A CROSS-VALIDATION OF PRIMARY PERSONALITY STRUCTURE IN THE 16 P.F. BY TWO PARCELLED FACTOR ANALYSES

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ABSTRACT

Since most recent factorings of primary personality structure as based on the 16 P.F. item domain have been by scales, they just conceivably could yield a simple structure perpetuating a past structure. Two new analyses have been made, one here, one prior, setting scales aside and using 46 "radial parcel" variables covering the 184 items. Radial parcels of four items each have built up quite regardless of scale groupings.

The present analysis on 250 students and the prior one on 780 adults yielded 22 and 21 factors respectively, statistically significant simple structures in both and a substantial invariance (matching) of factors between them by congruence and salient variable similarity coefficients. Further analysis, beyond the present, shows that the factors found here (a) match when converted to loadings on items, the recognized 16 P.F. factor patterns, and (b) yield the patterns of second-order factors that would be expected from the identification by item content given them as primaries. Additionally, new primaries appear here beyond those in the 16 P.F., and they also match from the present to the prior parcelled analysis. However, one or two poor matches — E, H, M, Q_2, and Q_4 — also result, which could be due to the substantial differences of population in the studies used in the cross validation.

CLARIFICATION OF THE ISSUES IN DEBATE

For over 25 years, Cattell's theory (1946, 1947, 1950, 1957) of personality structure has been that there are between 20 and 25 primary and 8 to 12 second-order factorial source traits in items of "normal" behavior. The theory further asserts that the particular concepts involved — which have received much experimental development — appear equally in Q- and L-data. That is to say, the same instrument-free factors can be found from jointly factoring questionnaire and life ratings by observers (Cattell, 1973). Although more than a dozen other investigators have supported these general conclusions, a minority — Eysenck (1971), Comrey and Duffy (1968), Howard and Browne (1971) — have categorically disagreed, asserting that there are only 3 to 10 primaries, several of them alleged-
ly different in nature from the 16 P.F. concepts. Sells, Demaree and Will (1970), on larger samples, believed 11 factors to exist and took an intermediate position identifying most with one or another of the primaries A through Q6 located by Cattell.

Naturally, the number of factors discovered will increase somewhat with larger item coverage encompassed, and it is fortunate for sanity and progress in this often methodologically crude and chaotic realm that the chief protagonists have recognized the need for referring to a standard basis — in fact, the personality sphere of variables as initially defined (Cattell, 1946, 1950) and now represented in 184 items in the 16 P.F. As further research shows (Cattell & Gibbons, 1968; Sells, Demaree & Will, 1970), much the same domain is covered in Guilford’s questionnaire (Guilford & Zimmerman, 1949-55) and in the “normal” part of the MMPI (Cattell & Bolton, 1969; Delhees & Cattell, 1971). The disagreement does not arise primarily about the basis of variables, therefore, but about (a) the required dimensionality and (b) the rotation within it.

The debate between the two groups of questionnaire researches is thus not merely one of number of factors, but also of nature of factors, because the rotational positions with underfactoring (Eysenck, Howarth & Browne, 1970) are unable to find a really tight simple structure (Cattell, 1966) and lead to conglomerates of primaries and second orders. The chief argument that recurs on the opposite side, when Comrey, Howarth, and Eysenck have been unable to find and support the primaries in the 16 P.F., the High School Personality Questionnaire, etc., is that the programmatic research that for 20 years and 40 researches has checked the structure and increased the scale validity for these 16 to 20 primaries is in some way bound by the traditions of an “establishment.” Since the same span of actual items has now been covered by both groups, this can no longer be ascribed to a choice of items favoring one structure, and the second group has therefore concentrated its attention on objections to factoring of scales as variables, instead of items.

It is, prima facies, a possible argument that a set of 64 scales (as in the 16 P.F., forms A, B, C, and D), growing out of simple structure found in items, will tend to perpetuate any falsities in the original simple structure position. However, this argument has perhaps been sufficiently refuted by (a) Cattell’s point that the number of alternative possible 60 variable hyperplanes in 64 variables is enormous, permitting other structure to emerge if they are, in fact, better; (b) the demonstration (Cattell & Vaughan, in press) that factoring by items and factoring by parcels yields the same (congruent) conclusions, as theory would expect; and (c) the psychometric argument that factoring by parcels, such as has predominated in this last 10 years, can lead, by progressive rectification (Cattell, 1973), to item substitutions that lead to the same improvements as would result from item factoring.

Faced with this accumulating refutation of attacks, there remains for the opposition, i.e., those opposing the 20- and primary-factor solution, with the identities given to traits A, B, C, etc., on the 16 P.F., only one possible far-out style of attack. It is to assert that the blind rotations to simple structure have achieved their matching coherence by some kind of accidental recognition of variables in the rotation process. In all our practice, variables at the rotation stage are represented only by numbers, and in the plots only by points. Of course, in a
familiar domain one sometimes saves time by Procrustes, or knowledge of the
variables, by getting near where the final search for simple structure can best
begin; but in all vital exploratory work in the early 16 P.F. rotations were entirely
blind. Nevertheless, when possibilities exist in accompanying tables for identifying
items or scales, common sense and laboratory experience tell us that instances
are known in science where research assistants have tried to "help" or where
accidental and unconscious leads occur. (One does not have to invoke ESP!).

Accordingly, it has seemed desirable to break away completely from opera-
tions with the recognizable items and familiar scales and begin as freshly as in the
1946-1956 studies that gave the original view of about 20 primaries and the 16 of
them used in the 16 P.F. At the same time, we planned to have the data in such
form that there could be no way whatever of sensing in the rotation the identities
of variables.

To keep up with what is happening in the dispute about the dimensionality, we
should note that while it has been going on Cattell and his coworkers have added
new questionnaire areas that, by their analyses, add 7 more normal primaries and
12 other primaries found only in pathological-behavior items. The former, for
some time known as the "seven missing factors," have been recognized in rela-
tively ill-defined, low-loaded form for some time as the factors in the personality
sphere of 184 or more normal items that cause the number of primaries to be
found in the 16 P.F. to indicate approach totals of 19 to 23 primary factors, by
various factor number tests and with various additions of items. They have recent-
ly been tied down with precision (Cattell & DeVoogd, 1973; Marshall & Cattell,
1974). The 12 new primaries in pathological responses have similarly been preci-
sioned in three recent researches (Delhees & Cattell, 1971; Cattell, 1973; Cattell
& Sells, in preparation). Within this framework, the present research confines
itself to checking on the number and nature of factors in the core of items before
the 7 and the 12 dimensions were added, though it is recognized that some of the
7 normals are likely to appear with small variance.

Objections to factoring homogeneous parcels or short scales (of, say, 4 to 12
items) have been made principally by Eysenck and by Howarth and Browne
(1971), though on no explicit statistical argument that the present writers can
discover. An argument of some apparent merit raised by others against it is that
scales might fix and perpetuate a simple structure previously reached. Comrey
and Duffy (1968), Nunnally (1967) and others have, on the other hand, objected
to the instability of items, and Comrey has based all his work on homogeneous
parcels. Cattell has argued (1973) that theoretically, statistically, the structure
from parcels and items should be the same, provided parcels are constructed on
correct principles as radial parcels (Cattell & Burdsal, 1973) and all of the same
size; the rotation has been shown to be more exact by parcels (Cattell, 1973a).
Both the present study and that with which it is compared will be found, inciden-
tally, to support this.

Even the at-first persuasive argument that factoring scales as variables may
tend to perpetuate a scale structure that is wrong has been questioned by Cattell
(1973), who points out that in successive factorings of, say, 64 variables consti-
tuted of 4 equivalent scales for each of 16 factors, there should, by the progressive
rectification principle, be progressive movement toward better simple structure.
Nevertheless, the nature of the personality primaries is of such vital importance to
personality theory that their exact nature should be, like Caesar's wife, above suspicion. It happens that their definition in the 16 P.F., though it began in item factoring, has for the last eight refactorings rested on parcels or scales, and the suspicion may exist in psychologists unfamiliar with the above arguments that the scales have prejudiced the rotation. Consequently, it has seemed timely to refactor with small parcels put together by radial homogeneity, quite independent of and more numerous than the existing scale variables. The vital decisions in rotation would here be inevitably blind, because the parcel contents would be unknown to the rotator, and only when the factors thus fixed are projected on the items will the agreement or disagreement with past work be revealed.

EXECUTION OF THE EXPERIMENT

The design to be described here introduces technical innovations needing adequate description. Fortunately, the details of what we call radial-parcelling double-factoring design can be set out elsewhere (Cattell & Burdsal, 1973). The principle of radial parcelling means putting those items (in this case, 4) in one parcel, the vectors of which lie angularly close together in the common factor space. It has been shown (Cattell, 1973) that putting together those items merely with acceptably significant mutual intercorrelations is not satisfactory, that it will reduce definition of simple structure and, without special precautions, may obliterate it altogether. The index of radial closeness has values substantially different from the correlation coefficient.

In this experiment, the 184 items (3 buffer items dropped) of Form A of the 16 P.F. were given to 257 undergraduates (240 after dropouts) as described elsewhere (Burdsal & Vaughan, 1974), the main condition being the subjects' understanding that results would be anonymous and not officially used. The grouping of the 184 items into 46 radial parcels was carried out, as part of the cross-validation concept, by the relations among items found in the prior group of 780 general adults that we shall hereinafter call group A to distinguish from the sample B that comprises the present study.

Although the full rationale of radial parcelling per se is set out elsewhere (Cattell & Burdsal, 1973), the essential steps in our procedure must be briefly listed. They include:

1. a principal-components analysis of the 184 items and application of the scree test to determine the number of factors (as shown in Figure 1, the results at the item level from populations A and B agreed in coming close to the number obtained in other studies on the 16 P.F. — 18 to 20 — and in being mutually close — 19 for group A and 20 for our present group B).

2. iterating communalities to 19 factors (since clusters are to be based on A) and extracting the unrotated V_0 matrices (access to these matrices and the original R matrix is supplied in Cattell, 1972; Burdsal & Vaughan, 1974).

3. applying the radial-parcelling program to yield cosines of angles among the 184 variables \( \cos \theta_{ab} = \frac{r_{ab}}{h_a h_b} \) and then grouping them first in 92 parts, assigning each item to another with which it has the highest cosine (or highest remaining cosine).
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(4) Calculating the mean vector for each pair, proceeding to a 92 x 92 inter-pair cosine matrix and again putting closest vector together, finally yielding 46 radial parcels of 4 items each.

Scores for these 46 radial parcels (item contents listed in Cattell & Burdual, 1973) were now calculated giving equal nominal weight to each item, and a 46 x 46 correlation matrix among parcels was calculated as a basis for factoring, as described below.

The goal of cross-validation was by this design not the limited one of comparing structures for factoring, as described below. Instead, the goal of cross-validation was by this design not the limited one of comparing structures for two samples from the same population but aimed at examining the degree of invariance across different populations. Thus, group A was conceived as a general-population mature-adult, male and female population, ranging in age from 16 to 28 and averaging 17.2 years. Although not precisely stratified for age and status, this group of 780 is as near to being so as any group yet factored. Group B was conceived as a decidedly selected group (though male and female) ranging in age only from 17 to 24 (mean = 20) and having the socio-educational selection of university undergraduates.

OUTCOME OF FACTOR ANALYSIS OF THE 46 PARCELED VARIABLES

A scree test was applied to the principal component’s latent roots from the two 46 x 46 correlation matrix of sample B. Although the theory of the scree test (Cattell, 1966) has not yet been extensively discussed, the test has a record of almost perfect empirical performance in locating the number of factors wherever that number has been first firmly established by ulterior or prior evidence. The need for a test of this degree of reliability needs particularly to be recognized if any integration is to be reached by the “underfactoring” psychologists — Eyssenck, Howarth, and Browne, and perhaps Comrey — who have hitherto decided the number of factors subjectively, or by unchecked methods or even by immediate convenience. The number of factors in the parcelled analysis B is virtually the same as that in group A (namely, 21, Vaughan & Finkbeiner, 1973) and in the two item factorings, namely, 19 and 20).

The present indication of a slightly greater number being consistently indicated for parcels than for items fits theoretical expectations. For the summation of common factor variance in parcels should occasionally bring into visibility factors of too small variance to pass the threshold of significance when isolated in items. Thus, another, though slight, advantage in using parcels instead of items comes to light beyond that merely of higher reliability of the variables.

Communalities were now iterated to the 22 factors thus indicated, and the rotation process for maximum simple structure was begun on the Vo’s (46 x 20 and 46 x 22) obtained. In accordance with the point made in several examinations of rotations elsewhere — that no automatic program, especially no analytical program, can directly reach acceptable maximum simple structure — the Vo was rotated from an initial oblimax solution by Rotogram. Sometimes called Rotoplot, this facilitates by computer aid the process of blind rotation to simple structure.
using human skill. Nineteen overall rotations sufficed to reach a maximum, which, incidentally, is 13 shifts fewer than for the item factoring.

The hyperplane count — 81% at ± .10 and 58% at ± .05, was remarkably close to that for the prior larger sample (81% and 59%). The significances of the simple structure for the factors in each, as calculated by the Cattell, Finkbeiner, and Vaughan (1973) method, were all above \( p < .01 \).

The care required in hand rotations and the length of time given in some 20 rotations are perhaps justified by the significances of hyperplane counts reached. It is noteworthy that by parcels here, compared to the corresponding item factorizations elsewhere (Cattell, 1972; Burdsal & Vaughan, 1974), the hyperplane percentage counts are decidedly better, and particularly at the desired narrower hyperplane values of ± .05. The above substantial and quantifiable answer to the suspicion cast by Eysenck and Howarth on factoring parcels and scales rather than items will hopefully once and for all remove this irrelevance from sound technical discussion of personality structure.

The factor pattern reached at this simple structure position is shown in Table 2. The R, Vo and L matrices that are available at ASIS.

THE MATCHING AND INTERPRETATION OF PRIMARY SOURCE TRAIT FACTORS

The 46 item parcels are the same in the two studies we have called A and B. Conceivably, this use of A-item groupings in the B data could be responsible for some slight impairment of structure in B (see below) relative to A, but it is the only way in which precise matching can be evaluated between the two studies by applying the congruence coefficient or the salient variable similarity index.

Since this is the first time such a two-population blind parcel factoring has been rigorously tested for matching, it has seemed desirable to examine all \( 22 \times 21 = 463 \) possible interfactor matches across the two studies. The reader should be careful not to judge the significance of the absolute values of the congruences by the familiar significances for correlations, since congruences are significant at noticeably lower values. Actually, the congruences for the factors considered matched in Table 1 are all significant at the \( P < .05 \) level except for the new factor 22.

There is a rough agreement of \( r_c \) and \( s \) that the earlier factors are matched, with higher significance and that some doubt exists on E, H, M, Q1 and Q2.

SUMMARY

The present experiments present the results of a sample (B) of 240 students, and to the matching of its results with those from the same parcels on a sample (A) of 780 general adults. However, for the sake of making meaningful psychological reference we have finally invoked the interpretations of the factors set out elsewhere (Vaughan & Finkbeiner, 1973; Cattell, 1973). In these last studies, further processes have been carried out that project these factors on to items and adequately identify them thereby with the standard index coding for Q-data.


TABLE 1

CONGRUENCE COEFFICIENTS

| Part (a) | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| 1        | 11| 09| 07| 14| 02| 06| 00| 04| 01| 25| 40| 01| 05| 15| 07| 19| 34| 14| 03| 08| 33| 02|   |   |   |   |   |   |
| 2        | -00| 43| -00| -06| 06| 30| -34| -03| 06| -40| 10| 16| 10| 17| -15| 24| 09| 10| 02| 07| 11| -07|   |   |   |   |   |   |
| 3        | 22| 12| 22| 23| -16| 00| 13| -03| -34| 02| -06| -03| 17| 15| -15| 00| 06| -17| -12| -24| 11| 14|   |   |   |   |   |   |
| 4        | -19| 01| -33| 26| -23| 03| -02| 02| -13| -18| -24| -05| -04| -04| -01| 11| -04| 22| 08| -14| 06| 25| -16|   |   |   |   |   |
| 5        | 07| 24| -12| 16| -11| 05| 11| 02| 31| 02| -34| 15| 02| 00| 04| 04| -04| 04| -09| -03| 17| 05| -02|   |   |   |   |   |
| 6        | -04| 01| -20| 00| 01| 22| -02| 04| 04| -02| 08| -26| 13| -19| 15| 14| -11| -44| -15| -12| -00| 07|   |   |   |   |   |
| 7        | 01| -11| -18| 27| 32| -02| 53| 13| 06| -05| -07| 10| 12| -08| 00| 03| -00| 15| -17| 03| -27| 03|   |   |   |   |   |
| 8        | 06| -01| -18| -03| 14| 14| -08| 79| -12| 12| 14| 10| -05| -04| -02| 09| -00| 15| 08| -16| -08| 04|   |   |   |   |   |
| 9        | -00| -04| -13| 35| -04| -46| -00| -07| 50| -28| 10| 03| -08| 03| 14| 27| 07| 12| 07| -38| 10| -16|   |   |   |   |   |
| 10       | -43| -02| 05| 14| -15| -01| -06| -01| 02| 13| -13| 08| -24| 03| -09| 07| 02| -06| 12| -12| 05| 04|   |   |   |   |   |
| 11       | -03| -07| 03| 10| -05| 13| 02| 02| 12| -05| 09| -05| -05| -04| 25| -01| -11| -02| -05| -00| 06|   |   |   |   |   |
| 12       | -05| -07| -26| -30| -14| 19| -39| -04| -15| 06| -14| 51| -26| 01| -03| 30| 06| 26| 18| 25| 01| 07|   |   |   |   |   |
| 13       | -34| 09| -15| -10| 12| -18| 02| 03| 34| -03| -06| 12| 05| 05| 09| 09| -09| 03| -01| -24| 07| 08| -07|   |   |   |   |
| 14       | 05| 07| 07| -04| -14| -15| -07| 01| 00| -10| -07| -20| -06| 35| 03| 20| -31| -05| 22| -40| -25| 02|   |   |   |   |   |
| 15       | -37| -01| 16| -06| -09| 11| 01| -17| -08| -06| -01| -18| 06| -01| 66| 03| -00| -11| 07| 02| 00| -09|   |   |   |   |   |
| 16       | -15| -02| 14| -16| -02| -02| 03| -10| -12| 01| -13| -02| 13| 17| 25| 03| 01| 18| 10|   |   |   |   |   |   |
| 17       | 36| 38| 08| 23| 24| -29| -05| 09| 08| -11| -37| -10| 05| -02| -00| -09| 48| -13| 03| -17| 47| -09|   |   |   |   |   |
| 19       | -20| -07| -25| -07| -21| -19| 08| -07| 06| -05| -02| 08| -09| 06| -04| 26| -12| -07| 52| 22| -09| 08|   |   |   |   |   |
| 20       | -27| -03| -05| -09| -18| -13| 13| 05| -01| 22| 10| -13| 32| 25| -06| 11| 17| 23| 24| 19| 16| -03|   |   |   |   |   |

Taking congruences over .30 as significant, we find the highest \( r_{ij} \) in the column is also the highest in the row for factors A, B, C, F, G, I, L, N, G, Q, and Q. This leaves E, H, M, Q, and Q less confirmed.
primaries: A, B, C, etc. With this reference of the present results to the customary titles, we may summarize thus:

1. The same 184 items from 16 P.F. Form A were grouped into the same 46 radial parcels as were determined in a prior sample from a different population. The factoring indicated 22 factors as against 21 in the other sample. After unique simple structure of a high degree of significance had been attained by 19 overall rotations results, we evaluated matches with the prior research.

2. The cross-validation of factor patterns was tested by both the congruence coefficient, $r_c$, and the salient variable similarity index, $s$, which in general agreed very well. However, the highest $r_c$ in the row was the highest $r_c$ in the column only in 11 of the 21 comparisons. This could well happen despite all matchings being correct due to the sampling error on $r_c$ when only 46 entries are involved. The matching verdict accordingly rests for the remaining 8 (?) pairs on a composite evaluation requiring (a) existence of a $P < .05$ or better significance of $r_c$ match; (b) simultaneous presence of a significant $s$ index match, and (c) in the case of completion of matching of the four comparisons: a with b and c with d, against a with d and b with c by finding a higher total matching score in the matching foursome arrangement adopted.

3. The present analysis shows the same kind of superiority to item analysis in hyperplane significance and in penetrating to one or two factors of smallest variance as in the other parcelled study. However, its properties are not quite as good as in the other parcelled study, first, in simple structure, and, secondly, as shown elsewhere (Vaughan & Finkbeiner, 1973) in less clear matching with the "ideal" (hypothetical as defined by the 16 P.F. key representing best researches). This could be due to the smaller sample (250 versus 780) and/or to the radial parcels being less than the optimal ones, because the four items in each had to be kept identical with those in the cross-validating sample. For the moment, we suspect the smallness of sample, because similar blurring occurs in the "ideal" matching when the data are factored by items (Burdsal & Vaughan, 1974). However, still more important is probably the real difference of population — general adults averaging 30-plus years instead of selected students averaging 20 years. When this is considered, the agreement of these two entirely independent and blindly rotated factor analyses is striking evidence of the persistence of identity and functional unity of the 16 primaries with which psychologists have for 20 years been most familiar. The evidence extends also to adequate invariance of, at any rate, four others not yet identified (but probably among the seven missing factors recently confirmed by Delhees & Cattell, 1971, and Cattell & DeVoogd, 1973).

4. The direct matching procedures here are not, however, the end of the examination of the identity of these primaries. Further analyses are reported elsewhere (Cattell & Vaughan, 1973) that take the correlations among primaries obtained at these unique simple structure rotations and find the second-order structures. The conclusion can be stated here, when one incorporates the detailed reports elsewhere, that both further second-order results and the more extended four-cornered direct matching of the "ideal" keyed 16 P.F. factors (representing the eight researches in its construction), the item A, the item B, the parcel A, and the present parcel B that a substantial inner consistency has been revealed. The latter is a quality essential to any general personality theory. However, perhaps the unique value of the present research has been that it abandons the scale struc-
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tures of the 16 P.F. and starts afresh with items, grouped objectively into radial parcels, yet finds essentially the same number and nature of factors as in the original programmatic researches on the test.

REFERENCES


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