SEMI-INCLUSIVE NEUTRAL CURRENT NEUTRAL PION PRODUCTION
SELECTION AT THE NO\(\nu\)A (NUMI OFF-AXIS ELECTRON NEUTRINO
APPEARANCE) NEAR DETECTOR USING PRONG LEVEL CONVOLUTIONAL
NEURAL NETWORKS

A Thesis by
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the requirements for the degree of
Master of Science

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The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.

Mathew Muether, Committee Chair

Meyer Holger, Committee Member

Abu Asaduzzaman, Committee Member
I dedicate this thesis to my family, friends, faculty and everyone who’s support has been very valuable to me during the past 2 years. It is very difficult, especially in the field of physics, to be able to write a thesis like this, one that without the support of incredible people would have never been conceived. My gratitude to everyone of you who helped me, guided me, and convinced me to get through the incredible amount of work that required to write this thesis, will always be on my mind. I can’t find the words really, to express how thankful I am to all of you.
I want to first acknowledge Dr. Muether. I must thank him for offering me a GRA position during my second semester to work on neutrino physics. He opened up opportunities in my career that I thought impossible. I will always remember the times I traveled with him to Fermilab in Chicago, IL during the NOvA collaboration meetings to present my analysis updates every semester. I will always remember the times that he had to take me out to dinner too because I felt like I didn’t performed under the required expectations to the NOvA collaboration. Without his emotional support I would of not achieved this thesis. These two years I struggled with my difficulties, Dr. Muether always cheered me up whenever I thought I was not the right man for the job, whenever I thought I wasn’t going to complete a certain graduate physics course, or whenever I thought I wasn’t able to make plots for my research study.

Every month was hard, specially during mid way the last semester at WSU as I was writing my abstract. Ideas in particle physics that I’ve never seen before became more clear due to Dr. Muether’s incredible amount of knowledge in the field. All the hard work made me more disciplined, to work even harder on the tasks that gradually became more clear. As every day went by, Dr. Muether was to me nothing but an amazing listener, an amazing friend, mentor, and graduate advisor. A person with outstanding class. I’m certainly grateful for all his support. This thesis entails a very crucial part of my growth, life experience, as well as the the great amount of excelling required ever done by me, in order to prepare for my thesis. It is my pleasure to also acknowledge the many who have contributed to the NuMI facility for the years of work, to create a beam that would allow for this thesis to exists, and many other theses that will follow up in the future of neutrino physics. I want to thank everyone from the Fermilab Accelerator, Technical, and Particle Physics Divisions, as well as from the NOvA Collaboration.

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I’ll like to acknowledge Dr. Jonathan Paley for his incredible amount of knowledge
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"Oh neutral pion... pesky and unattenuated item, thou rate vetoes the insight of the thou swing of the misunderstood neutrino, decaying onto two noumenon prongs of light, we nail a tandem baryon." - Alan Javier Cedeno
ABSTRACT

The NOνA neutrino experiment based in Fermilab is designed to measure $\nu_\mu \rightarrow \nu_e$ neutrino oscillations. This experiment will give us insight into the properties of massive neutrinos. Neutral current (NC) $\nu_{\mu,e}$ neutral pion production events can mimic the $\nu_\mu \rightarrow \nu_e$ oscillation signal and therefore are an important background for NOνA to understand. Neutral pions decay into two photons which can fake a single electron shower ($\nu_e$ appearance signal) in two ways: either the 2 photons can merge together or one of them may escape detection. In order to constrain this background, NOνA utilizes the Near Detector to measure neutral current $\pi^0$ neutrino interactions. In this analysis, neutrino-Nucleus ($\nu_\mu \rightarrow N$) NC $\pi^0$ interactions with total pion energy greater than 0.3 GeV are studied by selecting two prong events with two final state photons as determined by prong based Convolutional Visual Networks (CVN). The analysis is performed on 3.54x10^{21} Protons On Target (POT) of NOνA Near Detector simulated data and compared to 8.09x10^{20} POT of data. Optimization of the selection based on fractional cross-section uncertainty and an initial energy resolution study of the final sample are presented. The final 2 prong selection using prong based CVN gave a purity of 74%, selection efficiency of $\sim 1.8\%$, and an expected (NC) $\nu_{\mu,e}$ neutral pion cross section of $\sim 15.2\%$. 

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LIST OF SYMBOLS

\( \nu \) \hspace{1cm} \text{Neutrino}

\( \nu_e \) \hspace{1cm} \text{Electron Neutrino}

\( \nu_\tau \) \hspace{1cm} \text{Tau Neutrino}

\( \nu_\mu \) \hspace{1cm} \text{Muon Neutrino}

\( k, k' \) \hspace{1cm} \text{Neutrino Flavor}

\( P(\nu_k \rightarrow \nu_{k'}; E, L) \) \hspace{1cm} \text{\( \nu_k \rightarrow \nu_{k'} \) Oscillation or Transition Probability}

\( \pi \) \hspace{1cm} \text{Pion}

\( \pi^0 \) \hspace{1cm} \text{Neutral Pion}

\( \pi^+ \) \hspace{1cm} \text{Positively Charged Pion}

\( \pi^- \) \hspace{1cm} \text{Negatively Charged Pion}

\( \mu \) \hspace{1cm} \text{Muon}

\( \mu^+ \) \hspace{1cm} \text{Positively Charged Muon}

\( \mu^- \) \hspace{1cm} \text{Negatively Charged Muon}

\( \tau \) \hspace{1cm} \text{Tau Lepton}

\( \tau^+ \) \hspace{1cm} \text{Positively Charged Tau}

\( \tau^- \) \hspace{1cm} \text{Negatively Charged Tau}

\( \gamma \) \hspace{1cm} \text{Photon/Gamma}

\( e^+ \) \hspace{1cm} \text{Electron}

\( e^- \) \hspace{1cm} \text{Positron}

\( c \) \hspace{1cm} \text{Speed of Light}
CHAPTER I

INTRODUCTION

Neutrino-nucleus cross sections are important for measurement of neutrino properties. Since the original postulation of the neutrino by Wolfgang Pauli in 1930, how neutrinos interact with matter has been an active topic in particle physics. With the advent of new precision neutrino experiments, the demand on our understanding of neutrino interactions is becoming even greater [4]. Experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidence for the existence of neutrino oscillations, transitions in flight between different flavored neutrinos caused by non-zero neutrino masses and neutrino mixing. The data implies the existence of 3-neutrino mixing in vacuum [5]. This neutrino mixing provides a scheme in the lepton sector to compare with the structure in the quark sector that has been studied for more than 25 years. An intriguing possibility is that CP violation exists in the lepton sector and is somehow related to the fundamental matter-antimatter asymmetry of our universe [6].

Neutrinos and antineutrinos interact via Charged Current (CC) and Neutral Current (NC) weak interactions and appear in three flavors: electron, $\nu_e$ and $\bar{\nu}_e$, muon, $\nu_\mu$ and $\bar{\nu}_\mu$, and tau, $\nu_\tau$ and $\bar{\nu}_\tau$. The neutrino flavor is related to a charged lepton partner: for example $\nu_e$ is the neutrino that is produced with $e^+$, or produces an $e^-$, in CC weak interaction processes. The flavor of a given neutrino is Lorentz invariant. No neutrino/antineutrino among the three different flavors are identical. The states that describe different neutrino flavors are therefore orthogonal (within the precision of current data): $\langle \nu_{k'}|\nu_k \rangle = \delta_{k'k}$, $\langle \bar{\nu}_{k'}|\bar{\nu}_k \rangle = \delta_{k'k}$, and $\langle \bar{\nu}_{k'}|\nu_k \rangle = 0$ [5].

Neutrinos, $\nu_k/\bar{\nu}_k$, are produced in weak interaction processes in a state that is predominantly left-handed (LH). This is the reason why $\nu_k/\bar{\nu}_k$ are described in the Standard Model (SM) by a chiral LH flavor neutrino field $\nu_{kL}(x)$, where $k=e,\mu,\tau$. For massless $\nu_k$, the state $\nu_k/\bar{\nu}_k$ in which the field $\nu_{kL}(x)$ annihilates/creates is assigned helicity
If $\nu_k$ has a non-zero mass $m(\nu_k)$, the state of $\nu_k/\bar{\nu}_k$ is a linear superposition of the helicity -1/2 and +1/2 states, but the helicity $(+1/2)/(-1/2)$ state enters the superposition with a coefficient $\propto m(\nu_k)/E$, $E$ being the neutrino energy, and thus is strongly suppressed. Besides the chiral LH flavor neutrino field, there is another field for leptons, the LH charged lepton field $l_L(x)$. Both $\nu_{kL}(x)$ and $l_L(x)$ fields form an $SU(2)_L$ doublet. In the absence of neutrino mixing and assuming zero neutrino masses, both the flavor neutrino field or the charged lepton field can be assigned to one unit of the additive lepton charge (or flavor) $L_l$ and therefore the three charges are conserved by the weak interaction [5]. Massive "Dirac neutrinos" emerge in theories in which at least one additive lepton is conserved. The properties of these massive Dirac neutrinos depend strongly on the type of the lepton charge conserved in the theory. Massive Majorana neutrinos may emerge in theories in which the total lepton charge $L$ is not conserved. In such cases, the existence of relativistic neutrinos/antineutrinos states that are predominantly RH, $\nu_R/\bar{\nu}_L$, implies their couplings preserve no lepton charge. In other words, observations of neutrinos with at least one of the neutrinos containing a non-zero mass would imply that either the total lepton charge $L$ is violated (for example in cases where $\mu^-\rightarrow e^-+\gamma$) or $\nu_R(\bar{\nu}_L)$ should be a new type of non interacting neutrinos/antineutrinos. [4][7][5]. Deciding whether neutrinos are Dirac fermions, or Majorana fermions is fundamental for understanding the origin of $\nu$-masses, mixing, and the underlying symmetries of particle interactions.

Neutrino scattering studies have recently increased due to the need of such information for interpretation of neutrino oscillation data. For example see Figures 2, 3; which show cross-section uncertainty as a major error source on the latest NOvA oscillation results. In recent years, these studies done by many experiments have measured the total inclusive (interactions where some of the products are left unmeasured) cross section for neutrino $\nu_\mu+N\rightarrow\mu^-+X$ or antineutrino $\bar{\nu}_\mu+N\rightarrow\mu^++X$ scattering off nucleons covering a broad range of neutrino energies. High energy neutrino interactions are typically dominated by Deep Inelastic Scattering (DIS), and at lower neutrino energies, a complex combination of Quasi Elastic (QE) scattering and pion production processes dominate [5]. See Figure 1
for details of existing inclusive cross section measurements. More on this in section 2.3.

Figure 1: Total neutrino and antineutrino per nucleon CC cross sections (for an isoscalar target) divided by neutrino energy and plotted as a function of energy. In the Zeller’s paper, data are the same as in Figures 28, 11, and 12 with the inclusion of additional lower energy CC inclusive data from ▲ (Baker et al., 1982), * (Baranov et al., 1979), ■ (Ciampolillo et al., 1979), and * (Nakajima et al., 2011). These contributions include quasi-elastic scattering (dashed), resonance production (dot-dash), and deep inelastic scattering (dotted). Example predictions for each are provided by the NUANCE generator (Casper, 2002). Note that the quasi-elastic scattering data and predictions have been averaged over neutron and proton targets and hence have been divided by a factor of two for the purposes of this plot [4].
Figure 2: (a) Uncertainties on the measured $32$ mass splitting in 2017 $\nu_\mu$ analysis. Values taken from the error chart constructed by Luke Vinton. (b) Uncertainties on the measured $\theta_{23}$ mixing angle in the 2017 $\nu_\mu$ analysis. Values taken from the error chart constructed by Luke Vinton[8].
The NOνA neutrino experiment based at Fermilab is designed to measure $\nu_\mu \rightarrow \nu_e$ neutrino oscillations. Neutral current (NC) $\nu_{\mu,e}$ neutral pion production events (events where there is no outgoing charged lepton) can mimic the $\nu_\mu \rightarrow \nu_e$ oscillation signal and
therefore are an important background for NO\textsubscript{v}A to understand. To understand the impact of this background NO\textsubscript{v}A relies on models, like the Rein and Sehgal model\cite{9}. These models have free parameters constrained to experimental data sets. A cross section measurement of the NC $\pi^0$ events will improve uncertainty in the model. This in turn improves sensitivities of oscillation measurements. In this thesis I present work towards prong-level CVN selector for a cross-section measurement of the NC $\pi^0$ interaction in NO\textsubscript{v}A.
CHAPTER II

THE NO$\nu$A EXPERIMENT

NuMI Off-Axis $\nu_e$ Appearance Experiment (NO$\nu$A) is a long baseline neutrino experiment designed to search for appearance of $\nu_e$ in a beam of muon neutrinos ($\nu_\mu \rightarrow \nu_e$) by comparing electron neutrino rates at Fermilab, to those 810 km away in northern Minnesota. Furthermore, not only does NO$\nu$A measure the electron neutrino appearance rate in the Neutrinos at the Main Injector (NuMI) beam line but it also measures muon neutrino disappearance ($\nu_\mu \rightarrow \nu_\mu$) [10]. From these measurements, NO$\nu$A hopes to accomplish three things: Determine the $\theta_{23}$ octant ($\theta_{23} > 45^\circ$ or $\theta_{23} < 45^\circ$), measure the CP-violating phase $\delta$, and determine the neutrino mass hierarchy.

2.1 Neutrino Mixing

2.1.1 Three Neutrino Mixing

Most existing data on neutrino oscillations can be described assuming 3-flavor neutrino mixing, or lepton mixing in vacuum. This is the minimal neutrino mixing scheme which is compatible with the currently available data on oscillations. In the formalism of local quantum field theory, used to construct the Standard Model, the LH flavor neutrino field $\nu_{kL}(x)$ (see Appendix A), which enter into the expression for the lepton current in the CC weak interaction Lagrangian,

$$L_{CC} = -\frac{g}{\sqrt{2}} \sum_{k=e,\mu,\tau} \tilde{\nu}_L(x) \gamma^\alpha \nu_{kL}(x) W^{\alpha\dagger}(x) + h.c$$

are linear combinations of the fields of three (or more) neutrinos $\nu_k$ having masses $m_k \neq 0$:

$$\nu_{kL}(x) = \sum_{i=1}^{3} U_{ki} \nu_{iL}(x), \ k = e, \mu, \tau$$

\[1\text{hermitian conjugate}\]
where \( \nu_{iL}(x) \) is the LH component of the field of \( \nu_i (\nu_1, \nu_2, \nu_3) \) possessing a mass \( m_i \) and \( U \) is a unitary 3 by 3 matrix (the neutrino mixing matrix). \( U \) is often called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) or Maki-Nakagawa-Sakata (MNS) mixing matrix. Equation 2 implies that the individual lepton charges \( L_k, k = e, \mu, \tau \), are not conserved[5].

Expanding equation 2

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

(3)

where \( U \) can be parametrized\(^2\) by 3 angles. If these are Dirac massive neutrinos, then neutrino mixing is similar to quark mixing, see Ref.[11]; and for \( n = 3 \) there is just one CP violating phase\(^3\) in \( U \). Assuming massive neutrinos \( \nu_i \) are Dirac or Majorana particles, the unitary mixing matrix looks like:

\[
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{13} + c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}s_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13}
\end{pmatrix} \times diag(1, e^{i\alpha_{21}}, e^{i\alpha_{31}})
\]

(4)

where \( c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij} \), the angles \( \theta_{ij} = [0, \pi/2] \), \( \delta = [0, 2\pi] \) is the Dirac CP violation phase and \( \alpha_{21}, \alpha_{31} \) are two Majorana CP violation (CPV) phases.

As we know it, the fundamental parameters characterizing the 3-neutrino mixing are: 1) the 3 angles \( \theta_{12}, \theta_{23}, \theta_{13} \), 2) depending on the nature of massive neutrinos \( \nu_{i-1} \) Dirac

\(^2\)The number of massive neutrinos, \( n \) neutrino flavors and \( n \) massive neutrinos, the \( n \) by \( n \), \( U \) can be parametrized by \( n(n-1)/2 \) Euler angles and \( n(n+1)/2 \) phases. Considering that these are Dirac particles only \( (n-1)(n-2)/2 \) are physical and can be responsible for CP violation in the lepton sector[5].

\(^3\)CP invariance holds if \( U \) is real, \( U^* = U \).
(δ), or 1 Dirac + 2 Majorana (δ, α_{21}, α_{31}, CPV phases), and 3) the neutrino masses, m_1, m_2, m_3. Therefore, depending on whether the massive neutrinos are Dirac or Majorana particles, this makes 7 or 9 additional parameters in the minimally extended Standard Model of particle interactions with massive neutrinos [5]. θ_{12}, θ_{23}, θ_{13} are defined via the elements of the U:

\[ c_{12}^2 \equiv \cos^2 \theta_{12} = \frac{|U_{e1}|^2}{1 - |U_{e3}|^2}, \quad s_{12}^2 \equiv \sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}; \]

(5)

\[ s_{23}^2 \equiv \sin^2 \theta_{23} = |U_{e3}|^2, \quad s_{12}^2 \equiv \sin^2 \theta_{12} = \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2}, \]

\[ c_{23}^2 \equiv \cos^2 \theta_{23} = \frac{|U_{\tau3}|^2}{1 - |U_{e3}|^2}. \]

(6)

Eigenstates with different eigenmasses propagate through space at different frequencies and velocities. The mass eigenstates are a linear superposition of flavor eigenstates and conversely flavor eigenstates a superposition of mass eigenstates. The difference in frequencies and velocities produce a time varying interference between the neutrino mass states and thus, the probability of a neutrino flavor "surviving" is non-zero:

\[ P_{k \rightarrow k'} = |\langle \nu'_{k'}(t) | \nu_k \rangle|^2 = \left| \sum_i U_{ki}^* U_{k'j} e^{-im_i^2 L/2E} \right|^2 \]

(7)

more precisely written,

\[ P_{k \rightarrow k'} = \delta_{kk'} - 4 \sum_{i<j} Re(U_{ki}^* U_{k'i} U_{kj} U_{k'j}) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \]

\[ + 2 \sum_{i<j} Im(U_{ki}^* U_{k'i} U_{kj} U_{k'j}) \sin \left( \frac{\Delta m^2 L}{4E} \right) \]

(8)

where the phase of oscillation is \( \frac{\Delta m^2 L}{4E} \). The neutrino oscillation probabilities depend, in
In general, on the neutrino energy $E$, the source-detector distance $L$, on the elements of $U$ and, for relativistic neutrinos used in all neutrino experiments performed so far, on $\Delta m_{ij}^2 \equiv (m_i^2 - m_j^2)$, $i \neq j$ [5].

In the case of 3-neutrino mixing, $|\Delta m_{21}^2|$ is the smaller of the two neutrino mass squared differences, which, as it follows from the data, is responsible for solar $\nu_e$ disappearance and, observed by KamLAND, reactor $\bar{\nu}_e$ oscillations [12]. We will number (just for convenience), the massive neutrinos in such a way that $m_1 < m_2$, so that $\Delta m_{21}^2 > 0$. With these choices made, there are two possibilities: either $m_1 < m_2 < m_3$, or $m_3 < m_1 < m_2$. Then the larger neutrino mass square difference $\Delta m_{31}^2$ or $\Delta m_{32}^2$, can be associated with the experimentally observed oscillations of the atmospheric and accelerator $\nu_\mu$ and $\bar{\nu}_\mu$, as well as of the reactor $\bar{\nu}_e$ at $L \sim 1$ km.

Global analyses of the neutrino oscillation data available by the second half of 2014 allowed us to determine the 3-neutrino oscillation parameters $\Delta m_{21}^2$, $\theta_{21}$, $|\Delta m_{31}^2|$ ($|\Delta m_{32}^2|$), $\theta_{23}$ and $\theta_{13}$ with a relatively high precision [5].

Recent results of the Chooz experiment, see Ref.[13] with reactor $\bar{\nu}_e$ and from the more recent data of the Daya Bay, RENO, Double Chooz and T2K experiments, see Ref. [14][15][16][17], have measured, show that the element $|U_{e3}| = \sin^2 \theta_{13}$ of the neutrino mixing matrix $U$ is relatively small. This makes it possible to identify the angles $\theta_{12}$ and $\theta_{23}$ as the neutrino mixing angles associated with the solar $\nu_e$ and the dominant atmospheric $\nu_\mu$ (and $\bar{\nu}_\mu$) oscillations, respectively. The angles $\theta_{12}$ and $\theta_{23}$ are sometimes denoted as $\theta_{12} = \theta_\odot$ and $\theta_{12} = \theta_A$ (or $\theta_{\text{atm}}$), while $\Delta m_{21}^2$ and $\Delta m_{31}^2$ are often referred to as the "solar" and "atmospheric" neutrino mass squared differences and are often denoted as $\Delta m_{21}^2 \equiv \Delta m_{\odot}^2$, $\Delta m_{31}^2 \equiv \Delta m_A^2$ (or $\Delta m_{\text{atm}}^2$) [5].

Analyses by three different authors\(^4\) report practically the same result within $1 \sigma$ on $\Delta m_{21}^2$, $\sin^2 \theta_{12}$, $|\Delta m_{31}^2|$ and $\sin^2 \theta_{13}$. The results obtained in Ref. [18] on $\sin^2 \theta_{23}$ show, in particular, that for $|\Delta m_{31(32)}^2| > 0$ ($|\Delta m_{31(32)}^2| < 0$), i.e, for $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$), the best fit value of $\sin^2 \theta_{23} = 0.437$ (0.455). At the same time, the best fit values of $\sin^2 \theta_{23}$

\(^4\)see Refs.[18][19][20]
reported for $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$) in Ref. [19] and in Ref. [20] read, respectively: $\sin^2 \theta_{23} = 0.452 (0.579)$ and $\sin^2 \theta_{23} = 0.567 (0.573)$.

Furthermore on these three analyses, it was also determined by these authors that the best fit value of the Dirac CPV phases $\delta \approx 3\pi/2$. According to Ref. [18], the CP conserving values $\delta = 0 (2\pi)$ and $\pi (\delta = 0 (2\pi))$ are disfavored at 1.6$\sigma$ to 2.0$\sigma$ (at 2.0$\sigma$) for $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$). In the case of $\Delta m^2_{31(32)} > 0$, the value $\delta = \pi$ is statistically 1$\sigma$ away from the best fit value $\delta \approx 3\pi/2$ [5].

In August 2015 the first results of the NO$\nu$A neutrino oscillation experiment were announced [10][21]. These results together with the latest neutrino and the first antineutrino data from T2K experiment [15][22] were included, in particular, in the latest analysis of the global neutrino oscillation data performed in Ref. [23]. Thus in Ref. [23] the authors updated the results obtained earlier in [18][19][20][5].

Table 1 shows the best fit values and the 99.73 % CL allowed ranges of the neutrino oscillation parameters found in [23] using, in particular, the more "conservative" LID NO$\nu$A data from Ref. [21]. The best fit value of $\sin^2 \theta_{23}$ found for $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$) in Ref. [21] reads: $\sin^2 \theta_{23} = 0.437 (0.569)$. The authors of Ref. [21] also find that the hint for $\delta \approx 3\pi/2$ is strengthened by the NO$\nu$A $\nu_\mu \rightarrow \nu_e$ and T2K $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation data. The values of $\delta = \pi/2$ and $\delta = 0 (2\pi)$ are disfavored at 3$\sigma$ CL and 2$\sigma$ CL, respectively, while $\delta = \pi$ is allowed at approximately 1.6$\sigma$ CL (1.2$\sigma$) for $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$) [5].

Figure 4 and figure 5 shows the 2017-18 results from NO$\nu$A from 8.85x10$^{20}$ POT which concluded that competitive measurements of $\Delta m^2_{32}$ prefer mixing near maximal when it came down to $\nu_\mu$ disappearance [24].
Table 1: The best-fit values and 3σ allowed ranges of the 3-neutrino oscillation data from [23]. For the Dirac phase δ we give the best fit value and the 2σ allowed ranges; at 3σ no physical values of δ are disfavored. The values (values in brackets) correspond to $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$). The definition of $\Delta m^2$ used is: $m_3^2 - (m_2^2 + m_1^2)/2$. Thus, $m^2 = \Delta m^2_{31} - \Delta m^2_{31}/2 > 0$, if $m_1 < m_2 < m_3$, and $m^2 = \Delta m^2_{32} - \Delta m^2_{21}/2 < 0$ for $m_3 < m_1 < m_2$ [5].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>best-fit</th>
<th>3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2_{31}$ [$10^{-5}$ eV$^2$]</td>
<td>7.37</td>
<td>6.93 - 7.97</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2</td>
<td>[10^{-3}$ eV$^2]$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.297</td>
<td>0.250 - 0.354</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}, \Delta m^2 &gt; 0$</td>
<td>0.437</td>
<td>0.379 - 0.616</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}, \Delta m^2 &lt; 0$</td>
<td>0.569</td>
<td>0.383 - 0.637</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}, \Delta m^2 &gt; 0$</td>
<td>0.0214</td>
<td>0.0185 - 0.0246</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}, \Delta m^2 &lt; 0$</td>
<td>0.0218</td>
<td>0.0186 - 0.0248</td>
</tr>
<tr>
<td>$\delta/\pi$</td>
<td>1.35 (1.32)</td>
<td>(0.92 - 1.99)</td>
</tr>
</tbody>
</table>

Figure 4: Full joint fit with appearance analysis. Feldman Cousins corrections in 2D and 1D limits. Constrain $\theta_{13}$ using world average PDG, $\sin^2 2\theta_{23} = 0.082$. Best fit: $\Delta m^2_{32} = 2.444 \times 10^{-3}$, moved from previous result of 2.6σ to 0.8σ see Ref. [24].
Figure 5: (a) 1, 2 and 3 $\sigma$ allowed regions for $\sin^2 \theta_{23}$ by $\delta_{CP}$ and IH for the 2017 joint $\nu_e$ and $\nu_\mu$ analysis. Feldman-Cousins corrections are applied. Profiling over $\sin^2 2\theta_{13}, \Delta m^2_{32}$ and systematics. Using reactor constraint $\sin^2 \theta_{13} = 0.082 \pm 0.004$ (PDG 2017). Best fit points for $\nu_e + \nu_\mu +$ reactor are NHUO (1.21 $\pi$, 0.558, 2.445, 0.082, LL = 84.57); NHLO (1.46$\pi$, 0.474, 2.435, 0.082, LL = 84.70); IH (1.47$\pi$, 0.558, 2.510, 0.083, LL = 87.11) (b) 1, 2 and 3 $\sigma$ allowed regions for $\sin^2 \theta_{23}$ by $\delta_{CP}$ and NH for the 2017 joint $\nu_e$ and $\nu_\mu$ analysis. Feldman-Cousins corrections are applied. Profiling over $\sin^2 2\theta_{13}, \Delta m^2_{32}$ and systematics. Using reactor constraint $\sin^2 \theta_{13} = 0.082 \pm 0.004$ (PDG 2017). Best fit points for $\nu_e + \nu_\mu +$ reactor are NHUO (1.21 $\pi$, 0.558, 2.445, 0.082, LL = 84.57); NHLO (1.46$\pi$, 0.474, 2.435, 0.082, LL = 84.70); IH (1.47$\pi$, 0.558, 2.510, 0.083, LL = 87.11)[25].

and best joint fit measurements set limits on $\delta_{CP}$.

2.1.2 Majorana Neutrinos and Mass Hierarchy

Only neutral fermions can be of Majorana type, although not relevant to the study of the NC $\pi^0$ it is worth noting this type of fermion, since experimental evidence has yet to determine whether the neutrino is a Dirac or Majorana type fermion, following the
discovery of neutrino oscillations and hence the neutrino mass. At present time it hasn’t been determined yet weather the $\nu_3$ neutrino mass eigenstate is heavier or lighter then the $\nu_1$ and $\nu_2$ neutrino mass eigenstates in nature. The scenario in which $\nu_3$ is heavier, is referred to as the normal mass hierarchy (NH). The other scenario, in which the $\nu_3$ eigenstate is lighter is referred to as the inverted mass hierarchy (IH). The ultimate goals of most experiments and theories centered on neutrinos is to formulate the particle physics model that explains the observed neutrino masses and mixing patterns, and relates them to the well known charged lepton masses. Most theoretical models of neutrino mass assume that neutrinos are massive Majorana fermions. The best way to test such hypothesis is to search for the neutrinoless double beta decay. If (IH) is realized in nature, the next generation of double beta decay experiments, see Ref.[26], can decide whether neutrinos are Majorana fermions or not. Lets first explain the nomenclature. Since the neutrinos $\nu_1$ has the largest component of the electron neutrino $\nu_e$ while $\nu_3$ has the smallest component of $\nu_e$, the normal hierarchy in some crude sense resembles the mass ordering of the charged leptons, hence it is denoted as ‘normal’. Obviously, the inverted hierarchy represents the opposite situation [27]. Figure 6 illustrates the neutrino mass hierarchy,

![Neutrino mass hierarchy diagram](image)

Figure 6: Pattern of neutrino masses for the normal and inverted hierarchies is shown as mass squared. Flavor composition of the mass eigenstates as the function of the unknown CP phase $\delta_{CP}$ is indicated. $\Delta m^2_{atm} \sim |\Delta m^2_{31}| \sim |\Delta m^2_{32}|$ and $\Delta m^2_{sol} \sim |\Delta m^2_{21}|$ stands for the atmospheric and the solar mass-squared splitting, respectively.[27]

Today, the experiments studying flavor neutrino oscillations cannot provide
information on the nature of the massive neutrinos Dirac or Majorana nature. Establishing whether the neutrinos with definite mass $\nu_k$ are Dirac fermions possessing distinct antiparticles, or Majorana fermions, i.e, spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the origin of $\nu$-masses and mixing and the underlying symmetries of particle interactions\cite{5}.

The massive neutrinos are predicted to be of Majorana nature by the seesaw mechanism of neutrino mass generation. The observed patterns of neutrino mixing and of neutrino mass squared differences can be related to Majorana massive neutrinos and the existence of an approximate flavor symmetry in the lepton sector. The Majorana nature of massive neutrinos $\nu_k$ manifests itself in the existence of processes in which the total lepton charge $L$ changes by two units: $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$, $\mu^- + (A,Z) \rightarrow \mu^+ + (A,Z-2)$, etc. see Ref. \cite{5}. Extensive studies have shown that the only feasible experiments having the potential of establishing that the massive neutrinos are Majorana particles are at present the experiments searching for $(\beta\beta)_{0\nu}$-decay: $(A,Z) \rightarrow e^- + e^- + (A,Z+2)$ \cite{5}. The observation of $(\beta\beta)_{0\nu}$-decay and measurement of the corresponding half live-time with sufficient accuracy, would not only be a proof that the total lepton charge is not conserved, but might also provide unique information on the 1) type of neutrino mass spectrum \cite{6}, 2) Majorana phases in unitary matrix \cite{7} $U$, and 3) the absolute scale neutrino masses \cite{8}. Under assumptions of 3-$\nu$ mixing, of massive neutrinos $\nu_i$ being Majorana particles, and of $(\beta\beta)_{0\nu}$-decay generated via (V-A) charged weak interactions for the three Majorana neutrinos $\nu_i$ having masses $m_i \lesssim$ few MeV, the $(\beta\beta)_{0\nu}$-decay amplitude has the form\cite{9}: $A(\beta\beta)_{0\nu} \cong <m> M$, where $M$ is the corresponding nuclear matrix element which does not depend on the neutrino mixing parameters, and

$$| <m> | = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2| = |(m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\alpha_{21}}) c_{13}^2 + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta)}|$$

is effective Majorana mass in $(\beta\beta)_{0\nu}$-decay \cite{5}. 

\textsuperscript{5}see e.g. reference 88 in PDG \cite{5}
\textsuperscript{6}see e.g. reference 89 in PDG \cite{5}
\textsuperscript{7}see e.g. reference 66,90 in PDG \cite{5}
\textsuperscript{8}see e.g. reference 88 to 91 and references quoted therein in PDG \cite{5}
\textsuperscript{9}see, e.g. reference 46 and 88 in \cite{5}
2.2 The Physics of NO\(\nu\)A

The existence of flavor neutrino oscillations implies that if a neutrino of a given flavor say \(\nu_\tau\) is located at a large distance \(L\) i.e NO\(\nu\)A Far Detector (FD) from the \(\nu_\mu\) source i.e NO\(\nu\)A Near Detector (ND), the probability to find a different flavor, say \(\nu_\tau\), \(P(\nu_\mu \rightarrow \nu_\tau; E,L)\) is nonzero. This is called the oscillation or transition probability \(P(\nu_\mu \rightarrow \nu_\tau)\), where \(E\) is the initial neutrino energy available during the weak interaction process. However, if such probability does not equal to zero (in cases where the probability that \(\nu_\mu\) will not change into a neutrino of different flavor say \(\nu_e\), \(\nu_\tau\)) the probability of survival is smaller than one. If only \(\nu_\mu\) are detected in the experiment such as NO\(\nu\)A and they take part in oscillations, one could observe a disappearance of \(\nu_\mu\) on the way from the \(\nu_\mu\) NO\(\nu\)A Near Detector (source) to the NO\(\nu\)A Far Detector [5]. Due to the nature of neutrino oscillations being a linear combination of neutrino mass eigenstates \((\nu_1, \nu_2, \nu_3)\) that constitute the neutrino flavor states \((\nu_e, \nu_\mu, \nu_\tau)\). The superpositions are then parametrized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix in terms of three three mixing angles \((\theta_{12}, \theta_{13}, \theta_{23})\) as previously derived in section 2.1.2 [28].

The dominant component in the NO\(\nu\)A neutrino flux are the \(\nu_\mu/\bar{\nu}_\mu\). This flux depends on a number of factors e.g., energy and intensity of the primary proton beam, material and geometry of the target, selection of the momentum and charge of the secondary mesons that are focused, and production angle of the secondary mesons with respect to the primary beam. In other words, it is possible to control the peak energy, energy spread, and dominant neutrino flavor, of the NO\(\nu\)A NuMI beam.

NO\(\nu\)A’s primary focus is neutrino oscillations of the \(\nu_\mu \rightarrow \nu_e\) type. NO\(\nu\)A employs a two detector configuration in a accelerator long baseline experiment, a "near" detector measures non-oscillated neutrino flux (which is the analysis of this thesis), and the "far" detector measures the oscillated neutrino flux. Both detectors are discussed in detail in section 2.3\(^{10}\) [5].

\(^{10}\)It should be noted, that the near detector does not see the same neutrino flux as the far detector sees, because the neutrino source looks like a line source for the near detector, while it looks as point source for the far detector.
The energy of the emitted neutrino \( E_\nu \) from the the target source at NOvA contains an angle parameter associated with the parent pion direction which is given by:

\[
E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos \theta)},
\]

(10)

where \( E_\pi \) and \( p_\pi \) are the energy and momentum of the parent pion, \( m_\pi \) is the charged pion mass, \( m_\mu \) is the muon mass. An ideal case is that the pions are completely focused in parallel. For \( \theta = 0 \), it can be seen from the above equation that \( E_\nu \) is proportional to the \( E_\pi \) for \( E_\pi \gg m_\pi \). As the secondary pions have a wide spectrum a "wide-band beam" is produced [5]. For a given \( \theta \), differentiating the above expression with respect to \( E_\pi \), it can be shown that \( E_\nu \) takes a minimum value \( E_{\nu}^{max} = \frac{m_\pi^2 - m_\mu^2}{2E_\pi^2 \sin^2 \theta} \) at \( E_\pi^o = m_\pi / \sin \theta \). Numerical calculations show that a wide range of \( E_\pi \), in particular that of \( E_\pi \geq E_\pi^o \), contributes to a narrow range\(^{11}\) of \( E_\nu \leq E_{\nu}^{max} \). It is expected, therefore, that a narrow neutrino spectrum peaked at around \( E_{\nu}^{max} \) can be obtained at off-axis positions to the beam see Figure 7.

Figure 8 shows a simulated example of neutrino fluxes at 14 mrad, NOvA’s medium energy beam produces a narrow energy beam with approximately five times more neutrinos at 2 GeV compared to on-axis. This peak is well matched to the oscillation maximum which is expected to be 1.6 GeV for \( \Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2 \) [29].

\(^{11}\)see reference 113 in Ref. [5]
Figure 7: Left: The neutrino flux from a pion of energy $E_\pi$ as viewed from a site located at an angle $\theta$ from the beam axis. The flux has been normalized to a distance of 800 km. Right: The energy of the neutrinos produced at an angle $\theta$ relative to the pion direction as a function of the pion energy [29].

Figure 8: Charged-current $\nu_\mu$ event rates prior to oscillations calculated for a distance of 810 km from Fermilab and at various off-axis locations in the NuMI beam. The spectra are for the NuMI low-energy (left) and medium-energy (right) configurations [29].

In addition to the increased flux, the narrowness of the off-axis spectra enhances background rejection. One important source of background events are neutral-current events where the outgoing lepton (the neutrino) is not observed. The topologies of these
events can fake the electron showers produced by $\nu_e$ charged-current events. As the neutrino carries much of the event energy away, the visible energies of neutral-current events tends to feed down to lower energies. In a wide band beam this feed down into the signal region is much larger than it is in a narrow band off-axis beam where the feed down tends to push the neutral-current events outside the signal energy window. Figure 9 shows the number of neutral-current events as a function of their visible energy, illustrating this effect [29].

Figure 9: Simulated energy distributions for the $\nu_e$ oscillation signal, intrinsic beam $\nu_e$ events, neutral-current events and $\nu_\mu$ charged-current events with and without oscillations. The simulation used $\Delta m_{32}^2 = 2.5 \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{23} = 1.0$, and $\sin^2 2\theta_{13} = 0.10$. An off-axis distance of 12 km at 810 km was assumed.[29].

Another important source of backgrounds to the electron-neutrino appearance search is the intrinsic $\nu_e$ component of the NuMI beam which arise from muon and kaon decay. As these neutrinos result from three-body decays they are more broadly distributed in energy than the $\nu_\mu$'s produced by two-body decays. The spectra of the $\nu_e$'s in the NuMI beam off-axis are shown in Figure 9. The relative narrowness of the off-axis $\nu_\mu$ beam compared to the $\nu_e$ beam is an additional advantage of the off-axis approach [29].
2.3 NOνA Detectors

2.3.1 The Neutrinos at the Main Injector (NuMI) Beam

The proton accelerator cycle for the NuMI Beam starts with the Linac (see Figure 10) and is followed by the Booster and then the Main Injector. The Tevatron was operational during most of the Main injector neutrino oscillation search (MINOS) run but was not used in neutrino production. The Recycler is used in the follow-up, post-MINOS experiments. A large number of beam lines shown in Figure 10 were constructed for other experiments and are no longer in use or their function has changed. The AP1, AP2 and AP3 beam lines, AP0 target station and the ring named Muon formed the antiproton source which is no longer active and in the future some of these will be used for muon experiments. The P1 and A1 lines are proton and antiproton injection lines from the Main Injector to the Tevatron and are also no longer in use. The P2 and P3 lines use original Main Ring magnets and were part of the fixed target extraction complex. The squares labeled MI surrounding the Main Injector are various Main Injector service buildings.

The NuMI Facility produces an intense 700 kW beam of neutrinos to enable a new generation of experiments whose primary scientific goal is to definitively detect and study neutrino oscillations. The beam has sufficient intensity and energy so that experiments capable of identifying $\nu_\mu \rightarrow \nu_e$ oscillations, as well as other possibilities, are feasible. A beam of protons from Fermilab’s Main Injector is used to produce the neutrino beam. Interactions of the proton beam with a production target produces mesons, which decay to muons and neutrinos during their flight through a decay tunnel, see Figure 15. A hadron absorber downstream of the decay region will remove the remaining protons and mesons from the beam (see Figure 11). The muons will be absorbed by an intervening rock shield while the neutrinos continue through it to the near experimental hall and beyond to the far detector in Ash River, Minnesota. Inside the experimental halls of Fermilab, the Near Detector for the NOνA is designed to observe the relatively large sample of neutrinos that will interact in it [30][31].
2.3.2 Flux

Knowledge of the neutrino flux produced by the Neutrinos at the Main Injector (NuMI) beamline is essential to the neutrino oscillation and neutrino interaction measurements of the NO$
u$A experiment (see Figure 12) at Fermi National Accelerator Laboratory. The result is the most precise flux achieved for a neutrino beam in the one to
tens of GeV energy region. The NuMI focusing system has an \( \sim 200 \) kA current being pulsed through two aluminum horns to create a toroidal magnetic field. The current passes through a conductor (Al). The inner conductor is 2mm-4mm thick. Every charged particle traveling by the horns feel a \( P_T \) kick, see Figure 12 and Figure 14 for flux uncertainties [33].

![NOvA Simulation](image)

Figure 12: Neutrino beam split in types for the forward horn current at the NOvA near detector. The beam is mostly muon neutrinos peaked at 2 GeV. The electron neutrino shows a peak as well [34].

### 2.3.3 Target

The NOvA target as seen in Figure 13 is made of 48 graphite fins of the type ZXF-5Q (POCO graphite). It is housed in a canister with beryllium windows at the entrance and exit. The target canister is filled with helium gas to prevent the target graphite material from oxidation as well as reduce differential pressure on the Be Windows [1].
Figure 13: Carbon Target [1].

Figure 14: NOvA Target [1]. Flux uncertainties using PPFX.
Figure 15: Schematic of the NuMI Beam. The individual components of the NuMI beam (not to scale) are shown together with the relevant dimensions. All the important elements are shown, including the target, the horns, the decay pipe, the hadron absorber, and the so-called muon shield which consists of the dolomite rock preceding the MINOS Near Detector [30].

2.3.4 Detectors

Two different detectors have been built, one; a 222 ton Near detector placed in a small new underground cavern adjacent to the existing NuMI tunnel on the Fermilab site measures the inherent NuMI beam backgrounds relevant to a search of electron neutrino appearance and muon neutrino disappearance in the NuMI muon neutrino beam. The other detector is a 14 kiloton Far detector 810 km and 14.4 mrad off-axis from the NuMI beam source. Both are composed of cells of extruded PVC plastic in a cellular structure. Each cell is 3.9 cm wide by 6 cm deep and is 15.5 meter long. The cells are filled with liquid scintillator. The detector is read out via optical wave-shifting fiber into an avalanche photodiodes with associated electronics [29]. The readout from the fiber optics are then calibrated. Avalanche Photodiodes APD register the amount of energy deposited in the detector expressed in units (PECorr). The calibration provides factors so that the energy deposits can be expressed in physically meaningful units (MIP’s and GeV). Stopping muon are used because their energy deposit should be known from the Bethe Bloch formula, see Figure 16 for a view of the detectors [35].
Figure 16: The NOvA detectors are constructed from planes of PVC modules alternating between vertical and horizontal orientations. The far detector is 15.6 x 15.6 meters in size and 78 meters long. In addition to the far detector, this detector sees the NuMI beam at an angle of 110 milliradians. For the physics run, NOvA constructed a near detector placed underground at Fermilab at the same off-axis angle as the far detector. All detector are identical in their construction, but differ in size. [36].

2.4 Neutral Current $\pi^0$ Production

2.4.1 Brief History of the Pion

In this section, I will show content of the events that happened half a century ago of the pion pioneers, since this thesis is all about pions, I felt a brief history of the pion deserves a subs-section in this thesis, the following pages are from the CERN courier article posted in June 1997 pages 2-6:

While the classic discoveries of Thomson and Rutherford opened successive doors to subatomic and nuclear physics, particle physics may be said to have started with the discovery of the positron in cosmic rays by Carl Anderson at Pasadena in 1932, verifying Paul Dirac’s almost simultaneous prediction of its existence. Anderson used a cloud chamber, expanded at random, in a high magnetic field at the same time, Patrick Blackett at Cambridge was joined by an inventive young Italian, Giuseppe Occhialini, sent by a master of counter coincidence techniques, Bruno Rossi, then in Florence, to learn about cloud chambers. Very soon Blackett and Occhialini had built a counter-controlled chamber with which they discovered electron-positron pair production, a key prediction of Dirac’s ideas [37].

Cloud chambers played a major role in cosmic ray studies in the following years, leading to the
discovery of the ‘mesotron’ in 1937, originally identified as the nuclear force carrier postulated by Hideki Yukawa in 1935. However, several difficulties soon arose with this hypothesis, even though pictures of its decay to an electron, as postulated by Yukawa to explain beta-decay, were observed in cloud chamber pictures in 1940. In particular, the mesotron appeared to have a very weak nuclear interaction with matter, conclusively demonstrated by the counter experiments of Marcello Conversi, Ettore Pancini and Oreste Piccioni in Rome from 1943-1947 [37].

A possible explanation of these difficulties had been put forward in Japan in 1942 and 1943 by Yasutaka Tanikawa and by Shoichi Sakata and Takeshi Inoue, who suggested a two-meson hypothesis with a Yukawa-type meson decaying to a weakly interacting mesotron. Because of the war their ideas were not published in English until 1946 and 1947, the journals in question not reaching the USA until the end of 1947 [37].

Unaware of the Japanese work, Robert Marshak had put forward a similar hypothesis in June 1947, at a conference of American theoreticians on Shelter Island (off Long Island), and which he published later that year with Hans Bethe. None of the scientists at the conference knew that such two-meson decay events had already been observed some weeks earlier by Cecil Powell and his collaborators in Bristol, using the then little known photographic emulsion technique, but which in Powell’s hands became a powerful research tool. Powell had been a research student under C.T.R. Wilson at the Cavendish Laboratory in Cambridge, before joining the H.H. Wills Physics Laboratory, (also known as the Royal Fort), at Bristol University in 1928 as an assistant to the Director, Arthur Tyndall. They worked together on the mobility of ions in gases until 1935 when Powell became interested in nuclear physics, inspired by the discoveries in Rutherford’s Cavendish Laboratory. Together with a young lecturer, Geoffrey Fertel, he embarked on the construction of a 750 keV Cockcroft-Walton accelerator, which they brought in to operation in 1939. The original intention was to study low energy neutron scattering using a Wilson cloud chamber. However, in 1938 the theoretician Walter Heitler (then in Bristol) mentioned to Powell that in 1937 two Viennese physicists, Marietta Blau and Herta Wambacher, had exposed photographic emulsions for five months at 2,300 m in the Austrian Alps and had seen the tracks of low energy protons as well as ‘stars’ or nuclear disintegrations, probably caused by cosmic rays. Heitler commented that the method was so simple that ‘even a theoretician might be able also to do it’. This intrigued Powell and Heitler traveled to Switzerland with a batch of Ilford half-tone emulsions, 70 microns thick, and exposed them on the Jungfraujoch at 3,500 m. In a letter to ‘Nature’ in August 1939, they were able to confirm the observations of Blau and Wambacher [37].

The half-tone emulsions could only record the tracks of low energy protons and alpha particles and Powell realized that to do useful work it was necessary to increase their sensitivity by increasing the concentration of silver bromide [37].

World War II interrupted the work, but with the existing emulsions Powell was able to show that for scattering studies they gave results superior to cloud chambers, as well as being much faster. Blackett (who had been a contemporary of Powell in the Cavendish Laboratory) then played a decisive role through his influence with the Ministry of Supply of the 1945 UK Labour Government. He was largely responsible for the setting up of two panels, one to plan accelerator building in the United Kingdom (which he chaired) and one to encourage the development of sensitive emulsions (chaired by Joseph Rotblat, recently awarded the Nobel Peace Prize for his Pugwash work) [37].

Towards the end of the war, Blackett had invited his erstwhile collaborator Occhialini, then in Brazil, to join the British team working with the Americans on the atomic bomb. Occhialini arrived in the United Kingdom in mid-1945, only to learn that, as a foreign national, he could no longer work on the project. Instead, he joined Powell in Bristol, becoming a driving force behind the development of the new emulsion technique. He was joined by one of his research students, Cesare Lattes, towards the end of 1946. Photographic manufacturers Ilford were soon able to supply ‘Nuclear Research Emulsions’ and in autumn 1946 Donald Perkins, then at Imperial College, London, exposed some at 9,100 m in an RAF aeroplane, while Occhialini took several dozen plates to the Pic du Midi at 2,867 m in the French Pyrenees. At that time access to the Pic was by a rough track in summer and by ski in winter, a small
Examination of the emulsions in Bristol and in London revealed, as Powell later wrote, "a whole new world. It was as if, suddenly, we had broken into a walled orchard, where protected trees flourished and all kinds of exotic fruits had ripened in great profusion". This new world became a subject of intensive investigation. Occhialini has well described the atmosphere at Bristol:- "Unshaved, sometimes I fear unwashed, working seven days of the week till two, sometimes four in the morning, brewing inordinately strong coffee at all hours, running, shouting, quarrelling and laughing, we were watched with humorous sympathy by the war-worn native denizens of the Royal Fort".... "It was a reality of intense, arduous and continuous work, of deep excitement and incredibly fulfilled dreams. It was the reality of discovery....". Perkins was the first to observe a clear example of what appeared to be the nuclear capture of a meson in the emulsion and producing a nuclear disintegration. Measurements of the multiple scattering as a function of residual range indicated a mass between 100 and 300 times that of the electron. Perkins’ observations, published in January 1947, were confirmed by Occhialini and Powell, who published details of six such events only two weeks later [37].

Mesons were easily distinguished from protons in the emulsion because of their much larger scattering and by their variation of grain density with range. Yet another exotic fruit followed. In the spring of 1947 one of Powell’s team of microscope observers, Marietta Kurz, found a meson stopping and giving rise to a second meson, which left the emulsion when nearly at the end of its range. Powell and a young Bristol graduate, Hugh Muirhead, were the first physicists to look at the event, which they immediately recognised as being two related mesons. Within a few days a similar event was found by Irene Roberts, the wife of the group technician, Max Roberts, who later worked at CERN for many years. In this event the secondary meson ended in the emulsion, with a range of 610 microns. The two events gave such convincing evidence for a two-meson decay chain that Lattes, Muirhead, Occhialini and Powell published their findings in ‘Nature’ in the issue of 24 May, 1947 (see Figure 17) [37].

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Figure 17: In studying photographic plates exposed to the cosmic rays, Dr. C. P. S. Occhialini and Dr. C. F. Powell from H. H. Wills Physical Laboratory at University of Bristol found a number of multiple disintegrations each of which appears to have been produced by the entry of a slow charged particle into a nucleus. It is seen that associated with the ‘star’ there is one track, marked m, which shows frequent changes in direction. The points of scattering are most frequent near the ‘star’ and become progressively fewer in moving away from it, along the trajectory [38].

Commenting on the problems surrounding the identification of the cosmic ray mesotron with the
Yukawa nuclear force meson, they wrote: "Since our observations indicate a new mode of decay of mesons, it is possible that they may contribute to a solution of these difficulties". More evidence was needed to justify such a radical conclusion. For sometime no more two-meson events were found in the Pic du Midi emulsions and it was decided to make exposures at much higher altitudes. Lattes proposed going to Mount Chacaltaya in the Bolivian Andes, near the capital La Paz, where there was a meteorological station at 5,600 m. Arthur Tyndall recommended that Lattes should fly BOAC to Rio de Janeiro. Lattes preferred to take the Brazilian airline Varig, which had a new plane, the Super Constellation, thereby avoiding a disaster when the British plane crashed in Dakar and all on board were killed. Examination of the plates from Bolivia quickly yielded ten more two-meson decays in which the secondary particle came to rest in the emulsion. The constant range of around 600 microns of the secondary meson in all cases led Lattes, Occhialini and Powell, in their October 1947 paper in 'Nature', to postulate a two-body decay of the primary meson, which they called p or pion, to a secondary meson, m or muon, and one neutral particle. Subsequent mass measurements on twenty events gave the pion and muon masses as 260=130 and 205=120 times that of the electron respectively, while the lifetime of the pion was estimated to be short lived with a mean lifetime in the interval from $10^{-6}$ to $10^{-11}$ s [39]. Present-day values are 273.31 and 206.76 electron masses respectively and $2.6 \times 10^{-8}$ s.

Figure 18: It is shown here, an example in which a single proton is ejected. Back then it was already thought that nuclear interaction between mesons and nucleons was assumed to be of short range, and since the Coulomb repulsion would tend to prevent a slow positively charged meson from approaching a nucleus, C.M.G Lattes along with G.P.S Occhialini and C.F. Powell regarded the observation of the disintegrations produced by "$\sigma$-mesons" as providing strong evidence that they are negatively charged. [39].

The number of mesons coming to rest in the emulsion and causing a disintegration was found to be approximately equal to the number of pions decaying to muons. It was, therefore, postulated that the latter represented the decay of positively-charged pions and the former the nuclear capture of negatively-charged pions. Clearly the pions were the particles postulated by Yukawa. This led to the conclusion that most of the mesons observed at sea level are penetrating muons arising from the decay in flight of pions created in nuclear disintegrations higher up in the atmosphere.

Powell was awarded the 1950 Nobel Physics Prize for his development of the emulsion technique and for the discovery of the pion; Occhialini was awarded the 1979 Wolf Prize (shared with George Uhlenbeck) for his contribution both to the pion discovery and to that of pair production with Blackett, who obtained the 1948 Nobel Physics Prize [37].
The Impact of NC $\pi^0$ Physics On Neutrino Nucleus Interactions

In NC lepton-nucleus scattering, a neutrino with unknown energy enters the NOvA detector made of mineral oil/PVC and interacts with a single nucleon in the heavier nuclei. On occasion, the exchanged boson interacts with a pair of correlated nucleons. The final state $\nu_\mu$ escapes the nucleus as the initially produced hadronic shower undergoes significant further effects as it proceeds through the dense nuclear matter within the nucleus [40]. In this section, we will discuss the physics of NC $\pi^0$ production since it is crucial for NOvA to understand, being the largest background to the $\nu_\mu \rightarrow \nu_e$ neutrino oscillation. However, before delving into the physics, let’s remember a prominent American physicist that provided an understanding of how subatomic particles interact with one another. The paper by R.P. Feynman "Space-Time Approach to Quantum Electrodynamics" was received in May 9, 1949 to the department of physics at Cornell University in Ithaca, New York. In this paper, three things were accomplished. 1) Feynman showed that a considerable simplification can be attained in writing down matrix elements for complex processes in electrodynamics, 2) Feynman recognized that electrodynamics is modified by altering the interaction of electrons at short distances. 3) Feynman facilitated for the first time in history, a description of interactions of subatomic particles using diagrams which were more clear than the mathematical description [41]. Feynman diagrams remain a treasured asset in physics, because they often provide good approximations to reality. They help us bring our powers of visual imagination to bear on worlds we can’t actually see.

Current calculations for NOvA would have been literally unthinkable without Feynman diagrams. For further illustration of these scattering processes, Feynman diagrams are a good way to see why the NC $\pi^0$ background is so prominent for the $\nu_\mu \rightarrow \nu_e$ oscillation. At intermediate neutrino energy $E_\nu \approx 2$ GeV, the neutrino inelastic interaction produces a baryon resonance ($N^*$). The baryon resonance quickly decays, most often to a nucleon and single $\pi^0$ final state [4].

$$\nu_\mu N \rightarrow \mu^- N^*$$

(11)
then,

\[ N^* \rightarrow \pi^0 N' \]  \hspace{1cm} (12)

where \( N, N' = n, p \). In scattering off of free nucleons, there are seven possible resonant single pion reaction channels (seven each for neutrino and antineutrino scattering), three charged current (CC):

\[ \nu_\mu p \rightarrow \mu^- p \pi^+, \quad \bar{\nu}_\mu p \rightarrow \mu^+ p \pi^- \] \hspace{1cm} (13)

\[ \nu_\mu n \rightarrow \mu^- p \pi^0, \quad \bar{\nu}_\mu p \rightarrow \mu^+ n \pi^0 \] \hspace{1cm} (14)

\[ \nu_\mu p \rightarrow \mu^- p \pi^+, \quad \bar{\nu}_\mu p \rightarrow \mu^+ p \pi^- \] \hspace{1cm} (15)

and four neutral current (NC):

\[ \nu_\mu p \rightarrow \nu_\mu p \pi^0, \quad \bar{\nu}_\mu p \rightarrow \bar{\nu}_\mu p \pi^0 \] \hspace{1cm} (16)

\[ \nu_\mu p \rightarrow \nu_\mu n \pi^+, \quad \bar{\nu}_\mu n \rightarrow \bar{\nu}_\mu n \pi^0 \] \hspace{1cm} (17)

\[ \nu_\mu n \rightarrow \nu_\mu n \pi^0, \quad \bar{\nu}_\mu n \rightarrow \bar{\nu}_\mu n \pi^0 \] \hspace{1cm} (18)

\[ \nu_\mu n \rightarrow \nu_\mu p \pi^-, \quad \bar{\nu}_\mu n \rightarrow \bar{\nu}_\mu p \pi^- \] \hspace{1cm} (19)

To describe such resonance production processes, neutrino experiments most commonly use calculations from the Rein and Sehgal model (R. P. Feynman, 1971; Rein, 1987; Rein and Sehgal, 1981) with the additional inclusion of lepton mass terms. This model gives predictions for both CC and NC resonance production and a prescription for handling interferences between overlapping resonances [4].

NC \( \pi^0 \) interactions described under this model, are produced via three scattering processes where \( \pi^0 \) production takes place. Whenever a neutrino scatters via neutral boson exchange with the nucleon, the interaction may then produce at least one single \( \pi^0 \) via RES (Resonance) scattering (see Figure 19), deep inelastic scattering (DIS) (see Figure
20), or coherent scattering (COH) (see Figure 21). Each decay process between equations 13 to 19 are mainly RES scattering producing one single $\pi^0$.

Figure 19: RES

NC $\pi^0$ deep inelastic scattering results from a $Z^0$ scattering a quark inside $N$ and fragmenting the nucleon into many hadrons and producing a $\pi^0$ in the process.

Figure 20: DIS

The process involving scattering of a neutrino with an entire nucleus is the Coherent scattering, which results in the production of forward going pions in both charged current ($\nu_\mu + N \rightarrow \mu^- + \pi^+ + N$) and neutral current ($\nu_\mu + N \rightarrow \nu_\mu + \pi^0 + N$) channels. Coherent neutrino-nucleus interactions is also modeled according to the Rein-Sehgal model [42].
Figure 21: The characteristic of neutral current coherent scattering with one single $\pi^0$ final state is a single, forward-going $\pi^0$, with no other pions or nucleons or vertex activity [43]. $P$ is the so-called pomeron (or pomeron-like particle). Simply put, $P$ should carry the quantum number of vacuum and a small momentum transfer.

The identification of particular scattering processes in neutrino scattering experiments, such as resonance production, relies on the identification of one or more hadrons. Furthermore, reconstruction of the neutrino energy depends on the observed energies of these hadrons. It is therefore imperative to model as accurately as possible the effects of hadron formation and propagation in the nucleus. Data from semi-inclusive lepton scattering provides experimental observables which can directly confront models of hadronization and propagation utilized for neutrino scattering [40]. The semi inclusive neutral current neutral pion production model, is a collection of decays with at least one $\pi^0$ or more produced after the $\nu_\mu$ exchanges a $Z^0$ with the nucleon $N$ (where $N = n, p$) via the semi inclusive process $\nu_\mu + N \rightarrow \nu_\mu + N + \pi^0 + X$ where $X$ denotes anything else in the hadronic states (see Figure 22).
Figure 22: Semi inclusive decay of a single $\pi^0$ and anything else in the hadronic states ($X$).

The total energy produced by the incident secondary $\pi^0$ particle then decays into two gammas, see Figure 23.

The NC $\pi^0 \rightarrow \gamma \gamma$ is the most common type of $\pi^0$ decay\(^\text{12}\) (at least 98% of the time) which can mimic the $\nu_\mu \rightarrow \nu_e$ signal whenever there are two gammas merging together as

\(^{12}\text{the other common decay modes are } \pi^0 \rightarrow e^+e^-\gamma (< 2\%), \text{ and } \pi^0 \rightarrow \gamma \text{Ps} (< 2.0 \times 10^{-9}\%), \text{ where Ps is denoted as Positronium}\)
illustrated in Figure 24.

\[ \pi^0 \rightarrow \gamma + \gamma \]

Figure 24: A \( \pi^0 \) decaying onto two "merged" gammas.

If at least one decay photon is not detected, an NC \( \pi^0 \) event can be indistinguishable from a charged current \( \nu_e \) interaction, leading to an irreducible background in \( \nu_\mu \rightarrow \nu_e \) oscillation measurements [44].

In short, an important distinction in studying various classes of lepton-nucleus reactions should be made clear: one should distinguish inclusive reactions, where only the scattered lepton is presumed to be detected, from more exclusive reactions where, in addition to the final-state lepton, additional particles are presumed to be detected. Modeling neutrino-bound-nucleon cross sections not only at the lepton semi-inclusive cross section level, but also in the full phase space for all the exclusive channels that are kinematically allowed will improve our understanding of the role played by effects like nucleon-nucleon correlations in interactions and implementing this understanding in MC (Monte Carlo) generators, in order to avoid double counting [40]. Figure 25 shows some of the most common RES semi-inclusive NC \( \pi^0 \) Feynman diagram interactions,
Figure 25: RES $\pi^0$ decay and other common interaction channels.

2.4.3 Other NC $\pi^0$ Measurements

A few measurements of neutrino induced neutral current $\pi^0$ production have been done by previous experiments. One of the most recent ones within the past 2 years was a measurement of single $\pi^0$ production\(^{13}\) for the NOvA collaboration. Other experiments with measurements are the K2K, T2K, SciBooNE, and MiniBooNE experiments, see Table 2 and Figure 28.

Currently within the NOvA collaboration, two independent cross section measurements of NC $\pi^0$ production using two novel techniques (selectors) are being worked on. The first, which is the analysis of this paper, is Prong Level CVN (PL-CVN), and the second is Boosted Decision Tree Gradient (BDTG). BDTG is a multivariate method using the GradientBoost method (a type of BDT) using different sets of variables to determine which variables gives the best event selector for the NC $\pi^0$ with kinetic energy $> 0.5$ GeV. This independent measurement is performed using 9 input variables and a cut on BDTG is optimized by minimizing the fractional uncertainty on the cross section. The study has a

\(^{13}\)Measurement of neutrino-induced coherent $\pi^0$ production using high statistics NOvA data. A data-driven method is developed to constrain the non-coherent background. The total uncertainty is 16.7% including systematic and statistical uncertainties. This is one of the most precise measurements of coherent $\pi^0$ production in the world.[43]
fiducial volume of X(-130,160), Y(-150,120), Z(225,950) and a containment volume of X(-130,140), Y(-150,150), Z(300,1025). It also uses ND MC with high statistics (x4 than the ND data). All MC distributions and number of events are normalized to $8.09 \times 10^{20}$ that reflects the NOvA ND data. Like PL-CVN, a muon particle identifier is used to reject most of the CC background, as well as log-likelihoods to identify particle types. The systematic samples used for this study include flux, cross-section, detector response with systematic shifts of $\pm 1\sigma$. The analysis uses re-weighted mechanism to tune the flux and cross-section uncertainties.

While not finished, it is expected the BDT method will obtain a measurement uncertainty at a level of 10-15%, the current state of the analysis provided a purity of 73%, and 1.3% efficiency w.r.t the events in true fiducial. Figure 27 shows the fractional uncertainty on the total cross section [34]. The BDTG analysis also has studied the resolution and bias in kinetic energy and angle variables that show a bias by using a fixed $\pi^0$ mass in calculating reconstructed kinetic energy of a $\pi^0$, see Figure 26.

![Figure 26: Comparison of bias distributions using reconstructed $\pi^0$ mass to get reconstructed kinetic energy show comparatively less bias with somewhat better resolution in the lower energy bins [45].](image_url)
Figure 27: Cut at $> 0.6$ NC $\pi^0$ ID at the fractional uncertainty minima [34].

Table 2: SciBooNE Collaboration (Improved measurement of neutral current coherent $\pi^0$ production on carbon in a few-GeV neutrino beam) [46], (Measurement of Inclusive Neutral Current Neutral $\pi^0$ Production on Carbon in a Few-GeV Neutrino Beam) [47]. T2K Collaboration (Measurement of the single $\pi^0$ production rate in neutral current neutrino interactions on water) [44], MiniBooNE Collaboration (Measurement of $\nu_\mu$ and $\bar{\nu}_\mu$ induced neutral current single $\pi^0$ production cross sections on mineral oil at $E_\nu$ (1 GeV)) [48], NOvA Collaboration (Measurement of Neutral Current Coherent $\pi^0$ Production In The NOvA Near Detector) [43], K2K Collaboration (Measurement of single $\pi^0$ production in neutral current neutrino interactions with water by a 1.3-GeV wide band muon neutrino beam) [49], POT (Protons On Target) Experiment (Protons On Target)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Target</th>
<th>$\sigma_{meas.}$</th>
<th>$\langle E_\nu \rangle$</th>
<th>POT</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2K</td>
<td>Mineral oil</td>
<td>$7.63 \times 10^{-39} \text{ cm}^2 \text{ nucleon}^{-1}$</td>
<td>1.3 GeV</td>
<td>$3.49 \times 10^{20}$</td>
<td>Semi-Inc.</td>
</tr>
<tr>
<td>K2K</td>
<td>Water</td>
<td>$6.40 \times 10^{-2} \frac{\text{NC}\pi^0 \text{Rel.} \sigma}{\text{CC}\nu_\mu \text{Tot.} \sigma}$</td>
<td>1.3 GeV</td>
<td>$3.20 \times 10^{19}$</td>
<td>Semi-Inc.</td>
</tr>
<tr>
<td>SciBooNE</td>
<td>Polystyrene ($C_8H_8$)</td>
<td>$7.70 \times 10^{-2} \frac{\text{NC}\pi^0 \text{Rel.} \sigma}{\text{CC}\nu_\mu \text{Tot.} \sigma}$</td>
<td>1.1 GeV</td>
<td>$2.52 \times 10^{20}$</td>
<td>Semi-Inc.</td>
</tr>
<tr>
<td>SciBooNE</td>
<td>Carbon</td>
<td>$1.16 \times 10^{-2} \frac{\text{NC}\pi^0 \text{Rel.} \sigma}{\text{CC}\nu_\mu \text{Tot.} \sigma}$</td>
<td>0.8 GeV</td>
<td>$0.99 \times 10^{20}$</td>
<td>Inclusive</td>
</tr>
<tr>
<td>MiniBooNE</td>
<td>Water</td>
<td>$4.76 \times 10^{-40} \text{ cm}^2 \text{ nucleon}^{-1}$</td>
<td>808 MeV</td>
<td>$2.64 \times 10^{20}$</td>
<td>Exclusive</td>
</tr>
<tr>
<td>NOvA$^{14}$</td>
<td>Mineral Oil/PVC</td>
<td>$14.0 \times 10^{-40} \text{ cm}^2 \text{ nucleon}^{-1}$</td>
<td>2 GeV</td>
<td>$3.54 \times 10^{21}$</td>
<td>Semi-Inc.</td>
</tr>
</tbody>
</table>
Figure 28: Existing measurements of the cross section for the NC process, $\nu_\mu \ p \rightarrow \nu_\mu \ p \ \pi^0$, as a function of neutrino energy. Also shown is the prediction from Reference (Casper, 2002) assuming $M_A = 1.1$ GeV. The Gargamelle measurement comes from a more recent re-analysis of this data (Hawker, 2002). [4].

2.5 NO$\nu$A Near Detector NC $\pi^0$ Analysis Steps

This paper presents the PL-CVN Method to generate a $\pi^0$ invariant mass plot after optimizing cuts using systematics, as well as determining a containment, vertex, and $\pi^0$ total energy threshold. In particle physics, the invariant mass $m_0$ is equal to the mass in the rest frame of the particle, and can be calculated by the particle’s energy $E$ and its momentum $p$ as measured in any frame, by the energy-momentum relation: $m_0^2 \ c^2 = (\frac{E}{c})^2 - |p|^2$ or in natural units where $c = 1$, $m_0^2 = E^2 - |p|^2$.

First a pre-selection is made with fiducial vertex, containment, and 2 "prong" event cuts. The analysis continues by rejecting background using Reconstructed Muon Identifier (RemID) distributions. After the RemID cut, the next thing is to apply a Gamma identifier ("prong" based CVN algorithm (PL-CVN)). This allows production of a
distribution to pick a cut value to reject "prong" events that don’t decay into 2 γ’s (the π⁰ signature). The analysis also includes a 2 dimensional distribution between NC π⁰ invariant mass and true energy to evaluate the optimal total energy threshold of the π⁰. Lastly, an optimization for the gamma identifier is done by minimizing on total cross section for "prong" 1 gamma ID, and "prong" 2 gamma ID. Once the PL-CVN cuts are optimize, a resolution study is performed to see the bias vs. reconstructed total π⁰ energy, as well as the absolute resolution vs. reconstructed total π⁰.
CHAPTER III

NEUTRAL CURRENT EVENT SELECTION

3.1 Simulation

Neutrino interactions in the NOνA detectors are simulated using the GENIE event generator\textsuperscript{15}. The GENIE Monte Carlo Simulator has an implementation using the modified PCAC formula described in a recent revision of the Rein-Sehgal model that includes lepton mass terms that allows for the analysis of resonance production processes. The studies presented here were done with the ND Monte Carlo (MC) data sets. As the NOνA ND is located close to the target source (approximate 1 km away from target source), it has high statistics and thus provides an excellent opportunity for the measurement of various neutrino interactions mainly cross-section measurements. ND data used here were collected in the FHC\textsuperscript{16} configuration (all periods), tier caf, name of the data\textsuperscript{17} [42][50]. The distributions here, in this paper, reflect the data available at the NOνA ND, corresponding to \(8.09 \times 10^{20}\) Protons On Target (POT).

3.2 Data Quality Fiducialization and Containment

Standard NOνA data quality and timing cuts are used to select beam neutrino events under the normal beam and detector conditions. To start we select all events that happen in a containment volume of the NOνA detector. In the case of the ND, there are areas where the response of the detector is poorly understood. This is because a poorly modeled class of background events interact in the outer edges of the detector. Therefore, it is reasonable to think, that outer limits of the detector must be ignored in order to get a reliable result. For event identification the most reliable area in the NOνA detector is at its

\textsuperscript{15}The GENIE Neutrino Monte Carlo Generator: Physics and User Manual.
\textsuperscript{16}Forward Horn Current
\textsuperscript{17}Provided by the Fermilab nusoft NOνA production website: prod_caf_R17-03-01-prod3reco.d_nd_genie_nonswap_fhcnova_v08_full_v1
core. To ensure a reliable neutrino energy reconstruction and to reject background from entering particles, events in the ND are required first to be contained in an active volume by vetoing events with activity in the outer detector’s volume region.

The primary reason for containing, is because we want track/"prongs" where we capture the energy. If there is no containment we would get energy leaving the detector that we can’t reconstruct. For containment, we restrict the distance of the primary shower from the detector edges by using start and stop positions of the primary shower, see Figure 29. Once we define a containment volume, a fiducial volume is necessary, where the $\nu_k \rightarrow N$ interaction occurs, the tool that identifies these interactions uses an algorithm that defines a vertex based on the direction of the "prongs".

Figure 29: 2 "prong" events. Each prong represents a photon, both point to the vertex the decayed neutral pion. Fiducial and containment volumes are shown.

Clusters of hits that correspond to the same shower are reconstructed as "prongs". The intersection of the paths used to find a neutrino interaction vertex (a point where the neutrino interacts with a nucleus). The leading "prong" is the most energetic prong and the sub-leading "prong" is the second-most energetic "prong" and so on. A first pre-selection is made requiring the reconstructed vertex to be in a ND fiducial volume as well as all the reconstructed showers to be contained in the ND containment volume. These
first two cuts produce an spectrum with reliable events in this analysis and are known as pre-selection. This analysis requires 2 "prong" events, see Figure 31. The pre-selection shower containment and vertex fiducial cuts are defined with the standard length, width, and height of the NOνA ND detector with dimensions X(-180 cm ,180 cm), Y(-180 cm ,180 cm), and Z(200 cm, 1200 cm). See 30 for a brake down of the events in the containment and fiducial volumes.

Figure 30: The analysis begins by plotting POT-normalized distributions broken down into each individual interaction type which includes low and high energy neutral pion invariant mass with no included cuts.
Figure 31: First distribution (blue) is a simulated pre-selection distribution with only fiducial and containment volume. The second distribution (red) shows the full selection containing 2 "prongs" to reconstruct the invariant mass.

Containment and vertex pre-selection broken down by all interaction types in the ND are presented. A study with 1D vertex and "prong" stop position allowed for a final containment and fiducial pre-selection determination. Also a 2 dimensional discriminant study [51] between the $\pi^0$ reconstructed mass and vertex and stop coordinates in the detector, is shown [52]. The xy-coordinates of the ND have a length between -200 to 200 cm, and the z-coordinate goes from 0 to 1200 cm. Based on these parameters and the 1 and 2 dimensional discriminant plots we are able to constraint our background by getting rid of rapidly falling rates near the edges of the detector that could clutter our signal, and only keep the stable behavior. The best containment and fiducial values picked for the study were $X(-160\ cm\ ,120\ cm), Y(-180\ cm\ ,155\ cm), Z(200\ cm, 1200\ cm)$ with respect to containment, and $X(-80\ cm\ ,120\ cm), Y(-148\ cm\ ,60\ cm), Z(225\ cm, 950\ cm)$ with respect to fiducial volume. See the following 1D and 2 D figures,
Figure 32: (a) shows the stacked x direction shower containment, figure (b) shows the same plot, unstacked.
Figure 33: (a) shows the stacked y direction shower containment, figure (b) shows the same plot, unstacked.
Figure 34: (a) shows the stacked z direction shower containment, figure (b) shows the same plot, unstacked.
Figure 35: (a) shows the stacked x direction vertex, figure (b) shows the same plot, unstacked and zoomed in.
Figure 36: (a) shows the stacked y direction vertex, figure (b) shows the same plot, unstacked and zoomed in.
Figure 37: (a) shows the stacked z direction vertex, figure (b) shows the same plot, unstacked and zoomed in.
Figure 38: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of all interactions at the stop position of leading prongs in the containment volume.
Figure 39: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of all interactions vertex.
Figure 40: (a) shows the x direction, right (b) shows the y-direction, and figure (c) shows the z-direction of all non $\pi^0$ CC (charged current) interactions at the stop position of leading prongs in the containment volume.
Figure 41: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of CC Non $\pi^0$ vertex.
Figure 42: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of all CC (charged current) $\pi^0$ interactions at the stop position of leading prongs in the containment volume.
Figure 43: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of CC (charged current) $\pi^0$ vertex.
Figure 44: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of all non neutral current $\pi^0$ interactions at the stop position of leading prongs in the containment volume.
Figure 45: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of NC (neutral current) non $\pi^0$ vertex.
Figure 46: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of all NC (neutral current) $\pi^0$ interactions of pions with total energy $> 0.3$ GeV at the stop position of leading prongs in the containment volume.
Figure 47: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of NC (neutral current) $\pi^0$ vertex of pions with total energy $> 0.3$ GeV.
Figure 48: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of all NC (neutral current) $\pi^0$ interactions of pions with total energy $< 0.3$ GeV at the stop position of leading prongs in the containment volume.
Figure 49: (a) shows the x direction, figure (b) shows the y-direction, and figure (c) shows the z-direction of NC (neutral current) $\pi^0$ vertex of pions with total energy < 0.3 GeV.
3.3 RemID Discrimination

After finalizing our pre-selection (fiducial + containment volumes). The next thing to do is to reject charged-current muon interactions using an already developed algorithm. This algorithm is called RemID. RemID\(^{18}\) scores Kalman tracks based on a muon hypothesis, and therefore allows for CC interactions to be rejected. A score value of RemID < 0.36 was picked based on the distribution in Figure 50:

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\(^{18}\)Reconstructed Muon Identification (ReMID) algorithm

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Figure 50: On the lower end of the distribution, there is a clear bump containing most CC interactions.
CHAPTER IV

PION IDENTIFICATION

Following the steps from above, we have a sample that is pretty pure or enriched in neutral current events, now we need to select $\pi^0$ and in order to do that I am going to try to find two prongs that look like photons as shown in Figure 51. To obtain the purest signal sample, it is necessary to understand the particular type of events that are being studied. A good method to understand NC $\pi^0$ events is by using machine learning. Identification of neutrino interactions based on their topology without the need for detailed reconstruction is done by means of a new technique derived from CNN called Prong Level Convolutional Visual Networks (CVN).

4.1 General Introduction To Convolutional Neural Networks (CNN) and High Energy Physics

In experimental high energy physics for many years there has been a problem with the correct categorization of the particle interactions recorded by the detectors as a signal.
or background. Usually, the characterization is based on reconstructing high level objects such as clusters, tracks, showers, jets, and rings associated with a particle interaction recorded by detectors and summarizing the energies, directions, and shapes of these objects with a handful of quantities randomly generated[53]. These variables can be fed into machine learning algorithms like K-Nearest Neighbors, Boosted Decision Trees, or Multilayer Perceptions, generating quantities used to separate signal from background. These techniques have been wildly successful over the years, however, for these selectors features used to characterize the events are limited to those which have already been imagined and implemented for the NOvA experiment and may lead to sub optimal effectiveness of identification [53].

Computer vision, can extract discriminating features from raw data. The HEP community has extensively done studies using these methods to study jets for classifying jet-images derived using Fisher discriminant analysis, see Ref. [54]. Particle physicists at SLAC National Accelerator Laboratory, have explored many approaches to extract specific features from images to enable categorization. The effectiveness of the technique is shown within the context of identifying boosted hadronic W boson decays with respect to a background of quark- and gluon- initiated jets. In 2015, using Monte Carlo simulations, physicists Josh Cogan, Michael Kagan, Emanuel Strauss, Ariel Schwartzman demonstrated that the performance of this technique introduces additional discriminating power over other sub-structured approaches, and gave significant insight into the internal structure of jets [54].

Thanks to advancements such as the jet-images, and other studies done over recent years, the NOvA collaboration, went from using specifically constructed features, to the extraction of features using a machine learning algorithm known as convolutional visual networks (CVN), see Ref. [53]. This type of technique, is rapidly becoming the dominant selector type in recent analyses see, Ref [55][53][21] et.al. Extending this, it is possible to train a network for single particle identification using clusters of hits ("prongs").

MLP or multilayer perceptron (traditional neutral network) is a machine learning
algorithm consisting of an input layer, one or more hidden layers, and an output layer. The goal is to approximate a function \( f : \mathbb{R}^n \rightarrow \mathbb{R}^m \) where \( n \) is the dimensionality of the input \( \vec{x} \) and \( m \) is the dimensionality of the output \( \vec{f} \). From NO\textsuperscript{a}s consistent research on CNN, see Ref. [53], it is clear that MLP is a powerful technique but it has deficiencies. First, it scales poorly to a number of large raw inputs. Second, the number of nodes in a single hidden layer may approach infinity. Partially this is due to the fact that sigmoid\(^{19}\) (crescent shape) functions are saturating, that is, as the input to the sigmoid approaches \( \pm \infty \), the gradient approaches zero. Third, the large number of free parameters in a large network runs the risks of over-training in which the networks learns to reproduce the training sample too well and fails to generalize inputs it has not seen [53][57]. Deep learning [53][58], the use of architecture with many layers, has had considerable success in tasks like image recognition [53][59][60] and natural language processing [53][61] and has been made possible by several advances that mitigate the deficiencies of traditional MLPs. Instead of relying on engineering features as inputs, the developments of structures like CNNs have made it possible to robustly and automatically extract learned features. To allow for the efficient training of deep structures, saturating non-linearities are frequently replaced by rectified linear unit (ReLU)[53][62], defined as \( f(x) = \max(0,x) \), which is non-saturating. Finally, over-training is mitigated in fully connected layers using the regularization technique called dropout [53][63] in which, at every training iteration, each weight is set to zero with a probability \( r \) while the remaining weights are scaled up by a factor of \( 1/(1-r) \) to roughly maintain the overall scale of the value passed through the non-linearity [53].

CNNs have been highly successful in the field of computer vision [53][59][64]. The technique was inspired by studies of the visual cortex of animals [53][65]. In these studies, it was found that the visual cortex contains simple cells, which are sensitive to edge-like features within small regions of the retina, and complex cells, which are receptive to collections of simple cells and are sensitive to position independent edge-like features.

\(^{19}\)A sigmoid function is a mathematical function having a characteristic "S"-shaped curve or sigmoid curve.
CNNs mimic this structure using a series of convolutional layers that extract a set of features from the input image and pooling layers that perform dimensionality reduction and add translational invariance [53] see Figure 52.

Figure 52: Each kernel we create stays the same as we apply it across the image. Weight sharing reduces the number of free parameters. Each convolutional layer trains an array of kernels which produce corresponding feature maps. Weights going from layer to the next are a 4D tensor of N by M by H by W: 1) N is number of incoming feature maps, 2) M is the number of outgoing feature maps, 3) H and W are the height and width of the outgoing convolutional kernels. The next layer applies kernels to combine the information in a receptive field across feature maps in the previous layer to create new feature maps. [2],

The data passed from layer to layer in a CNN has a three dimensional structure - height, width, and channel number. Height and width refer to the dimensions of the input image, and channel number is defined in analogy with the RGB channels of color images. For an n by m convolutional layer, the input data is transformed according to,

\[
(f \ast g)_{p,q,r} = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{c} f_{i,j,k,r} g_{p+i,q+j,k}
\]

where \((f \ast g)_{p,q,r}\) refers to the \((p,q)\) pixel of the \(r\) channel of the transformed image, \(n\) and \(m\) are the height and width of the convolutional kernel, \(c\) is the number of channels of the
input image, \( f \) is a filter, and \( g \) is an array corresponding to pixel intensities of the input image. The filter \( f \) is a four dimensional tensor where \( i \) and \( j \) index the height and width of the filter, \( k \) indexes the input channel, and \( r \) indexes the output channel, and it is trained to identify features within the image. For a fixed \( k \) and \( r \), the filter, \( f \), can be thought of as an \( n \) by \( m \) convolutional kernel. After applying a separate convolutional kernel to each channel and performing a weighted sum across channel, the resulting output image is known as a feature map. The range of the \( r \) dimension determines the number of \( c \) stacks of \( n \) by \( m \) convolutional kernels that are trained. Each of these stacks of kernels produces a feature map which are stored in the channel dimension of the layer output. Finally, each output pixel is operated on by a non-linear function [53]. In this way, convolutional layers [58] produce many alternative representations of the input image, each serving to extract some feature which is learned from the training sample [53] (see Figure 53).

![Diagram](image)

**Figure 53:** Effectively, these cells are applying convolutional kernels across the visual field[2].

### 4.2 Event Identification Using CVN in NOvA

With minimal event reconstruction, NOvA has been able to build and train a single algorithm which achieved excellent separation of signal and background for both of the NOvA \( \nu_e \) appearance and the \( \nu_\mu \) disappearance oscillation channels. This algorithm, CVN, is powerful against the problem of event classification and represents a good demonstration
that CNNs can work extremely well with non-natural images like the readout of a sampling calorimeter. CVN also opened up other possibilities for event classification and reconstruction which we have just started to explore. For example, the same training and architecture reported here can be expanded to include identification of $\nu$ NC and CC interactions which are used by NOvA in sterile neutrino and exotic physics searches [53].

4.2.1 Event Reconstruction Using Prong Level CVN

The reconstruction technique using prongs for particle IDs with CVN is a very recent technique developed by the NOvA collaboration. The method is very similar to the regular CVN particle identification but the difference is that we feed prongs into the neural network. Reconstruction begins by separating full spill of events into individual particle interactions, known as NOvA "slices" see Ref.[3]. Slices intended to collect together all hits from a single neutrino interaction and serve as the foundation for all later reconstruction stages. The first step to slicing begins by collecting 500$\mu$s of data from the arrival of the neutrino beam. That’s what we call an event, see Figure 54.

Figure 54: Example of cosmic ray distribution throughout the 550 $\mu$s time windowing the Far Detector. The charge distribution of the raw hits is shown [66]. The top panel shows the vertically oriented planes and the bottom panel the horizontal planes.
Secondly, on that data, groups of hits are combined to give a slice, see Figure 55,

![Image](image.png)

Figure 55: Example of cosmic ray distribution throughout the 550 μs time windowing the Far Detector. The reconstructed slices (slicer4d) are drawn. [66]. Hits are colored in bold with the reconstructed slice they are a member of, un-bolded hits are not associated with a physics slice [3].

Thirdly, a modified Hough transform, see Ref.[3] is applied to identify prominent straight-line features in a slice, see Figure 56,
Figure 56: Example filtered slice in the Far Detector. The golden lines are the reconstructed hough lines. [66]. The Hough lines are used as seeds to an algorithm to determine the global 3D vertex for the slice under the assumption that all activity in the slice has a common origin [3].

Fourthly, after we have done the slicing and found a global 3D vertex, we can now look at individual slices to look for prong formation. Prongs are produced by using an adapted version of the classic fuzzy k-means clustering algorithm where $k$ refers to the number of clusters and "fuzzy" allows an object to have a membership in multiple clusters. This algorithm and prong formation is described in sections 5.6.1 and 5.6.5 in Evan Niner’s Ph.D thesis "Observation Of Electron Neutrino Appearance In The NuMI Beam With The NOvA Experiment". Prong formation is done in each detector view separately and then 3D prongs are formed through a matching process. To find the prongs, each cell hit is converted to an angle with respect to the vertex using the cell centers[3], see Figure 57 and 58 for a general depiction of how the prongs are obtained,
Figure 57: Example filtered slice in the Far Detector. The blue, green and red regions are the reconstructed fuzzyk prongs. The red cross is the reconstructed elasticarms vertex. Where the term elasticarms it’s the name of the algorithm that creates the vertices in novasoft [66].

Figure 58: Flowchart of the reconstruction algorithms used to identify $v_e$ CC interactions [3].

Lastly, once we have the prong objects, we feed them into the convolutional visual network (CVN). Clearly, the CVN technique implementation has expanded to the application of particle identification on a prong-by-prong basis, see Ref.[2]. The framework used for CVN is based out off Caffe, a deep learning framework made by Berkeley AI
Research (BAIR) and by community contributors which implements CNNs. The training architecture is adapted from GoogLeNet which uses "inception modules" which fan out previous layer into a set of convolutional layers of different kernel size and then down samples the associated features. The NOνA collaboration adapted a trimmed down version of this network for neutrino identification. Therefore, the same architecture is used as the starting point for prong-ID[2].

The input to training and evaluation are simulated pixel maps, each map is 200 planes (100 per view) long and 80 cells wide. The maps starts at upstream end of slice activity and centered on median cell (network results position independent). Maps we produced for each slice and each 2D and 3D FuzzyK prong (currently only 3D prongs in training) in the slice. The prong maps use the same boundary coordinates as the slice map so the slice context can be incorporated. The training selection uses 3D FuzzyK Prongs with a minimum of 5 hits per view. Prongs are matched by purity to true particle ID, the mother particle ID of the best match is the training category (i.e activity from neutron daughters is a neutron). Prongs with purity > 50% are kept while providing approximately 3.5 million prongs for training. The sample is divided into 80% training, and 20% testing in LevelDB files, these files are used for training the network. LevelDB is a database file format, we take standard NOνA files, extract the pixel images of the events we want and convert them to this format which is what was used in the software the network trains in. They only get used for the actual training. Training categories include Electron, Muon, Proton, Neutron, Pion, Photon, and other PDG particles available to the simulation. It also includes photons from $\pi^0$ and other sources in one category. CVN ID values for each FuzzyK prong are produced under seven particle assumptions [2].
4.3 Prong CVN Gamma ID Discrimination

As previously discussed, NOvA’s Prong Level CVN technique is a new type of identifier on a prong-by-prong basis. This analysis starts by observing how prong level CVN behaves with the pre-selection and RemID cuts obtained in chapter 3 through a partial analysis before systematic optimization. To continue with this process, one important thing to note is that after discriminating charged-current muon particles with RemID and rejecting most of the CC $\mu^\pm$ particles in the pre-selection, a lot of other non-$\pi^0$ events are still present in the background. This is taken care of with the proposed selection of NC $\pi^0$ signal events with prong "CVN Gamma ID". This variable allowed rejection of most non $\pi^0$'s remaining after pre-selection and RemID cuts by discriminating on prong 1. This process is then again repeated with the second most energetic prong CVN Gamma ID with the same pre-selection and RemID cuts, however, for the second prong we only care to observe the number of events with the highest probability of being photons and get an idea of how much signal events we might end up loosing in the process. Plots shown in this section contain Prong 1 CVN Gamma ID and Prong 2 CVN Gamma ID optimization based on systematics, while already implementing the final pre-selection and pion energy threshold (see section 4.3.1), as well as including ND data. Note: ND data was only compared after finalizing selection from MC. See Figure 59,
Figure 59: (a) Implementation of pre-selection and RemID cuts to select best cut for the first most energetic gamma ID with respect to Prong CVN Gamma ID, is obvious that an aggressive cut at Prong CVN Gamma ID > 0.7 (eye balling it) is the most ideal cut that would allow for most of the signal events. (b) Same pre-selection with RemID against the second most energetic Prong 2 CVN Gamma ID.

Now we explore a cut on Prong 2 CVN Gamma ID (second most energetic prong) to make sure to reconstruct a π⁰ invariant mass with the 2 prongs that resemble photons the most, see section 4.5 and figure 60,
Figure 60: Pre-selection, RemID, and Prong CVN Gamma ID > 0.7 cuts are being implemented vs. the second most energetic prong CVD Gamma ID variable, as you can see in this plot a possible cut could be at > 0.9 to obtain the prong 2 CVN Gamma ID cut. Keep in mind this plot is also already optimized based on systematic uncertainties.

4.3.1 Pion Energy Selection

Before delving onto NC $\pi^0$ event reconstruction, we must first do a partial analysis of the meson to how effective prong CVN Gamma ID is when it comes to the total energy of the $\pi^0$. From these partial analyses of prong CVN selections, we are able to determine a pion energy threshold to be able to obtain a clean NC $\pi^0$ reconstructed signal. By plotting reconstructed NC $\pi^0$ events vs. true energy ($E = p$ from the PDG), see figure 61, one is able to have an idea as to what is the best signal threshold for the pion energy. We set a cut value of $E_{\pi^0} > 0.3$ GeV.
Figure 61: (a) 2 dimensional plot implementing pre-selection, RemID, Prong CVN Gamma ID > 0.7, and Prong 2 CVN Gamma ID > 0.9 cuts w/ respect to invariant mass vs. true energy. Notice how true energy at 0.3 GeV, shows how NC $\pi^0$ mass is still being reconstructed. (b) Logarithmic version of the 2 dimensional plot.

From Figure 61, we try to get rid of poor reconstruction at low energy. The goal is to find pions that are well reconstructed at their known mass of $\sim 0.135$ GeV. As you can see in the Figure 61, a big quantity of pions are produced at $E_{\pi^0} > 0.3$ GeV. Separating low total
energy NC $\pi^0$ from high energy NC $\pi^0$ would allow to discriminate where is the most suitable $\pi^0$ true energy threshold to increase the signal, see Figures 62 and 63.

Figure 62: (a) $\pi^0$ true energy with just pre-selection cuts, (b) $\pi^0$ true energy with pre-selection and RemID cuts. (c) $\pi^0$ true energy with pre-selection, RemID, and Prong CVN Gamma ID $> 0.7$ cuts, (d) $\pi^0$ true energy with pre-selection, RemID, Prong CVN Gamma ID $> 0.7$, and Prong 2 CVN Gamma ID $> 0.9$ cuts.
Also, notice from Figure 62 to 63 how CC $\pi^0$ events begin to minimize significantly as the progression of the cuts are implemented (as expected). Also, based on the low energy $\pi^0$ distribution, our threshold at 0.3 GeV could maintain signal statistics.

### 4.4 Prong CVN Gamma ID Systematics Optimization

There are a lot of uncertainties in NOvA measurement techniques like effects of neutrino flux mismodeling, uncertainties in cross sections, and detector efficiencies. This analysis is based on the nominal Monte Carlo for the ND, and attempts the best approximation of how the world is. If the MC is correct, this analysis should be able to extract true parameters as its best fit point given sufficient statistics.

A systematic parameter or systematic error is a "knob" that can be "turned" to modify the details of the simulation by setting the magnitude of the adjustments that are allowed in the real world by some value of the parameter that we believe in. To make the optimization, the ND MC data set varied via systematic parameters and the analysis is repeated using a multi-universe class, more on this in section 4.4.1. These uncertainties introduced into the best fit is called the systematic uncertainties. In our scenario, we
disregard the target and flux since they will be both flat w.r.t CVN ID. We use MC-derived data from the ND as previously mentioned, generated with systematic shifts applied, and process it through the analysis chain. This analysis seeks to minimize the uncertainty in the cross section and impact of systematic shifts by optimizing Prong CVN Gamma ID and Prong 2 CVN Gamma ID cuts on the total cross section uncertainty [67].

4.4.1 Cross Section and Systematics

This optimization is achieved using the "CAFAna approach". Systematics in this package are based on altering the details of "SRProxy objects" as they are loaded in. These shifted events are then passed on the rest of the framework, most of which remains unaware it is dealing with modified MC [67].

In the end, the cross section will be calculated using data, but we do not know the background numbers in reality, so we have to scale the MC background numbers,

\[
\sigma = \frac{N_{Sel,Data} - s \cdot N_{Bkg,MC}}{T \cdot \phi \cdot \epsilon}
\]

where

\[
s = \frac{POT_{Data}}{POT_{MC}} \approx \frac{8.09 \times 10^{20}}{3.54 \times 10^{21}} \approx 0.2285.
\]

For a function of three variables,

\[
\delta F(x_1, x_2, x_3) = \left[\left(\frac{\partial F}{\partial x_1} \cdot \delta x_1\right)^2 + \left(\frac{\partial F}{\partial x_2} \cdot \delta x_2\right)^2 + \left(\frac{\partial F}{\partial x_3} \cdot \delta x_3\right)^2\right]^{1/2}.
\]

So for our cross section,

\[
\delta \sigma = \left[\left(\frac{\partial \sigma}{\partial N_{Sel,Data}} \cdot \delta N_{Sel,Data}\right)^2 + \left(\frac{\partial \sigma}{\partial N_{Bkg,MC}} \cdot \delta N_{Bkg,MC}\right)^2 + \left(\frac{\partial \sigma}{\partial \epsilon} \cdot \delta \epsilon\right)^2\right]^{1/2}
\]

\[
= \left[\left(\frac{1}{T \phi \epsilon} \cdot \delta N_{Sel,Data}\right)^2 + \left(-\frac{s}{T \phi \epsilon} \cdot \delta N_{Bkg,MC}\right)^2 + \left(-\frac{(N_{Sel,Data} - s \cdot N_{Bkg,MC})}{T \phi \epsilon^2} \cdot \delta \epsilon\right)^2\right]^{1/2}.
\]
Then, the fractional uncertainty on the cross section is,

\[
\frac{\delta \sigma}{\sigma} = \left[ (\frac{T \varphi \epsilon}{N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}}})^2 \cdot (\delta \sigma)^2 \right]^{1/2} \tag{26}
\]

\[
= \left[ (\frac{\delta N_{\text{Sel,Data}}}{N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}}})^2 + \left( \frac{s \cdot \delta N_{\text{Bkg,MC}}}{N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}}} \right)^2 + \left( \frac{\delta \epsilon}{\epsilon} \right)^2 \right]^{1/2} \tag{27}
\]

Here, \( \delta N_{\text{Sel,Data}} = \sqrt{N_{\text{Sel,Data}}} \) and \( (\delta N_{\text{Bkg,MC}})^2 = (\sqrt{s \cdot N_{\text{Bkg,MC}}})^2 + (\delta N_{\text{Bkg,MC}})^2 \). Then the fractional uncertainty becomes,

\[
\frac{\delta \sigma}{\sigma} = \left[ \frac{N_{\text{Sel,Data}}}{(N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}})^2} + \frac{s \cdot (s \cdot (N_{\text{Bkg,MC}}))}{(N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}})^2} \right. \\
+ \left( \frac{\delta N_{\text{Bkg,Syst}}}{N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}}} \right)^2 + \left( \frac{\delta \epsilon}{\epsilon} \right)^2 \right]^{1/2} \tag{28}
\]

Since in this analysis we used the whole MC dataset, we need the POT normalized \( N_{\text{Sel}} \), so we let \( N_{\text{Sel,Data}} = s \cdot N_{\text{Sel,MC}} \):

\[
\frac{\delta \sigma}{\sigma} = \left[ \frac{s \cdot N_{\text{Sel,MC}}}{(N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}})^2} + \frac{s \cdot (s \cdot (N_{\text{Bkg,MC}}))}{(N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}})^2} \right. \\
+ \left( \frac{\delta N_{\text{Bkg,Syst}}}{N_{\text{Sel,Data}} - s \cdot N_{\text{Bkg,MC}}} \right)^2 + \left( \frac{\delta \epsilon}{\epsilon} \right)^2 \right]^{1/2} \tag{29}
\]

The weights used in this analysis out the CAFAna package were

kXSecSCVWgt2017*kPPFXluxCVWgt, the knobs used were the same knobs from Daisy Kalra’s NC \( \pi^0 \) analysis, see Ref.[68]. Systematics samples used\textsuperscript{20}: GENIE, PPFX flux,

\textsuperscript{20} Light level variations: prod_caf_R17-03-01-prod3reco.l_phy_nonswap_fhc_nova_v08_full_lightmodel - lightdowncalibup_v1, prod_caf_R17-03-01-prod3reco.l_phy_nonswap_fhc_nova_v08_full_lightmodellightupcalibdown_v1 Cherenkov variations: prod_caf_R17-03-01-prod3reco.l_phy_genie_nonswap_fhc_nova_v08_full_enhbck - proton - shiftdown_v1 Calibration shape variations: prod_caf_R17-03-01-prod3reco.h_phy_genie_nonswap_fhc_nova_v08_periods1325_calib - shiftndxyviewneg - offset_v1, prod_caf_R17-03-01-prod3reco.h_phy_genie_nonswap_fhc_nova_v08_periods1325_calib - shiftndxyviewpos - offset_v1 Calibration offset: prod_caf_R17-03-01-prod3reco.j_phy_genie_nonswap_fhc_nova_v08_full_calib - shift - nd - func_v1
absolute calibration offset, calibration shape variations, light level variations, Cherenkov variations.

In order to obtain a GENIE simulated cross section, systematic uncertainties with the multi-universe approach in CAFAna are implemented. This approach draws the NOvA systematic error band from GENIE cross section models while varying the GENIE tunable physics parameters by $N\sigma$’s, where $N \in [-2,-1,1,2]$, then input $N$ through the shift parameters of the CAFAna Spectrum class, and obtain shifted spectra as the boundaries of the error band. Only a few of the GENIE knobs having the biggest effects are varied, these are ManCel, NormCCQE, NormCCRES, NormNCRES, MaNCRES, MvNCRES, MaCOHpi, R0COHpi, RvpCC1pi, RvpCC2pi, RvpNC1pi, RvpNC2pi, RvnCC1pi, RvnCC2pi, RvnNC1pi, RvnNC2pi, RvbarpCC1pi, RvbarpCC2pi, RvbarpNC1pi, RvbarpNC2pi, RvbarnCC1pi, RvbarnCC2pi, RvbarnNC1pi, RvbarnNC2pi, AhtBY, BhtBY, CV1uBY, CV2uBY, NC, AGKY_xF1pi, AGKY_pT1pi, MFP_pi, FrElas_pi, FrInel_pi, FraAbs_pi, FrPiProd_pi, FrCEx_N, FrElas_N, FrInel_N, FrAbs_N, FrPiProd_N, BR1gamma, BR1eta, Theta_Delta2Npi. However, these knobs are varied at the same time under the required number of universes in the model. In our particular case, we used 100 universes to assess the uncertainties. One can think of these universes as universes with different GENIE knob values for the physics parameters [69].

Our input is the ND selected MC scaled to data and related multiverse uncertainties, to which we output the fractional cross-section $\frac{\delta\sigma}{\sigma}$ to optimize Prong CVN Gamma ID cut. In addition to this, we also cross-check the individual inputs of $\frac{\delta\sigma}{\sigma}$ such as the number of selected, background, signal, and true signal events per POT (true signal events are necessary to calculate selection efficiency). We also do a cross-check on the purity of our signal, the statistical, and systematic uncertainties on background and selected events, as well as fractional efficiency uncertainties $\frac{\delta\epsilon}{\epsilon}$ with respect to Prong CVN Gamma ID and Prong 2 CVN Gamma ID cut value. The results for the fractional cross section uncertainties are optimized at $\sim 15.2\%$, see Figure 64,
Figure 64: (a) Minima of the curvature with Prong CVN Gamma ID $> 0.7$ where the fractional uncertainty $\sim 18\%$ (b) For Prong 2 CVN Gamma ID there were a few problems, first the bin located at 0.92 improved fractional by $\sim 0.7\%$ from bin number 0.9, bin number 0.95 improved the fractional uncertainty $\sim 1\%$ from bin number 0.9. So, the minima of the curvature then, is chosen at bin number 0.9 providing a total fractional uncertainty of $\sim 15.2\%$

For the statistical uncertainty on background events to Prong CVN Gamma ID and Prong 2 CVN Gamma ID cut value, see Figure 65 and 66,
Figure 65: (a) First component of statistical uncertainty on background events for Prong CVN Gamma ID
(b) Second component of statistical uncertainty on the background events for Prong CVN Gamma ID
(c) Statistical uncertainty on background events at Prong CVN GammaID > 0.7 is $\sim 6.5\%$
Figure 66: (a) First component of statistical uncertainty on background events for Prong 2 CVN Gamma ID (b) Second component of statistical uncertainty on the background events for Prong 2 CVN Gamma ID (c) Statistical uncertainty on background events at Prong 2 CVN GammaID > 0.9 is $\sim 7.5\%$
For the systematic uncertainty on background events to Prong CVN Gamma ID and Prong 2 CVN Gamma ID cut value, see Figure 67,

Figure 67: (a) Systematic uncertainty on background events is $\sim 13\%$ for Prong CVN Gamma ID $> 0.7$ (b) Systematic uncertainty on background events is $\sim 6.8\%$ for Prong 2 CVN Gamma ID $> 0.9$. 
For selection efficiency and fractional efficiency with respect to Prong CVN Gamma ID and Prong 2 CVN Gamma ID cut value, see Figures 68, and 69,

![Graph (a)](image1)

![Graph (b)](image2)

Figure 68: (a) Selection efficiency is signal events passing pre-selection cuts, divided by true signal events with just fiducial cut, for Prong CVN Gamma ID > 0.7 selection efficiency is \(\sim 6\%\) (b) Fractional efficiency uncertainty for Prong CVN Gamma ID > 0.7 is \(\sim 13\%\).
Figure 69: (a) Selection efficiency is signal events passing pre-selection cuts, divided by true signal events with just fiducial cut, for Prong 2 CVN Gamma ID > 0.9 selection efficiency is $\sim 1.8\%$ (b) Fractional efficiency uncertainty for Prong CVN Gamma ID > 0.7 is $\sim 14\%$.

For statistical uncertainty on selected events with respect to Prong CVN Gamma ID and Prong 2 CVN Gamma ID cut value, see Figures 70, and 71,
Figure 70: (a) First component of statistical uncertainty on selected events for Prong CVN Gamma ID (b) Second component of statistical uncertainty on the background for Prong CVN Gamma ID (c) Statistical uncertainty on background events at Prong CVN GammaID > 0.7 is $\sim 0.9\%$
Figure 71: (a) First component of statistical uncertainty on background for Prong 2 CVN Gamma ID (b) Second component of statistical uncertainty on the background for Prong 2 CVN Gamma ID (c) Statistical uncertainty on background events at Prong 2 CVN Gamma ID $> 0.9$ is $\sim 0.15\%$
For signal purity with respect to Prong CVN Gamma ID and Prong 2 CVN Gamma ID cut value, where purity is defined to be $\frac{N_{Sel}}{N_{Sel}-N_{Bkg}}$, see Figures 72,

Figure 72: (a) Purity with respect to Prong CVN Gamma ID $> 0.7$ is $\sim 55\%$ (b) Purity with respect to Prong 2 CVN Gamma ID $> 0.9$ is $\sim 74\%$.

4.4.2 Prong CVN Gamma ID Distributions and Systematic Error Band

Additional distributions for Prong CVN Gamma ID illuminate the fractional cross section including the cumulative plots, see Figures 73, 74, 75, 76:
Figure 73: (a) Prong CVN Gamma ID distribution for selected events (b) Prong CVN Gamma ID distribution with systematic error band for selected events.
Figure 74: (a) Prong CVN Gamma ID distribution for background events (b) Prong CVN Gamma ID distribution with systematic error band for background events.
Figure 75: (a) Prong CVN Gamma ID distribution for signal events (b) Prong CVN Gamma ID distribution with systematic error band for signal events.
4.4.3 Prong 2 CVN Gamma ID Distributions and Systematic Error Band

Additional distributions for Prong 2 CVN Gamma ID illuminate the fractional cross section including the cumulative plots, see Figures 77, 78, 79, 80:
Figure 77: (a) Prong 2 CVN Gamma ID distribution for selected events (b) Prong 2 CVN Gamma ID distribution with systematic error band for selected events.
Figure 78: (a) Prong 2 CVN Gamma ID distribution for background events (b) Prong 2 CVN Gamma ID distribution with systematic error band for background events.
Figure 79: (a) Prong 2 CVN Gamma ID distribution for signal events (b) Prong 2 CVN Gamma ID distribution with systematic error band for signal events.
Figure 80: (a) Prong 2 CVN Gamma ID distribution for true signal events (b) Prong 2 CVN Gamma ID distribution with systematic error band for true signal events.

4.5 Invariant Mass After Prong Based CVN Gamma ID Optimization

After the partial analysis on Prong CVN Gamma ID in section 4.3, the pion energy threshold analysis in sub-section 4.3.1, and the optimization of Prong CVN Gamma ID and Prong CVN Gamma ID in 4.4.1. Now an observation of the accuracy in the NC $\pi^0$ invariant mass reconstruction is necessary. To have a sense on how these new cuts affect the signal, two plots were made. The first one is an implementation of pre-selection, RemID cut, NC $E_{\pi^0} > 0.3$ GeV and Prong CVN Gamma ID > 0.7 with respect to invariant mass while comparing to data, see Figure 81.
Figure 81: (a) Stacked plot with respect to $\pi^0$ reconstructed invariant mass (b) Un-stacked plot with respect to $\pi^0$ reconstructed invariant mass.
The second one, is the implementation of all cuts or final selection, see Figure 82,

Figure 82: (a) Stacked plot with respect to $\pi^0$ reconstructed invariant mass (b) Un-stacked plot with respect to $\pi^0$ reconstructed invariant mass.
CHAPTER V

RESOLUTION ANALYSIS

Comparing reconstructed total energy to true energy I studied the absolute resolution and bias of the final selected sample. The study of energy resolution in selected, background and signal events will allow the study of the final cross section extension, to a possible double differential measurement. In this study we used the final selection cuts.

Figure 83: (a) Bias with respect to reconstructed total energy. Point (0.3,0) shows very little bias between selection, background, and signal at $E_{Reco,\pi^0} = 0.3$ GeV. (b) Absolute resolution with respect to reconstructed total energy shows $\sim 1.5$ units from the origin at $E_{Reco,\pi^0} = 0.3$ GeV.
Figure 84: $\Delta E$ 2 dimensional plot shows most of the $\pi^0$ containing very little bias.

In the region from 0.3 to 0.6 GeV in Figure 84 where most of the events are, the absolute resolution is about 100 MeV. The absolution resolution was calculated with the r.m.s and its error from Figure 84 and taking its projection along the y axis while the bias takes the mean and its error also from Figure 84 and takes its projection along the y axis.
CHAPTER VI

CONCLUSION

To conclude this study, a few details must be pointed out. First, Prong CVN Gamma ID > 0.7 has a fractional cross section uncertainty of $\sim 18\%$ with a purity of $\sim 55\%$, while Prong 2 CVN Gamma ID > 0.9 has a fractional cross section uncertainty of $\sim 15\%$ with a purity of $\sim 74\%$, see Table 3. A study with Prong 2 CVN Gamma ID > 0.95 as seen in Figure will only provide $\sim 1\%$ improvement in the fractional cross section uncertainty.

Table 3: Cut flow table for Prong CVN Gamma ID > 0.7 and Prong 2 CVN Gamma ID > 0.9.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>#Signal</th>
<th>#Background</th>
<th>Efficiency</th>
<th>Purity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Prong + Fid. (Not Pre-Sel Cut)</td>
<td>2051438</td>
<td>21150733</td>
<td>1</td>
<td>0.09</td>
</tr>
<tr>
<td>Pre-Sel + 2 Prongs</td>
<td>196268</td>
<td>1320499</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>PreSel + 2 Prongs + RemID &lt; 0.36</td>
<td>186928</td>
<td>709468</td>
<td>0.09</td>
<td>0.21</td>
</tr>
<tr>
<td>PreSel + 2 Prongs + RemID &lt; 0.36 + Png 1</td>
<td>94074</td>
<td>76755</td>
<td>0.05</td>
<td>0.55</td>
</tr>
<tr>
<td>Gamma ID &gt; 0.7</td>
<td>28630</td>
<td>9979</td>
<td>0.014</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Small differences in energy on how the detectors picks up the signal may explain the difference between MC/Data in the invariant mass plot. This could already be giving a strong hint of what is going on in the cross section, it could be telling us that the real cross
section is lower than in the model. The invariant mass is just including statistical uncertainties, a systematic error band needs to be included, yet, based on the systematic error band from signal events in section 4.4.3 gives a hint of a pretty good agreement with the data. It is important to note that there are large systematic uncertainties not being shown in the invariant mass plot. However, the resolution shows better than or comparable to world leading data. We are on track to produce the best measurement on ν NC pion production on Carbon to date using prong CVN. Ultimately, we want to make a cross section measurement as a function of π⁰ energy. In order to achieve this, we would need to finalize the resolution studies to understand the energy binning of the final energy dependent cross-section, perform work to constrain background uncertainties using data driven techniques, and study the energy unfolding to convert reconstructed energy to true energy space.
BIBLIOGRAPHY
BIBLIOGRAPHY


[67] “Cafana systematics.”


APPENDIXES
APPENDIX A

Introduction to Chirality and the Dirac Equation

First predicted by Bruno Pontecorvo in 1957, oscillations of neutrinos are a consequence of the presence of flavor neutrino mixing, or lepton mixing, in vacuum. To understand this, let’s talk about Dirac zero-mass free particles by introducing his equation which satisfies the relatively massless behavior of the neutrino particle. Let’s begin with the differential operators:

\[ E \rightarrow i\hbar \frac{\partial}{\partial t} \]  \hspace{1cm} (30)

\[ p \rightarrow -i\hbar \nabla \]  \hspace{1cm} (31)

and consider the relativistic energy-momentum relation (use \( c = \hbar = 1 \) from here on)

\[ E^2 = p^2 + m^2 \]  \hspace{1cm} (32)

Making the appropriate operator substitutions we obtain,

\[ -\frac{\partial^2 \phi}{\partial t^2} + \nabla^2 \phi = m^2 \phi. \]  \hspace{1cm} (33)

This equation was first introduced by Oskar Benjamin Klein a Swedish physicist born in the late 18th century in the town of Danderyd in 15 September 1894 and died 5 February 1977 of unknown causes. Klein’s equation allows an interpretation of negligible mass neutrinos, like the neutrinos observed by NO\( \nu \)A produced at energies between 1 GeV and 2 GeV traveling at relativistic velocities observed by NO\( \nu \)A. Is important to note that neutrinos don’t have a zero invariant mass (the inertia of the particle measured when it is at rest) and as far as we know all neutrinos respect Lorentz symmetries (special relativity implying that the laws of physics stay the same for all observers that are moving with respect to one another within an inertial frame). From equation 33 and multiplying by \(-i\phi^*\)
and subtracting its complex conjugate equation \(-i\phi\), gives,

\[
\frac{\partial}{\partial t} \left[ i \left( \phi^* \frac{\partial \phi}{\partial t} - \phi \frac{\partial^* \phi}{\partial t} \right) \right] + \nabla \cdot \left[ -i \left( \phi^* \nabla \phi - \phi \nabla \phi^* \right) \right] = 0
\]

(34)

where

\[
\rho = i \left( \phi^* \frac{\partial \phi}{\partial t} - \phi \frac{\partial^* \phi}{\partial t} \right)
\]

(35)

\[
J = -i \left( \phi^* \nabla \phi - \phi \nabla \phi^* \right)
\]

(36)

are the probability and the flux densities. Free neutrinos, like those at NO\(\nu\)A, have a solution to this:

\[
\phi = Ne^{ip \cdot x - iEt}
\]

(37)

From this solution it is clear that the probability density is proportional to the relativistic energy \(E\) of the particle:

\[
\rho = -i(2iE)|N|^2 = 2E|N|^2
\]

(38)

\[
J = -i(2i\mathbf{p})|N|^2 = 2\mathbf{p}|N|^2.
\]

(39)

With this the Klein-Gordon equation using four-momentum notation and by using the D’Alembertian operator becomes

\[
\Box^2 + m^2 = 0
\]

(40)

and in turn forms a probability and flux density four-vector

\[
J^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)
\]

(41)

which satisfies the covariant continuity relation

\[
\partial_\mu J^\mu = 0.
\]

(42)
Testing things out from the free particle solution in equation 37 we get,

\[ J^\mu = 2p^\mu |N|^2. \] (43)

However, if equation 37 is to be substituted into equation 40 it returns,

\[ E = \pm \sqrt{p^2 + m^2}. \] (44)

It is quite clear that a intriguing event has just happened, since it appears to be that not only \( E > 0 \) are possible solutions to equation 40 but now there are also \( E < 0 \) solutions associated with a negative probability density because transitions can occur to lower energies, creating seemingly non-physical difficulties in Klein’s model. Without loss of generality, a complete set of states must not be ignored[70].

In 1927 Paul Dirac devised a relativistic wave equation linear in \( \frac{\partial}{\partial t} \) and \( \nabla \). In which he succeeded in overcoming the problem of the negative probability density. Using the Dirac equation,

\[ H\psi = (\alpha \cdot P + \beta m)\psi \] (45)

where the four coefficients \( \alpha_i \) and \( \beta \) are determined by the requirement that a free particle must satisfy the relativistic energy-momentum relation

\[ H^2\psi = (P^2 + m^2)\psi. \] (46)

However, note that beta is not involved in the case of zero-mass particles and that we need only satisfy where we sum over repeated indices, with the condition \( i > j \) on the second term,

\[ H^2\psi = (\alpha_i P_i + \beta m)(\alpha_j P_j + \beta m)\psi. \] (47)

Comparing with 46, it is clear that alpha one, alpha two, alpha three, and beta, all anti commute\(^{21}\) with each other. This means that the root of each one of them equal to 1 or more precisely \( \alpha_1^2 = \alpha_2^2 = \alpha_3^2 = \beta^2 = 1 \). Therefore, they cannot simply be numbers since all

\(^{21}x * y = -(y * x)\)
of them anti-commute with each other, and we are led to consider matrices operating on a wave function \( \psi \), which is a multicomponent column vector[70].

To state the dimensionality of these matrices, a non unique \((\alpha, \beta)\) 4 by 4 matrix representation,

\[
\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}
\] (48)

is used. These are the Dirac-Pauli matrix representation where \( I \) are the 2 by 2 identity matrices typically written as 1, and \( \sigma \) are the Pauli matrices

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\] (49)

Multiplying equation 45 by \( \beta \) will allow for a covariant form of the Dirac equation

\[
i\beta \frac{\partial \psi}{\partial t} = m\psi - i\beta \cdot \nabla \psi,
\] (50)
or

\[
(i\gamma^\mu \partial_\mu - m)\psi = 0
\] (51)

where \( \gamma^\mu \equiv (\beta, \beta\alpha) \) are the Dirac \( \gamma \)-matrices and \( \gamma^0, \gamma^1, \gamma^2, \gamma^3 \) are to be regarded as a four vector. Since the Dirac equation are really four differential equations it can be rewritten as

\[
\sum_{k=1}^{4} \left[ \sum_{\mu} i\left( \gamma^\mu \right)_{jk} \partial_\mu - m\delta_{jk} \right] \psi = 0.
\] (52)

The Dirac \( \gamma \)-matrices in equation 51 also satisfies the anti-commutation relations the following way,

\[
\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}
\] (53)

where \( g^{\mu\nu} \) is the metric tensor, and as a further matter since \( \gamma^0 = \beta \) this means that \( \gamma^{0\dagger} = \gamma^0 \) and \( (\gamma^0)^2 = I \). Thus, \( \gamma^{k\dagger} = (\beta\alpha^k)^\dagger = \alpha^k\beta = -\gamma^k \) and \( (\gamma^k)^2 = \beta\alpha^k\beta\alpha^k = -I \). In summary, \( \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0 \).
Every differential in equation 51, contains a four-momentum eigensolution

$$\psi = u(p)e^{-ip \cdot x}$$  \hspace{1cm} (54)

where \( u \) is a four component "spinor" independent of \( x \). While seeking the energy eigenvector, one again finds that the Dirac equation has two \( E > 0 \) solutions describing the neutrino and two \( E < 0 \) solutions describing the antineutrino by substituting the wavefunction in equation 46 with the wavefunction in equation 54. Note, although neutrinos do have a mass the neutrino is maybe considered massless due to its high momentum. Note, that \( \beta \) in this case shouldn't be involved since \( m = 0 \) and therefore neutrinos only need to satisfy

$$\alpha_i \alpha_j + \alpha_j \alpha_i = 2 \delta_{ij}$$  \hspace{1cm} (55)

and \( \alpha_i = \alpha_i^\dagger \). The relations can be accomplished by the 2 by 2 Pauli matrices by taking \( \alpha_i = -\sigma_i \) and \( \alpha_i = \sigma_i \) to allow the Dirac equation to be divided into two decoupled equations for the two-component spinors

$$E\chi = -\sigma \cdot p\chi$$  \hspace{1cm} (56)

$$E\phi = +\sigma \cdot p\phi$$  \hspace{1cm} (57)

The decoupled equations contain one positive and one negative solution since they are based out of the energy-momentum relations \( E^2 = p^2 \). Assuming equation 56 is for a "massless" neutrino, the positive energy solution should be \( E = |p| \) in order to be satisfied. In other words, \( \chi \) describes left-handed neutrino (helicity \( \lambda = -1/2 \)) of energy \( E \) and momentum \( p \). The same goes for the negative energy solutions with right handed antineutrino (helicity \( \lambda = +1/2 \)) of negative energy and momentum. Purely in terms of what is being represented or implied, equation 56 describes \( \nu_L \) and \( \bar{\nu}_R \). Equation 57 describes the other helicity states \( \nu_R \) and \( \bar{\nu}_L \). The wave equation 56 was first proposed by Hermann Weyl in 1929 but it was rejected because of noninvariance under the parity.
operation \( P \) which takes \( \nu_L \rightarrow \nu_R \). Putting this result onto four-component form

\[
\alpha = \begin{pmatrix}
-\sigma & 0 \\
0 & \sigma
\end{pmatrix}, \quad \nu = \begin{pmatrix}
\chi \\
\phi
\end{pmatrix}
\] (58)

and expressing \( \gamma \) in the Weyl (chiral) representation, we obtain

\[
\gamma = \begin{pmatrix}
0 & \sigma \\
-\sigma & 0
\end{pmatrix}, \quad \gamma^0 = \begin{pmatrix}
0 & I \\
I & 0
\end{pmatrix}, \quad \gamma^5 = \begin{pmatrix}
-I & 0 \\
0 & I
\end{pmatrix}.
\] (59)

Quite a few experiments agree that leptons enter the CC weak interaction in a special combination of two bilinear definitions,

\[
J^\mu = \bar{\psi}_e \gamma^\mu (1/2)(1 - \gamma^5) \psi_{\nu}
\] (60)

which is the V-A (vector minus axial vector) form of the weak current \( J^\mu \) and \( \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \). The \( (1/2)(1 - \gamma^5) \) is of concern because it violates parity maximally, in the case of the electron and its neutrino

\[
(1/2)(1 - \gamma^5)u_{\nu} = \begin{pmatrix}
I & 0 \\
0 & 0
\end{pmatrix} \begin{pmatrix}
\chi \\
\phi
\end{pmatrix} = \begin{pmatrix}
\chi \\
0
\end{pmatrix}
\] (61)

in which it projects only \( \nu_L \) and \( \bar{\nu}_R \) and indicates that only LH neutrinos \( \nu_k \) and RH antineutrinos \( \bar{\nu}_k \) are coupled to charged leptons by the weak interactions. Up to this day there still no empirical evidence for RH neutrinos \( \nu_k \) or LH antineutrinos \( \bar{\nu}_k \). Keep in mind that the mass of the neutrino is assumed strictly to be zero for the \( \nu_L \) and \( \bar{\nu}_R \) assertion to be made[70].

For a massive fermion, the projection definition of \( (1/2)(1\pm\gamma^5)u \) has both right- and left-handed components of \( u \). The fact that \( (1/2)(1 - \gamma^5) \) projects the negative helicity neutrinos at high energy energies does not depend on the choice of representation. But for this case the best representation would be the Dirac-Pauli because it allows for diagonalization of the neutrino energy on the nonrelativistic limit with \( \gamma^0 \) in the diagonal,
or in our particular case diagonalization of the neutrino energy in the relativistic limit with $\gamma^5$.

Ultimately, based on the assumption for the neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) to have nonzero mass the handedness is not a good quantum number. That, is $\gamma^5$ does not commute with the Hamiltonian. However, helicity (a combination of the spin and the linear motion of a subatomic particle) is conserved but is frame dependent, see equation 62 and 63,

\[
[(\alpha \cdot P + \beta m), \gamma^5] = [\gamma^5, (\alpha \cdot P - \beta m)]
\] (62)

whereas

\[
[(\alpha \cdot P + \beta m), \sigma \cdot \hat{p}] = [\sigma \cdot \hat{p}, (\alpha \cdot P + \beta m)]
\] (63)

In other words, in order to have massive neutrinos and still ensure that weak interactions couple only to $\nu_L$ and $\bar{\nu}_R$ Majorana neutrinos described above must be introduced in the picture.
APPENDIX B

Brief History of Ettore Majorana

Ettore Majorana was born in Catania, on the eastern coast of Sicily, on August 5, 1906, at 8:51 pm. A child genius in his early days with a precocious ability with numbers. He would do cubic roots in his head while the other kids were out playing marbles. Ettore was best known for his 1938 stunt. On the night of March 26, 1938, when Ettore was 31 years old, he boarded a ship in Palermo, Sicily, and was never seen again. He left behind a series of suicide notes and was known to have been depressed for at least five years; but what prevents the case from being closed is that his body was never recovered, and over the next few decades, he was allegedly sighted on numerous occasions. He also took with him the equivalent of 70,000 dollars, as well as his passport. Indeed, after such a feat, it is hard to deny him human status after that. Over the years, many theories have been proposed as to what happened to Ettore, some believe he was abducted by aliens, others think he joined a monastery in Calabria, still some also think he might of gotten into trouble with the Mafia, and my personal favorite is that he was kidnapped by other political powers due to his nuclear knowledge. There is strong evidence that the Majorana neutrino was first conceived in a 1932 paper that he left sitting in his drawer until he was done writing another paper "Relativistic Theory of Elementary Particles with Arbitrary Spin" so it would become nothing but a minor appendix[71].
APPENDIX C

Extraction

The 120 GeV protons are taken out of the Main Injector accelerator using resonant extraction techniques, with regulation developed specifically for fast beam pulses. An instability is induced in the orbit of the protons using in concert the main quadrupole circuit, one of the correction quadrupole families and the zero harmonic octupoles. At a predetermined point of maximum excursion from the nominal orbit, the unstable protons encounter an electric field that kicks them toward an extraction channel. Three Lambertsons and one C-magnet complete the extraction. In this manner, the entire set of circulating protons is extracted in roughly 100 turns taking on the order of 1 millisecond. The machine repetition cycle for this mode of extraction is expected to be 1.9 seconds. To produce the NuMI beam with the intensities required to achieve the NOvA experimental goals will require extraction of at least $8.09 \times 10^{20}$ protons per pulse [31].
APPENDIX D

Transport of Primary Protons

The extracted protons are focused and bent strongly downward by a string of quadrupoles and bending magnets so that they enter the pre-target hall located 150 feet downstream of the NuMI Stub, a specially constructed appendage to the MI enclosure\textsuperscript{22}. Another set of bend magnets brings the protons to the correct pitch (-58 mrad) for the zero targeting angle (maximum intensity) beam directed toward the NO\textsubscript{v}A experiment. The size and angular dispersion of the proton beam is controlled by a final set of quadrupoles and is matched to the diameter of the production target [31].

\textsuperscript{22}For conventional construction reasons the pre-target and target halls are located in the dolomite rock formation, requiring that the initial trajectory be bent down more than is actually required to aim the neutrino beam to Soudan