STRENGTH CHARACTERIZATION OF CHOPPED-FIBER COMPOSITES USING IOSIPESCU TEST METHOD

A Thesis by

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STRENGTH CHARACTERIZATION OF CHOPPED-FIBER COMPOSITES USING IOSIPESCU TEST METHOD

The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science, with a major in Aerospace Engineering.

Nicholas Smith, Committee Chair

C. Charles Yang, Committee Member

Ramazan Asmatulu, Committee Member
DEDICATION

To my parents, family members, and friends,
and to my advisor, Dr. Nicholas Smith
ACKNOWLEDGEMENTS

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ABSTRACT

In the aviation industry, composites are considered high-performance structural materials. Composite materials are mainly used in fuselages, wings, propellers, interior components, landing gears, doors, etc. Their main advantages are high strength and stiffness, and low density, in comparison to heavy weight materials such as metals, thereby producing a low weight in the finished part. The major drawback with composite materials is their inability to meet complex design requirements, hence the reason for investigating discontinuous-fiber composites. These composites have relatively low strength and stiffness, but can be easily formed into complex shapes and sizes, thus meeting the design requirements of many industries.

The strength properties of discontinuous-fiber composites can reach that of continuous-fiber composites if their aspect ratios are high and their fibers are aligned, but in actual practice it is difficult to maintain good alignment with discontinuous fibers. Nevertheless, discontinuous-fiber composites are typically more affordable than continuous-fiber composites. The main objective of this research was to analyze this problem by characterizing the shear strength properties of discontinuous-fiber composites that have a suitable alignment in order to satisfy the industrial needs for developing effective products with high efficiency and low cost.

In this thesis, the in-plane shear strength properties of quasi-isotropic discontinuous composites were calculated using the Iosipescu shear test method. The experimental results were analyzed using digital image correlation (DIC) analysis to determine the strain field in the region of interest (ROI), which provides the shear strain of the specimen. Thus, shear stress vs shear strain plots were mapped, and from these plots, shear stress, stiffness, and modulus were determined and characterized. The theoretical results of failure thus obtained were compared with the experimental results.
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<tr>
<td>2D</td>
<td>Two-Dimensional</td>
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<td>3D</td>
<td>Three-Dimensional</td>
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<td>CFRP</td>
<td>Carbon Fiber-Reinforced Polymer</td>
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<td>DIC</td>
<td>Digital Image Correlation</td>
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<td>FPF</td>
<td>First Ply Failure</td>
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<td>FRP</td>
<td>Fiber-Reinforced Plastic</td>
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<td>PC</td>
<td>Personal Computer</td>
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<td>ROI</td>
<td>Region of Interest</td>
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A composite material is a material produced using two or more constituent materials with essentially distinctive physical or chemical properties that, when consolidated, produce a material with characteristics not the same as the individual components. The individual parts/components stay separate and distinct within a completed structure. The new materials might be preferred for some reasons: common examples incorporate materials which are stronger, lighter, or less costly when contrasted with conventional materials.

A considerable amount of research is being done on the strength properties and mechanical behavior of materials reinforced with discontinuous fibers. The main advantages of composite materials are their high strength and stiffness, and low density, in comparison to heavy materials such as metals, thereby producing a low weight in the finished part. Their major drawback is the inability to meet complex design requirements. Hence, a number of industrial suppliers like Hexcel and Solvay Composites have lately proposed a resolution to this problem with a newfound class of fiber reinforcements made with carbon fiber tows that produce discontinuous, yet aligned, fiber segments. These relatively long, overlapping fiber lengths (on the order of several inches) are held together with filaments or a binder and can move with respect to each other, thus yielding the necessary reinforcement to more quickly and effortlessly stretch and move for greater, wrinkle-free conformability and faster layup [1]. This is how the idea of discontinuous-fiber composites originated. These composites have relatively low strength and stiffness, but they can be easily formed into complex shapes and sizes, thus meeting the structural and strength requirements of many industries.
The idea of taking continuous carbon tows and breaking them into short fibers and then aligning and recombining them into yarns first appeared in the mid-1980s in products from Courtaulds’ Heltra Division, ICI Fiberite (Tempe, Arizona), and DuPont (Wilmington, Delaware), among others [1]. These materials were however not widely accepted despite their strong shear strength, smooth surface finish, etc., due to the fear of predictability in fiber lengths and their high production costs.

In the last couple of years, aligned discontinuous-fiber methods have made a rebound, to some extent because of the growing demand for quicker and automated manufacturing. Remarkably, the materials’ ability to stretch and deform while still producing sound machines is opening the door for automated vacuum forming and planned forming of complex curved parts [1].

The strength properties of discontinuous-fiber composites can reach that of continuous-fiber composites if their aspect ratios are superior and the fibers are aligned, but in actual practice, it is difficult to maintain good alignment with discontinuous fibers. Nevertheless, discontinuous-fiber composites are typically more affordable than continuous-fiber composites. Some of the other benefits of discontinuous-fiber composites are the ease to recycle and create three-dimensional (3D) printings.

This thesis deals with two types of quasi-isotropic fiber composites (including four different fiber orientations). The composite panels for this research were manufactured using a hand layup process. Five composite panels were constructed: two were continuous panels—namely, quasi-isotropic and unidirectional, and the remaining three were discontinuous panels, including two quasi-isotropic panels, each with two different chip sizes, and one unidirectional hybrid panel with two different chip sizes.
The main objective of this research was to compare the in-plane shear strength characteristics of a specifically oriented discontinuous-fiber composite specimen with the corresponding continuous-fiber composite specimen and characterize the strength properties. In this research, the discontinuous-fiber panels required plies with the same size and shape of chips that are uniform throughout the geometry. The chips on the consecutive plies were cut in such a way that they offset exactly at the center of the top and bottom plies. The specimens cut from all the panels were v-notched with dimensional specifications as mentioned in the ASTM D5379 test standard. All specimens were subjected to the Iosipescu shear test, which is the most common and widely used shear test method for v-notched specimens. Images were captured through a two-dimensional (2D) capture camera while testing the specimens so that digital image correlation (DIC) analysis could be conducted to obtain the strain field.
CHAPTER 2
BACKGROUND

Manufacturing airframe components from composite materials is typically an expensive alternative to aluminum construction. The fundamental task facing the aerospace industry is to fully obtain the performance advantages of composite materials while intensely decreasing the fabrication cost [2, 3, 4]. To address this issue lately, structural design engineers are concentrating more on the usage of short-fiber-reinforced thermoplastics as structural materials in aerospace, mechanical, and automotive applications, due to the high specific strength and stiffness properties of discontinuous-fiber thermoplastic composites [5].

In geometrically three dimensional components, where the stress state is either difficult to determine or is known to be distributed roughly equal in all directions, such as in structural fixtures, the use of discontinuous-fiber composites will significantly decrease the cost and structural weight for medium or large volume production. For such applications, the use of unidirectional composite prepreg is highly expensive because of its demand for an enormous amount of manual labor and its inability to be wrapped over complex forms [2].

2.1 Composite Manufacturing Processes

Various methods are used to fabricate composite components. To determine the appropriate method for a specific part requires that the part materials, part design, and part application meet the fabricating challenges. Typically, composite manufacturing processes include some type of molding in order to shape the fiber-reinforced resin material. A mold tool is required to give the unformed resin/fiber combination its shape prior to and during cure.
2.1.1 Hand Layup Process

The most common fabrication method for thermoset composites is the hand layup process, which involves manually laying down individual dry fabric layers, referred to as “plies” or prepreg plies, onto a tool to form a laminate stack. In this research, the composite prepreg plies used were discontinuous (chopped) unidirectional AS4/8552 plies, as shown in Figure 1. This prepreg comprises a large number of AS4 fibers pre-impregnated with an 8552 resin system, wrapped into tows, and then arranged in a single unidirectional ply. The layup process involves manually engineering each ply into the required shape and then firmly affixing it to the previous layer or mold surface, as shown in Figure 2. For this research, after the layup of every four plies, the laminate stack was subjected to a debulking process to ensure that there were no voids or air entrapped between the plies.

Figure 1: Ply design
2.1.1.1 Vacuum Bagging Process

Once the layup process is complete, the laminate stack is subjected to vacuum bagging, as shown in Figure 3. In this research, the composite layup was placed on its tool, and a release film was spread on the tool, which is mounted on the bagging table. Thermocouples were attached at one edge of each of the layups. A two-layered tacky tape was fixed around the edges of the tool creating a boundary. Then another layer of release film was placed on the laminate stacks, and a dam was placed around it to prevent resin flow between the laminate stacks. A caul plate was placed on each of the laminate stacks. A layer of breather/bleeder material was spread on the laminates, and a vacuum suction port fitting the hose was placed on one side. Finally, a bagging film was spread and affixed tightly to the tacky tape on the edges to seal and make it airtight. In some places, especially at the edges of the layup, pleats were created to ensure more room for vacuuming. The port was then connected to a hose, and the vacuum valve was turned on. Once the vacuum leakage was checked inside the sealed area and all the possibilities for leakage were
checked and fixed, the laminates were placed in an autoclave, which was programmed according to the prepreg manufacturers curing cycle. After the curing cycle of 5–6 hours, the composite plate was removed from the autoclave.

2.1.2 Composite Molding Processes

Composite molding processes are grouped into two general categories: open and closed. With open molding, the gel coat and laminate are exposed to the atmosphere in the course of the fabrication procedure. In closed molding, the composite is prepared in a two-sided mold or set inside a vacuum sack. An assortment of handling techniques within the open and closed molding classifications are employed.

2.1.2.1 Open Molding

The open-molding process utilizes a single-sided mold that acts as the configuration and functional surface of the part. A gel coat is applied to the fabricated mold surface. Reinforcements
are then applied either by hand (and then wet-out with catalyzed resin) or by using a spray-up procedure where catalyzed resin and chopped fiberglass are splashed onto the previously gel-coated surface. The focus of the open-molding process is to saturate the fiber reinforcement with resin and then, utilizing manual roll-out methods, to unite the laminate and evacuate ensnared air by using fiber-reinforced plastic (FRP) rollers, paint rollers, or squeegees. A central point in this operation is the transfer of resin from a drum or capacity tank to the mold. The methods used to transport the resin, by and large, describe the particular procedure technique.

2.1.2.1.1 Hand Layup

In the hand layup process, in order to obtain an excellent surface, initially a pigmented gel coat is manually poured onto the mold, as shown in Figure 4. At the point when the gel coat has cured, a roll stock fiberglass reinforcement mat and/or woven roving is put on the mold manually, and the catalyzed resin is poured, brushed, or sprayed on. Manual rolling then expels captured air, compacts the composite, and meticulously wets the reinforcement with the resin. Subsequent layers of fiberglass reinforcement are added for thickness. A catalyst or accelerating agent starts curing in the resin system, which solidifies the composite without external heat.

![Figure 4: Schematic diagram of hand layup process](image)
2.1.2.1.2 Spray Layup

Like the hand layup process, in the spray layup process, a gel coat is initially sprayed onto the mold preceding spray up of the substrate laminate. A continuous-strand, glass-winding catalyzed resin is fed through a chopper/spray gun, which stores the resin-saturated “chop” on the mold. As with hand layup, the manual rolling removes the entrapped air and wets the fiber reinforcement. In the spray layup procedure, the administrator controls thickness and consistency; thus, this procedure is more administrator-dependent than the hand layup process. Extra layers of chop laminate are included as required for thickness.

![Figure 5: Schematic diagram of spray layup process](image)

2.1.2.2 Closed Molding

In the closed molding process, natural/raw materials (fibers and resin) cure inside a two-sided mold or inside a vacuum sack (closed off from air). This process is generally mechanized and requires very specialized gear, so it is mainly used in large plants that produce enormous volumes of material—up to 500,000 parts per year.
2.1.2.2.1 Injection Molding

The three primary units of an injection molding machine (as shown in Figure 6) are the feed hopper, heaters, barrel, and ram ("screw"). The composite material, generally pultruded fibers chopped into pellets is placed in the hopper, where they are fed into the screw. In the screw, the material is heated and compressed. Once in hot fluid form, the ram ("screw") compels the fluid into the firmly braced mold, and the fluid sets. More viscous liquid plastics require higher pressures (and higher press loadings) to compel the material into each hole and corner. The composite material cools as the metal mold conducts heat away, and after that the press is cycled to expel the molding. For thermosetting plastics, the mold will be heated to set the plastic. The injection velocity is typically 1–5 seconds, and this process can produce as many as 2,000 parts/hour, in some multiple crevice molds.

![Figure 6: Schematic diagram of injection molding process](image)

2.1.2.2 Compression Molding

In the compression molding process, composites are rammed between two matching molds, as shown in Figure 7. The designed composite layup is placed in the open mold cavity. Intense pressure and heat ranging from 250° to 400°F is then applied on the closed mold all the way through the cure cycle, which typically takes place in an oven. The heat and pressure produces a composite part that is very close to the final shape.
2.1.2.2.3 Transfer Molding

A schematic diagram of the transfer molding process is shown in Figure 8. It includes a plunger, transfer pot with runners, sprue, mold cavity, and ejector pin. The plunger applies pressure on the charge in the transfer pot where it is heated. The pressure applied by the plunger forces a charge through the sprue into the mold cavity where it is given the desired shape. The molded part is then released through the ejector pin at the bottom of the setup. As shown in Figure 8, the plunger consists of heaters that cure the product in the cavity under heat and pressure.
2.1.2.2.4 Chopped Prepreg Molding

The molding of chopped prepregs is most commonly done at isothermal temperatures in matched metal dies. Isothermal curing eliminates the gradient heating and cooling, which is usually associated with composite prepregs. In contrast to prepreg autoclave molding, the use of matched metal dies provides a dimensionally controlled surfaces on both sides of the part. The fundamental principle in the molding of chopped prepreg-based compounds is to apply pressure when both the resin and the fiber move as a single unit. If the resin is thick enough to flow or too thin to move, then the molding compound will not viably fill the mold, resulting in a low-quality part. This is contrary to the autoclave processing of continuous-fiber prepregs, where a least-resin viscosity is preferred so that entangled air can be expelled at low pressures. The key point-resin division occurs when the resin consistency is too low, thus forcing the press to follow up on incompressible fibers that cannot be made to stream.

2.2 Strength of Composites

The study of the mechanical properties of carbon fiber-reinforced polymer (CFRP) laminates, fracture behavior, and response to stress elevation began almost half a century ago [6]. A significant amount of experimental investigation has been done on the damage mechanisms in CFRP laminates subjected to different types of loading, such as tension, compression, shear, impact, and fatigue. A better understanding of these failure mechanisms will contribute to improving composite laminate designs. This chapter explains our present degree of knowledge on the damage and failure mechanisms in notched CFRP laminates.

2.2.1 Shear Testing

Prabhakaran [7] investigated the determination of an ideal test method for shear testing of composites based on the specimen, notch geometry, and loads applied. In this section, an Iosipescu
specimen is considered. The load acting on the test specimen, as shown in Figure 9(a), determines its strength. Considering the test specimen to be a laminate, the strength can be determined by using the first-ply failure analysis method. The shear and moment distributions of an Iosipescu specimen subject to an idealized load are shown in Figures 9(b) and (c).

The notches are machined to 90° angles such that the parabolic shear stress distribution observed in beams of a constant cross section is transformed to a constant shear stress distribution in the region between the notches. Also, in Iosipescu specimens, regions between the notches are usually assumed to be free from bending stress [7]. The presence of transverse compressive stresses is observed to be significant because of the applied loads, whereas the longitudinal stress due to bending is observed to be insignificant over the test section [7]. This means that the loading surfaces must be located away from the notched region of the specimens. The increase in the radius
at the notch tips from sharp to 0.1 inch results in the increase in the uniformity of the shear stress distribution and reduces the shear stress concentration, especially for materials with a high degree of orthotropy [7].

2.2.2 Failure Mechanisms

Hollmann examined the shear failure of edge-notched multi-orientated composites using the Iosipescu and double-rail shear fixtures along with X-ray radiography [8, 9]. It was observed that an unidentifiable damage developed in the angled (−45°, 45°, and 90°) plies from the roots of the notches, along with delamination. Hollmann observed similar failure modes when a CFRP spar web encompassing an eccentric semi-symmetric cutout was subjected to shear load [8, 9]. Totry et al. [10] analyzed the damage propagation in CFRP laminates under shear load using optical microscopy, which detected only the surface damage of these specimens. Therefore, a reliable method for characterization of shear damage was not accomplished. Tan et al. [11] attempted to investigate the failure mechanism in notched composites subject to multi-axial loading. Typically, the compressive strength of a unidirectional lamina is assumed to be lower than its tensile strength; hence, the critical load-bearing ply in the quasi-isotropic specimens under substantial shear loading is the −45° ply [11]. The damage propagation is instantaneous, and hence, the total failure is catastrophic. The dominant damage mechanisms are observed to be splitting/delamination and micro-buckling at the notch tips in the load bearing −45° plies, as shown in Figure 10. The splits initiate micro-buckling in the 45° plies and diminish the effect of the notch upon the load-bearing −45° plies. The damage sequence is illustrated as follows:

- Initiation of anti-symmetrical splits in the 90° plies.
- Development of splits in the ±45° plies.
• Initiation and stable growth of micro-buckles in the –45° plies (direction of micro-buckling is prompted by splits in the adjacent 90° plies).

• Delamination surrounding the micro-buckle.

• Catastrophic failure due to unstable growth of the micro-buckling.

Figure 10: Dominant damage and failure mechanism

When a quasi-isotropic v-notched specimen of dimensions 90 mm × 25 mm × 2 mm was subject to pure shear loading, the failure in each ply is shown in Figure 11 [11].

Figure 11: Ply-by-ply damage of central notch at 90% failure load
2.2.3 Strength Characterization

Jianmei et al. [12] investigated the Iosipescu shear test of hybrid composites composed of carbon and glass fiber tows with an epoxy matrix to determine strength characteristics such as shear strength and shear modulus. This study involved FEA analysis of the fiber tow to evaluate the effect of varied microstructures in hybrids on the shear stress-strain states. Based on the theoretical and experimental results obtained during testing, this paper concludes by saying that the Iosipescu v-notch shear test method is most reliable for obtaining the elastic shear modulus but not the shear strength, because the specimens did not fail in the predicted test region. In this study, the obtained shear strength of 0° specimens and 90° specimens with low carbon and high carbon are $6.03 \pm 0.28$ GPa, $5.34 \pm 0.19$ GPa and $5.90 \pm 0.37$ GPa, $5.48 \pm 0.19$ GPa, respectively. This study also suggests that more detailed investigations on the shear strength measurements of hybrid composites are needed.

Brown and Los [13] conducted a study to validate the previous studies conducted to determine the shear properties of composites using the Iosipescu specimen. An IM7/8551-7 unidirectional fiber-reinforced composite was studied. The shear moduli of 0° specimens and 90° specimens are $6.38 \pm 0.11$ GPa and $5.17 \pm 0.88$ GPa, respectively.

Odegard and Kumosa conducted 10° off-axis tests and Iosipescu tests to determine the shear strength of unidirectional carbon/epoxy composites [14]. Shear strength values for the analytical, linear/elastic, and non-linear finite element models of specimens subjected to 10° off-axis tests are 83.6, 76.9, and 77.0 MPa, respectively. The shear stress values of specimens subjected to Iosipescu tests are 34.5 MPa for models with splits and 41.3 MPa for models without splits.
Yokoyama and Nakai [15] tested unidirectional T700/2521 carbon/epoxy laminated composites of 10.05 mm thickness to evaluate the interlaminar (out-of-plane) and in-plane shear properties at high deformation rates using the standard split Hopkinson pressure bar. The in-plane shear strength of specimens subjected to a double-notch shear test and short-beam shear test are 77.4±2.4 MPa and 78.5±0.6 Mpa, respectively.

Ferabol et al. [2] studied the failure mechanism and elastic behavior of discontinuous carbon fiber/epoxy laminates obtained from compression molding of randomly oriented pre-impregnated unidirectional tape. The specimens here were subjected to tension, compression, and flexural tests to investigate the influence of chip length.

2.3 Image Correlation

Lin [16] analyzed the strain measurement using the digital image correlation method. The image correlation software setup that is involved plays a vital role in the analysis, adjusting whether the result will be closer to the actual value [16]. During analysis, the step size is significant in obtaining accurate results. Step size defines a calculation grid, in which the speckle pattern is traced, and the distance from speckle dots to the center of each grid is calculated [16]. Therefore, the smaller the grid, the better the result. On the other hand, the size of dots will influence the tracking procedure. In the case of large displacement, if the size of dots is larger than the subset size, then the grid cannot locate dots within this search area and hence loses the displacement information. Thus, a small grid with a large search area and small subset size can yield excellent results [16].

All experiments conducted thus far have dealt with the failure analysis and strength characterization of continuous-fiber composites subjected to shear load. This indicates a
reasonable gap in the literature relative to the strength characterization of discontinuous-fiber composites.

2.4 Motivation

This thesis was motivated by existing research on the strength characterization of discontinuous-fiber composites. The previous studies were conducted by manufacturing composites using injection molding or compression molding that produce randomly oriented chips. The main objective of this study was to bridge the gap in the literature relative to aligned discontinuous fibers. The goal was to isolate the manufacturing complexities (such as resin-rich areas, resin-starved areas, voids, etc) and noise caused due to randomly oriented fibers, and study the impact of fiber length on the strength characteristics. This paper deals with the fabrication of aligned short-fiber composites of varying chip sizes using the hand layup process, whereby short-fiber chips can be arranged in such a manner as to create a properly aligned composite, which significantly affects its strength properties. We have carefully controlled specific orientation states such that it will be helpful for general use in widely used manufacturing processes. More studies on strength characterization of discontinuous-fiber composites would be beneficial in creating a database to assist structural engineers in choosing materials based on their structural and design complexity.
CHAPTER 3
LAMINATE ANALYSIS

3.1 Introduction

A laminate is an assembly of multiple-layered lamina (plies) organized in a desired manner. Each adjoining lamina may be of the same or different materials, and their fiber orientations can be arbitrary with respect to a reference axis (by convention, the x-axis). Even though, a lamina, or ply, of fiber-reinforced composite is light in weight, it is strangely strong along the fiber direction. However, the same lamina is considerably weaker in all off-fiber directions [17]. In order to deal with this problem and sustain loadings from different angles, a laminate is preferably made by a number of laminae oriented at different directions [17].

Laminate failures are usually catastrophic. Even after the failure of one lamina, the composite laminate continues to take more load until all plies fail. Each failed ply contributes to the strength and stiffness of the laminate.

3.2 Classical Laminate Theory

Classical lamination theory (CLT) is analogous to classical plate theory, except for the material properties (stress-strain relations) involved. Classical plate theory assumes the material to be isotropic, although a multiple-layered fiber-reinforced laminate may have more complicated stress-strain relations [17]. Classical lamination theory, as introduced here, is applicable to orthotropic continuous-fiber laminated composites only. The methodology utilized in detailing CLT is similar to that utilized in developing load stress relationships in the basic strength of materials. An initial displacement field predictable with applied loads is presumed. Through the strain-displacement fields and an appropriate constitutive relationship, a state of stress
is characterized. By fulfilling the conditions of static equilibrium, a load-strain relation is defined, and consequently, a state of stress is defined for each lamina.

Stress-strain relationships are obtained from the four foundation equations of lamination theory: kinematic, constitutive, force resultant, and equilibrium. The effect of each of these segments is summarized as below [17].

3.2.1 Kinematics Equations

Displacement fields can be expressed in terms of the distances by which the composite plate’s middle plane moves from the static/unloaded position \( u_0, v_0, w_0 \) and the rotations of the middle plane \( k_x, k_y, k_{xy} \). Based on Kirchhoff’s hypothesis that material normals remain straight and unstretched, it can be assumed that the shear strain in the \( z \) direction is negligible:

\[
\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \approx 0 \equiv \frac{\partial v}{\partial z} \approx -\frac{\partial w}{\partial y} \quad (1)
\]

\[
\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \approx 0 \equiv \frac{\partial u}{\partial z} \approx -\frac{\partial w}{\partial x} \quad (2)
\]

Using Kirchhoff’s hypothesis assumption that the normals always remain at right angles (i.e., normal) to the mid-plane, the \( x \) and \( y \) dependent variables in \( u(x, y, z) \) and \( v(x, y, z) \) can be made definite through the following geometric expressions:

\[
u = v_0 + z \frac{\partial v_0}{\partial z} + O(2) \approx v_0 - z \frac{\partial w_0}{\partial y} \quad (4)\]

The mid-plane strains are

\[
\varepsilon_x^0 = \frac{\partial u_0}{\partial x} \quad (5)
\]

\[
\varepsilon_y^0 = \frac{\partial v_0}{\partial y} \quad (6)
\]
\[
\gamma_{xy}^0 = \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \quad (7)
\]

The mid-plane curvatures (rotations) are

\[
k_x = -\frac{\partial^2 w_0}{\partial x^2} \quad (8)
\]

\[
k_y = -\frac{\partial^2 w_0}{\partial y^2} \quad (9)
\]

\[
k_{xy} = -\frac{\partial^2 w_0}{\partial x \partial y} \quad (10)
\]

Combining the two equations results in

\[
\varepsilon_x = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \quad (11)
\]

\[
\varepsilon_y = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial y^2} \quad (12)
\]

\[
\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w_0}{\partial x \partial y} \quad (13)
\]

Therefore, the kinematics equations become

\[
\varepsilon_x = \varepsilon_x^0 + zk_x \quad (14)
\]

\[
\varepsilon_y = \varepsilon_y^0 + zk_y \quad (15)
\]

\[
\gamma_{xy} = \gamma_{xy}^0 + 2zk_{xy} \quad (16)
\]

where

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + z \begin{bmatrix}
k_x \\
k_y \\
2k_{xy}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial u_0}{\partial x} \\
\frac{\partial v_0}{\partial y} \\
\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}
\end{bmatrix} - z \begin{bmatrix}
\frac{\partial^2 w_0}{\partial x^2} \\
\frac{\partial^2 w_0}{\partial y^2} \\
2\frac{\partial^2 w_0}{\partial x \partial y}
\end{bmatrix} \quad (17)
\]

where \(u_0, v_0, w_0\) are the displacement fields in the x, y, and z directions, respectively.
### 3.2.2 Constitutive Equations

Constitutive equations describe how stresses and strains relate within each lamina (Hooke’s law). For a lamina of any orientation, the coordinate transformation matrix is

\[
[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2\sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & -2\sin \theta \cos \theta \\
-\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
\]  

(18)

Therefore,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix}_k = [T]_k^{-1} [C]_k [T]_k^T \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}_k
\]

where \([C]\) is the principal stiffness matrix.

Conversely,

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}_k = [T]_k^T [S]_k [T]_k \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix}_k
\]

where the subscript \(k\) indicates the \(k^{th}\) layer counting from the top of the laminate, and \([S]\) is the principal compliance matrix.

### 3.2.3 Resultant Equations

Force and moment resultants are the expedient quantities for tracing the important stresses in plates. For a classical plate, the moment and force resultants can be expressed in terms of the normal and shear stresses in integral forms, respectively, as

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = \int_{-t/2}^{t/2} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_{k}} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}_k dz
\]

(21)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \int_{-t/2}^{t/2} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} zdz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_{k}} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}_k zdz
\]

(22)
where “n” indicates the total number of layers. In equations (21) and (22), the integration is moved inside the summation because perfect bonding is presumed [17].

### 3.2.4 Equilibrium Equations

Equilibrium equations depict how the plate carries pressure loads with its internal stresses. Six equilibrium equations (three for the forces and three for the moments) must be satisfied. The equations of force equilibrium are as follows:

- **X-direction:**
  \[
  \frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} + p_x = 0
  \]  
  (23)

- **Y-direction:**
  \[
  \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + p_y = 0
  \]  
  (24)

- **Z-direction:**
  \[
  \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} + p_z = 0
  \]  
  (25)

where \( N_x, N_y, N_{xy}, N_{yx}, Q_{xz}, \) and \( Q_{yz} \) are force resultants, whereas \( p_x, p_y, \) and \( p_z \) are distributed external loads applied on the plate.

The equations of moment equilibrium are as follows

- **X-direction:**
  \[
  -\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_{yz} + m_x = 0
  \]  
  (26)

- **Y-direction:**
  \[
  \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_{xz} + m_y = 0
  \]  
  (27)

- **Z-direction:**
  \[
  N_{xy} - N_{yx} + m_z = 0
  \]  
  (28)

where \( M_x, M_y, M_{xy}, M_{yx}, N_{xy}, \) and \( N_{yx} \) are moment resultants, and \( m_x, m_y, \) and \( m_z \) are distributed external moments applied on the plate.

In a real-life scenario, the applied external loading is usually limited to a distributed pressure so that all forces and moments \( (p_x, p_y, m_x, m_y, \) and \( m_z) \) are zero, excluding \( p_z \), which is non-zero.
Therefore, the above force equilibrium equations become

X-direction: \( \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \) \hspace{1cm} (29)

Y-direction: \( \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \) \hspace{1cm} (30)

Z-direction: \( \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} = -p_z \) \hspace{1cm} (31)

The moment equilibrium equations yield

\( Q_{yz} = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} \) \hspace{1cm} (32)

\( Q_{xz} = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} \) \hspace{1cm} (33)

Substituting the values of \( Q_{xz} \) and \( Q_{yz} \) in the force equilibrium equation corresponding to the z-direction yields

\( \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -p_z \) \hspace{1cm} (34)

Since all external force components excluding \( p_z \) are zero, the shear stresses at any given point are

\( \sigma_{xy} = \sigma_{yx} \) \hspace{1cm} (35)

\( \sigma_{yz} = \sigma_{zy} \) \hspace{1cm} (36)

\( \sigma_{zx} = \sigma_{xz} \) \hspace{1cm} (37)

Therefore,

\( N_{xy} = N_{yx} \) \hspace{1cm} (38)

\( M_{xy} = M_{yx} \) \hspace{1cm} (39)
3.2.5 Stiffness Matrices A, B, D

The relationship between resultants (forces $N$ and moments $M$) and strains (strains $e$ and curvatures $k$) are of utmost interest in a practical sense [17]. Replacing the stresses in the force and moment resultants with strains via the constitutive equations yields

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \begin{bmatrix}
\tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{16} \\
\vdots & \tilde{C}_{22} & \tilde{C}_{26} \\
\text{sym} & \cdots & \tilde{C}_{66}
\end{bmatrix}_k \begin{bmatrix}
\int_{z_{k-1}}^{z_k} \epsilon_x^0 \, dx \\
\int_{z_{k-1}}^{z_k} \epsilon_y^0 \, dx \\
\int_{z_{k-1}}^{z_k} \gamma_{xy}^0 \, dx
\end{bmatrix} + \begin{bmatrix}
k_x \\
k_y \\
2k_{xy}
\end{bmatrix} z \, dz
\]

(40)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \begin{bmatrix}
\tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{16} \\
\vdots & \tilde{C}_{22} & \tilde{C}_{26} \\
\text{sym} & \cdots & \tilde{C}_{66}
\end{bmatrix}_k \begin{bmatrix}
\int_{z_{k-1}}^{z_k} \epsilon_x^0 \, dx \\
\int_{z_{k-1}}^{z_k} \epsilon_y^0 \, dx \\
\int_{z_{k-1}}^{z_k} \gamma_{xy}^0 \, dx
\end{bmatrix} \, dz + \begin{bmatrix}
k_x \\
k_y \\
2k_{xy}
\end{bmatrix} z^2 \, dz
\]

(41)

The laminate stiffness matrix $[\tilde{C}_{ij}]$ can be more precisely expressed in terms of $[Q_{ij}]$, i.e., material stiffness referred to in terms of laminate coordinates. Thus, equations (40 and 41) become

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
\vdots & Q_{22} & Q_{26} \\
\text{sym} & \cdots & Q_{66}
\end{bmatrix}_k \begin{bmatrix}
\int_{z_{k-1}}^{z_k} \epsilon_x^0 \, dx \\
\int_{z_{k-1}}^{z_k} \epsilon_y^0 \, dx \\
\int_{z_{k-1}}^{z_k} \gamma_{xy}^0 \, dx
\end{bmatrix} + \begin{bmatrix}
k_x \\
k_y \\
2k_{xy}
\end{bmatrix} z \, dz
\]

(42)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
\vdots & Q_{22} & Q_{26} \\
\text{sym} & \cdots & Q_{66}
\end{bmatrix}_k \begin{bmatrix}
\int_{z_{k-1}}^{z_k} \epsilon_x^0 \, dx \\
\int_{z_{k-1}}^{z_k} \epsilon_y^0 \, dx \\
\int_{z_{k-1}}^{z_k} \gamma_{xy}^0 \, dx
\end{bmatrix} + \begin{bmatrix}
k_x \\
k_y \\
2k_{xy}
\end{bmatrix} z^2 \, dz
\]

(43)

Equations (42) and (43) can be further simplified into a matrix form to obtain the force and moment resultants by applying the summation and integration operations to their respective components.

Thus, according to the work of Black [1],

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
\vdots & A_{22} & A_{26} \\
\text{sym} & \cdots & A_{66}
\end{bmatrix} \begin{bmatrix}
\epsilon_x^0 \\
\epsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + \begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
\vdots & B_{22} & B_{26} \\
\text{sym} & \cdots & B_{66}
\end{bmatrix} \begin{bmatrix}
k_x \\
k_y \\
2k_{xy}
\end{bmatrix}
\]

(44)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
\vdots & B_{22} & B_{26} \\
\text{sym} & \cdots & B_{66}
\end{bmatrix} \begin{bmatrix}
\epsilon_x^0 \\
\epsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + \begin{bmatrix}
D_{11} & D_{12} & D_{16} \\
\vdots & D_{22} & D_{26} \\
\text{sym} & \cdots & D_{66}
\end{bmatrix} \begin{bmatrix}
k_x \\
k_y \\
2k_{xy}
\end{bmatrix}
\]

(45)

Combining equations (44) and (45) yields
where $A$ is the in-plane stiffness, $B$ is the in-plane and bending deformation coupling stiffness, and $D$ is the bending/twisting stiffness of the laminate. These three stiffness matrices are briefly defined, respectively, as follows:

$$A_{ij} = \sum_{k=1}^{N} (Q_{ij})_{k} (z_{k} - z_{k-1}) = \sum_{k=1}^{N} (Q_{ij})_{k} t_{k}$$  \hspace{1cm} (47)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (Q_{ij})_{k} (z_{k}^{2} - z_{k-1}^{2}) = \sum_{k=1}^{N} (Q_{ij})_{k} t_{k} \bar{z}_{k}$$  \hspace{1cm} (48)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (Q_{ij})_{k} (z_{k}^{3} - z_{k-1}^{3}) = \sum_{k=1}^{N} (Q_{ij})_{k} \left[ t_{k} \bar{z}_{k}^{2} + \frac{t_{k}^{3}}{12} \right]$$  \hspace{1cm} (49)$$

where $t_{k}$ is the thickness of the $k^{th}$ layer, and $\bar{z}_{k}$ is the distance from the mid-plane to the centroid of the $k^{th}$ layer and

$$[Q_{12}] = \begin{bmatrix} E_{1} & \frac{\theta_{21}E_{1}}{1-\theta_{12}\theta_{21}} & 0 \\ 1-\theta_{12}\theta_{21} & 1 & 0 \\ \vdots & \vdots & \ddots \\ E_{2} & \frac{\theta_{21}E_{2}}{1-\theta_{12}\theta_{21}} & 0 \\ 1-\theta_{12}\theta_{21} & 1 & \ddots \end{bmatrix}$$  \hspace{1cm} (50)$$

The principal stiffness values are defined as follows:

$$Q_{xx} = Q_{11} m^{4} + 2(Q_{12} + 2Q_{66})m^{2}n^{2} + Q_{22} n^{4}$$

$$Q_{yy} = Q_{11} n^{4} + 2(Q_{12} + 2Q_{66})m^{2}n^{2} + Q_{22} m^{4}$$

$$Q_{xy} = (Q_{11} + Q_{22} - 4Q_{66})m^{2}n^{2} + Q_{12}(m^{4} + n^{4})$$

$$Q_{ss} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})m^{2}n^{2} + Q_{66}(m^{4} + n^{4})$$

$$Q_{xs} = (Q_{11} - Q_{12} - 2Q_{66})m^{4}n + (Q_{12} - Q_{22} + 2Q_{66})m^{3}n$$

$$Q_{ys} = (Q_{11} - Q_{12} - 2Q_{66})mn^{3} + (Q_{12} - Q_{22} + 2Q_{66})m^{3}n$$

where $m = \cos \theta$, $n = \sin \theta$, $\theta$ = lamina orientation.

The strain-resultant relations can be derived with appropriate matrix operations:

$$\{\varepsilon^{0}\} = [A^{*} \quad B^{*} \quad D^{*}] \{N\}$$  \hspace{1cm} (52)$$
where
\[ A^* = A^{-1} - [-A^{-1}B(D - BA^{-1}B)^{-1}BA^{-1}] \]  
(53)

\[ B^* = -A^{-1}B(D - BA^{-1}B)^{-1} = -(D - BA^{-1}B)^{-1}BA^{-1} \]  
(54)

\[ D^* = (D - BA^{-1}B)^{-1} \]  
(55)

Note that A, B, D, and \( A^*, B^*, D^* \) are all symmetric matrices [17].

### 3.3 Laminate Failure Theories

The micromechanical strength predictions are accurate with regard to failure initiation at critical points. They are only approximate as far as the global failure of the lamina is concerned. From a macro-mechanical perspective, the strength of a lamina is an anisotropic property, i.e., it fluctuates with the fiber orientation. A lamina may be characterized by a number of basic strength parameters. For an in-plane loading, a lamina may be characterized by five strength parameters: longitudinal tensile \( (F_{1T}) \), longitudinal compressive \( (F_{1C}) \), transverse tensile \( (F_{2T}) \), transverse compressive \( (F_{2C}) \) and in-plane shear \( (F_6) \).

Macro-mechanical failure theories for composites are generally an augmentation and implementation of isotropic failure theories to represent anisotropy in strength and stiffness of the composite. The four commonly used failure criteria theories for fiber-reinforced plastics are as follows:

- Maximum Stress Theory
- Maximum Strain Theory
- Deviatoric Strain Energy Theory (Tsai-Hill)
- Interactive Tensor Polynomial Theory (Tsai-Wu)

#### 3.3.1 Maximum Stress Theory

The maximum stress hypothesis is commonly attributed to C. F. Jenkins and is an extension of the maximum normal stress theory for the failure of orthotropic materials (such as wood). For
a failure to occur according to the maximum stress theory, one of three possible conditions must be met:

\[
\sigma_1 \geq X, \sigma_2 \geq Y, \tau_{12} \geq S \tag{56}
\]

Consider a lamina subjected to uniaxial tension, as shown in Figure 12.

![Figure 12: Uniaxial tension for unidirectional lamina.](image)

Performing the transformation produces the following:

\[
\sigma_1 = \sigma_x \cos^2 \theta \tag{57}
\]

\[
\sigma_2 = \sigma_x \sin^2 \theta \tag{58}
\]

\[
\tau_{12} = -\sigma_x \cos \theta \sin \theta \tag{59}
\]

To ensure that failure does not occur under the conditions embodied in equation (56), the stresses in the principal material directions must be less than the respective applied stresses (X, Y, S) in those directions, such that

\[
\sigma_x \leq \frac{X}{\cos^2 \theta}, \sigma_x \leq \frac{Y}{\sin^2 \theta}, \sigma_x \leq \frac{S}{\sin \theta \cos \theta} \tag{60}
\]

If the applied stresses are compressive, then the X and Y would be replaced by X’ and Y’, so that the failure condition becomes \( \sigma_1 > X’ \), \( \sigma_2 > Y’ \). The failure criterion for shear remains unchanged, since S is independent of the sign of the applied shear stress.

In cases of multi-axial stress, the principal direction stresses and condition for failure to occur are as follows:

\[
\sigma_1 = \cos^2 \theta \sigma_x + \sin^2 \theta \sigma_y + 2 \cos \theta \sin \theta \tau_{xy} \geq X \tag{61}
\]
\[ \sigma_2 = \cos^2 \theta \sigma_y + \sin^2 \theta \sigma_x - 2 \cos \theta \sin \theta \tau_{xy} \geq Y \quad (62) \]
\[ \tau_{12} = (\sigma_y - \sigma_x) \cos \theta \sin \theta + (\cos^2 \theta - \sin^2 \theta) \tau_{xy} \geq S \quad (63) \]

### 3.3.2 Maximum Strain Theory

The maximum strain failure criteria is an expansion of St. Venant’s maximum strain theory that accommodates orthotropic material behavior. Material or its constituents undergo fracture when a critical separation is produced as a result of stress; therefore, a more realistic criterion for failure is strain. The maximum strain failure theory is expressed as follows

\[
\begin{align*}
\frac{-X'}{E_1} < \varepsilon_1 & < \frac{X}{E_1} \\
\frac{-Y'}{E_2} < \varepsilon_2 & < \frac{Y}{E_2} \\
\frac{-S'}{G_{12}} < \gamma_{12} & < \frac{S}{G_{12}}
\end{align*}
\quad (64)
\]

Using Hooke’s law,

\[
\begin{align*}
\varepsilon_1 &= \frac{\sigma_1}{E_1} - \frac{\theta_{12} \sigma_2}{E_1} \\
\varepsilon_2 &= \frac{\sigma_2}{E_2} - \frac{\theta_{21} \sigma_1}{E_2} \\
\gamma_{12} &= \frac{\tau_{12}}{G_{12}}
\end{align*}
\quad (65, 66, 67)
\]

For a uniaxial tension as shown previously in Figure 8, the principal material direction stresses are

\[
\sigma_1 = \cos^2 \theta \sigma_x, \sigma_2 = \sin^2 \theta \sigma_x, \tau_{12} = -\cos \theta \sin \theta \sigma_x
\quad (68)
\]

Therefore,

\[
\begin{align*}
\varepsilon_1 &= \frac{\sigma_x}{E_1} (\cos^2 \theta - \theta_{12} \sin^2 \theta) \\
\varepsilon_2 &= \frac{\sigma_x}{E_2} (\sin^2 \theta - \theta_{21} \cos^2 \theta) \\
\gamma_{12} &= \frac{-\sigma_x}{G_{12}} (\cos \theta \sin \theta)
\end{align*}
\quad (69, 70, 71)
Now, equation (2) becomes

\[
\frac{-X'}{\cos^2 \theta - \dot{\theta}_{12} \sin^2 \theta} < \sigma_x < \frac{X}{\cos^2 \theta - \dot{\theta}_{12} \sin^2 \theta} \tag{72}
\]

\[
\frac{-Y'}{\sin^2 \theta - \dot{\theta}_{12} \cos^2 \theta} < \sigma_x < \frac{Y}{\sin^2 \theta - \dot{\theta}_{12} \cos^2 \theta} \tag{73}
\]

\[
\frac{-S'}{\cos \theta \sin \theta} < \sigma_x < \frac{S}{\cos \theta \sin \theta} \tag{74}
\]

### 3.3.3 Deviatoric Strain Energy Theory (Tsai-Hill Failure Criterion)

The Tsai-Hill criterion is an adoption of Von-Mises/distortional energy criteria for isotropic materials under plane stress:

\[
\sigma^2_{p1} - \sigma_{p1}\sigma_{p2} + \sigma^2_{p2} = Y^2 \tag{75}
\]

For anisotropic ductile material, Hill modified the above criterion as

\[
A\sigma_1^2 + B\sigma_2^2 - C\sigma_1\sigma_2 + D\tau_6^2 = Y^2 \tag{76}
\]

where A, B, C, and D are constants and dependent on the material and nature of the anisotropy.

The above criterion in equation (76) was simplified by Azzi and Tsai for orthotropic material with the constants A, B, C, and D, and are related to the basic material strength parameters: \(F_{1T}, F_{1C}, ..., F_6\).

For uniaxial loading to failure in the longitudinal/fiber (1) direction, \(\sigma_1 = F_1, \sigma_2 = 0, \tau_6 = 0\) yields \(AF_1^2 = 1\) at failure, i.e.,

\[
A = \frac{1}{F_1^2} \tag{77}
\]

For uniaxial transverse (2) loading to failure, \(\sigma_1 = 0, \sigma_2 = F_2, \tau_6 = 0\) yields \(BF_2^2 = 1\) at failure, i.e.,

\[
B = \frac{1}{F_2^2} \tag{78}
\]
For pure shear loading to failure in the 1–2 plane, $\sigma_1 = 0, \sigma_2 = 0, \tau_6 = F_6$ yields $DF_6^2 = 1$ at failure i.e.,

$$D = \frac{1}{F_6^2} \quad (79)$$

The notation “C” represents coupling between the normal stresses $\sigma_1$ and $\sigma_2$, so the value of $C$ can be obtained from a biaxial test, such that $\sigma_1 = \sigma_2 = F_2, \tau_6 = 0$. At failure

$$A\sigma_1^2 + B\sigma_2^2 + C\sigma_1\sigma_2 = 1 \quad (80)$$

Substituting the values of $A$ and $B$, equation (80) becomes

$$(\frac{1}{F_1^2})F_2^2 + (\frac{1}{F_2^2})F_2^2 + CF_2F_2 = 1 \quad (81)$$

Therefore,

$$C = \frac{-1}{F_1^2} \quad (82)$$

Hence, the Tsai-Hill criterion becomes

$$(\frac{\sigma_1}{F_1})^2 + (\frac{\sigma_2}{F_2})^2 - \frac{\sigma_1 \sigma_2}{F_1^2} + \left(\frac{\tau_6}{F_6}\right)^2 \leq 1 \quad (83)$$

At failure,

$$(\frac{\sigma_1}{F_1})^2 + (\frac{\sigma_2}{F_2})^2 - \frac{\sigma_1 \sigma_2}{F_1^2} + \left(\frac{\tau_6}{F_6}\right)^2 = 1 \quad (84)$$

where $F_1$ and $F_2$ are the strengths along the 1 and 2 directions, depending on the type of loading (tension/compression).

### 3.3.4 Interactive Tensor Polynomial Theory (Tsai-Wu Failure Theory)

The Tsai-Wu criterion accounts for the interaction of stress components and is also known as “quadratic interaction theory”. Tsai-Wu uses the general form of the tensor polynomial criterion expanded to

$$f_1\sigma_1 + f_2\sigma_2 + f_6\tau_6 + f_{11}\sigma_1^2 + f_{22}\sigma_2^2 + f_{66}\tau_6^2 + 2f_{12}\sigma_1\sigma_2 \leq 1 \quad (85)$$
For uniaxial longitudinal tensile loading, \( \sigma_1 = F_{1T}, \sigma_2 = 0, \tau_6 = 0 \) yields

\[
f_1 F_{1T} + f_{11} F_{1T}^2 = 1 \tag{86}
\]

For compressive longitudinal loading to failure, \( \sigma_1 = F_{1C}, \sigma_2 = 0, \tau_6 = 0 \) yields

\[
-f_1 F_{1C} + f_{11} F_{1C}^2 = 1 \tag{87}
\]

Solving equations (86) and (87) simultaneously yields

\[
f_1 = \frac{1}{F_{1T}} - \frac{1}{F_{1C}} \quad \text{and} \quad f_{11} = \frac{1}{F_{1T} F_{1C}} \tag{88}
\]

Similarly, for uniaxial transverse tensile loading, \( \sigma_1 = 0, \sigma_2 = F_{2T}, \tau_6 = 0 \) yields

\[
f_2 F_{2T} + f_{22} F_{2T}^2 = 1 \tag{89}
\]

For compressive transverse loading to failure, \( \sigma_1 = 0, \sigma_2 = F_{2C}, \tau_6 = 0 \) yields

\[
-f_2 F_{2C} + f_{22} F_{2C}^2 = 1 \tag{90}
\]

Solving equations (89) and (90) simultaneously yields

\[
f_2 = \frac{1}{F_{2T}} - \frac{1}{F_{2C}} \quad \text{and} \quad f_{22} = \frac{1}{F_{2T} F_{2C}} \tag{91}
\]

From the pure shear test,

\[
f_{66} F_6^2 = 1 \tag{92}
\]

which yields

\[
f_{66} = \frac{1}{F_6^2} \tag{93}
\]

To determine the interaction term \( f_{12} \) requires the strength of lamina subjected to a combined state of stress, i.e., applying measured biaxial tensile strength, \( \sigma_1 = \sigma_2 = F_X, \tau_6 = 0 \):

\[
f_{12} = \frac{1}{2F_X} [1 - F_X (f_1 + f_2) - F_X^2 (f_{11} + f_{22})] \tag{94}
\]

Or, if the biaxial strength is not available, then \( f_{12} \) can be empirically expressed as

\[
f_{12} \approx F_{12}' \sqrt{f_{11} f_{22}} \tag{95}
\]
where $F_{12}'$ is between $-0.5$ and $0$. Thus,

$$f_{12} \cong \frac{-1}{2} \sqrt{f_{11}f_{22}}$$

(96)

3.4 Anisotropic Strength of Laminates

3.4.1 Laminate Failure Analysis

First ply failure (FPF) occurs when the stress resultants ($N_x, N_y \ldots$) are increased until the strength ratio for one of the plies is reduced to a value of $1$. Failure loads ($N_x, N_y \ldots$) corresponding to FPF are conservative in nature. The laminate could carry additional load after one (or more) ply(ies) has failed.

Total laminate failure occurs when the stress resultants are increased until the strength ratios for all plies are reduced to a value of $1$ (or lower). This can be predicted by the progressive application of FPF while appropriately eliminating the failed plies.

3.4.1.1 Failure Analysis of Panels

The material considered in the failure analysis of panels is AS4 (12k)/8552 composite. The properties of this material are described in Tables 1, 2, and 3.

<table>
<thead>
<tr>
<th>MATERIAL ELASTIC PROPERTIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Modulus ($E_1$)</td>
</tr>
<tr>
<td>Transverse Modulus ($E_2$)</td>
</tr>
<tr>
<td>In-Plane Shear Modulus ($G_{12}$)</td>
</tr>
</tbody>
</table>
TABLE 2
POISSON’S RATIO

<table>
<thead>
<tr>
<th>$\nu_{12}$</th>
<th>$\nu_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.03</td>
</tr>
</tbody>
</table>

TABLE 3
MATERIAL STRENGTH PROPERTIES

<table>
<thead>
<tr>
<th>Property</th>
<th>Ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Tensile Strength ($F_{1t}$)</td>
<td>300</td>
</tr>
<tr>
<td>Longitudinal Compressive Strength ($F_{1c}$)</td>
<td>222</td>
</tr>
<tr>
<td>Transverse Tensile Strength ($F_{2t}$)</td>
<td>9.27</td>
</tr>
<tr>
<td>Transverse Compressive Strength ($F_{2c}$)</td>
<td>38.9</td>
</tr>
<tr>
<td>In-Plane Shear Strength ($F_{6}$)</td>
<td>11.6</td>
</tr>
</tbody>
</table>
CHAPTER 4
SHEAR TESTING METHOD

4.1 Introduction

Measuring the shear properties of unidirectional polymer-matrix composites is not a straightforward task. These properties are obtained by subjecting a unidirectional composite material to a pure and uniform shear stress state. Though it appears to be simple, the challenging task is to avoid complications caused by machining and the influence of other stresses.

The determination of composite strengths and elastic constants are very useful for designing structures. An orthotropic composite consists of five in-plane elastic constants \( E_1, E_2, G_{12}, \vartheta_{12} \) and \( \vartheta_{21} \) and five strength parameters (described previously in section 2.3). Moduli \( E_1, E_2 \) are the two Young’s moduli with respect to the in-plane material symmetry L-T (longitudinal-transverse) axes, \( G_{12} \) is the shear modulus, and \( \vartheta_{12} \) and \( \vartheta_{21} \) are Poisson’s ratios. For orthotropic composites, stiffness and strength properties along the material symmetry axes can easily be determined using standard tension or compression specimens.

Over the years, numerous test methods have been introduced for depicting the shear strength of composite materials, each with its own constraints. These test methods include the following: Iosipescu, torsional tube, slotted tensile, \( \pm 45^\circ \) tensile, two-rail, crossbeam sandwich, picture-frame panel, and \( 10^\circ \) off-axis shear. The most prevalent of these methods is the Iosipescu shear test method, the popularity of which is based on the relative ease of fabrication and testing of the specimen at a low cost, and the ability to determine accurate shear strength values.

4.2 ASTM Test Standard for Iosipescu Specimen

According to standard ASTM D5379, the standard test specimen (as shown in Figure 13) is 76 mm (3 inches) long, 20 mm (0.75 inch) wide, and up to 12 mm (0.5 inch) thick [18]. Typically
a specimen on the order of 2.5 mm thick is utilized. The testing apparatus will also accommodate shorter, smaller, and narrower specimens, if desired. A 90° notch is machined into every edge of the specimen, and a strain gage can be mounted on the specimen surface in the indented/notch region to supervise the shear strain. Either a standard ±45° biaxial gage or an extraordinary Iosipescu gage is used.

![Figure 13: Iosipescu test specimen dimensions](image)

4.3 **Iosipescu Test**

The Iosipescu shear test was introduced by Nicolae Iosipescu in the 1960s, and the method used today is originally based on his work with homogeneous and isotropic materials such as metals. The test fixture, as shown in Figure 14, has its one side dislodged vertically while the other side remains stationary. Furthermore, restricting force couples forestall in-plane bending and twisting of the specimen [19]. The Iosipescu shear test comprises a V-notched specimen mounted in both sides of the Iosipescu test fixture.

![Figure 14: Iosipescu test fixture](image)
The specimens used in this research are made of AS4/8552 composite materials, and the shape and size of the specimens are as specified in ASTM D5379 test standard. The test fixture is balanced by using an adjusting tool. First, the crosshead is moved to a position so that the adjusting tool can be inserted to make the left and right parts of the jigs parallel. Once the parallelism is obtained, the specimen is fixed horizontally in the central region using the centering-pin. In order to achieve vertical alignment, the rear face of the specimen is kept in contact with the fixture. The space between the specimen and the fixture can be closed by simply finger tightening the wedge-adjusting screws. It is not necessary to fully tighten the wedges because they accommodate any tolerance in the width dimension of the specimen.

In analyzing the specimen, it was first believed that a condition of uniform shear stress would exist in the test section between the notches. However, analysts have since demonstrated that non-uniform normal and shear stresses exist anywhere in the test section of the specimen, and stress concentrations exist at the notch tips. A compression load is applied on the upper half of the fixture by attaching a suitable adapter to the crosshead of the testing machine. When the load is applied, there may be occurrences of out-of-plane bending and twisting, so to avoid this, the top and bottom edges of the fixture are carefully machined flat, i.e., parallel to each other and perpendicular to the faces of the specimen. The stress-strain curves can be easily obtained by attaching a rosette strain gage on either one or both faces of the specimen between the notches in the central region of the specimen. The use of a strain gauge is optional. The load applied and the stress-strain signals are recorded until the specimen reaches failure.

The average shear stress of the specimen across the notch region is calculated as

\[ \tau = \frac{P}{A} \]  

(97)
where $P$ is the force or load applied, and $A$ is the cross sectional area of the specimen between the notches. In this research, the strain properties were obtained using the digital image correlation analysis method.

The small size of the specimen, the ease of performing tests, and also the capability to obtain all three shear stress states as well as in-plane and transverse shear data have made the Iosipescu test method the most popular method since its first application in the 1990s.
CHAPTER 5
DIGITAL IMAGE CORRELATION ANALYSIS

Digital image correlation is a non-contact optical technique for measuring strains. On a basic level, DIC compares a series of binary images of specimens at different points of deformation, tracks pixels movement in the region of interest (ROI) [20], and estimates displacement and strain by the use of correlation algorithm. Typically, DIC requires a digital camera with zooming feature and personal computer (PC) software. The DIC arrangement is shown in Figure 15.

![Figure 15: Schematic representation of DIC arrangement](image)

Before proceeding with the mechanical tests, visible marks are of great importance and should be applied on the specimen surface in order to let the camera capture and track their movement. These marks and can be obtained by spray painting, engraving, or applying surface roughness (or texture). During testing, consecutive pictures are taken at regular intervals by one or more cameras while the specimens are subjected to deformation tests [21]. The number of cameras required depends on whether the analysis is 2D or 3D, and the preferred level of accuracy. The images are later transferred to a PC and analyzed through a correlation algorithm by using software such as MATLAB and ARAMIS [22].
5.1 Basic Principle of Digital Image Correlation

Using a stereoscopic sensor setup, each recipient point is centered on a particular pixel in the image plane of the respective sensor [23]. By determining the intrinsic parameters (interior geometric and optical attributes of the camera and positions between lens and the charge-coupled image device) and extrinsic parameters (exterior geometric relation between camera and specimen) of the sensor, the positions of each recipient point in 3D can be calibrated [23]. With the calibration data, the DIC system can interpret the image coordinates to the geometric coordinates [24].

A random intensity pattern is used on the specimen surface such that the position of each object point in the two images can be identified by applying a correlation algorithm [23]. Consider a planar specimen that is illuminated by a light source so that the camera captures the deformations of the specimens clearly. Let “R” be the undeformed configuration of the region of interest, and let $\bar{R}$ be the deformed configuration of the region of interest. The light intensity patterns of regions $R$ and $\bar{R}$ are defined as $I(x)$ and $\bar{I}(y)$, respectively, where $x \in R; y \in \bar{R}$. Both $I(x)$ and $\bar{I}(y)$ are assumed to be in unique correspondence with the respective specimen surface, and they are integer functions ranging from 0 to 255 when using an 8-bit gray-scale (binary) digital camera [13]. During the deformation process, if the intensity pattern undergoes only deformation but retains its local value, then the following should result [9]:

$$\bar{I}(y) = \bar{I}(\tilde{y}(x)) = I(x), \forall x \in R$$  \hspace{1cm} (98)

Thus, by using the DIC technique, measurement of the displacement field can be framed as follows: The mapping relation between the two intensity patterns ($I(x)$ and $\bar{I}(y)$) is described as

$$y = \tilde{y}(x)$$  \hspace{1cm} (99)

Hence,

$$\bar{I}(\tilde{y}(x)) - I(x) = 0, \forall x \in R$$  \hspace{1cm} (100)
For homogeneous deformation,

\[ \hat{y}(x) =Fx + b \]  
(101)

In 2D deformation,

\[ F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \]  
(102)

And \( \det F = f_{11}f_{22} - f_{12}f_{21} > 0 \)

\[ b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \]  
(103)

where \( F \) is a constant tensor, and \( b \) is a constant vector.

Substituting equation (101) into equation (100) yields

\[ \bar{I}(Fx + b) - I(x) = 0, \forall x \in R \]  
(104)

Once the deformation gradient tensor \( F \) for 2D deformation is determined, the strain within the region \( R \) can be calculated.

5.2 Best Features of Digital Image Correlation

The best features of the DIC technique are as follows:

- Non-contact optical measurement.
- Data set compilation from thousands of points on the surface.
- Analysis done in post-processing.
- High-precision data with respect to shape, deformation and strain, and displacement field.
- Unaffected by rigid-body motion.
- Unaffected by physical attributes of the specimen, such as surface geometry or temperature.
5.3 Disadvantages of Digital Image Correlation

Disadvantages of the DIC technique are as follows:

- Unable to detect or measure inherent damage.
- Must have a clear line of sight by the cameras to the test specimen.

5.4 Applications of Digital Image Correlation Technique

DIC is a very effective method for determining deformations on the surface of a specimen. It is widely used in many fields, some of which are the following:

- Failure and fatigue studies.
- Material testing and characterization.
- Static and dynamic measurement of strain or body motion.

5.5 ARAMIS Software

In this research, ARAMIS software was used to conduct the DIC analysis. The ARAMIS DIC system is a sophisticated three-dimensional measurement tool used to study the material meticulously and to understand the component behavior [24]. This non-contact, material, unconventional computing system provides high-resolution images and exceptionally precise data relative to a component’s 3D shape, in addition to the strain and displacement fields for a test specimen subject to a load or force [25].

ARAMIS uses the DIC principle to apply a series of digital images for optical measurements to provide material properties of the test specimen [26]. The images are post-processed using the subset method, which is a computerized image-processing method that can track features from digital images. Photogrammetry is used to trace those features in 3D in order to assign them to a 3D coordinate system. The initial set of 3D coordinates in the undeformed image is said to be the reference, and all deformations are then compared to this reference point.
The system performs highly accurate measurements, and detects and indicates even small changes in the characteristics of the test specimen with 3D measurements in the sub-micrometer range, irrespective of the specimen’s geometry and temperature. The 3D measuring data generated by ARAMIS is also used to validate simulation results in prototype and component testing in order to precisely optimize simulations. This method is inexpensive and less tedious than other methods.

For components subjected to static and dynamic loading, ARAMIS provides accuracy in the following [25]:

- 3D coordinates.
- 3D displacements, velocities, and accelerations.
- Surface strain.
- Evaluations of six degrees of freedom.
6.1 Manufacturing of Panels

All panels were manufactured using the hand layup and vacuum bagging processes. In this research, all panels were balanced symmetric laminates, and the specimens obtained from these panels were identical in shape and size. The thickness of the panels (i.e., specimens) was chosen to be 0.13 inch, in order for panels to be balanced and symmetric. A total of 24 plies was required for all panels.

Ply Details

Ply details are as follows:

- Material: AS4 12k/8552
- Ply cutting depth: 0.125 inch thick + 10% of backing material thickness
- Chip cutting depth: 0.018 inch
- Final ply cutting depth: 0.035 inch
- Ply thickness: 0.0065 inch (0.165 mm)
- Total panel thickness: 0.156 inch (3.9 mm)
- Total panel thickness after curing: 0.170 inch (4.3 mm)
- Number of plies required: 24

Panel 1: Continuous Quasi-Isotropic $[45/-45/0/90]_3s$ Panel

The plies in this panel were arranged in $[45/-45/0/90]_3s$ order. Three specimens were cut from this panel, each with $0^\circ$ orientation. The layup order of plies is described in Table 4.
<table>
<thead>
<tr>
<th>Ply Number</th>
<th>Ply Orientation (degrees)</th>
<th>Ply Dimensions (inches*inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45</td>
<td>6*6</td>
</tr>
<tr>
<td>2</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>6*6</td>
</tr>
<tr>
<td>6</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>8</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>9</td>
<td>45</td>
<td>6*6</td>
</tr>
<tr>
<td>10</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>12</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>13</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>15</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>16</td>
<td>45</td>
<td>6*6</td>
</tr>
<tr>
<td>17</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>19</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
<td>6*6</td>
</tr>
<tr>
<td>21</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>23</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>24</td>
<td>45</td>
<td>6*6</td>
</tr>
</tbody>
</table>
Panel 2: Discontinuous Quasi-Isotropic \([45/–45/0/90]_{3s}\) Panel with 0.75-Inch Chip Size

The discontinuity for plies in Panel 2 was obtained by cutting 0.75 inch × 0.75 inch chips on these plies. The layup and bagging process were exactly the same as for Panel 1. Three specimens were cut from this panel, each with 0° orientation.

Panel 3: Discontinuous Quasi-Isotropic \([45/–45/0/90]_{3s}\) Panel with 1-Inch Chip Size

The manufacturing and bagging process of this panel was exactly the same as for Panel 2, but the chips were cut to a size of 1 inch × 1 inch. Three specimens were cut from this panel, each with 0° orientation.

The layups of both discontinuous Panels 2 and 3 are shown in Table 5. The main objective of laying consecutive plies with two different dimensions for the discontinuous panels was to ensure that chips on the consecutive plies were offset exactly at the center of each chip in the preceding and succeeding plies.

### TABLE 5
LAYUP OF PLIES IN DISCONTINUOUS QUASI-ISOTROPIC \([45/–45/0/90]_{3s}\) PANELS

<table>
<thead>
<tr>
<th>Ply Number</th>
<th>Ply Orientation (degrees)</th>
<th>Ply Dimensions (inches*inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>((0.75<em>0.75)) chip ((1</em>1)) chip</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>6*6</td>
</tr>
<tr>
<td>2</td>
<td>–45</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>6*6</td>
</tr>
<tr>
<td>6</td>
<td>–45</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>8</td>
<td>90</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>9</td>
<td>45</td>
<td>6*6</td>
</tr>
</tbody>
</table>
### TABLE 5 (continued)

<table>
<thead>
<tr>
<th>Ply Number</th>
<th>Ply Orientation (degrees)</th>
<th>Ply Dimensions (inches*inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>–45</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>6*6</td>
</tr>
<tr>
<td>12</td>
<td>90</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>13</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>15</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>16</td>
<td>45</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>17</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>19</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>21</td>
<td>90</td>
<td>6*6</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>5.25*5.25</td>
</tr>
<tr>
<td>23</td>
<td>–45</td>
<td>6*6</td>
</tr>
<tr>
<td>24</td>
<td>45</td>
<td>5.25*5.25</td>
</tr>
</tbody>
</table>

**Panel 4: Continuous Unidirectional Panel**

The arrangement of plies and their dimensions for the continuous unidirectional panel are described in detail in Table 6. Three specimens with 0° orientation, three specimens with 90° orientation, and three specimens with 45° orientation were cut from this panel. Therefore, a total of nine specimens was obtained from this panel.
TABLE 6
LAYUP OF PLIES IN CONTINUOUS UNIDIRECTIONAL PANEL

<table>
<thead>
<tr>
<th>Ply Number</th>
<th>Ply Orientation (degrees)</th>
<th>Ply Dimensions (inches*inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>9*9</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>9*9</td>
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<tr>
<td>3</td>
<td>0</td>
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<td>9*9</td>
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<tr>
<td>5</td>
<td>0</td>
<td>9*9</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>9*9</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>9*9</td>
</tr>
<tr>
<td>8</td>
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<td>9*9</td>
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<td>9*9</td>
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<td>9*9</td>
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<td>9*9</td>
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<td>0</td>
<td>9*9</td>
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<td>23</td>
<td>0</td>
<td>9*9</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>9*9</td>
</tr>
</tbody>
</table>
Panel 5: Discontinuous Unidirectional Panel

This panel consisted of discontinuous prepreg plies. The plies were cut in 10 inches × 10 inch dimensions, of which the actual chopped-fiber (discontinuous-fiber) area was 9 inches × 9 inches. The reason for leaving extra space on either side was to ensure that the cutter did not pull the material along while cutting the flakes on the plies. In this panel, both the 0.75 inch × 0.75 inch flakes ply and 1 inch × 1 inch flakes ply were stacked on top of each other. The number of specimens obtained was nine, including three specimens each of 0°, 90° and 45° orientations.

6.2 Cutting of Specimens

The specimens from all the panels were cut down by using a water jet cutting machine. First, the coupons were ground into a rectangular shape, and then using a v-notch grinding machine and a diamond-coated blade, v-notches were cut in the central region of the rectangular coupons. The coupons were cut to size according to the ASTM D5379 test standard. The symmetrical notches were located at the central region in order to maintain a uniform shear distribution. It must be noted that the purpose of cutting the coupons directly from the cured panels with v-notches would be to prevent meeting the notch-to-notch tolerance requirements and also reduce the risk of matrix cracking and delamination. The coupons were then face ground to obtain uniform thickness.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Number of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
TABLE 8

NUMBER OF SPECIMENS OBTAINED FROM UNIDIRECTIONAL PANELS

<table>
<thead>
<tr>
<th>Panel 4</th>
<th>Panel 5</th>
<th>Cut orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>0°</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>90°</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>45°</td>
</tr>
</tbody>
</table>

6.2.1 Labeling of Specimens

The specimens obtained from the panels were labelled such that the test data and the analysis reports corresponding to each specimen were organized in the best possible way. Labeling the specimens helped in distinguishing the specimens, tracking them back to the panel from which they were obtained as well as comparing and better understanding the similarities among the specimen failures. Thus, conclusions from the test data and image correlation analysis reports were able to be drawn easily.

6.3 Iosipescu Shear Testing of Specimens

The 45° and –45° plies would be very strong under shear; hence, the failure of the specimens may not necessarily be at the notch region. To avoid this issue, all specimens were tabbed. After tabbing, the specimens were spray-painted with speckles on one of their surfaces. The quality of the speckles plays a very important role in shear testing analysis. Preferably, the size of the speckles is as small as possible so that millions of speckles can be formed on the specimen surface. Each speckle acts as a reference point in the digital image analysis. The specimens were then subjected to shear testing using the Iosipescu shear test fixture at a standard test speed of 0.01 in/min. The images of the specimens were captured at regular intervals to analyze testing with the digital image correlation method. Figures 16 to 19 show the experimental laboratory setup and testing of the specimens.
Figure 16: Experimental laboratory setup

Figure 17: Side view of camera and light source
Figure 18: Front view of camera and light source

Figure 19: Test fixture
CHAPTER 7
RESULTS AND DISCUSSIONS

Iosipescu testing was carried out on quasi-isotropic continuous-fiber and discontinuous-fiber specimens with two different chip sizes and unidirectional continuous-fiber and discontinuous (chopped) fiber specimens. Images were captured at regular intervals during testing, and load and displacement data were obtained. Shear strength values were then calculated at the ultimate failure load. Digital image correlation analysis was performed using the images captured during testing. From DIC analysis, each specimen’s strain field was obtained.

Testing was done on specimens with 0.75-inch square chips and 1-inch square chips, and continuous specimens at different orientation states to investigate the effect of chipping on the strength and stiffness values of the composites. The final results obtained in this investigation are explained here.

7.1  Quasi-Isotropic Panels

Quasi-isotropic specimens are considered to be very strong for shear stress due to the presence of 45° and −45° plies. The strength properties for these specimens are described in the following sections.

7.1.1  Shear Stress-Strain Response

The effects of chips on quasi-isotropic specimens are described here. The shear stress values were obtained from the Iosipescu testing of the specimens, and the shear strain field was obtained by conducting digital image correlation analysis. The shear stress-strain response for quasi-isotropic specimens is show in Figures 20 to 22.
Figure 20: Shear stress-strain response of continuous-fiber quasi-isotropic specimens

Figure 20, illustrates the shear stress-shear strain response of quasi-isotropic specimens with continuous-fibers arranged in [45/-45/0/90]s order. A total of three specimens were tested for shear. The plot depicts that the three specimens exhibit similar failure pattern confirming the successful prediction of failure pattern for quasi-isotropic composites.
Figure 21: Shear stress-strain response of quasi-isotropic specimens with 1 inch × 1 inch chips

The plot in Figure 21, illustrates the shear stress- shear strain response of quasi-isotropic specimens with 1 inch × 1 inch square chips. We see that the stress concentration of these specimens has decreased by 42% in comparison to continuous-fiber specimens.
Figure 22: Shear stress-strain response of quasi-isotropic specimens with 0.75 inch \( \times \) 0.75 inch chips

The plot in Figure 22, illustrates the shear stress- shear strain response of quasi-isotropic specimens with 0.75 inch \( \times \) 0.75 inch square chips. A total of three specimens were tested for shear. We see that the stress concentration of these specimens has further reduced by 55\% in comparison to the continuous-fiber specimens.
7.1.2 Strength Characterization

The calculated shear strength and shear modulus values are illustrated in Table 9. The shear strength values were calculated at failure load. The shear modulus values were obtained from the linear section in stress-strain curves.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Shear Strength (psi)</th>
<th>Shear Modulus (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous-Fiber [45/-45/0/90]24</td>
<td>48180.38 ± 250</td>
<td>2170.59 ± 57</td>
</tr>
<tr>
<td>Chopped-Fiber 0.75 in × 0.75 in [45/-45/0/90]24</td>
<td>21392.33 ± 420</td>
<td>2041.60 ± 37</td>
</tr>
<tr>
<td>Chopped-Fiber 1 in × 1 in [45/-45/0/90]24</td>
<td>27959.09 ± 1800</td>
<td>2083.19 ± 100</td>
</tr>
<tr>
<td>Predicted Values</td>
<td>50113.338 (Tsai-Hill)</td>
<td>51108.232 (Tsai-Wu)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2350.336</td>
</tr>
</tbody>
</table>

TABLE 9

STRENGTH CHARACTERIZATION OF QUASI-ISOTROPIC SPECIMENS

With the quasi-isotropic specimens, it can be seen that as the fiber continuity is reduced to 1-inch square chips and 0.75 inch square chips, the shear strength results are reduced by 42% and 55%, respectively, while the decrease in shear modulus is observed to be just 4% and 6%, respectively. It can be concluded that specimens with larger chip size exhibit larger strength values than specimens with smaller chip size, while there is little effect on modulus values.

The fact that the strength properties of continuous-fiber specimens are within 4% and 6% of the values predicted using Tsai-Hill and Tsai-Wu criterion respectively, indicates important factors that the testing and analysis were done in accordance to the standard test methods, and that the shear strength of continuous-fiber composites for quasi-isotropic configuration was successfully confirmed.
7.2 Unidirectional Panels

Unidirectional panels are considered to be the basis for determining the strength properties of composites. The variation or deviation in the strength properties based on the change in laminate configuration can be compared easily with results obtained for the unidirectional specimens.

7.2.1 Shear Stress-Strain Response of Continuous-Fiber Unidirectional Panel

The variations in strength properties obtained by testing unidirectional specimens with continuous fibers and 0.75-inch square chips and 1-inch square chips are shown here. The stress-strain responses are plotted in Figures 23, 24, and 25.

Figure 23: Shear stress-strain response of continuous-fiber $[0]_{24}$ specimens
The plot in Figure 23 illustrates the shear stress-shear strain response of continuous-fiber unidirectional [0]_{24} specimens subjected to shear testing. We see that all the three specimens exhibit similar failure pattern, confirming the successful prediction of failure pattern for 0° composites.

![Figure 24: Shear stress-strain response of continuous-fiber [45]_{24} specimens](image-url)
The plot in Figure 24 illustrates the shear stress-shear strain response of continuous-fiber unidirectional [45]_{24} specimens subjected to shear testing. We see that all the three specimens exhibit similar failure pattern, confirming the successful prediction of failure pattern for 45° composites.

Figure 25: Shear stress-strain response of continuous-fiber [90]_{24} specimens
The plot in Figure 25 illustrates the shear stress-shear strain response of continuous-fiber unidirectional \([90]_{24}\) specimens subjected to shear testing. The failure pattern of these specimens could not be predicted due to the occurrence of twisting.

7.2.2 Shear Stress-Strain Response of Discontinuous-Fiber Unidirectional Panel

Similar to continuous-fiber specimens, the stress-strain values for discontinuous (chopped)-fiber specimens obtained from testing and DIC analysis are illustrated in Figures 26, 27, and 28.

![Diagram](image.png)

Figure 26: Shear stress-strain response of chopped-fiber \([0]_{24}\) specimens
The plot in Figure 26 illustrates the shear stress-shear strain response of unidirectional [0]_{24} chopped-fiber specimens subjected to shear testing. These specimens consist of both 1 inch square chips and 0.75 inch square chips. We see from the plot that the shear strength has reduced by 12% when compared to the shear strength of continuous fiber [0]_{24} specimens.

Figure 27: Shear stress-strain response of chopped-fiber [45]_{24} specimens.

The plot in Figure 27 illustrates the shear stress-shear strain response of unidirectional [45]_{24} chopped-fiber specimens subjected to shear testing. These specimens consist of both 1 inch
square chips and 0.75 inch square chips. We see from the plot that the shear strength has reduced by 2% when compared to the shear strength of continuous-fiber [45] specimens.

The plot in Figure 28 illustrates the shear stress-shear strain response of unidirectional [90] chopped-fiber specimens subjected to shear testing. These specimens consist of both 1 inch square chips and 0.75 inch square chips.

Figure 28: Shear stress-strain response of chopped-fiber [90] specimens

The plot in Figure 28 illustrates the shear stress-shear strain response of unidirectional [90] chopped-fiber specimens subjected to shear testing. These specimens consist of both 1 inch square chips and 0.75 inch square chips.
7.2.3 Strength Characterization

Similar to the quasi-isotropic specimens, the strength properties of unidirectional specimens are illustrated in Tables 10, 11, and 12. The shear strength values were calculated at failure load. The modulus values were obtained from the linear section in stress-strain curves.

**TABLE 10**

STRENGTH CHARACTERIZATION OF [0]$_{24}$ SPECIMENS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Shear Strength (psi)</th>
<th>Shear Modulus (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous-Fiber [0]$_{24}$</td>
<td>16862.056 ± 79</td>
<td>936.682 ± 6</td>
</tr>
<tr>
<td>Chopped-Fiber [0]$_{24}$</td>
<td>14709.027 ± 440</td>
<td>912.063 ± 25</td>
</tr>
<tr>
<td>Predicted Values</td>
<td>17245.58</td>
<td>955.42</td>
</tr>
</tbody>
</table>

**TABLE 11**

STRENGTH CHARACTERIZATION OF [45]$_{24}$ SPECIMENS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Shear Strength (psi)</th>
<th>Shear Modulus (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous-Fiber [45]$_{24}$</td>
<td>9720.696 ± 360</td>
<td>1141.348 ± 4.9</td>
</tr>
<tr>
<td>Chopped-Fiber [45]$_{24}$</td>
<td>9513.940 ± 710</td>
<td>1105.715 ± 27</td>
</tr>
<tr>
<td>Predicted Values</td>
<td>12042.735 (Tsai-Hill)</td>
<td>1222.378</td>
</tr>
<tr>
<td></td>
<td>12120.356 (Tsai-Wu)</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 12**

STRENGTH CHARACTERIZATION OF [90]$_{24}$ SPECIMENS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Shear Strength (psi)</th>
<th>Shear Modulus (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous-Fiber [90]$_{24}$</td>
<td>9671.443 ± 3900*</td>
<td>1026.592 ± 2.7</td>
</tr>
<tr>
<td>Chopped-Fiber [90]$_{24}$</td>
<td>12072.829 ± 760</td>
<td>1032.065 ± 58</td>
</tr>
<tr>
<td>Predicted Values</td>
<td>12305.965 (Tsai-Hill &amp; Tsai-Wu)</td>
<td>1035.52</td>
</tr>
</tbody>
</table>
The asterisk (*) in Table 12 indicates that the test conducted with continuous-fiber [90]24 specimens was incomplete. This value was obtained by testing just two specimens, unlike the others. Also, the thickness of the unidirectional continuous-fiber specimens was significantly less than the chopped-fiber specimens. This might be due to face grinding of the specimens to obtain uniform thickness throughout the specimen geometry and occurrence of twisting while testing the specimens. Hence, the result varies from published and expected values.

It can be seen that in [0]24 specimens, the decrease in shear strength and modulus values from continuous-fiber to chopped-fiber specimens is 12.76% and 2.6%, respectively. Thus, it can be said that for specimens with 0° configuration, continuity affected the strength values more expressively than it affected the modulus values. Although there is a difference in the strength values, it can be seen that this is not large. The strength and modulus values of the continuous-fiber specimens are within 2.2% and 1.9%, respectively, of predicted values, indicating that the strength property of continuous-fiber specimens has been successfully confirmed.

In the [45]24 specimens, it can be seen that the decrease in shear strength and modulus values from continuous-fiber to chopped-fiber specimens is just 2.01% and 3.12%, respectively. Thus, there is an insignificant impact on the specimens due to the change in fiber continuity, thereby indicating that the 45° configuration remains strong, even under the discontinuous-fiber condition. The shear strength values of the continuous specimens are within 15% of predicted values, confirming that the prediction was successful.

A significant difference in the thickness of the specimens and the occurrence of twisting dramatically affected the strength properties of continuous-fiber unidirectional [90]24 specimens. It can be seen that as the continuity was reduced from the continuous-fiber to the chopped-fiber specimens, instead of reduced values of shear strength and modulus, greater values were obtained.
This was not intuitively expected. Therefore, the prediction of strength properties for 90° specimens was not successful.

7.3 Conclusions

From the above plots, it was observed that the shapes of the stress-strain curves of specimens with the same orientation exhibit a similar failure pattern, indicating that the failure pattern for these configurations was successfully predicted. The shear strength and shear modulus values were highly influenced by the specimen thickness, on which the strength and elastic properties strongly rely, thus implying that the distinctive nature of the specimens may demand requirements for standardized measurements or devices for successful production of discontinuous-fiber composites. Hence, the accuracy of overall averaging of the properties may most likely occur only for thicker specimens.

The occurrence of twisting during loading significantly affected the strength results, especially the calculation of stress and modulus. Twisting may occur for various reasons, such as out-of-plane tolerance fixture, improper installation of specimens in the fixture, usage of unstable and thin specimens, specimen configuration with an extremely low tolerance to twist, poor specimen preparation, etc.

Specimen preparation/face grinding can penetrate into the matrix system of the fiber-reinforced composites, thereby causing damage to the reinforcing fibers and resulting in improper failure of specimens. Thus, the surface preparation process of the reinforcing fibers should be done very carefully.

For specimens with 0.75-inch square chips and 1-inch square chips, determining the effect of aspect ratio on the strength properties would be much more valuable at this point. Plotting a strength versus fiber length (as shown in figure 29) would provide a clear explanation of how chips
behave when subjected to loading, thereby detailing the expected impact of chipping in real-world structures.

![Figure 29: Shear Strength vs Fiber Length](image)

An appreciable reduction in strength properties was observed as the continuity of fibers was reduced, which was intuitively expected. Factors that greatly contribute to the reduction of strength properties are the length of the fibers subjected to load, the reduced ability of short fibers to overcome all load, and the possible formation of gaps while manufacturing, which is commonly caused by the movement of chips during the curing process.

In the present study, the strength knock-down factor as an effect of chipping on the strength properties was observed to be greater as chip size was reduced. It was observed that specimens with a larger chip size exhibited greater strength values than specimens with a smaller chip size. Hence, the size of the chip should be chosen relative to the size of the composite structure such
that high strength values are obtained. The impact of chipping on the modulus and stiffness values was not too high. An average of 2–3% reduction was seen.

The combination of various-sized chips, as shown in the unidirectional chopped specimens in this study, indicated a diminishing effect on the strength knock-down factor. There was less than 5% deviation in strength results between the continuous-fiber specimens and chopped-fiber specimens, indicating that a combination of chips with varying size provide greater ultimate strength, which is close to that observed in continuous-fiber specimens.

Other studies deal with tensile testing, compressive testing, flexural testing, and impact testing of chopped-fiber composites. All of these tests have been conducted on randomly oriented discontinuous-fiber composites, similar to that for quasi-isotropic specimens. The strength knock-down factor in compression, tensile and flexural test studies conducted by Feraboli [2], range from 20% in compression, 30% in tension to 43% - 50% in flexure which is in great agreement with the strength knock-down factor in shear observed in the present study.

Tsai-Hill and Tsai-Wu theories are the most common methods used to predict the failure properties of composites. The values predicted using Tsai-Wu are either greater or lower than Tsai-Hill depending on the nature of testing and the orientation of the composite. Both values are the same for composites of unidirectional (0º and 90º) nature. In shear they are assumed to be almost similar for angled laminates with Tsai-Wu values being slightly greater than Tsai-Hill values due to the addition of linear terms. However, Tsai-Hill criterion proves to be the best method to predict the ultimate values under shear due to the isolation of the stress terms in the equation, making it easier to use for predicting values under pure shear. While, Tsai-Wu proves to be best to distinguish between the tensile and compressive strengths of lamina.
The strength properties of continuous-fiber specimens were within 5% of the predicted values for all configurations. This indicates that testing and analysis were done in accordance to standard test methods and that the strength properties of continuous-fiber specimens were successfully confirmed.
CHAPTER 8
FUTURE WORK

From the conclusions drawn in Chapter 7, the following are recommendations for future work in order to extend this current study.

Different arrangements of chips in the layup should be studied. The present study focuses on only one type of arrangement, which is offsetting the chips at one-half distance from the edge of preceding and succeeding plies. Future studies that reduce the distance of offset to either one quarter or less should be conducted. The reduction of offset distance will arrange the chips closer to each other, thereby reducing the formation of large gaps, which are either resin-rich areas (areas where the concentration of resin system is more than the fiber concentration) or physical voids between the chips after curing. On the other hand, with greater offset distance, the resulting composite obtained after curing might have large voids due to lack of material or large resin-rich areas. This may result in determining the effect of chipping on highly resin-rich composites. Studying various offsetting measures will produce data that briefly describe the effect of the chip arrangement on strength properties, in order to understand the elastic behavior and strength properties of chips.

Studying different chip sizes in the scientific research would be valuable in understanding the effect of fiber aspect ratio on the strength properties of composites. The ultimate goal at any point in time is to determine a composite model with easily obtained and described behavioral characteristics. Conducting many more studies will help form a database of models and their behavior. This database will be helpful for industrialists to choose a model based on property requirements.
In the present study, the strength properties found as an effect of fiber length could possibly be a function of the volume fraction, and further study should be conducted to identify whether the results obtained were influenced by this. Studying various materials with different material properties and volume fractions would provide an understanding of this factor.

Industries follow several manufacturing processes, such as injection molding, compression molding, transfer molding, etc., and each process produces composites with different fiber orientations. In the present study, composite panels were manufactured with fibers arranged in quasi-isotropic and unidirectional 0°, 90°, and 45° orientations, which are frequently obtained in the most commonly used manufacturing processes. These orientations are very strong in shear flow. Thus, accentuating the effect of chipping on these orientation states will be helpful for general use in other manufacturing methods.

Using a suitable strain gage while testing the coupons could be effective in comparing the results obtained from the digital image correlation process. Image analysis is purely based on the images captured during testing, which means that image quality and pixel size affect the results in different ways. A strain gage is needed to ensure that results obtained from the digital image analysis properly correspond to actual data.
REFERENCES
LIST OF REFERENCES


LIST OF REFERENCES (continued)


