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A NOTE ON TESTING THE SPHERICITY HYPOTHESIS WITH BARTLETT'S TEST

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ABSTRACT

This study employs computer simulations of correlation matrices to evaluate sample size requirements with different numbers of variables for testing departures from sphericity, implying the presence of group factors. Inductive approximations of average off-diagonal correlations due to Lawley (1940) and to Baggaley (1982) were found to be biased for correlation matrices with homogeneous elements. This bias led to some error in estimation by Baggaley of the appropriate sample size for evaluating departures from sphericity. Lawley's approximation was clearly inferior to Baggaley's approximation.

INTRODUCTION

In a recent issue in this journal Baggaley (1982) used the formula for Bartlett's (1954) sphericity test for a correlation matrix to estimate the average absolute off-diagonal correlation. This estimate was derived inductively from an analysis of seven empirical correlation matrices. This estimate was then used to solve for the ratio of observations to variables required to reject the sphericity hypothesis with the number of variables ranging from 20 through 100, and the estimated average absolute off-diagonal correlation ranging from 0.10 to 0.50.

Baggaley's estimate of the average absolute correlation was based on the natural logarithm of the determinant of the correlation

$$Q = [-\ln \det(R)]/p,$$

where p is the number of variables. Another related approach of estimating the average correlation from -ln det(R) would be to use Lawley's (1940) estimate of this quantity from the sum of squared lower off-diagonal correlations. The resulting estimate of the average absolute off-diagonal correlation is the square root of minus one times the natural logarithm of the determinant after dividing

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by the number of lower off-diagonal correlations (i.e., the degrees of freedom for Bartlett's test)

$$L = \sqrt{[-\ln \det(R)]/p(p-1)/2}.$$

Both Baggaley's and Lawley's estimates of the average absolute off-diagonal correlations are approximations. The accuracy of these approximations will depend on the number of variables and the size and pattern of the off-diagonal correlations.

In the present study, rather than determine the average absolute off-diagonal correlation from the determinant of the correlation matrix, the problem was approached in the reverse direction. The determinant of the correlation matrix was calculated from the off-diagonal correlations. The analysis employs a patterned matrix that yields an exact expression of the determinant from the average absolute off-diagonal correlation. In addition to revealing the ratio of sample size to number of variables required to reject the sphericity hypothesis for the matrices studied, the present study also gives some indication of the amount of bias to be expected in employing Baggaley's and Lawley's approximations.

METHOD

Following Baggaley (1982), correlation matrices were analyzed with average absolute off-diagonal correlations of 0.10, 0.15, 0.20, 0.30, 0.40, and 0.50, and number of variables investigated of 20, 30, 50, 70, and 100. All matrices analyzed were equi-correlation matrices (i.e., all off-diagonal correlations were equal). The largest eigenvalue of an equi-correlation matrix is equal to $1 + (p - 1)r$, where p is the number of variables and r is the constant off-diagonal correlation. All the other eigenvalues are equal to $1 - r$. The determinant for Bartlett's sphericity test was obtained from the product of the eigenvalues. For each matrix analyzed the sample size required to reject the sphericity hypothesis was obtained iteratively by using subroutines from the International Mathematical and Statistical Libraries (1982) for the percentage points of the chi square distribution, and for the chi square value for a given degrees of freedom and percentage points. A test size of 0.05 was employed in the analysis. All computations were done on a CYBER 170/835 with a 60 bit word length.

RESULTS AND DISCUSSION

The results are reported in Table 1 organized by size of off-diagonal correlation. The amount of bias across all levels of off-diagonal correlations and number of variables for Baggaley's and for Lawley's approximations are reported at the bottom of Table 1 in terms of root mean square error (RMSE). Lawley's approximation is clearly inferior to Baggaley's approximation since the RMSE associated with Lawley's approximation is more than three times as large as the RMSE associated with Baggaley's approximation. The bias in terms of RMSE across the equi-correlation matrices with off-diagonal correlations of 0.10, 0.15, 0.20, 0.30, 0.40, and 0.50 for Baggaley's approximation was 0.04, 0.05, 0.04, 0.03, 0.05, and 0.11, and for Lawley's approximation was 0.05, 0.08, 0.12, 0.19, 0.26, and 0.33. The amount of bias associated with Lawley's approximation increased monotonically with the size of the off-diagonal correlation, whereas the bias associated with Baggaley's approximation was generally quite acceptable for all equi-correlation matrices except the ones with off-diagonal correlations equal to 0.50. The bias across the equi-correlation matrices with 20, 30, 50, 70, and 100 variates was 0.15, 0.18, 0.20, 0.22, and 0.23 for Lawley's approximation and 0.06, 0.05, 0.06, 0.06, and 0.07 for Baggaley's approximation. This result suggests that the bias associated with Lawley's approximation increases monotonically with the number of variates but that Baggaley's approximation is not particularly influenced by the number of variates.

Overall, the amount of bias present for the matrices investigated suggests that Lawley's approximation is clearly inferior to Baggaley's approximation. The bias associated with Baggaley's approximation suggests that this approximation should be viewed with some caution especially when the variates are highly correlated. In general, the larger of the two sample sizes required to reject the sphericity hypothesis suggested by Bagagley's study and the present study should be used for research purposes.

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TABLE 1

RATIO OF OBSERVATIONS (N) TO VARIATES (P) REQUIRED TO REJECT THE SPHERICITY HYPOTHESIS WITH BARTLETT'S TEST ($\alpha = 0.05$)

r	Q	L	p	N/p
.50	.54	.24	20	1.50
.50	.58	.20	30	1.33
.50	.62	.16	50	1.24
.50	.63	.14	70	1.19
.50	.65	.11	100	1.15
.40	.38	.20	20	1.95
.40	.41	.17	30	1.73
.40	.44	.13	50	1.56
.40	.46	.12	70	1.50
.40	.47	.10	100	1.45
.30	.24	.16	20	2.75
.30	.27	.14	30	2.40
.30	.29	.11	50	2.16
.30	.31	.09	70	2.04
.30	.32	.08	100	1.96
.20	.13	.12	20	4.65
.20	.15	.10	30	3.97
.20	.17	.08	50	3.44
.20	.18	.07	70	3.21
.20	.19	.06	100	3.04
.15	.07	.10	20	6.85
.15	.10	.08	30	5.73
.15	.12	.07	50	4.86
.15	.13	.06	70	4.49
.15	.13	.05	100	4.19
.10	.05	.07	20	12.35
.10	.06	.06	30	9.93
.10	.07	.05	50	8.10
.10	.07	.05	70	7.31
.10	.08	.04	100	6.72
RMSE	.06	.20		

Note: Q-Baggaley's and L-Lawley's approximations of the average, absolute off-diagonal correlation, RMSE-root mean square error.