OPTIMIZATION OF ELECTRIC VEHICLE CHARGING SCHEDULE USING DISTRIBUTED NETWORK COMPUTING

A Thesis by

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The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Electrical Engineering.

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DEDICATION

To my family, my friends, and the city I love
ACKNOWLEDGEMENTS

I would like to thank my advisor Dr. Visvakumar for providing many opportunities to learn and challenge myself.

I would like to thank my family for giving me unconditional support and allowing me to finish this degree.

I also thank all my friends for helping me complete this thesis and for inspiring me. I would like to thank all members of the power system group for sharing many great ideas and their knowledge with me. I also thank Arun Manoharan for providing many comments about my projects and this thesis. Finally, I wish to thank Suresh Koppisetty and Ateh Atayo for helping me with this project.
ABSTRACT

In recent years, the number of electric vehicles has increased dramatically, and the forecast shows that this speed of growth will accelerate even more over the next ten years. The significant growth in the number of electric vehicles indicates a large energy demand in the power distribution system. Without a well-organized schedule for charging electric vehicles, users will typically apply immediate charging, which burdens the system and damages equipment. To reduce the peak demand of the system, a process of scheduling electric vehicle charging should be established for reducing the peak demand.

This thesis proposes a distributed computing process for solving the electric vehicle charging schedule problem. This process models the electric vehicle charging availability to determine the charging rate. Also, this process can ensure that the charging schedule meets the energy demand of individual vehicles and the power limit of the power distribution system.
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<th>Abbreviation</th>
<th>Full Form</th>
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<tr>
<td>BEV</td>
<td>Battery Electric Vehicle</td>
</tr>
<tr>
<td>DER</td>
<td>Distributed Energy Resource</td>
</tr>
<tr>
<td>EIA</td>
<td>Energy Information Administration</td>
</tr>
<tr>
<td>EV</td>
<td>Electric Vehicle</td>
</tr>
<tr>
<td>ICE</td>
<td>Internal Combustion Engine</td>
</tr>
<tr>
<td>G2V</td>
<td>Grid to Vehicle</td>
</tr>
<tr>
<td>GEMS</td>
<td>Grid Energy Management System</td>
</tr>
<tr>
<td>GHG</td>
<td>Green House Gas</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HEMS</td>
<td>Home Energy Management System</td>
</tr>
<tr>
<td>HEV</td>
<td>Hybrid Electric Vehicle</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
</tr>
<tr>
<td>MST</td>
<td>Minimum Spanning Tree</td>
</tr>
<tr>
<td>NN</td>
<td>Neural Network</td>
</tr>
<tr>
<td>PHEV</td>
<td>Plug-in Hybrid Electric Vehicle</td>
</tr>
<tr>
<td>SOC</td>
<td>State of Charge</td>
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<tr>
<td>V2G</td>
<td>Vehicle to Grid</td>
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<tr>
<td>V2H</td>
<td>Vehicle to Home</td>
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CHAPTER 1

INTRODUCTION

1.1 Background

The concern about global warming has been raised in recent years. Since electric vehicles (EVs) can reduce the emission of greenhouse gases (GHGs) [1,2], the public is considering EVs as an alternative to replacing conventional internal combustion engine (ICE) vehicles, which is one of the main sources of GHG. A report [3] from the Energy Information Administration (EIA) estimates that the projected sales of battery electric vehicles (BEVs) and plug-in hybrid electric vehicles (PHEVs) will reach 1.3 million or 8% of projected total vehicle sales in 2025.

On the other hand, due to the increase in EV penetration, the impact of charging an EV on the power system becomes more significant. The integration of EVs into the power grid can increase the power consumption in residential areas [4]. This extra power consumption leads to a higher operating cost involving generation and maintenance. Moreover, a high penetration of EVs can cause overload problems in the power distribution system. If many EVs charge within a short period time without controlling their charging rate, then the power distribution system cannot function normally, thereby resulting in damage to equipment.

To ensure the safety of the power system, some possible solutions would be to redesign the power distribution system for satisfying the increased energy demand and scheduling EV charging in order to reduce the maximum load during peak demand hours. However, the former solution is too expensive for deploying new equipment to the system. Therefore, this thesis proposes a distributed network computing approach for optimizing the charging schedule of each electric vehicle.
1.2 Power System

The conventional power system consists of three main parts: generation, transmission, and power distribution [5]. Each part has a unique and independent function. The generation system produces power to supply the demand of the whole system. The transmission system transfers energy from the generation side to the power distribution system. The power distribution system delivers energy to commercial and residential areas.

In this thesis, it is assumed that there are no distributed energy resource (DERs), such as photovoltaic panels and wind turbines, located in the power distribution system. Therefore, the power distribution system can be considered as a load, and can be modeled separately from the generation system and the transmission system.

1.3 Electric Vehicles

An electric vehicle, or vehicle that is installed with an electric motor, can be classified into three types: battery electric vehicles, plug-in hybrid electric vehicles, and hybrid electric vehicles (HEVs). A BEV is one in which power is fully supplied by the batteries. Both the battery and ICE are installed in PHEVs and HEVs, but the HEV’s battery is generated by the braking system, while the PHEV’s battery is charged by an external power source.

Some of the regulated standards of EV charging [6-8] include IEEE 1547, SAE-J2894, IEC 1000-3-2, and the U.S. National Electric Code 690, which limit the allowable harmonic and DC current injection into the grid. SAE J1772 and JARI/TEPCO are standards for the connector of an EV charger. The standards for voltage and current in the type of EV charging is shown in Table 1.
TABLE 1
TYPES OF EV CHARGING

<table>
<thead>
<tr>
<th>Level</th>
<th>Phase</th>
<th>Setting</th>
<th>Voltage (V)</th>
<th>Maximum Current (A)</th>
<th>Maximum Power (kW)</th>
<th>Charging Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Residential</td>
<td>120 (AC)</td>
<td>20</td>
<td>1.9</td>
<td>11–36</td>
</tr>
<tr>
<td>2</td>
<td>1 or 3</td>
<td>Residential/Commercial</td>
<td>208/240 (AC)</td>
<td>80</td>
<td>19.2</td>
<td>2–3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>Commercial</td>
<td>208-600 (AC/DC)</td>
<td>400</td>
<td>100</td>
<td>0.2–1</td>
</tr>
</tbody>
</table>

The three EV charging operations are grid to vehicle (G2V), vehicle to home (V2H), and vehicle to grid (V2G), as shown in Figure 1. The definitions for these three models are as follows:

- **G2V**: the vehicle can be considered as a load that only consumes energy from grid.
- **V2H**: the vehicle can be considered either a load or a source, and can supply power to the house when the power grid is demanding energy.
- **V2G**: the vehicle can be considered either a load or a source, and in addition to providing energy to the house, it can provide energy to the grid [9-10].

![Figure 1](image)

**Figure 1. Charging operations**: (a) grid to vehicle, (b) vehicle to house, and (c) vehicle to grid.

This thesis assumes that EVs can only consume energy from a power grid that does not provide any source of power. Therefore, the G2V mode is considered here.
1.4 Distributed System

As shown in Figure 2, power system architecture can be classified into two types: centralized and distributed. In the centralized architecture, the EV charging schedule is determined by a direct aggregator that has all the information and requirements of EVs [11]. This decision often achieves global optimization. However, if a single failure at the aggregator occurs, then the entire system potentially collapses. Also, this requires complete information from all EVs, which can be obstructed by the bandwidth limitation of the communication channel, thereby possibly limiting the scalability.

![Figure 2. Power system architecture: (a) centralized system, and (b) distributed system.](image)

In contrast to the centralized system, the distributed system does not have a central aggregator. Here, the decision is determined by a local controller or a local aggregator. Each local aggregator is only allowed to connect to partial EVs. All controllers and aggregators are independent decision-makers that schedule EV charging based on local information as well as received information from the neighbor controllers and aggregators. Since each controller and aggregator does not have complete information, the final solution may not be the global optimal solution. The advantage of the distributed system is that it is highly scalable and effectively uses distributed computational resources. Also, because most of the information used for computation
is stored in the local controller, the privacy of users and the security of their information is increased.

1.5 Contribution

To reduce the peak demand, it is necessary to calculate the optimal EV charging schedule. Although many studies have addressed this problem, most approaches model the EV demand and availability simply using historical data; they cannot adapt to the real-time change of EV availability. Also, they cannot be applied to a large-scale EV system. Therefore, this thesis proposes a 15-minute ahead EV charging schedule using distributed computation. The contributions of this thesis are the following:

- Development of a state transition model to predict the availability of EV charging.
- Formulation of the problem by introducing the cost function, local constraints, and global constraints.
- Transformation of the global optimization problem into a distributed optimization problem.
- Development of a process to obtain the optimal solution using distributed computing.
- Analysis of the performance of the proposed approach.

1.6 Organization of Thesis

This thesis consists of six chapters. Chapter 1 introduces the objective of the thesis and explains the important concept. A literature review of the user driving model, EV charging schedule, and distributed system are discussed in Chapter 2. Chapter 3 formulates the problem by introducing the cost function and constraints. Chapter 4 discusses the process of scheduling the EV charging using the distributed network. Chapter 5 presents the numerical analysis. Chapter 6 concludes this thesis and discusses future work.
CHAPTER 2
LITERATURE REVIEW

The EV charging schedule problem has been studied for more than a decade. This chapter summarizes previous studies of the EV charging schedule. The first section introduces the models of EV demand and the battery. The second section discusses different approaches and strategies for managing the EV charging. The last section provides some work on the distributed system algorithm.

2.1 EV Demand and Battery Models

Wang et al. introduced unconstrained traffic assignment models using the Bureau of Public Road function [11]. Their model can estimate the traffic flow and travel time of a trip. Also, it presents the transportation network graph, which is used to model the EV’s state of charge (SOC) on the road.

Tianheng et al. modeled EV energy demand of the trip using the neural network (NN) and traffic features [12]. This model requires road velocity and position data from the intelligent traffic system, and the global positioning system (GPS) for training and testing the radial basis function of the NN model. This model guarantees that the error of prediction is less than 10%.

Wu et al. proposed a stochastic optimal EV charging schedule for three different operating models: V2G, G2V, and V2H [13]. This work also modeled the EV charging availability represented by the Markov chain model and the impact on the SOC. However, this paper hypothesizes that all houses can charge an EV at the maximum rate at anytime without considering limitations of the power distribution system.

On the another side, Dubey and Santoso analyzed the impact of EV charging on the power distribution system voltage [14]. EV charging on the street and EV charging in a parking
lot are considered in the paper. Their results show that if more than 10 EVs are charging on the same phase in a parking lot, then this will violate the voltage limit. This paper also suggests a practical solution for mitigating the EV load by changing the infrastructure and controlling the time-of-use price.

Neyestani et al. provided an algorithm to find the optimal parking lot located on the power distribution system by considering installation costs, power loss costs, voltage deviation costs, and system reliability costs [15]. This paper provides a detailed model of the EVs, and the parking lot’s SOC constraints and status.

2.2 EV Charging Schedule

Some approaches to scheduling EV charging have been summarized in terms of economics, cost, profit, demand response, and consumer aspects [16, 17]. These approaches can also be classified by the time complexity, solving technique, and type of system. Techniques include convex optimization, game theory, particle swarm optimization, and dynamic programming [18]. This section summarizes prior research work that is based on the two types of control systems: centralized and distributed.

2.2.1 Centralized System

Song et al. used a convex power flow model to optimize the EV charging substation through minimizing the system energy cost [19]. Hoog et al. proposed a method that can optimize the charging schedule and maintain the voltage drop within the boundary of normal conditions [20]. To reduce the complexity of the voltage drop calculation, They used a simple linear approximation for the constraints. Kang et al. proposed a hybrid algorithm of particle swarm optimization and genetic algorithm to solve EV charging during large-scale utilization [21]. Their proposed algorithm first initializes more than one possible option. Then the algorithm
chooses the optimal option by comparing the objective function values. This study also shows that the optimal result approaches to the global optimal solution.

Yao et al. determined the charging priority of EVs by measuring the capacity of individual EV demand and the remaining charging time [22]. In order to reduce the complexity of the problem, they suggest using the on-off strategy that an EV can only be charged by a constant. Then, the problem can be solved by linear programming in real time.

Nguyen et al. integrated solar and wind energy into the problem. Their proposed approach determines the optimal charging and discharging rate of vehicles in G2V and V2G [23] scenarios. This approach models the EV demand using information on the EV’s SOC and the departure time, which can ensure that the optimal schedule meets the desired SOC.

Similarly, Veldman, and Verzijlbergh optimized the charging cost in different wind power scenarios [24]. The electricity price in their model is time-varying, in that the price is affected by EV demand, so the simulation result can reflect the financial impact due to this demand.

The limitation of these models discussed here [19-24] is that the EV demand and availability predictions are very simple. The variables are either considered as a constant or a numerical value based on historical data.

2.2.2 Distributed System

Liu et al. advocated an aggregative game model that optimizes the day-ahead EV charging schedule [25]. In this approach, all EVs are independent in that they only need to meet their local driving requirements. However, the controller makes decision by predicting the change of market price due to the influence of the demand of other EVs.
Mou et al. adopted a water-filling technique in the decentralized system. They solve the residential EV charging problem on the low-voltage transformer side [26]. This algorithm requires initializing the non-EV power demand information in local controllers. Then, the information is sent to the transformer in the power distribution system. The transformer aggregates information on the demand and modifies the total demand based on the system requirement. Then the calculated result is returned to the local controllers, who individually determine the optimal charging rate using the water-filling technique. The drawback of this algorithm is that the optimal solution is a local optimal and may not be unique.

Kikusato et al. proposed a decentralized EV charging optimization using the home energy management system (HEMS) [27]. The HEMS controller collects the historical EV power profile for predicting the EV demand and then decides the EV daily charging schedule, sending the decision to the grid energy management system (GEMS). The GEMS aggregates the charging schedule from all HEMS controllers to determine the power curtailment according to the limits of the power distribution system. The HEMS will modify the charging schedule based on the curtailment.

Likewise, Rivera et al. aggregated the EV charging profile to solve the EV charging optimization problem [28]. One critical difference between the work of Kikusato et al. [27] and Rivera et al. [28] is that the latter uses an iterative method to solve the problem.

You et al. studied the EV charging schedule of the charging station by separating the problem into a primal problem in the aggregator and a subproblem in the EV controller [29]. The subproblem determines the lower bound and the high bound of the optimal solution of the primal problem, and the primal problem determines the total cost.
The disadvantage of using a distributed system is that the solutions of most of the proposed methods are not optimized.

2.3 Distributed System Algorithm

The rapid development of modern technology increases the complexity of the system with respect to the network size and the control difficulty. To tackle the change, the distributed system raises the interest of researchers. Some researchers apply the distributed algorithms into communication, robotics, control, and optimization problems [30].

Movric and Lewis dealt with the optimal cooperative regulator and cooperative tracker problem in the distributed system [31]. They considered the fixed topology directed graph and the linear time-invariant agent dynamics, proposing a distributed a linear-quadratic regulator controller to achieve global optimal performance.

Liu and Wang solved the convex optimization problem with a bounded solution set. The network requires bidirectional communication for all agents in the distributed network [32]. Zhu et al. studied the convex optimization problem with a more general situation that includes equality and inequality constraints [33]. The cost functions in both of these studies must be convex.

Nedić and Olshevsky proposed an algorithm for distributed optimization over time-varying directed graphs [34]. They proved the convergence of the solution and analyzed the speed of the convergence rate.
CHAPTER 3

PROPOSED MODELING

In this chapter, the system’s cost function and constraints are established to model the vehicle’s behavior in the power distribution system. The cost function represents the overall predicted cost for the next 24-hour period and involves three uncertainties: the energy consumption of the house, the vehicle’s daily energy demand, and the vehicle’s availability. To estimate the impact of these three uncertainties, some statistical results and state transition models are applied and presented here.

This paper makes a critical assumption. The approach here assumes that the EV charging schedule of all houses is completely determined by controllers. Also, EV users must follow the charging schedule in that they must charge their EV at the determined power rate in the given time intervals.

3.1 Vehicle Demand Model and House Energy Consumption Model

The estimated demand for an EV is the product of the average travel distance of the last thirty days and the energy required per mile. The average travel distance should be updated every day for tracking the EV demand influenced by the user’s driving pattern.

Similarly, the energy consumption of a house can be estimated by calculating the average energy of the last thirty days, excluding the energy consumption of the EV. Then, the total energy consumption is the sum of the energy consumption of the charging EV and the energy consumption of the house:

\[ p_{total} = p^{EV} + p^{House} \]  

where \( p^{EV} \) is the energy consumption of the EV, and \( p^{House} \) is the energy consumption of the house, excluding the energy consumption of the EV.
3.2 Vehicle Availability Model

In this study, the state transition model was adopted to model vehicle availability, as shown in Figure 3. Vehicle availability can be classified into two statuses: 0 representing the vehicle out of the house, and 1 representing the vehicle at the house that is available to charge. The two statuses are connected by a set of probabilistic functions that represent the chance of transition from one status to another status. The collection of probabilistic functions is the transition function, defined as

\[
p(s_{k+1}|s_k) = \begin{cases} 
1 - p_{01}(t) & s_k = 0 \text{ and } s_{k+1} = 0 \\
p_{01}(t) & s_k = 0 \text{ and } s_{k+1} = 1 \\
p_{10}(t) & s_k = 1 \text{ and } s_{k+1} = 0 \\
1 - p_{10}(t) & s_k = 1 \text{ and } s_{k+1} = 1 
\end{cases}
\] (2)

where \(0 \leq p_{01}(t), p_{10}(t) \leq 1\).

The probabilistic functions can be obtained from the controller by statistical calculation. The controller in each house counts the number of arrivals and departures in every time interval. After the controller collects data for a significant number of days, the probabilistic function can be simply found by

\[
p_{01}(t) = \frac{\text{number of arrival at interval } t}{\text{total number of days}} \quad (3)
\]

\[
p_{10}(t) = \frac{\text{number of departure at interval } t}{\text{total number of days}} \quad (4)
\]
According to the definition, the chance of status $s_{k+1}$ depends on the previous status $s_k$. In other words, the probability of status $s_{k+1}$ can be calculated recursively starting from the initial status $s_{t_0}$. Therefore, the probability of a vehicle at home ($s_t = 1$) for any given time $t_k$ can be predicted from the status $s_{t_0}$ as

$$p(s_t = 1|s_{t_0}) = \begin{cases} 
p(s_t = 1|s_{t-1} = 1) * p(s_{t-1} = 1) + p(s_t = 1|s_{t-1} = 0) * p(s_{t-1} = 0) & \text{if } t \neq t_0 \\
0 & \text{if } t = t_0 \text{ and } s_{t_0} = 0 \\
1 & \text{if } t = t_0 \text{ and } s_{t_0} = 1
\end{cases} \tag{5}$$

### 3.3 Cost Function and Constraints

This section formulates the problem by introducing the cost function, local constraints, and global constraints.

#### 3.3.1 Cost Function

The demand of the power system usually reflects the market price. As a result, minimizing the peak demand is equivalent to minimizing the overall cost of the system. Then, the cost function can be defined as

$$\min \sum_{i=1}^{N} \sum_{t=t_0}^{kT+t_0} (C_{i,t}^{total})^2 = \min \sum_{i=1}^{N} \sum_{t=t_0}^{kT+t_0} \left(\Delta t^2 C_t^2 * (P_{t,t}^{total})^2\right) \tag{6}$$

where $C_{i,t}^{total}$ represents the total cost of the $i$th house in the time interval $t$, $\Delta t$ is the time step defined as 15 minutes, $C_t$ is the day-ahead market price for the 15-minute interval, and $P_{t,t}^{total}$ is the controllable variable indicating the scheduled total energy consumption of the $i$th house in the time interval $t$. Moreover, since the time step $\Delta t$ is a constant, it can be eliminated without loss of generality. Then, the cost function becomes

$$\min \sum_{i=1}^{N} \sum_{t=t_0}^{kT+t_0} \left(C_t^2 * (P_{t,t}^{total})^2\right) \tag{7}$$

After the cost function is minimized, the calculated $P_{t,t}^{total}$ can be used to determine the scheduled EV charging energy $P_{t,t}^{EV}$ in the $i$th house in the time interval $t$ as
\[ E(P_{i,t}^{EV}) = P_{i,t}^{total} - P_{i,t}^{House} \]  

(8)

where \( E(P_{i,t}^{EV}) \) is the expected charging EV energy of the \( ith \) house during time interval \( t \), and \( P_{i,t}^{House} \) is the energy consumption of the \( ith \) house in time interval \( t \). Also, \( E(P_{i,t}^{EV}) \) is equal to the product of the probability of \( ith \) EV availability in time interval \( t \) and \( P_{i,t}^{EV} \):

\[ E(P_{i,t}^{EV}) = p_{i,t}(s_t = 1 | s_{t_0}) \cdot P_{i,t}^{ EV} \]  

(9)

Then, \( P_{i,t}^{EV} \) can be found by

\[ P_{i,t}^{EV} = \frac{P_{i,t}^{total} - P_{i,t}^{House}}{p_{i,t}(s_t = 1 | s_{t_0})} \]  

(10)

where \( p_{i,t}(s_t = 1 | s_{t_0}) \) is a function of \( s_{t_0} \). Since \( p_{i,t}(s_t = 1 | s_{t_0}) \) is the probabilistic function that depends on the daily driving pattern and the given status of vehicle availability \( s_{t_0} \), the numerical result of the function \( \rho_{i,t}(s_{t_0}) \) can be calculated before the simulation is processed. Therefore, \( \rho_{i,t}(s_{t_0}) \) can be considered as a constant during the computation.

3.3.2 Local Constraints

The charge controller must maintain the charging rate at any time within the charging limits of each EV:

\[ 0 \leq P_{i,t}^{EV} \leq P_{i,t}^{max}, i \in N \cup t \in t_0 + kT, k \in \mathbb{N} \]  

(11)

where \( P_{i,t}^{max} \) is the maximum charging rate of the \( ith \) EV. This constraint can be rewritten in terms of \( P_{i,t}^{total} \) and \( P_{i,t}^{House} \) as

\[ 0 \leq P_{i,t}^{total} \leq \rho_{i,t}(s_{t_0}) \cdot P_{i,t}^{max} + P_{i,t}^{House}, i \in N \cup t \in t_0 + kT, k \in \mathbb{N} \]  

(12)

In addition, the second local constraints can ensure that the total charging energy of each day can fulfill the demand of driving as follows:
\[ \sum_{t=t_0}^{kT+t_0} E(P_{l,t}^{EV}) = D_i^{EV}, \ i \in 1, ..., N \quad (13) \]

Then, (8) can be substituted into (13) as

\[ \sum_{t=t_0}^{kT+t_0} P_{l,t}^{total} - P_{l,t}^{House} = D_i^{EV}, \ i \in 1, ..., N \quad (14) \]

The constraint of total energy consumption over all intervals can be written in terms of \( D_i^{EV} \) and \( P_{l,t}^{House} \) as

\[ \sum_{t=t_0}^{kT+t_0} P_{l,t}^{total} = D_i^{EV} + \sum_{t=t_0}^{kT+t_0} P_{l,t}^{House}, \ i \in N \quad (15) \]

where \( D_i^{EV} \) is the daily energy demand of the \( i \)th EV.

### 3.3.2 Global Constraints

To guarantee the voltage regulated within the system limit, the total supplied energy in each interval should be limited under the designed maximum energy as

\[ \sum_{i=1}^{N} P_{i,t}^{Total} \leq P_{global}^{MAX}, t \in t_0, t_0 + 1, ..., t_0 + T \quad (16) \]

where \( P_{global}^{MAX} \) is the maximum energy in every interval, which can be computed by

\[ P_{global}^{MAX} = \text{real} \left( \frac{\%V_{drop}^{MAX}}{K_{drop} \cdot d_{sub}} \right) \cdot 15 \cdot 60 \quad (17) \]

where \( \%V_{drop}^{MAX} \) is the maximum percentage voltage drop, \( K_{drop} \) is the K-drop factor, and \( d_{sub} \) is the distance from the substation to the node.
CHAPTER 4

DISTRIBUTED COMPUTATION APPROACH

4.1 Networking Setting

For achieving the global optimization of the EV charging, each local controller is required to connect to some controllers in the neighborhood, such that the information can be shared between the connected controllers. By sharing information between controllers, the information can be spread to the entire network. For example, as shown in Figure 4, agent 1 wants to transmit a local message to all other agents in the network. In the first iteration, agent 1 can transmit the local message to its connected controllers (agents 9 and 2). Then, in the second iteration, since agent 9 and agent 2 already have the message of agent 1, they can also transmit that message to their connected controllers, which are agent 0, and agents 5 and 3, respectively. By repeatedly sharing the message to connected controllers, the message reaches every agent in the network.

Figure 4. Distributed network setting.
For spreading the message to every agent in the network, the network setting must have two requirements. First, the communication channels between agents must be bidirectional so that each agent is capable of receiving and transmitting the message to local connected agents. Second, the network can form a minimum spanning tree (MST), which guarantees that any agent’s message can transmit to all other agents, and the maximum iteration of transmission between any two agents is the number of agents minus one.

4.2 Distributed Optimization Problem

According to Zhu et al. [33], the distributed optimization problem with general constraints can be expressed as

\[
\min f(x) = \sum_{i} f_i(x)
\]

\[
s.t. \quad L_i x = 0
\]

\[
A_i x = b_i
\]

\[
g_i(x) \leq 0, \quad i \in I
\]

\[
x \in X_0 = \bigcap_{i=1}^{N} \Omega_i
\]

where \(x = (x(1), x(2), x(3), \ldots, x(4))\) is the collection of the scheduled energy rates in each house, \(x(i) \in R^{96}\) is the 24-hour charging schedule of the \(i\)th EV with a 15-minute interval, \(N\) is the total number of houses, \(f(x): R^{96N} \to R\) is the global cost function or sum of all local cost functions stored in each agent, \(f_i(x): R^{96N} \to R\) is local cost function that is required to be convex, \(L_i \in R^{1 \times 96N}\) is the row vector of Laplacian matrix representing the network connection, \(A_i \in R^{m_i \times 96N}\) is a constant matrix, \(b_i \in R^{m_i}\) is the output vector for the equality constraint, \(m_i\) is the number of equality constraints in the \(i\)th agent, \(g_i = (g_{i1}, g_{i2}, \ldots, g_{ij_i})^T: R^{96N} \to R^{j_i}\) is the
function for inequality constraints, $J_i$ is the number of inequality constraints in the $i$th agent, and $X_0$ is a closed convex set representing the collection of local boundary requirements $\Omega$ for each agent.

In the distributed optimization problem, each agent only has access to the local information, $x_i,f_i(x_i),L_i,A_i,b_i,g_i$, and $\Omega_i$, and the adjoining computed schedule $x_j$. This guarantees the privacy of each user because it can prevent others from obtaining their private information, such as the user’s driving pattern.

### 4.2.1 Conversion of Global Constraint to Local Constraint

To transform from the problem outlined in section 3.3 to the format of the distributed optimization problem, the global constraint is necessary to be localized and stored in one local agent. Therefore, agent 0 is set up in the substation for controlling the total energy output. For agent 0, the local objective function is

$$f_0(p_{\text{Total}}) = 0$$

Then, the global constraint can be converted to the local inequality constraint as

$$g_0(p_{\text{Total}}) = \sum_{i=1}^{N} p_{i,t}^{\text{Total}} - p_{\text{MAX}}^{\text{global}} \leq 0, t \in t_0 + kT, k \in \mathbb{N}$$

The local agent also has the local boundary set as

$$\Omega_0 = \{ p_{k,t}^{\text{Total}} | 0 \leq p_{i,t}^{\text{Total}} \in \mathbb{R}, i \in 1, \ldots, N \cup t \in t_0 + kT, k \in \mathbb{N} \}$$

### 4.2.2 Local Constrains of Each Agent

Since each house can be consider an agent, the local objective function for agent $i$ can be written as

$$f_i(p_{\text{Total}}) = \sum_{t=t_0}^{kT+t_0} \left( C_t \ast (p_{i,t}^{\text{Total}})^2 \right)$$

The local constraints become
\[ \Omega_i = \{ p_{i,t}^{Total} | 0 \leq p_{i,t}^{Total} \leq p_{i,t}(s_{t_0}) \cdot P_{i,max} + P_{i,House}, k = i \cup t \in t_0 + kT, k \in \mathbb{N} \text{ and } 0 \leq p_{i,t}^{Total} \in \mathbb{R}, (k \in 1, ..., N \cap k \neq i) \cup t \in t_0 + kT, k \in \mathbb{N} \} \] (23)

\[ \sum_{t=t_0}^{kT+t_0} p_{i,t}^{Total} = D_i^{EV} + \sum_{t=t_0}^{kT+t_0} p_{i,t}^{House} \] (24)

### 4.3 Distributed Subgradient Algorithm

The distributed optimization problem has already been studied for more than a decade. Zhu et al. proposed a distributed subgradient algorithm to solve the convex optimization problem with general constraints [33]. The general subgradient method can be expressed as

\[ x^{k+1} = x^k - \alpha_k \nabla F(x^k) \] (25)

where \( x^k \) is the solution in the \( k \)th iteration, \( \alpha_k \) is the step size in the \( k \)th iteration, and \( \nabla F(x^k) \) is the subgradient of the convex cost function \( F \) at \( x^k \). Also, \( \nabla F(x^k) \) can be viewed as the estimated error between \( x^k \) and the optimal solution \( x^* \). By subtracting the error, the solution \( x^k \) gradually approaches the optimal solution. From the perspective of the distributed optimal problem, the error in agent \( i \) is caused by the subgradient of the local solution \( x_i \) and the inconsistency of the result between the directly connected agents. Then, the subgradient method can be rewritten as

\[ x_i^{k+1} = x_i^k - \alpha_k E(x_i^k, \sum_{j \in N_i} a_{ij}(x_i^k - x_j^k)) \] (26)

where \( x_i^k \) is the computed solution in the \( k \)th iteration in agent \( i \), and \( E \) is the error function that depends on the local solution \( x_i^k \) and the solution computed by the directly connected agent \( x_j^k \).

If the error is significantly large, such as larger than a defined threshold, then the subgradient method will compute the solution for the next iteration. Otherwise, the computational process will be terminated. Since the error also depends on the computation of neighbor agents, the small error indicates that the difference of the computed solution between neighbor agents is small.
Therefore, all agents in the system can reach an agreement; therefore, the algorithm is referred to as consensus algorithm [35].

According to Zhu et al. [33], the error function $E$ can be further expressed as

$$E = \left[ -x_i^k + P_i (x_i^k - \partial f_i (x_i^k)) - \sum_{j \in N_i} a_{ij} (x_i^k - x_j^k) - \sum_{j \in N_i} a_{ij} (\lambda_i^k - \lambda_j^k) + A_i^T \mu_i^k \\
- A_i^T (A_i x_i^k - b_i) - \partial g_i (x_i^k) \left( \gamma_i^k + g_i (x_i^k) \right)^+ \right]$$

$$\lambda_i^{k+1} = \lambda_i^k + x_i^k$$

$$\mu_i^{k+1} = \mu_i^k - A_i x_i^k + b_i$$

$$\gamma_i^{k+1} = \left( \gamma_i^k + g_i (x_i^k) \right)^+$$

$$P_X (u) = arg \min_{v \in X} ||u - v||$$

$$f^+ = (\max (0, f (1)), \max (0, f (2)), \ldots, \max (0, f (3)))$$

where $\lambda_i^k, \mu_i^k$ and $\gamma_i^k$ are the Lagrangian multipliers that can be chosen before the computational process begins, $\mu_i^k$ and $\gamma_i^k$ ensure that the equality constraints and inequality constraints, respectively, comply, operation $f^+$ maps the vector $f$ to positive space thereby ensuring that the solution is consistent of an inequality constraint, and function $P_X (u)$ is a projector operator that maps the input to the closed points in the constraint set $X$.

A challenge of using the subgradient method is to control the convergence rate. Therefore, to reduce the overall computational time, it is necessary to design the step size in every iteration and the initial solution $P_{Total}^0$. 

20
4.3.1 Step Size

In the subgradient method, the choice of step size is very important because it directly controls the convergence speed and the maximum error between the calculated solution and the optimal solution, based on the following equation:

$$\lim_{k \to \infty} f^k - f^* < \epsilon(\alpha)$$

(28)

where $f^k$ is the calculated solution in the $k$th iteration, $f^*$ is the optimal solution of the problem, and $\epsilon(s)$ is the error function of the step size $\alpha$. If $\alpha$ is large, then the convergence speed is fast, but the resulting error is also large. In opposition to this, if $\alpha$ is small, then the convergence speed is slow, but the resulting error is small. To balance the convergence speed and the resulting error, the initial step size needs to be large, and the step size should be decreasing at every iteration. Therefore, the step size function is chosen as

$$\alpha(i) = \frac{\gamma}{\text{iteration}+\beta}$$

(29)

where $\gamma$ and $\beta$ are constants that are chosen before the simulation begins.

4.3.2 Initial Solution

The choice of the initial solution is also critical and can determine the number of iterations required for the simulation. For example, if the initial solution is also the global optimal solution and meets all constraints, then the subgradient process can be stopped in the first iteration. In contrast, if the distance between the initial solution and the global optimal solution is large, then numerous iterations are required to complete the simulation.

One of the reasonable choices of initial solution is the collection of the local optimal solution $P_i^{Local\ Total}$ from agent 1 to agent $N$ because the collection certainly fits the equality constraints in agent 1 to agent $N$. Also, the collection has minimized all local cost functions, so the global cost function is also minimized because the global cost function is the sum of all local
cost functions. The only condition that must be verified is the inequality constraints in agent 0. If
the collection also matches the requirement of the inequality constraints, then the collection is
certainly the global optimal solution. Therefore, the required iteration is one that checks the
global constraint. Even though the collection does not match the inequality constraints, the
collection is close enough to the optimal solution, so the required iteration is small.

Therefore, the initial solution can be obtained by solving the local optimal problem in
agent \(i, i \in 1, ..., N\), as follows:

\[
f_i(P_i^{Local\ Total}) = \sum_{t=t_0}^{kT+t_0} \left( C_t^2 \ast (P_i^{Local\ Total})^2 \right)
\]

\[
\Omega_i = \{P_{i,t}^{Local\ Total} \mid 0 \leq P_{i,t}^{Local\ Total} \leq \rho_{i,t}(s_{t_0}) \cdot P_i^{max} + P_i^{House}, t \in t_0 + kT, k \in \mathbb{N} \}
\] (30)

\[
\sum_{t=t_0}^{kT+t_0} p_i^{Local\ Total} = D_i^{EV} + \sum_{t=t_0}^{kT+t_0} p_i^{House}
\]

where \(P_i^{Local\ Total} \in \mathbb{R}^{96}\) is the optimal solution of the local optimal problem. This problem is a
simple quadratic programming problem, which can be solved by Karush-Kuhn-Tucker (KKT)
conditions:

\[
\nabla f_i(P_i^{Local\ Total}) + \sum_{t=t_0}^{kT+t_0} \mu_i \nabla (-P_i^{Local\ Total}) + \sum_{m=t_0}^{kT+t_0} \nu_m \nabla (P_i^{Local\ Total} - \rho_{i,m}(s_{t_0})) \cdot P_i^{max} + P_i^{House}) + \lambda \nabla \left( \sum_{t=t_0}^{kT+t_0} P_i^{Local\ Total} - D_i^{EV} - \sum_{t=t_0}^{kT+t_0} P_i^{House} \right) = 0
\] (31)

After finding \(P_i^{Local\ Total}\) for all agents, each agent can transmit its local optimal solution
like the example shown in section 4.1. After \(N - 1\) iterations, each agent has the collection of the
local optimal solution \((P_1^{Local\ Total}, P_2^{Local\ Total}, ..., P_N^{Local\ Total})\), which is also the initial solution
of the subgradient algorithm.
4.4 EV Charging Scheduling Process

The complete EV charging scheduling process is shown in Figure 5. Every 15 minutes, the scheduling process will run once to determine the EV charging schedule for the next 15 minutes. Steps in the EV charging scheduling process are as follows:

Step 1: The local controller monitors the state of EV at $t_0$.

Step 2: Based on the status at $t_0$, the local controller computes the probability of EV availability, updates the data, and formulates the local cost function and local constraints.

Step 3: The local controller initializes the attempted solution by minimizing the local cost function. Then, the local controller spreads the solution to agents in the network and also resets the step function.

Step 4: The local controller applies the subgradient method shown in section 4.3 and computes the change of the solution.

Step 5: If the change of solution is less than a threshold, then the controller stops the computing process and outputs the EV charging schedule of time $t_0$; otherwise, step 4 is repeated.

Step 6: The controller sleeps and repeats step 1 after 15 minutes.
Figure 5. EV scheduling process.
5.1.1 Electric Vehicle

The electric vehicle data was sampled from a total of 923,573 pieces of data from the National Household Travel Survey. Figures 6 to 8 show one of the samples. Figures 6 and 7 show the probability of arrival to departure and probability of departure to arrival, respectively. Figure 8 shows the real EV charging status, where 1 means that the EV is available to charge, and 0 means that the EV is unavailable to charge. The battery demand is calculated based on the travel data. It is assumed that 3.6 kW is consumed every mile.

Figure 6. Probability of arrival to departure.
Figure 7. Probability of departure to arrival.

Figure 8. Real EV charging status.
5.1.2 Cost of Electricity

Figure 9 shows the day-ahead market price of one day. The maximum price is $0.05909/kWh at 18:00, and the minimum price is $0.03026/kWh at 3:00.

![Market Price Graph](image)

Figure 9. Market price.

5.1.3 House Load

Figure 10 shows the load multiplier, which represents the energy consumption pattern of houses. The maximum load of houses is in the range of 1.5 kW to 7 kW. The average and standard derivation of the maximum load of the houses are 4 kW and 1.28 kW, respectively.
5.1.4 Step Size and Charging Setting

The step size was chosen to be \( \alpha = 1000/(\text{iteration} + 2000) \). The charging set was assumed to be level 2 charging, which can provide 19,200 W of maximum power. The threshold of the calculated error is 0.025 kWh per 15 minutes, which is approximately 0.5% of the maximum charging power rate.

In the follow sections, three cases (5 EVs, 20 EVs, and 100 EVs) are used to examine the performance of the proposed computing process.

5.2 5 EV Case

Figure 11 shows the network connection of the 5 EVs system. Figure 12 shows the result of immediate EV charging and the result of scheduled EV charging. It shows that the peak demand is reduced from 9.976 kWh to 2.637 kWh. Most of the demand is shifted to the period
between 0 a.m. to 7 a.m. Figure 13 also shows convergence plot. It shows that the error between agents is gradually decreasing.

Figure 11. Network connection of 5 EV system.

Figure 12. Total energy consumption of 5 EV system.
5.3 20 EV Case

In the 20 EV case, the results of different types of networks are compared. Figures 14 and 15 show the network connections of maps 1 and 2, respectively. The network connection of map 2 is that of a managed service provider (MSP), which simulates the situation that more than one communication links have failed to function. Figure 16 shows the EV charging schedules of maps 1 and 2. The computed result of the two situations is nearly identical. However, the simulation in map 2 requires more iterations to converge, according to Figure 17. The maximum iteration of the simulation in map 1 is 647, while the maximum iteration of the simulation in map 2 is 1738. The average iteration of the simulation in map 1 is 118.54, while the average iteration of the simulation in map 2 is 301.76. This observation explains that due to the communication failure, a message transmitting from one agent to other agents travels along a longer path,
thereby requiring more iterations to complete the task. Therefore, the number of overall iterations is increased. In the 20 EV case, the simulation still shows good performance in the results. In this case, the peak demand is reduced from 13.76 kWh to 12.51 kWh.

Figure 14. Network connection of map 1.

Figure 15. Network connection of map 2.
Figure 16. EV charging schedule of 20 EV system.

Figure 17. Number of iterations of the simulation.
Figure 18. Total energy consumption of the 20 EV system.

5.4 100 EV Case

Figure 19 shows the connection of the 100 EV system. Figure 20 shows the result of immediate EV charging and the results of scheduled EV charging, indicating that the total energy consumption is reduced from 75.33 kWh to 58.96 kWh. Figure 21 shows that the result is sensitive to vehicle availability due to the change of vehicles’ initial statutes. The graph shows that the standard deviation is between 0.1306 and 7.45. The minimum point at the 84th interval is an expectational point on the graph which has a nearly zero standard deviation because the houses’ electricity consumption reaches the peak during the period. Therefore, all calculations agree that no EV should be charged during the 84th interval.
Figure 19. Network connection of the 100 EV system.

Figure 20. Total energy consumption of the 100 EV system.
Figure 21. Sensitivity plot of the result with different initial statuses.
CHAPTER 6
CONCLUSION AND FUTURE WORK

6.1 Conclusion

This thesis proposed a distributed computing process to obtain the optimal electric vehicle charging schedule. In the numerical results, data from the National Household Travel Survey was used to test the performance of the proposed control process. It was shown that the proposed control process can effectively reduce the maximum peak load.

6.2 Future Work

Future work in this area can be the following:

- Integrate a more complex model of the EV availability, EV demand, and cost of electricity to the proposed process.
- Consider the impact of the charging schedule on the market price in the process.
- Address the optimal step size of the process, which was not done in this thesis.
- Since convergence rate highly depends on the network structure, improve the design of the network structure, which can reduce the overall computational time.
REFERENCES


REFERENCES (continued)


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