Modeling and Optimization of a Health Care System

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Abstract. In many applications, the number of elements in the system (i.e., machines, jobs, customers) is either constant or remain fixed. This type of a system is modeled as a closed queuing system. Closed queuing systems are those in which there is no exogenous arrival to the system and no element leaves the system. In these systems, departure from one state is an arrival to the next state. Those cases in which the system is always full and any departure from the system is immediately replaced by a new arrival are also considered closed systems. This closeness of the system changes the nature of the modeling process in which the well known queuing models need to be modified accordingly. In this research, a health care center with a limited number of beds which are used at their maximum capacity is considered. In this closed queuing system, available and reserved nurses are also modeled by means of closed queuing systems. Considering the interaction of the resulting networks, the optimal level of reserved nurses as well as the patients discharge and admission rates is determined in such a way that the incurred cost is minimized.

1. Introduction
In recent years, increased number of patients and augmented health care costs have made the hospitals to be more efficient and productive. One of the factors which has an important influence on the efficiency of health care systems is the flow of patients in the system. Christodoulou and Taylor [1] studied the bed occupancy patterns of elderly people. Categorizing the geriatric patients into short-stay and long-stay patients, they modeled the length of stay by means of Markov models. Gorunescu, McClean, and Millard [2] proposed a queuing model for geriatric patients’ length of stay by which the optimal number of beds in the health care department can be determined. The cost parameters considered in their model included the average cost of patients, cost of empty beds, and the cost of loosing demand. In both of these models, the arrivals to the system and the service times in each phase were exponentially distributed. Faddy and McClean [3] utilized Markov chain to model the length of stay of old people in hospital incorporating the effect of age and the year of admission onto the length of stay. Chaussalet, Xie, and Millard [4] proposed a closed queuing model for the flow of geriatric patients in health care departments. Closed queuing systems are queuing systems in which the number of entities flowing through the system is constant at every point of time. This is the case when there is no incoming/outgoing flow to the system or the system is always working with its maximum capacity. In the latter case, whenever an entity goes out of the system it is replaced by a new one in the input. Therefore it can be assumed that the entity has never left the system. They imposed this restriction on the model to make it more realistic since in actual case the capacity of hospital is limited. They used this model to investigate the effect of different “What if” scenarios on the system like changing the discharge rate in different phases of care process or the flow rate between different phases. In a recent study, Shaw and Marshall [5] employed Markov chain to model the flow and progress of heart failure patients in a health department. They divided the care process into three separate phases and fitted a phase-type distribution to the length of stay of patients in the system. The discharge rate in each phase was constant while the overall system was a closed queuing network.

2. Approach
To model the progress of patients through the health care system, we assume that the care process can be categorized into some number of phases like “diagnosis and acute care”, “rehabilitation”, and “longer stay”. Each patient who enters a phase can be discharged form the system after that phase or may proceed to the next phase with a probability (Figure 1). In the last phase, all of the patients are discharged from the system eventually. It is assumed that the healthcare system is working with full (limited) capacity. It means that all of the beds (say S) are occupied and whenever a patient goes out of the system, it is replaced by a patient from the waiting list. Therefore this system can be modeled by means of Closed Queuing Systems. All the models
reviewed in literature have considered exponential arrivals and service times in different phases. Unlike the models in literature, we assume a generally distributed service time in each phase. In this case we are interested in the number of patients in each phase and their length of stay in that phase. Since the overall number of patients in the system is constant, therefore if the numbers of patients in two phases are known, then the number of patients in the third phase can be determined easily. Hence the State Space of the problem will be a two dimensional state space as shown in figure 2.

Figure 1. Progress of patients through different phases of health care system (Left) and the closed network representation of the system (Right)

Suppose $\mu_i$ is the rate at which a patients goes out of a phase and $p_{ij}$ is the probability by which a patient goes from phase $i$ to phase $j$. Defining $N$ as the overall number of beds in the hospital and $n_i$ as the number of patients in phase $i$, $n_2$ and $n_3$ are selected to represent the state space of the problem (Figure 2). In this model $\mu_i$ is dependent to the number of nurses working throughout the system and the focus of this research is to find the optimal number of nurses regarding the admission and discharge rate of patients in each phase.

Figure 2. The state space of the problem

References