E[f_{ij}^{kl}]$	**Expected Value**
$\left[\max(E[f_{15,21}^{kl}])\right]\forall f_{kl}$	3	$\left[\max(E[f_{7,8}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{21,25}^{kl}])\right]\forall f_{kl}$	3	$\left[\max(E[f_{8,9}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{20,14}^{kl}])\right]\forall f_{kl}$	69	$\left[\max(E[f_{9,10}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{11,13}^{kl}])\right]\forall f_{kl}$	79	$\left[\max(E[f_{10,11}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{1,2}^{kl}])\right]\forall f_{kl}$	219	$\left[\max(E[f_{11,12}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{2,3}^{kl}])\right]\forall f_{kl}$	219	$\left[\max(E[f_{12,13}^{kl}])\right]\forall f_{kl}$	219
$\left[\max(E[f_{3,4}^{kl}])\right]\forall f_{kl}$	219	$\left[\max(E[f_{21,22}^{kl}])\right]\forall f_{kl}$	136
$\left[\max(E[f_{4,5}^{kl}])\right]\forall f_{kl}$	219	$\left[\max(E[f_{22,23}^{kl}])\right]\forall f_{kl}$	136
$\left[\max(E[f_{5,6}^{kl}])\right]\forall f_{kl}$	219	$\left[\max(E[f_{24,25}^{kl}])\right]\forall f_{kl}$	136

17	15	25	23	24
13	19	21	22	18
11	12	20	14	16
5	6	4	10	2
7	8	3	9	1

Figure 4.10. Risk-based layout for twenty-five-department single-period case study.

TABLE 4.20

SUMMARY OF RISK RANKING AND RISK REDUCTION FOR TWENTY-FIVE-DEPARTMENT SINGLE-PERIOD CASE STUDY

Flow Intensity		**Maximum Risk**		**Reduction in Material Handling Cost**	**Reduction**
		Layout Based on Projected Demand	**Risk-Based Layout**		
x_1	**15-21**	$241 (18)*	$241	$0	0%
	21-25	$481 (17)*	$120	$361	75%
x_2	**20-14**	$5,498 (16)*	$2,749	$2,749	50%
x_3	**11-13**	$6,322 (15)*	$3,161	$3,161	50%
x_5	**1-2**	$43,879 (1)*	$8,678	$35,200	80%
	2-3	$17,552 (5)*	$26,327	-$8,776	-50%
	3-4	$17,552 (6)*	$8,776	$8,776	50%
	4-5	$17,552 (7)*	$17,552	$0	0%
	5-6	$17,552 (8)*	$8,776	$8,776	50%
	7-8	$17,552 (9)*	$8,776	$8,776	50%
	8-9	$35,103 (2)*	$17,552	$17,552	50%
	9-10	$17,552 (10)*	$8,776	$8,776	50%
	10-11	$35,103 (3)*	$35,103	$0	0%
	11-12	$35,103 (4)*	$8,776	$26,327	75%
	12-13	$17,552 (11)*	$17,552	$0	0%
x_7	**21-22**	$16,356 (12)*	$5,452	$10,904	67%
	22-23	$10,904 (13)*	$5,452	$5,452	50%
	24-25	$10,904 (14)*	$10,904	$0	0%

*Number in parenthesis indicates ranking of risk.

Seven scenarios were considered for analysis. With respect to material handling costs, these seven scenarios had the highest risks:

1. Projected demands are true.
2. $x_1 \geq x_2 + x_6 = 153\, units.$
3. $x_2 \geq x_6 = 349\, units.$
4. $x_3 \geq x_6 = 319\, units.$
5. $x_5 \geq x_1 = 269\, units.$
6. $x_7 \geq x_6 = 286\, units.$

7. $x_1 \geq x_2 + x_6 = 153 units$, $x_2 \geq x_6 = 349 units$, $x_3 \geq x_6 = 319 units$, $x_5 \geq x_1 = 269 units$, and $x_7 \geq x_6 = 286 units.$

The analysis was done with respect to material handling cost for the layouts based on projected demand value, and the risk-based layout. Table 4.21 shows the material handling cost comparison of the layout based on projected demand values and the risk-based layout under defined scenarios. Results indicate that when all projected demand values are true, the risk-based layout incurred an additional $16,000 in overall material handling cost (3.4 percent increase). On the other hand, for scenario 7 (0.064 percent probability of occurrence), the risk-based layout resulted in a reduction of material handling cost of $83,400 (10.1 percent reduction). The risk-based layout performed worst when scenario 3 occurred (62.06% probability of occurrence), which will incur an additional material handling cost of $24,400 when compared with the best layout generated based on projected demand values. To the contrary, it performed best when scenario 5 (78.48 percent probability of occurrence) occurred, which reduces the material handling cost by $72,000 when compared with the best layout generated, based on projected demand values.

TABLE 4.21

SUMMARY OF MATERIAL HANDLING COST COMPARISON FOR TWENTY-FIVE-DEPARTMENT SINGLE-PERIOD CASE STUDY

Scenario	**Likelihood**	**Material Handling Cost**		**Increases in Material Handling Cost**
		Layout Based on Projected Demand	**Risk Based Layout**	
1	-	$466,400	$482,400	$16,000
2	1.5%	$468,640	$484,480	$15,840
3	62.06%	$494,400	$518,800	$24,400
4	54.68%	$472,800	$485,600	$12,800
5	78.48%	$739,200	$667,200	-$72,000
6	15.98%	$515,720	$515,280	-$440
7	0.064%	$825,160	$741,760	-$83,400

4.7 Conclusion

This chapter describes the development of the risk-based facility layout approach for a single period. This approach aims to minimize the maximum risk of layout due to uncertainty in the products demand. Both mathematical formulation and the heuristic procedure are detailed for the solution procedure, along with three case studies for the procedure validation. Results show that the proposed approach is efficient in reducing the risk of the facility layout when the product demands are subjected to variability. In the next chapter, an extension of the risk-based facility layout approach to consider the multi-period case is presented.

CHAPTER 5

RISK-BASED FACILITY LAYOUT APPROACH FOR MULTI-PERIOD PROBLEM

5.1 Introduction

In Chapter 3, the development of the facility layout risk assessment method is presented. Through experimentation, this method proved to be an effective tool to assess the impact of uncertainty of product demand to the facility layout in terms of risk. Chapter 4 presented the risk-based facility layout design approach, which aims to reduce the maximum risk of the facility layout for a single period. The approach demonstrated that it can be used to reduce the risk of facility layout effectively. To further enhance the application of the risk-based facility layout approach, this chapter presents the risk-based facility layout approach for a multi-period case. The following sections detail the development of this approach along with several tested case studies.

5.2 Notations

The notions used in the risk-based facility layout approach for a multi-period case are the following:

L	Sum of transition costs from one period to next and increased cost due to risk
D_{pq}^{t}	Distance between any two locations p and q in period t
X_{ip}^{t}	Binary variable, 1 if department i is allotted to location p or location p is allotted to department i in period t^{th}, 0 otherwise
D_{pq}^{*t}	Distance between any two locations p and q for risk-based layout in period t

X_{ip}^{*t}	Binary variable, 1 if department *i* is allocated to location *p* in period t, 0 otherwise for risk-based layout
F_{ij}^{t}	Predicted flow intensity between departments *i* and *j* at period t^{th}
F_{ij}^{*t}	Flow intensity between departments *i* and *j* considering risk in period *t*
$\left(R_{ij}^{kl}\right)^t$	Risk that $f_{ij} > f_{kl}$ *in* period *t*
Z_t^*	Minimum total expected loss at t^{th} period
$TC_{t-1,t}$	Risk-based layout transition cost from period *(t – 1)* to *t*
$TC_{t-1,t}^{C}$	Layout based on projected demand values' transition cost from period *(t – 1)* to *t*
R_t^C	Expected loss in period *t*, given that layout based on projected demand values is used
R_t	Expected loss in period *t*, given that risk-based layout is used
T	Current period
M	Cost of moving a department per unit distance ($/unit distance)

5.3 Risk-Based Facility Layout Approach (Multi-Period Case)

In order to generate an effective layout for the multi-period scenario the risk-based facility layout design procedure was modified, taking into consideration the transition cost involved in changing the layout from one period to the next. Krishnan et al. (2008) describe three situations by which the transition cost is applied: (1) when transition cost is negligible, (2) when transition cost is high, and (3) when transition cost is medium. Their method used discrete production scenarios.

5.3.1 Negligible Transition Cost

In the case where the transition cost is negligible, the best solution for layout design was obtained by solving the risk-based facility layout approach for each period separately. The model for the multi-period layout problem with negligible transition cost was derived by modifying the single-period model to include the time period index "*t*". Similar to the single-period approach, the optimal layout based on projected demand was generated for all time periods. The modified mixed-integer model is shown below:

$$\left[\text{Min } Z = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{p=1}^{P} \sum_{q=1}^{P} F_{ij}^{t} \, C \, D_{pq}^{t} \, X_{ip}^{t} X_{jq}^{t} \right] \forall t \tag{5.1}$$

Subject to the following:

$$\left(\sum_{i=1}^{N} X_{ip}^{t} = 1 \right) \forall \; p \tag{5.2}$$

$$\left(\sum_{p=1}^{P} X_{ip}^{t} = 1 \right) \forall \; i \tag{5.3}$$

$$\{ i=1 \ldots N\text{-}1, j=i+1 \ldots N, p=1 \ldots P, \; q=1 \ldots P, \text{and } t=1 \ldots T \}$$

where constraint equations (5.2) and (5.3) are used to ensure that there is only one department per location in time period *t*. After the best layout based on the projected demand for each period is generated, the layout that minimizes the maximum risk can be obtained using the following formulation:

$$\left[\text{Min } R = \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{p=1}^{P} \sum_{q=1}^{P} F_{ij}^{*t} C D_{pq}^{*t} X_{ip}^{*t} X_{jq}^{*t} \right) - Z^{t} \right] \forall t \tag{5.4}$$

Subject to the following:

$$\left(\sum_{i=1}^{N} X_{ip}^{*t} = 1 \right) \forall \; p \tag{5.5}$$

$$\left(\sum_{p=1}^{P} X_{ip}^{*t}=1\right)\forall\ i \tag{5.6}$$

$$\{i=1...N\text{-}1, j=i+1...N, p=1...P,\ q=1...P, and\ t=1...T\}$$

where constraint equations (5.5) and (5.6) are similar to constraint equations (5.2) and (5.3). To estimate F_{ij}^{*t} , the following equation was used:

$$F_{ij}^{*t} = \begin{cases} \left(\left(\max(E[f_{ij}^{kl}]^t)\,\forall\,(f_{kl})^t\right) + F_{ij}^t\right), & \text{when } (p_{f_{ij}}^{f_{kl}})^t > 0, \text{for at least one } (f_{kl})^t \\ F_{ij}^t, \text{ when } (p_{f_{ij}}^{f_{kl}})^t = 0, \text{for all } (f_{kl})^t \end{cases} \tag{5.7}$$

The same GA procedure for layout generation based on the projected demand was used to obtain the risk-based facility layout design. The cost function in equation (5.4) and fitness function in equation (4.9) were used for individual layout designs for each period.

5.3.2 High Transition Cost

For the situation in which transition cost is high, a single robust layout that minimizes the overall material handling risk for the multiple periods is preferred. In order to use the risk-based facility layout approach to solve this problem, the distributions of product demand from the periods under consideration were aggregated. Once the aggregated *pdf* of all products demand were obtained, this problem could be solved using the risk-based facility layout approach for a single period.

5.3.3 Medium Transition Cost

When considering the situation where transition cost is neither high nor low, losses can occur due to tradeoff between the transition cost from one layout to the next, and the increased material handling cost due to risk reduction must be addressed. To incorporate this tradeoff into the risk-based approach, the generic risk-based mixed-integer programming for a multi-period scenario can be represented as follows:

$$\left[\text{Min } L=\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sum_{p=1}^{P}\sum_{q=1}^{P}F_{ij}^{*t}\,C\,D_{pq}^{*t}\,X_{ip}^{*t}X_{jq}^{*t}\right)-Z^{t}+\left(\sum_{i=1}^{N}\sum_{p=1}^{P}\sum_{q=1}^{P}X_{ip}^{(t-1)}X_{iq}^{*t}D_{pq}^{*t}M\right)\right]\forall t \qquad (5.8)$$

Subject to the following:

$$\left(\sum_{i=1}^{N}X_{ip}^{*t}=1\right)\forall\ p \qquad (5.9)$$

$$\left(\sum_{p=1}^{P}X_{ip}^{*t}=1\right)\forall\ i \qquad (5.10)$$

$$\{i=1...N\text{-}1, j=i+1...N, p=1...P,\ q=1...P, \text{and } t=1...T\}$$

The objective function is the summation of the risk associated with the facility layout in the current period and the transition cost needed for changing the layout from the previous period to the current. Constraint equations (5.9) and (5.10) were used for the same purpose as that of constraint equations (5.5) and (5.6). To estimate F_{ij}^{*t}, equation (5.7) was used for each time period t.

To solve the multi-period problem with the medium transition cost scenario, the same GA procedure used previously was used to obtain the risk-based layout with the exception of cost function and fitness function. To minimize the risk of layout while considering the transition cost, the modified cost function in equation (5.8) was used. The fitness function is given as

$$\text{Fitness Value} = v = K(L_i)^{\alpha-1}e^{-((\frac{L_i-L^*}{L^*})\beta)} \qquad (5.11)$$

To obtain the Z^t value for the medium transition cost scenario, the best layout based on projected demand values considering the transition cost is required, and it can be formulated as the following mixed-integer programming problem:

$$\left[\text{Min } Z=\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sum_{p=1}^{P}\sum_{q=1}^{P}F_{ij}^{t}\,C\,D_{pq}^{t}\,X_{ip}^{t}X_{jq}^{t}\right)+\left(\sum_{i=1}^{N}\sum_{p=1}^{P}\sum_{q=1}^{P}X_{ip}^{(t-1)}X_{iq}^{t}D_{pq}^{t}M\right)\right]\forall t \tag{5.12}$$

Subject to the following:

$$\left(\sum_{i=1}^{N}X_{ip}^{t}=1\right)\forall\ p \tag{5.13}$$

$$\left(\sum_{p=1}^{P}X_{ip}^{t}=1\right)\forall\ i \tag{5.14}$$

$$\{i=1\ldots N\text{-}1, j=i+1\ldots N, p=1\ldots P,\ q=1\ldots P, and\ t=1\ldots T\}$$

where constraint equations (5.13) and (5.14) were used for the same purpose as that of constraint equations (5.5) and (5.6). The objective function in equation (5.12) aims to minimize material handling cost while considering transition cost. The same GA procedure used previously can also be used to obtain the layout, with the exception of cost function and fitness function. The cost function is given in equation (5.12) and the fitness function given in equation (4.9).

In addition, similar to the case study shown in Chapter 4 (section 4.5.1), there is a situation where the risk of facility layout is absent. This situation occurs when the uncertainties in product demand are not large enough to cause the flow intensity between a pair of departments to exceed others; hence, the relationship between all pairs of departments remains unchanged. When this situation occurs, the same GA procedure used to generate Z^t for the medium transition cost can be used to obtain the best layout, which minimizes the material handling cost while considering transition cost.

5.3.4 Risk-Based Facility Layout Design Procedure (Multi-Period Case)

In this section, the procedure for solving the multi-period scenario using the risk-based facility layout approach is presented. The GA procedure used for the single-period problem was used to develop the best layout for each of the periods in terms of minimizing the maximum risk in each period. If transition costs are ignored, this solution set represents the lower bound or the best layouts for each period with respect to reduction in risk. On the other hand, by adding the cost of transition from the best layout in period *t* to the best layout in period *t+1*, the upper bound solution can be determined. By considering transition cost and risk of the layout simultaneously, the best layout that balances transition cost and risk of layout can be obtained. To consider transition cost and risk of layout concurrently, the cost function and fitness function were modified (equations (5.8) and (5.11)) to include the transition cost from period *t* to *t+1*.

In addition, the layout based on projected demand was evaluated against the risk-based layout in terms of loss, since loss in the risk-based facility layout approach for the multi-period includes both risk and transition cost simultaneously. The tradeoff between the risk reduction and the transition cost were analyzed in order to select the best layout for a period under consideration. The best layout based on projected demand considering transition cost was generated using the proposed GA with the modified cost function and fitness function (equation (5.12) and (4.9)).

The procedure for solving the multi-period risk-based facility layout approach for layout design is given below:

Step 1: Determine number of periods (T) to be considered $\{t = 1, 2, \ldots, T\}$.

Step 2: Set $t = 0$ and $Z_0^* = 0$.

Step 3: Execute the GA procedures for risk-based layout (medium transition cost).

Step 4: Calculate total expected loss using the following expression:

$$L_T^* = \min \begin{cases} L_{(T-1)}^* + TC_{t-1,t}^C + R_t^C \\ L_{(T-1)}^* + TC_{t-1,t} + R_t \end{cases} \tag{5.15}$$

where $L^*_{(T-1)}$ is the loss until period (*T-1*), R^C_t is the risk of the layout based on projected demand considering transition cost, and R_t is the risk associated with the risk-based layout considering transition cost.

Step 4: Increment t by one.

Step 5: If t < T, go to Step 3; otherwise, go to Step 6.

Step 6: Stop the procedure.

5.4 Nine-Department Case Study (Multi-Period)

To demonstrate the use of the risk-based facility layout approach for the multi-period scenario, a case study consisting of nine departments with five products is given below. In this case study, the assessment of $E[f^{kl}_{ij}]^t$ values was conducted through a mathematical formulation method. The summary of product information is shown in Table 5.1. In addition, the following assumptions were made for this nine-department multi-period case study:

- All departments are equal in size (40 × 40).
- Process sequence of each product is known.
- Probability density functions of product demand follow uniform distribution, are of a continuous type, and are independent.
- Material handling cost is $1/unit distance.
- Distances between departments are calculated based on a centroid to centroid procedure.
- Department rearrangement cost is $30/unit distance.
- Time horizon (T) considered is 3

TABLE 5.1

PRODUCT DEMAND AND PROCESS SEQUENCE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY

Product	t = 1		t = 2		t = 3		Sequence
	Distribution (uniform)	Projected Demand	Distribution (uniform)	Projected Demand	Distribution (uniform)	Projected Demand	
1	(100, 180)	170	(100, 220)	200	(100, 220)	200	1-3-5-7-9
2	(250, 350)	280	(200, 300)	230	(125, 170)	150	1-2-7-4-6
3	(200, 370)	300	(200, 350)	280	(300, 650)	350	4-5-6
4	(30, 130)	100	(150, 200)	175	(200, 370)	330	3-5-7-8-6
5	(80, 400)	100	(100, 300)	130	(200, 600)	300	1-8

5.4.1 Nine-Department Multi-Period Case Study (t = 1)

The first step of the risk-based facility layout approach for the multi-period case was to generate the best layout based on projected demand. By using the GA detailed in Chapter 4 with the modified cost function in equation 5.12 and fitness function in equation 4.9, the best layout for projected demand considering transition cost was generated and is shown in Figure 5.1.

9	2	1
7	5	3
4	6	8

Figure 5.1. Layout based on projected demand for nine-department multi-period case study (t = 1).

Once the layout for projected demand was generated, the second step was to perform the facility layout risk assessment method. For the first period, f_{18} and f_{27} were identified as the significant flow intensity, and the associated $E[f_{ij}^{kl}]^1$ for these two flow intensity is summarized in Table 5.2. The risk-flow table was then generated and is shown in Table 5.3. The resulting risk rankings are $R^*_{f_{18}}$ and $R^*_{f_{27}}$, which correspond to \$13,067 and \$2,608, respectively.

TABLE 5.2

SUMMARY OF $E[f_{ij}^{kl}]^1$ VALUES FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t =1)

$E[f_{ij}^{kl}]^1$	**Expected Value**
$\left[\max(E[f_{18}^{kl}]^1)\right]\forall(f_{kl})^1$	163
$\left[\max(E[f_{27}^{kl}]^1)\right]\forall(f_{kl})^1$	46

TABLE 5.3

RISK-FLOW TABLE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 1)

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{12}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{27}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{45}}$	
$R_{f_{18}}^{f_{kl}}$	\$5,854	\$13,067	\$5,854	\$12,697	\$5,854	
$R_{f_{27}}^{f_{kl}}$	-	-	-	-	\$2,608	
$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{46}}$	$R_{f_{ij}}^{f_{47}}$	$R_{f_{ij}}^{f_{56}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{f_{79}}$	$R_{f_{ij}}^{*}$
$R_{f_{18}}^{f_{kl}}$	\$7,704	\$5,854	\$7,704	\$12,697	\$13,067	\$13,067
$R_{f_{27}}^{f_{kl}}$	\$2,608	-	-	-	-	\$2,608

Subsequently, to generate the risk-based layout, equation (5.7) was used to tabulate the from-between chart, which is shown in Table 5.4 and the generated layout is shown in Figure 5.2.

Based on the risk-based layout, the risk associated with flow intensity between departments 1 and 8 was reduced from \$13,067 to \$6,533, which is a 50 percent reduction in risk. The risk associated with flow intensity between departments 2 and 7 was decreased from

$2,608 to $1,847, which is an approximately 30 percent reduction. Table 5.5 summarizes the results of risk ranking and reduction offered by the risk-based layout.

TABLE 5.4

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t =1)

Dept.	1	2	3	4	5	6	7	8	9
1		326	170					263	
2							326		
3					270				
4					300	326	326		
5						300	270		
6								100	
7								100	170
8									
9									

9	8	1
4	7	2
6	5	3

Figure 5.2. Risk-based layout for nine-department multi-period case study (t = 1)

TABLE 5.5

SUMMARY OF RISK RANKING AND RISK REDUCTION FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t =1)

Flow intensity		Maximum Risk		Reduction in Material Handling Cost	Reduction
		Layout Based on Projected Demand	Risk-Based Layout		
x_5	1-8	$13,067 (1)*	$6,533	$6,533	50%
x_2	2-7	$2,608 (2)*	$1,847	$762	30%

*Number in parenthesis indicates ranking of risk.

For the material handling cost comparison, four scenarios were considered:

1. Projected demands are true.
2. $x_5 \geq x_1,\ x_5 = 263\,units.$
3. $x_2 \geq x_3, x_2 = 326\,units.$
4. $x_5 \geq x_1,\ x_5 = 263\,units$ and $x_2 \geq x_3, x_2 = 326\,units.$

Table 5.6 shows the total loss comparison of the layout based on projected demand values and the risk-based layout under defined scenarios. Results show that if all projected demands occurred, the risk-based layout would incur an additional $985 in the overall material handling cost (0.7 percent increase). On the other hand, in the event that scenario 2 (81.92 percent probability of occurrence) or 4 (49.04 percent probability of occurrence) occurred, the risk-based layout will reduce overall loss by $5,535 (4.2 percent reduction) and $6,294 (3 percent reduction), respectively.

TABLE 5.6

SUMMARY OF TOTAL LOSS COMPARISON FOR NINE-DEPARTMENT MULTI-PERIODCASE STUDY (t = 1)

Scenario	Likelihood	Loss		Increase in Material Handling Cost (*) or Decrease in Loss
		Layout Based on Projected Demand	Risk-Based Layout	
1	-	$0	$985	$985*
2	81.92%	$13,040	$7,505	$5,535
3	59.86%	$8,119	$8,345	-$226
4	49.04%	$21,159	$14,865	$6,294

When the risk-based layout (Figure 5.2) was selected, the associated total risk was $14,865. If the layout based on projected demand values was used, the associated risk was $21,155. Hence, the minimum total expected loss in period 1 (L_1^*) can be calculated using equation (5.15) as

$$L_1^* = \min \begin{cases} TC_{0,1} + R_1^C = 0 + 21{,}155 = \$21{,}155 \\ TC_{0,1} + R_1 = 0 + 14{,}865 = \$14{,}865 \end{cases}$$

Therefore, for the first period, the risk-based layout is selected and the total expected loss is $14,865.

5.4.2 Nine-Department Multi-Period Case Study (t = 2)

For the second iteration, first the layout based on projected demand values (Figure 5.3) was generated. Based on the results of risk assessment, the summary of $E[f_{ij}^{kl}]^2$ values is shown in Table 5.7, and the resulting risk-flow table is presented in Table 5.8. Results indicate that three flow intensities were exposed to risk: f_{18}, f_{27}, and f_{47}. The risk rankings are $R^*_{f_{18}}$ ($5,707), $R^*_{f_{27}}$ ($1,413), and $R^*_{f_{47}}$ ($1,413), respectively.

9	8	2
6	7	1
4	5	3

Figure 5.3. Layout based on projected demand for nine-department multi-period case study (t = 2).

TABLE 5.7

SUMMARY OF $E[f_{ij}^{kl}]^2$ VALUES FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

$E[f_{ij}^{kl}]^2$	**Expected value**
$\left[\max(E[f_{18}^{kl}]^2)\right]\forall(f_{kl})^2$	101
$\left[\max(E[f_{27}^{kl}]^2)\right]\forall(f_{kl})^2$	25
$\left[\max(E[f_{47}^{kl}]^2)\right]\forall(f_{kl})^2$	25

TABLE 5.8

RISK-FLOW TABLE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{12}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{27}}$	$R_{f_{ij}}^{f_{45}}$	$R_{f_{ij}}^{f_{46}}$
$R_{f_{18}}^{f_{kl}}$	\$1,883	\$5,707	\$1,883	\$1,413	\$1,883
$R_{f_{27}}^{f_{kl}}$	-	-	-	\$1,413	-
$R_{f_{47}}^{f_{kl}}$	-	-	-	\$1,413	-

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{47}}$	$R_{f_{ij}}^{f_{56}}$	$R_{f_{ij}}^{f_{78}}$	$R_{f_{ij}}^{f_{79}}$	$R_{f_{ij}}^{f_{86}}$	$R_{f_{ij}}^{*}$
$R_{f_{18}}^{f_{kl}}$	\$1,883	\$1,413	\$3,826	\$5,707	\$3,826	\$5,707
$R_{f_{27}}^{f_{kl}}$	-	\$1,413	-	-	-	\$1,413
$R_{f_{47}}^{f_{kl}}$	-	\$1,413	-	-	-	\$1,413

Table 5.9 is the from-between chart generated using equation (5.7) for the risk-based facility layout design (Figure 5.4).

TABLE 5.9

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		255	200					231	
2							255		
3					375				
4					280	255	255		
5						280	375		
6								175	
7								175	200
8									
9									

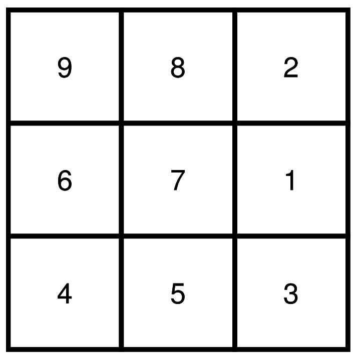


Figure 5.4. Risk-based layout for nine-department multi-period case study (t = 2).

However, in this case, the risk-based layout did not provide any risk reduction for the second period, since the risk associated with the possible changes in product demands for the second period were not large enough to cause the layout based on projected demands to lose its optimality. When the risk-based layout (Figure 5.4) was used, the associated expected loss was \$10,531 and the transition cost from period 1 to 2 was \$4,800. Thus, the total expected loss for period 2 (L_2^*) can be calculated using equation (5.15) as

$$L_2^* = \min \begin{cases} L_1^* + TC_{1,2}^C + R_2^C = 14,865 + 4,800 + 10,531 = \$30,196 \\ L_1^* + TC_{1,2} + R_2 = 14,865 + 4,800 + 10,531 = \$30,196 \end{cases}$$

5.4.3 Nine-Department Multi-Period Case Study (t = 3)

For the third period, the layout based on projected demand values is shown in Figure 5.5. Based on the results of risk assessment, the summary of $E[f_{ij}^{kl}]^3$ values is shown in Table 5.10, and the resulting risk-flow table is presented in Table 5.11. Results indicate that three flow intensities were exposed to risk: f_{18}, f_{56}, and f_{86}. The risk rankings are $R_{f_{56}}^*$ ($13,929), $R_{f_{18}}^*$ ($13,499), and $R_{f_{86}}^*$ ($2,831), respectively.

9	8	2
6	7	1
4	5	3

Figure 5.5. Layout based on projected demand for nine-department multi-period case study(t = 3).

TABLE 5.10

SUMMARY OF $E[f_{ij}^{kl}]^3$ VALUES FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

$E[f_{ij}^{kl}]^3$	**Expected value**
$\left[\max(E[f_{18}^{kl}]^3)\right]\forall(f_{kl})^3$	169
$\left[\max(E[f_{56}^{kl}]^3)\right]\forall(f_{kl})^3$	247
$\left[\max(E[f_{86}^{kl}]^3)\right]\forall(f_{kl})^3$	50

TABLE 5.11

RISK-FLOW TABLE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{45}}$	$R_{f_{ij}}^{f_{56}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{f_{68}}$	$R_{f_{ij}}^{f_{78}}$	$R_{f_{ij}}^{*}$
$R_{f_{18}}^{f_{kl}}$	$13,499	$2,707	$2,707	$13,499	$7,178	$7,178	$13,499
$R_{f_{56}}^{f_{kl}}$	$13,929	-	-	$13,929	-	-	$13,929
$R_{f_{86}}^{f_{kl}}$	-	$2,831	$2,831	-	-	-	$2,831

The from-between chart generated using equation (5.7) for the risk-based layout is shown in Table 5.12. To minimize the risk, the risk-based layout was generated and is shown in Figure 5.6.

TABLE 5.12

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		150	200					469	
2							150		
3					530				
4					597	150	150		
5						597	530		
6								380	
7								380	200
8									
9									

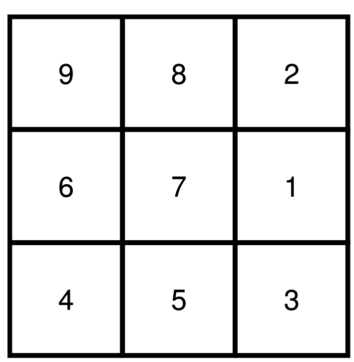


Figure 5.6. Risk-Based layout for nine-department multi-period case study (t = 3).

The risk-based layout did not provide any risk reduction, since the obtained risk-based layout had the same configuration as the layout based on projected demand. The total expected loss for this iteration (L_3^*) which includes losses up to period 3 and the loss during transition from period 2 to period 3 can be calculated using equation (5.15) for both risk-based layout and layout based on projected demand values:

$$L_3^* = \min \begin{cases} L_2^* + TC_{2,3}^C + R_3^C = 30,196 + 0 + 38,209 = \$68,405 \\ L_2^* + TC_{2,3} + R_3 = 30,196 + 0 + 38,209 = \$68,405 \end{cases}$$

Therefore, for this period, the layout based on projected demand, which is also the same as the layout selected in period 2 is chosen.

5.5 Sixteen-Department Case Study (Multi-Period)

For the sixteen-department multi-period case study, the same assumptions used in the nine-department multi-period case study were applied. However, the distances between departments were calculated using the rectilinear method. In addition, the assessment of $E[f_{ij}^{kl}]^t$ values was conducted via simulation method.

5.5.1 Sixteen-Department Multi-Period Case Study (t = 1)

For the first iteration, results of the sixteen-department single-period case study shown in Chapter 4 were used. When the generated risk-based layout (Figure 4.6) was used, the total expected loss associated with this layout was $99,719. Hence, the minimum total expected loss in period 1 (L_1^*) was used. This loss can be calculated as using equation (5.15) as

$$L_1^* = TC_{0,1} + R_1 = 0 + 99,719 = \$99,719$$

5.5.2 Sixteen-Department Multi-Period Case Study (t = 2)

A summary of the case study setting for the sixteen-department case for the second period is presented in Tables 5.13 and 5.14, and the best layout based on projected demand values considering transition cost is shown in Figure 5.7.

TABLE 5.13

PRODUCT DEMAND AND PROCESS SEQUENCE FOR SIXTEEN-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

Product	Product Demand Distribution	Projected Demand	Process Sequence
1	Normal (140, 10)	150	1-3-5-7-9-11-13-15
2	Normal (260, 20)	280	2-4-6-8-10-12-14-16
3	Uniform (200, 310)	240	7-9-11-13
4	Uniform (200, 300)	220	1-3-5-7-9
5	Uniform (150, 400)	200	1-2-3-4-5-6-10-14-15

TABLE 5.14

FROM-BETWEEN CHART FOR SIXTEEN-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		200	370													
2			200	280												
3				200	370											
4					200	280										
5						200	370									
6								280		200						
7									610							
8										280						
9											390					
10												280		200		
11													390			
12														280		
13															150	
14															200	280
15																
16																

7	5	4	2
9	6	3	1
11	8	10	12
13	15	14	16

Figure 5.7. Best layout based on projected demand values for sixteen-department multi-period case study (t =2).

In this case study, a total of twenty two flow intensities were considered in the second period. By checking the risk conditions, among these flow intensities, five flow intensities were at risk (f_{35} , f_{46} , $f_{12,14}$, f_{23} ,and $f_{6,10}$). A summary of corresponding $E[f_{ij}^{kl}]^2$ values is shown in Table 5.15, and the risk-flow table is shown in Table 5.16.

TABLE 5.15

SUMMARY OF $E[f_{ij}^{kl}]^2$ VALUES FOR SIXTEEN-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

$E[f_{ij}^{kl}]^2$	**Expected values**
$\left[\max(E[f_{35}^{kl}]^2)\right]\forall(f_{kl})^2$	25
$\left[\max(E[f_{46}^{kl}]^2)\right]\forall(f_{kl})^2$	0
$\left[\max(E[f_{12.14}^{kl}]^2)\right]\forall(f_{kl})^2$	0
$\left[\max(E[f_{23}^{kl}]^2)\right]\forall(f_{kl})^2$	85
$\left[\max(E[f_{6,10}^{kl}]^2)\right]\forall(f_{kl})^2$	85

TABLE 5.16

RISK-FLOW TABLE FOR SIXTEEN-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{24}}$	$R_{f_{ij}}^{f_{46}}$	$R_{f_{ij}}^{f_{68}}$	$R_{f_{ij}}^{f_{8,10}}$	$R_{f_{ij}}^{f_{10,12}}$	$R_{f_{ij}}^{f_{12,14}}$	
$R_{f_{35}}^{f_{kl}}$	-	-	-	-	-	-	
$R_{f_{23}}^{f_{kl}}$	$6,798	$6,798	$6,798	$6,798	$6,798	$6,798	
$R_{f_{6,10}}^{f_{kl}}$	$6,798	$6,798	$6,798	$6,798	$6,798	$6,798	
$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{14,16}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{f_{9,11}}$	$R_{f_{ij}}^{f_{11,13}}$	$R_{f_{ij}}^{*}$
$R_{f_{35}}^{f_{kl}}$	-	-	-	-	$1,939	$1,939	$1,939
$R_{f_{23}}^{f_{kl}}$	$6,798	$1,123	$1,123	$1,123	$1,134	$1,134	$6,798
$R_{f_{6,10}}^{f_{kl}}$	$6,798	$1,123	$1,123	$1,123	$1,134	$1,134	$6,798

Table 5.17 shows the from-between chart generated using equation (5.7) for the risk-based layout (Figure 5.8).

The risk-based layout did not provide any risk reduction for the second period, since the risk associated with the possible changes in product demands for the second period were not large enough to cause the layout based on projected demands to loss its optimality. The total expected loss associated with this risk-based layout (Figure 5.8) was $37,440, and the transition cost from period 1 to 2 was $4,800. Hence, the minimum total expected loss in period 2, given that the risk-based layout was used (L_2^*), was calculated using equation (5.15) as

$$L_2^* = \min \begin{cases} L_1^* + TC_{1,2}^C + R_2^C = 99,719 + 4,800 + 37,440 = \$141,959 \\ L_1^* + TC_{1,2} + R_2 = 99,719 + 4,800 + 37,440 = \$141,959 \end{cases}$$

TABLE 5.17

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR SIXTEEN-DEPARTMENT MULTI-PERIODCASE STUDY (t = 2)

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		285	395													
2			285	280												
3				285	395											
4					285	280										
5						285	395									
6								280		285						
7									610							
8										280						
9											390					
10												280		285		
11													390			
12														280		
13															150	
14															285	280
15																
16																

7	5	4	2
9	6	3	1
11	8	10	12
13	15	14	16

Figure 5.8. Risk-based layout for sixteen-department multi-period case study (t = 2).

5.5.3 Sixteen-Department Multi-Period Case Study (t = 3)

For the third period, the summary of the case study settings are presented in Table 5.18 and 5.19 and the best layout based on projected demand values is shown in Figure 5.9.

TABLE 5.18

PRODUCT DEMAND AND PROCESS SEQUENCE FOR SIXTEEN-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

Product	Product Demand Distribution	Projected Demand	Process Sequence
1	Normal (60, 5)	60	1-3-5-7-9-11-13-15
2	Normal (300, 20)	290	2-4-6-8-10-12-14-16
3	Uniform (100, 250)	155	7-9-11-13
4	Uniform (200, 250)	210	1-3-5-7-9
5	Uniform (100, 500)	150	1-2-3-4-5-6-10-14-15

TABLE 5.19

FROM-BETWEEN CHART FOR SIXTEEN-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		150	270													
2			150	290												
3				150	270											
4					150	290										
5						150	270									
6								290		150						
7									425							
8										290						
9											215					
10												290		150		
11													215			
12														290		
13															60	
14															150	290
15																
16																

7	5	4	2
9	6	3	1
11	8	10	12
13	15	14	16

Figure 5.9. Best layout based on projected demand values for sixteen-department multi-period case study (t =3).

A total of twenty two flow intensities were considered. By checking the risk conditions, among these flow intensities, five flow intensities were at risk (f_{35} , f_{46} , $f_{12,14}$, f_{23} ,and $f_{6,10}$). A summary of corresponding $E[f_{ij}^{kl}]^3$ values is shown in Table 5.20, and the risk-flow table is shown in Table 5.21.

TABLE 5.20

SUMMARY OF $E[f_{ij}^{kl}]^3$ VALUES FOR SIXTEEN-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

$E[f_{ij}^{kl}]^3$	**Expected values**
$\left[\max(E[f_{35}^{kl}]^3)\right]\forall(f_{kl})^3$	10
$\left[\max(E[f_{46}^{kl}]^3)\right]\forall(f_{kl})^3$	0
$\left[\max(E[f_{12,14}^{kl}]^3)\right]\forall(f_{kl})^3$	0
$\left[\max(E[f_{23}^{kl}]^3)\right]\forall(f_{kl})^3$	141
$\left[\max(E[f_{6,10}^{kl}]^3)\right]\forall(f_{kl})^3$	141

TABLE 5.21

RISK-FLOW TABLE FOR SIXTEEN-DEPARTMENT
MULTI-PERIOD CASE STUDY (t = 3)

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{24}}$	$R_{f_{ij}}^{f_{46}}$	$R_{f_{ij}}^{f_{68}}$	$R_{f_{ij}}^{f_{8,10}}$	$R_{f_{ij}}^{f_{10,12}}$	$R_{f_{ij}}^{f_{12,14}}$	$R_{f_{ij}}^{f_{14,16}}$
$R_{f_{35}}^{f_{kl}}$	$800	$800	$800	$800	$800	$800	$800
$R_{f_{23}}^{f_{kl}}$	$9,680	$9,680	$9,680	$9,680	$9,680	$9,680	$9,680
$R_{f_{6,10}}^{f_{kl}}$	$9,680	$9,680	$9,680	$9,680	$9,680	$9,680	$9,680
$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{f_{79}}$	$R_{f_{ij}}^{f_{9,11}}$	$R_{f_{ij}}^{f_{11,13}}$	$R_{f_{ij}}^{*}$
$R_{f_{35}}^{f_{kl}}$	-	-	-	-	-	-	$800
$R_{f_{23}}^{f_{kl}}$	$9,840	$9,840	$9,840	$3,280	$11,280	$11,280	$11,280
$R_{f_{6,10}}^{f_{kl}}$	$9,840	$9,840	$9,840	$3,280	$11,280	$11,280	$11,280

Table 5.22 shows the from-between chart generated using equation (5.7) for the risk-based layout (Figure 5.10).

TABLE 5.22

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR SIXTEEN-DEPARTMENT
MULTI-PERIOD CASE STUDY (t = 3)

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1		291	280													
2			291	290												
3				291	280											
4					291	290										
5						291	290									
6								290		291						
7									425							
8										290						
9											215					
10												290		291		
11													215			
12														290		
13															60	
14															291	290
15																
16																

7	5	4	2
9	6	3	1
11	8	10	12
13	15	14	16

Figure 5.10. Risk-based layout for sixteen-department multi-period case study (t = 3).

However, in this case study, the risk-based layout did not provide any risk reduction for the third period, since the risk associated with the possible changes in product demands for the third period were not large enough to cause the layout based on projected demands to lose its optimality. The total expected loss associated with this risk-based layout was $58,000, and the transition cost from period 2 to 3 was zero since the generated risk-based layout for period 3 was the same as that of period 2. Hence, the minimum total expected loss in period 3 given that the risk-based layout was used (L_3^*) can be calculated using equation (5.12) as

$$L_3^* = \min \begin{cases} L_2^* + TC_{2,3}^C + R_3^C = 141,959 + 0 + 58,000 = \$199,959 \\ L_2^* + TC_{2,3} + R_3 = 141,959 + 0 + 58,000 = \$199,959 \end{cases}$$

Therefore, the layout used in period two was used for period three as well.

5.6 Twenty-Five-Department Case Study (Multi-Period)

For the twenty-five-department multi-period case study, the same assumptions used in the sixteen-department multi-period case study were applied. In addition, similar to the sixteen-department case study, the simulation method was also used to obtain the required $E[f_{ij}^{kl}]^t$ values.

5.6.1 Twenty-Five-Department Multi-Period Case Study (t = 1)

For the first iteration of the twenty-five-department multi-period case study, the result of twenty-five-department single-period case study, shown in Chapter 4 was used. When the generated risk-based layout (Figure 4.8) was used, the total expected loss associated with this layout was $275,360. Hence, the minimum total expected loss in period 1 is used and L_1^* can be calculated using equation (5.12) as

$$L_1^* = TC_{0,1} + R_1 = 0 + 275,360 = \$275,360$$

5.6.2 Twenty Five-Department Multi-Period Case Study (t = 2)

A summary of the case study setting for the twenty-five-department multi-period case for the second period is presented in Table 5.23 and the best layout based on projected demand values is shown in Figure 5.11. The from-between chart for this case study can be found in Appendix F.

TABLE 5.23

PRODUCT DEMAND AND PROCESS SEQUENCE FOR TWENTY-FIVE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

Product	Product Demand Distribution	Projected Demand	Process Sequence
1	Normal (140, 30)	150	1-9-3-7-5-11-19-13-17-15-21-25-23
2	Normal (100, 15)	105	2-10-4-8-6-12-20-14-18-16-24-22
3	Uniform (100, 400)	150	5-11-13-19-21
4	Uniform (40, 120)	60	6-12-14-16-18
5	Uniform (200, 500)	300	1-2-3-4-5--6-7-8-9-10-11-12-13
6	Triangular (40, 200, 500)	120	13-19-21-6-12-14
7	Triangular (100, 170, 500)	110	20-21-22-23-24-25

17	21	25	23	22
15	13	19	24	1
14	20	10	8	2
16	12	11	9	3
18	6	5	7	4

Figure 5.11. Layout based on projected demand values for twenty-five-department multi-period case study (t =2).

Based on the case study setting, a total of forty five flow intensities were considered. Among these flow intensities, fifteen flow intensities were at risk. The corresponding $E[f_{ij}^{kl}]^2$ of these flow intensities are summarized in Table 5.24 and the risk-flow table can be found in Appendix G. The from-between chart generated using equation (5.7) for the risk-based layout can be found in Appendix H, and the risk-based layout is shown in Figure 5.12.

17	15	25	24	16
19	21	22	23	18
13	14	20	4	2
12	10	6	5	3
11	9	8	7	1

Figure 5.12. Risk-based layout for twenty-five-department multi-period case study (t = 2).

TABLE 5.24

SUMMARY OF $E[f_{ij}^{kl}]^2$ VALUES FOR TWENTY-FIVE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

$E[f_{ij}^{kl}]^2$	**Expected Value**	$E[f_{ij}^{kl}]^2$	**Expected Value**
$\left[\max(E[f_{19}^{kl}]^2)\right]\forall(f_{kl})^2$	7	$\left[\max(E[f_{11,13}^{kl}]^2)\right]\forall(f_{kl})^2$	115
$\left[\max(E[f_{37}^{kl}]^2)\right]\forall(f_{kl})^2$	7	$\left[\max(E[f_{45}^{kl}]^2)\right]\forall(f_{kl})^2$	34
$\left[\max(E[f_{11,19}^{kl}]^2)\right]\forall(f_{kl})^2$	7	$\left[\max(E[f_{67}^{kl}]^2)\right]\forall(f_{kl})^2$	34
$\left[\max(E[f_{13,17}^{kl}]^2)\right]\forall(f_{kl})^2$	7	$\left[\max(E[f_{78}^{kl}]^2)\right]\forall(f_{kl})^2$	34
$\left[\max(E[f_{15,21}^{kl}]^2)\right]\forall(f_{kl})^2$	7	$\left[\max(E[f_{9,10}^{kl}]^2)\right]\forall(f_{kl})^2$	34
$\left[\max(E[f_{2,10}^{kl}]^2)\right]\forall(f_{kl})^2$	8	$\left[\max(E[f_{12,13}^{kl}]^2)\right]\forall(f_{kl})^2$	34
$\left[\max(E[f_{10,4}^{kl}]^2)\right]\forall(f_{kl})^2$	8	$\left[\max(E[f_{21,6}^{kl}]^2)\right]\forall(f_{kl})^2$	138
$\left[\max(E[f_{48}^{kl}]^2)\right]\forall(f_{kl})^2$	8	$\left[\max(E[f_{20,21}^{kl}]^2)\right]\forall(f_{kl})^2$	156
$\left[\max(E[f_{86}^{kl}]^2)\right]\forall(f_{kl})^2$	8	$\left[\max(E[f_{21,22}^{kl}]^2)\right]\forall(f_{kl})^2$	156
$\left[\max(E[f_{14,18}^{kl}]^2)\right]\forall(f_{kl})^2$	8	$\left[\max(E[f_{24,25}^{kl}]^2)\right]\forall(f_{kl})^2$	156
$\left[\max(E[f_{16,24}^{kl}]^2)\right]\forall(f_{kl})^2$	8	$\left[\max(E[f_{12,14}^{kl}]^2)\right]\forall(f_{kl})^2$	192
$\left[\max(E[f_{24,22}^{kl}]^2)\right]\forall(f_{kl})^2$	8	$\left[\max(E[f_{19,21}^{kl}]^2)\right]\forall(f_{kl})^2$	94

Table 5.27 shows a summary of risk reduction and risk ranking. Based on the results, the risk-based layout provides risk reduction for thirteen out of twenty-two flow intensities. However, three flow intensities show an increase in risk. The maximum risk reduction provided by the risk-based layout is 75 percent. On the other hand, the maximum increase in risk is 100 percent. In general, the risk-based layout provides a 31 percent reduction in overall risk.

TABLE 5.25

SUMMARY OF RISK RANKING AND RISK REDUCTION FOR TWENTY-FIVE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

Flow Intensity		Maximum Risk		Reduction in Material Handling Cost	Reduction
		Layout Based on Projected Demand	Risk-Based Layout		
x_1	1-9	$840 (20)*	$840	$0	0%
	3-7	$560 (21)*	$560	$0	0%
	11-19	$560 (22)*	$840	-$280	-50%
	13-17	$560 (23)*	$560	$0	0%
	15-21	$560 (24)*	$280	$280	50%
x_2	2-10	$8,880 (11)*	$17,760	-$8,880	-100%
	10-4	$17,760 (4)*	$13,320	$4,440	25%
	4-8	$13,320 (8)*	$13,320	$0	0%
	8-6	$17,760 (5)*	$4,440	$13,320	75%
	14-18	$8,880 (12)*	$17,760	-$8,880	-100%
	16-24	$22,200 (1)*	$4,440	$17,760	80%
	24-22	$8,880 (13)*	$8,880	$0	0%
x_3	11-13	$13,800 (7)*	$9,200	$4,600	33%
x_5	4-5	$2,720 (15)*	$1,360	$1,360	50%
	6-7	$2,720 (16)*	$2,720	$0	0%
	7-8	$2,720 (17)*	$1,360	$1,360	50%
	9-10	$2,720 (18)*	$1,360	$1,360	50%
	12-13	$2,720 (19)*	$1,360	$1,360	50%
x_6	21-6	$22,080 (2)*	$16,560	$5,520	25%
x_7	20-21	$12,480 (9)*	$12,480	$0	0%
	21-22	$18,720 (3)*	$6,240	$12,480	67%
	24-25	$12,480 (10)*	$6,240	$6,240	50%
$x_4 + x_6$	12-14	$15,360 (6)*	$15,360	$0	0%
$x_3 + x_6$	19-21	$7,520 (14)*	$3,760	$3,760	50%

*Number in parenthesis indicates ranking of risk.

For the material handling cost comparison, seven scenarios are considered:

1. Projected demand values are true.
2. $x_1 > x_2 + x_4,\ x_1 = 157\, units.$
3. $x_2 > x_1,\ x_2 = 113\, units.$
4. $x_5 > x_1 + x_3,\ x_5 = 334\, units.$
5. $x_7 > x_1, x_7 = 266\, units.$

6. $x_3 > x_1, x_3 = 115\,units$, $x_6 > x_1, x_6 = 138\,units$, $x_3 + x_6 > x_2 + x_4 + x_6, x_3 + x_6 = 364\,units$ and

$x_4 + x_6 > x_1 + x_3, x_4 + x_6 = 372\,units.$

7. Scenarios 2, 3, 4, 5, and 6 are true.

Table 5.26 shows a summary of the material handling cost and loss comparison results. Results show that in the case where all projected demands are true, the risk-based layout will incur an additional \$5,400 in loss. However, for all of the risk scenarios (0.14 percent probability of occurrence), risk-based layout helps to reduce the overall loss by \$27,800 (18 percent reduction). The risk-based layout performs worst when scenario 2 occurs (16.16 percent probability of occurrence); this will incur an additional loss of \$6,800 when compared with the best layout generated based on projected demand values. It performs best when scenario 5 occurs (90.8 percent probability of occurrence), which reduces the loss by \$13,320 when compared with the best layout generated based on projected demand values.

TABLE 5.26

SUMMARY OF TOTAL LOSS COMPARISON FOR TWENTY-FIVE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

Scenario	Likelihood	Loss		Increase in Material Handling Cost (*) or Decrease in Loss
		Layout Based on Projected Demand	Risk-Based Layout	
1	-	0	5,400	5,400*
2	16.16%	4,480	11,280	-6,800
3	11.1%	7,680	12,440	-4,760
4	38.36%	23,120	27,160	-4,040
5	90.8%	56,160	42,840	13,320
6	21.92%	58,760	50,280	8,480
7	0.14%	150,200	122400	27,800

The total expected loss for this iteration (L_2^*), which included losses up to period 2 and the loss during transition from period 1 to period 2 was calculated using equation (5.15) for both risk-based layout and layout based on projected demand values, and it is as follows:

$$L_2^* = \min \begin{cases} L_1^* + TC_{1,2}^C + R_2^C = 275,360 + 64,800 + 118,400 = \$458,560 \\ L_1^* + TC_{1,2} + R_2 = 275,360 + 38,400 + 117,000 = \$430,760 \end{cases}$$

Therefore, for this second period, the risk-based layout was selected.

5.6.3 Twenty Five-Department Multi-Period Case Study (t = 3)

A summary of the case study setting for the twenty-five-department case for the third period is presented in Table 5.27 and the best layout based on projected demand values is shown in Figure 5.13. The from-between chart for this case study can be found in Appendix I.

TABLE 5.27

PRODUCT DEMAND AND PROCESS SEQUENCE FOR TWENTY-FIVE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

Product	Product Demand Distribution	Projected Demand	Process Sequence
1	Triangular (25, 30, 40)	32	1-9-3-7-5-11-19-13-17-15-21-25-23
2	Uniform (110, 140)	138	2-10-4-8-6-12-20-14-18-16-24-22
3	Triangular (310, 320, 350)	335	5-11-13-19-21
4	Uniform (150, 180)	178	6-12-14-16-18
5	Uniform (50, 100)	63	1-2-3-4-5--6-7-8-9-10-11-12-13
6	Uniform (400, 590)	420	13-19-21-6-12-14
7	Triangular (200, 250, 300)	295	20-21-22-23-24-25

17	15	24	23	16
2	1	25	22	18
10	9	19	21	20
3	11	13	12	14
4	5	8	6	7

Figure 5.13. Layout based on projected demand values for twenty-five-department multi-period case study (t =3).

Once the facility layout risk assessment method was performed, results indicated that there was no risk associated with the layout based on projected demand. Therefore, this layout was used in period three and its associated transition cost was \$66,000 and the total expected loss was calculated as

$$L_3^* = L_2^* + TC_{2,3} + R_3 = 430,760 + 66,000 + 0 = \$496,760$$

5.7 Conclusion

In this chapter, the risk-based facility layout approach for the multi-period problem was presented. This approach was designed specifically for the situation where the transition cost was at the medium range. Both transition cost and risk reduction were taken into consideration in the method in order to generate the layout, which balanced the tradeoff between the two. This essentially led to minimizing the total expected loss for all periods under consideration. Results from the case studies confirmed that the method was effective in reducing the total expected loss.

CHAPTER 6

ADDITIONAL CASE STUDIES

6.1 Introduction

In the previous two chapters, the risk-based facility layout approach for single period and multi-period problem were presented along with numerous case studies. In those case studies, the projected demand values were taken to be different from the expected demand. This chapter presents additional case studies that consider the projected demand equal to expected demand. Triangular distribution is utilized to represent the projected demand uncertainty. Since triangular distribution requires the minimum, most likely, and maximum values to define the distribution, it can be easily used to represent the projected demand uncertainty when the actual projected demand distribution is unknown. The following sections demonstrate single-period and multi-period case studies.

6.2 Single-Period Case Study

Based on the data provided in Table 6.1 and 6.2 for the nine-department single-period case study, the best layout based on the projected demand values is shown in Figure 6.1. A total of thirteen flow intensities were considered (f_{12}, f_{13}, f_{18}, f_{27}, f_{35}, f_{45}, f_{46}, f_{47}, f_{56}, f_{57}, f_{68}, f_{78}, *and* f_{79}) in the from-between chart in Table 6.2.

TABLE 6.1

PRODUCT DEMAND AND PROCESS SEQUENCE FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY

Product	Product Demand Distribution	Most likely	Product Sequence
1	Triangular (100, 150, 200)	150	1-3-5-7-9
2	Triangular (275, 280, 320)	280	1-2-7-4-6
3	Triangular (200, 240, 310)	240	4-5-6
4	Triangular (40, 60, 120)	60	3-5-7-8-6
5	Triangular (40, 50, 400)	50	1-8

TABLE 6.2

FROM-BETWEEN CHART FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		280	150					50	
2							280		
3					210				
4					240	280	280		
5						240	210		
6								60	
7								60	150
8									
9									

6	5	3
4	7	1
8	9	2

Figure 6.1. Best layout based on projected demand for nine-department single-period case study.

In this case study, only f_{18} and f_{45} were exposed to risk. Table 6.3 shows a summary of the calculated $E[f_{ij}^{kl}]$ values. The remaining $R_{f_{ij}}^{f_{kl}}$ values were calculated and are presented in the risk-flow table (Table 6.4). Table 6.5 is the input from-between chart generated using equation (4.8) for risk-based layout generation.

TABLE 6.3

SUMMARY OF $E[f_{ij}^{kl}]$ VALUES FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY

$E[f_{ij}^{kl}]$	**Expected value**
$\left[\max(E[f_{18}^{kl}])\right] \forall f_{kl}$	157
$\left[\max(E[f_{45}^{kl}])\right] \forall f_{kl}$	6

TABLE 6.4

RISK-FLOW TABLE FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{13}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{f_{79}}$	$R_{f_{ij}}^{f_{12}}$	$R_{f_{ij}}^{f_{27}}$
$R_{f_{45}}^{f_{kl}}$	-	-	-	-	\$283	\$283
$R_{f_{18}}^{f_{kl}}$	\$8,234			\$8,234	\$2,238	\$2,238

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{74}}$	$R_{f_{ij}}^{f_{46}}$	$R_{f_{ij}}^{f_{45}}$	$R_{f_{ij}}^{f_{56}}$	$R_{f_{ij}}^{f_{78}}$	$R_{f_{ij}}^{f_{86}}$	$R_{f_{ij}}^{*}$
$R_{f_{45}}^{f_{kl}}$	\$283	\$283	-	-	-	-	\$283
$R_{f_{18}}^{f_{kl}}$	\$2,238	\$2,238	\$3,849	\$3,849	\$9,129	\$9,129	\$9,129

TABLE 6.5

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		280	150					152	
2							280		
3					210				
4					245	280	280		
5						245	210		
6								60	
7								60	150
8									
9									

Utilizing information presented in Table 6.5, the facility layout design that minimizes risk was developed. For the layout generation, the GA approach discussed in the previous chapters was used. The layout that minimizes risk is presented in Figure 6.2. The resulting $R^*_{f_{18}}$ value was \$4,080, which is a 55 percent reduction in risk. However, $R^*_{f_{45}}$ remained the same.

6	4	9
5	7	2
3	8	1

Figure 6.2. Risk-based layout for nine-department single-period case study.

In this case study, f_{18} and f_{45} were identified as significant flows, which had the highest risk to the layout, and they corresponded to $R^*_{f_{18}}$ and $R^*_{f_{45}}$, respectively. For the analysis of the performance of the layout designs with respect to material handling costs, four scenarios which result in maximum risks were:

1. Projected demands are true.
2. $x_3 \geq x_2, x_3 = 245\, units.$
3. $x_5 \geq x_4, x_5 = 152\, units.$
4. $x_5 \geq x_1 + x_4,\ x_5 = 152\, units$ and $x_3 \geq x_2,\ x_3 = 245\, units.$

The analysis was performed with respect to the material handling costs for layouts based on projected demand values and risk-based layout. Table 6.6 shows the material handling cost comparison of the layout based on projected demand values and the risk-based layout for these scenarios. Based on the results, given that the projected demand values are true, the risk-based layout incurred an additional $941 in overall material handling cost (0.8 percent increase). For scenario 2 (9.5 percent probability of occurrence), the risk-based layout also incurred an additional cost of $941. On the other hand, for both scenarios 3 (84.84 percent probability of occurrence) and 4 (8.1 percent probability of occurrence), the risk-based layout resulted in a reduction of material handling cost by $4,108 (5.3 percent reduction).

TABLE 6.6

SUMMARY OF MATERIAL HANDLING COST COMPARISON FOR NINE-DEPARTMENT SINGLE-PERIOD CASE STUDY

Scenario	Likelihood	Material Handling Cost		Increase in Material Handling Cost
		Layout Based on Projected Demand	Risk-Based Layout	
1	-	$114,103	$115,044	$941
2	9.5%	$114,586	$115,527	$941
3	84.84%	$123,232	$119,124	-$4,108
4	8.1%	$123,715	$119,607	-$4,108

6.3 Multi-Period Case Study

For the multi-period case study, the assumptions used in section 5.4 were also applied. The summary of product information for the first period is shown in Table 6.7.

TABLE 6.7

PRODUCT DEMAND AND PROCESS SEQUENCE FORNINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 1)

Product	Product Demand Distribution	Most likely	Product Sequence
1	Triangular (100, 170, 180)	170	1-3-5-7-9
2	Triangular (250, 280, 350)	280	1-2-7-4-6
3	Triangular (200, 300, 370)	300	4-5-6
4	Triangular (30, 100, 130)	100	3-5-7-8-6
5	Triangular (80, 100, 400)	100	1-8

The best layout based on projected demand generated using the GA detailed in Chapter 4 with the modified cost function in equation 5.12 and fitness function in equation (4.9) was generated and is shown in Figure 6.3

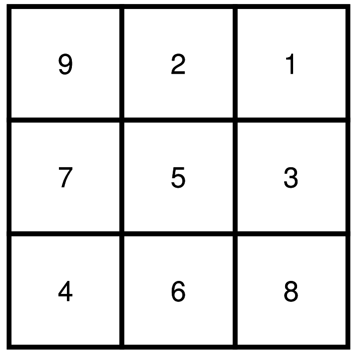


Figure 6.3. Layout based on projected demand for nine-department multi-period case study (t = 1).

Once the layout for projected demand was generated, the second step was to perform the facility layout risk assessment method. For the first period, f_{18} and f_{27} were identified as the significant flow intensities, and the associated $E[f_{ij}^{kl}]^1$ for these two flow intensities is summarized in Table 6.8. Then risk-flow table is generated, which is presented in Table 6.9. The resulting risk rankings, $R^*_{f_{18}}$ and $R^*_{f_{27}}$, correspond to \$8,560 and \$1,358, respectively.

TABLE 6.8

SUMMARY OF $E[f_{ij}^{kl}]^1$ VALUES FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t =1)

$E[f_{ij}^{kl}]^1$	**Expected value**
$\left[\max(E[f_{18}^{kl}]^1)\right]\forall(f_{kl})^1$	163
$\left[\max(E[f_{27}^{kl}]^1)\right]\forall(f_{kl})^1$	46

TABLE 6.9

RISK-FLOW TABLE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 1)

$R^{f_{kl}}_{f_{ij}}$	$R^{f_{13}}_{f_{ij}}$	$R^{f_{35}}_{f_{ij}}$	$R^{f_{57}}_{f_{ij}}$	$R^{f_{79}}_{f_{ij}}$	$R^{f_{12}}_{f_{ij}}$	$R^{f_{27}}_{f_{ij}}$	
$R^{f_{kl}}_{f_{27}}$	-	-	-	-	-	-	
$R^{f_{kl}}_{f_{18}}$	\$8,080	\$5,200	\$5,200	\$8,080	\$2,400	\$2,400	
$R^{f_{kl}}_{f_{ij}}$	$R^{f_{74}}_{f_{ij}}$	$R^{f_{46}}_{f_{ij}}$	$R^{f_{45}}_{f_{ij}}$	$R^{f_{56}}_{f_{ij}}$	$R^{f_{78}}_{f_{ij}}$	$R^{f_{86}}_{f_{ij}}$	$R^{*}_{f_{ij}}$
$R^{f_{kl}}_{f_{27}}$	-	-	\$1,358	\$1,358	-	-	\$1,358
$R^{f_{kl}}_{f_{18}}$	\$2,400	\$2,400	\$2,720	\$2,720	\$8,560	\$8,560	\$8,560

Subsequently, in order to generate the risk-based layout, equation (5.7) was used to tabulate the from-between chart, which is shown in Table 6.10, and the generated layout is shown in Figure 6.4.

TABLE 6.10

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 1)

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		304	170					207	
2							304		
3					270				
4					300	304	304		
5						300	270		
6								100	
7								100	170
8									
9									

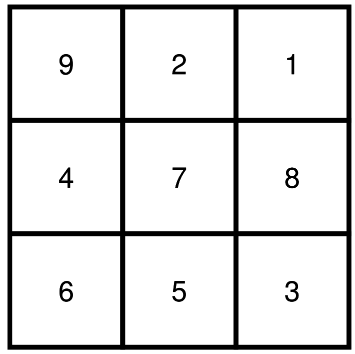


Figure 6.4. Risk-based layout for nine-department multi-period case study (t = 1).

Based on the risk-based layout, the risk associated with flow intensity between departments 1 and 8 was reduced from $8,560 to $4,280, which is a 50 percent reduction in risk. The risk associated with the flow intensity between departments 2 and 7 was decreased from $1,358 to $960, which is an approximate 30 percent reduction. Table 6.11 summarizes the results of risk ranking and risk reduction offered by the risk-based layout.

TABLE 6.11

SUMMARY OF RISK RANKING AND RISK REDUCTION FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 1)

Flow Intensity		**Maximum Risk**		**Reduction in Material Handling Cost**	**Reduction**
		Layout Based on Projected Demand	**Risk-Based Layout**		
$\mathbf{x_2}$	**2-7**	$1,358	$960	$398	29.33%
$\mathbf{x_5}$	**1-8**	$8,560	$4,280	$4,280	50.00%

*Number in parenthesis indicates ranking of risk.

For the material handling cost comparison, four scenarios were considered:

1. Projected demands are true.
2. $x_5 \geq x_1 + x_4,\ x_5 = 207\, units.$

3. $x_2 \geq x_3, x_2 = 304\, units.$

4. $x_5 \geq x_1 + x_4,\ x_5 = 207\, units$ and $x_2 \geq x_3, x_2 = 304\, units.$

Table 6.12 shows the total loss comparison of the layout based on projected demand values and the risk-based layout under defined scenarios. Results show that if all projected demands occurr, the risk-based layout will incur an additional $974 in overall material handling cost (0.7 percent increase). On the other hand, in the event that scenarios 3 (44.76 percent probability of occurrence) or 4 (42.77 percent probability of occurrence) occur, the risk-based layout will reduce overall loss by $3,306 (29 percent reduction) and $3,704 (39 percent reduction) respectively.

TABLE 6.12

SUMMARY OF TOTAL LOSS COMPARISON FOR 9-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 1)

Scenario	**Likelihood**	**Loss**		**Increase in Material Handling Cost (*) or Decrease in Loss**
		Layout Based on Projected Demand	**Risk-Based Layout**	
1	-	$0	$974	$974
2	95.56%	$4,238	$4,814	-$576
3	44.76%	$8,560	$5,254	$3,306
4	42.77%	$12,798	$9,094	$3,704

When the risk-based layout (Figure 6.4) was selected, the associated total risk was $9,094. If the layout based on projected demand values was used, the associated risk was $12,798. Hence, the minimum total expected loss in period 1 is used. L_1^* can be calculated using equation (5.15) as

$$L_1^* = \min \begin{cases} TC_{0,1} + R_1^C = 0 + 12,798 = \$12,798 \\ TC_{0,1} + R_1 = 0 + 9,094 = \$9,094 \end{cases}$$

Therefore, for the first period, the risk-based layout was selected, and the total expected loss was $9,094.

For the second iteration, the summary of product information is presented in Table 6.13, and the layout based on projected demand values is shown in Figure 6.5. Based on the results of risk assessment, the layout based on projected demand has zero risk.

TABLE 6.13

PRODUCT DEMAND AND PROCESS SEQUENCE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 2)

Product	Product Demand Distribution	Most likely	Product Sequence
1	Triangular (100, 200, 220)	200	1-3-5-7-9
2	Triangular (200, 230, 300)	230	1-2-7-4-6
3	Triangular(200,280, 350)	280	4-5-6
4	Triangular (150, 175, 200)	175	3-5-7-8-6
5	Triangular (100, 130, 300)	130	1-8

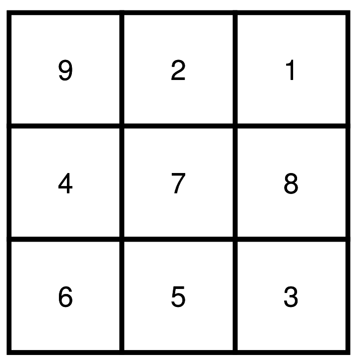


Figure 6.5. Layout based on projected demand for nine-department multi-period case study (t = 2).

In addition, the layout based on projected demand value for this period (Figure 6.5) was also the same as the layout selected in period one, hence the minimum total expected loss for period 2 was calculated as

$$L_2^* = L_1^* + TC_{1,2}^C + R_2^C = 0 + 0 + 9,094 = \$9,094$$

In the third period, the product demands and process sequence are summarized in Table 6.14 and the layout based on projected demand values is shown in Figure 6.4. Based on the results of risk assessment, the summary of $E[f_{ij}^{kl}]^3$ values is shown in Table 6.15, and the resulting risk-flow table is presented in Table 6.16. Results indicate that two flow intensities were exposed to risk: f_{56}, and f_{86}. The risk rankings were $R^*_{f_{56}}$ ($4,415) and $R^*_{f_{86}}$ ($113), respectively.

TABLE 6.14
PRODUCT DEMAND AND PROCESS SEQUENCE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

Product	Product Demand Distribution	Most likely	Product Sequence
1	Triangular (100, 200, 220)	200	1-3-5-7-9
2	Triangular (125, 150, 170)	150	1-2-7-4-6
3	Triangular (300, 350, 650)	350	4-5-6
4	Triangular (200, 330, 370)	330	3-5-7-8-6
5	Triangular (200, 300, 600)	300	1-8

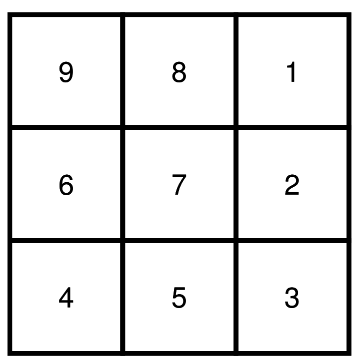


Figure 6.6. Layout based on projected demand for nine-department multi-period case study (t = 3).

TABLE 6.15

SUMMARY OF $E[f_{ij}^{kl}]^3$ VALUES FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

$E[f_{ij}^{kl}]^3$	**Expected value**
$\left[\max(E[f_{86}^{kl}]^3)\right]\forall(f_{kl})^3$	2
$\left[\max(E[f_{56}^{kl}]^3)\right]\forall(f_{kl})^3$	78

TABLE 6.16

RISK-FLOW TABLE FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

$R_{f_{ij}}^{f_{kl}}$	$R_{f_{ij}}^{f_{45}}$	$R_{f_{ij}}^{f_{56}}$	$R_{f_{ij}}^{f_{35}}$	$R_{f_{ij}}^{f_{57}}$	$R_{f_{ij}}^{*}$
$R_{f_{86}}^{f_{kl}}$	-	-	$113	$113	$113
$R_{f_{56}}^{f_{kl}}$	$4,415	$4,415	-	-	$4,415

The from-between chart generated using equation (5.7) for the risk-based layout is shown in Table 6.17. To minimize the risk, the risk-based layout was generated and is shown in Figure 6.7.

TABLE 6.17

FROM-BETWEEN CHART FOR RISK-BASED LAYOUT FOR NINE-DEPARTMENT MULTI-PERIOD CASE STUDY (t = 3)

Dept.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		150	200					300	
2							150		
3					530				
4					428	150	150		
5						428	530		
6								332	
7								332	200
8									
9									

9	8	1
6	7	2
4	5	3

Figure 6.7. Risk-based layout for nine-department multi-period case study (t = 3).

The risk-based layout did not provide any risk reduction, since the obtained risk-based layout had the same configuration as the layout based on projected demand. The total expected loss for this iteration (L_3^*), which includes losses up to period 3 and the loss during transition from period 2 to period 3 was calculated using equation (5.15) for both risk-based layout and layout based on projected demand values as

$$L_3^* = \min \begin{cases} L_2^* + TC_{2,3}^C + R_3^C = 9,094 + 5,796 + 7,728 = \$22,618 \\ L_2^* + TC_{2,3} + R_3 = 9,094 + 5,796 + 7,728 = \$22,618 \end{cases}$$

6.3 Conclusion

In this chapter, additional case studies were conducted to address the scenario where the projected demand was given in a range with the most likely value. The case studied showed that the risk-based facility layout approach can be used to address this type of projected demand as well. In addition, results of the case studies indicate that the higher the uncertainty in demand, the higher the risk of the layout. In the next chapter, the conclusion of this research and possible future work are presented.

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH

7.1 Conclusions

In the manufacturing environment where product demands are highly volatile, assessing the future product demand is one of the critical issues in the design of a facility layout. In general, facility layout design deals with allocations of machines/departments in a facility with the objective of reducing material handling cost. These allocations are determined based upon product flow intensities between pairs of departments. Flow intensity is defined as the total product flow between a pair of departments. The higher the flow intensity, the closer the departments should be placed to reduce material handling cost. Hence, a fluctuation in any of the products demand will modify the strength of relationship between departments in which the product is processed. When the change in demand is large enough to cause the flow intensity between one pair of departments to exceed or fall below the flow intensity between another pair of departments, the layout could be compromised. Forecasting is one of the most widely used techniques to project future product demands. However, forecasts are inaccurate, and therefore, the uncertainty of these projected product demands should be considered during the design stage of the facility layout.

In the past two decades, many researchers have attempted to address both the static facility layout problem (SFLP) and dynamic facility layout problem (DFLP) in a stochastic environment where the product demands are subjected to variability. In most of this research, probabilistic scenario-based methods are used. This method induces the variability or uncertainty of product demands to facility layout problem by establishing multiple production scenarios and assigning each of these scenarios with their corresponding probability of occurrence. Once all scenarios are constructed and the probability of occurrence is assigned, then the compressed flow matrix is used to combine multiple flow matrices into one single matrix in order to generate a layout that minimizes the material handling cost. With such a method, the dependency of product demand within each scenario is assumed. However, in an environment where each product demand by itself demonstrates high variability and is independent from one another, the scenario-based method may not be suitable for addressing overall uncertainty of demands. Hence, a new approach, which considers uncertainty of each product demand independently, is needed. To incorporate the uncertainty in product demand into the facility layout problem (FLP), the measure of the impact of this uncertainty is required. In this research, the impact of product demand uncertainty to facility layout was quantified in terms of risk. Therefore, the objectives of this dissertation were to investigate the models required for quantifying risk in facility layout design and develop the FLP approach for single-period and multi-period case studies, which minimize risk of the facility layout design.

To achieve the objectives of this dissertation, the facility layout risk assessment method was developed and detailed in Chapter 3. In this research, the risk of facility layout design was defined as the potential increase in material handling cost due to uncertainty in product demand. Traditionally, the material handling cost is the summation of the product of the material flow between departments, the distance between pairs of departments, and material handling cost per unit distance. The risk assessment method quantifies the product demand uncertainty by estimating the maximum expected increase in product demand volume between one pair of departments to exceed the other pair of departments. This is inspired by the fact that the placement of departments is directly driven by the total amount of material flow between them. The higher the material flow, the closer the departments should be placed. Hence, a change in the demand of any product will modify the strength of relationship between the relevant departments. Thus, if the change is large enough to cause the flow intensity between one pair of departments to exceed or fall below that of another pair of departments, the layout could be compromised, which subsequently results in an increase in material handling cost. In Chapter 3, two methods of estimating this expected increase in product demand were proposed: (1) The mathematical formulation method and (2) The simulation method. Once the expected increase in product demand was obtained, the risk of facility layout can be quantified.

With the estimated risk of layout, the FLP approach, which attempt to minimize the risk of the layout called the risk-based facility layout approach, is presented in Chapter 4. And as with any FLPs, the problem is formulated as a quadratic assignment problem (QAP), which is considered an NP-hard problem. To provide an alternative solution approach, a genetic algorithm (GA) was developed. A dynamically adjusted fitness function was used in the GA procedure. Based on the experimental work, results show that using the dynamic fitness function improved the speed and the solutions obtained from the GA procedure. Numerous case studies were conducted, and results show that the risk assessment procedure was beneficial in estimating the risk of the facility layout, while the layout generation procedure reduced the overall risk of the generated layout. Several case studies were tested, and a risk reduction of up to 68 percent was observed.

To further enhance the range of application of the risk-based facility layout approach, for a multi-period case study was detailed in Chapter 5. Typically, three situations occur where the transition cost is applied: (1) Negligible transition cost, (2) Medium transition cost and (3) High transition cost. The medium transition cost was the focus of this study. To apply the transition cost in the risk-based facility layout approach, the tradeoff between risk reduction and cost of rearranging the machine between periods was analyzed. The objective of the risk-based approach was to minimize the risk of the layout while considering the transition cost into consideration. A layout that balances both risk and transition cost is desired. As in the single-period case, the GA procedure that minimized the loss (sum of transition cost and risk) was used for multi-period problems. In the multi-period scenario, it was observed that by reducing risk in the first period, the changes in layout required in future periods could be reduced.

7.2 Discussion

The numerous case studies tested confirmed that the risk-based layout is an effective FLP approach to reduce the risk of the layout. In many cases, risk reduction can be as high as 80 percent for specific flow intensities. However, in certain cases, although the results of the facility layout risk assessment method indicated that the risk of the layout was high, the layout based on the projected demand value still remained optimal. In the risk-based facility layout approach, the expected increase for a flow intensity to exceed the other was added to its corresponding projected demand value in the from-between chart to generate the layout that minimized risk. However, with this new from-between chart, if the change in all flow intensities move in the same direction, and the magnitude and increases are proportional, then the generated risk-based layout will be the same as the layout based on projected demand. This situation can often occur in the multi-period cases also. Given that the changes in projected demand values in the from-between chart for the previous and current periods moved in the same direction and with proportional increases in magnitude, the layout for the previous period was a best layout based on the projected demand values for the current period as well.

7.3 Future Research

The risk-based facility layout approach was proven to be an effective tool to incorporate the uncertainty of each product demand into the FLP approach and also to reduce the impact of this uncertainty in terms of minimizing the risk of the layout. It can also be extended in numerous aspects. The first aspect would be to incorporate the utility analysis into the risk-based facility layout approach. The risk of the layout would be assessed through the probabilistic method, and the decision of selecting the layouts would be dependent on the risk attitude of the decision-maker. Thus, the risk values obtained through the risk assessment method would have a different weight for each individual decision-maker. By incorporating the utility analysis into the approach, the decision-maker's risk attitude could be factored into the FLP.

As shown in the case studies, when the problem size is larger, the identification of significant flows becomes extremely complex, and the number of calculations required for completing the risk assessment increases substantially. To reduce the complexity for large sized problems, a divide-and-conquer approach could be incorporated into the risk-based facility layout approach. By dividing a large sized problem into several small-sized problems, the identification of significant flow intensity process could be reduced. In addition, since the solution quality of the GA procedure is dependent on time and problem size, the quality of the solution could be improved within the same total amount of time by using the divide-and-conquer approach.

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APPENDICES

APPENDIX A

Design Matrix

Run	Time (minutes)	Problem Size (Departments)	% crossover	% deviation from the best solution
26	0.5	16	0	3.07%
36	0.5	16	0	5.83%
72	0.5	16	0	9.16%
90	0.5	16	0	0.00%
110	0.5	16	0	3.58%
4	1	16	0	10.81%
43	1	16	0	0.00%
99	1	16	0	9.40%
124	1	16	0	6.64%
133	1	16	0	0.00%
55	5	16	0	5.83%
95	5	16	0	3.88%
106	5	16	0	9.14%
116	5	16	0	6.64%
136	5	16	0	7.46%
21	10	16	0	6.64%
22	10	16	0	3.58%
40	10	16	0	3.07%
70	10	16	0	2.76%
111	10	16	0	3.07%
18	60	16	0	9.71%
44	60	16	0	0.00%
80	60	16	0	9.14%
100	60	16	0	8.88%
147	60	16	0	7.46%
30	0.5	25	0	14.97%
87	0.5	25	0	8.79%
114	0.5	25	0	2.16%
131	0.5	25	0	3.70%
150	0.5	25	0	5.01%
76	1	25	0	5.06%
102	1	25	0	14.98%
113	1	25	0	6.29%
142	1	25	0	11.20%
145	1	25	0	7.71%
5	5	25	0	10.10%
46	5	25	0	6.23%
65	5	25	0	12.70%
107	5	25	0	5.07%
130	5	25	0	7.73%

Run	Time (minutes)	Problem Size (Departments)	% crossover	% deviation from the best solution
7	10	25	0	6.85%
64	10	25	0	13.30%
75	10	25	0	10.89%
119	10	25	0	11.34%
143	10	25	0	10.48%
25	60	25	0	10.08%
57	60	25	0	9.58%
58	60	25	0	12.86%
71	60	25	0	6.07%
141	60	25	0	5.06%
62	0.5	16	50	8.88%
83	0.5	16	50	3.07%
125	0.5	16	50	3.88%
129	0.5	16	50	6.95%
137	0.5	16	50	6.64%
29	1	16	50	6.95%
33	1	16	50	0.00%
89	1	16	50	3.88%
101	1	16	50	3.58%
140	1	16	50	2.76%
2	5	16	50	4.99%
11	5	16	50	13.32%
23	5	16	50	0.00%
42	5	16	50	0.00%
68	5	16	50	3.07%
16	10	16	50	7.46%
54	10	16	50	2.76%
67	10	16	50	5.83%
108	10	16	50	6.64%
148	10	16	50	2.76%
37	60	16	50	6.64%
47	60	16	50	6.64%
77	60	16	50	3.07%
79	60	16	50	7.46%
93	60	16	50	5.83%
45	0.5	25	50	17.90%
48	0.5	25	50	10.07%
74	0.5	25	50	10.51%
121	0.5	25	50	2.59%
132	0.5	25	50	15.54%
14	1	25	50	7.04%
19	1	25	50	4.51%

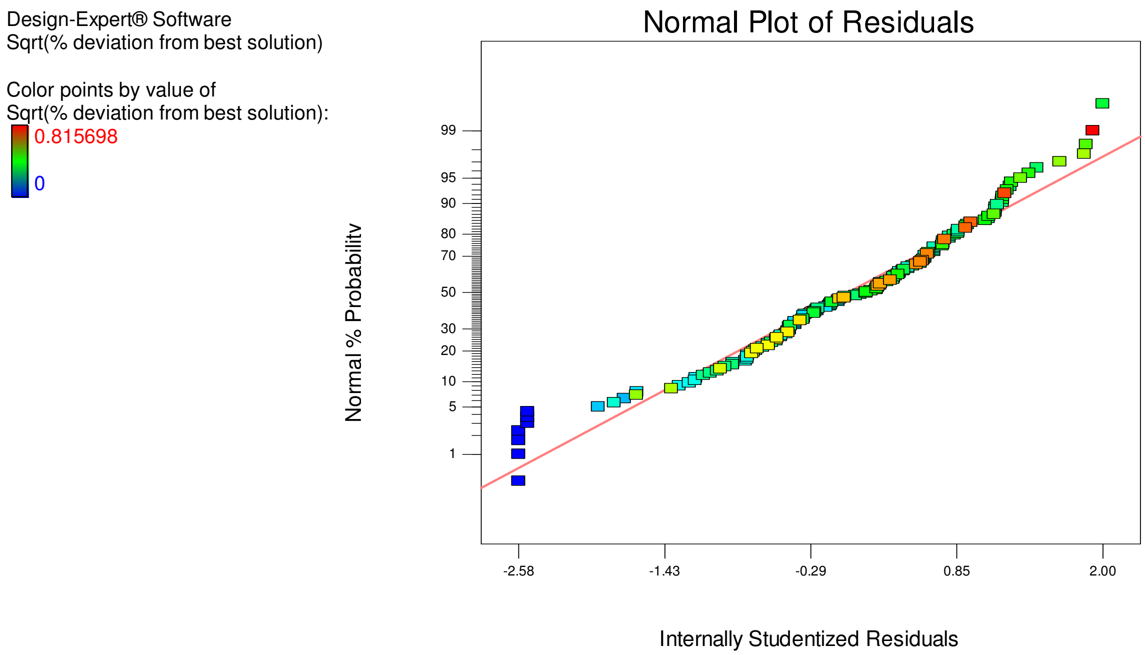
Run	Time (minutes)	Problem Size (Departments)	% crossover	% deviation from the best solution
39	1	25	50	6.07%
94	1	25	50	12.21%
103	1	25	50	11.43%
31	5	25	50	17.67%
51	5	25	50	22.14%
52	5	25	50	7.23%
98	5	25	50	3.44%
149	5	25	50	16.52%
13	10	25	50	14.38%
61	10	25	50	12.59%
118	10	25	50	6.80%
128	10	25	50	4.78%
146	10	25	50	9.02%
1	60	25	50	16.70%
27	60	25	50	4.95%
69	60	25	50	12.13%
105	60	25	50	7.50%
117	60	25	50	18.85%
17	0.5	16	100	23.26%
34	0.5	16	100	14.40%
73	0.5	16	100	17.17%
82	0.5	16	100	17.77%
120	0.5	16	100	8.88%
32	1	16	100	27.47%
41	1	16	100	16.34%
53	1	16	100	24.93%
60	1	16	100	16.09%
109	1	16	100	16.62%
28	5	16	100	13.59%
91	5	16	100	11.64%
122	5	16	100	10.02%
135	5	16	100	9.71%
138	5	16	100	12.47%
63	10	16	100	9.40%
66	10	16	100	19.43%
84	10	16	100	13.55%
92	10	16	100	9.14%
123	10	16	100	5.83%
38	60	16	100	29.12%
56	60	16	100	16.10%
59	60	16	100	20.24%
112	60	16	100	13.04%

APPENDIX A (continued)

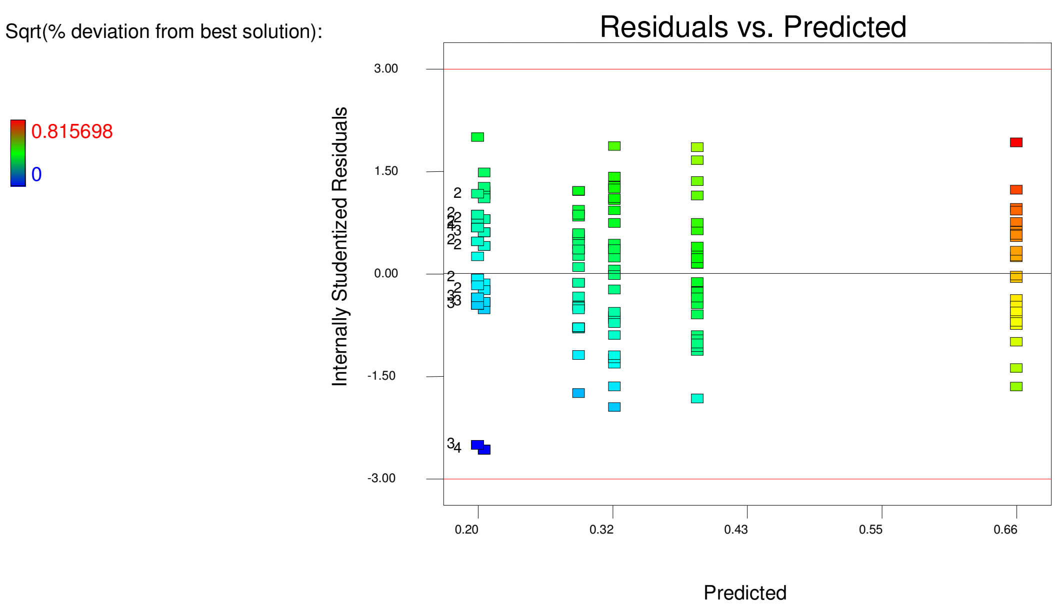
Run	Time (minutes)	Problem Size (Departments)	% crossover	% deviation from the best solution
144	60	16	100	16.63%
12	0.5	25	100	39.68%
24	0.5	25	100	54.49%
86	0.5	25	100	46.14%
134	0.5	25	100	37.80%
139	0.5	25	100	66.54%
3	1	25	100	30.06%
6	1	25	100	36.04%
85	1	25	100	57.71%
96	1	25	100	46.34%
115	1	25	100	37.17%
8	5	25	100	33.56%
9	5	25	100	52.06%
15	5	25	100	49.50%
104	5	25	100	42.89%
126	5	25	100	27.67%
49	10	25	100	43.27%
50	10	25	100	50.48%
88	10	25	100	35.89%
97	10	25	100	47.22%
127	10	25	100	50.03%
10	60	25	100	37.85%
20	60	25	100	53.99%
35	60	25	100	49.88%
78	60	25	100	36.33%
81	60	25	100	38.69%

APPENDIX B

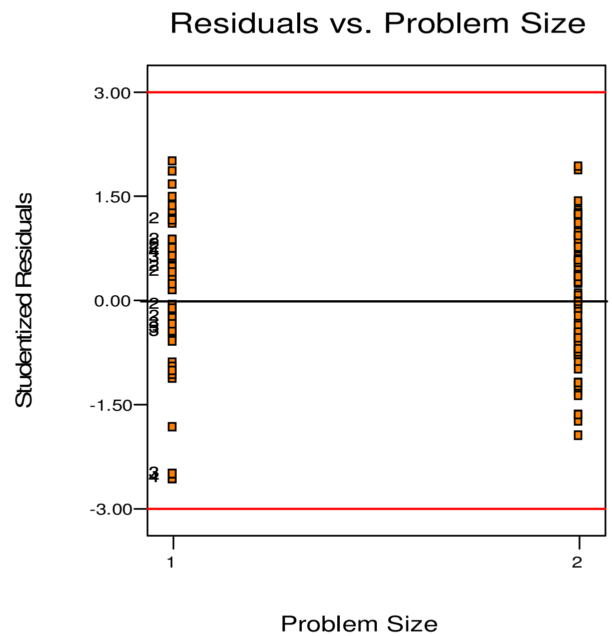
Normal Probability Plot, Residual Plots, and Outlier Plot



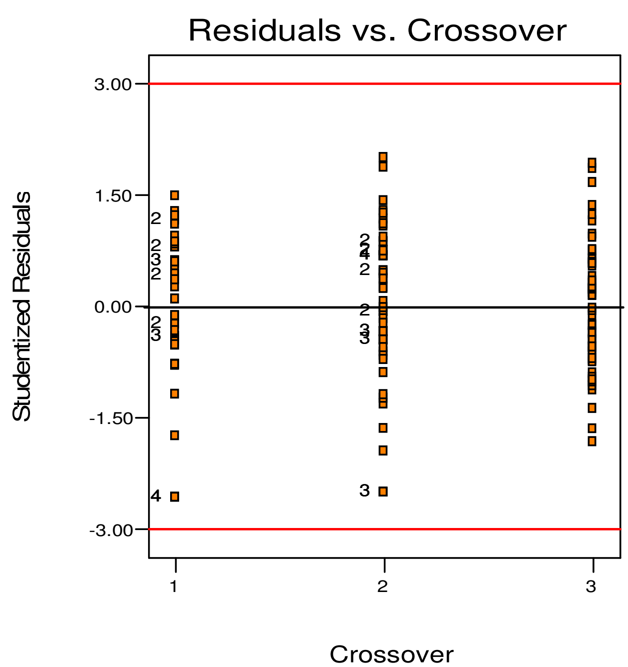
Normal Probability Plot



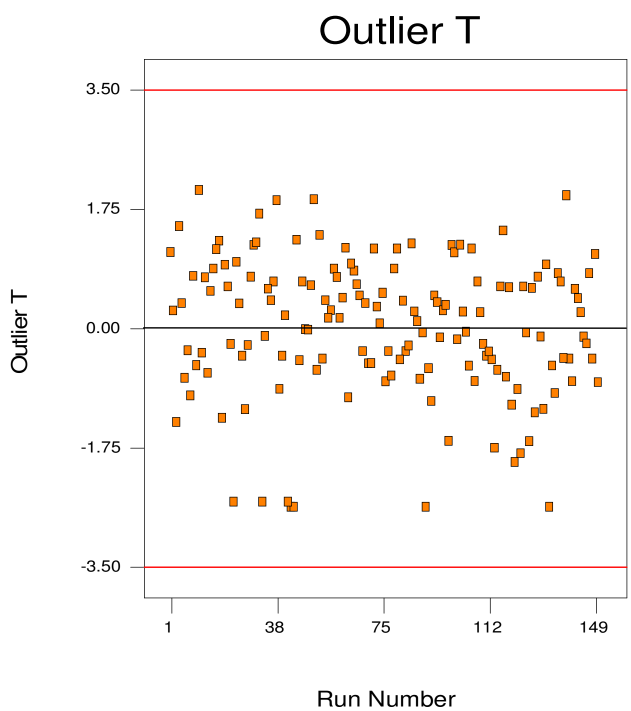
Residule VS. Predicted plot.



Residule VS. Problem Size plot.



Residule VS. Percentage Crossover Operation plot.



Outlair plot.

APPENDIX C

From-Between Chart for Twenty-Five-Department Single-Period Case Study

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1		50							150																
2			50							280															
3				50			150		150																
4					50			280		280															
5						50	150				390														
6							50	280				640									300				
7								50																	
8									50																
9										50															
10											50														
11												50	240						150						
12													50	360						280					
13																	150		690						
14																60		280		280					
15																	150				150				
16																		340						280	
17																									
18																									
19																					540				
20																					150				
21																						150			150
22																							150	280	
23																								150	150
24																									150
25																									

APPENDIX D

Risk-Flow Table for Twenty-Five-Department Single Period Case Study

		Flow Intensity					
		x_2					
		2-10	**10-4**	**4-8**	**8-6**	**6-12**	**12-20**
	1-2	$33,652	$33,652	$33,652	$33,652	$33,652	$33,652
	2-3	$13,461	$13,461	$13,461	$13,461	$13,461	$13,461
	3-4	$13,461	$13,461	$13,461	$13,461	$13,461	$13,461
	4-5	$13,461	$13,461	$13,461	$13,461	$13,461	$13,461
	5-6	$13,461	$13,461	$13,461	$13,461	$13,461	$13,461
x_5	**7-8**	$13,461	$13,461	$13,461	$13,461	$13,461	$13,461
	8-9	$26,922	$26,922	$26,922	$26,922	$26,922	$26,922
	9-10	$13,461	$13,461	$13,461	$13,461	$13,461	$13,461
	10-11	$26,922	$26,922	$26,922	$26,922	$26,922	$26,922
	11-12	$26,922	$26,922	$26,922	$26,922	$26,922	$26,922
	12-13	$13,461	$13,461	$13,461	$13,461	$13,461	$13,461

		Flow Intensity				
		x_2				
		20-14	**14-18**	**18-16**	**16-24**	**24-22**
	1-2	$33,652	$33,652	$33,652	$33,652	$33,652
	2-3	$13,461	$13,461	$13,461	$13,461	$13,461
	3-4	$13,461	$13,461	$13,461	$13,461	$13,461
	4-5	$13,461	$13,461	$13,461	$13,461	$13,461
	5-6	$13,461	$13,461	$13,461	$13,461	$13,461
x_5	**7-8**	$13,461	$13,461	$13,461	$13,461	$13,461
	8-9	$26,922	$26,922	$26,922	$26,922	$26,922
	9-10	$13,461	$13,461	$13,461	$13,461	$13,461
	10-11	$26,922	$26,922	$26,922	$26,922	$26,922
	11-12	$26,922	$26,922	$26,922	$26,922	$26,922
	12-13	$13,461	$13,461	$13,461	$13,461	$13,461

		Flow Intensity				
		x_7				
		20-21	**21-22**	**22-23**	**23-24**	**24-25**
	1-2	$25,115	$25,115	$25,115	$25,115	$25,115
	2-3	$10,046	$10,046	$10,046	$10,046	$10,046
	3-4	$10,046	$10,046	$10,046	$10,046	$10,046
	4-5	$10,046	$10,046	$10,046	$10,046	$10,046
	5-6	$10,046	$10,046	$10,046	$10,046	$10,046
x_5	**7-8**	$10,046	$10,046	$10,046	$10,046	$10,046
	8-9	$20,092	$20,092	$20,092	$20,092	$20,092
	9-10	$10,046	$10,046	$10,046	$10,046	$10,046
	10-11	$20,092	$20,092	$20,092	$20,092	$20,092
	11-12	$20,092	$20,092	$20,092	$20,092	$20,092
	12-13	$10,046	$10,046	$10,046	$10,046	$10,046

APPENDIX D (continued)

		Flow Intensity					
		x_3	x_4	x_6	$x_1 + x_3$	$x_2 + x_4$	$x_4 + x_6$
		11-13	**14-16**	**21-6**	**5-11**	**16-18**	**12-14**
x_5	**1-2**	$33,580	$40,335	$43,392	$18,258	$23,108	$32,201
	2-3	$13,432	$16,134	$17,357	$7,303	$9,243	$12,881
	3-4	$13,432	$16,134	$17,357	$7,303	$9,243	$12,881
	4-5	$13,432	$16,134	$17,357	$7,303	$9,243	$12,881
	5-6	$13,432	$16,134	$17,357	$7,303	$9,243	$12,881
	7-8	$13,432	$16,134	$17,357	$7,303	$9,243	$12,881
	8-9	$26,864	$32,268	$34,713	$14,606	$18,486	$25,761
	9-10	$13,432	$16,134	$17,357	$7,303	$9,243	$12,881
	10-11	$26,864	$32,268	$34,713	$14,606	$18,486	$25,761
	11-12	$26,864	$32,268	$34,713	$14,606	$18,486	$25,761
	12-13	$13,432	$16,134	$17,357	$7,303	$9,243	$12,881

		Flow Intensity				
		x_1				
		1-9	**9-3**	**3-7**	**3-7**	**11-19**
x_5	**1-2**	$43,879	$43,879	$43,879	$43,879	$43,879
	2-3	$17,552	$17,552	$17,552	$17,552	$17,552
	3-4	$17,552	$17,552	$17,552	$17,552	$17,552
	4-5	$17,552	$17,552	$17,552	$17,552	$17,552
	5-6	$17,552	$17,552	$17,552	$17,552	$17,552
	7-8	$17,552	$17,552	$17,552	$17,552	$17,552
	8-9	$35,103	$35,103	$35,103	$35,103	$35,103
	9-10	$17,552	$17,552	$17,552	$17,552	$17,552
	10-11	$35,103	$35,103	$35,103	$35,103	$35,103
	11-12	$35,103	$35,103	$35,103	$35,103	$35,103
	12-13	$17,552	$17,552	$17,552	$17,552	$17,552

		Flow Intensity				
		x_1				
		13-17	**17-15**	**15-21**	**21-25**	**25-23**
x_5	**1-2**	$43,879	$43,879	$43,879	$43,879	$43,879
	2-3	$17,552	$17,552	$17,552	$17,552	$17,552
	3-4	$17,552	$17,552	$17,552	$17,552	$17,552
	4-5	$17,552	$17,552	$17,552	$17,552	$17,552
	5-6	$17,552	$17,552	$17,552	$17,552	$17,552
	7-8	$17,552	$17,552	$17,552	$17,552	$17,552
	8-9	$35,103	$35,103	$35,103	$35,103	$35,103
	9-10	$17,552	$17,552	$17,552	$17,552	$17,552
	10-11	$35,103	$35,103	$35,103	$35,103	$35,103
	11-12	$35,103	$35,103	$35,103	$35,103	$35,103
	12-13	$17,552	$17,552	$17,552	$17,552	$17,552

		Flow Intensity					
		x_2					
		2-10	**10-4**	**4-8**	**8-6**	**6-12**	**12-20**
x_3	**11-13**	$1,544	$1,544	$1,544	$1,544	$1,544	$1,544

		Flow Intensity					
		x_2					x_6
		20-14	**14-18**	**18-16**	**16-24**	**24-22**	**21-6**
x_3	**11-13**	$1,544	$1,544	$1,544	$1,544	$1,544	$6,322

		Flow Intensity					
		x_2					
		12-20	**20-14**	**14-18**	**18-16**	**16-24**	**24-22**
x_7	**21-22**	$9,547	$9,547	$9,547	$9,547	$9,547	$9,547
	22-23	$6,365	$6,365	$6,365	$6,365	$6,365	$6,365
	24-25	$6,365	$6,365	$6,365	$6,365	$6,365	$6,365

		Flow Intensity				
		x_2				
		2-10	**10-4**	**4-8**	**8-6**	**6-12**
x_7	**21-22**	$9,547	$9,547	$9,547	$9,547	$9,547
	22-23	$6,365	$6,365	$6,365	$6,365	$6,365
	24-25	$6,365	$6,365	$6,365	$6,365	$6,365

		Flow Intensity				
		x_3	x_6	$x_1 + x_3$	$x_2 + x_4$	$x_4 + x_6$
		11-13	**21-6**	**5-11**	**16-18**	**12-14**
x_7	**21-22**	$9,703	$16,356	$3,112	$4,806	$10,525
	22-23	$6,469	$10,904	$2,075	$3,204	$7,016
	24-25	$6,469	$10,904	$2,075	$3,204	$7,016

		Flow Intensity				
		x_2				
		2-10	**10-4**	**4-8**	**8-6**	**6-12**
x_1	**15-21**	$0	$0	$0	$0	$0
	21-25	$0	$0	$0	$0	$0

		Flow Intensity				
		x_2				
		12-20	**20-14**	**14-18**	**18-16**	**16-24**
x_1	**15-21**	$0	$0	$0	$0	$0
	21-25	$0	$0	$0	$0	$0

		Flow Intensity				
		x_2	x_3	$x_2 + x_4$	$x_4 + x_6$	$x_2 + x_4 + x_6$
		24-22	**11-13**	**16-18**	**12-14**	**6-12**
x_1	**15-21**	$0	$0	$0	$241	$0
	21-25	$0	$0	$0	$481	$0

		Flow Intensity		
		x_6	**$x_1 + x_3$**	**$x_1 + x_3 + x_6$**
		21-6	**5-11**	**13-19**
x_2	**20-14**	$5,498	$0	$0

APPENDIX E

From-Between Chart for Risk-Based Layout for Twenty-Five-Department Single-Period Case Study

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1		270							154																
2			270							350															
3				270			154		154																
4					270			350		350															
5						270	154				390														
6							270	350				640									300				
7								270																	
8									270																
9										270															
10											270														
11												270	320						154						
12													270	360						350					
13																	154		690						
14																60		350		350					
15																	154				154				
16																		340						350	
17																									
18																									
19																					540				
20																					287				
21																						287			154
22																							287	350	
23																								287	154
24																									287
25																									

APPENDIX F

From-Between Chart for Twenty-Five-Department Multi-Period Case Study (t = 2)

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1		300							150																
2			300							105															
3				300			150		150																
4					300			105		105															
5						300	150				300														
6							300	105				285									120				
7								300																	
8									300																
9										300															
10											300														
11												300	150						150						
12													300	180						105					
13																	150		420						
14																60		105		105					
15																	150				150				
16																		165						105	
17																									
18																									
19																					270				
20																					110				
21																						110			150
22																							110	105	
23																								110	150
24																									110
25																									

APPENDIX G

Risk-Flow Table for Twenty-Five-Department Multi-Period Case Study when t = 2

		Flow Intensity					
		x_5					
		1-2	**2-3**	**3-4**	**4-5**	**5-6**	**6-7**
x_1	**1-9**	$120	$120	$120	$120	$120	$120
	3-7	$80	$80	$80	$80	$80	$80
	11-19	$80	$80	$80	$80	$80	$80
	13-17	$80	$80	$80	$80	$80	$80
	15-21	$80	$80	$80	$80	$80	$80
		Flow Intensity					
		x_5					
		7-8	**8-9**	**9-10**	**10-11**	**11-12**	**12-13**
x_1	**1-9**	$120	$120	$120	$120	$120	$120
	3-7	$80	$80	$80	$80	$80	$80
	11-19	$80	$80	$80	$80	$80	$80
	13-17	$80	$80	$80	$80	$80	$80
	15-21	$80	$80	$80	$80	$80	$80
		Flow Intensity					
		x_3	$x_2 + x_4$	$x_4 + x_6$	$x_2 + x_4 + x_6$		
		11-13	**16-18**	**12-14**	**6-12**		
x_1	**1-9**	$600	$840	$240	$120		
	3-7	$400	$560	$160	$80		
	11-19	$400	$560	$160	$80		
	13-17	$400	$560	$160	$80		
	15-21	$400	$560	$160	$80		
		Flow Intensity					
		x_3	x_6				
		11-13	**21-6**				
x_2	**2-10**	$160	$160				
	10-4	$320	$320				
	4-8	$240	$240				
	8-6	$320	$320				
	14-18	$160	$160				
	16-24	$400	$400				
	24-22	$160	$160				

		Flow Intensity				
		x_1				
		1-9	**9-3**	**3-7**	**7-5**	**11-19**
x_2	**2-10**	$640	$640	$640	$640	$640
	10-4	$1,280	$1,280	$1,280	$1,280	$1,280
	4-8	$960	$960	$960	$960	$960
	8-6	$1,280	$1,280	$1,280	$1,280	$1,280
	14-18	$640	$640	$640	$640	$640
	16-24	$1,600	$1,600	$1,600	$1,600	$1,600
	24-22	$640	$640	$640	$640	$640
		Flow Intensity				
		x_1				
		13-17	**17-15**	**15-21**	**21-25**	**25-23**
x_2	**2-10**	$640	$640	$640	$640	$640
	10-4	$1,280	$1,280	$1,280	$1,280	$1,280
	4-8	$960	$960	$960	$960	$960
	8-6	$1,280	$1,280	$1,280	$1,280	$1,280
	14-18	$640	$640	$640	$640	$640
	16-24	$1,600	$1,600	$1,600	$1,600	$1,600
	24-22	$640	$640	$640	$640	$640
		Flow Intensity				
		x_7				
		20-21	**21-22**	**22-23**	**23-24**	**24-25**
x_2	**2-10**	$80	$80	$80	$80	$80
	10-4	$160	$160	$160	$160	$160
	4-8	$120	$120	$120	$120	$120
	8-6	$160	$160	$160	$160	$160
	14-18	$80	$80	$80	$80	$80
	16-24	$200	$200	$200	$200	$200
	24-22	$80	$80	$80	$80	$80
		Flow Intensity				
		x_1				
		1-9	**9-3**	**3-7**	**7-5**	**11-19**
x_3	**11-13**	$13,800	$13,800	$13,800	$13,800	$13,800
		Flow Intensity				
		x_1				
		13-17	**17-15**	**15-21**	**21-25**	**25-23**
x_3	**11-13**	$13,800	$13,800	$13,800	$13,800	$13,800

		Flow Intensity					
		x_5					
		1-2	**2-3**	**3-4**	**4-5**	**5-6**	**6-7**
x_3	**11-13**	$5,760	$5,760	$5,760	$5,760	$5,760	$5,760

		Flow Intensity					
		x_5					
		7-8	**8-9**	**9-10**	**10-11**	**11-12**	**12-13**
x_3	**11-13**	$5,760	$5,760	$5,760	$5,760	$5,760	$5,760

		Flow Intensity					
		x_5					
		1-2	**2-3**	**3-4**	**4-5**	**5-6**	**6-7**
x_6	**21-6**	$8,960	$8,960	$8,960	$8,960	$8,960	$8,960

		Flow Intensity					
		x_5					
		7-8	**8-9**	**9-10**	**10-11**	**11-12**	**12-13**
x_6	**21-6**	$8,960	$8,960	$8,960	$8,960	$8,960	$8,960

		Flow Intensity				
		x_1				
		1-9	**9-3**	**3-7**	**7-5**	**11-19**
x_6	**21-6**	$22,080	$22,080	$22,080	$22,080	$22,080

		Flow Intensity				
		x_1				
		13-17	**17-15**	**15-21**	**21-25**	**25-23**
x_6	**21-6**	$22,080	$22,080	$22,080	$22,080	$22,080

		Flow Intensity		
		x_3	$x_1 + x_3$	$x_2 + x_4$
		11-13	**5-11**	**16-18**
x_6	**21-6**	$10,560	$5,920	$18,080

		Flow Intensity		
		$x_2 + x_4$	$x_4 + x_6$	$x_2 + x_4 + x_6$
		16-18	**12-14**	**6.12**
x_3	**11-13**	$11280	$3840	$2160

		Flow Intensity	
		$x_1 + x_3$	$x_1 + x_3 + x_6$
		5-11	**13-19**
x_5	**4-5**	$2,720	$0
	6-7	$2,720	$0
	7-8	$2,720	$0
	9-10	$2,720	$0
	12-13	$2,720	$0

APPENDIX G (continued)

		Flow Intensity					
		x_5					
		1-2	**2-3**	**3-4**	**4-5**	**5-6**	**6-7**
x_7	**20-21**	$4,960	$4,960	$4,960	$4,960	$4,960	$4,960
	21-22	$7,440	$7,440	$7,440	$7,440	$7,440	$7,440
	24-25	$4,960	$4,960	$4,960	$4,960	$4,960	$4,960

		Flow Intensity					
		x_5					
		7-8	**8-9**	**9-10**	**10-11**	**11-12**	**12-13**
x_7	**20-21**	$4,960	$4,960	$4,960	$4,960	$4,960	$4,960
	21-22	$7,440	$7,440	$7,440	$7,440	$7,440	$7,440
	24-25	$4,960	$4,960	$4,960	$4,960	$4,960	$4,960

		Flow Intensity					
		x_1					**$x_1 + x_3$**
		1-9	**9-3**	**3-7**	**7-5**	**11-19**	**5-11**
x_7	**20-21**	$12,480	$12,480	$12,480	$12,480	$12,480	$3,280
	21-22	$18,720	$18,720	$18,720	$18,720	$18,720	$4,920
	24-25	$12,480	$12,480	$12,480	$12,480	$12,480	$3,280

		Flow Intensity					
		x_1					**$x_1 + x_3 + x_6$**
		13-17	**17-15**	**15-21**	**21-25**	**25-23**	**13-19**
x_7	**20-21**	$12,480	$12,480	$12,480	$12,480	$12,480	$240
	21-22	$18,720	$18,720	$18,720	$18,720	$18,720	$360
	24-25	$12,480	$12,480	$12,480	$12,480	$12,480	$240

		Flow intensity				
		x_3	**x_6**	**$x_2 + x_4$**	**$x_4 + x_6$**	**$x_2 + x_4 + x_6$**
		11-13	**21-6**	**16-18**	**12-14**	**6-12**
x_7	**20-21**	$6,000	$5,120	$10,240	$3,760	$2,080
	21-22	$9,000	$7,680	$15,360	$5,640	$3,120
	24-25	$6,000	$80,864	$10,240	$3,760	$2,080

		Flow Intensity		
		$x_1 + x_3$	**$x_3 + x_6$**	**$x_1 + x_3 + x_6$**
		5-11	**19-21**	**13-19**
$x_4 + x_6$	**12-14**	$30,720	$320	$160

		Flow Intensity					
		x_5					
		1-2	**2-3**	**3-4**	**4-5**	**5-6**	**6-7**
$x_4 + x_6$	**12-14**	$15,040	$15,040	$15,040	$15,040	$15,040	$15,040
		Flow Intensity					
		x_5					
		7-8	**8-9**	**9-10**	**10-11**	**11-12**	**12-13**
x4 + x6	**12-14**	$15,040	$15,040	$15,040	$15,040	$15,040	$15,040

		Flow intensity
		$x_2 + x_4 + x_6$
		6-12
$x_3 + x_6$	**19-21**	$15,040

APPENDIX H

From-Between Chart for Risk-Based Layout for Twenty-Five-Department Multi-Period Case Study (t = 2)

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1		334							157																
2			334							113															
3				334			157		157																
4					334			113		113															
5						334	157				300														
6							334	113				285									258				
7								334																	
8									334																
9										334															
10											334														
11												334	265						157						
12													334	372						113					
13																	157		420						
14																60		113		113					
15																	157				157				
16																		165						113	
17																									
18																									
19																					364				
20																					266				
21																						266			157
22																							266	113	
23																								266	157
24																									266
25																									

APPENDIX I

From-Between Chart for Twenty-Five-Department Multi-Period Case Study (t = 3)

Dept.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1		63							32																
2			63							138															
3				63			32		32																
4					63			138		138															
5						63	32				367														
6							63	138				736									420				
7								63																	
8									63																
9										63															
10											63														
11												63	335						32						
12													63	598						138					
13																	32		787						
14																178		138		138					
15																	32				32				
16																		316						138	
17																									
18																									
19																					755				
20																					295				
21																						295			32
22																							295	138	
23																								295	32
24																									295
25																									