MINIMAL LEAD TIME QUOTATION UNDER SERVICE LEVEL CONSTRAINT

A Dissertation by

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ABSTRACT

In today's competitive market, companies that offer a high quality service at a reasonable cost, survive the competition. A major factor contributing to high service quality is the ability to meet customers' demand with a short lead time. Short lead time and on time delivery are two conflicting objectives. A reasonable balance between these objectives is necessary. The published research works on due date quotation make restrictive assumptions regarding the production process, considering only the production lead time, ignoring suppliers lead times, and/or rejecting some orders. No model can be found in the literature that considers all of these characteristics simultaneously. The proposed research considers a system to which orders arrive over time. Every order has a desired range of delivery date as defined by the customer. We consider order cost of delay that represents the cost of quoting due dates greater than requested. A two phase model is considered that assumes that production is constrained by a supply process. At arrival, each order is to be assigned a due date based on the status of the system. The system status is affected by internal factors including the level of WIP as well as external factors such as the supplier process. The due dates are to be assigned so that company's objectives, defined by customer satisfaction and retention are achieved. It is desired to accept all orders, and to maximize on-time delivery of orders without placing any restriction on the behavior of the production process. The comparative results indicate that the proposed procedure is effective in quoting minimum lead times to achieve a given level of customer satisfaction.

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LIST OF ABBREVIATIONS / NOMENCLATURE / NOTATIONS

NOTATIONS

n_i	Number of jobs in system at i^{th} job arrival
$f(n_i)$	Mean flow time when ni jobs are in the system
$sf(n_i)$	Safety lead time to ensure achieving service level for n_i jobs in the system
L_i	Lead time for i^{th} job arriving job
T_o	Raw process time (i.e. average time for one job to traverse empty system)
W_o	Critical WIP
r_b	Bottleneck Rate
p_1, p_2	Parameters that must be estimated
q_1, q_2	Parameters that must be estimated
n_o	Number of jobs in the system that the curve of function changes to straight line
α	Target service level
Z_α	α-percentile of the standard normal distribution
$\sigma(n_i)$	Standard deviation of flow time when n_i jobs in the system
K	Adjustment factor (in manufacturing model)
λ	Processing rate of the machine
f(.)	Density function
F(.)	Distribution function
Δd	Difference between quoted due date and the requested
d_u	Upper date requested by customer
d_l	Early shipment in window requested by customer

C_h	Cost of holding cost
C_r	Cost of reimbursement
C_t	Cost of tardiness
$L_{integrated}$	Integrated lead time
$L_s(i)$	Lead time of supply process for each i^{th} component of an order
η	Integrated adjustment factor
C	Total cost
K	Purchase order
L_s	Supply lead time (total)
$F_{M(k)}(.)$	Distribution function of lead time of purchase order
$t(k)$	Time of issue an order
$F_{Ls}(.)$	Distribution function of ready time
d_q	Quoted due date
D_q	Optimum quoted due date
L_m	Manufacturing lead time in integrated due date quoting model

ABBREVIATIONS

ANN	Artificial Neural Network
CON	Constant Flow Allowance
DDA	Due Date Assignment
EDD	Earliest Due Date
FIFO	First In First Out
JIQ	Jobs in Queue
JIS	Jobs in System
LTQ	Lead Time Quotation
M/M/1	Single Class Queuing System
M/G/1	Multi Class Queuing System
NOP	Number of Operations
NP-hard	Non-deterministic Polynomial-time hard
ORR	Order Review/Release
PPW	Processing Plus Wait
RDM	Random Allowance
SLK	Slack Flow Allowance
SPTA	Shortest Processing Time Among Available Jobs
TWK	Total Work Flow Allowance
WIP	Work In Process

CHAPTER 1

INTRODUCTION

In today's competitive market, companies with high level of customer satisfaction stay ahead of the competition. An important measure of customer satisfaction is the degree to which the promised services are delivered within the quoted lead time (also referred to as service level). Short lead times and on time delivery are fundamentally in conflict with each other. Short lead times are desired but may result in late delivery of services while longer lead times, though not desirable, will ensure on time delivery. A reasonable balance between these two is necessary.

The increasing importance of setting achievable lead times (due dates) has resulted in the development of various models for addressing this problem. Cheng and Gupta (1989) in a survey divided the methods on setting due dates into exogenous and endogenous methods. Exogenous due date methods, also referred to as externally imposed due dates, do not consider any information regarding the arriving job, jobs already in the system, future job arrivals, shop congestion or shop structure when setting due dates. Due dates are typically set by order-entry, marketing, or other non-production departments. A common example of an exogenous due date setting method is a uniform lead time quoted as part of a company's product catalog. Endogenous due date methods, also known as internally set due dates, do take into account the information about the arriving job, shop congestion and flow time estimates. Generally, these due dates are assigned by the production department. A copy shop, an auto dealer, a mail order Computer Company, or any firm that quotes differential lead times on the basis of work backlog, makes use of endogenous due dates. Most modeling approaches in the literature address setting endogenous due dates.

The most common due date assignment methods in the literature are Constant flow allowance (CON) that assign a constant due date for all jobs. Slack (SLK) which a flow allowance that reflects equal waiting time or slack assigned to the job processing time. Total Work (TWK) due date assignment, due dates are based on total work content and are equal to a multiple of the job processing times. The Processing plus Wait (PPW) due date assignment combines SLK and TWK due date assignment rules. Number of Operations (NOP) model, due dates are determined on the basis of the number of operations to be performed on each job. Random allowance (RDM) method, each job receive a random due date, usually following a probability distribution. In the Jobs in Queue (JIQ) and in the Jobs in System (JIS) methods, due dates are determined based on, respectively, the current job queue length and the number of jobs in the system (Gorden et al., 2002). Among all the due date assignment methods, CON and RDM are categorized under exogenous method and TWK, SLK, NOP, JIQ, JIS and PPW are under endogenous method (Chand, et al. 1992).

Extensive research has been carried out in the area of due date assignment. The objective in these works varies depending on the point of view from which the problem is being approached. In this regard two general points of view exist: Point of view of the shop manager, and point of view of the customer placing the order (Seidmann et al. (1981) and Panwalker et al. (1982)). When the shop viewpoint is taken, the objective is usually related to the minimization of one or more cost functions associated with the state of the shop (work in process, machine load, etc.), while it is more likely related to the due date from the customer viewpoint (Gorden et al., 2002).

Review of the literature indicates that the current research in setting due dates suffer from the following problems: (1) they do not necessarily accept all jobs. For example, the manager may decide whether or not to accept an order, depending on the possibility and profitability of processing the job (Keskinocak, 1997); (2) they make restrictive assumptions about the production process and require particular distribution of underlying production system; (3) they only consider the manufacturing process ignoring the uncertain supply process.

A two phase production model is assumed; manufacturing and supply process. It is assumed that the behavior of the production system is mainly affected by the level of WIP as a major internal factor as well as uncertain supply process. Every order has a desired range of delivery date as defined by the customer. A simple model is developed to compute the minimal lead time quotation for a single demand independent of other demands. We show that model can be extended with multiple simultaneous orders. This research presents a methodology for endogenous due dates quoting under a target service level constraint. Flow time for each arriving job is predicted from the historical data available in the system based on the regression technique and lead time is adjusted until the service level is achieved. All orders are accepted and on-time delivery is maximized (i.e., late delivery as well as early delivery is minimized) of orders. There is no assumption about the distribution of process times. We introduce cost of delay to represent the difference between the quoted due date and the requested demand order. The presented simple integrate model considering production mode is applicable to a wide range of real-world systems where safety lead times are necessary for the process.

The reminder of this research is organized as follows: Chapter 2 provides state of the art review literature and also presents the lead time quotation for manufacturing process, Chapter 3 discusses the lead time quotation for supply process, Chapter 4 show an integrated model considering both manufacturing and supply process and Chapter 5 summarizes the conclusions and future work.

CHAPTER 2

LITERATURE REVIEW AND LEAD TIME QUOTATION CONSIDERING MANUFACTURING PROCESS

Most of the literature on scheduling problems focuses on sequencing decisions only (see Panwalkar, et al., 1982), assuming that the due dates are exogenously assigned; that is due dates are usually treated as "given" information which takes the form of input to a scheduling problem. In actual practice, however, the due date can be a decision variable within the boundary of the scheduling problem. Denote the due date d_i, lateness l_i (= C_i-d_i), tardiness t_i (= max {0, C_i-d_i}), and earliness e_i (= max {0, d_i-C_i}) for job *i*, where c_i is the completion time. Minimizing (weighted) tardiness, numbers of tardy jobs, lateness, average flow times and completion times are among common objectives.

Researchers have focused on developing a variety of scheduling models with due date decision variables. The significance of assigning accurate due dates to jobs in a production system is well recognized by academic researchers and practicing managers. Several models of assigning due dates are considered in the literature; some of them are mentioned below. The simplest is the model in which all jobs have a common due date (i.e., d_j=d, j=1… n). Such a model corresponds, for instance, to an assembly system in which the components of the product should be ready at the same time, or to a shop where several jobs constitute a single customer's order. More generally, this model corresponds to any system in which, for some reasons, appointment, technical constraints, etc., several tasks should be completed at the same time. This method of due date assignment is known in the scheduling literature as Constant flow allowance (CON) model. It is also called the common due date model.

In the Slack (SLK) due date assignment, a flow allowance that reflects equal waiting time or slack, denoted by q, is assigned to the jobs. Thus, $d_j = p_j + q$. In this model, the goal is to find an optimal value of the slack q with respect to the criterion to be optimized.

In the Total Work (TWK) due date assignment, due dates are based on total work content and are equal to a multiple of the job processing times, i.e., $d_j = kp_j$. The optimal due date assignment consists of finding the value of the common multiplier k.

The Processing Plus Wait (PPW) due date assignment combines SLK and TWK due date assignment rules. In this model, $d_j = kp_j + q$. The optimal due date assignment consists of finding the optimal values of k and q.

In the Number of Operations (NOP) model, due dates are determined on the basis of the number of operations n_j to be performed on job j: $d_j = kn_j$. The goal, in this case, is to find a value of k which yields optimal value of the objective function.

In the Random allowance (RDM) method, each job receive a random due date, usually following a probability distribution. In the Jobs in Queue (JIQ) and in the Jobs in System (JIS) methods, due dates are determined based on, respectively, the current job queue length and the number of jobs in the system (Gorden et al., 2002).

Among above-mentioned methods, CON and RDM are categorized under exogenous method which entirely ignores any information about the arriving job, jobs already in the system, future jobs, or the structure of the shop itself. TWK, SLK, NOP, JIQ, JIS and PPW are under endogenous method that the due dates are set internally by the scheduler as each job arrives on the basis of job characteristics, shop status information and an estimate of the flow time (Chand, et al. 1992).

Some of the due date assignment models assume that orders will be placed by customers regardless of the length of the quoted lead time (see Cheng, et al. 1989 for a survey)

(Keskinocak, et al., 1997). The objective in this case is to minimize average quoted lead time subject to some constraints on the number tardy jobs. Most of the papers based on these models have been simulation based. For example, Eilon and Chowdhury (1976), Weeks (1979), Miyazaki (1981), Baker and Bertrand (1981), and Bertrand (1983) consider various due date assignment and sequencing policies. In general, the results indicate that quoting due dates based on estimates of shop congestion (like current queue lengths, number of jobs and waiting time in the system) and the estimated duration of the processing time of jobs leads to significant lower average due dates.

Wein (1991) studied a multi-class M/G/1 queuing system for the problem of simultaneous due date setting and priority sequencing.He formulated two problems as minimizing the weighted average lead time of jobs subject to the constraints of the maximum fraction of tardy jobs and the maximum average tardiness.

Duenyas and Hopp (1995) also considered problem of quoting lead times, using a queuing model, and provided effective heuristics under various problem characteristics such as infinite, restricted and finite capacity. In their model, there is a single class of customers; the net revenue per customer is constant, customers have the same preferences for lead times, and the arrival and processing times follow the same distribution. They showed the optimality of different forms according to different sequences. Duenyas (1995) extended their results to multiple customer classes, with different net revenues and lead time preferences.

Eilon and Chowdhury (1976) considered several due-date assignment procedures with both a First-In-First-Out sequencing rule, and sequencing rules which consider the expected consequence of job processing time on waiting times, earliness, and tardiness. Their simulation results support the expected conclusion that setting due dates based on job content and simple estimates of shop congestion leads to better shop performance than setting them based solely on job content.

Weeks (1979) (cited in Cheng et al., 1989) concluded that due dates assigned based on expected job flow time (a function of the required job processing time and expected job delay time) and shop congestion information may provide more attainable due dates than rules based only on job characteristics. In his study, mean lateness, mean earliness, and mean missed due dates are used as measures of shop performance. Miyazaki (1981) with some experiments compared due-date assignment policies based on shop conditions combined with different dispatching rules for reducing job tardiness. Their comparison shows that the efficiency of the new proposed dispatching rules can be better than conventional scheduling systems.

Bertrand (1983) investigated the performance with respect to controlling mean lateness and reducing the standard deviation of lateness by taking advantage of the proposed due-date assignment policy and sequencing rule; both of them based on a time-phased representation of workload and the machine capacity in the shop. In this study, two parameters, a minimum allowance for waiting and a maximum fraction of the available capacity allowed for loading, are used, and the effectiveness of the proposed due-date setting strategy is shown.

Baker and Bertrand (1981) compared three parametric due-date setting strategies particularly for a single-machine model with preemption and a fixed set of jobs. The three rules set a job's due date equal to its arrival time plus a constant, its arrival time plus expected processing time, and its arrival time plus a constant multiplied by expected processing time. The authors combined SPTA (Shortest Processing Time among Available jobs) and EDD (Earliest Due Date) as the sequencing rules for a static model and dynamic model respectively, to solve the problem of minimizing average due date subject to no jobs being tardy under the assumption of known processing times and arrival times in advance. Their experimental results showed that the first rule is dominated by the other two.

Considering a common due date for all jobs is not always practically appropriate. In fact, a group of several jobs might have the same due date, but different groups can be due at different times. Chand and Chhajed (1992) generalized this problem for the simultaneous determination of optimal due dates and optimal sequence for n-job single machine problem with multiple due dates. They assumed penalty function which is a linear function of the due date and the earliness/tardiness for the job. All jobs are assumed to be available at time 0. The objective is to minimize the total penalty for all jobs. The number of distinct due dates, m, to be assigned to the jobs is assumed to be pre-specified and known. The results introduced above hold for the common due date setting situations, $m=1$. If each job has a different due date assigned, $m = n$, Panwalkar, Seidmann and Smith (1982) showed that the SPTA rule gives an optimal sequence. The result of Chand and Chhajed (1992) focuses on a generalized situation in which m is allowed to take any value in the set $\{1, 2,..., n\}$.They presented an algorithm to determine the optimal due dates and sequence in $O(n \ln n)$ time.

Many formulations of the customer lead time problem found in the literature have attempted to quote the shortest due date; i.e. minimize lead time. These formulations typically model a random manufacturing lead time and include a service level or tardiness constraint (see Cheng and Gupta (1991) for a survey). For example, Bookbinder and Noor (1985) solved the due date setting problem subject to a service level constraint for a single server with distinct exponentially distributed production times. Similarly work of Wein (1991) or Spearman and Zhang (1999) that minimized lead time, but considered the constraints of service level and average job tardiness.

Hsu and Sha (2004) described a method of using artificial neural network (ANN) to predict lead time. They presented simulation and statistical analysis along with an ANN rule for due date assignment. Their method shows a better sensitivity and variance with a smaller tardiness rate than the other rules. They presented some suggestions for several combinations of review/release (ORR) and dispatching rules.

Gordon et al., (2002) provide a review of the results in due date assignment. Their study provides a unified framework of the due date assignment and scheduling decisions for the static production settings. They show the most challenging and open problems where algorithms are not proposed or, for NP-hard problems, where it is not clear whether the problem is NP-hard, for the single machine.

Hopp and Sturgis (2000) propose a due date policy for problem of lead time assignment under a service level constraint. Their policy is based on control chart method to adjust model parameters over time. They break the lead time quote into two stages, the flow time mean and a safety lead time. They develop quadratic functions to estimate the mean and standard deviation of flow time based on number of jobs in the system. They compare their method to JIS rule and show that their method has a better performance to JIS rule under service level constraint.

In this research, we use the work of Hopp and Sturgis (2000) to develop approximation functions for flow time but different polynomial functions for flow time standard deviation. Similar to their work we propose a safety lead time and we consider an adjustment factor attached to it. We develop a procedure to change this factor to achieve the target service level. They have used control chart to adjust the model parameters. Similarly we compare our results to the analytical model they have used and study the model performance due to changes in the system. The model in this research is not a simulation based but a regression based functions. In addition to their work, we study the effect of service level on the safety allowance and show the flexibility of our model based on the number of jobs in the system and different service levels.

Panwalkar, Seidmann et al. (1982) were the first who considered the optimal due date assignment problems together with scheduling decisions and tackled the problem analytically. They noted that job scheduling can be approached either from the viewpoint of the shop manager or from the viewpoint of the customer placing the job orders. When the shop viewpoint is taken, the objective of the scheduling is usually related to the minimization of one or more cost functions associated with the state of the shop (work-in-process, machine load, etc.), while it is more likely related to the due date from the customer viewpoint (Gorden et al., 2002).

In some other models, not all possible jobs are processed. In some of these models, a maximum lead time is associated with each customer. The firm decides whether or not to accept a specific offer, and if the offer is accepted, a perfectly reliable lead time (which must be less than the customer's maximum lead time) is quoted. In other words, an accepted job has to start processing within its quoted lead time. The objective typically involves a revenue function which decreases with increasing quoted lead times. This is known as the Lead Time Quotation (LTQ) problem (see Keskinocak et al., 1997 and the references there). It seems that some effective algorithms exist for special cases of this problem, but not for the most general cases.

Fry et al. (1989) developed due date assignment rules based on job and shop characteristics using the factor that has the most influence on the deviation of planned lead times from actual flow time. Adam et al. (1993) used Little's law to obtain coefficients used in the traditional due date assignment procedures of constant allowance, total work content, and critical path processing time. Philipoom et al. (1994) assigned due dates based on nonlinear independent variable regression models. They concluded that these regression models outperform linear rules in the literature (Cheng and Gupta, 1989; Raman and Talbot, 1993; Udo, 1990).

Wein and Chevalier (1992) developed a queuing network to assign due dates, to release jobs to the shop floor from the backlog and to sequence jobs while minimizing work-in process. They also imposed an upper constraint on their estimates of lead times that is proportional to the number of tardy jobs. Dellaert (1991) looked at finding the best combination of rules to set due dates for a single product with limited setup costs. In particular, he examined rules to generate due dates acceptable to most customers while limiting storage and penalty costs resulting from due date deviations. His method is based on a queuing model with Poisson arrivals and exponential distributions for service times and setup times. He addressed simultaneously the problem of setting lead times and determining the minimum number of jobs between setups.

Bookbinder and Noor (1985) solved the due date setting problem subject to a service level constraint for a single server with distinct exponentially distributed production times. Enns (1994) set due dates which consist of flow time estimates and forecast errors in a dynamic forecasting model.

Duenyas and Hopp (1995) outlined a modeling framework for quoting customer lead times when the demand is sensitive to the lead time quote. Actually, they considered the impact of the quoted due dates on the customer's decisions to place an order in addition to due date quotation and scheduling decisions. They modeled the problem of quoting customer lead time under a variety of modeling assumptions incorporating the status of the manufacturing facility.

In the following, we review the due date setting models considering simultaneous supply and production processes.

Wang (2007) considered the general, no-wait and no-idle flow shop scheduling problems with deteriorating jobs, respectively. The normal processing time proportional to its decreasing rate is assumed and some dominant relationships between machines can be satisfied. It is shown that for the problems to minimize the make span or the weighted sum of completion time, polynomial algorithms still exist, although these problems are more complicated than the classical ones. When the objective is to minimize the maximum lateness, the solutions of a classical version may not hold.

Song, Hicks and Earl (2002) considered an uncertain manufacturing and assembly process times in a multi-stage assembly environment and presented a method for due date assignment. Their method decomposes the complex product structure into two sub systems. They developed exact and approximate completion time distributions for both sub stages. Product due dates are assigned based on minimizing earliness and tardiness costs under a service target. They verified the results by simulation.

Cheng, Chen and Shakhlevich (2002) considered the problem of scheduling a set of non-simultaneously available jobs on one machine. Each job has a ready time only at or after which the job can be processed. All the jobs have a common due date, which needs to be determined. The problem was to determine a due date and a schedule so as to minimize a total penalty depending on the earliness, tardiness and due date. They gave an efficient algorithm that finds an optimal due date and schedule when either the job sequence is predetermined or all jobs have the same processing time.

Cheng, Ding and Lin (2004) presented some scheduling problems that machine process time is dependent on its start time. They developed a framework to compare this class of scheduling problems to other classical models. They presented a complexity boundary for each problem, solution algorithm, heuristics and performance analysis.

Cheng and Kang (2004) studied a due date assignment and single machine scheduling with deteriorating jobs, that is jobs whose processing times are an increasing function of their start time like ingot preheating process in steel mills. They consider the case of a single machine and linear job- independent deterioration. They found an optimal combination of the due date and schedule so as to minimize the sum of due date, earliness and tardiness penalties.

Some other researcher considered the scheduling problems with due date window assignment. A due window is a time interval associated with a job. The concept of a due window is a generalization of the concept of the due date. In the due window scheduling model, no cost is incurred if a job is completed within its due window. Otherwise, an earliness or tardiness cost is incurred. Two types of due windows considered, hard and soft. In the hard due window case, a job completion before or after its due window, is prohibited. In contrast, the soft due window concepts assumes that earlier or later job completion is possible at an additional cost (Biskup and Feldmann (2005)). Scheduling problems with due window assignment can be used for mathematical modeling of product delivery dates negotiation between the manufacturer and the customers in a make-to-order manufacturing system (Wang et. al., 1998 and 1999). Biskup and Feldmann (2005) pointed out that small buffer stocks exist in most companies even in those which have adopted the just-in-time principle to adjust to a slightly earlier or later delivery of ordered goods. Lee (1991) observed due window scheduling problems in metal cutting companies and electronic companies. Koulamas (1997) discussed the applications of a due window model in scheduling chemical reactions and satellite photography and Yeung et al. (2004) in scheduling assembly lines. A bibliography of the results on scheduling problems, with various models of due date and due window assignment with earliness and tardiness penalties, is provided by Gordon et al. (2002). Most of the results on scheduling with fixed or assignable job due windows were obtained for a single machine processing environment. A combination of parallel machine scheduling with due window assignment was studied only by Kramer and Lee (1994), Mosheiov (2001), Mosheiov and Oron (2004). Kramer and Lee (1994) studied a special case of the constant due window model, in which the due window size is given. They proved the NP-hardness of this problem, presented a dynamic programming algorithm for the two-machine case and a heuristic for the general case. Mosheiov (2001), Mosheiov and Oron (2004) considered an extension of the problem, in which a cost for the due window starting time is included in the objective function. Mosheiov (2001) proposed a heuristic algorithm and a lower bound for this problem, while Mosheiov and Oron (2004) suggested a constant-time solution algorithm for the case of unit job processing times. Janiak and Marek (2001 and 2003), Janiak and Winczaszek (2003) and Janiak et al. (2007) studied problems of scheduling n jobs on m identical parallel machines, in which a due window has to be assigned to each job. If a job is completed within its due window, then it incurs no scheduling cost. Otherwise, it incurs earliness or tardiness cost. Two soft due window models are considered. In both models, the due window size is a decision variable common for all jobs. In the first model, called a constant due window, the due window starting time is a decision variable common for all jobs, and in the second, called a slack due window, the due window starting time is equal to the job processing time plus a decision variable common for all jobs. The objective is to find a job schedule as well as the size and location(s) of the due window(s) such that a weighted maximum or sum of costs associated with job earliness, job tardiness, and due window size is minimized. They established the properties of optimal solutions of these minmax and minsum problems. For a constant due window model, we prove that the minmax problem with arbitrary weights and the minsum problem with equal weights are polynomially equivalent to the classical parallel machine scheduling problem to minimize the makespan. They further show that the problems for a constant due window model and slack due window model with the same objective function are reversible in the sense that their optimal solutions are mirror images of each other. These results imply O(n) and O(n log n) time algorithms for the considered problems when m=1.

Most of the due date and customer lead time research have used production models that considered the manufacturing process. In some procurement lead time setting literature, models have focused on the supply process, but formulations have generally focused on inventory holding cost instead lead time. For example, Yano (1987) addressed the lead time setting problem in assembly systems with stochastic procurement and production lead times. Hope and Spearman (1993) also considered setting procurement lead times in assembly systems. Most of the literature has made use of steady state procurement lead time models. Song and Zipkin (1996) considered the state of the supply process through a Markovian model that updates lead times as the system changes for a single item system. There is not any literature considered the costs of delay quoting such a discount or break price to retain the customer and to out knowledge, no due date setting considered modeling these costs.

In the following sections, we describe the problem of lead time quotation considering manufacturing process. We describe some existing literature and the extension and modification we have added.

2.1 Problem Definition of Manufacturing Lead Time Quotation

Consider a system to which jobs arrive over time. The sequence of the operations is the same for each job, such as flow shop. Every order has a desired range of delivery date as defined by the customer. At arrival, each order is to be assigned a due date based on the status of the system. The system status is affected by internal factors including the level of work in process jobs (WIP) as well as external factors such as the supplier process. The due dates are to be assigned so that company's objectives defined by customer satisfaction and retention are achieved. It is desired to accept all orders, and to maximize on-time delivery (i.e., minimize late delivery as well as early delivery) of orders. There are no assumptions about distribution of the production process like processing time distribution. Based on the above discussion, the goal of this section is to implement the general endogenous due date assignment model based on work of Hopp and Sturgis (2000) for manufacturing process that requires no assumptions about the distribution of the production process under desired service level constraint. The objective is to quote a due date for an incoming job such that a certain level of customer service is achieved. Service level, defined as the percentage of orders filled on time, is used as a measure of customer service.

This modified model will be used as an input to the proposed model in Chapter 3.

2.2 Model Development

The objective is to quote a due date for an incoming job such that a certain level of customer service is achieved. Service level is used as a measure of customer service. Moreover, only the production lead time is considered here. The general approach is to approximate the lead time at the time of arrival of a job and adjust that until the desired level of service is achieved. Lead time approximation can be done in a number of ways as follows.

a) If the arrival time and the flow time of each job in the system are known, a lead time can be stated and adjusted for each job until the service constraint is met. In other words, each lead time is a decision variable (control) that must be adjusted. This approach, however, is not very effective because in addition to not having the flow time of each job processed in advance, for each job, lead times must be adjusted. Therefore, there are simply too many variables (lead times) to consider and control for this approach to be effective and it is not practical.

b) Since flow time is a function of the number of jobs in the system, lead time can be calculated based on the number of work in process. Considering this fundamental structure, another example of how to calculate lead times is to maintain decision variables on WIP not on each job. Thus, the number of variables (lead times) to control is reduced to the number of jobs in the system. However, this lead time calculation approach is still ineffective because the number of variables (lead times) to adjust to meet the service level remains large, especially for large and highly congested systems such as real world industry and it is not practical.

c) An alternative approach to approximate lead time is to develop a function defining the relationship between lead time and work in process (WIP) for the system under study. Based on this approach, as a new job arrives based on the number of jobs in the system, one can use this function to approximate the lead time. The challenge in this method is to develop a general functional relationship between lead time and WIP for a given system. It is well known that developing a generic mathematical relationship that can be applied to all situations is very difficult if not impossible. Hence, we propose to use regression to approximate this relationship. This technique requires that historical date on the relationship between WIP and lead time (flow time) must exist. It is also assumed that the system behavior does not change radically over time hence allowing the utilization of regression. Given the historical data, regression is used to model the relationship between WIP and lead time. This model can then be used to approximate lead time based on number of jobs in the system.

The model discussed in (c) does not consider service level constraint. The traditional method to ensure certain level of confidence in lead time quotation has been to add a safety lead time. This method, however useful is not flexible. To allow for some flexibility and to achieve different levels of confidence and hence different levels of customer satisfaction, safety lead time will be added to the approximation function. An adjustment factor is attached to the safety lead time function. This adjustment factor allows one to adjust the lead time until an acceptable level of service level is achieved.

Hence, based on the above discussion, two functions will be introduced:

i) The first function is to estimate the flow time for each arriving job based on the WIP.

ii) The second function is to estimate the standard deviation of flow time for each job based on WIP. This function will be used in determining the safety lead time.

2.3 Model Details

This section describes the model details for manufacturing due date quotation under a service level constraint. First the relationship between manufacturing flow time and WIP in order to quote lead time is described in details. Quoting a reasonable manufacturing due date for a job to achieve a certain customer service level is critically dependent on how to describe job flow time. It is well known that flow time is a function of work in process (WIP). The more the WIP, the higher average flow time is. Hopp and Spearman (1996) showed this relationship for tandem lines by stating "Best" and "Worst" cases for systems with given values of bottleneck rate (rate of slowest server) and raw process time (average time for one job to traverse an empty system). They also stated a "Practical Worst" case that relates to a balanced, single machine station line with exponential processing times. Typical flow time curves lie between the Best case and Practical Worst Case, as shown in Figure 2.1.

Figure 2.1 implies that WIP and flow time and therefore WIP and lead time are strongly related. To more accurately predict lead time, it is useful to decompose lead time into an estimate of average flow time plus a safety lead time in order to have flexibility to assure achieving the target fill rate.

Therefore, we can write the lead time quotation formula as follows,

$$\text{lead time} = \text{average of flow time} + \text{safety lead time}$$

$$L_i = f(n_i) + sf(n_i) \qquad (2.1)$$

Where

n_i = number of jobs in system at i^{th} job arrival (including job i)

$f(n_i)$ = mean flow time when n_i jobs are in the system

$sf(n_i)$ = safety lead time to ensure achieving service level for n_i jobs in the system

L_i = lead time for i^{th} job arriving job

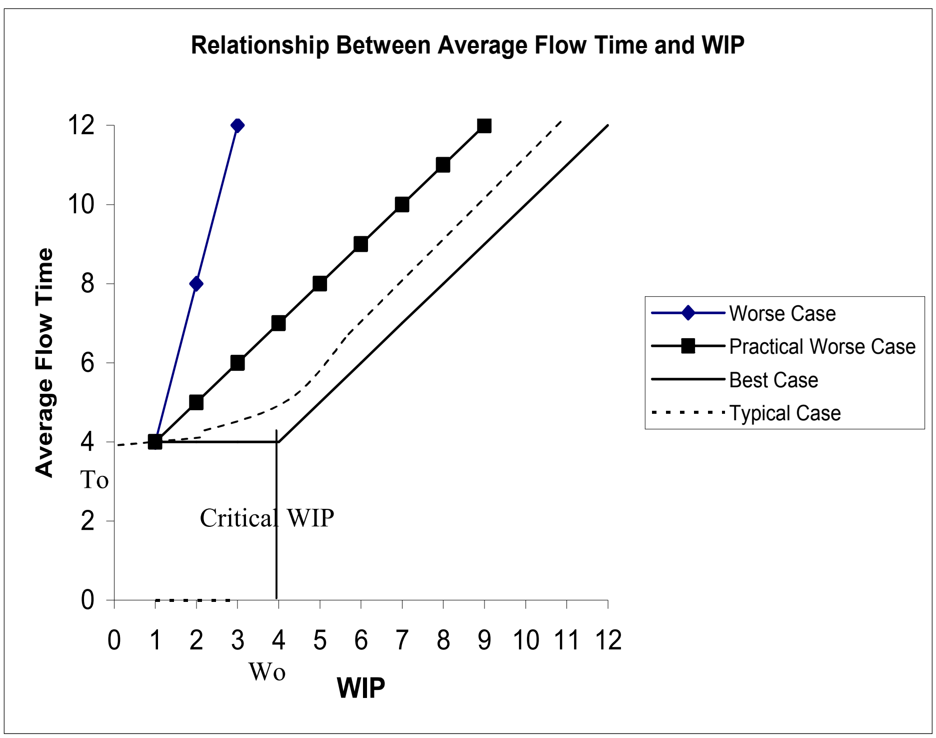


Figure 2.1: Relationship between average flow time and number of Jobs in the system (Hopp and Spearman, 1996)

Following sections will show how to derive safety lead time and also flow time estimation function to determine lead times for a given service level constraint. In order to do this, we use the work of Hopp and Sturgis (2000) that considers a due date policy for problem of lead time assignment under a service level constraint. They proposed a lead time quote into two stages, the mean flow time and a safety lead time. They developed quadratic functions to estimate the mean and standard deviation of flow time based on number of jobs in the system. We use similar approximation functions for flow time and different polynomial function for flow time standard deviation. Similar to their work we propose a safety lead time and we consider an adjustment factor attached to it. We develop a procedure to change this factor to achieve the target service level. Similar to their work, we compare our results to an analytical model and study the model performance due to changes in the system. Our model is not a simulation based but a regression based functions. We study the effect of service level on the safety allowance and show how our model is more flexible in safety lead time based on the number of jobs in the system and different service levels.

2.3.1 Flow Time Estimation Function

The flow time function is estimated by fitting a curve to the historical data showing the relationship between flow time and number of jobs in the system (see figure 1). We assume a two part function for the flow time approximation function. The first part is a quadratic polynomial is to estimate the lowest part of the curve in order to enable f (n_i) to shape the curved part of the average flow time function and the second part is a straight asymptote.

$$f(n_i) = \begin{cases} T_o + p_1(n-1) + p_2(n-1)^2 & for\ n \le n_o \\ \dfrac{T_o}{W_0} n & for\ n > n_o \end{cases} \tag{2.2}$$

Where

T_o = raw process time (i.e. average time for one job to traverse empty system)

$\frac{T_o}{W_o}$ = inverse of bottleneck rate of system (W_o is the critical WIP which is the minimum number of jobs in the system that achieves the maximum throughput, computed as $W_o = r_b T_o$.)

p_1, p_2= parameters that must be estimated

n_o = is the number of jobs in the system that the curve part of the function changes to straight line.

Similarly, we assume an approximation function for flow time standard deviation ($\sigma(n_i)$),

$$\sigma(n_i) = \begin{cases} q_2 + q_1 n & for\, n \le n_o \\ \sum_{i=0}^{6} a_i n^i & for\, n > n_o \end{cases} \qquad (2.3)$$

We assume n_o is approximately $2W_o$. q_1, q_2 and a_i are parameters to be estimated.

In section 2.3.3, we will show how to find p_1, p_2, q_1 and q_2. After finding these parameters for the flow time function, the adjustment factor, K, in safety lead time formula can be changed to have a minimum average lead time to meet the target service level. It is obvious that the precision of this model is a function of the estimation of $f(n_i)$, and the safety lead time ($KZ_\alpha\sigma(n_i)$).

2.3.2 Safety Lead Time Derivation

Safety lead time is used as a buffer against variability in the production process. As already mentioned, to allow for some flexibility, safety lead time will be added to the approximation function. It is assumed that safety lead time is a function of:

a) flow time variation (standard deviation)

b) percentile of normal distribution at a given target service level (fraction of on time orders), i.e., for 90% service level, $Z_{90\%.}$

c) an adjustment factor to allow achieving the target service level.

Hence, we express safety lead time for job i as

$$sf(n_i) = KZ_\alpha \sigma(n_i) \tag{2.4}$$

Where

α = target service level

Z_α = α-percentile of the standard normal distribution

$\sigma(n_i)$ = standard deviation of flow time when n_i jobs in the system

K = adjustment factor

We could say that if flow times were independent normal random variables, K=1 would result in the target service level of α. However, in a general case, we have to adjust the factor to achieve minimum average lead times under a target service level.

Figure (2) illustrates the procedure to calculate the adjustment factor. First, mean and standard deviation of flow time for each WIP level is calculated. For a given service level, Z value is considered from table. As a start point, assume K=1 and find the lead time through the lead time formula (2.1) for each level WIP level. Compare these quoted lead time to the date and find the number of jobs that are not early or late which give the service level. The idea is to have the quoted lead times as small as possible. Hence, if the service level is met, try a small K value and stop and use this K for the future lead time quotation. If the service level is not met, greater safety lead time is needed. There for K should be increased to meet the service level. K is assumed with three decimals with incremental value of 0.001.

2.3.3 Determination of Parameters In The Flow Time Function

In the following, we show through an example how to calculate the parameters of the proposed functions for flow time (equations 2.2 and 2.3). Since, we do not have any real historical data; we use simulation to generate data for various systems. Then, we use these data to estimate the parameters in the flow time functions using the regression technique. Furthermore, we compare the actual average flow time of some examples and the mean estimated flow times of the proposed model.

2.3.4 Example

As an example, consider a production system (referred to as system 1) consisting of three serial balanced machines. Processing times are normally distributed with a mean of one hour and a standard deviation of 0.1 hour. Jobs arrive to the system exponentially with average inter-arrival times of 1.0 hours (Figure 2.3).

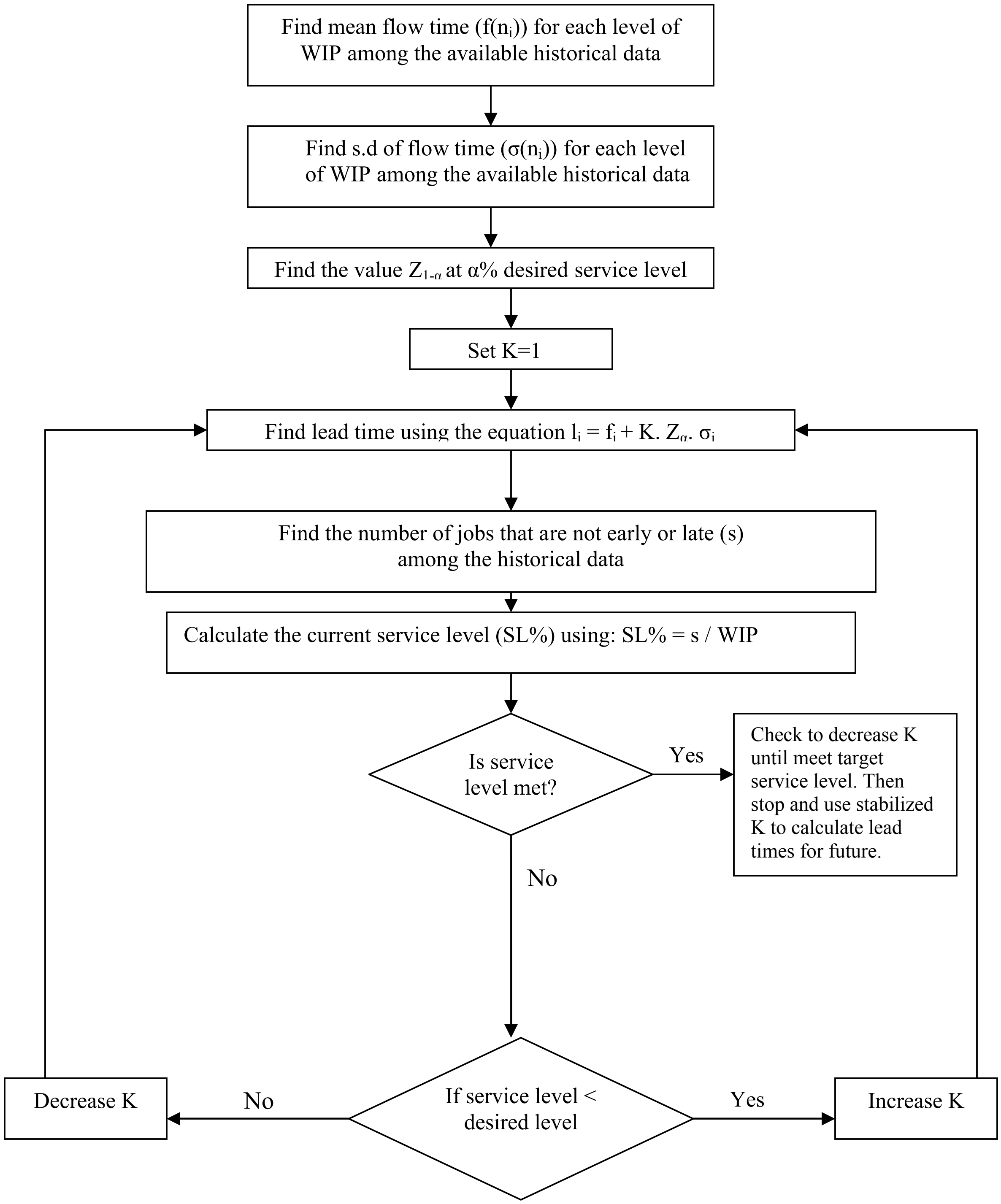


Figure 2.2 Procedure to calculate adjustment factor

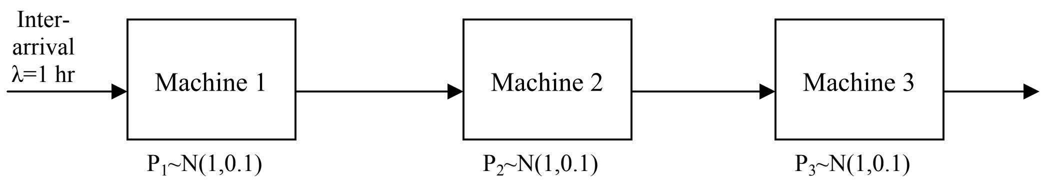


Figure 2.3 System 1 – three serial machines

WITNESS simulation software is used to model this simple line and to find the value of flow times based on WIP in the system. Six different seeds are used to generate random data for processing and inter-arrival times. Simulation is run for 50,000 hours and the value of WIP is changed from 1 to 30 at the increment of one and the corresponding flow time is calculated. The result of this simulation is tabulated and shown in Table 3.1.

In order to find the parameters of equations (2.2) and (2.3), values of T_o, r_b and n_o are needed which are calculated as follows:

$T_o = 1 + 1 + 1 = 3$ (hr)

$r_b = 1$ (job/hr)

$W_o = r_b T_o = 1 * 3 = 3$ (job)

$n_o = 2W_o = 2*3 = 6$

Microsoft Excel is used to develop the regression function of flow time (2.2) for data in Table 2.1. The following approximation function is found:

$$f(n_i) = \begin{cases} 0.0223(n-1)^2 + 0.6938(n-1) + 3.0407 & \text{if } n \le 6 \\ \dfrac{1}{1.10963} n & \text{if } n > 6 \end{cases}$$

TABLE 2.1

SIMULATED DATA – FLOW TIME BASED ON WIP FOR SYSTEM 1

	number of jobs in the system									
	1	2	3	4	5	6	7	8	9	10
Seed 1	2.89	3.65	4.17	5.42	6.48	7.48	8.11	8.89	10.09	11.14
Seed 2	3.09	4.06	4.14	5.71	6.86	7.55	8.8	9.62	9.7	10.29
Seed 3	2.93	3.37	4.06	4.65	5.61	6.84	7.25	7.73	8.85	9.95
Seed 10	2.96	4.05	4.98	5.85	6.78	7.85	8.75	9.98	10.76	10.89
Seed 20	3	3.48	4.53	4.91	5.4	5.77	7.25	7.95	9.08	9.91
Seed 50	3.38	4.08	4.94	5.29	6.34	6.71	7.25	8.3	9.3	9.78
Mean	3.041667	3.781667	4.47	5.305	6.245	7.033333	7.901667	8.745	9.63	10.32667
Std of Flow Time	0.163648	0.293338	0.37674	0.420228	0.55467	0.691777	0.688487	0.833881	0.647199	0.515515

	number of jobs in the system									
	11	12	13	14	15	16	17	18	19	20
Seed 1	11.47	12.76	13.68	14.99	15.89	16.66	16.91	18.21	19.3	20.25
Seed 2	10.53	11.31	11.82	12.31	13.17	14.72	15.8	17.22	17.22	18.48
Seed 3	11.06	11.49	11.65	12.73	13.08	13.78	14.2	15.3	16.62	17.48
Seed 10	11.27	11.57	13	13.74	14.8	15.1	15.82	16.67	17.84	18.58
Seed 20	10.85	11.4	11.72	13.08	13.51	14.54	15.45	16.29	16.84	18.19
Seed 50	11.07	12.11	13.22	13.55	14.88	15.99	16.73	17.01	17.4	17.66
Mean	11.04167	11.77333	12.515	13.4	14.22167	15.13167	15.81833	16.78333	17.53667	18.44
Std of Flow Time	0.298687	0.510316	0.81166	0.856894	1.038194	0.950446	0.892364	0.88825	0.879956	0.902755

	number of jobs in the system									
	21	22	23	24	25	26	27	28	29	30
Seed 1	21.06	21.3	22.54	22.76	23.23	23.96	25.28	25.47	26.69	27.69
Seed 2	18.74	19.68	19.74	20.3	20.58	21.83	21.99	23.04	23.49	23.72
Seed 3	18.9	20.16	21.23	21.63	22.7	23.73	24.15	25.05	26.5	26.62
Seed 10	19.06	19.5	20.58	21.42	22.94	23.28	24.26	25.38	25.53	25.94
Seed 20	18.56	18.85	20.02	20.24	21.43	21.9	22.79	23.71	24.16	24.52
Seed 50	18.64	19.71	20.13	21.33	21.68	22.66	22.79	23.31	23.66	24.83
Mean	19.16	19.86667	20.70667	21.28	22.09333	22.89333	23.54333	24.32667	25.005	25.55333
Std of Flow Time	0.865486	0.749214	0.947763	0.854108	0.9379	0.83258	1.113967	1.000777	1.301803	1.341935

In order to test the goodness of fit for approximated mean flow time function, coefficient of determination (R-square) is used. Therefore, we found R^2 between the output of approximated function and the simulated data. R^2 is calculated for both portion $n \leq n_o$ and $n > n_o$ ($n_o = 2W_0$). Values of 0.97 and 0.99 represent that the functional form in equation (2.2) presents an excellent fit to the mean flow time.

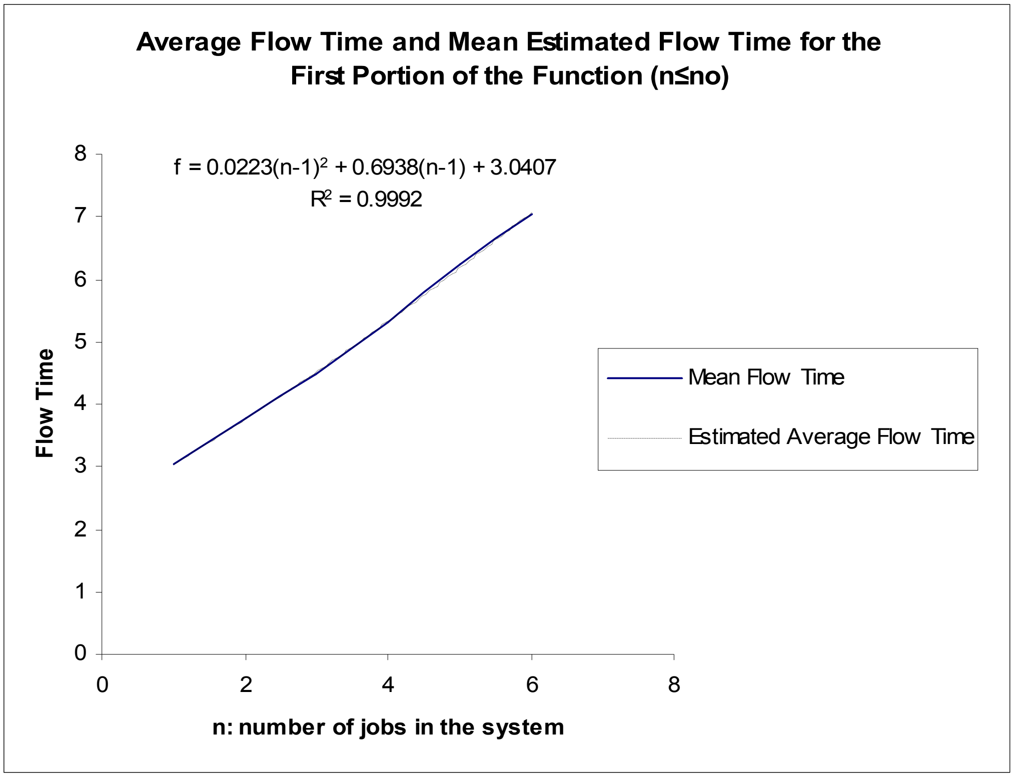
Figure 2.4 shows the actual and estimated flow time versus number of jobs in the system for system 1.

To calculate safety lead time, σ and K need to be calculated. Regression technique is used for data in Table 2.1 to find the equation between standard deviation functions of flow time and WIP (2.3). The following approximation function is found:

$$\sigma(n_i)= \begin{cases} 0.0699 + 0.0991n & if\ n \leq 6 \\ -3*10^{-7}n^6 + 3*10^{-5}n^5 - 0.0012n^4 + 0.0177n^3 - 0.0859n^2 - 0.3687n + 3.8122 & if\ n > 6 \end{cases}$$

The R^2 value for both portion of the estimated function is significant. That is, 98% of the variance in the standard deviation of flow time is explained by the regression when $n \leq n_o$ and 77% is explained for $n > n_o$.

Figure 2.5 shows both actual and estimated standard deviation of flow time versus number of jobs in the system for the system 1.



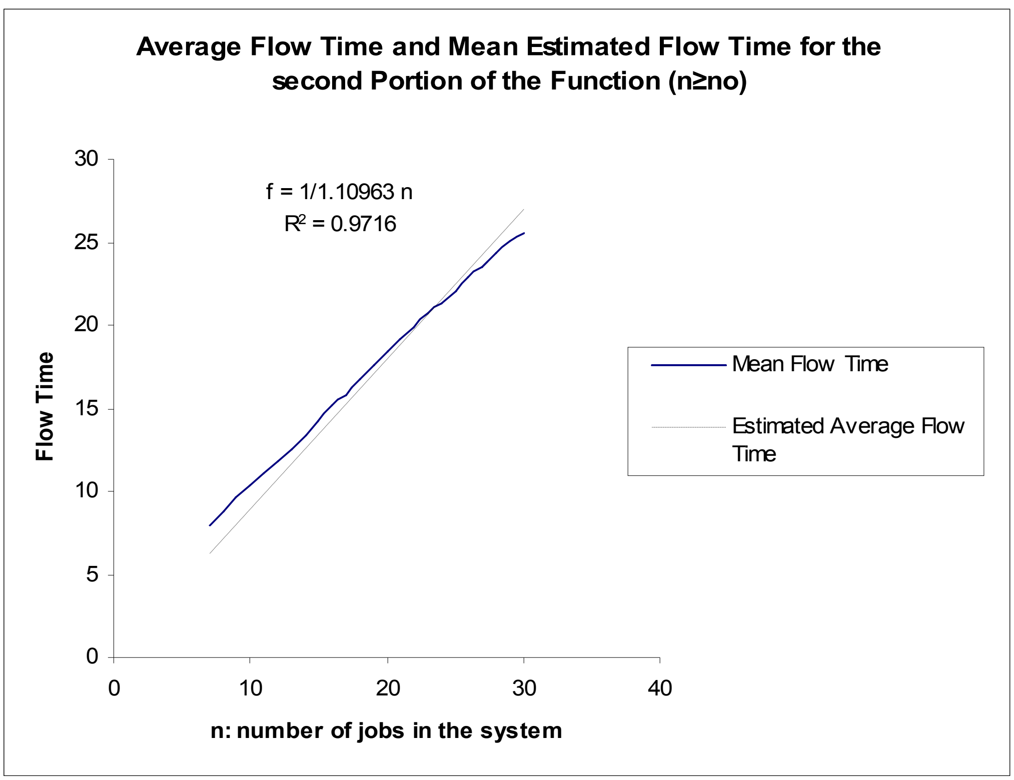
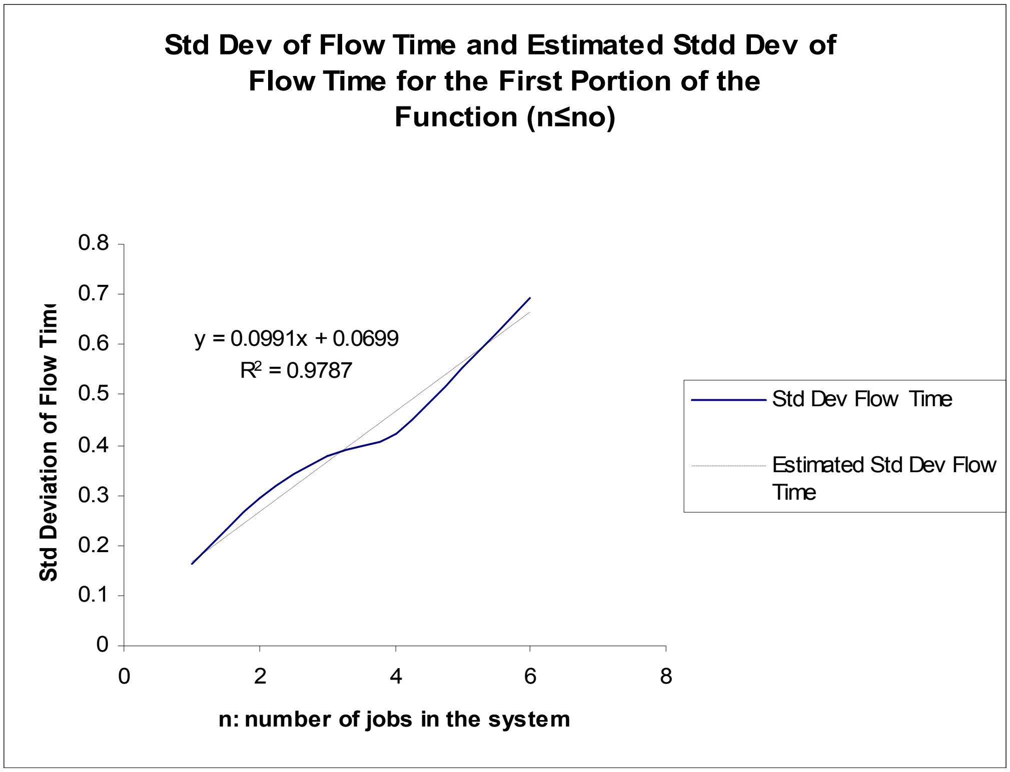


Figure 2.4 Average flow times as a function of the number of jobs in the system 1



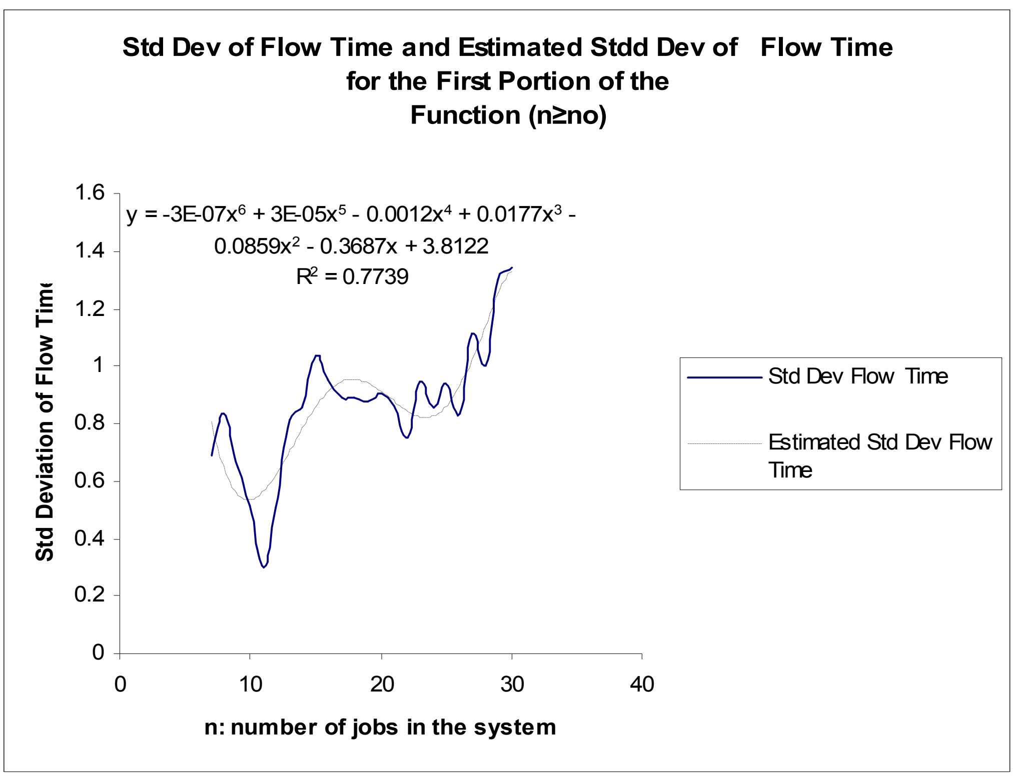


Figure 2.5 Std Dev of flow time as a function of the number of jobs in the system 1

Value of the adjustment factor K is found through trial and error. Its value could be smaller or larger than 1. In some cases, it must be more than one, in increasing safety lead time to achieve the desired service level and in other cases, with a K value of less than one, shorter quoted lead time can be found at the same service level.

To demonstrate the application of adjustment factor (K), consider system 1 in which the desired service level was 90%. The Z-value from normal table is 1.645. Assuming K=1, the lead time is quoted for each WIP level and the achieved service level is calculated which is 73.33%. This means that the quoted lead times are too small resulting in missing due dates. The adjustment factor K is therefore increased to ensure that the desired service level is achieved. For each n, the adjustment factor is changed by trial and error to have the minimum safety lead time to meet the desired service level. The suitable adjustment factor is found to be 1.627. This increased K value increases the safety lead time to achieve a service level of 90%. Table 2.2 shows the quoted lead times for K=1 and K= 1.627.

Another service level of 95% is also considered for system 1. The related Z-value from normal table is 1.96. Adjustment factor (K) is calculated to be1.452. The safety lead time is added to the estimate of flow time (see equation 2.4) and the quoted lead times for manufacturing process is calculated. Tables 2.3 and 2.4 show the quoted manufacturing lead times based on WIP for system 1 at 90% and 95% service level.

TABLE 2.2

QUOTED LEAD TIMES WITH K=1 AND K=1.627 AT 90% SERVICE LEVEL FOR SYSTEM1

WIP	1	2	3	4	5	6	7	8	9	10
Quoted Lead time with Z90% and K=1	2.6472	3.4817	4.3608	5.2846	6.2529	7.2658	7.5448	8.1536	8.9284	9.8924
Quoted Lead time with Z90% and K=1.627	2.8215	3.7582	4.7396	5.7655	6.8360	7.9512	8.3200	8.7455	9.4410	10.4444

WIP	11	12	13	14	15	16	17	18	19	20
Quoted Lead time with Z90% and K=1	11.0895	12.5932	14.5163	17.0208	20.3285	24.7318	30.6042	38.4125	48.7279	62.2386
Quoted Lead time with Z90% and K=1.627	11.8271	13.7086	16.2723	19.7820	24.5987	31.1978	40.1872	52.3262	68.5443	89.9612

WIP	21	22	23	24	25	26	27	28	29	30
Quoted Lead time with Z90% and K=1	79.7622	102.2582	130.8419	166.7971	211.5909	266.8873	334.5621	416.7178	515.6992	634.1088
Quoted Lead time with Z90% and K=1.627	117.9070	153.9430	199.8835	257.8176	330.1321	419.5342	529.0761	662.1784	822.6562	1014.7434

TABLE 2.3

QUOTED MANUFACTURING LEAD TIMES BASED ON THE WIP FOR 90% SERVICE LEVEL

WIP	1	2	3	4	5	6	7	8	9	10
Lead Time at 90%	2.82	3.76	4.74	5.77	6.84	7.95	8.32	8.75	9.44	10.44
WIP	11	12	13	14	15	16	17	18	19	20
Lead Time at 90%	11.83	13.71	16.27	19.78	24.60	31.20	40.19	52.33	68.54	89.96
WIP	21	22	23	24	25	26	27	28	29	30
Lead Time at 90%	117.91	153.94	199.88	257.82	330.13	419.53	529.08	662.18	822.66	1014.74

TABLE 2.4

QUOTED MANUFACTURING LEAD TIMES BASED ON THE WIP FOR 95% SERVICE LEVEL

WIP	1	2	3	4	5	6	7	8	9	10
Lead Time at 95%	2.85	3.80	4.80	5.84	6.93	8.06	8.45	8.84	9.53	10.54
WIP	11	12	13	14	15	16	17	18	19	20
Lead Time at 95%	11.95	13.89	16.56	20.24	25.30	32.26	41.76	54.61	71.80	94.52
WIP	21	22	23	24	25	26	27	28	29	30
Lead Time at 95%	124.18	162.44	211.23	272.78	349.61	444.62	561.04	702.52	873.10	1077.30

2.3.5 Approximation Function for Flow Time Standard Deviation

The presented function (2.3) has a linear function for the first portion and a six degree polynomial function for the second portion. The goal is to have simple function as much as possible and a good approximation due to R^2 as well. About the second portion function, different polynomials are used and Table 2.5 shows the results. In overall, the six degree is the best fit and therefore will be used in the equation.

TABLE 2.5

DIFFERENT POLYNOMIAL FUNCTION FOR FLOW TIME STD ($N>N_0$)

	Approximation Function for Flow Time Std ($n>n_0$)	R^2
1	$\sum_{i=0}^{2} a_i n^i$	54.13%
2	$\sum_{i=0}^{3} a_i n^i$	56.62%
3	$\sum_{i=0}^{4} a_i n^i$	72.57%
4	$\sum_{i=0}^{5} a_i n^i$	76.75%
5	$\sum_{i=0}^{6} a_i n^i$	77.39%

2.3.6 Adjustment Factor Incremental Increase

Table 2.6 shows the service level obtained for different Adjustment Factor until the desired met (See the procedure in figure 2.2). Higher adjustment factors observed for higher service levels.

TABLE 2.6

DIFFERENT ADJUSTMENT FACTOR FOR SYSTEM 1 AT 90% DESIRED SERVICE LEVEL AND OBSERVED SERVICE LEVELS

System (1) at 90% Desired Service Level	
Adjustment Factor	Service Level Observed
1.000	73.33%
1.200	80.00%
1.300	83.33%
1.500	86.67%
1.600	86.67%
1.610	86.67%
1.620	86.67%
1.625	86.67%
1.626	86.67%
1.627	90.00%

2.4 Model Effectiveness Evaluation

In order to study the effectiveness of the proposed model, similar to the work of Hopp and Sturgis (2000), we compare it to an M/M/1 system for which the exact due date quotes can be found analytically.

In an M/M/1 system, arrival and processing times are independent exponentially distributed random variables. Note that an exponential distribution is just a special case of the more general gamma function. We can write an analytical representation for the flow time distribution for n jobs in the system considering that the time to process n jobs has n-Erlang distribution. The Erlang distribution is a special case of the gamma distribution where the shape parameter is an integer (see Appendix F). From Erlang distribution, we can derive the distribution of the flow time given n jobs in the system, F_n:

$$
\begin{aligned}
P(F_n \leq f) &= 1 - P(F_n > f) \\
&= 1 - \sum_{k=0}^{n-1} \frac{(\lambda f)^k e^{-\lambda f}}{k!} \\
&= 1 - e^{-\lambda f} \sum_{k=0}^{n-1} \frac{(\lambda f)^k}{k!}
\end{aligned}
$$

Or we can write the function,

$$
P(F_n \leq f) = 1 - e^{-\lambda f} - \frac{e^{-\lambda f}(\lambda f)^1}{1!} - \ldots - \frac{e^{-\lambda f}(\lambda f)^{n-1}}{(n-1)!} \qquad (2.5)
$$

where λ is the processing rate of the machine.

Therefore, for the problem of lead time quotation to meet a target service level constraint, we must find the value of f in equation (2.5) such that the probability is equal to or larger than the user's target service level (for example, 90% or 95%), or

$$
P(F_n \leq f^*) \geq \text{service level} = \alpha \qquad (2.6)
$$

Actually f^* is the quoted lead time. The computed values of f^* (processing rate $\lambda = 1$) using analytical method is shown in table 2.7.

We simulated an M/M/1 system with processing and inter-arrival rates of λ=1 using three different pseudo-random numbers (seeds) and setting target service levels at 90% and 95%. Then, we compared the result of exact model in equation 2.5 with our due date quoting model. We changed the adjustment factor, K, in the proposed model to have the minimum average flow time in order to achieve the target service level. Table 2.7 shows the quoted lead times at different WIP level for both analytical and proposed models.

To compare the efficiency of the quoted due dates, Hopp and Sturgis (2000) in their work considered several measures such as mean earliness, mean tardiness, and mean missed due date . Mean earliness (tardiness) is defined as the average of the absolute time between the quoted due date and the flow time for all jobs that completed early (late). Mean missed due date is defined as the mean absolute value of the time difference between the quoted due date and flow time for all jobs. Mean missed due date is the sum of mean earliness and mean tardiness (see Weeks, 1979).

We use similar measures to their work and Table 2.8 shows the result of performance measures calculated as the averages of the three different runs for the two usual target service levels of 90% and 95%.

From these, we can conclude that:

1. The results of Table 2.8 show that for 90% service level, mean lead time quote is just 3.58% higher than the analytical method. The proposed model has a better mean tardiness than the analytical method. For 95% service level, mean lead time quote is 17.18% higher than the analytical method. The proposed model has a better mean tardiness than the analytical method. In overall, the results of indicate that the performance of the proposed model is good in comparison to the analytical method.

2. The performance measures show that the proposed method is better at 90% service level in comparison to the 95% service level. It seems that the proposed model perform better for the lower service level.
3. The proposed method for quoting lead time to meet target service level is simple, practical and effective. It makes no assumptions about the distribution of processing times which makes it proper to many real world systems.
4. It is easy to use because it uses data that is already collected like number of jobs in the system and bottleneck rate.

TABLE 2.7

QUOTED LEAD TIMES FOR M/M/1 SYSTEM – ANALYTICAL AND PROPSED METHOD AT TWO SERVICE LEVELS

Service Level	n	1	2	3	4	5	6	7	8	9	10
Service Level= 90%	Quoted Lead Time in Analytical Method	2.26	3.83	5.26	6.61	7.91	9.19	10.43	11.67	12.89	14.10
	Quoted Lead Time in Proposed Method	2.00	2.95	3.94	5.43	7.14	8.86	10.47	11.92	13.20	14.33
	n	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	Quoted Lead Time in Analytical Method	15.30	16.48	17.66	18.84	20.00	21.16	22.32	23.47	24.62	25.76
	Quoted Lead Time in Proposed Method	15.35	16.34	17.34	18.43	19.64	21.02	22.58	24.32	26.20	28.21
	n	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
	Quoted Lead Time in Analytical Method	26.90	28.03	29.17	30.30	31.43	32.55	33.67	34.79	35.91	37.03
	Quoted Lead Time in Proposed Method	30.26	32.31	34.26	36.05	37.61	38.90	39.89	40.62	41.16	41.69

Service Level	n	1	2	3	4	5	6	7	8	9	10
Service Level= 95%	Quoted Lead Time in Analytical Method	2.91	4.63	6.17	7.62	9.00	10.35	11.67	12.97	14.25	15.51
	Quoted Lead Time in Proposed Method	2.73	3.49	4.40	6.18	8.32	10.49	12.47	14.17	15.57	16.72
	n	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	Quoted Lead Time in Analytical Method	16.76	18.00	19.23	20.45	21.66	22.86	24.06	25.26	26.44	27.63
	Quoted Lead Time in Proposed Method	17.69	18.59	19.52	20.60	21.90	23.48	25.36	27.56	30.02	32.67
	n	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
	Quoted Lead Time in Analytical Method	28.80	30.00	31.15	32.32	33.47	34.63	35.79	37.00	38.09	39.24
	Quoted Lead Time in Proposed Method	35.41	38.14	40.70	42.99	44.88	46.30	47.21	47.67	47.82	47.93

TABLE 2.8

EFFECTIVENESS COMPARISON OF THE ANALYTICAL AND PROPOSED LEAD TIME QUOTATION METHODS

	M/M/1 Analysis	Proposed Method	Theoretical Method
Service Level= 90%	Actual Mean Flow Time	17.770	17.770
	Mean Lead Time Quote	22.081	21.318
	Mean Earliness	4.956	3.738
	Mean Tardiness	1.454	1.850
	Mean Missed Due date	6.410	5.588
Service Level= 95%	Actual Mean Flow Time	17.770	17.770
	Mean Lead Time Quote	25.698	21.931
	Mean Earliness	8.214	5.358
	Mean Tardiness	0.234	0.440
	Mean Missed Due date	8.448	5.798

2.5 Performance Tests

Hopp and Sturgis (2000) in order to examine the performance of their proposed model, studied the model behavior in systems where parameters change over time due to continuous improvement in the system or change in equipment, work methods, etc. Similarly we perform the following tests,

Two systems with four machines in tandem, each with normal processing times are considered. In the first case, we increased the capacity by increasing the production rate at the bottleneck (machine two) from 0.85 jobs per hour to 1 job per hour. The mean processing time for the other machines was set to 1 hour and the standard deviation of processing time for all four machines was set to 0.3 hours. Appendices B and C shows the data used for case 1 before and after change.

In the second case, we decreased the mean time to repair on machine two from 2 to 0.5

hours. Machine two was the only machine that failed and failures occurred exponentially with a mean of 40 hours. All four machines had mean processing times of 1 hour and standard deviation equal to 0.3 hours. We considered this system as a representative of a continuous improvement effort. Appendices D and E shows the data used for case 2 before and after change. Figure 2.6 and 2.7 show both case 1 and case 2 before and after changes.

Each system was run using three different seeds. Jobs arrived to the system exponentially with average inter-arrival times of 1.1 hours. Table 2.9 summarizes the behavior of the method before and after changes in the system. It lists the systems under review and identifies averages for service level, flow time, quoted lead time, safety lead time and adjustment factor, K. Note that in both cases, the method reduced average mean lead time and safety lead time in response to the system improvement, that is, for improvement in case 1, the quoted lead time reduced from 22.323 (hr) to 18.909 (hr) and for case 2, the quoted lead time reduced from 22.608 (hr) to 21.528 (hr).

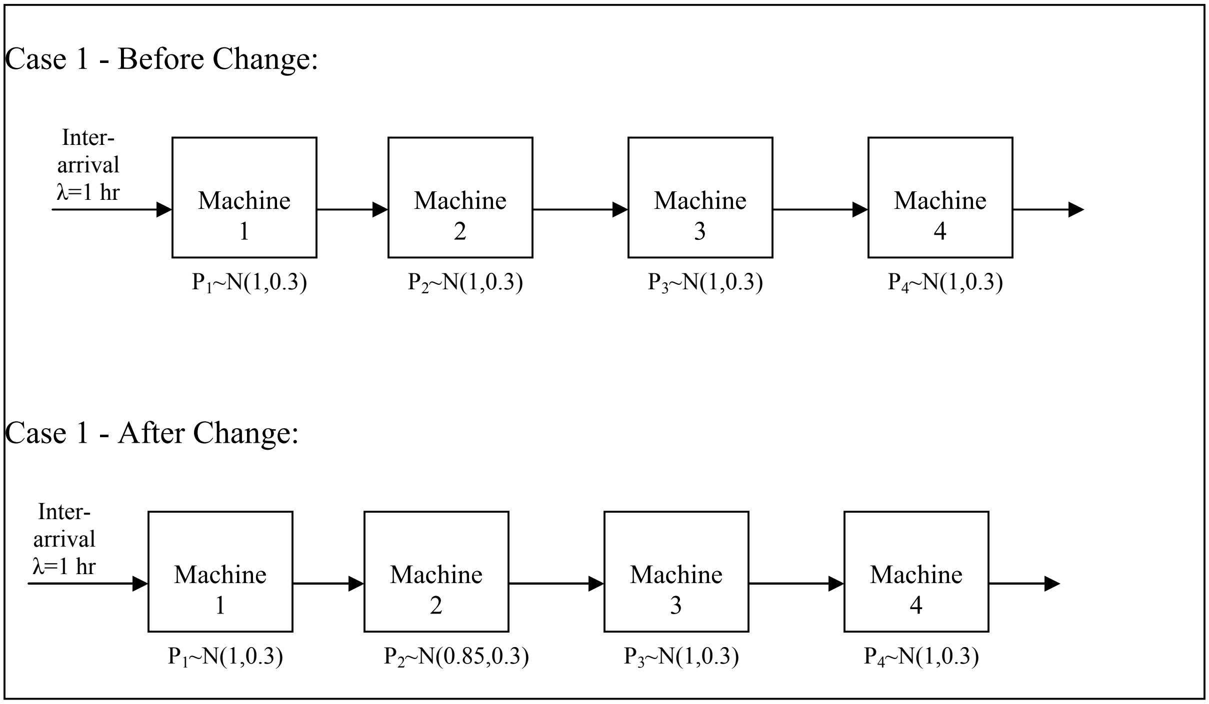


Figure 2.6 – Case 1 before and after change

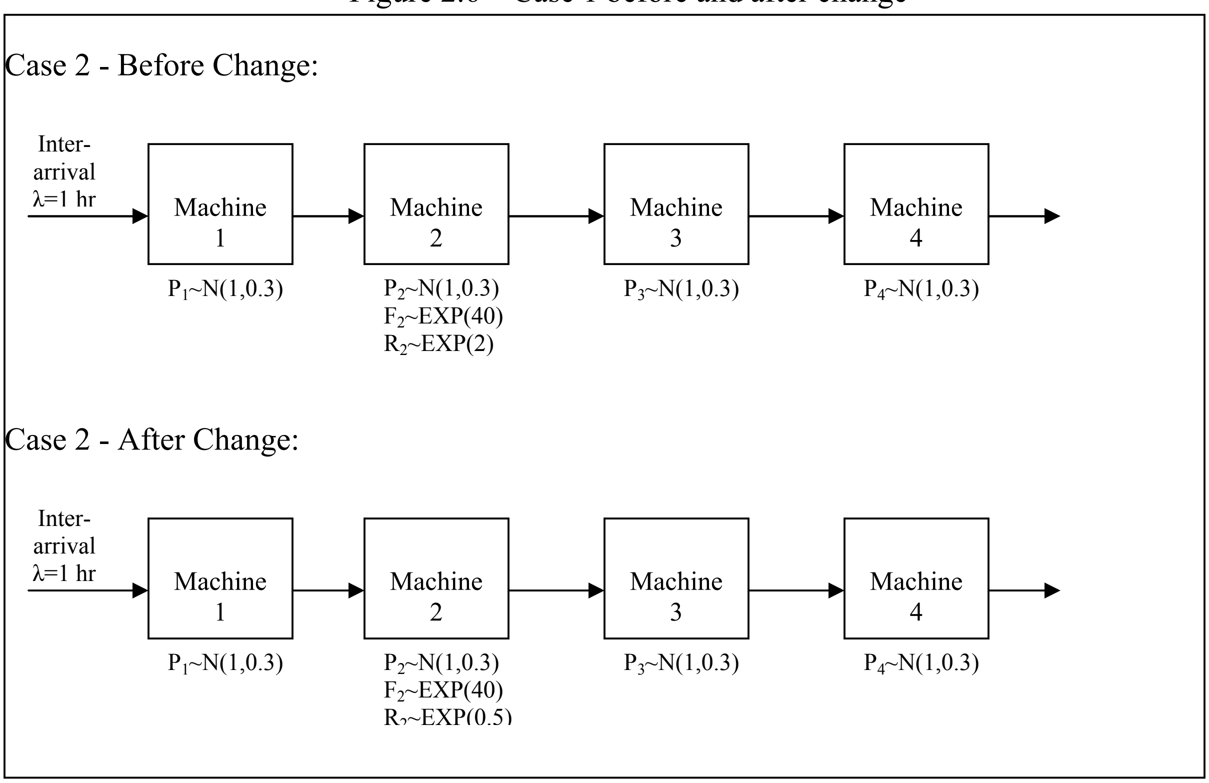


Figure 2.7 – Case 2 before and after change

TABLE 2.9

BEFORE AND AFTER LEAD TIME COMPARISONS FOR DYNAMIC PERFORMANCE TESTS

Dynamic Performance Tests Target Service Level = 90%					
		Average Flow Time	Average Quoted Lead Time	Average Safety Lead Time	Adjustment Factor "K"
Case 1*	Before	21.165	22.323	1.183	0.399
	After	19.326	18.909	1.159	0.523
Case 2*	Before	21.336	22.608	1.790	0.166
	After	20.469	21.528	1.100	0.65

Case 1:Increase Capacity
Case 2:Decrease Repair Time

2.6 Conclusions

The modified model of Hopp and Sturgis (2000) for setting due dates to achieve target service level is simple, practical and effective. It makes no assumptions about the distribution of processing times which makes it very useful for real-world systems. It is easy to practice and implement because it just uses data that the already collected in the company, like number of jobs in the system and rate of bottleneck. Also, we tested the effectiveness because the results of due date quotes meet target service level like due date quotes from a theoretical model for a case where an analytical solution is known (i.e., M/M/1 system). Furthermore, our proposed model is effective because it can adjust the due date quotes for systems that faces to changes over time such as efforts to process improvement through process improvement projects; such as increases in capacity, decreases in repair time.

CHAPTER 3

INTEGRATED SUPPLY AND MANUFACTURING LEAD TIME QUOTATION UNDER SERVICE LEVEL CONSTRAINT

Chapter 2 presented a modified methodology based on the work of Hopp and Sturgis (2000) to quote due dates for incoming jobs, taking into account the status of the manufacturing system. In real world applications, manufacturing system status is also affected by the supply chain. To be more effective in satisfying customer service requirements, a due date quotation technique should consider supply chain performance as well. Realizing the fact that an ordered job usually consists of a number of components, Section 3.1 presents a model to find the supply process lead time of components of an order through the supply process. The problem is divided into two categories: single order (Section 3.1.1) and multiple orders (Section 3.1.2). In the single order problem, it is assumed that orders are independent of each other, and in the multiple order problem, quoted due dates to the customer are set simultaneously. Section 3.1.3 presents a numerical example to calculate supply ready time.

Section 3.2 describes a cost minimization formulation of the problem of setting the integrated due date quotation considering both manufacturing and supply processes. Section 3.2.1 presents the model not considering service level while Section 3.2.2 considers service level. The model is divided into categories: Section 3.2.2.1 for a single order with a requested window date, and Section 3.2.2.2 shows an extension of the model to a multiple orders problem.

Section 3.3 presents some numerical examples of the proposed model. Section 3.4 describes effectiveness of the integrated model, which is a comparison of the model to a system for which the exact due date quotes can be found analytically. This effectiveness evaluation is divided to two sections. Section 3.4.1 evaluates the integrated model using the manufacturing lead time as proposed in Chapter 2 while in Section 3.4.2, the integrated method is evaluated using an average manufacturing lead time. Section 3.5 compares the integrated model using some proposed different safety lead time methods. Section 3.6 analyzes the characteristics of the solution to the problem for some special cases and finally Section 3.6 presents the results and conclusions.

3.1 Supply Lead Time Calculation

In this section, a supply lead time model to calculate the ready time of the components of a specific order is developed. In modeling the supply process, it is assumed that to begin production of a specific job, all components, including raw materials and parts, must be ready. In real world applications, procurement could be divided into two general categories: supplies available within the plant's inventory, and supplies that must be purchased from outside the plant. "Purchase order" is the term used for outside orders (see Hegedus and Hopp (2001)). For a job to be ready for manufacturing, all purchase orders must be available. In the following model, we only consider the supplies that must be provided from outside.

To model the procurement process, purchase orders associated with each job must be on-hand. This is done by using the information in the company to determine which purchase orders are necessary for a job to be released to the manufacturing department.

3.1.1 Supply Lead Time Procedure for a Single Order

To model the supply lead time, L_s, it is necessary to first model the arrival time of the all purchased components for the job. The time of issuing the purchase order as well as the supply lead time should be considered (in practice, purchase order for different components may have been placed at different times). Similar to the work of Hegedus and Hopp (2001), the lead time of purchase order k is denoted by $l_s(k)$ with a distribution function of $F_{ls(k)}(.)$. Also, if K is the set of purchased components for the job, and t(k) is the time of issuing order k, then arrival time of purchase order k is given as:

$$t(k) + l_s(k)$$

As stated previously, when the last component in set K arrives, the job is ready for production. This shows the importance of both the time a purchase order is placed and the supply process lead time for all components associated with a job. In other words, the procurement lead time for a job is the arrival time of the last component in that order, or,

$$\text{Supply Lead Time } (L_s) = \text{Max}_{k \in K} (t\,(k) + l_s\,(k)). \qquad (3.1)$$

Since all purchased components must be ready to start manufacturing a job, the distribution function of the supply lead time F_{Ls} (.) is therefore the multiplication of purchase order arrival-time distribution functions:

$$F_{Ls}\,(t) = \{F_{Ls\,(1)}\}\ \{F_{L\,(2)}\}\ldots\ \{F_{L\,(k)}\}\ \text{ For all } k \in K \qquad (3.2)$$

In section 3.2, using this supply lead time distribution function for the components as well as manufacturing lead time, the completion time distribution function is developed and a model to quote the due date for each job is presented.

3.1.2 Supply Lead Time Procedure for Multiple Orderss

In practice, it is possible that more than one orders require due dates to be set simultaneously. In order to find the supply lead time for each order a simple procedure is developed:

Orders should be sorted based on the requested due date from customer, then for each job, based on the company information components and purchase orders should be found. Using equation (3.2), the supply lead time distributions for each job can be found. Figure (3.1) shows the procedure how to find the supply lead time for multiple orders.

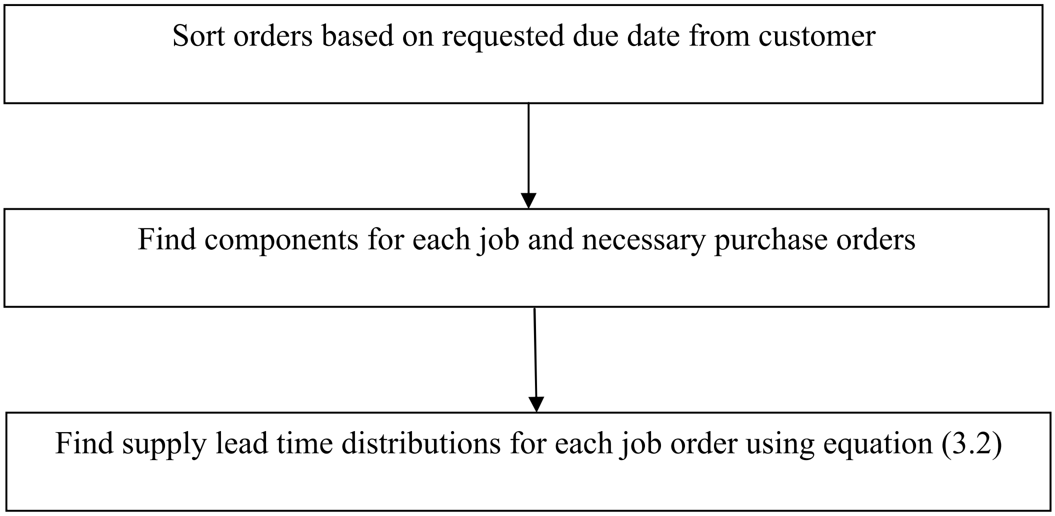


Figure 3.1. Supply lead time procedure for multiple orders

3.1.3 Numerical Example for Supply Lead Time

A numerical example is presented to clarify the procedure for single order supply lead time.

Assume a customer has requested a product to be delivered. Based on the available information, this order requires to two purchase orders and other components are available in the inventory. One purchase order is placed at the present time and has an exponentially distributed lead time with an expected lead time of 10 days. The second order will be issued in five days and has an exponentially distributed lead time with an expected lead time of five days. The problem can be summarized as follows:

$t(1) = 0, F(L_s(1)) \sim EXP(x, 10)$

$t(2) = 5, F(L_s(2)) \sim EXP(x, 5)$

Note that for exponential distribution, density function is $f(x, \lambda) = \lambda e^{-\lambda x}$, and distribution function is $F(x, \lambda) = 1 - e^{-\lambda x}$, for $x \geq 0$. Therefore, for these exponential distribution functions, the supply lead time is

$$F_{Ls}(t) = (1- e^{-(1/5)*(t-5)})\,(1- e^{-(1/10)*(t-0)})$$

This will be used in the next proposed integrated model to calculate the quoted due date.

3.2 Integrated Due Date Quotation

This section describes a cost minimization formulation of the problem of setting the integrated due date considering both manufacturing and supply processes. Section 3.2.1 presents the model not considering service level while Section 3.2.2 considers service level. The model is divided into categories: Section 3.2.2.1 for a single order with a requested due date window by customer, and Section 3.2.2.2 shows an extension of the model to a multiple order problem.

Let a random variable, Ls, represent the supply lead time of an order. Since there is a window requested by customer, let du be the upper requested due date for an order, dl the lower due date allowed by the customer for shipment, and Δd the difference between the quoted due date, dq, and the upper requested due date, or delay (Δd = dq – du). Since usually the requested window from customer is tight, in the following model, it is assumed that the quoted due date is never earlier than the upper requested due date, so Δd is always a non-negative number. Therefore, the quoted due date dq is

$$d_q = \Delta d + d_u \tag{3.1}$$

Since the delay is defined as the difference between the quoted due date and the upper requested due date, the quoted due date is a function of the delay (Figure 3.1).

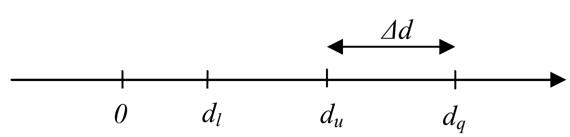


Figure 3.1. Requested window by customer and the quoted due date

3.2.1 Integrated Due Date Quotation (Not Considering Service Level Constraint)

The following describes a cost minimization formulation for the problem of setting the due date for a single order with a requested due date window, independent of other demands. Extend this formulation for multiple order problems is discussed at the end of this section.

Let a random variable $L_{integrated}$ represent the total completion time of job(s) used to satisfy an order, with corresponding density and distribution functions f(.) and F(.), respectively. For an order with an upper requested due date, d_u, in order to find the quoted due date, d_q, the approach is to develop the total cost function and minimize it to find the quoted due date.

Iyer et al. (2003) presented demand postponement as strategy to handle potential demand surges. Under this method, a fraction of the demands from the regular period are postponed and satisfied during a postponement period. This permits capacity to be produced to satisfy the postponed demands. A reimbursement per unit is paid to customers whose demands are postponed. We use a similar concept to introduce a cost associated to each job because of difference between quoted due date and upper due date requested by customer. It is a like a discount or reimbursement to customer per unit per difference day.

To develop the total cost function, the following new notations are defined,

C_h = Cost of holding inventory for finished parts (in dollars per unit per day)

C_t = Cost of tardiness (cost or penalty for "not fulfilling the quoted due date," in dollars per unit per day)

C_r = Cost of reimbursement (cost for the difference of quoted due date d_q, later than the upper requested date by customer, d_u, in dollars per unit per day)

To derive the cost equation, the following assumptions are made:

- Orders cannot be delivered before the lower requested due date by the customer, d_l, so jobs completed early are subject to cost of holding inventory, C_h.
- Upper requested due dates are after the lower requested due date (early shipment allowed date), that is $d_u \geq d_l$. Since the request is received at the present time, it follows that $d_u \geq 0$; since $d_q - d_l = d_u + \Delta d - d_l = (d_u - d_l) + \Delta d$, therefore $(d_q - d_l)$ is non-negative. Next section considers a special case where this condition is not met. The delivery (shipment range) can then occur between d_l and $(d_q - d_l)$.

  Total cost, C, is the summation of all three defined costs, C_h (cost of holding inventory), C_r (cost of reimbursement) and C_t (cost of tardiness) as follows:

$$C = C_h \int_0^{d_q - d_l} (d_q - d_l - t)\, f(t)\, dt + C_r \Delta d + C_t \int_{d_q}^{\infty} (t - d_q)\, f(t)\, dt \qquad (3.2)$$

The derivative of equation (3.2) can be shown as,

$$C_h F(d_q - d_l) + C_r - C_t (1 - F(d_q)) \qquad (3.3)$$

The second order derivation of this equation is,

$$C_h f(d_q - d_l) + C_t f(d_q)$$

which is a positive function. Therefore, the total cost function has a minimum (see appendix H).

Setting equation (3.3) to zero, will provide optimal value of the quoted due date denoted as (D_q) that minimizes the cost function. Hence,

$$C_t (1 - F(D_q)) = C_h F(D_q - d_l) + C_r \qquad (3.4)$$

Note that if we use this function considering both supply and manufacturing, the function could be considered as,

$$C_t (1 - F_{integrated}(D_q)) = C_h F_{integrated}(D_q - d_l) + C_r$$

Equation (3.4) could be written as

$$C_t F(D_q) + C_h F(D_q - d_l) = C_t - C_r$$

Since the distribution function of supply lead time is available, in order to find D_q, relationship between integrated total completion time and manufacturing lead time is necessary. Following describes this matter,

If $L_{integrated}$ is a random variable representing the integrated total completion time of a job (both supply and manufacturing), L_s is a random variable for the supply lead time of components, and L_m is the manufacturing lead time, then considering both the supply and manufacturing processes, we have the following simple model for the production completion time of the job:

$$L_{integrated} = L_m + L_s \tag{3.5}$$

In order to find the distribution function of the integrated total completion time, the property of sums of independent random variables is used (see http://en.wikipedia.org/wiki/Probability_density_function).

The probability density function of the sum of two independent random variables U and V is

$$f_{U+V}(x) = \int_{-\infty}^{\infty} f_U(y) f_V(x-y)\,dy$$

Therefore, we could write

$$f_{\text{Lintegrated}}(t) = \int_0^{\infty} f_{Lm}(l_m) f_{L_s}(t - l_m)\,dl_m$$

Assuming a constant number for the manufacturing lead time (from the proposed method in Chapter 2); hence, f_{Lf} (l_f) is replaced by one, and the distribution function of the integrated completion time is

$$F_{integrated}(t) = F_{Ls}(t - l_m) \tag{3.6}$$

Note that $F_{integrated}$ is independent of d_u and Δd. That is, in the presented model for single order problem, the total integrated completion time of the job is not a function of the requested window or quoted due date. However, the total integrated completion time is dependent on upper requested due date by customer for the multiple order problem, because orders are first sorted based on the upper requested due date and the supply lead time is related to this allocation. However, the method as already mentioned above is an extension of solving the single order problem sequentially.

3.2.2 Integrated Lead Time Quotation under Service Level Constraint

This section describes an integrated model to quote the lead time for the supply and manufacturing processes under a desired service level. First, the model is for a single order with a requested window due date, and the end of this section discusses how to extend this formulation to the problem where multiple orders need due dates set simultaneously.

3.2.2.1 Model for Single Order

In order to consider the service level, the quoted lead time is expressed as,

Integrated lead time = supply lead time + manufacturing lead time + safety lead time

$$L_{integrated}(i) = L_s(i) + L_m(i) + sf(i) \tag{3.7}$$

where

$L_{integrated}(i)$ = integrated lead time for i^{th} arriving order (considers both manufacturing and supply processes)

$L_s(i)$ = supply lead time for i^{th} arriving order

$L_m(i)$ = manufacturing lead time

$sf(i)$ = safety lead time to ensure achieving service level

Note that manufacturing lead time could be from different methods such section 2.1 based on the number of jobs in the system (n_i) at i^{th} order arrival (including job i).

The safety lead time for job order i is defined as

$$sf\,(i) = \eta Z_\alpha \sigma_m(i) \tag{3.8}$$

where

α = target service level

Z_α = α-percentile of the standard normal distribution

$\sigma_m(i)$ = standard deviation of manufacturing flow time when n_i jobs are in the system

η = integrated adjustment factor (used to adjust the buffer against variability for both manufacturing and supply process)

The integrated adjustment factor must be adjusted to achieve minimum average lead time under a target service level. Figure 3.2 shows how to calculate the integrate adjustment factor.

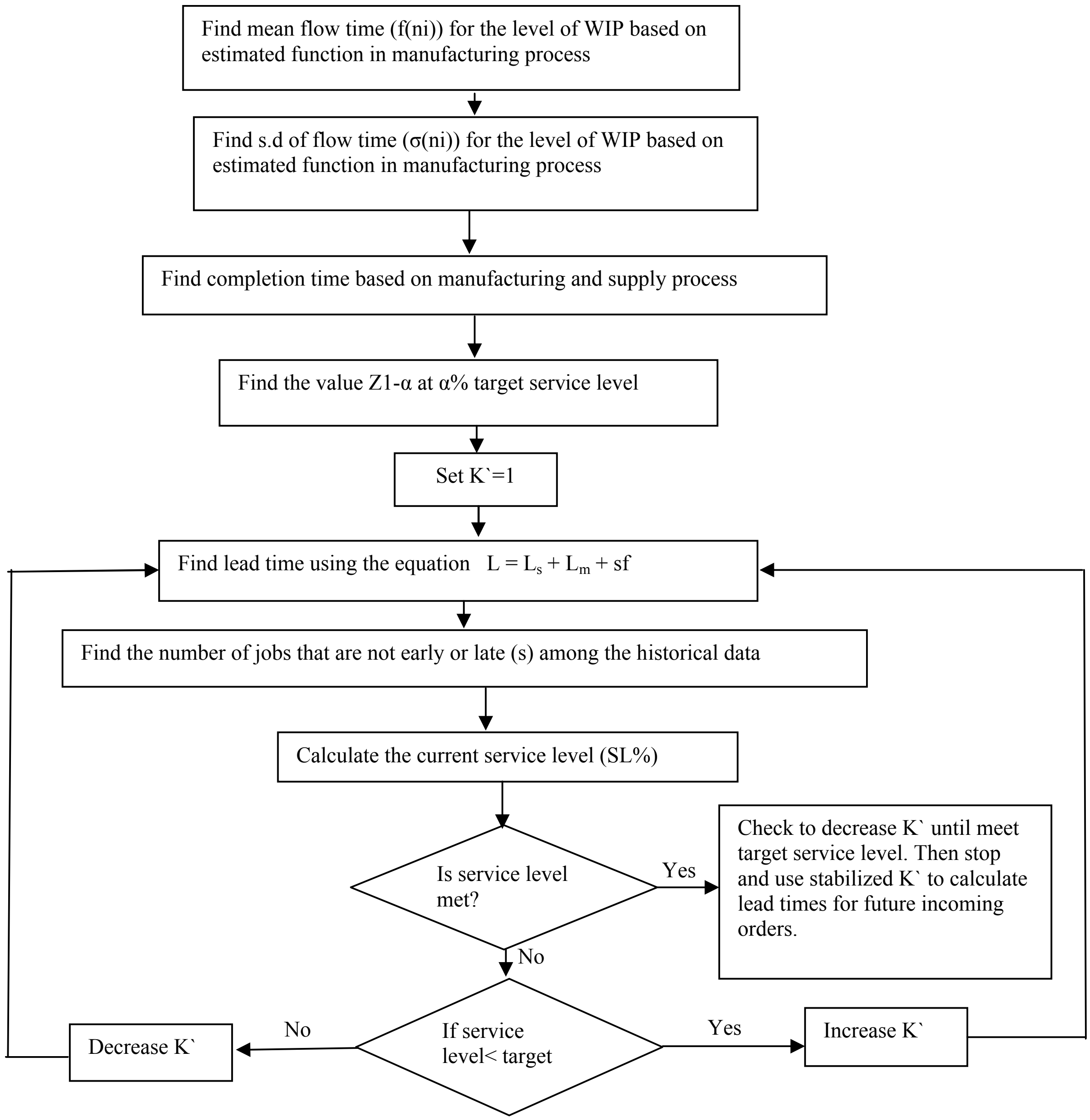


Figure 3.2. The process to calculate integrated adjustment factor.

3.2.2.2 Model for Multiple Orders

The procedure for the multiple order problems is as follows:

- Find the supply lead time based on the procedure in section 3.1.2.

- Find the estimated manufacturing lead time based on the procedure in Chapter 2 (or any other methods).
- Calculate the total integrated completion time for each demand.

Note that the manufacturing model assumes First In First Out (FIFO). Therefore, no re-sequencing is considered for the current jobs in the manufacturing system. Thus, the sorting of demands is significant because supply is allocated first come, first served as well. Note also that other manufacturing lead times could be used in place of equation (2.13), as long as it maintains the sequence used in the supply lead time allocation. The details are shown through a numerical example.

3.3 Numerical Examples:

The following is a simple numerical example to clarify the proposed procedure. For the manufacturing process, we use a similar system in the example of Chapter 2 (Section 2.3.4).

3.3.1 Integrated Lead Time Quotation under Service Level Single Order Example

A customer has requested a product to be delivered within 3 to 12 days. Based on the available information, this order needs two purchased orders and some other components that are available in inventory. The purchase order for one of the components is placed at present time and has an exponentially distributed lead time with an expected lead time of 10 days. The other order will be issued in 5 days and has an exponentially distributed lead time with an expected lead time of 5 days. Also assume that cost of inventory holding for the finished products is $0.1 per unit per day. The cost of reimbursement is $1 per unit per day and the cost of tardiness is $5 per unit per day. The manufacturing process (referred to as system 1 in Chapter 2.1) consists of three serial balanced machines (see complete details of the manufacturing system in Chapter 2.1).There are 3 jobs in the system and the target service level is 90%. We need to find the quoted due date.

We can summarize the problem as follows,

$d_u = 12$ (days)

$d_l = 3$ (days)

$t(1) = 0$, $F(L_s(1)) \sim EXP(x, 10)$

$t(2) = 5$, $F(L_s(2)) \sim EXP(x, 5)$

$C_h = 0.1$ (\$/unit/day)

$C_r = 1$ (\$/unit/day)

$C_t = 5$ (\$/unit/day)

Note that for exponential distribution, the density function is $f(x, \lambda) = \lambda e^{-\lambda x}$, and the distribution function is $F(x, \lambda) = 1 - e^{-\lambda x}$, for $x \geq 0$. Therefore, using equation (3.2) for these exponential distribution functions, the supply lead time distribution function is

$$F_{Ls}(t) = (1 - e^{-(1/5)*(t-5)})(1 - e^{-(1/10)*(t-0)})$$

Referring to Chapter 2.1, the manufacturing lead time estimation function based on the WIP level is found for the system. From equation (3.4)

$$(5.0)(1 - F_{integrated}(12 + x)) = (0.1) F_{integrated}(12 + x - 3) + 1.0$$

Manufacturing lead time can be calculated as (from Chapter 2.1):

$$l_m(i) = 0.0223(n-1)^2 + 0.6938(n-1) + 3.0407$$

Therefore,

$$l_m(3) = 4.5$$

From equation (3.6), the integrated total completion time random variable has the distribution of

$$F_{integrated}(t) = F_{Ls}(t - l_m) = F_{Ls}(t - 4.5)$$

Hence,

$$F_{integrated}(12 + x) = F_{Ls}(12 + x - 4.5) = F_{Ls}(7.5 + x)$$

$$F_{integrated}(9 + x) = F_{Ls}(9 + x - 4.5) = F_{Ls}(4.5 + x)$$

Therefore, by replacing into the equation from (3.4),

$$5 (1 - F_{Ls}(7.5 + x)) = 0.1 F_{Ls}(4.5 + x) + 1$$

The F_{Ls} functions are replaced, and LINGO is used to solve for x to minimize the function (see appendix I).

Results show the x (Δd) as 10.9 days and a quoted due date (D_q) of 22.9 days. Since no real data is available to calculate the adjustment factor, an integrated adjustment factor of 1.1 is assumed for this example. Using equation (3.6) and the estimation function for standard deviation of flow time from the manufacturing example

$$\sigma_m (i) = 0.0699 + 0.0991n$$

Therefore,

$$\sigma_m (3) = 0.37$$

The integrated quoted lead time is

$$L_{integrated} = 22.9 + 1.1 * 1.645 * 0.37 = 23.6 \text{ (days)}$$

Therefore, the company commits to a due date 23.6 days that includes both supply and manufacturing processes at 90 percent service level.

3.3.2 Effect of Cost Parameters on Quoted Due Date for Single Order Integrated Lead Time Quotation:

The presented model has considered three types of costs: cost of holding inventory, cost of reimbursement, and cost of tardiness. Estimating cost of inventory holding is a relatively straightforward matter. However, reimbursement and tardiness costs are non-defined costs. For the example in Section 3.3.1, cost parameters have been changed and the quoted due date (D) is calculated as follows,

Case No.	C_t ($/unit/day)	C_h ($/unit/day)	C_r ($/unit/day)	D_q (day)
1	5	0.1	1	22.9
2	7	0.1	1	25.7
3	8	0.1	1	26.8
4	5	0.1	0.5	28.1
5	5	0.1	0.7	25.6
6	5	0.1	1.5	19.9
7	5	0.3	1	32
8	5	0.5	1	21.3
9	5	0.3	0.5	26.1
10	5	0.3	1.5	15.6

The results in the above different cases show that increasing cost of tardiness (Ct) has significant increase on the committed due date to the customer. On the other hand, for a constant tardiness cost, quoted due date is less even with increased holding cost (Ch) and cost of reimbursement (Cr).

In real industries, companies have goals or agreements involving these measures. However, a method for matching penalty costs as a solid measure is worthwhile to study in the future.

3.3.3 Effect of Upper Requested Date (d_u) on Quoted Due Date for Single Order Integrated Lead Time Quotation:

In the presented model, all three types of costs associated to an order are affected by the upper requested due date (d_u) from customer. Therefore d_u has effect on the quoted due date (d_q). For the example in Section 3.3.1, for different d_u, quoted due date (d_q) is calculated as follows,

Case No.	d_u (day)	d_q (day)
1	8	26.9
2	12	22.9
3	14	20.9
4	16	18.9
5	20	14.9

The numerical results show that for a constant parameters, it is possible to quote shorter due dates for wider requested due date by the customer. However, a method for matching requested due date and also penalty costs as a solid measure is worthwhile to study in the future.

3.3.4 Integrated Lead Time Quotation under Service Level Multiple Order Example

Now suppose that the manufacturing company has two orders: one order is described in the example above and the other has an upper requested due date of 13 days from now. As already described, the first step in the procedure for multiple order problems is to sequence the order by upper requested date. Since the order described in the previous example has an earlier upper due date, the same due date as described above would be quoted for the order.

To determine a due date for the second order, it is necessary to determine the components purchase orders. Suppose that the second order has the same purchase orders as the first order. Manufacturing lead time can be calculated as

$$L_m\,(i) = 0.0223(n-1)^2 + 0.6938(n-1) + 3.0407$$

Therefore,

$$l_m(4) = 5.3$$

From equation (4.4)

$$(5.0)\,(1- F_{Ls}\,(13 + x)) = (0.1)\,F_{Ls}\,(13 + x -3) + 1.0$$

Hence,

$$F_{integrated}\,(13 + x) = F_{Ls}\,(13 + x - 5.3) = F_{Ls}\,(7.7 + x)$$

$$F_{integrated}(10 + x) = F_{Ls}(10 + x - 5.3) = F_{Ls}(4.7 + x)$$

Therefore,

$$5\,(1- F_{Ls}(7.7 + x)) = 0.1\, F_{Ls}(4.7 + x) + 1$$

The F_{Ls} functions and LINGO are used to solve for x to minimize the function (see appendix I).

Results indicate that x (Δd) is 10.8 days and the quoted due date (D_q) is 23.8 days. Similarly with an integrated adjustment factor of 1.1 for this example and using equation (3.6) and the estimation function for standard deviation of the flow time from the manufacturing example,

$$\sigma_m(i) = 0.0699 + 0.0991n$$

Therefore,

$$\sigma_m(4) = 0.5$$

and the integrated quoted lead time is

$$L_{integrated} = 23.8 + 1.1 * 1.645 * 0.5 = 24.7 \text{ (days)}$$

Therefore, the company commits a due date of 24.7 days, which includes both supply and manufacturing processes at 90 percent service level.

3.4 Integrated Model Effectiveness Evaluation

Chapter 2 presented a modified methodology based on the work of Hopp and Sturgis (2000) to quote manufacturing lead time (l_m) taking into account the status of the system (WIP). The estimated manufacturing lead time is used in the integrated model that considers the supply process as well.

In order to study the effectiveness of the integrated model, a single order example is compared to an M/M/1 system for which the exact due date quotes can be found analytically (see details of the exact method in Section 2.4).

It is desired to develop another type of manufacturing lead time estimation to be used in the integrated model and compare them. One possible method could be using average flow time for different levels of WIP based the data available in the system. Section 3.4.1 shows a model which uses manufacturing lead time based on the model in Chapter 2 and Section 3.4.2 shows an integrated model which uses the average flow time. A comparison between these two models is presented for the M/M/1 integrated model.

3.4.1 Model Effectiveness using Manufacturing Lead Time from Chapter 2

A customer has requested a product to be delivered within 10 to 15 days. Based on the available information, the demand has one purchase order and other components that are available in inventory. The purchase order is placed at the present time and has an exponentially distributed lead time with an expected lead time of 10 days. Assume that the cost of holding inventory for finished products is \$0.1 per unit per day. The cost of reimbursement is \$1 per unit per day, and the cost of tardiness is \$5 per unit per day. The manufacturing process (see section 3.4) consists of an M/M/1 system. The number of jobs in the system (WIP level) is assumed to be from 1 to 30. Target service levels are 95, 90, and 85 percent. Therefore, the problem can be summarized as follows:

$d_u = 15$ (days)

$d_l = 10$ (days)

$t(1) = 0$, $F(L_s(1)) \sim EXP(x, 10)$

$C_h = 0.1$ (\$/unit/day)

$C_r = 1$ (\$/unit/day)

$C_t = 5$ (\$/unit/day)

Using equation (2.15) for the exponential distribution function, the supply lead time is

$$F_{Ls}(t) = (1- e^{-(1/10)*(t-0)})$$

Referring to Chapter 2, the manufacturing lead time estimation function is found for the system based on the WIP level. And from equation (3.6), the integrated total completion time random variable has the distribution,

$$F_{integrated}(t) = F_{Ls}(t - l_m)$$

From equation (3.4)

$$(5.0)\ (1- F_{integrated}(15 + x)) = (0.1)\ F_{integrated}(15 + x -10) + 1.0$$

Hence,

$$F_{integrated}(15 + x) = F_{Ls}(12 + x - l_m)$$

$$F_{integrated}(5 + x) = F_{Ls}(5 + x - l_m)$$

Replacing the F_{Ls} functions and using LINGO to solve for x, Table 3.1 shows the distribution functions and Δd for different WIP levels for M/M/1 system.

TABLE 3.1

DISTRIBUTION FUNCTIONS AND DELAY FOR DIFFERENT WIP LEVELS FOUND BY SOLVING EQUATION IN LINGO FOR M/M/1 SYSTEM AND USING THE INTEGRATED METHOD

n	1	2	3	4	5	6	7	8	9	10
Manufacturing flow time (l_m)	1.00	2.20	3.30	4.40	5.51	6.61	7.71	8.81	9.91	11.01
$F_{integrated}(t)= F_{Ls}(t - l_m)$	$F_{Ls}(t-1)$	$F_{Ls}(t-2.2)$	$F_{Ls}(t-3.3)$	$F_{Ls}(t-4.4)$	$F_{Ls}(t-5.51)$	$F_{Ls}(t-6.61)$	$F_{Ls}(t-7.71)$	$F_{Ls}(t-8.81)$	$F_{Ls}(t-9.91)$	$F_{Ls}(t-11.01)$
$F_{integrated}(15+x)= F_{Ls}(15+x-l_m)$	$F_{Ls}(x+14)$	$F_{Ls}(x+12.80)$	$F_{Ls}(x+11.7)$	$F_{Ls}(x+10.6)$	$F_{Ls}(x+9.5)$	$F_{Ls}(x+8.39)$	$F_{Ls}(x+7.29)$	$F_{Ls}(x+6.19)$	$F_{Ls}(x+5.09)$	$F_{Ls}(x+3.99)$
$F_{integrated}(5+x)= F_{Ls}(5+x-l_m)$	$F_{Ls}(x+4)$	$F_{Ls}(x+2.8)$	$F_{Ls}(x+1.7)$	$F_{Ls}(x+0.6)$	$F_{Ls}(x-0.51)$	$F_{Ls}(x-1.61)$	$F_{Ls}(x-2.71)$	$F_{Ls}(x-3.81)$	$F_{Ls}(x-4.91)$	$F_{Ls}(x-6.01)$
Δd or x	1.67	2.87	3.97	5.07	6.17	7.28	8.38	9.48	10.58	11.68

n	11	12	13	14	15	16	17	18	19	20
Manufacturing flow time (l_m)	12.11	13.21	14.31	15.41	16.52	17.62	18.72	19.82	20.92	22.02
$F_{integrated}(t)= F_{Ls}(t - l_m)$	$F_{Ls}(t-12.11)$	$F_{Ls}(t-13.21)$	$F_{Ls}(t-14.31)$	$F_{Ls}(t-15.41)$	$F_{Ls}(t-16.52)$	$F_{Ls}(t-17.62)$	$F_{Ls}(t-18.72)$	$F_{Ls}(t-19.82)$	$F_{Ls}(t-20.92)$	$F_{Ls}(t-22.02)$
$F_{integrated}(15+x)= F_{Ls}(15+x-l_m)$	$F_{Ls}(x+2.89)$	$F_{Ls}(x+1.79)$	$F_{Ls}(x+0.69)$	$F_{Ls}(x-0.41)$	$F_{Ls}(x-1.52)$	$F_{Ls}(x-2.62)$	$F_{Ls}(x-3.72)$	$F_{Ls}(x-4.82)$	$F_{Ls}(x-5.92)$	$F_{Ls}(x-7.02)$
$F_{integrated}(5+x)= F_{Ls}(5+x-l_m)$	$F_{Ls}(x-7.11)$	$F_{Ls}(x-8.21)$	$F_{Ls}(x-9.31)$	$F_{Ls}(x-10.41)$	$F_{Ls}(x-11.52)$	$F_{Ls}(x-12.62)$	$F_{Ls}(x-13.72)$	$F_{Ls}(x-14.82)$	$F_{Ls}(x-15.92)$	$F_{Ls}(x-17.02)$
Δd or x	12.78	13.88	14.98	16.08	17.18	18.28	19.38	20.48	21.58	22.68

n	21	22	23	24	25	26	27	28	29	30
Manufacturing flow time (l_m)	23.12	24.22	25.32	26.42	27.53	28.63	29.73	30.83	31.93	33.03
$F_{integrated}(t)= F_{Ls}(t - l_m)$	$F_{Ls}(t-23.12)$	$F_{Ls}(t-24.22)$	$F_{Ls}(t-25.32)$	$F_{Ls}(t-26.42)$	$F_{Ls}(t-27.53)$	$F_{Ls}(t-28.63)$	$F_{Ls}(t-29.73)$	$F_{Ls}(t-30.83)$	$F_{Ls}(t-31.93)$	$F_{Ls}(t-33.03)$
$F_{integrated}(15+x)= F_{Ls}(15+x-l_m)$	$F_{Ls}(x-8.12)$	$F_{Ls}(x-9.22)$	$F_{Ls}(x-10.32)$	$F_{Ls}(x-11.42)$	$F_{Ls}(x-12.53)$	$F_{Ls}(x-13.63)$	$F_{Ls}(x-14.73)$	$F_{Ls}(x-15.83)$	$F_{Ls}(x-16.93)$	$F_{Ls}(x-18.03)$
$F_{integrated}(5+x)= F_{Ls}(5+x-l_m)$	$F_{Ls}(x-18.12)$	$F_{Ls}(x-19.22)$	$F_{Ls}(x-20.32)$	$F_{Ls}(x-21.42)$	$F_{Ls}(x-22.53)$	$F_{Ls}(x-23.63)$	$F_{Ls}(x-24.73)$	$F_{Ls}(x-25.83)$	$F_{Ls}(x-26.93)$	$F_{Ls}(x-28.03)$
Δd or x	23.78	24.88	25.98	27.08	28.18	29.28	30.38	31.48	32.58	33.68

Random numbers for the supply process are generated (see Appendix G for how to generate a random exponential distribution in Excel). Using equations (3.7) and (3.8) and a similar procedure from Chapter 2, the following integrated adjustment factors (η) are found:

1.596 at 95 percent, 0.135 at 90 percent, and 0.008 at 85 percent service levels.

Similar to the procedure in Chapter 2, the theoretical method manufacturing lead times for all service levels are found. An average supply lead time of ten days is assumed, and the total lead time for different WIP levels is calculated. Table 3.2 shows a summary of quoted integrated lead times for an M/M/1 system for both an analytical method and the method described here.

TABLE 3.2

QUOTED LEAD TIMES FOR M/M/1 SYSTEM USING MANUFACTURING LEAD TIME FROM CHAPTER 2 —ANALYTICAL AND INTEGRATED MODELS AT THREE SERVICE LEVELS

Service Level= 95%	n	1	2	3	4	5	6	7	8	9	10
	Quoted Lead Time in Analytical Method	12.91	14.63	16.17	17.62	19.00	20.35	21.67	22.97	24.25	25.51
	Quoted Lead Time in Integrated Method	16.46	14.46	13.01	23.28	26.27	29.31	32.01	34.19	35.84	37.02
	n	11	12	13	14	15	16	17	18	19	20
	Quoted Lead Time in Analytical Method	26.76	28.00	29.23	30.45	31.66	32.86	34.06	35.26	36.44	37.63
	Quoted Lead Time in Integrated Method	37.88	38.62	39.42	40.47	41.93	43.89	46.42	49.50	53.06	56.97
	n	21	22	23	24	25	26	27	28	29	30
	Quoted Lead Time in Analytical Method	38.80	40.00	41.15	42.32	43.47	44.63	45.79	47.00	48.09	49.24
	Quoted Lead Time in Integrated Method	61.05	65.09	68.84	72.09	74.63	76.30	77.06	77.00	76.37	75.67

Service Level= 90%	n	1	2	3	4	5	6	7	8	9	10
	Quoted Lead Time in Analytical Method	12.26	13.83	15.26	16.61	17.91	19.19	20.43	21.67	22.89	24.10
	Quoted Lead Time in Integrated Method	13.55	12.30	11.17	20.30	21.53	22.78	23.99	25.17	26.31	27.41
	n	11	12	13	14	15	16	17	18	19	20
	Quoted Lead Time in Analytical Method	25.30	26.48	27.66	28.84	30.00	31.16	32.32	33.47	34.62	35.76
	Quoted Lead Time in Integrated Method	28.50	29.57	30.65	31.75	32.87	34.03	35.23	36.48	37.75	39.05
	n	21	22	23	24	25	26	27	28	29	30
	Quoted Lead Time in Analytical Method	36.90	38.03	39.17	40.30	41.43	42.55	43.67	44.79	45.91	47.03
	Quoted Lead Time in Integrated Method	40.36	41.67	42.96	44.21	45.41	46.55	47.63	48.65	49.62	50.60

TABLE 3.2

QUOTED LEAD TIMES FOR M/M/1 SYSTEM USING MANUFACTURING LEAD TIME FROM CHAPTER 2 – ANALYTICAL AND INTEGRATED MODELS AT THREE SERVICE LEVELS
(continued)

	n	1	2	3	4	5	6	7	8	9	10
Service Level= 85%	Quoted Lead Time in Analytical Method	11.87	13.33	14.68	15.96	17.21	18.43	19.64	20.83	22.01	23.20
	Quoted Lead Time in Integrated Method	13.34	12.14	11.04	20.08	21.19	22.31	23.41	24.52	25.62	26.72
	n	11	12	13	14	15	16	17	18	19	20
	Quoted Lead Time in Analytical Method	24.33	25.47	26.63	27.77	28.90	30.03	31.16	32.28	33.40	34.52
	Quoted Lead Time in Integrated Method	27.82	28.92	30.01	31.11	32.22	33.32	34.42	35.53	36.64	37.75
	n	21	22	23	24	25	26	27	28	29	30
	Quoted Lead Time in Analytical Method	35.63	36.74	37.85	38.95	40.06	41.16	42.26	43.36	44.45	45.55
	Quoted Lead Time in Integrated Method	38.86	39.97	41.08	42.19	43.30	44.40	45.50	46.59	47.69	48.78

The efficiency of the quoted due dates using the integrated model are compared using several measures, such as mean earliness, mean tardiness, and mean missed due date (similar to work by Hegedus and Hopp (2000)). Mean earliness (tardiness) is defined as the average of the absolute time between the quoted due date and the flow time for all jobs that are completed early (late). Mean missed due date is defined as the mean absolute value of the time difference between the quoted due date and flow time for all jobs. Mean missed due date is the sum of mean earliness and mean tardiness (see Weeks, 1979).

Table 3.3 shows the result of performance measures calculated as the averages of the three different runs for the target service levels of 95, 90, and 85 percent.

TABLE 3.3

EFFECTIVENESS COMPARISON OF ANALYTICAL AND INTEGRATED LEAD TIME QUOTATION (USING MANUFACTURING LEAD TIME FROM CHAPTER 2) METHODS

	M/M/1 Analysis	Integrated Method	Theoretical Method
Service Level= 95%	Mean Lead Time Quote	47.803	31.931
	Mean Earliness	18.142	2.377
	Mean Tardiness	2.616	1.621
	Mean Missed Due date	20.758	3.998
Service Level= 90%	Mean Lead Time Quote	33.268	30.320
	Mean Earliness	3.563	1.238
	Mean Tardiness	2.914	1.699
	Mean Missed Due date	6.477	2.937
Service Level= 85%	Mean Lead Time Quote	32.215	29.255
	Mean Earliness	2.651	0.816
	Mean Tardiness	2.082	1.680
	Mean Missed Due date	4.733	2.496

From these, the following can be concluded:

- The results of Table (3.3) show that for 90 and 85 percent service levels, the mean lead time quote is just 9.72 and 10.12 percent, respectively, higher than the analytical method. The integrated model has higher mean tardiness and earliness than the analytical method. Overall, results indicate that the performance of the integrated model is higher than the analytical method, but it is good.
- Performance measures show that the integrated method is better at the 85 percent service level in comparison to 95 and 90 percent service levels. It seems that the integrated model performs better at the lower service level.

- Using the integrated method for quoting lead time to meet target service level is simple, practical, and effective. It makes no assumptions about the distribution of processing times, which makes it appropriate to many real-world systems.
- The integrated method is easy to use because it employs data that is already collected, such as the number of jobs in the system and bottleneck rate.

3.4.2 Model Effectiveness using Average Manufacturing Flow Time

For the example in Section 3.4.1, instead of using the manufacturing lead time from Chapter 2, the average of manufacturing lead time is used (32.158 days over WIP level from 1 to 30). Using the method from Figure 3.3, the following integrated adjustment factors (η) are found: 0.669 at 95 percent, 0.618 at 90 percent, and 0.531 at 85 percent service levels

Using a similar procedure, quoted integrated lead times is found. Table 3.4 shows the result of performance measures for analytical method, the integrated model using manufacturing lead time from Chapter 2 and integrated model using the average flow time.

TABLE 3.4

EFFECTIVENESS COMPARISON OF ANALYTICAL, INTEGRATED LEAD TIME QUOTATION USING MANUFACTURING LEAD TIME FROM CHAPTER 2 AND AVERAGE FLOW TIME METHODS

	M/M/1 Analysis	Integrated Model Using Manufacturing Lead time from Chapter 2	Integrated Model Using Average Manufacturing Lead time	Theoretical Model
Service Level= 95%	Mean Quoted Lead Time	47.803	38.716	31.931
	Mead Earliness	18.142	8.654	2.377
	Mean Tardiness	2.616	0.088	1.621
	Mean Missed Due Date	20.758	8.742	3.998
Service Level= 90%	Mean Quoted Lead Time	33.268	37.242	30.320
	Mead Earliness	3.563	7.940	1.238
	Mean Tardiness	2.914	2.572	1.699
	Mean Missed Due Date	6.477	10.572	2.937
Service Level= 85%	Mean Quoted Lead Time	32.215	35.982	29.255
	Mead Earliness	2.651	7.156	0.816
	Mean Tardiness	2.082	4.301	1.680
	Mean Missed Due Date	4.733	11.457	2.496

The following can be concluded:

- The results of Table (3.4) shows that for 90 and 85 percent service levels, the integrated lead time quote using average manufacturing lead time has higher performance measures than the integrated lead time quote using manufacturing lead time from method presented in Chapter 2 and both have higher measures than the analytical method. The integrated model using average manufacturing lead time has higher mean earliness and missed due date than the method using the method from Chapter 2. Overall, results indicate that for 90% and 85% service levels, the performance of the integrated model using method in Chapter 2 is better than using average manufacturing lead time. It could be because of using the estimated regression functions of flow time versus WIP which is more accurate than using the average flow time.

- It seems that for a high service level of 95%, using average flow time is more efficient. Some more studies are needed.

3.5 Integrated Lead Time Quotation Model using Different Safety Lead Times

The safety lead time model in function (3.8) only considers the standard deviation of manufacturing flow time assuming that since the supply lead time in the functions is considered through distributions, there is no need to have its standard deviation.

It is desired to propose a safety lead time for the integrated model that considers both variations of manufacturing and supply process and also use the procedure to adjust adjustment factor (Figure 3.3) to achieve the service level and compare the results to the already proposed safety lead time. The following safety lead time is proposed,

$$sf(i) = \eta Z_{\alpha} \sigma_m(i) (1/\lambda_s)$$

where

α = target service level

Z_{α} = α-percentile of the standard normal distribution

$\sigma_m(i)$ = standard deviation of manufacturing flow time when n_i jobs are in system

$1/\lambda_s$ = standard deviation of supply lead time

η = integrated adjustment factor

On the other hand, it is desired to compare the safety lead times from function (3.8) and the above one with a real world model. It could be a safety lead time that just includes an adjustment factor not including any Z value or standard deviations. This is what actually some companies use.

Therefore, a real world safety lead time model such as $sf(i) = \eta$ will be compared with both above safety lead times. The summary of the all safety lead times are as follows,

a) $sf(i) = \eta Z_\alpha \sigma_m(i)$

b) $sf(i) = \eta\ Z_\alpha\ \sigma_m(i)\ (1/\lambda_s)$

c) $sf(i) = \eta$

For the example in Section 3.4.1, using the manufacturing lead time from Chapter 2, for three different service level 95%, 90% and 85%, the quoted integrated lead time is calculated. For the safety lead time (c), the average of the three adjustment factors in example of Section 3.4.1 (1.596 at 95 percent, 0.135 at 90 percent, and 0.008 at 80%), 0.580 is used. Using this constant safety lead time, a service level of 86.67% is achieved.

Table 3.5 shows the results of performance measures for the three different safety lead times.

TABLE 3.5

EFFECTIVENESS COMPARISON OF INTEGRATED LEAD TIME QUOTATION USING DIFFERENT SAFETY LEAD TIMES

		(a) Integrated Model Using Safety Lead Time: sf(i) = η Zα σm(i)	(b) Integrated Model Using Safety Lead Time: sf(i) = η Zα σm(i) (1/λs)	(c) Integrated Model Using Safety Lead Time: sf(i) = η
Service Level= 95%	Mean Quoted Lead Time	47.803	33.722	32.738
	Mead Earliness	18.142	4.051	3.066
	Mean Tardiness	2.616	2.772	2.045
	Mean Missed Due Date	20.758	6.824	5.111
Service Level= 90%	Mean Quoted Lead Time	33.268	32.269	32.738
	Mead Earliness	3.563	2.607	3.066
	Mean Tardiness	2.914	2.581	2.045
	Mean Missed Due Date	6.477	5.188	5.111
Service Level= 85%	Mean Quoted Lead Time	32.215	32.163	32.738
	Mead Earliness	2.651	2.593	3.066
	Mean Tardiness	2.082	2.105	2.045
	Mean Missed Due Date	4.733	4.698	5.111

The following can be concluded:

In the above examples, for safety lead time method (c), an average of adjustment factors from Section 3.4.1 (0.580) is used. In real world, making decision about this constant safety lead time is vital. Although it has better performance measure than to the other two safety lead time, a service level of 86.67% is achieved. It could be leading to not satisfy the customer retention.

Between safety lead times in methods (a) and (b), method (b) which includes the supply process standard deviation has better performance measures at all service levels; for instance for mean missed due dates, method (b) 67.1%, 19.9% and 0.7% is better than method (a) at 95%, 90% and 85% service level respectively.

Overall, it seems that using a safety lead time that includes both variations of supply and manufacturing process has better performance because of less mean missed due dates. Using a constant safety lead time (method c) could be easy and simple in practice, however some pre-studies to find a better η value for this method is always essential in practice otherwise, the service level achieved is not the one desired.

3.6 Analysis of Some Special Cases

This section analyzes the characteristics of the solution to the problem for some special cases (equation (3.4)): (a) in real world applications it is possible that the customer does not accept early delivery because of lack of storage space, etc.; (b) there is not any restrictions on early delivery of orders; (c) the customer requests a due date less than the early delivery limit. Before explaining the special cases, we rewrite equation (3.4) as

$$C_h \, F_{integrated} \, (D_q - d_l) + C_t \, F_{integrated} \, (D_q)) = C_t - C_r$$

The left side of the function is positive, therefore $(C_t - C_r)$ must be positive. This implies that the cost of tardiness must be greater than or equal to the cost of reimbursement ($C_t \geq C_r$).

3.6.1 Early Delivery Is Not Accepted by Customer

Considering a special case of the problem where early deliveries to customers are not allowed, this problem is modeled by setting $d_l = 0$. The solution to the problem found in equation (3.4) for this case then reduces to:

$$F_{integrated}(D_q) = \frac{Ct - Cr}{Ct + Cr} \tag{3.9}$$

Comparing this equation to equation (3.4), it can be seen that due dates for environments that do not allow early shipments will be lower than those environments that allow early shipment. This is because expected inventory holding costs are greater under these conditions. Therefore, it is possible to reduce the quoted due dates and consequently inventory holding costs.

3.6.2 Early Delivery Does Not Have Limitation by Customer

Next is a special case where there is no limit on early deliveries of orders satisfying customer orders. This is modeled by setting $d_l = d_q$ and $C_h = 0$. Under this circumstance, a manufacturer would never hold the finished order in inventory, so the inventory holding cost term can be removed from the formulation of the problem. This causes the solution to reduce to

$$F_{integrated}(D_q) = \frac{Ct - Cr}{Ct} \tag{3.10}$$

or equivalently

$$F_{integrated}(D_q) = 1 - \frac{Cr}{C_t} \tag{3.11}$$

From equation (3.4)

$$F_{integrated}(D_q) + C_h/C_t\, F_{integrated}(D_q - d_l) = 1 - C_h/C_t \tag{3.12}$$

Comparing equation (3.12) to equation (3.11), it is clear that due dates for environments that maintain a finished products inventory is lower than those environments that do not. A practical result is that a customer can make early delivery constraints shorter to reduce quoted lead times.

3.6.3 Customer Requests Due Dates Less Than Early Delivery Limit

The proposed problem formulation assumed that the requested due date was greater than or equal to the delivery limit ($d_u \geq d_l$). Now consider the case where this condition is not met. If ($d_u < d_l$), inventory holding cost can only occur when ($\Delta d > d_l - d_u$). Thus the formulation of the problem becomes

$$C = C_h \int_{d_l - d_u}^{\Delta d} (d_q - d_l - t)\, f(t)\, dt \quad + C_r \Delta d + C_t \int_{d_q}^{\infty} (t - d_q)\, f(t)\, dt \qquad (3.13)$$

Similarly, for $d_u < d_l$, the equation can be written as,

$$C_t\, (1 - F_{integrated}\, (D_q)) = C_h\, (F_{integrated}\, (\Delta d) - F_{integrated}\, (d_l - d_u)) + C_r \qquad (3.14)$$

3.6.4 Operational Issues

The presented problem in function (3.4) has three cost coefficients: cost of inventory holding, cost of reimbursement, and cost of tardiness. Estimating the cost of holding inventory is a relatively straightforward matter. However, reimbursement and tardiness costs are non-defined costs. A method for matching penalty costs rates as a solid measure is worthwhile to study in the future.

3.7 Conclusions

A method for quoting due dates for demand orders with requested due dates was developed. This method incorporates a two stage production model that describes inventory, reimbursement and tardiness costs, and service level issues. The underlying production model assumes that production is constrained primarily by the procurement process, as is the case in many assembly systems in real industries. For this reason, the approximation method of the manufacturing stage of the production process is used, as presented in Chapter 2. A simple optimal policy for both single and multiple order problems is obtained. This method is practically well suited to any environment for production planning and procurement.

The formulation of the problem includes a limit on lower date requested by customer (for deliveries) which introduces a finished products inventory holding cost. It was shown that limiting early deliveries results shorter quoted due dates in an effort to avoid inventory costs. Hence, if a manufacturing company limits early deliveries, they should realize shorter lead times but may maintain higher component costs. On the other hand, when there is no limit on early deliveries, longer lead times can be quoted.

The distinguishing feature of the formulation of this due date quoting problem over other models in the literature is its emphasis on the production supply process and the handling of requested window dates in the form of cost of reimbursement. Additional supply information can significantly improve results, such as some studies have shown on delay and tardiness costs.

CHAPTER 4

CONCLUSIONS AND FUTURE WORK

4.1 Conclusions

This study presented a method for quoting lead time for job orders with requested due dates and a window delivery range. This method integrates a two phase production model that considers service level, WIP, costs of inventory, tardiness, and reimbursement. The presented integrated production model assumes the supply process constraint as well as manufacturing process similar to that in real world industries. For this reason, a method is used to approximate manufacturing lead time based on the WIP. This approximation method uses the available data in the system to approximate manufacturing flow time through an endogenous method. A cost minimization formula to quote due date that consider both production phases is developed. The function considers three kinds of costs such as earliness, tardiness and reimbursement as well as a customer requested deliver window. With the use of the cost minimization formula, it is possible to obtain a simple lead time quotation methodology for both single and multiple orders. The supply process lead time distribution for all components is also extracted from the available historical data not considering any specific assumptions. It was shown that limiting early deliveries results in shorter quoted due dates to avoid holding costs. On the other hand, when there is no limit on early deliveries, longer lead times are quoted. Results indicate that the effectiveness of the integrated model is good in comparison to the analytical method.

Features of this method of lead time quotation that distinguishes it from other models in the literature is as follows: it makes no assumptions about the distribution of manufacturing processing times, which makes it very useful for real world systems; it is easy to practice and implement because it simply uses data that is already collected by the company, like the number of jobs in the system and the rate of bottleneck; it emphasizes the supply process when considering requested window dates by customer and cost of reimbursement; most of the research problems make use of an average lead time for the supply process.

4.2 Future Work

This research leaves open the opportunity for additional related work. Here, the First In First Out was assumed for the manufacturing process. It would be important to study other sequencing or re-sequencing methods. There is also opportunity to study other adjustment factor procedures such as start point other than one, and different incremental increase of the factor. Since in this research results showed that the models works better for lower service levels, a study about effect of different service level on the results also can be done to find a range for service level that can provide better results. Since no real data was available for this research, simulation was used to generate data for the manufacturing process, and some distributions were assumed for the supply process. It would be important to use the method studied here with real data. In the presented cost minimization formula a major factor is the customer requested deliver range. It would be great to use a real range and verify the model.

The model in this study has three cost coefficients: cost of holding inventory, cost of reimbursement, and cost of tardiness. It is a relatively straightforward matter to estimate holding cost. However, costs of reimbursement and tardiness are non-defined costs. A method for matching penalty costs as a solid measure would be worth studying in the future. In fact, the relation between customers' requested range and all associated three types of costs should be studied. This can be done by using some real data and using different techniques such as regression to find the correlation between these factors and effect on the quoted due date.

BIBLIOGRAPHY

BIBLIOGRAPHY

Baker, K.R. and J.W.M. Bertrand (1981), A Comparison of Due-Date Selection Rules. *AIIE Transactions* 13, pp. 123-131.

Bertrand, J.W.M. (1983), The Effect of Workload Dependent Due-Dates on Job Shop Performance. *Management Science* 29, pp. 799-816.

Biskup, D. and M. Feldmann (2005), On Scheduling Around Large Resterictive Common Due Windows, *Eur. J. Oper. Res.* Vol. 162, no. 3, pp. 740-761.

Bookbinder, J. and A. Noor (1985),Setting Job-Shop Due-dates with Service-Level Constraints. *Journal of Operation Research Society 36*, pp. 1017-1026.

Brucker, P. (1998), *Scheduling Algorithms.* Springer, New York.

Chand, S. and D. Chhajed (1992), A Single Machine Model for Determination of Optimal Due Date and Sequence. *Operations Research* 40, pp. 596-602.

Cheng, T.C.E. and M.C. Gupta (1991), Survey of Scheduling Research Involving Due Date Determination Decisions. *European Journal of Operational Research* 38, pp.156-166.

Cheng, T.C.E, Chen, Z.L. and Shakhlevich, N.V. (2002), Common Due Date Assignment and Scheduling with Ready Times. *Computer and Operations Research* 29, pp. 1957-1967.

Cheng, T.C.E., Ding, Q. and Lin, B.M.T. (2004), A Concise Survey of Scheduling with Time-Dependent Processing Times. *European Journal of Operational Research*, v 152, pp. 1-13.

Cheng, T.C.E., Kang, L., and Ng, C.T. (2004), Due-Date Assignment and Single Machine Scheduling With Deteriorating Jobs. *Journal of the Operational Research Society* 55, pp. 198-203.

Duenyas, I. (1995), Single Facility Due Date Setting with Multiple Customer Classes. *Management Science* 41, pp. 608-619.

Duenyas, I and W.J. Hopp (1995), Quoting Customer Lead Times*. Management Science* 41, pp. 43-57.

Eilon, S. and I.G. Chowdhury (1976), Due Dates in Job Shop Scheduling. *International Journal of Production Research* 14, pp. 223-237.

Gorden, V., Proth J.M., Chu C. (2002), Due Date Assignment and Scheduling: SLK, TWK and Other Due Date Assignment Models. *Production Planning and Control, 13,* pp.117-132.

Gorden, V., Proth J.M., Chu C. (2002), A Survey of the State-of-the-art of Common Due Date Assignment and Scheduling Research. *European Journal of Operational Research, 139,* pp.1-25.

Hall, N.G. and M.E. Posner (1991), Earliness-Tardiness Scheduling Problems, I: Weighted Deviation of Completion Times about a Common Due Date. *Operations Research* 39, pp. 836-846.

Hariri, A.M.A. and C.N. Potts (1983), An Algorithm for Single Machine Sequencing with Release Dates to Minimize Total Weighted Completion Time. *Discrete Applied Math* 5, pp. 99-109.

Hegedus, M. and Hopp, W.J. (2001), Setting Procurement Safety Lead Times for Assembly Systems. *International Journal of Production Research* 39, pp. 3459-3478.

Hopp, W. and Spearman, M. (1993), Setting Safety Lead times for Purchased Components in Assembly Systems. *IIE Transactions 2*5, pp. 2-11.

Hopp, W. and Spearman, M. (2001), Factory Physics. 2nd edition, *Mc Graw Hill*.

Hopp, W. and Sturgis, M.L Roof (2000), Quoting Manufacturing Due Dates Subject to a Service Level Constarint. *IIE Transactions 32*, pp. 771-784.

Hsu, S. Y. and Sha, D. Y. (2004), Due Date Assignment Using Artificial Neural Networks Under Different Shop Floor Control Strategies, *International Journal of Production Research* 42, pp. 1727-1745.

Iyer, A.V., Deshpande, V. and Wu, Z. (2003), A Postponement Model for Demand Management, *Management Science* 49 (8), pp. 983–1002.

Janiak, A. and Marek, M. (2001), Multi-machine Scheduling Problem with Optimal Due Interval Assignment Subject to Generalized Sum Type Criterion, *In Proc. Oper. Res.*, pp. 207 -212.

Janiak, A. and Marek, M. (2003), Multi-processor Scheduling Problem with Due Interval Assignment Subject to Generalized Min-Max Type Criterion, *In Proc. 9th IEEE Int. Conf. Methods and Models Autom and Robot., Miedzyzdroje, Poland,* vol 2, pp. 1169 -1173.

Janiak, A. and Winczaszek, M. (2003), An Optimal Algorithm for a Single Processor Scheduling Problem with a Common Due Window, *In Proc. 9th IEEE Int. Conf. Methods and Models Autom and Robot., Miedzyzdroje, Poland,* pp. 1213 -1216.

Janiak, A., Kovalyov, M.Y., and Marek, M. (2007), Soft Due Window Assignment and Scheduling on Parallel Maachine, *IEEE Transactions on Systems, MAN, and Cybernetics –Part A* 37, pp. 614 -620.

Kahlbacher, H. (1992), Termin-und Ablaufplanung - ein analytischer Zugang, *Ph.D. thesis,* University of Kaiserslautern, cited in [3]

Keskinocak, P., R. Ravi, and S. Tayur (1997), Algorithms for Reliable Lead Time Quotation. *GSIA Working Paper,* Carnegie Mellon University, Pittsburgh, PA.

Koulamas, C. (1997), Maximizing the Weighted Number of On-Time Jobs in a Single-machine Scheduling with Time Windows, *Math. Comput. Model, vol* 25, no10, pp. 57-62.

Kramer, F.J., and Lee, C.Y. (1994), Due Window Scheduling for Parallel Machines, *Math. Comput. Model,* vol 20, no. 2, pp. 69-89.

Lee, C.Y. (1991), Earliness-Tardiness Scheduling Problems with Constant Size of Due Window, *Dept. Ind. and Syst. Eng., Univ. Florida, Gainsville, FL, Res. Rep.* 91-17.

Lenstra, J.K., A.H.G. Rinnooy Kan, and P. Brucker (1977), Complexity of Machine Scheduling Problems. *Annals of Discrete Math* 1, pp. 343-362.

Miyazaki, S. (1981), Combined Scheduling System for Reducing Job Tardiness in a Job Shop. *International Journal of Production Research* 19, pp. 201-211.

Mosheiov, G. (2001), A Due-Window Determination in Minmax Scheduling Problems. *INFOR,* vol 39, no 1, pp. 107-123.

Mosheiov, G. and Oron, D. (2004), Due-Window Assignment with Unit Processing-time Jobs. *Nav. Res. Logist,* vol 51, no 7, pp. 1005-1017.

Panwalkar, S.S., M.L. Smith, and A. Seidmann (1982), Common Due Date Assignment to Minimize Total Penalty for the One Machine Scheduling Problem. *Operations Research* 30, pp. 391-399.

Pinedo, M. (2002), *Scheduling: Theory, Algorithms* and *Systems,* Second Edition, Prentice Hall, Inc. Englewood Cliffs, New Jersey.

Potts, C.N. (1980), Analysis of a Heuristic for One Machine Sequencing with Release Dates and Delivery Times. *Operations Research* 28, pp. 1436-1441.

Schrage, L. (1968),A proof of the optimality of the shortest remaining processing time discipline, *Operations Research 16*, pp. 687-690.

Seidmann, A., S.S. Panwalker, and M.L. Smith (1981), Optimal Assignment of Due Dates For a Single Processor Scheduling Problem. *International Journal of Production Research* 19, pp. 393-399.

Smith, W.E. (1956), Various Optimizers for Single-stage Production. *Naval Res. Logist. Quart.* 3, pp. 59-66.

Song, J. and Zipkin, P. (1996), Inventory Control with Information about Supply Conditions. *Management Science, 42*, pp. 1409-1419.

Song, D.P, Hicks, C. and Earl, C.F. (2002), Product Due Date Assignment for Complex Assemblies. *Int J. Production Economics* 76, pp. 243-256.

Spearman, M., and R.Q. Zhang (1999), Optimal Lead Time Policies. *Management Science* 45, pp. 290-295.

Wang, D. and Fang, S. and Hodgson, T. (1998), A Fuzzy Due-Date Bargainer for the Make-to-order Manufacturing Systems, *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev., Vol* 28, no 3, pp. 492-497.

Wang, D. and Fang, S. and Nuttle, H. (1999), Soft Computing for Multicustomer Due-date Bargaining, IEEE *Trans. Syst., Man, Cybern. C, Appl. Rev., Vol* 29, no 4, pp. 566-575.

Wang, Ji-Bo. (2007), Flow Shop Scheduling Problems With Decreasing Linear Deterioration Under Dominant Machines. *Computer and Operations Research* 34, pp. 2043-2058.

Weeks, J.K. (1979), A Simulation Study of Predictable Due-Dates. *Management Science* 25, pp. 363-373.

Wein, L. M. (1991); Due-date Setting and Priority Sequencing in a Multiclass M/G/1 Queue. *Management Science* 37, pp. 834-850.

Yano,C. (1997); Demand Fulfillment Rates in an Assemble-to-order System with Multiple Products and Dependent Demands. *Production and operations Management 6*, pp. 309-324.

Yeung, W.K., Oguz, C. and Cheng, T.C.E (2004), Two-Stage Flowshop Earliness and tardiness Machine Scheduling Involving a Common Due Window, *Int. J. Prod. Econ.,* vol 90, no. 3, pp. 421-434.

APPENDICES

APPENDIX A

SIMULATED DATA- FLOW TIME BASED ON WIP IN THE SYSTEM 1

	number of jobs in the system									
	1	2	3	4	5	6	7	8	9	10
Seed 1	2.89	3.65	4.17	5.42	6.48	7.48	8.11	8.89	10.09	11.14
Seed 2	3.09	4.06	4.14	5.71	6.86	7.55	8.8	9.62	9.7	10.29
Seed 3	2.93	3.37	4.06	4.65	5.61	6.84	7.25	7.73	8.85	9.95
Seed 10	2.96	4.05	4.98	5.85	6.78	7.85	8.75	9.98	10.76	10.89
Seed 20	3	3.48	4.53	4.91	5.4	5.77	7.25	7.95	9.08	9.91
Seed 50	3.38	4.08	4.94	5.29	6.34	6.71	7.25	8.3	9.3	9.78
Std of Flow Time	0.163648	0.293338	0.37674	0.420228	0.55467	0.691777	0.688487	0.833881	0.647199	0.515515
Mean	3.041667	3.781667	4.47	5.305	6.245	7.033333	7.901667	8.745	9.63	10.32667

	number of jobs in the system									
	11	12	13	14	15	16	17	18	19	20
Seed 1	11.47	12.76	13.68	14.99	15.89	16.66	16.91	18.21	19.3	20.25
Seed 2	10.53	11.31	11.82	12.31	13.17	14.72	15.8	17.22	17.22	18.48
Seed 3	11.06	11.49	11.65	12.73	13.08	13.78	14.2	15.3	16.62	17.48
Seed 10	11.27	11.57	13	13.74	14.8	15.1	15.82	16.67	17.84	18.58
Seed 20	10.85	11.4	11.72	13.08	13.51	14.54	15.45	16.29	16.84	18.19
Seed 50	11.07	12.11	13.22	13.55	14.88	15.99	16.73	17.01	17.4	17.66
Std of Flow Time	0.298687	0.510316	0.81166	0.856894	1.038194	0.950446	0.892364	0.88825	0.879956	0.902755
Mean	11.04167	11.77333	12.515	13.4	14.22167	15.13167	15.81833	16.78333	17.53667	18.44

	number of jobs in the system									
	21	22	23	24	25	26	27	28	29	30
Seed 1	21.06	21.3	22.54	22.76	23.23	23.96	25.28	25.47	26.69	27.69
Seed 2	18.74	19.68	19.74	20.3	20.58	21.83	21.99	23.04	23.49	23.72
Seed 3	18.9	20.16	21.23	21.63	22.7	23.73	24.15	25.05	26.5	26.62
Seed 10	19.06	19.5	20.58	21.42	22.94	23.28	24.26	25.38	25.53	25.94
Seed 20	18.56	18.85	20.02	20.24	21.43	21.9	22.79	23.71	24.16	24.52
Seed 50	18.64	19.71	20.13	21.33	21.68	22.66	22.79	23.31	23.66	24.83
Std of Flow Time	0.865486	0.749214	0.947763	0.854108	0.9379	0.83258	1.113967	1.000777	1.301803	1.341935
Mean	19.16	19.86667	20.70667	21.28	22.09333	22.89333	23.54333	24.32667	25.005	25.55333

APPENDIX B

SIMULATED DATA- FLOW TIME BASED ON WIP - CASE 1- BEFORE CHANGE

	number of jobs in the system									
	1	2	3	4	5	6	7	8	9	10
Seed 1	3.68	5.35	5.79	6.74	7.12	9.35	9.63	10.85	12.17	11.94
Seed 3	3.55	4.36	5.09	5.59	6.63	7.83	7.18	8.26	10.11	11
Seed 5	5	5.06	6.94	9.46	10.3	10.86	12.13	13.81	13.43	13.76
Std of Flow Time	0.655049	0.415559	0.762671	1.62268	1.6269057	1.236995	2.0208634	2.267456	1.368438	1.145697
Mean	4.076667	4.923333	5.94	7.263333	8.0166667	9.346667	9.6466667	10.97333	11.90333	12.23333

	number of jobs in the system									
	11	12	13	14	15	16	17	18	19	20
Seed 1	13.89	15.67	16.7	17.6	19.52	19.73	19.82	20	21.91	23.51
Seed 3	12.09	14.26	16.59	17.94	18.33	18.84	21.1	25.2	29.97	30.69
Seed 5	16.51	17.01	18.9	19.55	23.3	23.48	24.28	26.08	27.63	34.96
Std of Flow Time	1.814779	1.122804	1.063965	0.850503	2.1188414	2.010644	1.8750526	2.682884	3.385551	4.724496
Mean	14.16333	15.64667	17.39667	18.36333	20.383333	20.68333	21.733333	23.76	26.50333	29.72

	number of jobs in the system									
	21	22	23	24	25	26	27	28	29	30
Seed 1	24.28	26.37	27.34	29.86	32.13	31.48	33.53	32.3	35.2	35.44
Seed 3	30.75	31.7	33.83	34.11	34.68	34.31	34.84	36.04	36.27	37.45
Seed 5	35.74	35.43	36.73	37.32	38.24	37.33	37.82	37.56	37.66	41.07
Std of Flow Time	4.691512	3.717906	3.92573	3.055381	2.5057312	2.388672	1.7950735	2.210219	1.007119	2.329554
Mean	30.25667	31.16667	32.63333	33.76333	35.016667	34.37333	35.396667	35.3	36.37667	37.98667

APPENDIX C

SIMULATED DATA- FLOW TIME BASED ON WIP - CASE 1- AFTER CHANGE

	number of jobs in the system									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Seed 1	3.53	5.2	5.47	7.27	7.14	9.28	9.55	10.77	12.04	11.84
Seed 3	3.4	4.21	4.94	5.44	7.71	8.61	9.75	9.92	10.42	11.77
Seed 5	4.85	4.91	6.45	7.14	10.05	10.15	10.7	15.36	14.81	14.51
Std of Flow Time	0.655049	0.415559	0.625513	0.83372	1.2591267	0.630467	0.5016639	2.389426	1.812592	1.275469
Mean	3.926667	4.773333	5.62	6.616667	8.3	9.346667	10	12.01667	12.42333	12.70667

	number of jobs in the system									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Seed 1	14.62	16.39	17.48	20.35	22.18	22.5	22.37	24.7	25.64	26.23
Seed 3	13.11	14.89	15.32	16.02	17	17.28	17.72	18.74	20.64	22.14
Seed 5	15.28	15.98	16.02	20.87	20.3	20.11	21.27	22.7	24.32	26.01
Std of Flow Time	0.908271	0.633	0.899827	2.174136	2.1410485	2.133578	1.9842435	2.476629	2.115677	1.878339
Mean	14.33667	15.75333	16.27333	19.08	19.826667	19.96333	20.453333	22.04667	23.53333	24.79333

	number of jobs in the system									
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Seed 1	27.05	27.1	27.83	29.94	33.67	33.64	33.59	35.42	36.43	37.07
Seed 3	23.52	24.82	25.52	26.09	27.22	26.9	29.01	29.05	30.72	31.93
Seed 5	26.61	26.29	27.96	28.58	29.05	30.79	31.6	31.76	31.92	32.88
Std of Flow Time	1.570654	0.943716	1.120843	1.594163	2.7140744	2.762491	1.8751178	2.610164	2.458188	2.23304
Mean	25.72667	26.07	27.10333	28.20333	29.98	30.44333	31.4	32.07667	33.02333	33.96

APPENDIX D

SIMULATED DATA- FLOW TIME BASED ON WIP - CASE 2- BEFORE CHANGE

	number of jobs in the system									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Seed 1	3.68	7.35	6.97	7.72	8.38	8.18	8.97	11.78	12.02	14.26
Seed 3	3.55	6.32	7.06	7.91	8.5	7.92	10.19	10.91	11.35	12.62
Seed 5	5	7.06	8.48	8.21	10.17	9.99	11.2	15.24	14.72	15.27
Std of Flow Time	0.655049	0.433667	0.691584	0.201715	0.8169999	0.920664	0.9117383	1.870157	1.456625	1.092001
Mean	4.076667	6.91	7.503333	7.946667	9.0166667	8.696667	10.12	12.64333	12.69667	14.05

	number of jobs in the system									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Seed 1	15.78	16.13	17.55	20.09	21.95	22.24	22.36	24.71	25.85	26.51
Seed 3	14.01	16.23	18.26	18.39	20.31	21.22	20.92	21.94	22.48	24.15
Seed 5	16.24	16.55	19.15	19.96	20.8	23.33	24.64	25.74	27.69	28.5
Std of Flow Time	0.96133	0.179134	0.654574	0.772571	0.6873621	0.861562	1.5315352	1.604639	2.157329	1.77802
Mean	15.34333	16.30333	18.32	19.48	21.02	22.26333	22.64	24.13	25.34	26.38667

	number of jobs in the system									
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Seed 1	27.84	29.63	30.58	33.21	37.5	37.33	36.54	39.87	40.55	40.09
Seed 3	24.81	27.14	29.69	30.3	30.71	30.81	31.68	31.61	33.39	34.67
Seed 5	28.17	28	29.91	31.31	33.25	35.54	37.56	40.21	39.71	43.93
Std of Flow Time	1.512151	1.032613	0.378506	1.206381	2.8011545	2.750503	2.5654629	3.976363	3.195719	3.798678
Mean	26.94	28.25667	30.06	31.60667	33.82	34.56	35.26	37.23	37.88333	39.56333

APPENDIX E

SIMULATED DATA- FLOW TIME BASED ON WIP - CASE 2- AFTER CHANGE

	number of jobs in the system									
	1	2	3	4	5	6	7	8	9	10
Seed 1	3.68	5.85	6.02	7.37	7.2	9.55	9.8	11.01	12.3	12.06
Seed 3	3.55	4.82	5.56	5.59	7.15	7.54	10.18	10.33	10.76	12.59
Seed 5	5	5.56	6.98	7.43	10.48	10.54	11.05	15.68	15.11	15.63
Std of Flow Time	0.655049	0.433667	0.59157	0.853594	1.5581257	1.248119	0.5232165	2.377996	1.800932	1.572945
Mean	4.076667	5.41	6.186667	6.796667	8.2766667	9.21	10.343333	12.34	12.72333	13.42667

	number of jobs in the system									
	11	12	13	14	15	16	17	18	19	20
Seed 1	14	15.77	16.79	17.69	19.6	19.8	19.89	20.07	21.97	23.57
Seed 3	12.93	13.34	14.56	16.83	18.42	18.54	20.26	21.05	21.4	25.35
Seed 5	16.56	16.84	19.42	20.22	21.03	23.56	24.85	25.93	27.88	28.67
Std of Flow Time	1.522987	1.464385	1.986325	1.43885	1.0671561	2.132437	2.2560191	2.562863	2.929607	2.11347
Mean	14.49667	15.31667	16.92333	18.24667	19.683333	20.63333	21.666667	22.35	23.75	25.86333

	number of jobs in the system									
	21	22	23	24	25	26	27	28	29	30
Seed 1	24.34	26.43	27.39	29.91	32.17	31.52	33.57	32.34	35.24	35.48
Seed 3	27.85	28.96	29.48	29.9	32.04	33.21	33.16	34.97	34.32	37.81
Seed 5	28.34	28.16	30.06	31.46	33.39	35.68	37.69	40.33	39.83	44.05
Std of Flow Time	1.781391	1.055872	1.146657	0.733045	0.6080753	1.708235	2.0456838	3.324766	2.410039	3.618032
Mean	26.84333	27.85	28.97667	30.42333	32.533333	33.47	34.806667	35.88	36.46333	39.11333

APPENDIX F

ERLANG DISTRIBUTION

Erlang Distribution:

Given a Poisson distribution with a rate of change λ, the distribution function $D(x)$ giving the waiting times until the hth Poisson event is

$$D(x) = 1 - \frac{\Gamma(h, x\lambda)}{\Gamma(h)}$$

for $x \in [0, \infty)$, where $\Gamma(x)$ is a complete gamma function, and $\Gamma(a, x)$ an incomplete gamma function. With h explicitly an integer, this distribution is known as the Erlang distribution, and has probability function

$$P(x) = \frac{\lambda(\lambda x)^{h-1}}{(h-1)!} e^{-\lambda x}.$$

It is closely related to the gamma distribution, which is obtained by letting $\alpha \equiv h$ (not necessarily an integer) and defining $\theta \equiv 1/\lambda$. When $h = 1$, it simplifies to the exponential distribution.

From: www.mathworld.com

APPENDIX G

GENERATE RANDOM DATA FROM AN EXPONENTIAL DISTRIBUATION IN EXCEL

Excel cannot directly generate data from an exponential; however, the following procedure can be used to obtain random observations from an exponential distribution.

1. The first step is to create a set of uniform random numbers between 0 and 1, see Uniform for more information.
2. To obtain the exponential random numbers, we need to use the following formula:

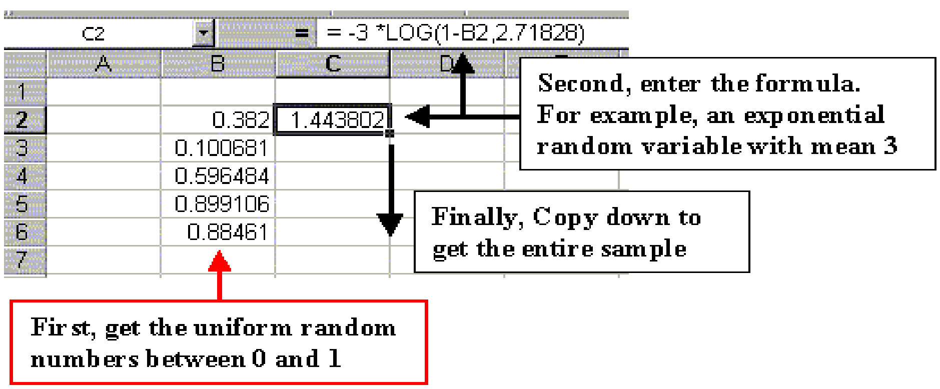
exponential random number = - *mean* *log(1 - *unif* , 2.71828)

where
mean is the mean for the exponential distribution
unif is a uniform random number

Note: Don't forget the - in front of the mean.

3. Copy the formula down for all observations. See "How do I edit my work?" for more information on copy and pasting.

An example, one sample of 5 observations from a exponential distribution with a mean of 3:



APPENDIX H

SECOND DERIVATE TEST

A stationary point is a point X_0 at which the derivative of a function $f(x)$ gets to zero,

$$f'(x) = 0$$

A stationary point may be a minimum, maximum, or inflection point.

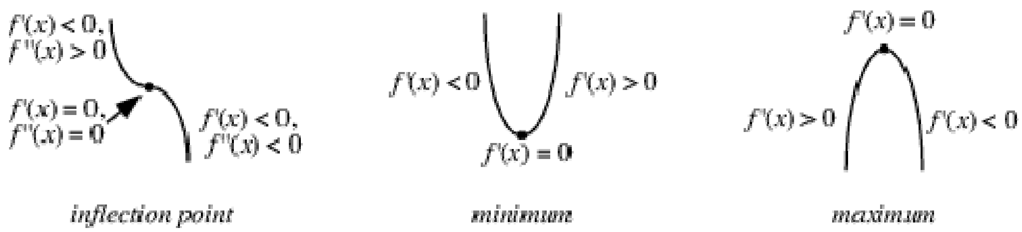


Figure Different stationary points for second derivative test

The test states: If the function f is twice differentiable in a neighborhood of a stationary point x, then:

- If $f''(x) < 0$ then f has a maximum at x.
- If $f''(x) > 0$ then f has a minimum at x.

Note that if $f''(x) = 0$ the second derivative test says nothing about the point x and it is an inflection point (http://mathworld.wolfram.com/SecondDerivativeTest.html)

APPENDIX I

LINGO FORMULATIONS

Example 5.2.1

```
MIN = @ABS(5*(1-(1-@exp(-0.2*(x+2.5)))*(1-@exp(-0.1*(x+7.5))))-0.1*(1-@exp(-0.2*(x-0.5)))*(1-@exp(-0.1*(x+4.5)))-1);
0<=x;
x<=100;
END
```

Example 5.2.3

```
MIN = @ABS(5*(1-(1-@exp(-0.2*(x+2.7)))*(1-@exp(-0.1*(x+7.7))))-0.1*(1-@exp(-0.2*(x-0.3)))*(1-@exp(-0.1*(x+4.7)))-1);
0<=x;
x<=100;
END
```