LDPC-LIKE APPROACH FOR DISTRIBUTED DETECTION IN WIRELESS SENSOR NETWORKS

A Thesis by

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I have examined the final copy of this thesis for form and content and recommend that it to be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

Kamesh Namuduri, Committee Chair

We have read this thesis and recommend its acceptance:

M. E. Sawan, Committee Member

Zhiren Jin, Committee Member

إِنَّ فِي خَلْقِ ٱلسَّمَـٰوَٰتِ وَٱلْأَرْضِ وَٱخْتِلَـٰفِ ٱلَّيْلِ وَٱلنَّهَارِ لَأَيَـٰتٍ لِّأُوْلِي

ٱلْأَلْبَـٰبِ (١٩٠) ٱلَّذِينَ يَذْكُرُونَ ٱللَّهَ قِيَـٰمًا وَقُعُودًا وَعَلَىٰ جُنُوبِهِمْ

وَيَتَفَكَّرُونَ فِي خَلْقِ ٱلسَّمَـٰوَٰتِ وَٱلْأَرْضِ رَبَّنَا مَا خَلَقْتَ هَـٰذَا بَـٰطِلاً

سُبْحَـٰنَكَ فَقِنَا عَذَابَ ٱلنَّارِ (١٩١)

(Translation of the meaning)

Verily! In the creation of the heavens and the earth, and in the alternation of night and day, there are indeed signs for men of understanding.

Those who remember Allâh (always, and in prayers) standing, sitting, and lying down on their sides, and think deeply about the creation of the heavens and the earth, (saying): "Our Lord! You have not created (all) this without purpose, glory to You! (Exalted be You above all that they associate with you as partners). Give us salvation from the torment of the Fire

Quran: AlImran, 190,191

DEDICATION

To my great parents

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Mutaz Z. Shukair
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ABSTRACT

LDPC codes have many applications in channel and source coding. In this thesis, we apply LDPC-like belief propagation algorithm to address distributed detection problem in Wireless Sensor Network. The objective is to achieve convergence to a weighted average and to make decision at all sensor nodes. We also consider the design of WSN structure, which achieves convergence asymptotically, and guarantees the consensus of all sensor nodes. We study the network structures that can achieve the fastest convergence.

The results show analogy to the performance of LDPC codes in channel coding. Introducing certain range of irregularity results in an improvement in the network performance in terms of the rate of convergence.

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Chapter 1: Introduction

1.1 Overview: Wireless Sensor Networks and Brief Introduction

Wireless Sensor networks are one of the emerging technologies that have many applications in different fields. This technology is empowered by recent advances in microelectronics and fiber fabrication, which enabled the manufacture of small size and low cost wireless sensors [1].

In general, wireless sensors consist of a sensing and communication modules, and a small processor for information processing purposes. The whole wireless sensor is powered by a non-chargeable small battery attached to it. Once this battery is exhausted then the whole unit is lost. This is the reason why energy consumption decides the network life time.

Data exchange is the backbone feature in Wireless Sensor Network (WSN) [11]-[13]. Sensors cooperate to deliver a consensus about certain event or phenomena; either to a centralized fusion center or to one another. While the first case is an optimum solution in detection problems, in many practical applications, the existence of a single fusion center is not feasible [1]-[3], [6].

In distributed detection schemes, the sensor nodes combine information from neighboring nodes to arrive at the consensus about the event of interest [1]. The communication cost is a crucial factor in deciding the network life time, which is reflected by the number of messages passed between neighbor nodes before reaching the final decision about the sensed phenomena. Different routing methods results in different number of iterations (message passes), and hence different communication costs and accuracy levels. [3]

In this thesis, message routing between sensors nodes is considered in a distributed WSN. An analogy to the general LDPC (Low-Density Parity-Check Code) decoding algorithm is made, and a sum product-like algorithm is deployed to exchange information between the sensor nodes to reach a global decision for a binary hypothesis testing problem. Though no coding/decoding work is made in this work, Tanner graph is used to present the message routing between sensor nodes. We will refer to this graph as the *routing graph* through this thesis. The routing graph will be also presented in a matrix form analogue to the parity-check matrix in LDPC codes, and we will call it the *routing matrix*.

1.2 LDPC Codes

LDPC (Low-density parity-check) codes or Gallagar's codes are an error correction codes discovered by Robert Gallagar around 1960[15]. They were forgotten for about 30 years due to their high complexity. In 1990's these codes were rediscovered, and being used in many applications especially in channel encoding.

Any LDPC code is described by its parity check matrix, which is usually a sparse matrix, where few ones are spread over huge pool of zeros. In other words, the number of ones is much less than the number of zeros.

LDPC codes belong to the category of iterative decoding. They outperform any rate ½ code and approach the Shannon limit with in 0.0045 dB in white Gaussian noise channel. [14].

Gallagar provided an algorithm to solve the log likelihood in iterative way; by passing the log-likelihood or the probabilities between the variable nodes, through the check nodes. This iterative algorithm is known as the sum-product algorithm [15].

The parity check matrix in LDPC codes can have either a regular structure or irregular structure. Interestingly, irregular codes outperform the regular codes. The LDPC codes whose performance approaches the Shannon limit closely are “Gallager codes” based on irregular parity check matrix structure. The uniform codes are those codes which have uniform weight t_c per columns, and uniform weight t_r per row [5].

LDPC codes are very popular in coding field. In this thesis, we apply their structure in wireless sensor network. The sum-product algorithm is deployed to exchange information between sensor nodes, for detecting a binary physical phenomenon. The regularity of the wireless network structure is studied, and the wireless network performance is investigated under different irregular patterns.

1.3 Tanner Graph

Tanner Graphs are bipartite graphs used for constructing long error-correcting codes from smaller error-correcting codes. A Tanner Graph describes the set of equations or constrains that express error correcting codes [4].

Low density parity check (LDPC) codes can be described using Tanner's graphs and the iterative sum-product algorithm also can be implemented on this kind of graphs. For linear codes, Tanner graphs can be also presented as a parity check matrix, where the rows describe constrains or equations that form the code, and the columns represents variable nodes, which are the codeword bits received on the decoder side.

In this thesis, we model WSN analogous to the LDPC code structure. In LDPC codes, each check equation can be presented as a check node in Tanner graph, and each bit can be presented as a variable node in Tanner graph. Connection lines show which bits are involved in specific parity check equation (or check node).

Chapter 2: Literature Review

In this chapter, review the literature of wireless sensor network, LDPC codes and belief propagation in distributed nodes.

- Xiao and Boyd in [7] studied and the problem of computing the average in distributed nodes starting from some initial values. They also considered an iterative algorithm which asymptotically computes the average with out having a central node.
- Xiao, Boyd, and Kim [8] studied distributed averaging in the presence of additive noise. They measured the quality of consensus by the total mean square deviation of the individual variables. They showed that this problem is a convex optimization problem and the global solution can be found.
- Dai and Zhang [9] considered consensus estimation in distributed networks to estimate a source with certain distribution, where nodes collect the corrupted observations independently, and the communication graph between the nodes is treated as Markov random field.
- MacKay, Wilson, and Davey in [5] proposed methods to construct irregular LDPC codes. The encoding time and memory requirements at the encoder were investigated for both regular and irregular LDPC codes, and the performance of the proposed codes was compared to Gallager original codes.
- Chair and Varshney [6] considered quantized data fusion problem, where sensor nodes sends their decisions as ones and zeros to indicate target presence or

absence, respectively. And the optimum fusion rule was derived with weights assigned according to the sensor detection reliability.

- Niu, Chen, and Varshney [16] solved the fusion rule problem in a noisy and fading environment. The knowledge of channel statistics is assumed to be available, and the knowledge of the actual fading coefficients is not necessary. Likelihood ratio (LR) based fusion rule is derived, and it is shown that the derived rule outperforms the equal gain combiner (EGC).
- Chamerland and Veeravalli [17] considered a binary detection problem when the sensor observations are correlated. Sensor nodes are assumed to work as analog relay amplifiers. they studied the case when sensor nodes are highly dense in the observation area, and found that this dense structure provides better performance at the fusion center.

Chapter 3: System Model

3.1 System Model Overview:

We model the observation noise as additive white Gaussian noise (AWGN), where sensors experience uncorrelated observation noise. It is assumed that the WSN is composed of N sensor nodes which cooperate with each other to reach a global decision about the observed phenomena. Deterministic signal x is considered, where $x = -\mu$ under the null hypothesis H_0 and $x = \mu$ under the alternate hypothesis H_1. (For simplicity, no noise is assumed on the communication channels, and consequently no error is associated by exchanging the information.)

The priors of H_0 and H_1 are π_0 and π_1 respectively $\tilde{H}$ denotes global consensus decision at all sensor nodes. The n^{th} sensor observation (y_n) is given by,

$$\begin{aligned} H_o: \quad & y_n = -\mu + n_n \\ H_1: \quad & y_n = \quad \mu + n_n \end{aligned} \tag{3.1}$$

where $n_n \sim \mathcal{N}(0, \sigma^2)$ is the observation noise at the n^{th} sensor node which is assumed to be i.i.d (independent and identically distributed) zero mean, additive white Gaussian noise. Each sensor node measures the observation (y_n) and computes the log-likelihood before relaying this value to the associated virtual fusion center(s). The observation y_n has the following distribution,

$$y_n \sim \mathcal{N}(x_n, \sigma^2) \tag{3.2}$$

where x_n is the deterministic signal to be detected, $x = -\mu$ under the null hypothesis H_0 and $x = \mu$ under the alternate hypothesis H_1.

From the above model, the conditional probability density function (PDF) of the observation is given by.

$$H_0 : P(y_n / x = -\mu) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-(y_n + \mu)^2 / 2\sigma^2)$$

$$H_1 : P(y_n / x = \mu) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-(y_n - \mu)^2 / 2\sigma^2) \tag{3.3}$$

Equations (3.3) expresses the probability density functions of the observation at each sensor node, where the true deterministic signal is known.

Let us define the following probabilities:

$P(x_n = \mu / y_n)$: The probability that the true phenomenon is H_1 given the observation y_n n^{th} sensor node.

$P(x_n = -\mu / y_n)$: The probability that the true phenomenon is H_0 given the observation y_n at n^{th} sensor node.

Then we can write,

$$P(y_n) = P(y_n / x_n = \mu).P(x_n = \mu) + P(y_n / x_n = -\mu).P(x_n = -\mu) \tag{3.4}$$

Using the above equation and applying Bayes's rule, we get the following probability distribution functions:

$$P(x_n = \mu / y_n) = \frac{P(y_n / x_n = \mu).P(x_n = \mu)}{P(y_n)} \tag{3.5}$$

and

$$P(x_n = -\mu / y_n) = \frac{P(y_n / x_n = -\mu).P(x_n = -\mu)}{P(y_n)} \tag{3.6}$$

Substituting equations of (3.3) into (3.4), and substituting the posteriors, $P(y)$ can be expressed as.

$$P(y) = \frac{1}{\sqrt{2\pi}\sigma}\left(e^{\frac{-(y_n-\mu)^2}{2\sigma^2}}.\pi_1 + e^{\frac{-(y_n+\mu)^2}{2\sigma^2}}.\pi_0\right) \qquad (3.7)$$

Then probabilities in (3.5) and (3.6) can be expressed as follows.

$$P(x_n = \mu / y_n) = \frac{e^{\frac{-(y_n-\mu)^2}{2\sigma^2}}.\pi_1}{\left(e^{\frac{-(y_n-\mu)^2}{2\sigma^2}}.\pi_1 + e^{\frac{-(y_n+\mu)^2}{2\sigma^2}}.\pi_0\right)} \qquad (3.8)$$

or in more simple form,

$$P(x_n = \mu / y_n) = \left(1 + \frac{e^{\frac{-(y_n+\mu)^2}{2\sigma^2}}.\pi_0}{e^{\frac{-(y_n-\mu)^2}{2\sigma^2}}.\pi_1}\right)^{-1} \qquad (3.9)$$

The conditional probability $P(x_n = -\mu / y_n)$ can also be expressed in a similar manner.

Let l_n be the log-likelihood ratio at each sensor node, l_n can be computed from the following log-ratio.

$$l_n = \ln\left(\frac{P(y_n / x_n = \mu)}{P(y_n / x_n = -\mu)}\right) \qquad (3.10)$$

And by substituting from (3.3) we get the following.

$$l_n = \ln\left(\frac{e^{-(y_n-\mu)^2/2\sigma^2}}{e^{-(y_n+\mu)^2/2\sigma^2}}\right) \qquad (3.11)$$

Equation (3.11) can be simplified to:

$$l_n = \frac{4y_n\mu}{2\sigma^2} = \frac{2y_n\mu}{\sigma^2} \qquad (3.12)$$

It can be shown that l_n is Gaussian with the following distributions.

$$\begin{aligned} H_0 &: l_n \sim \mathcal{N}\left(\tfrac{-2\mu x_n}{\sigma^2}, \quad \tfrac{4\mu^2}{\sigma^2}\right) \\ H_1 &: l_n \sim \mathcal{N}\left(\tfrac{2\mu x_n}{\sigma^2}, \quad \tfrac{4\mu^2}{\sigma^2}\right) \end{aligned} \tag{3.13}$$

From the likelihood ratios, the decision at each sensor node is:

$$l_n \underset{\tilde{H} = H_0}{\overset{\tilde{H} = H_1}{\gtrless}} \ln(\tau) \tag{3.14}$$

where the threshold τ is given by $\tau = \frac{\pi_0}{\pi_1}$

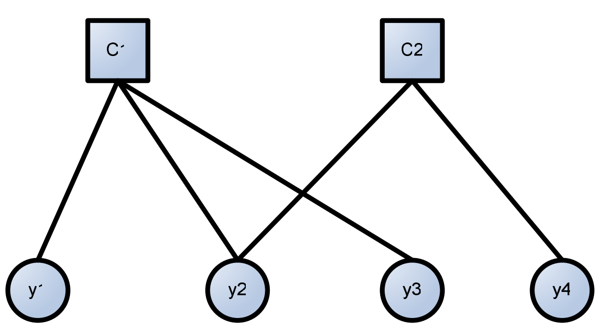
3.2 Connection Diagram, General Wiring Scheme

3.2.1 Routing Graph

In this thesis, WSN is modeled analogous to the LDPC code structure, using Tanner graph. In LDPC codes, each check equation can be presented as a check node, and each bit can be presented as a variable node in Tanner graph. Connection lines show bits which are involved in specific parity check equations (or check nodes).

Each sensor node can be represented as a variable node in Tanner graph. Sensor nodes will exchange their information with the virtual fusion centers (VFC), which are represented as check nodes in Tanner graph. We refer to the Tanner graph that represents the WSN as the routing graph. For message exchange, we will also apply the same rules applied in LDPC codes; the information messages are not allowed to be exchanged between the same type of nodes i.e., sensor nodes can only talk to those VFC nodes to which they are connected and VFC nodes are allowed to talk to only those sensor nodes to which they are connected.

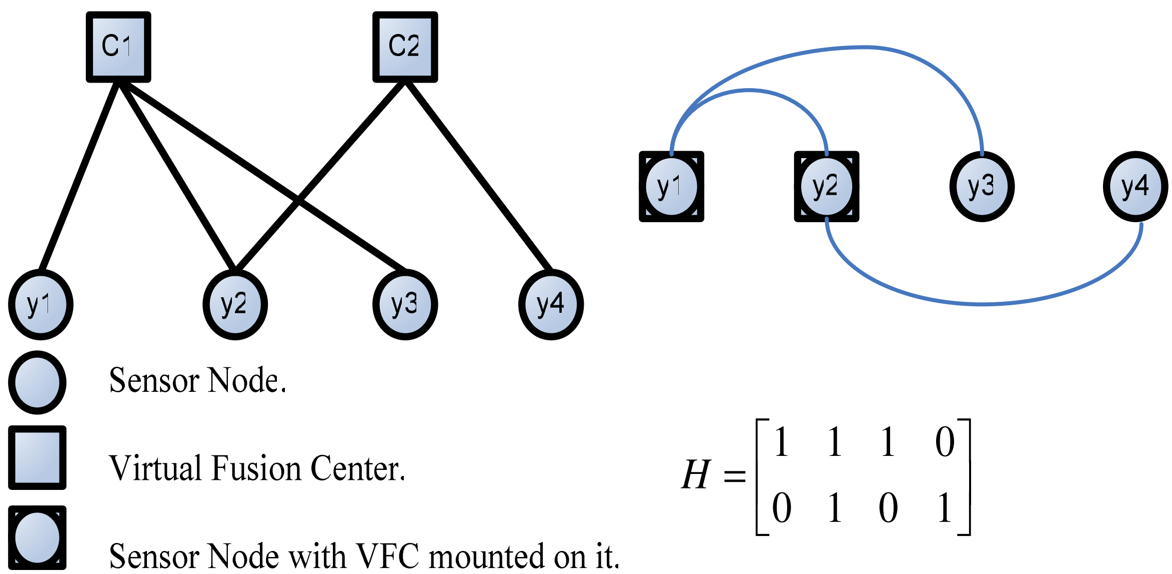
Figure 1 shows an example of representation of a WSN using Tanner graph and the corresponding parity check matrix.



$$H = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Figure 1 Representation of WSN using Tanner graph and parity check matrix.

Separation of VFC from normal sensing nodes is done just to simplify the graphical model. However, in practice, the functionality of a VFC node can be implemented in any sensor node without any change to the mathematical model. Figure 2 shows an example of WSN represented using Tanner graph. This representation is not limited to a small number of nodes. It can be applied to more complex and general network structures.



$$H = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Figure 2 Representation of WSN using Tanner graph. In this graph, C's represent VFC nodes and Y's represent sensor nodes. The matrix *H* denotes the routing graph.

As in LDPC decoders (or any other decoder) the first step is to sense the initial bit values, which are usually corrupted by external noise, and from these initial observations the decoder tries to map these observations to the closest codeword in the code book [19].

In the same way, the first step in WSN is to obtain sensor observations about the physical phenomenon which serves as a common source for all sensors. The goal is to utilize all the observations at all sensor nodes to reach a common understanding about the physical phenomena [10].

In centralized detection, sensor nodes cooperate to reach a final decision by conveying their observations independently to a central node which formulates the final decision based on all the received observations [9]. However, the assumption of having a central node is not possible in many practical cases; where the WSN is required to have local decisions at each sensor node. In such cases, the sensor nodes themselves cooperate to find the true decision by exchanging the data among themselves. This problem is known in literature as distributed detection [3].

In this work, we consider a hybrid problem; where groups of sensor nodes deliver their observations to different VFCs (Virtual Fusion Centers). An initial step in the iterative algorithm, the VFCs receives all the observations from the associated sensor nodes, and then each VFC updates the soft values at each sensor node. Finally, the iterative algorithm converges to final value and the decision is taken at each sensor node.

The VFC in practice can be any sensor node which updates its consensus from the connected neighbors, and consequently updates the connected sensors. The consensus about the physical phenomena propagates in the WSN through the common sensor nodes between the VFC. These sensor nodes serve as common agents between VFCs.

In this work, special conditions are imposed on the network structure to ensure convergence, and the communication cost is considered as a constraint.

3.3 Convergence and global convergence and weight design

The proposed algorithm does not require the knowledge of the complete network graph, neither at the sensor nodes nor at the VFC. Each sensor node needs only to know to which VFCs it is connected, and the VFC nodes needs only to know the sensor nodes that belong to them. Sensors start to exchange messages according to the assigned connections. Finally all sensor nodes converge to a global decision.

Our goal is to achieve convergence to a global weighted average at every sensor. The weighted averaging algorithm gives more credibility to VFCs with higher number of connections, i.e. the average value at each VFC, will be weighted by the number of contributing sensor nodes. The same weighting method applies for sensor nodes, where sensor nodes which are involved with more VFCs are assigned higher weights proportional to the number of connections.

Different weight design methods to achieve distributed global average consensus were discussed in [7]. Other models that achieve global agreement different than the linear average were also studied in the literature [8].

In this section we consider the design of averaging weights to achieve global consensus of the weighted average, from initial observations. The weight design depends on the number of connections at each node (sensor or VFC). Consequently the weights will be known at each node once the connections are assigned for each node.

First, let's consider the final value to which the WSN asymptotically converges. This value is what would be achieved if we had complete knowledge at a fusion center, with complete access to all sensors observations.

$C = [c_1, c_2, ..., c_m]^T$ is the vector that has the degree of each VFC (i.e. number of connected sensor nodes at each VFC), $L = [l_{1,} l_2, ..., l_n]^T$ is the vector that has the initial log-likelihood value at each sensor node, and H is mxn routing matrix, where n is number of sensor nodes and m is number of VFCs.

The row-vector C can be obtained directly from the routing matrix; by summing each row individually (or equivalently counting the number of ones in each row).

To obtain C from the original matrix H, the following equation is used.

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \\ c_m \end{bmatrix} = H.1_{nx1} \tag{3.15}$$

Then, we can express our weighted average as:

$$s = \frac{1}{\sum_{i=1}^{m} c_i} C^T HL \tag{3.16}$$

This weight design divides the sensor network to sub communities, and gives more weight for communities of larger number of members (sensor nodes).

Let's first consider the view at each VFCs, where the collected values from all connected sensor nodes are averaged and weighted by the number of sensor nodes involved. To do so we define a normalized routing matrix $\hat{H}_c$, where "0" entries from the original H matrix are kept the same, and each "1" entry is replaced by a normalized weight corresponding to the number of connections at each VFC. For example, the first row is normalized by c_1, 2nd row by c_2, and so on.

$\hat{H}_c$ can be obtained as follows.

$$\hat{H}_c = D_c H \tag{3.17}$$

where D_c is an mxm diagonal matrix with each diagonal entry as $1/c_i$, i.e:

$$D_c = \begin{bmatrix} 1/c_1 & 0 & \cdots & 0 \\ 0 & \ddots & & 0 \\ & & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ & & & \vdots \\ 0 & 0 & \cdots & 1/c_m \end{bmatrix} \tag{3.18}$$

Figure 3 demonstrates an example of the averaging operation that happens at the first VFC. Same thing happens at all VFCs at the same time but for illustration purposes one VFC is shown

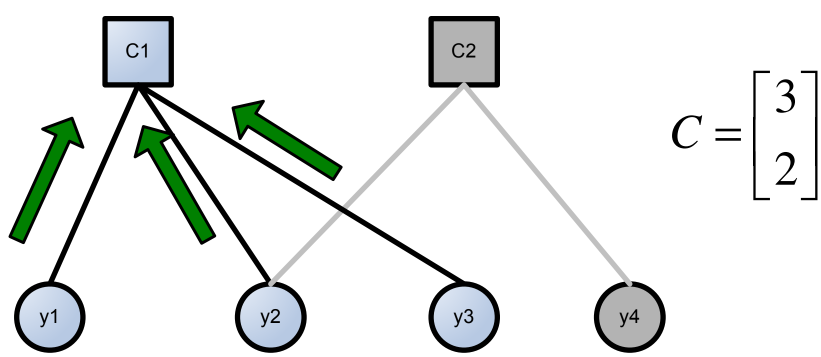


Figure 3 Example of the averaging operation at each VFC. The vector C represents the degree of each VFC

Now to define the average at each sensor node, we define,

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \\ v_m \end{bmatrix} = H^T.1_{mx1} \tag{3.19}$$

where v_i expresses the degree of each sensor node, i.e. the number of VFCs that updates the i^{th} sensor node. In the same way, we define $\hat{H}_v$

$$\hat{H}_v = D_v H \tag{3.20}$$

where D_v is an *mxm* diagonal matrix with each diagonal entry as $1/v_i$. i.e,

$$D_v = \begin{bmatrix} 1/v_1 & 0 & ... & 0 \\ . & & & . \\ . & & & . \\ 0 & & ... & 1/v_n \end{bmatrix} \tag{3.21}$$

Figure 4 illustrates the averaging that happens at the second sensor node.

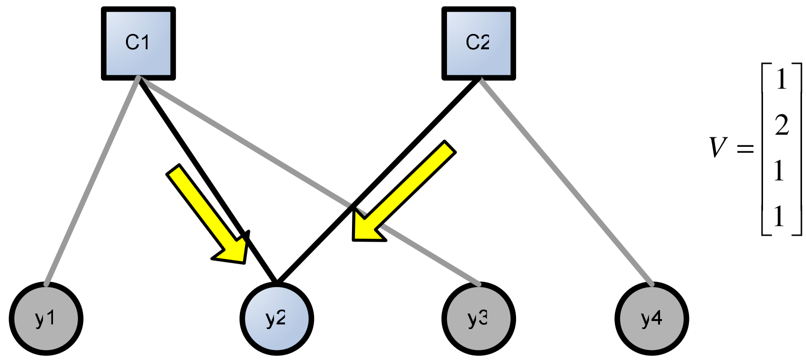


Figure 4 Example of the averaging operation at each sensor node. The vector V represents the degree of each sensor node.

After collecting the initial observations and calculating the log-likelihood, each sensor sends its likelihood value to the VFC to which it is connected to. After this initial step, each VFC calculates the weighted average and updates the sensor nodes connected to it. Finally each sensor node calculates the weighted average from all received messages. We refer to this process as *one iteration cycle*.

In matrix notation, one iteration cycle can be expressed as follows:

$$F(t) = \hat{H}_c L(t) \tag{3.22}$$

where, F is a *mx1* vector which has the computed averages at each VFC node. The following equation reflects the update of each sensor node from the VFCs

$$L(t+1) = \hat{H}_v F(t) \tag{3.23}$$

Combining (3.22) and (3.23) we get:

$$L(t+1)_{updated} = \hat{H}_v \hat{H}_c L(t)_{initial} \tag{3.24}$$

We can formulate the averaging matrix W as:

$$W = \hat{H}_v \hat{H}_c \tag{3.24}$$

Let $L(t)$ represent the current vector at *t*, and let $L(t+1)$ represent the vector at the next iteration. Then, (3.24) can be expressed as

$$L(t+1)_{updated} = WL(t)_{initial} \tag{3.25}$$

If we consider *t – number* of iteration cycles, then

$$\begin{aligned} L(t=1) &= WL(t=0) \\ L(t=2) &= WL(t=1) \\ &. \\ &. \\ &. \\ &. \\ L(t+1) &= WL(t) \end{aligned}$$

This is equivalent to applying the matrix W, "*t*" number of times starting form the initial vector $L(0)$, i.e.,

$$L(t+1) = WW...W_t L(t=0) \tag{3.26}$$

Let W^t represent the weighting matrix accumulated after "t" times of steps, then at any iteration "*t*", we can see that :

$$L(t) = W^t L(0) \tag{3.27}$$

Since the chosen matrices are linear averaging matrices, it can be shown they converge to global value equals ***s*** as in [7]. The matrix W corresponds to the weighting matrix H. It is computed using the degree of each VFC and each sensor node; sensor nodes converge asymptotically to the value in (3.16), i.e.,

$$\lim_{t \to \infty} W^t L(t=0) = \frac{1}{\sum_{i=1}^{m} c_i} CHL \tag{3.28}$$

Chapter 4: Restructuring Algorithm

4.1 Overview

In this section, we show how to alter the structure of the routing matrix, while persevering the constraints that guarantee the convergence. In the terminology of LDPC codes the regular code is the code in which, all parity check equations are involved with the same number of bits, and all the bits are involved in the same number of check equations, and irregular codes are those ones that don't satisfy this condition.

There is an important difference between the LDPC decoding model and the proposed detection model in WSN. In LDPC, each check node checks if its constituent variable nodes satisfy the check condition, while in WSN, each VFC node checks how far the variable nodes are from a weighted average. In LDPC decoder, each bit converges to its true value, while in WSN, each sensor node converges to the same (weighted average) value.

4.2 Structuring the routing matrix

It is known that irregular codes outperform regular codes in general. From this perspective we investigate the structure of the connection diagram of the wireless sensor network that result in good performance. We use the following definition for a regular network structure.

Regular network structure is the WSN network structure, in which all the virtual fusion centers have the same number of connections to sensor nodes.

Note that one sensor node may be connected to more than one virtual fusion center.

The following rules are imposed on any network structure:

- A link on the connection diagram represents a bidirectional communication channel between the fusion center and the sensor node. These links will be represented as value “1” in the connection matrix. If a link does not exist, it will be indicated with a value “0”.
- Sensor nodes can not communicate with each other directly, but instead they exchange the information via the virtual fusion centers.
- Virtual fusion centers can not communicate directly with each other, but instead they exchange the information via the sensor nodes.

Neither the sensor nodes nor the virtual fusion centers are required to know the whole structure of the network, each sensor node is only required to know to which virtual fusion centers to which it should relay its information. Each virtual fusion center is only required to know the sensor nodes to which it is required to update its observations.

The network structure for C number of virtual fusion centers and N number of sensor nodes is expressed in a (CxN) binary matrix (H), where binary “1” expresses the existence of a connection, and a binary zero expresses lack of connection.

The c^{th} row contains all the sensor nodes that report their information to the c^{th} virtual fusion center, and the n^{th} column contains all the virtual fusion centers that the n^{th} sensor node is connected to.

This is analogous to the parity-check matrix in LDPC code; where each row expresses the bits that are involved in a certain parity check equation, and each column express the parity check equations that a certain bit is involved in.

4.3 Varying the Network Structure: An Overview

A regular structure is considered as a starting point, and then gradually we start varying the regular pattern toward more irregular patterns in order to investigate the effect of the structure irregularity on the speed of convergence of the network. The rate of convergence reflects the total energy consumption for the WSN to achieve global agreement. We consider a special case of regular pattern which will be generalized. At each level of irregularity, the average performance in terms of the number of iterations will be measured.

4.4 Building Irregular Patterns

In this model, we start from a regular structure; where all the virtual fusion centers are connected to equal number of sensor nodes, and the number of common sensor nodes between any two successive virtual fusion centers is predefined.

Starting from this regular structure, each connection goes through a random test that decides if the connection needs to be changed or remain the same. In order to clarify this step, let us consider the connection matrix H. For each element in the H matrix with a binary value”1”, the following test is performed. A random value from a uniform distribution is picked up and compared with a threshold, if this random value is less than or equal to the threshold, then the position of this entry may be changed. This corresponds to changing the position of the connection, otherwise the element stays. This threshold can be defined as the probability of changing a connection.

Let us denote the probability of changing a connection as *Pr*. For small values of *Pr*, the network structure will be very similar to the starting structure, and for high values of *Pr* more changes are expected, which makes the structure more random.

Notice, when a connection is changed only one end of the connection is changed and the other end remains the same. The connection is changed from the virtual fusion center side while the sensor side is kept the same. This change corresponds to moving the value “1” entry within the same column.

Candidate positions are defined for matrix elements which pass the random test. These candidates denote the positions that a single element may be changed to. The set of candidates include all the “0” element positions in the same column, and the original position.

After defining the set of positions to which a sensor node can be reconnected to, a new position is selected randomly, with equable probability. The last step corresponds to, randomly selecting a virtual fusion center from all the available virtual fusion centers.

The *Pr* value is varied gradually from “0” to “1”, in small steps. For each value of *Pr*, the above procedure is repeated. For each generated structure, the convergence speed in terms of the number of iterations is measured, and the average rate of convergence is calculated.

In the above calculations, any network that results in a discontinuity into two more parts will be excluded. Such a discontinuity could result in divergence or huge jump in the number of iterations. The procedure could also be performed from the sensor node side, keeping the VFC connections intact.

Chapter 5: Results and Discussion

The proposed scheme is tested by simulating small and large networks. First, a reference network of eight sensor nodes and three VFC centers is considered. The starting pattern is a regular structure with two common sensor nodes shared between each VFC. At a fixed SNR of -10 dB irregular patterns are generated as described in chapter 4 and the number of iterations is computed for each Pr value over several possible generated structures.

As the number of iterations goes to infinity all sensors will converge asymptotically to the same value. In this simulation, the number of iterations is computed until the absolute discrepancy between consecutive iterations is less than 0.1%.

Figure 5 shows that the best performance is attained for Pr values ranges between 0.4 and 0.7. Such structures outperform the uniform structure (i.e.$P_r = 0$). This result is similar to the fact that irregular LDPC codes outperform regular LDPC codes, when all other system parameters are fixed.

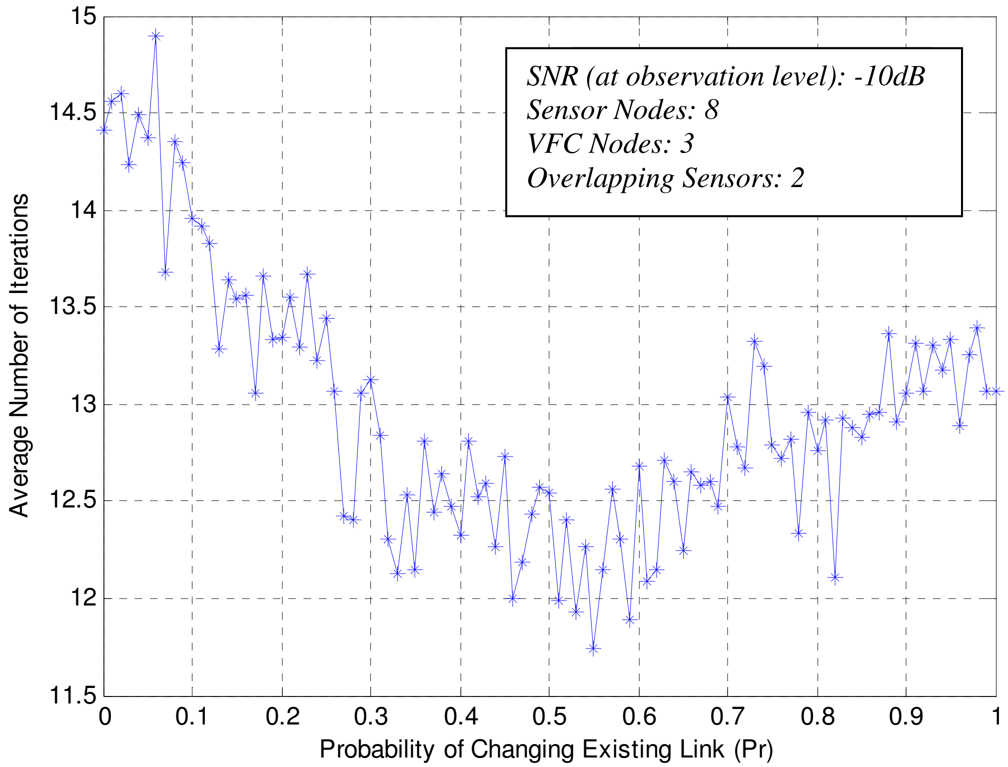


Figure 5 Number of Iterations at Different Pr values

The network performance is investigated at different levels of signal to noise ration (SNR) of the observations. For this purpose a reference network of 14 sensor nodes is considered, and the average number of iterations is computed at each SNR value.

Figure 6 shows a comparison of the network performance at different SNR levels, it is noticed that the general behavior is similar in different level of SNR. The lowest number of iterations is achieved when the network irregularity is medium (0.4 ~ 0.7).

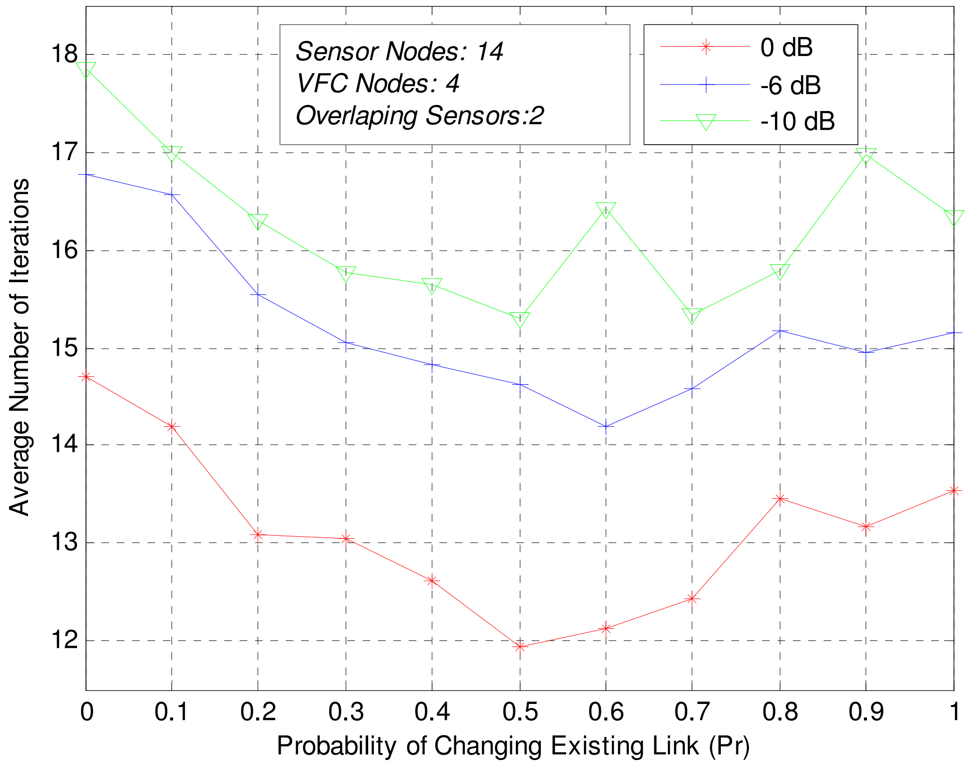


Figure 6 Comparison of WSN Performance at Different SNR Levels

A second reference network with 42 sensor nodes is considered, with 2 overlapping sensor nodes, and 4 connections per VFC. Figure 7 and 8 show similar results, at different Pr values. For large networks the performance seems to improve as the irregularity level is increased.

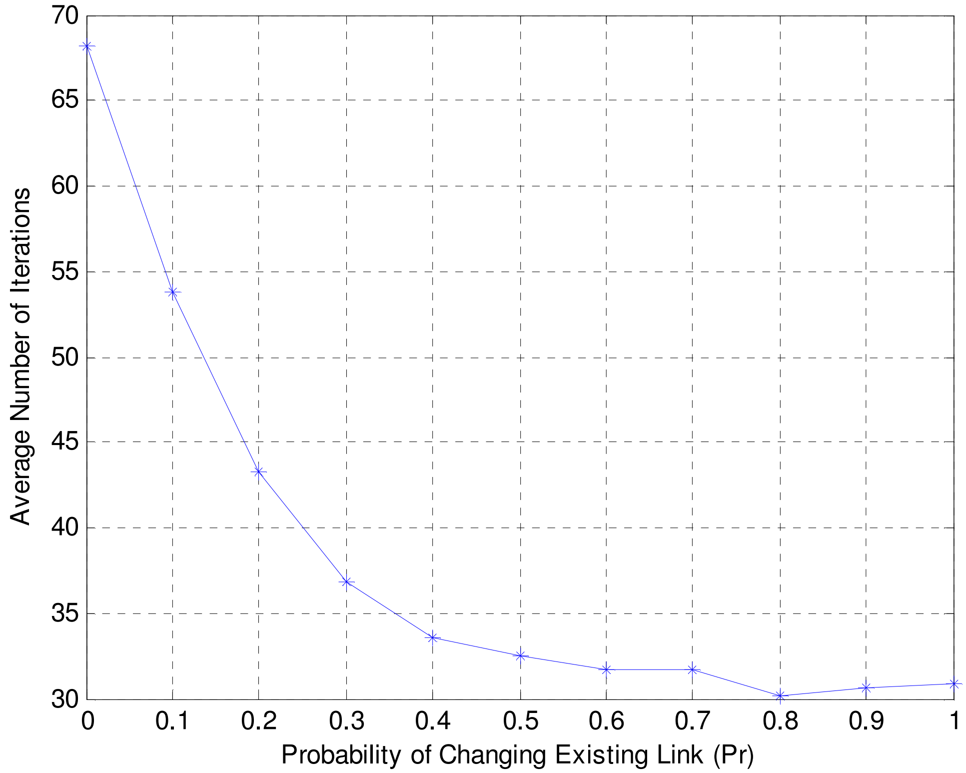


Figure 7 Number of Iterations at Different Pr Values for a Large Network

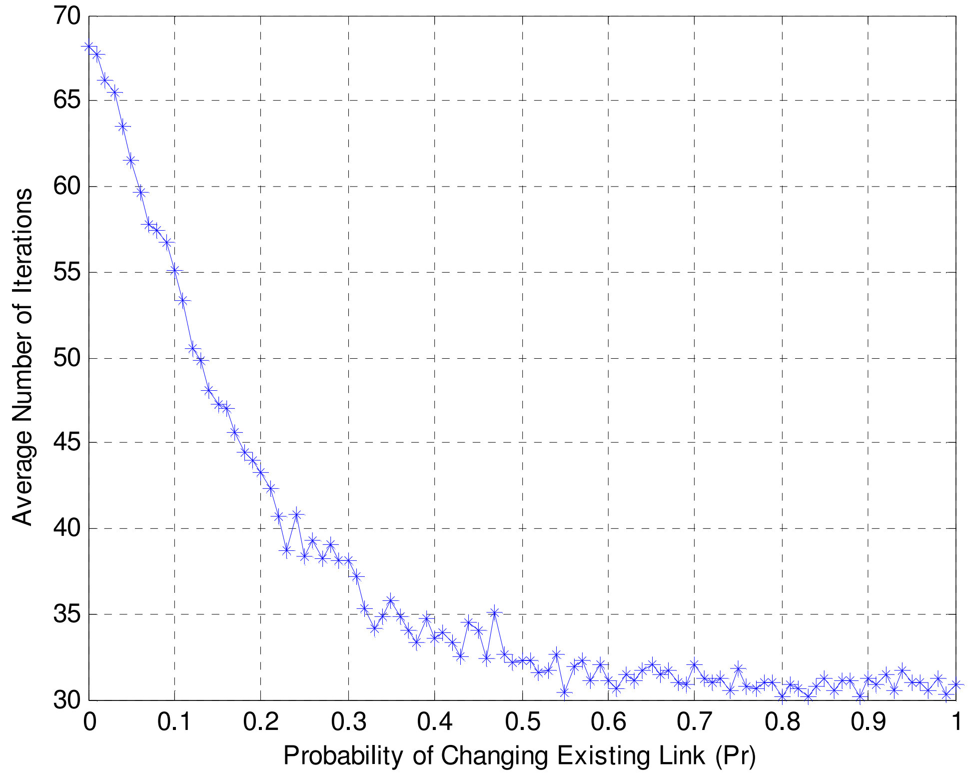


Figure 8 Number of Iterations at Different Pr Values for Large Network, at High Resolution

Chapter 6: Further work

In this thesis, we investigated WSN network structure for distributed detection. The communication links between sensor nodes are assumed to be ideal. The proposed algorithm can be investigated for its performance in the presence of channel noise and fading.

When communication links are affected by external noise, the convergence could be achieved when the noise level is low. As the noise power is increased, it is expected that the proposed algorithm may take more iterations to converge to the weighted average. If the noise power exceeds a certain limit it is expected that the WSN may not converge.

Studying noisy communication channels makes the proposed WSN more realistic. Different fading models for WSN are available in literature. To further pursue this work, the same model can be studied under noisy and fading environment.

In our model, we assumed a special case of observation noise which is i.i.d AWGN. In many cases, especially when the sensor nodes are placed very close to each other in the field, the noise may be correlated.

The spatial distribution for sensor nodes is not considered for the proposed model. However, in practical cases, the physical placement of the sensor nodes plays a major role in the performance of WSN. In large networks, the maximum distance of communication over which a desired quality of communication could be achieved, is an important aspect. Considering this fact, at high values of P_r (i.e. $P_r \sim 1$) the process of assigning communication links may be less random, and each link may have less candidate positions to be changed to.

Conclusion

In this work, we considered a distributed detection problem, where a distributed averaging algorithm is used to achieve global weighted average at every sensor node. A message passing algorithm similar to the one used in LDPC decoder used, to update the consensus about the true phenomena. Also in analogy to LDPC codes, the routing matrix is varied, resulting in different irregular patterns, and the average performance in terms convergence speed, was compared at each level of irregularity. Results showed that irregular patterns outperform regular patterns in a certain rang of irregularity. This result is also analogous to the fact that Gallager's codes whose performance approach Shannon limit are based on irregular structures and they outperform the codes based on regular parity check matrices [5].

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