CONTROL OF A PASSIVE DYNAMIC COMPASS GAIT WALKING ROBOT USING NEURAL NETWORK CENTRAL PATTERN GENERATOR

A Dissertation by

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CONTROL OF A PASSIVE DYNAMIC COMPASS GAIT WALKING ROBOT USING NEURAL NETWORK CENTRAL PATTERN GENERATOR

The following faculty members have examined the final copy of this dissertation for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Doctor of Philosophy with a major in Mechanical Engineering.

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To my loving wife and parents
In the beginning was the Word, and the Word was with God, and the Word was God. (John 1:1)
ACKNOWLEDGMENTS

I would like to thank my adviser, Dr. Behnam Bahr, and co-adviser Dr. Brian Driessen for their guidance, advice and support. I also wish to thank to my committee members, Dr. Hamid Lankarani, Dr. M Edwin Sawan, and Dr. John Watkins for their advice and effort. Finally, I wish to thank to my wife and parents for their prayer and support. This dissertation would not exist without their efforts.
Important features in human walking are the passive dynamic nature of its gait and its rhythmic nature, generated from the Central Pattern Generator (CPG) in a human’s spinal cord. The passive dynamic mechanism has been shown in literatures to be a key attribute of the human walking; moreover, passive dynamic walkers, similar to the human body, have been developed that could walk on a downward slope only using the gravity. Much research has been conducted on biped passive dynamic walking mechanisms. Many of these studies focus on either the stability or the control of a walking mechanism. Another feature in human walking is a CPG, a rhythmic movement signal generator. In walking, humans do not plan every step. Instead, a human learns walking patterns and the learned patterns are stored in the human’s spinal cord. This attribute has been studied in the past and has been integrated into some biped robots, lately.

In this study, a biped model with two features, a passive dynamic mechanism and a central pattern generator, are investigated. The multi-body dynamics equations of motion of biped are formulated and numerically solved. Control of the walking speed of these biped models is the main aim of this study. A model based on Goswami’s passive compass gait biped model is used in this study. A CPG-based gait generation method with a feed forward neural network serves as the CPG for the model. The neural network is trained using the acceleration pattern of the legs, rather than the movement pattern of the legs. The selection is based on the human method of learning to walk. Desired walking velocities are selected as commands for the control system. When based on the acceleration pattern of the legs, the control system with the CPG shows robust results in simulations. The results indicate that this CPG-based control system is capable of generating stable gaits and satisfactory results in terms of the walking velocity in a range which is at the outside of the trained data range.
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CHAPTER 1

INTRODUCTION

1.1 Humanoid Robot

Since humans first invented machines, we have envisioned machines that move as we do. However, the realization of this vision has developed slowly. The first attempt at accomplishing this feat of engineering dates back to ancient Rome (the wine pouring servant by Heron of Alexandria, 50 A.D.) [1, 2]. In the middle ages, a few machines were developed by Al-Jazari (a robot band and hand washing automation, 1206 A.D.) and by Leonardo da Vinci (the armored knight, 1495 A.D.). Moving machines of the early modern era include the creations of Jacques de Vaucanson (The Flute Player, 1738 A.D.) and Pierre Jacquet-Droz (the musician and writer automaton, 1774 A.D.) [1]. However, these machines were merely automata, not in any real sense robots. Real improvements in replicating human movement have come during the recent modern age with the invention of the computer. Many humanoid robots have been developed in past few decades. Waseda University and Honda are pioneers in humanoid robot development. Waseda University in Japan has developed the Wabot-1 (1973), Wabot-2 (1984), Wabian (1997), and Wabian-2 (current) robots [3, 4]. Honda has developed P1 (1993), P2 (1996), P3 (1997), and ASIMO (2000–current) [5, 6]. Currently, many other robots are in production or are being developed.

1.2 Walking

There are many ways to move from one place to another. An animal usually travels between places by crawling, walking, running, swimming, or flying; these constitute the natural methods of travel for an animal. However, humans have more ways to travel. Since inventing machines, humans have been able to travel by a machine such as, riding, driving, and flying by
using machine. (These are the modern ways to travel.) Travel over land is done by a wheeled machine, providing several benefits, the most important of which is the short traveling time. Wheeled machines can travel at much faster speeds than any human or any animal is capable of. On the other hand, this means of travel has several drawbacks. The biggest disadvantage is that a wheeled machine cannot travel over very rough terrain. For this reason, researchers have been studying nature's most common method of ground travel, walking.

1.3 Bipedalism

Many researchers are currently studying how to create robots that duplicate walking in the natural world. Some of these researchers have been employing the walking of insects as a model [7, 8, 9]. Insect-inspired walking robots are most commonly hexapods; furthermore, the majority are based on one insect species, thecockroach, which is one of the fastest in nature. The first walking robot to utilize the insect mode of walking was created by the Russian V. S. Gurfunkel [7]. Currently, the Poly-PEDAL laboratory at the University of California in Berkeley and the SPRAWL developer, the Biomimetics Laboratory at Stanford University are the leading research facilities active in the development of insect-inspired walking robots [8, 9].

Other researchers are focusing on the walking of animals [10, 11]. Robots modeled after the walking of animals are commonly quadruped robots. The first computer-controlled quadruped robot was Phoney Pony by A.A. Frank [10]. Currently, much research is being conducted in this area. One of the most famous quadrupeds is ‘Big Dog,’ developed by Boston Dynamics [11].

Still other investigators are trying to duplicate the walking of humans. Since Dr. Ichiro Kato of Waseda University in Japan developed the first biped walker controlled by a computer,
known as WAP-1 [3], much progress has been made in the development of bipedal robotics and, many studies have recently been conducted in this area.

The most important difference between the gait of insects and animals, on one hand, and the gait of humans. On the other, is that the gait of insects and animals is statically stable while the gait of humans is dynamically stable. This distinction means that if an insect or an animal is frozen in the middle of its walking sequence it will not fall, i.e., it is statically stable, but if a human is frozen while it is in the process of walking, it will fall down, being dynamically, but not statically, stable.

There are many reasons to study biped walking, gait, among which is the prospect of its application in the area of rehabilitation treatment. One of many examples of utilizing gait research in the rehabilitation therapy is the Locomat, a rehabilitation therapy machine from Hocoma Co. [12, 13]. Another goal is developing a service robot that embodies the precise meaning of the word, “robot.” One such service robot is the ASIMO from Honda [6].

1.4 Approach in this study

This study focuses on the biped walking robot. Many different physical models of this type of robot have been developed or are still in development. One of the simplest human gait models, the compass gait model, will be investigated in this study. The compass gait model has no knee and walks in the manner a compass moves as it draws a circle. A detailed explanation follows in Chapter 2.

Designing the physical model of a biped walking robot is one thing, but designing a controller for that physical model is another. While there are many methods of making such a robot walk, most of them are based on two control theories for biped robots, the Zero Moment Point method and the Central Pattern Generator method. These two methods will be discussed in
detail in Chapter 2. A modified Central Pattern Generator (CPG) method will be used in this research.

1.5 Contributions

The major contributions of this research are in three main areas involving biped robot technology:

- Simplicity of the controller. In this research, a modified Central Pattern Generator (CPG) is constructed with neural networks, which results in a more simplistic calculation and makes the robot control more efficient and robust.

- Pattern that the CPG follows. Patterns are chosen to mimic the sensor process and the actuation process of a human being. A detailed discussion follows the control section in Chapter 3.

- Integration of the passive dynamics biped model (which incorporates gravity and momentum) and the modified CPG. This combination achieves efficiency and a more natural appearance of walking.
CHAPTER 2

BACKGROUND

2.1. Human Gait

Reviewing human walking patterns is a very important step in biped robot design. In order to follow a discussion of human gait, one needs to be familiar with the terminology of human gait research. The term “gait” refers to a particular sequence of lifting and placing the feet during locomotion, which includes any type of movement carried out by legs, such as walking, running, galloping, trotting, hopping, etc. Each repetition of the sequence is called a “gait cycle.” The time taken to complete one cycle is the “gait period.” The inverse of the gait period is the “gait frequency” (1/period).

In comparison, animal gait and human gait exhibit both similarities and differences. The most important similarity is that the gait of all animals, humans, and insects, is controlled by a central pattern generator (CPG). The CPG is defined as "the set of interconnected neurons that are responsible for generating the rhythmic motor patterns that animals use in locomotion,"[14]. The most significant difference between human gait and four-legged animal gait is that an animal's gait is statically stable while a human's gait is dynamically stable.

2.1.1 Gait Cycle and Gait Phases

When a human walks, one leg supports his or her body and the other leg swings forward to a new support location. Then the legs switch their tasks. A single sequence of these functions by one limb is called a gait cycle. A gait cycle is divided into two gait phases, the stance phase and the swing phase, as shown in Figure 2-1.
The gait phases can be divided into five sub-phases as illustrated in Figure 2-2. The key determiner for this division is the way in which the focused leg contacts the ground.

**Figure 2-1.** Stance Phase and Swing Phase [15]

**Figure 2-2.** The subdivisions of the phases of a gait cycle focused on its relationship to the floor contact pattern [15]
Within the stance phase, there are three sub-phases. The first sub-phase is the “Initial Double Stance,” also called Double Limb Support. The initial double stance sub-phase occurs at the beginning of a gait cycle. The second sub-phase is the “Single Limb Support,” also known as Single Stance. The single limb support sub-phase occurs with the beginning of the swing phase of the opposite leg. During the single limb support sub-phase, one leg supports the whole body weight. The third sub-phase is called the “Terminal double stance”; it begins when the foot of the swinging leg touches the ground and ends when the supporting leg starts to swing. During the swing phase, the focused leg advances to the next support location by swinging; this swing phase starts when the foot of the stance leg leaves the ground (toe-off).

If we concentrate on functionality in a gait cycle, the stance phase and the swing phase can be divided into three tasks; these tasks can be further divided into eight sub-phases, also called periods. Figure 2-3 shows these subdivisions, called sub-phases or periods [15].

Figure 2-3. The subdivisions of the phases of a gait cycle, focusing on their functionalities [16]
Detailed illustrations of the subdivisions are shown in Figure 2-4.

![Sub-phases](image)

**Figure 2-4.** Detailed illustrations of the sub-phases [15]

### 2.1.1.1 Task 1: Weight Acceptance

Weight acceptance task is the most challenging task in the gait cycle. Three functional patterns, namely shock absorption, initial limb stability, and the preservation of progression are associated in this task. This task consists of two sub-phases, the initial contact phase and the loading response phase. During the weigh acceptance task, body weight is transferred from one limb to the other, which has just finished its swing phase and has an unstable alignment.

- **Phase 1: Initial Contact**

  The interval of this phase constitutes the first 0~2% of the gait cycle. The initial contact phase occurs as the foot contacts the ground. In functional terms, this phase represents the starting point of the stance with the heel rocker.
Phase 2: Loading Response

The interval of this phase constitutes the first 0~10% of the gait cycle. The loading response phase is the initial double stance period, beginning with initial floor contact. The period of the loading response phase starts with the initial ground contact of one foot and ends when the other foot starts to swing. The functions of this phase are shock absorption, weight-bearing stability, and preservation of progression. The reason that the beginning of the Phase 2 interval overlaps Phase 1, starting at 0% of the gait cycle is that in Phase 1, the initial contact is sometimes instantaneous. In other words, the body weight begins loading on the support leg as soon as the foot touches the ground.

2.1.1.2 Task 2: Single Limb Support

This task begins with the lifting of the other foot as it begins to swing and continues until the other foot touches the ground again. This task consists of two sub-phases, known as the mid-stance phase and the terminal stance phase. During this task, only one limb is supporting the entire body weight by itself.

Phase 3: Mid Stance

The interval of this phase constitutes the next 10~30% of the gait cycle. The mid-stance phase begins with the lifting of the other foot and ends with the alignment of the body weight over the fore-foot. The phase has two functions. The first function of this phase is to advance the swing foot over the support foot; the second function is to stabilize the leg and trunk.

Phase 4: Terminal Stance

The interval of this phase constitutes the next 30~50% of the gait cycle. The terminal stance phase begins when the heel of the support leg is lifted from the ground and finishes when the swing foot touches the ground to advance the body past the support leg.
2.1.1.3 Task 3: Limb Advancement

This stage the portion of the cycle during which the limb swings to advance the body. The limb advancement task consists of four sub-phases, namely the pre-swing phase, the initial swing phase, the mid-swing phase, and the terminal swing phase.

- Phase 5: Pre Swing
  Its interval of this phase constitutes the 50~60% segment of the gait cycle immediately following the terminal stance phase. The pre-swing phase begins when the other leg contacts the ground, and ends when the focused leg leaves the ground (toe-off). In other words, the other leg is in the initial contact sub-phase of its stance phase. The function of this sub-phase is to position the focused leg for its swing.

- Phase 6: Initial Swing
  The interval of this phase constitutes the next 60~73% of the gait cycle. The initial swing phase begins when the foot leaves the ground and ends when the swinging foot is opposite the support foot. The functions of this sub-phase are to make sure that the swinging foot does not collide with the ground and to move the leg forward from behind of the body.

- Phase 7: Mid-Swing
  The interval of this phase constitutes the next 73~87% of the gait cycle. The mid-swing phase starts as the swinging leg is opposite the support leg and finishes when the swinging leg is in front of the body with the shinbone vertical to the ground. The functions of this sub-phase are to advance the limb and to insure that the swinging foot clears from the ground.

- Phase 8: Terminal Swing
  The interval of this phase constitutes the final 87~100% of the gait cycle. The terminal swing phase begins when the shinbone is aligned vertically to the ground and ends when the foot
strikes the ground. The progression of the leg is complete when the shank is located in front of the thigh. The functions of this sub-phase are to finish the advancement of the leg and to ready the focused leg to take up the role of supporting the other leg. If there is no double limb stance in the gait cycle, the body is defined as no longer walking, but running.

2.1.2 Functions of Joints

2.1.2.1 Movement of Center of Mass

Learning about the way in which the center of mass moves when a human walks is essential to understanding human gait.

A few facts that are interested to know are

- While a human is walking, the center of his or her body mass does not stay fixed in one location, but its locus of movement is usually confined to the person's pelvis [16].
- When a human is walking at a normal speed on a level surface, if his or her center of mass is projected onto the progress plane, the projection of his or her center of mass will be appear as a smooth sinusoidal curve [16].

Displacements in the center of mass in the three planes of space during a single gait cycle are shown in Figure 2-5. In Figure 2-5, (a) represents the lateral displacement in the horizontal plane, (b) shows the vertical displacement, and (c) illustrates the combined displacements of the lateral displacement and the vertical displacement as projected onto a plane perpendicular to the plane of progression.
The effect of speed on the movement of the center of mass is also worth investigating. Figure 2-6 illustrates the effect of variations in speed on the displacement of the pelvis as projected onto a plane perpendicular to the plane of progression.

**Figure 2-5.** Displacement of the center of mass in three planes of space during a single stride [16]

**Figure 2-6.** Effect of variations in speed on displacement of the pelvis as projected onto the plane perpendicular to the plane of progression [16]
According to Figure 2-6, the vertical displacement becomes larger while the horizontal displacement becomes narrower as the speed increases.

**2.1.2.2 Functions of Joints**

To understand human gait, we need to look into the effect of the joints in the gait pattern. A series of bipedal walking models will illustrate the way each joint adds smoothness to the sinusoidal displacement pathway revealed while walking [16]. The effects of the vertical displacement by joints will be examined.

**Vertical Displacement**

First, we will see how a joint affects the vertical displacement of the center of mass. When a normal adult man walks at a typical speed, the total amount of vertical displacement is generally about 5 cm. During a single stride, the highest point of vertical displacement happens at about the middle of the stance phase of the supporting limb. During a gait cycle, the lowest point of vertical displacement occurs during the middle of the double weight bearing, double limb stance.

- Compass Gait

The compass gait is the simplest model representing bipedal gait. This model is constructed of a bar and levers, with the bar representing the pelvis and levers for the legs. The center of mass of this model is located in the middle of the bar, the pelvis, as illustrated in Figure 2-7 [16, 17]. In this model, the legs which are levers have no knee joints, no ankle joints, and no feet. The only joints in the legs in this model are hip joints which can only accomplish flexion and extension movements. If the trajectory of the center of mass of this model is drawn, the trajectory can be represented as a series of intersecting arcs as shown in Figure 2-7.
A model based on this configuration will be used in this study.

- Pelvic Rotation

If an average human walking at normal speed on level grounds, his or her pelvis rotates. During this type of movement, the rotation axis of the pelvis will follow the walking progression line. The average rotation is about 4 degree to each side, for total of approximately 8 degrees. This pelvic rotation elevates the end of the trajectory arc shown in Figure 2-7 and it flattens the trajectory arc of the center of mass in a compass gait model. Figure 2-8 illustrates the effect of pelvic rotation [16, 17]. Incorporating the pelvic rotation into the compass gait makes the center of mass of the model drop less during the double limb support phase. The solid line at top in Figure 2-7 is the trajectory of the center of mass of a compass gait without modification, while the dashed line is the trajectory of the center of mass of a compass gait with pelvic rotation.
Figure 2-8. Effect of pelvic rotation [16, 17]

- Pelvic List

In normal level walking, the pelvis of the swinging leg side moves downward about 5 degrees. This movement is called pelvic list. The effect of this list is to allow the center of mass of the model to move a shorter distance upward when body is passing over the support leg. Thus, pelvic list further flattens the arc of movement. The knee flexion of the swinging leg is needed to make sure that the swinging leg does not hit the ground due to lack of depth of the pelvis on the swinging leg side. The effects of pelvic list are illustrated in Figure 2-9 [16, 17].
The solid line overlapping the dashed line in Figure 2-9 is the trajectory of the center of mass of a compass gait with only pelvic rotation effect which is the dashed line in Figure 2-8.

- Knee Flexion in Stance Phase

At very slow walking speeds, the knees do not flex much. On the other hand, the swinging-leg-side knee bends about 15 degrees at the beginning of the swing phase; moreover, it remains in this bended position until the center of mass of the body passes the supporting leg at other walking speeds. The flexion of the support-leg knee absorbs the impact to the body from the heel-ground contact at the beginning of the supporting weight stage and lowers the center of mass when it passes the supporting leg. The effects of this knee flexion are shown in Figure 2-10 [16, 17].
Figure 2-10. Knee flexion in the supporting leg during stance [16, 17]

The solid line at the top in Figure 2-10 is the trajectory of the center of mass of a gait cycle, with the effects of the pelvic rotation and the pelvic list.

➢ Foot

The function of the ankle and foot is to change the trajectory of the center of mass from a series of intersecting arcs to a sinusoidal curve.

A foot changes the trajectory of the knee from one arc to two intersecting arcs as shown in Figure 2-11 [16, 17]. In Figure 2-11, Figure (A) shows the trajectory of the knee without a foot, figure (B) illustrates the effect of the foot on the knee trajectory without the ankle, and figure (C) shows the effect of a foot with flail ankle on the knee pathway. Figure 2-11 (A), (B), and (C) show the progression of the knee trajectory which resembles more and more of the knee trajectory of a normal human when walking.
Figure 2-11. Effect of the foot on pathway of the knee [16, 17]

- Ankle

Ankle motion also has an effect on the knee pathway. This effect is illustrated in Figure 2-12 [16].

Figure 2-12. Effect of ankle motion on pathway of knee [16]
In Figure 2-12, the top figures explain how the muscles work to move ankle of the supporting leg and the bottom figure show the result knee trajectory by the ankle motion. These ankle movements smooth and flatten the knee trajectory during the stance phase. These motions also restrain the foot from striking the ground.

Because of the involvement of the ankle and foot, the trajectory of knee becomes more flat than it would be without ankle and knee movement as shown in Figure 2-13 [16, 17].

**Figure 2-13.** Pathway of the knee in walking at moderate speed [16, 17]

The lateral displacement of the body will not discussed here, because a two dimensional model in the plane of progression is the subject in this study.
2.2.  **Physical Models**

Many different human gait models, all based on the actual human skeleton, have been or are currently being studied. Some of them focus primarily on the joints while other focus on the properties of the leg itself.

2.2.1  **Skeleton Model**

The skeleton model of a leg has been the subject of much research in the medical field, as well as in the field of robotics. The skeleton model has been developed primarily for gait research in medicine and for the creation of humanoid robots in robotics. Figure 2-14 illustrates the 12-DOF leg skeleton model of a humanoid robot.

![Figure 2-14. Leg model used in ASIMO [19]](image)

This type of model must be actively controlled if it is to imitate human walking. Scheduled joint actuations are used for each joint in order to replicate human walking. The most well known example of this type of physical model is the ASIMO from Honda [19].
2.2.2 Passive Dynamic Walking Models

Different kinds of physical models have emerged as the result of several decades of research aimed at developing more human-like walking, based on human leg skeleton models. These models, known as passive dynamic walking models, focus more on the leg configuration itself than the skeleton and joint models do. Passive dynamic walking models employ fewer controls, instead using the natural dynamic properties of the model itself, i.e. inertia, momentum, etc, to imitate human walking. A few notable studies have been conducted involving passive dynamic walking models. These will be discussed in following subsections.

2.2.2.1 Tad McGeer’s Passive Dynamic Walker

Some researchers, such as Mochon and McMahon, became aware of the passive dynamics in human walking [20]. However, research investigating passive dynamic walking has seen considerable progress since McGeer explained the passive dynamics of human walking [21, 22, 23].

McGeer’s passive dynamic walking research began with the question, “Why does a human have legs?” Wheels are much better than legs on an even ground if it is biologically possible to design an organism with them. Then McGeer contends that wheels and legs are not that different. Figures 2-16, 2-17, and, 2-18 show the process of developing a set of leg from a wheel [23].
The Synthetic Wheel

Figure 2-15. The Synthetic Wheel in a step [23]

Figure 2-15 shows the first form in the transition toward the leg and also the simplest of the walking models, the synthetic wheel [23]. The synthetic wheel consists of two straight legs connected by a hinge to semicircular feet, in which the foot radius is the leg length, incorporating a weightless prismatic knee joint for foot clearance. In addition, the weight “payload,” which is much heavier than the weight of legs sits on the top of the hinge. The mechanism of walking in the synthetic wheel model is that the support leg rolls forward steadily like a spoke in a wheel, while at the same time the other leg swings ahead like a pendulum. The role of the support leg changes when the free leg ends its swing, which is also the point in time at which the speeds of the legs are same. When this point is reached, the angles of legs become opposite to those at the beginning of the step. This cycle of the walking will repeat the naturally stable step continuously if it is pushed at the start. Therefore this walking motion synthesizes the motion of an ordinary wheel.
The Straight-Legged Biped

Figure 2-16. The Straight-Legged Biped in a step [23]

The second form of the transition is the straight-legged biped [23]. Figure 2-16 shows one gait cycle of progress for a straight-legged biped. To modify the synthetic wheel into more human-like form, we need to give it small feet such as those of a human and a much smaller payload than the synthetic wheel normally has in order to attain a distribution of mass similar to that of a human. Because this model has small feet, the transition between the support leg and the swing leg will not be smooth. This model will lose some energy during the transition period. In other words, the straight-leg biped model will disperse energy on each heel strike, i.e. at the end of a swing phase of the swing leg or at the beginning of a support phase of the support leg. If this model is to walk continuously, this dispersed, as thus depleted, energy must be replenished. The dissipated energy can be replenished by having model walk on a downward slope in which case gravity will replenish the lost energy due to the heel strike. This mechanism will insure that the straight-legged biped model walks continuously on a downhill slope.
The Knee-Jointed Biped

![Figure 2-17](image)

**Figure 2-17.** The Knee-Jointed Biped in a step [23]

Figure 2-17 illustrates a knee-jointed biped executing a step [23]. The difference between the straight-legged biped model and the knee-jointed biped model is the existence of knees on each leg which allow foot clearance but add additional energy-dispersing locations. The knee in this model will not go over the fully extended leg position, the way in which a human knee will. This model exhibits more dispersion of energy than the straight-legged biped model; however, the dispersed energy can be restored by walking on a steeper downward slope. The steeper slope amplifies the potential energy acting on the legs. This increase in energy will replenish the dissipated energy during the heel strike and during the full extension of the knee. In the end, the walking pattern of the knee-jointed biped model is similar to the straight-legged biped model as shown in Figure 2-17.

These models, the synthetic wheel, the straight-legged biped, and the knee-jointed biped, show how human walking is similar to the rolling of a wheel. These models have all been studied...
in two dimensional planes. Although human lives in a three dimensional world, these models, along with McGeer’ analysis, demonstrate that a human-like frame can walk without actuators on every joints, in other words, human have a passive dynamic frames.

2.2.2.2 Compass Gait Model

Compass gait models have been studied since the 1970s [21, 22, 23, 24, 25]. Frank [24] and Yamashita, et al [25] were among the first researchers to study compass gait models. McGeer demonstrates that a straight-legged biped model, i.e. also called a compass gait model, can be a passive dynamic model if the model properties were right [21, 22, 23]. During the late 1990s, the theory behind the compass gait model was studied intensively by Goswami and Espiau in collaboration with others [26, 27, 28, 29]. The compass gait studied by Goswami and et al [29] is illustrated in Figure 2-18.

![Figure 2-18. Model of a compass-like biped robot walking down a slope in swing stage [29]](image-url)
According to the Goswami and et al study [29], compass gait has two stages, the swing stage and transition stage. In the swing stage, the swing leg advances forward; the researchers assume that there is no slip at the tip of the support leg and the ground. In the transition stage, the swing leg touches the ground at the same time as the support leg leaves the ground; therefore, the period of the transition stage is instantaneous. In other words, the double stance phase in the compass gait occurs instantaneously unlike this phase in actual human gait. During the transition, the swing leg and the ground collide at the end of the swing stage. This description incorporates several assumptions of what occurs during the collision. The first assumption is that the collision is inelastic. The second assumption is that no sliding occurs while the swing leg is colliding with the ground. The third assumption is that the robot configuration does not change during the collision. The fourth assumption is that the both angular momentum of the robot about the impacting foot and the angular momentum of the pre-impact support leg about the hip are conserved.

While the swing leg is swinging, the swing leg without a knee will hit the ground. This is prevented by introducing weightless prismatic-joint knees in each leg. The prismatic-joint knee will contract the swing leg just enough to clear the ground while the support leg is fully extended.

Goswami and et al developed nonlinear equations of motion, along with transition equations, for use with this model, [26, 27, 28, 29].

2.2.2.3 Three-Dimensional Passive Walker

Prior to Steven Collins and his colleagues from Cornell University developing a three-dimensional passive dynamic walking robot [30], all such robots studied were modeled in a two-dimensional plane. It is appropriate to mention this research, as being the first and only use of a three-dimensional physical model in passive dynamic robotics. However, it will not be discussed
further detail because the present study focuses on the two-dimensional passive dynamic walking robot.

2.3. Control Methods

There are many control methods which imitate the walking of a human. However, most of them are derived from three concepts, the zero-moment point (ZMP), the central pattern generator (CPG) and the energy method. These concepts will be briefly discussed in subsections 2.3.1 and 2.3.2.

2.3.1 Zero-Moment Point

The zero-moment point (ZMP) concept was introduced by Miomir Vukobratović and his colleague in 1968 [31]. The ZMP concept has been developed specifically for biped walking mechanisms [31, 32, 33, 34, 35, 36, 37]. The concise definition of zero-moment point states that “ZMP is the point where the influence of all forces acting on the mechanism can be replaced by one single force” [36]. Figure 2-19 illustrates zero-moment point (ZMP) [38].

![Figure 2-19. Zero-Moment Point [38]](image)

The first practical application of the zero-moment point concept was developed in the laboratory of Dr. Ichiro Kato at Waseda University in Japan in 1984 [39]. Dr. Kato and his team
developed a biped walking robot, WL-10RD, and used the ZMP to dynamically balance the biped gait of the robot [39]. Since that time, most actively controlled biped robots have used the zero-moment point concept. For example, ASIMO from Honda and QRIO from Sony are utilize the ZMP to control their balance.

2.3.1.1 Applications of Zero-Moment Point Concept

The zero-moment point concept is important in two applications, gait synthesis and gait control.

Gait Synthesis

In gait synthesis, the proper dynamics of the biped robot above the foot are determined by using this concept to ensure a desired ZMP position [32, 34, 37].

Gait Control

In the gait control, the ZMP position for the given mechanism’s motion is determined by applying the concept [36]. The ZMP position shows whether the robot is dynamically stable or dynamically unstable in terms of gait control.

Locating the Zero-Moment Point Position

The ZMP position can be measured by placing force sensors on the sole of the robot’s foot. Figure 2-20 shows some examples of the position of force sensors on the bottom of the robot’s foot. These force sensors give information about the contact force between the ground and the robot [37].
Figure 2-20. Examples of the disposition of force sensors on the sole of the robot’s foot [36]

2.3.1.2 Procedure to Determine the ZMP Position

Step 1

The first step in determining the actual ZMP position is finding the computed ZMP position, which is Point ‘p’ in Figure 2-21 (a). The ZMP position can be calculated from the static equilibrium equation with respect to the ankle joint. In this step, the actual foot size is not counted when the ZMP location is computed. This ZMP position is not the final ZMP position.

Figure 2-21. Illustration of the determination of ZMP position: (a) Step 1, and (2) Step 2 [36]

Step 2

If the computed ZMP position is outside of the support polygon, then this ZMP position is called a fictitious zero-moment point (FZMP). The computed ZMP location from Step 1 must
be evaluated with the real support polygon. If the computed ZMP position is outside of the support polygon (FZMP), the actual ZMP position will be located at the edge of the foot (support polygon), in which case the robot will roll over at the actual ZMP position. This is illustrated in Figure 2-21 (b) [36].

2.3.2 Central Pattern Generator

The first recognition of the existence of the central pattern generator was come by the research of T. Graham Brown in 1911 [40]. Brown found the rhythmic patterns of the progression of a decerebrated cat [40] and a normal cat under deep narcosis [41] when stimulated. These researches showed that a cat could walk without the control of its brain. Based on these researches, Brown suggested that there was a rhythmic pattern generator in the spinal cord and now, we know that this is the central pattern generator (CPG). Since the publication of Brown’s studies, much research on the central pattern generator has been conducted in field of biology. Today it has come to be accepted that most of the rhythmic motions which a human performs are initiated by central pattern generators. Examples of movements that involve CPGs include walking, running, swimming, chewing, breathing, the beating of the heart, movements of the intestine, and many more. While vertebrate animals have central pattern generators, CPGs are not confined to this group, but are found in various invertebrates such as insects as well. Many different vertebrate animals have been used for the CPG research such as cats [40, 41], lampreys [42], salamanders [43], frog embryos [44], and in vitro mice (their spinal cords) [45]. The CPGs of vertebrates are slightly different from those of invertebrates. The locomotion CPG of vertebrates is usually located in the spinal cord while those of invertebrates can be found in each leg. The central pattern generator is a rhythmic movement generator. It can generate the same repetitive movement pattern without involving the brain, nor does it need any sensory
information to generate that movement. On the other hand, sensory inputs have a very important role in CPG functionalities, even if the sensory inputs are not needed for the CPG to function. Brown filmed a demonstration of his research on the locomotion of the decerebrate cat and presented it at the Cambridge Meeting of the Physiological Society in October 1941 [46]. In this film, the decerebrate cat was walking on a treadmill and when the treadmill gained speed, the cat changed its walking pattern from walking to trotting and later to galloping. This behavior indicates that the sensory inputs affect the CPG, implying that the movement pattern initiated by the CPG can be changed by modifying sensory input. This feature can be also implemented into a CPG controlled robot, which will have sensory inputs acting as a command to change the walking pattern. Many researchers have been trying to implement this design characteristic from nature in the field of robotics. Some studies have used the CPG of insects, such as cockroaches and stick insects, for hexapod robots. Other, quadruped robots have used the CPG of animals. Biped robots controlled by a CPG are based on human walking patterns.

2.3.2.1. Types of Central Pattern Generators

A central pattern generator can be built by many different means since the CPG only needs to fire rhythmic movement control. Among the many types of CPGs, the two CPG types which have been most effectively implemented use either the “Connectionist Model” or “Systems of Coupled Oscillators.”

Connectionist Model

One of the types of the central pattern generators used in robot locomotion control is the “Connectionist Model,” essentially is a network of neurons connected each other, i.e., a neural network. This CPG type is used in comparatively fewer studies in robotics [47, 48].
Systems of Coupled Oscillators

The technique of using systems of coupled oscillators is a much more common practice in developing a CPG for locomotion control of a robot [49, 50, 51]. The use of these systems is usually found in quadruped and hexapod robots. Figure 2-22 shows a CPG locomotion controlled robot using system of coupled oscillators [51].

![Diagram of a CPG model](image)

**Figure 2-22.** Salamander CPG model tested with an amphibious salamander-like robot [51]

### 2.3.2.2. Benefits of Using CPG based Locomotion Control

According to Ijsppert, there are several benefits to use CPG based locomotion control, as opposed to other locomotion controls [52]. These benefits include:
• A CPG based locomotion control is more robust against perturbations than other controls are.

• CPGs are much more suitable for modular robots than other controllers are because the CPG controllers are distributed among the limbs.

• In changing the pattern of locomotion, a CPG based locomotion controller usually has fewer control parameters.

• It is much simpler to add sensory feedback input to a CPG based locomotion controller.

• CPG models are generally better for use with learning and optimization algorithms.

2.3.3 Energy Method

The energy method is based on the principle of the passive walking robot. The principle behind this method is that a passive walking robot can walk on a downward slope utilizing the gravity provided by the slope, which constitutes the potential energy [21, 22, 23, 26, 27, 28, 29]. Moreover, this principle can be implemented in a passive robot in order to make it walk on level ground or even an upward incline. Energy loss will occur through the collision of the foot and the ground while the robot is walking. If this energy loss can somehow be replenished, the robot will walk on level ground or uphill. The typical energy diagram of a passive compass gait model is shown in Figure 2-23[27].
Figure 2-23. The KE vs. PE diagram of a compass gait [27]
3.1. Compass Gait Model

The physical model used in this study is a compass gait model as explained in Section 2.1.2.2 and Section 2.2.2.2. This model is based on the model of Goswami [13, 14, 15, 16]. This section will discuss the configuration of the compass gait used in this study and the derivation of the equations of motion, the equations of the transition period, and the equations in a slope.

3.1.1. Configuration of a Compass Gait Model

The physical compass gait model used in this study is illustrated in Figure 3-1.

![Figure 3-1. Configuration of the compass gait model](image-url)
Definitions of the symbols:

- \( m_H \): Mass of the hip
- \( m_L \): Mass of the leg
- \( l \): Length of the leg
- \( a \): Position of the center of the leg mass from the center of the hip mass
- \( b = l - a \)
- \( \theta_u \): Angle between the centerline and the support leg
- \( \theta_w \): Angle between the centerline and the swing leg

3.1.2. Equations of Motion

A compass gait can be treated as a double-linked inverted pendulum, with masses on each link and on the joint and pinned on the ground at the foot of the support leg. Figure 3-2 illustrates the configuration of a compass gait with the velocity directions and the gravity direction of each mass.

![Figure 3-2. Configuration with velocity and gravity direction](image)
The subscript ‘u’ refers to the support leg; the subscript ‘w’ refers to the swing leg.

First, we need to define the position vectors to solve for the equations of motion. These position vectors are shown in Figure 3-3.

![Figure 3-3. Position vectors on a compass gait model](image)

The equations of motion can be found by using the Lagrangian equation. The origin of the vectors will be set to the foot of the support leg as shown in Figure 3-3. The position vectors for each mass are

\[
\mathbf{r}_H = l \sin(\pi - \theta_u)\mathbf{i} + l \cos(\pi - \theta_u)\mathbf{j} \quad (3-1)
\]

\[
\mathbf{r}_{u,H} = a \sin(\theta_u - \pi)\mathbf{i} - a \cos(\theta_u - \pi)\mathbf{j} \quad (3-2)
\]

\[
\mathbf{r}_{w,H} = a \sin(\theta_w - \pi)\mathbf{i} - a \cos(\theta_w - \pi)\mathbf{j} \quad (3-3)
\]

The kinetic energy and the potential energy of this compass gait model are

\[
T = \frac{1}{2} m_H v_H^2 + \frac{1}{2} m_L v_u^2 + \frac{1}{2} m_L v_w^2 \quad (3-4a)
\]

\[
V = m_H g(\mathbf{r}_H)_y + m_L g(\mathbf{r}_u)_y + m_L g(\mathbf{r}_w)_y \quad (3-5a)
\]
and the solved energy equations are

$$T = \frac{1}{2} \left( m_l l^2 + m_a l^2 + m_w l^2 \right) \dot{\theta}_u^2 + \frac{1}{2} m_l a^2 \dot{\theta}_w^2 - m_l l a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w)$$  \hspace{1cm} (3-4b)$$

$$V = -(m_a l + m_w l + m_l l) g \cos \theta_u + m_l g a \cos \theta_w$$ \hspace{1cm} (3-5b)

The gravity constant, $g$, includes the direction of gravity and has the value $-9.81 \text{ m/s}^2$.

Given equation (3-4b) and equation (3-5b), the Lagrangian will be

$$L = T - V$$

$$= \frac{1}{2} \left( m_l l^2 + m_a l^2 + m_w l^2 \right) \dot{\theta}_u^2 + \frac{1}{2} m_l a^2 \dot{\theta}_w^2 - m_l l a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w)$$

$$+ (m_a l + m_w l + m_l l) g \cos \theta_u - m_l g a \cos \theta_w$$  \hspace{1cm} (3-6)

The Lagrangian equation for this compass gait will be

for the support leg, \hspace{1cm} \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_u} - \frac{\partial L}{\partial \theta_u} = 0 \hspace{1cm} (3-7a)

for the swing leg, \hspace{1cm} \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_w} - \frac{\partial L}{\partial \theta_w} = 0 \hspace{1cm} (3-8a)

The Lagrangian equation can be found by using equation (3-6).

The Lagrangian equation with respect to the support leg is

$$\left( m_l l^2 + m_a l^2 + m_w l^2 \right) \ddot{\theta}_u - m_l l a \cos(\theta_u - \theta_w) \dot{\theta}_w$$

$$- m_l l a \sin(\theta_u - \theta_w) \dot{\theta}_u^2 + (m_a l + m_w l + m_l l) g \sin \theta_u = 0$$  \hspace{1cm} (3-7b)

The Lagrangian equation with respect to the swing leg is

$$m_l a^2 \ddot{\theta}_w - m_l l a \cos(\theta_u - \theta_w) \dot{\theta}_u + m_l l a \sin(\theta_u - \theta_w) \dot{\theta}_u^2 - m_l g a \sin \theta_w = 0$$  \hspace{1cm} (3-8b)

The equations of motion can be derived from equation (3-7b) and equation (3-8b).
The matrix form of equation (3-9) is
\[
\begin{bmatrix}
  m_u l^2 + m_t b^2 + m_l l^2 & -m_t a \cos(\theta_u - \theta_w) \\
  -m_t a \cos(\theta_u - \theta_w) & m_t a^2
\end{bmatrix}
\begin{bmatrix}
  \dot{\theta}_u \\
  \dot{\theta}_w
\end{bmatrix}
\]
\[
+ \begin{bmatrix}
  0 \\
  m_t a \sin(\theta_u - \theta_w) \dot{\theta}_u
\end{bmatrix}
\begin{bmatrix}
  \dot{\theta}_u \\
  \dot{\theta}_w
\end{bmatrix}
\]
\[
+ \begin{bmatrix}
  (m_t b + m_l l + m_l l) \sin \theta_u \\
  -m_t g b \sin \theta_w
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0
\end{bmatrix}
\] (3-10a)

We can simplify this matrix as follows.
\[
M \ddot{\theta} + C \dot{\theta} + G = 0
\] (3-10b)

Detailed solution can be found in the Appendix.

### 3.1.3. Equations of Transition Period

The transition period is that period when the swing leg hits the ground and the support leg starts to swing. In actual human walking, there will be a double stance phase. However, this phase is assumed to be very short, in fact instantaneous, in this study.

A few assumptions need to be made before deriving the equations for the transition period. During the transition period, an impact between the foot of the pre-impact support leg and the ground will occur. The impact in this case will be assumed to be the inelastic collision. Another assumption concerning the transition period is that there will be no sliding when the pre-impact swing leg hits the ground.

The relationship between steps can be found using the conservation of angular momentum. For the transition period, the conservation of the angular momentum applies in two situations, namely

- The angular momentum of the biped about the collision point is conserved before and after the transition.
The angular momentum of the non-colliding foot is conserved about the hip joint during the collision.

Therefore, the number of equations using the conservation of angular momentum will be two. One equation is derived with respect to the collision point, and the other is derived with respect to the hip.

3.1.3.1. Equation derived with respect to the collision point

Before Impact

The configuration of a compass gait model before impact is illustrated in Figure 3-4.

\[
\begin{align*}
\mathbf{L}^- &= \mathbf{L}_H^- + \mathbf{L}_u^- + \mathbf{L}_w^-
\end{align*}
\]  

(3-11)

If we solve the equation using the configuration shown in Figure 3-4, the angular momentum before the heel strike will be
\[
L^{-1} = \left[ (m_H l^2 \cos(\theta_u^r - \theta_w^r) + 2 m_L b \cos(\theta_u^r - \theta_w^r) - m_L ab) \dot{\theta}_u^r - m_L ab \dot{\theta}_w^r \right] \cdot k \quad (3-12)
\]

After Impact

The configuration of a compass gait model after impact is shown in Figure 3-5.

![Figure 3-5. Configuration after collision with respect to the collision point](image)

The angular momentum after heel strike can be formulized as follows:

\[
L^+ = L_{HH}^+ + L_u^+ + L_w^+ \quad (3-13)
\]

The angular momentum specified for the compass gait model in this study can be derived using Figure 3-5.

\[
L^+ = \left[ (m_H l^2 + m_L l^2 + m_L b^2 - l a \cos(\theta_u^r - \theta_w^r)) \dot{\theta}_u^r + \{ m_L a^2 - m_L a \cos(\theta_u^r - \theta_w^r) \} \dot{\theta}_w^r \right] \cdot k \quad (3-14)
\]

The angular momentum relationship equation with respect to the collision point is

\[
L^- = L^+
\]

\[
= \left[ (m_H l^2 + m_L l^2 + m_L b^2 - l a \cos(\theta_u^r - \theta_w^r)) \dot{\theta}_u^r + \{ m_L a^2 - m_L a \cos(\theta_u^r - \theta_w^r) \} \dot{\theta}_w^r \right] \cdot k \quad (3-15)
\]
3.1.3.2. Equation derived with respect to the hip

The configuration to solve the other equation with respect to the hip is shown in Figure 3-6.

![Configuration with respect to the hip during transition period](image)

**Figure 3-6.** Configuration with respect to the hip during transition period

**Before Impact**

Using Figure 3-6a, the angular momentum with respect to the hip before collision can be derived as follows:

\[
L_{\text{before}} = m_L r_{u,\text{H}}^a \times v_{u}^a = -m_L a b \dot{\theta}_u^a k
\]

**After Impact**

The angular momentum with respect to the hip after collision can be found using a configuration of Figure 3-6b.

\[
L_{\text{after}} = m_L r_{w,\text{H}}^a \times v_{w}^a = \left[-m_L a \cos(\dot{\theta}_u^a - \dot{\theta}_w^a) \dot{\theta}_u^a + m_L a^2 \dot{\theta}_w^a \right] k
\]

The angular momentum relationship equation with respect to the hip is
\[ \mathbf{L}^- = \mathbf{L}^+ \]

\[ -m_t a b \dot{\theta}_w^+ \mathbf{k} = \left[ -m_t a \cos(\theta_u^+ - \theta_w^+) \dot{\theta}_u^+ + m_t a^2 \dot{\theta}_w^+ \right] \mathbf{k} \quad (3-18) \]

If we put equation (3-15) and (3-18) in the matrix form, these equation become as follow:

\[
\begin{bmatrix}
    m_t l^2 \cos(\theta_u^+ - \theta_w^-) + 2m_t lb \cos(\theta_u^+ - \theta_w^-) - m_t a b & -m_t a b & 0 \\
    -m_t a b & 0 & 0 \\
    m_t l^2 + m_t l^2 + m_t b^2 - la \cos(\theta_u^+ - \theta_w^+) & m_t a^2 - m_t a la \cos(\theta_u^+ - \theta_w^+) & m_t a^2
\end{bmatrix}
\begin{bmatrix}
    \dot{\theta}_u^- \\
    \dot{\theta}_w^- \\
    \dot{\theta}_u^+
\end{bmatrix}
= \begin{bmatrix}
    \dot{\theta}_u^- \\
    \dot{\theta}_w^- \\
    \dot{\theta}_u^+
\end{bmatrix} \mathbf{k} \quad (3-19a) \]

The simplified version of the transition equations is as follows:

\[ A^- \dot{\mathbf{0}}^- = A^+ \dot{\mathbf{0}}^+ \quad (3-19b) \]
3.1.4. Compass Gait on a slope

At this point, we have the equations of motion and the equations for the transition period. To simulate our robot’s passive walking; the next step is finding the equations for the compass gait model on a slope. To do this, we need to define the configuration of the compass gait model on a slope. This configuration is illustrated in Figure 3-7.

![Figure 3-7. Compass gait on a downward slope](Image)

In Figure 3-7, the angular velocities and the angular accelerations are not going to change if the slope is unchanged along the direction of walking.

This configuration gives the equation of motion on a slope which is
\[
\begin{bmatrix}
m_H l^2 + m_L b^2 + m_L l^2 & -m_L a \cos(\theta_u^* - \theta_w^*) & -m_L a \cos(\theta_u^* - \theta_w^*) \\
-m_L a \cos(\theta_u^* - \theta_w^*) & m_I a^2 & m_I a^2 \\
0 & -m_L a \sin(\theta_u^* - \theta_w^*) & -m_L a \sin(\theta_u^* - \theta_w^*) \\
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_u^* \\
\ddot{\theta}_w^* \\
\end{bmatrix}
+ \begin{bmatrix}
\left( m_L b + m_I l + m_L l \right) g \sin(\theta_u^* - \phi) \\
-m_L g b \sin(\theta_w^* - \phi)
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix}
\] (3-20)

As we can see, the slope only affects the gravity term in the original equations of motion.

In terms of the equations for the transition period, a constant slope will not affect the equation at all. For a constant slope, there is no change in angular velocity or angular acceleration. In terms of the angles, their difference is only counted in the transition equations.

The equation for the transition period is

\[
\begin{bmatrix}
m_H l^2 \cos(\theta_u^* - \theta_w^*) + 2m_L b \cos(\theta_u^* - \theta_w^*) - m_L a \sin(\theta_u^* - \theta_w^*) - m_L a \sin(\theta_u^* - \theta_w^*) \\
-m_L a \sin(\theta_u^* - \theta_w^*) & 0 & -m_L a \sin(\theta_u^* - \theta_w^*) & -m_L a \sin(\theta_u^* - \theta_w^*) \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_u^* \\
\dot{\theta}_w^* \\
\end{bmatrix}
= \begin{bmatrix}
m_H l^2 + m_L b^2 + m_L l^2 - m_L a \cos(\theta_u^* - \theta_w^*) - m_L a \cos(\theta_u^* - \theta_w^*) \\
-m_L a \cos(\theta_u^* - \theta_w^*) & m_I a^2 & m_I a^2 \\
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_u^* \\
\ddot{\theta}_w^* \\
\end{bmatrix}
\] (3-21)

The detailed solution for the dynamic equations will be shown in the Appendix chapter.

3.1.5. **Compass Gait on a slope with control input torques**

The dynamic equation of the compass gait with control torques can be found by adding the control torques \( (T_u, T_w) \) to the right side of equation (3-20).

\[
\begin{bmatrix}
m_H l^2 + m_L b^2 + m_L l^2 & -m_L a \cos(\theta_u^* - \theta_w^*) & -m_L a \cos(\theta_u^* - \theta_w^*) \\
-m_L a \cos(\theta_u^* - \theta_w^*) & m_I a^2 & m_I a^2 \\
0 & -m_L a \sin(\theta_u^* - \theta_w^*) & -m_L a \sin(\theta_u^* - \theta_w^*) \\
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_u^* \\
\ddot{\theta}_w^* \\
\end{bmatrix}
+ \begin{bmatrix}
\left( m_L b + m_I l + m_L l \right) g \sin(\theta_u^* - \phi) \\
-m_L g b \sin(\theta_w^* - \phi)
\end{bmatrix} = \begin{bmatrix} T_u \\
T_w \end{bmatrix}
\] (3-22)
3.2. **Control System of the Compass Gait Model**

The control system used in this study is based on the central pattern generator technique. Based on a connectionist model, the central pattern generator is used to walk a compass gait robot in this study. Much research has been conducted on the use of control systems based on a central pattern generator (CPG) to cause biped robots to walk. Many improvements have been tested, many different theories generated, and many contributions made in CPG controlled biped robots. Most, if not all of this research has been conducted on concentrating on either how to follow or how to make the exact movement patterns of the gait. This method worked pretty well. Currently, there are many robots which appear to walk virtually as humans do. These robots’ movements substantially resemble natural relaxed human gait.

In this study, the goal, i.e. natural looking and efficient human-like gait, is the same as in all this previous research; however, the control method is different. When humans and animals walk, they do not follow exact patterns for every joint. This lack of pattern precision functions effectively because of the central pattern generators with reflex circuits. Even the signals from the CPG are not always the same because the body, human or animal, is a biological mechanism, and therefore, does not function like a machine. Uncertainties will always be included in the signal from the CPG in the biological being even if the CPG tries to send same signal every time. The natural world around us is an ever-changing entity. This changing environment will always add some degree of uncertainty to the lives of the biological beings. To survive this ever-changing natural world, biological beings feel, i.e. sense, change in the environment, rather than measure change in the environment. However, our technology is limited, so we have to make out with what we have. Humans have developed many electrical and mechanical sensors to measure changes in the surroundings. If we compare the technical sensors to the biological sensors, a
human being in this case, the most biological-like technical sensors are the accelerometer and textile sensor (i.e. touch sensor). We can not sense the angle like a potentiometer. If we close our eyes, we can not measure what degree of our arm is with respect to our trunk, we can only feel gravity acting on our arm and we can estimate the angle. Human beings can not measure velocity, either. If we close our eyes in a quiet smooth-moving car, we can not feel the car moving. On the basis of this reasoning, we conclude that the central pattern generator based on accelerations acting on the robot body is biologically closer to the actual CPG of biological beings than a CPG based on the movement pattern. Therefore, the central pattern generator used in this study is based on accelerations acting on the robot body instead of choosing the movement pattern as the reference of the CPG.

What about the choice of a neural network as the central pattern generator structure? Humans can learn new movement patterns and train their central pattern generators. For example, swimming can be learned even at an adult age; and furthermore, after enough practice, humans do not need to think about the movement pattern of our arms and legs in accomplishing the task. This fact clearly shows that learning in the central pattern generator is on-going process throughout life. A technology that mimics the human learning process is the neural network, one of the intelligent control methods. Hence, a neural network has been chosen for the CPG structure of our robot.

Returning to the discussion of the control system for the compass gait robot in this study, Figure 3-8 shows the basic configuration of the composite system. At this point, the outputs of the central pattern generator go through the PID filter to provide input to the compass gait model. This PID filter can be changed to a different method in order to create more robust control system.
3.2.1. **Plant – Compass Gait Model**

The inputs of the compass gait model are torques applied to the each leg at the hip joint mass. The outputs from the compass gait model (plant) are angles from the potentiometers and the angular accelerations from the accelerometers on each leg. The angular velocities will be calculated. The angular velocity can be calculated, either with computational hardware or by internal calculation.

![Diagram of Compass Gait Model with CPG Control](image)

**Figure 3-8.** Compass gait model with CPG control

3.2.2. **Central Pattern Generator**

A neural network is used as the central pattern generator (CPG). The neural network is a feed forward network and has three inputs, the desired velocity \( v_d \) and two measured angles \( \theta_u, \text{measured} \) and \( \theta_w, \text{measured} \), in the input layer. This network has four outputs, two required velocities
(\dot{\theta}_u,_{required} \text{ and } \dot{\theta}_w,_{required})$, two required accelerations ($\ddot{\theta}_u,_{required}$ and $\ddot{\theta}_w,_{required}$) in the output layer, Furthermore, the neural network has three hidden layers with fifteen neurons in each hidden layer. The neural network is illustrated in Figure 3-9.

**Figure 3-9.** Neural network used as a CPG

### 3.2.3. PID Filter (PI or PD Controller)

The PID filter (or PID controller) is actually either a PD filter or a PI filter depending on which input is chosen for the proportional part. The PID filter has four inputs, two measured angles ($\theta_u,_{measured}$ and $\theta_w,_{measured}$) and two measured accelerations ($\ddot{\theta}_u,_{measured}$ and $\ddot{\theta}_w,_{measured}$) and two torques ($T_u, T_w$) as outputs. The PID filters can be expressed in the following equations:
\[ T_u = k_u e_{\dot{\theta}_u} + k_a e_{\ddot{\theta}_u} \]  
(3-23)

\[ T_w = k_v e_{\dot{\theta}_w} + k_a e_{\ddot{\theta}_w} \]  
(3-24)

### 3.2.4. Velocity Calculation

The angular velocities of the each leg can be derived from equation (3-22).

\[
\dot{\theta}_u^* = \sqrt{T_w + \left[ m_i a \cos(\theta_u^* - \theta_w^*) \ddot{\theta}_w^* - m_i a^2 \dddot{\theta}_w^* \right] + m_i g b \sin(\theta_w^* - \phi) \over m_i a \sin(\theta_u^* - \theta_w^*) \dot{\theta}_u^*} 
\]  
(3-25)

\[
\dot{\theta}_w^* = \sqrt{T_u + \left[ m_i I^2 + m_i b^2 + m_i l^2 \dddot{\theta}_u^* - m_i a \cos(\theta_u^* - \theta_w^*) \dddot{\theta}_w^* \right] + (m_i b + m_i l + m_i l) g \sin(\theta_u^* - \phi) \over m_i a \sin(\theta_u^* - \theta_w^*) \dot{\theta}_u^*} 
\]  
(3-26)
CHAPTER 4

SIMULATION

4.1. Passive Walking

The passive walking of the compass gait model can be performed by putting the compass gait model on a downward slope and giving the robot appropriate initial conditions. The compass gait model is simulated on a set of inclines. The simulated slope starts with a slope angle of 0.1° which increases to a final slope angle of 4.5°.

The velocity of the compass gait model increases along with the increase in the downward slope angle. Figure 4-1 shows the relationship between the downward slope angle and the velocity of the compass gait.

![Velocity vs. Slope angle](image)

**Figure 4-1.** Relationship between the slope angle and the velocity of the compass gait

The gait period also increases as the downhill slope angle increases. The relationship between the gait period and the downward incline is illustrated in Figure 4-2.
As shown in Figures 4-1 and 4-2, the compass gait model goes into the chaos gait region when the downhill slope angle reaches $4.4^\circ$.

The following series of plots demonstrate the characteristics of the compass gait. The simulation is performed on a $3.0^\circ$ downward incline. In this compass gait robot model, $180^\circ$ is the symmetry line, or the center line, of the body.
Figure 4-3. Time history of the support leg

Figure 4-4. Time history of the swing leg
**Figure 4-5.** Time history of the angular velocity of the support leg

**Figure 4-6.** Time history of the angular velocity of the swing leg
Figure 4-7. Time history of the angular acceleration of the support leg

Figure 4-8. Time history of the angular acceleration of the swing leg
Figure 4-9. Time history of the position of the center of mass of the hip

Figure 4-10. Position pattern of the swing leg on 3° downhill slope
Figure 4-11. Acceleration pattern of the support leg on 3° downhill slope

Figure 4-12. Acceleration pattern of the swing leg on 3° downhill slope
These plots clearly show the regular pattern of leg movement. It will be interesting to compare the pattern of the swing leg for different slopes.

Figure 4-13. Position pattern of the swing leg on 1° downhill slope
Figure 4-14. Position pattern of the swing leg on 4° downhill slope
Figure 4-13 is the position pattern of the swing leg on a $1^\circ$ downward slope, while Figure 4-14 shows the position of the swing leg on a $4^\circ$ downward slope. These two plots show the walking pattern changes as the speed of the model changes. The pattern is not scaled as the speed changes; on the contrary, distinctly different patterns emerge. The movement of the swing leg after it passes the symmetry line is bigger at the faster speed.

4.2. **Compass Gait Model with the Control System on Level Ground**

The object of this control system is to control the velocity of the compass gait biped robot model. To simulate controlled walking on level ground and to obtain the desired walking velocity, we need to train the central pattern generator (CPG) and choose the appropriate gains for the PID filter.

4.2.1. **Constructing the Control System**

The control system consists of two major parts, a central pattern generator and a PID filter.

4.2.1.1. **Central Pattern Generator**

The central pattern generator used in this study employs a connectionist model, which is a feed-forward neural network in this study. This neural network needs to be trained to generate a proper signal. The training data used for the neural network are from the passive walking simulation in Section 4.1. As mentioned in Section 3.2.2, the inputs of the neural network are the desired velocity ($v_d$) and the measured angles ($\theta_u, \text{measured}$ and $\theta_w, \text{measured}$). The outputs of the neural network are the required velocities ($\dot{\theta}_u, \text{required}$ and $\dot{\theta}_w, \text{required}$) and the required accelerations ($\ddot{\theta}_u, \text{required}$ and $\ddot{\theta}_w, \text{required}$).

The range of the desired velocity to which the neural network has been trained is from 0.2464 [m/sec] to 0.7933 [m/sec] which matches the velocity of the model walking on a slope.
from $0.1^\circ$ to $4.3^\circ$. The neural network is trained for 100,000 epochs. The mean square error (MSE) of the network is $0.79989 \times 10^{-7}$. Figure 15 shows part of the training process.

![Performance plot of the neural network used as a CPG](image)

**Figure 4-15.** Performance plot of the neural network used as a CPG

4.2.1.2. PID Filter

The parameters of the PID filters are $k_v$ and $k_a$. These parameters were obtained by trial-and-error. However, this PID gain set is not a unique set. In fact, different sets of the parameters are able to achieve same goal. Figure 4-16 shows this PID gain set. The PID gains found are illustrated versus the desired velocities. The trained desired velocity range is between 1 km/h and 4.0 km/h. However, the PID gains are in the range from 1.5 km/h to 6.5 km/h. These PID gains
are connected to one 3-D cubic spline. This gain spline will be used to find appropriate gains for the desired walking velocity input, which fall between the found PID gains.

![PID Gains vs. Desired Velocity](image)

**Figure 4-16.** PID Gains vs. Desired Velocity

4.2.2. Results

The initial condition for the simulation is the standing still position of the compass model. Therefore, the initial conditions of the model are zero radians per second for the angular velocities of the both legs, and \( \pi \) radians for the angular velocities of both legs, \((\pi \ [\text{rad}], \pi [\text{rad}], 0[\text{rad/sec}], 0[\text{rad/sec}])\).
By using the PID gains that are shown Figure 4-16, the resulting walking velocities are acquired. Figure 4-17 illustrates the resulting walking velocity versus the desired walking velocity, as well as the error between them in percentage. The resulting walking speeds are represented by a solid line and, while the errors in percentage between them are represented by a dashed line.

![Figure 4-17](image)

**Figure 4-17.** Resultant Walking Velocity and Error versus Desired Walking Velocities

When the desired walking speed is over 6.5 [km/h] (1.8056 [m/s] or 4.0389 [mile/h]), the control system does not give a satisfactory result. The biped robot is able to walk with stabilized
steps but the resulting walking speed does not reach the desired walking speed. The reason for this deficiency is that the biped robot has reached its maximum walking speed. In contrast the robot, a normal human can walk as fast as around 2.0 [m/s] (7.2 km/h or 4.4739 mile/h). The robot has no joint and it is fixed in two-dimensional plane. Therefore, this compass gait 2D biped robot can not reach the fastest walking speed of a normal human. The biped robot can achieve a walking speed of 6.5 km/h by running. Using a compass gait model to study running is an entirely different line of research and is not considered a part of this research.

Figure 4-18 shows the change in the final gait period over the desired walking speed input. As Figure 4-18 shows, the final gait period decreases as the desired walking speed increases.

Figure 4-18. Final Gait Period vs. Desired Walking Velocity
Two cases are presented here. For the first case, the desired velocity is 5.689 [km/h] = 1.5803 [m/s] = 3.535 [mph] while in other case, the desired velocity is set to 5.689 [km/h], initially, and then decreases to 2.25 [km/h] = 0.625 [m/s] = 1.3981 [mph].

The desired walking velocities are chosen between the PID gains obtained by trial-and-error.

### 4.2.2.1. Case 1: Fixed desired velocity

The desired walking velocity is fixed at $v_d = 5.689$ [km/h] and the compass gait model is initially standing still. The object of this simulation is finding to demonstrate that the PID gain spline extracts satisfactory results.

![Graph of Walking Speed vs. Step](image)

**Figure 4-19.** Walking Speed vs. Step
Figure 4-19 shows the change in the gait speed along the steps. This figure shows that the resulting walking speed converges with the desired walking speed at approximately the 48\textsuperscript{th} step. The resulting walking speed is 5.7043 [km/h] at the 78\textsuperscript{th} step. The error between the desired walking speed and the resulting walking speed is 0.2681 [%].

\textbf{Figure 4-20.} Angle trajectory of the Swing leg vs. Time
Figure 4-20 illustrates the angle trajectory of the swing leg point mass. Figure 4-21 shows the angle trajectory of the support leg point mass. These two figures (Figure 4-20 and 4-21) clearly show that the angle of the robot legs become wider as the robot tries to achieve the desired walking speed. These figures further demonstrate that the robot reaches the desired walking speed after about 10 seconds. The robot shows that the ranges of the angle changes of the both legs are fixed at about 10 seconds and demonstrates that the intervals between the steps
of the both legs are set by the same point in time. Figure 4-20 and 4-21 also illustrate that the
time interval between steps narrows as the robot tries to meet the desired walking speed.

![Step Width vs. Step](image)

**Figure 4-22.** Step width vs. Step

Figure 4-22 shows the changes in step width over time. As in Figures 4-20 and 4-21,
Figure 4-22 shows the step width of the biped robot increases as the step number increases.
Figure 4-23. Acceleration Change of the Swing leg in Time

Figure 4-24. Acceleration Change of the Support leg in Time
Figure 4-23 shows the acceleration change of the swing leg in time, while Figure 4-24 shows the acceleration change of the support leg in time. These two figures also show the increase in magnitude as the biped robot attempts to reach the desired walking speed.

Figure 4-25. Position of the Swing Leg Point Mass
Figure 4-25 illustrates the pattern of the position of the point mass in the swing leg along the X-Y plane. As the figure shows, the robot tries to meet the desired walking speed, it starts by kicking its swing leg higher and then the robot settles down into a regular pattern as the desired walking speed is achieved. In Figure 4-25, the pattern shows that the robot is kicking its swing leg much higher when the swing leg is in front of the body then it does otherwise. This behavior is the expected response of the swing leg because humans also kick their legs when they attempt to walk faster, even though we may not recognize the phenomenon.

![Figure 4-25](image)

**Figure 4-25. Control Torque Applied to the Support Leg**

![Figure 4-26](image)

**Figure 4-26. Control Torque Applied to the Support Leg**
The control torque applied to the both legs of the biped robot in time is shown in Figures 4-26 and 4-27. In both figures, the torque is settling at about 12 second.

The gait period is illustrated in Figure 4-28. The final gait period (step period) is 0.22 second at the 78th step, moreover, the gait period settles down at about the 22nd step. The step starts slowly and then settled down.
Figure 4-28. Gait Period Change in Steps
4.2.2.2. **Case 2: Varying desired velocity**

In this case, the robot is going to start from a standing still position and is going to walk in order to achieve the desired walking speed of 2.25 [km/h], then the robot is going to speed up to the desired walking speed, of 5.689 [km/h] at 15 [second].

The purpose of this case is to test how well the robot responds in case of changes in the desired input.

Figure 4-29 demonstrates the gait speed change of the robot according to the desired walking speed change.

![Figure 4-29. Gait Speed of the biped robot vs. Steps](image-url)
As Shown in Figure 4-29, the first desired walking speed is met, with an approximately 6.356 % error rate at about the 11\textsuperscript{th} step; the second desired walking speed is achieved with a 0.2637 % overshoot at the final step, the 83\textsuperscript{rd} step.

An examination of Figure 4-29 reveals that the robot takes only 25 steps during the first 15 second and 58 steps for the lest. The reason for this behavior is that the robot takes a longer time for each step when the walking speed is lower. This distinction is shown in Figure 4-30.

\textbf{Figure 4-30.} Gait Period vs. Step
As confirmed in Figure 4-30, the settle-downed gait period for the first part is 0.58 [seconds], while for the second part it is 0.22 [seconds]. The first gait period is longer than the final gait period, signifying that the step count of the robot is higher when its walking speed is faster.

![Graph showing the angle of the support leg over time](image)

**Figure 4-31. Support Leg Angle Change in Time**
Figure 4-32. Swing Leg Angle Change in Time

The angle change histories of both legs of the biped walking robot in time are shown in Figures 4-31 and 4-32. As these figures clearly show, the walking speed of the robot starts to change at 15 seconds even though the second desired walking speed has not yet been met at 15 second. The length of time between steps changes at 15 seconds, the step time interval becoming shorter after 15 seconds. This change means that the robot paces faster after 15 seconds when the desired walking speed increases, as should occur.

The position pattern of the swing leg of the walking robot is illustrated in Figure 4-33.
Figure 4-33. Position Pattern of the Swing Leg
In Figure 4-33, there are two patterns. The first pattern is for the first desired walking speed, 2.25 [km/h]. The first pattern shows that the highest point of the posterior of the robot body is about 0.497 [m] and the highest point of the front of the robot body is about 0.516 [m]. The second pattern is for the second desired walking speed, 5.689 [km/h]. The highest points of the second pattern are approximately 0.499 [m] for the posterior of the robot body and approximately 0.519 [m] for the front. The reason for the lack of clarity in the second pattern can be found in Figure 4-29. The simulation has ended without enough of a settled walking phase for the second pattern, in contrast with the first pattern.

For another reference, we can check the step width history, the results of which are shown in Figure 4-34.

Figure 4-34. Step Width History

The step width changes from 38.72 [cm] from the 24\textsuperscript{th} step to 34.86 [cm] at the 83\textsuperscript{rd} step as the desired walking speed changes from 2.25 [km/h] to 5.689 [km/h] at 15 second.
Figure 4-35. Angular Acceleration Change of the Support Leg

Figure 4-36. Angular Acceleration Change of the Swing Leg
Figure 4-37. Control Torque for the Support Leg

Figure 4-38. Control Torque for the Swing Leg
Studying the accelerations of both legs and the control torques applied to both legs can be helpful as a means to reinforce the previous results.

The angular acceleration time histories of both legs are shown in Figures 4-35 and 4-36. The time histories of the control torque for both legs are illustrated in Figures 4-37 and 4-38. These figures also show the similar variations of behavior as did previous figures for situation in which the desired walking speed is altered.
CHAPTER 5

CONCLUSION

5.1. Conclusion

The main objective is to develop a robust velocity controller for a biped 2-D walking robot, which would achieve a desired walking velocity. To meet this objective, the dynamics equations of motion of the biped walking robot system were studied and coded in MATLAB with graphical animation of the gait. A central pattern generator (CPG) based controller was developed and tested. This controller was tested using a compass gait model based on the model created by Goswami. The controller for the compass gait model used a neural network based CPG, which had been trained with the simulation data from the passive downhill walking of the compass gait model. The PID gain set for the model was found through the trial-and-error method. The PID gain spline for the model was constructed based on this PID gain set. Some of the data were chosen from this PID gain spline for the compass gait model and these data were used to verify the feasibility of the controller. Two cases were tested in simulations to show the robustness and adaptability of the controller. The first case was to investigate how well the controller performs when the fixed desired walking velocity was presented as an input. The second case was to discover how well the controller executed the task of walking when the desired walking velocity varied in time. This study demonstrated by the simulations that the controller performed well in both cases and shows the robustness of the controller.

5.2. Contributions

In this study, the following contributes are made to biologically inspired robotics.

- Incorporating the CPG with some modification to the controller of a compass gait biped model demonstrates robustness and adaptability.
• The controller in this study shows that the CPG, with some modification, can achieve efficiency by simplifying the calculation of the control.

• Inputs and outputs for the biped robot in this study are closer to the actual biological inputs and outputs of a human being which can make utilization of this technology with actual biological beings easier.

• Combining passive dynamics and modified CPG in this study shows a more natural looking and efficient biped robot during walking by incorporating gravity and momentum.

5.3. Future Recommendations

One recommendation for further study on this topic would be to completely eliminate the PID gain spline by replacing it using an intelligent control scheme such as a knowledge-based control, fuzzy logic control, etc.

Also, slopes that simulate walking uphill or downhill could be introduced with the control scheme. Adding knee joints, such as revolute joints or prismatic joints for the foot clearance, would also be an interesting and potentially productive topic for further study, because this would likely make the gait of the walking robot more human-like.

Other topics of interest for future research include the following: (1) adding sensors, such as a vision sensor, (2) extending this study to the three-dimensional walking robot, and (3) studying the aspect of running (even though it constitutes totally different dynamics). Finally, the construction and test of a prototype for this biped walking robot is recommended.
REFERENCES


REFERENCES (continued)


REFERENCES (continued)


1. Configuration of a Compass Gait

- $m_H$: Mass of the hip
- $m_L$: Mass of the leg
- $l$: Length of the leg
- $a$: Position of the center of the leg mass from the center of the hip mass
- $b = l - a$
- $\theta_u$: Angle between the centerline and the support leg
- $\theta_w$: Angle between the centerline and the swing leg
2. Equation of Motion

a. Configuration

The support leg of a compass gait can be treated as a double-linked inverted pendulum.

The subscript ‘,u’ represents the support leg and the subscript ‘,w’ represents the swing leg.

First, define the position vectors as follow.
\[ \mathbf{r}_H = l \sin(\pi - \theta_u) \mathbf{i} + l \cos(\pi - \theta_u) \mathbf{j} \]
\[ \mathbf{r}_{u,H} = a \sin(\theta_u - \pi) \mathbf{i} - a \cos(\theta_u - \pi) \mathbf{j} \]
\[ \mathbf{r}_{w,H} = a \sin(\theta_w - \pi) \mathbf{i} - a \cos(\theta_w - \pi) \mathbf{j} \]

b. Equation

1) Kinetic energy

\[ T = \frac{1}{2} m_H v_H^2 + \frac{1}{2} m_u v_u^2 + \frac{1}{2} m_w v_w^2 \]

a) Velocity at the center of mass of hip \((v_H)\)

\[ \mathbf{v}_H = \frac{d\mathbf{r}_H}{dt} \]
\[ = \frac{d}{dt}(l \sin(\pi - \theta_u)) \mathbf{i} + \frac{d}{dt}(l \cos(\pi - \theta_u)) \mathbf{j} \]
\[ = \frac{d}{dt}(l \sin \theta_u) \mathbf{i} + \frac{d}{dt}(-l \cos \theta_u) \mathbf{j} \]
\[ = l \dot{\theta}_u \cos \theta_u \mathbf{i} + l \dot{\theta}_u \sin \theta_u \mathbf{j} \]
\[ v_H^2 = \dot{x}_H^2 + \dot{y}_H^2 \]
\[ = \left(l \dot{\theta}_u \cos \theta_u\right)^2 + \left(l \dot{\theta}_u \sin \theta_u\right)^2 \]
\[ = l^2 \dot{\theta}_u^2 \cos^2 \theta_u + l^2 \dot{\theta}_u^2 \sin^2 \theta_u \]
\[ = l^2 \dot{\theta}_u^2 \left(\cos^2 \theta_u + \sin^2 \theta_u\right) \]
\[ = l^2 \dot{\theta}_u^2 \]
b) Velocity at the center of mass of the support leg ($r_u$)

$$
r_u = r_H + r_{u,H}
$$

$$
= [l \sin(\pi - \theta_u) + a \sin(\theta_u - \pi)]i + [l \cos(\pi - \theta_u) - a \cos(\theta_u - \pi)]j
$$

$$
= (l-a)\sin(\pi - \theta_u)i + (l-a)\cos(\pi - \theta_u)j
$$

$$
= b\sin(\pi - \theta_u)i + b\cos(\pi - \theta_u)j
$$

$$
= b\sin \theta_u i - b\cos \theta_u j
$$

$$
v_u = \frac{dr_u}{dt}
$$

$$
= \frac{d}{dt} (b\sin \theta_u)i + \frac{d}{dt} (-b\cos \theta_u)j
$$

$$
= b\dot{\theta}_u \cos \theta_u i + b\dot{\theta}_u \sin \theta_u j
$$

$$
v_u^2 = \dot{x}_u^2 + \dot{y}_u^2
$$

$$
= (b\dot{\theta}_u \cos \theta_u)^2 + (b\dot{\theta}_u \sin \theta_u)^2
$$

$$
= b^2\dot{\theta}_u^2 \cos^2 \theta_u + b^2\dot{\theta}_u^2 \sin^2 \theta_u
$$

$$
= b^2\dot{\theta}_u^2
$$

c) Velocity at the center of mass of the swing leg ($r_w$)

$$
r_w = r_H + r_{w,H}
$$

$$
= [l \sin(\pi - \theta_w) + a \sin(\theta_w - \pi)]i + [l \cos(\pi - \theta_w) - a \cos(\theta_w - \pi)]j
$$

$$
= [l \sin \theta_w + a(\sin \theta_w \cos \pi - \cos \theta_w \sin \pi)]i + [-l \cos \theta_w - a(\cos \theta_w \cos \pi + \sin \theta_w \sin \pi)]j
$$

$$
= (l \sin \theta_w - a \sin \theta_w)i + (-l \cos \theta_w + a \cos \theta_w)j
$$

$$
= (l \sin \theta_w - a \sin \theta_w)i - (l \cos \theta_w - a \cos \theta_w)j
$$

$$
v_w = \frac{dr_w}{dt}
$$

$$
= \frac{d}{dt} (l \sin \theta_w - a \sin \theta_w)i + \frac{d}{dt} (-l \cos \theta_w + a \cos \theta_w)j
$$

$$
= (l \dot{\theta}_w \cos \theta_w - a \dot{\theta}_w \cos \theta_w)i + (l \dot{\theta}_w \sin \theta_w - a \dot{\theta}_w \sin \theta_w)j
$$
APPENDIX (continued)

\[ v_w^2 = x_w^2 + y_w^2 \]
\[ = (\dot{\theta}_w \cos \theta_w - a \dot{\theta}_w \cos \theta_w)^2 + (\dot{\theta}_w \sin \theta_w - a \dot{\theta}_w \sin \theta_w)^2 \]
\[ = l^2 \dot{\theta}_w^2 \cos^2 \theta_w + a^2 \dot{\theta}_w^2 \cos^2 \theta_w - 2la \dot{\theta}_w \dot{\theta}_w \cos \theta_w \cos \theta_w \]
\[ + l^2 \dot{\theta}_w^2 \sin^2 \theta_w + a^2 \dot{\theta}_w^2 \sin^2 \theta_w - 2la \dot{\theta}_w \dot{\theta}_w \sin \theta_w \sin \theta_w \]
\[ = l^2 \dot{\theta}_w^2 + a^2 \dot{\theta}_w^2 - 2la \dot{\theta}_w \dot{\theta}_w (\cos \theta_w \cos \theta_w + \sin \theta_w \sin \theta_w) \]
\[ = l^2 \dot{\theta}_w^2 + a^2 \dot{\theta}_w^2 - 2la \dot{\theta}_w \dot{\theta}_w \cos(\theta_w - \theta_w) \]

\[ T = \frac{1}{2} m_H v_H^2 + \frac{1}{2} m_L v_L^2 + \frac{1}{2} m_J v_J^2 \]
\[ = \frac{1}{2} m_H \left( l^2 \dot{\theta}_u^2 + \frac{1}{2} m_J \left( b^2 \dot{\theta}_u^2 \right) + \frac{1}{2} m_J \left[ l^2 \dot{\theta}_u^2 + a^2 \dot{\theta}_u^2 - 2la \dot{\theta}_u \dot{\theta}_u \cos(\theta_u - \theta_w) \right] \right) \]
\[ = \frac{1}{2} m_H l^2 \dot{\theta}_u^2 + \frac{1}{2} m_J l^2 \dot{\theta}_u^2 + \frac{1}{2} m_J \dot{\theta}_u^2 + \frac{1}{2} m_J a^2 \dot{\theta}_u^2 - m_J la \dot{\theta}_u \dot{\theta}_u \cos(\theta_u - \theta_w) \]
\[ = \frac{1}{2} \left( m_H l^2 + m_J b^2 + m_J l^2 \right) \dot{\theta}_u^2 + \frac{1}{2} m_J a^2 \dot{\theta}_u^2 - m_J la \dot{\theta}_u \dot{\theta}_u \cos(\theta_u - \theta_w) \]

2) Potential energy

\[ V = m_H g(r_H) + m_J g(r_J) + m_L g(r_L) \]
\[ = m_H g(-l \cos \theta_u) + m_J g(-b \cos \theta_u) + m_J g(-l \cos \theta_u + a \cos \theta_w) \]
\[ = -m_H g \cos \theta_u - m_J g b \cos \theta_u - m_J g \cos \theta_u + m_J g a \cos \theta_w \]
\[ = -(m_H l + m_J b + m_J l) g \cos \theta_u + m_J g a \cos \theta_w \]

where the gravity acceleration includes the direction of the gravity (e.g., \( g = -9.81 \) m/s\(^2\))
3) Lagrangian

\[ L = T - V \]
\[ = \frac{1}{2} \left( m_L l^2 + m_L b^2 + m_L l^2 \right) \dot{\theta}_u^2 + \frac{1}{2} m_L a^2 \dot{\theta}_w^2 - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) \]
\[ + (m_L b + m_L l + m_L l) g \cos \theta_u - m_L a g \cos \theta_w \]

4) Equation of Motion

Lagrangian’s equations are

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_u} - \frac{\partial L}{\partial \theta_u} = 0 \]
\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_w} - \frac{\partial L}{\partial \theta_w} = 0 \]

a) With respect to \( \theta_u \)

i) \( \frac{\partial L}{\partial \theta_u} \)

\[ \frac{\partial L}{\partial \theta_u} = \frac{\partial}{\partial \theta_u} \left\{ \frac{1}{2} \left( m_L l^2 + m_L b^2 + m_L l^2 \right) \dot{\theta}_u^2 + \frac{1}{2} m_L a^2 \dot{\theta}_w^2 - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) \right\} \]
\[ + (m_L b + m_L l + m_L l) g \cos \theta_u - m_L a g \cos \theta_w \}
\[ = m_L a \dot{\theta}_u \dot{\theta}_w \sin(\theta_u - \theta_w) - (m_L b + m_L l + m_L l) g \sin \theta_u \]

ii) \( \frac{\partial L}{\partial \dot{\theta}_u} \)

\[ \frac{\partial L}{\partial \dot{\theta}_u} = \frac{\partial}{\partial \dot{\theta}_u} \left\{ \frac{1}{2} \left( m_L l^2 + m_L b^2 + m_L l^2 \right) \dot{\theta}_u^2 + \frac{1}{2} m_L a^2 \dot{\theta}_w^2 - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) \right\} \]
\[ + (m_L b + m_L l + m_L l) g \cos \theta_u - m_L a g \cos \theta_w \}
\[ = \left[ m_L l^2 + m_L b^2 + m_L l^2 \right] \dot{\theta}_u - m_L a \dot{\theta}_w \cos(\theta_u - \theta_w) \]
iii) \( d(\partial L/\partial \dot{\theta}_u)/dt \)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_u} \right) = \frac{d}{dt} \left[ (m_H l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \dot{\theta}_u \cos(\theta_u - \theta_w) \right] \\
= (m_H l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \dot{\theta}_u \cos(\theta_u - \theta_w) \\
- m_L a \dot{\theta}_u \left[ -\sin(\theta_u - \theta_w) \right] \ddot{\theta}_u - \dot{\theta}_w \\
= (m_H l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \cos(\theta_u - \theta_w) \ddot{\theta}_w \\
+ m_L a \sin(\theta_u - \theta_w) \ddot{\theta}_w \left[ \dot{\theta}_u - \dot{\theta}_w \right]
\]

iv) Lagrangian’s equation

\[
0 = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_u} - \frac{\partial L}{\partial \theta_u} = \frac{d}{dt} \left[ (m_H l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \cos(\theta_u - \theta_w) \ddot{\theta}_w + m_L a \sin(\theta_u - \theta_w) \ddot{\theta}_w (\dot{\theta}_u - \dot{\theta}_w) \right] \\
- \left[ m_L a \ddot{\theta}_w \sin(\theta_u - \theta_w) - (m_L b + m_H l + m_L) g \sin \theta_u \right] \\
= (m_H l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \cos(\theta_u - \theta_w) \ddot{\theta}_w \\
+ m_L a \sin(\theta_u - \theta_w) \ddot{\theta}_w \dot{\theta}_u - m_L a \sin(\theta_u - \theta_w) \dot{\theta}_w^2 \\
- m_L a \sin(\theta_u - \theta_w) \ddot{\theta}_u \dot{\theta}_w + (m_L b + m_H l + m_L) g \sin \theta_u \\
= (m_H l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \cos(\theta_u - \theta_w) \ddot{\theta}_w - m_L a \sin(\theta_u - \theta_w) \dot{\theta}_w^2 \\
+ (m_L b + m_H l + m_L) g \sin \theta_u
\]

\[
(m_H l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \cos(\theta_u - \theta_w) \ddot{\theta}_w \\
- m_L a \sin(\theta_u - \theta_w) \dot{\theta}_w^2 + (m_L b + m_H l + m_L) g \sin \theta_u = 0
\]
b) With respect to $\theta_w$

i) $\partial L/\partial \theta_w$

$$\frac{\partial L}{\partial \theta_w} = \frac{\partial}{\partial \theta_w} \left\{ \frac{1}{2} (m_H l^2 + m_L b^2 + m_L l^2) \dot{\theta}_u^2 + \frac{1}{2} m_a^2 \dot{\theta}^2 - m_la \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) ight\}$$

$$+ (m_l b + m_H l + m_l l)g \cos \theta_u - m_l g_a \cos \theta_w \}$$

$$= \frac{\partial}{\partial \theta_w} \left\{ - m_la \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) - m_l g_a \cos \theta_w \right\}$$

$$= -m_la \dot{\theta}_u \dot{\theta}_w [-\sin(\theta_u - \theta_w)(-1)] - m_l g_a (-\sin \theta_w)$$

$$= -m_la \dot{\theta}_u \dot{\theta}_w \sin(\theta_u - \theta_w) + m_l g_a \sin \theta_w$$

ii) $\partial L/\partial \dot{\theta}_w$

$$\frac{\partial L}{\partial \dot{\theta}_w} = \frac{\partial}{\partial \dot{\theta}_w} \left\{ \frac{1}{2} (m_H l^2 + m_L b^2 + m_L l^2) \dot{\theta}_u^2 + \frac{1}{2} m_a^2 \dot{\theta}^2 - m_la \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) ight\}$$

$$+ (m_l b + m_H l + m_l l)g \cos \theta_u - m_l g_a \cos \theta_w \}$$

$$= \frac{\partial}{\partial \dot{\theta}_w} \left\{ \frac{1}{2} m_a^2 \dot{\theta}^2 - m_la \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) \right\}$$

$$= m_a^2 \dot{\theta}_w - m_la \dot{\theta}_u \cos(\theta_u - \theta_w)$$

iii) $d(\partial L/\partial \dot{\theta}_w)/dt$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_w} \right) = \frac{d}{dt} \left\{ m_a^2 \dot{\theta}_w - m_la \dot{\theta}_u \cos(\theta_u - \theta_w) \right\}$$

$$= m_a^2 \ddot{\theta}_w - m_la \dot{\theta}_u \cos(\theta_u - \theta_w) - m_la \dot{\theta}_u \dot{\theta}_w [-\sin(\theta_u - \theta_w)] (\dot{\theta}_u - \dot{\theta}_w)$$

$$= m_a^2 \ddot{\theta}_w - m_la \dot{\theta}_u \cos(\theta_u - \theta_w) + m_la \dot{\theta}_u \sin(\theta_u - \theta_w)] (\dot{\theta}_u - \dot{\theta}_w)$$
iv) Lagrangian’s equation

\[
0 = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_w} - \frac{\partial L}{\partial \theta_w} = m_L a^2 \ddot{\theta}_w - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) + m_L a \dot{\theta}_u \sin(\theta_u - \theta_w)\left(\dot{\theta}_u - \dot{\theta}_w\right) - \left[-m_L a \dot{\theta}_u \dot{\theta}_w \sin(\theta_u - \theta_w) + m_L g a \sin \theta_w\right]
\]
\[
= m_L a^2 \ddot{\theta}_w - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) + m_L a \dot{\theta}_u \sin(\theta_u - \theta_w)\dot{\theta}_u^2 - m_L a \dot{\theta}_u \sin(\theta_u - \theta_w)\dot{\theta}_w
\]
\[
+ m_L a \dot{\theta}_u \dot{\theta}_w \sin(\theta_u - \theta_w) - m_L g a \sin \theta_w
\]
\[
= m_L a^2 \ddot{\theta}_w - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) + m_L a \dot{\theta}_u \sin(\theta_u - \theta_w)\dot{\theta}_u^2 - m_L g a \sin \theta_w
\]

\[
m_L a^2 \ddot{\theta}_w - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w)\dot{\theta}_u + m_L a \dot{\theta}_u \sin(\theta_u - \theta_w)\dot{\theta}_w^2 - m_L g a \sin \theta_w = 0
\]

c) Equations of motion

\[
\begin{align*}
\begin{cases}
(m_l l^2 + m_L b^2 + m_L l^2) \ddot{\theta}_u - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) \dot{\theta}_u - m_L a \dot{\theta}_u \sin(\theta_u - \theta_w) \dot{\theta}_w^2 - m_L g a \sin \theta_u = 0 \\
m_L a^2 \ddot{\theta}_u - m_L a \dot{\theta}_u \dot{\theta}_w \cos(\theta_u - \theta_w) \dot{\theta}_u + m_L a \dot{\theta}_u \sin(\theta_u - \theta_w) \dot{\theta}_w^2 - m_L g a \sin \theta_u = 0
\end{cases}
\end{align*}
\]

In terms of the matrix,

\[
\begin{bmatrix}
m_l l^2 + m_L b^2 + m_L l^2 & -m_L a \dot{\theta}_u \cos(\theta_u - \theta_w) \\
-m_L a \dot{\theta}_u \cos(\theta_u - \theta_w) & m_L a^2
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_u \\
\dot{\theta}_w
\end{bmatrix}
+ 
\begin{bmatrix}
0 & -m_L a \dot{\theta}_u \sin(\theta_u - \theta_w) \\
-m_L a \dot{\theta}_u \sin(\theta_u - \theta_w) & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_u \\
\dot{\theta}_w
\end{bmatrix}
+ 
\begin{bmatrix}
(m_L b + m_L l + m_L l) g \sin \theta_u \\
-(m_L g a \sin \theta_u)
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]
3. Transition period

The transition period is the period when the swing leg hit the ground and the support leg is started to swing. For a real human walking, there will be a double stance phase. However, this phase is assumed very short in this study.

a. Assumptions

The assumptions for the transition period are

3) Impact

The inelastic collision is assumed when the swing leg hits the ground.

4) Sliding

No sliding condition is assumed when the swing leg touches the ground.

b. Relationships

The relationship between steps can be found using the conservation of angular momentum.

The conservation of the angular momentum applies in two situations:

- The angular momentum of the biped about the collision point is conserved before and after the transition.
- The angular momentum of the non-colliding foot is conserved about the hip joint during the collision.
Therefore, there will be two equations using the conservation of angular momentum.

1) With respect to the collision point

   a) Before impact

\[
\mathbf{L}^- = \mathbf{L}_H^- + \mathbf{L}_u^- + \mathbf{L}_w^-
\]

i) \( \mathbf{L}_H^- \)

\[
\mathbf{L}_H^- = m_H \mathbf{r}_H^- \times \mathbf{v}_H^-
\]

\[
\mathbf{r}_H^- = l \sin(\pi - \theta_w^-) \mathbf{i} + l \cos(\pi - \theta_w^-) \mathbf{j} = l \sin \theta_w^- \mathbf{i} - l \cos \theta_w^- \mathbf{j}
\]

\( \mathbf{v}_H^- \) is the velocity of the hip just before impact. This is not same as \( \frac{d\mathbf{r}_H^-}{dt} \). This is rather similar to the velocity of the hip in the derivation of the equation of motion, \( \mathbf{v}_H \).
APPENDIX (continued)

\[ \mathbf{v}_H^- = l \dot{\theta}_u^- \cos \theta_w^- \mathbf{i} + l \dot{\theta}_u^- \sin \theta_w^- \mathbf{j} \]

\[ \mathbf{r}_H \times \mathbf{v}_H^- = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ l \sin \theta_w^- & -l \cos \theta_w^- & 0 \\ l \dot{\theta}_u^- \cos \theta_w^- & l \dot{\theta}_u^- \sin \theta_w^- & 0 \end{vmatrix} \]

\[ = (-1)^{i+3} \left[ (l \sin \theta_w^-) (l \dot{\theta}_u^- \sin \theta_u^-) - (-l \cos \theta_w^-) (l \dot{\theta}_u^- \cos \theta_u^-) \right] \mathbf{k} \]

\[ = l^2 \dot{\theta}_u^- \sin \theta_u^- \sin \theta_w^- + l^2 \dot{\theta}_u^- \cos \theta_u^- \cos \theta_w^- \]

\[ = l^2 \dot{\theta}_u^- \left( \cos \theta_u^- \cos \theta_w^- + \sin \theta_u^- \sin \theta_w^- \right) \]

\[ = l^2 \dot{\theta}_u^- \cos(\theta_u^- - \theta_w^-) \mathbf{k} \]

\[ \mathbf{L}_H^- = m_H (\mathbf{r}_H \times \mathbf{v}_H^-) = m_H l^2 \dot{\theta}_u^- \cos(\theta_u^- - \theta_w^-) \mathbf{k} \]

ii) \( \mathbf{L}_u^- \)

\[ \mathbf{L}_u^- = m_L \mathbf{r}_u^- \times \mathbf{v}_u^- \]

\[ \mathbf{r}_{u,H}^- = a \sin(\theta_u^- - \pi) - a \cos(\theta_u^- - \pi) \mathbf{j} \]

\[ = a(\sin \theta_u^- \cos \pi - \cos \theta_u^- \sin \pi) \mathbf{i} - a(\cos \theta_u^- \cos \pi + \sin \theta_u^- \sin \pi) \mathbf{j} \]

\[ = -a \sin \theta_u^- \mathbf{i} + a \cos \theta_u^- \mathbf{j} \]

\[ \mathbf{r}_u^- = \mathbf{r}_H^- + \mathbf{r}_{u,H}^- \]

\[ = (l \sin \theta_w^- \mathbf{i} - l \cos \theta_w^- \mathbf{j}) + \left[ -a \sin \theta_u^- \mathbf{i} + a \cos \theta_u^- \mathbf{j} \right] \]

\[ = (l \sin \theta_w^- - a \sin \theta_u^-) \mathbf{i} + \left( -l \cos \theta_w^- + a \cos \theta_u^- \right) \mathbf{j} \]

\[ \mathbf{v}_u^- = b \dot{\theta}_u^- \cos \theta_u^- \mathbf{i} + b \dot{\theta}_u^- \sin \theta_u^- \mathbf{j} \]
APPENDIX (continued)

\[ \mathbf{r}_w \times \mathbf{v}_u = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ l \sin \theta_w^c - a \sin \theta_u^c & -l \cos \theta_w^c + a \cos \theta_u^c & 0 \\ b \theta_u^c \cos \theta_u^c & b \theta_u^c \sin \theta_u^c & 0 \end{vmatrix} \]

\[ = (-1)^{i+3} \left[ l \sin \theta_w^c - a \sin \theta_u^c \left( b \theta_u^c \sin \theta_u^c \right) - \left( -l \cos \theta_w^c + a \cos \theta_u^c \right) \left( b \theta_u^c \cos \theta_u^c \right) \right] \mathbf{k} \]

\[ = \left[ b \theta_u^c \sin \theta_u^c \sin \theta_w^c - ab \theta_u^c \sin^2 \theta_u^c + lb \theta_u^c \cos \theta_w^c \cos \theta_u^c - ab \theta_u^c \cos^2 \theta_u^c \right] \mathbf{k} \]

\[ = \left[ l \cos \left( \theta_u^c - \theta_w^c \right) - ab \dot{\theta}_u \right] \mathbf{k} \]

\[ \mathbf{L}_u = m_L (\mathbf{r}_w^c \times \mathbf{v}_w^c) = m_L \left[ l \cos \left( \theta_u^c - \theta_w^c \right) - ab \dot{\theta}_u \right] \mathbf{k} \]

iii) \( \mathbf{L}_w \)

\[ \mathbf{L}_w = m_L \mathbf{r}_w^c \times \mathbf{v}_w^c \]

\[ \mathbf{r}_w^c = \sin \left( \theta_w^c - \pi \right) - a \cos \left( \theta_w^c - \pi \right) \mathbf{j} \]

\[ = a \sin \theta_w^c \cos \pi - a \cos \theta_w^c \sin \pi \mathbf{j} - a \left( \cos \theta_w^c \cos \pi + \sin \theta_w^c \sin \pi \right) \mathbf{j} \]

\[ = -a \sin \theta_w^c \mathbf{i} + a \cos \theta_w^c \mathbf{j} \]

\[ \mathbf{r}_w^c = \mathbf{r}_w^c + \mathbf{r}_{w,0} \]

\[ = \left( l \sin \theta_w^c \mathbf{i} - l \cos \theta_w^c \mathbf{j} \right) + \left( -a \sin \theta_w^c \mathbf{i} + a \cos \theta_w^c \mathbf{j} \right) \]

\[ = \left( l \sin \theta_w^c - a \sin \theta_w^c \right) \mathbf{i} + \left( -l \cos \theta_w^c + a \cos \theta_w^c \right) \mathbf{j} \]

\[ = \left( l - a \right) \sin \theta_w^c \mathbf{i} - \left( l-a \right) \cos \theta_w^c \mathbf{j} \]

\[ = b \sin \theta_w^c \mathbf{i} - b \cos \theta_w^c \mathbf{j} \]

\[ \mathbf{v}_w^c = \left( l \dot{\theta}_u \cos \theta_u^c - a \dot{\theta}_u \cos \theta_u^c \right) \mathbf{i} + \left( l \dot{\theta}_u \sin \theta_u^c - a \dot{\theta}_u \sin \theta_u^c \right) \mathbf{j} \]
APPENDIX (continued)

\[
\mathbf{r}_w^\times \mathbf{v}_w = \begin{bmatrix} i & j & k \end{bmatrix} = \begin{bmatrix} b \sin \theta_w^- & -b \cos \theta_w^- & 0 \\ (l \hat{\theta}_u^- \cos \theta_u^- - a \hat{\omega}_w^- \cos \theta_w^-) & (l \hat{\theta}_u^- \sin \theta_u^- - a \hat{\omega}_w^- \sin \theta_w^-) & 0 \end{bmatrix} = (-1)^{l+3} \left[ (b \sin \theta_w^-)(l \hat{\theta}_u^- \sin \theta_u^- - a \hat{\omega}_w^- \sin \theta_w^-) \right. \\
\left. -(-b \cos \theta_w^-)(l \hat{\theta}_u^- \cos \theta_u^- - a \hat{\omega}_w^- \cos \theta_w^-) \right] \mathbf{k} \\
= \left[ l b \hat{\theta}_u^- \sin \theta_u^- \sin \theta_w^- - ab \hat{\omega}_w^- \sin^2 \theta_w^- + lb \hat{\omega}_u^- \cos \theta_u^- \cos \theta_w^- - ab \hat{\omega}_w^- \cos^2 \theta_w^- \right] \mathbf{k} \\
= \left[ l b \hat{\theta}_u^- \left( \cos \theta_u^- \cos \theta_w^- + \sin \theta_u^- \sin \theta_w^- \right) - ab \hat{\omega}_w^- \left( \sin^2 \theta_w^- + \cos^2 \theta_w^- \right) \right] \mathbf{k} \\
= \left[ l b \hat{\theta}_u^- \cos \left( \theta_u^- - \theta_w^- \right) - ab \hat{\omega}_w^- \right] \mathbf{k}
\]

\[
\mathbf{L}_w^- = m_i (\mathbf{r}_w^- \times \mathbf{v}_w^-) \\
= m_i \left[ l b \hat{\theta}_u^- \cos \left( \theta_u^- - \theta_w^- \right) - ab \hat{\omega}_w^- \right] \mathbf{k} \\
= \left[ m_i l b \cos \left( \theta_u^- - \theta_w^- \right) \hat{\omega}_u^- - m_i ab \hat{\omega}_w^- \right] \mathbf{k} 
\]

iv) \( \mathbf{L}^- \)

\[
\mathbf{L}^- = \mathbf{L}_u^- + \mathbf{L}_w^- + \mathbf{L}_w^- \\
= m_i l^2 \cos \left( \theta_u^- - \theta_w^- \right) \hat{\theta}_u^- + m_i \left[ l \cos \left( \theta_u^- - \theta_w^- \right) - a \right] \hat{\omega}_w^- \mathbf{k} \\
+ \left[ m_i l b \cos \left( \theta_u^- - \theta_w^- \right) \hat{\omega}_u^- - m_i ab \hat{\omega}_w^- \right] \mathbf{k} \\
= \left[ m_i l^2 \cos \left( \theta_u^- - \theta_w^- \right) + m_i l b \cos \left( \theta_u^- - \theta_w^- \right) - m_i ab + m_i l b \cos \left( \theta_u^- - \theta_w^- \right) \right] \hat{\theta}_u^- \\
- m_i ab \hat{\omega}_w^- \mathbf{k} \\
= \left[ m_i l^2 \cos \left( \theta_u^- - \theta_w^- \right) + 2 m_i l b \cos \left( \theta_u^- - \theta_w^- \right) - m_i ab \right] \hat{\theta}_u^- - m_i ab \hat{\omega}_w^- \mathbf{k} 
\]
APPENDIX (continued)

b) After impact

\[ \mathbf{L}^+ = \mathbf{L}_H^+ + \mathbf{L}_u^+ + \mathbf{L}_w^+ \]

\[ \mathbf{r}_H^+ = l \sin\left(\pi - \theta_u^+\right) \mathbf{i} + l \cos\left(\pi - \theta_u^+\right) \mathbf{j} \]

\[ = l \sin \theta_u^- \mathbf{i} - l \cos \theta_u^- \mathbf{j} \]

\[ \mathbf{r}_{w,H}^+ = a \sin\left(\theta_w^+ - \pi\right) \mathbf{i} - a \cos\left(\theta_w^+ - \pi\right) \mathbf{j} \]

\[ = -a \sin \theta_w^+ \mathbf{i} + a \cos \theta_w^+ \mathbf{j} \]

\[ \mathbf{r}_{w,H}^+ = a \sin\left(\theta_w^+ - \pi\right) \mathbf{i} - a \cos\left(\theta_w^+ - \pi\right) \mathbf{j} \]

\[ = -a \sin \theta_w^+ \mathbf{i} + a \cos \theta_w^+ \mathbf{j} \]

i) \( \mathbf{L}_{H,i}^+ \)

\[ \mathbf{L}_{H,i}^+ = m_H \mathbf{r}_{H,i}^+ \times \mathbf{v}_{H,i}^+ \]

\[ \mathbf{r}_H^+ = l \sin \theta_u^- \mathbf{i} - l \cos \theta_u^- \mathbf{j} \]

\[ \mathbf{v}_{H,i}^+ = l \dot{\theta}_u^+ \cos \theta_u^+ \mathbf{i} + l \dot{\theta}_u^+ \sin \theta_u^+ \mathbf{j} \]
\[ \mathbf{r}_H^+ \times \mathbf{v}_H^+ = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ l \sin \theta_u^+ & -l \cos \theta_u^+ & 0 \\ l \dot{\theta}_u^+ \cos \theta_u^+ & l \dot{\theta}_u^+ \sin \theta_u^+ & 0 \end{vmatrix} \]

\[ = (-1)^{i+3} \left[ (l \sin \theta_u^+) (l \dot{\theta}_u^+ \sin \theta_u^+) - (-l \cos \theta_u^+) (l \dot{\theta}_u^+ \cos \theta_u^+) \right] \mathbf{k} \]

\[ = \left( l^2 \dot{\theta}_u^+ \sin^2 \theta_u^+ + l^2 \dot{\theta}_u^+ \cos \theta_u^+ \right) \mathbf{k} \]

\[ = l^2 \ddot{\theta}_u^+ \mathbf{k} \]

\[ \mathbf{L}_H^+ = m_H \left( \mathbf{r}_H^+ \times \mathbf{v}_H^+ \right) = m_H l^2 \dot{\theta}_u^+ \mathbf{k} \]

ii) \[ \mathbf{L}_u^+ = m_l \mathbf{r}_u^+ \times \mathbf{v}_u^+ \]

\[ \mathbf{r}_u^+ = \mathbf{r}_H^+ + \mathbf{r}_{u,H} \]

\[ = \left( l \sin \theta_u^+ \mathbf{i} - l \cos \theta_u^+ \mathbf{j} \right) + \left( -a \sin \theta_u^+ \mathbf{i} + a \cos \theta_u^+ \mathbf{j} \right) \]

\[ = \left( l \sin \theta_u^+ - a \sin \theta_u^+ \right) \mathbf{i} + \left( -l \cos \theta_u^+ + a \cos \theta_u^+ \right) \mathbf{j} \]

\[ = (l - a) \sin \theta_u^+ \mathbf{i} - (l - a) \cos \theta_u^+ \mathbf{j} \]

\[ = b \sin \theta_u^+ \mathbf{i} - b \cos \theta_u^+ \mathbf{j} \]

\[ \mathbf{v}_u^+ = b \dot{\theta}_u^+ \cos \theta_u^+ \mathbf{i} + b \dot{\theta}_u^+ \sin \theta_u^+ \mathbf{j} \]

\[ \mathbf{r}_u \times \mathbf{v}_u = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b \sin \theta_u^+ & -b \cos \theta_u^+ & 0 \\ b \dot{\theta}_u^+ \cos \theta_u^+ & b \dot{\theta}_u^+ \sin \theta_u^+ & 0 \end{vmatrix} \]

\[ = (-1)^{i+3} \left[ b \sin \theta_u^+ \left( b \dot{\theta}_u^+ \sin \theta_u^+ \right) - (-b \cos \theta_u^+) \left( b \dot{\theta}_u^+ \cos \theta_u^+ \right) \right] \mathbf{k} \]

\[ = \left( b^2 \dot{\theta}_u^+ \sin^2 \theta_u^+ + b^2 \dot{\theta}_u^+ \cos^2 \theta_u^+ \right) \mathbf{k} \]

\[ = b^2 \dot{\theta}_u^+ \mathbf{k} \]
APPENDIX (continued)

$$L_u^+ = m_c \left( r_u^+ \times v_u^+ \right) = m_c b^2 \dot{\theta}_w^+ k$$

iii) $L_w^+$

$$L_w^+ = m_c r_w^+ \times v_w^+$$

$$r_w^+ = r_{H_H}^+ + r_{w,H}^+$$

$$= (l \sin \theta_w^+ i - l \cos \theta_w^+ j) + (-a \sin \theta_w^+ i + a \cos \theta_w^+ j)$$

$$= (l \sin \theta_w^+ - a \sin \theta_w^+) i + (-l \cos \theta_w^+ + a \cos \theta_w^+) j$$

$$v_w^+ = \left( l \dot{\theta}_w^+ \cos \theta_w^+ - a \dot{\theta}_w^+ \cos \theta_w^+ \right) i + \left( l \dot{\theta}_w^+ \sin \theta_w^+ - a \dot{\theta}_w^+ \sin \theta_w^+ \right) j$$

$$\begin{vmatrix} i & j & k \\ (l \sin \theta_u^+ - a \sin \theta_w^+) & (-l \cos \theta_u^+ + a \cos \theta_w^+) & 0 \\ (l \dot{\theta}_u^+ \cos \theta_u^+ - a \dot{\theta}_w^+ \cos \theta_w^+) & (l \dot{\theta}_u^+ \sin \theta_u^+ - a \dot{\theta}_w^+ \sin \theta_w^+) & 0 \\ (-1)^{\nu_3} \left( l \sin \theta_u^+ - a \sin \theta_w^+ \right) \left( l \dot{\theta}_u^+ \sin \theta_u^+ - a \dot{\theta}_w^+ \sin \theta_w^+ \right) & + l^2 \dot{\theta}_u^+ \cos^2 \theta_u^+ - l a \dot{\theta}_u^+ \cos \theta_u^+ \sin \theta_u^+ - l \dot{\theta}_u^+ \cos \theta_u^+ \cos \theta_w^+ - \dot{\theta}_u^+ \cos \theta_u^+ \sin \theta_w^+ + a^2 \dot{\theta}_u^+ \cos \theta_u^+ \sin \theta_w^+ & k \\ l^2 \dot{\theta}_u^+ \sin^2 \theta_u^+ - la \dot{\theta}_u^+ \sin \theta_u^+ \sin \theta_w^+ - la \dot{\theta}_u^+ \sin \theta_u^+ \sin \theta_w^+ + a^2 \dot{\theta}_w^+ \sin^2 \theta_w^+ & + l^2 \dot{\theta}_u^+ \cos^2 \theta_u^+ - la \dot{\theta}_u^+ \cos \theta_u^+ \sin \theta_u^+ - la \dot{\theta}_u^+ \cos \theta_u^+ \cos \theta_w^+ - \dot{\theta}_u^+ \cos \theta_u^+ \sin \theta_w^+ & k \\ & + l^2 \dot{\theta}_u^+ \left( \sin^2 \theta_u^+ + \cos^2 \theta_u^+ \right) - la \dot{\theta}_u^+ \left( \cos \theta_u^+ \cos \theta_w^+ + \sin \theta_u^+ \sin \theta_w^+ \right) - la \dot{\theta}_u^+ \left( \cos \theta_u^+ \cos \theta_w^+ + \sin \theta_u^+ \sin \theta_w^+ \right) & k \\ & - la \dot{\theta}_u^+ \left( \cos \theta_u^+ \cos \theta_w^+ + \sin \theta_u^+ \sin \theta_w^+ \right) + a^2 \dot{\theta}_w^+ \left( \sin^2 \theta_w^+ + \cos^2 \theta_w^+ \right) & k \\ & + l^2 \dot{\theta}_u^+ - la \dot{\theta}_u^+ \cos \theta_u^+ - l a \dot{\theta}_u^+ \cos \theta_u^+ \cos \theta_w^+ & a^2 \dot{\theta}_w^+ \cos \theta_u^+ \sin \theta_w^+ + a^2 \dot{\theta}_w^+ \cos \theta_u^+ \sin \theta_w^+ \end{vmatrix}$$

$$L_w^+ = m_c \left( r_w^+ \times v_w^+ \right)$$

$$= m_c \left[ l^2 \dot{\theta}_u^+ - la \dot{\theta}_u^+ \cos \left( \theta_u^+ - \theta_w^+ \right) - l a \dot{\theta}_w^+ \cos \left( \theta_u^+ - \theta_w^+ \right) + a^2 \dot{\theta}_w^+ \right] k$$

$$= m_c \left[ l^2 - la \cos \left( \theta_u^+ - \theta_w^+ \right) \right] \dot{\theta}_u^+ + m_c \left[ a^2 - la \cos \left( \theta_u^+ - \theta_w^+ \right) \right] \dot{\theta}_w^+$$
iv) \( \mathbf{L} \)

\[
\mathbf{L}^+ = \mathbf{L}_{H}^+ + \mathbf{L}_u^+ + \mathbf{L}_w^+ \\
= m_H l^2 \dot{\theta}_u^+ \mathbf{k} + m_L b^2 \dot{\theta}_u^+ \mathbf{k} \\
+ \left[ m_L \left( l^2 - l a \cos(\theta_u^+ - \theta_w^+) \right) \dot{\theta}_u^+ + m_L \left( a^2 - l a \cos(\theta_u^+ - \theta_w^+) \right) \dot{\theta}_w^+ \right] \mathbf{k} \\
= \left\{ m_H l^2 + m_L l^2 + m_L b^2 - l a \cos(\theta_u^+ - \theta_w^+) \right\} \dot{\theta}_u^+ + \left\{ m_L a^2 - m_L l a \cos(\theta_u^+ - \theta_w^+) \right\} \dot{\theta}_w^+ \mathbf{k}
\]

2) With respect to the hip

\[
\mathbf{L}^- = \mathbf{L}^+
\]

**Before Impact**

**After Impact**

\[ \theta_u \]

\[ \theta_w \]

\[ r_{u,H}^- \]

\[ r_{w,H}^- \]

\[ \mathbf{v}_u^- = b \dot{\theta}_u^- \cos \theta_w^- \mathbf{i} + b \dot{\theta}_u^- \sin \theta_w^- \mathbf{j} \]

**d) Before impact**

\[ \mathbf{L}^- = m_L \mathbf{r}_{u,H}^- \times \mathbf{v}_u^- \]

\[ \mathbf{r}_{u,H}^- = a \sin(\theta_u^- - \pi) \mathbf{i} - a \cos(\theta_u^- - \pi) \mathbf{j} \]

\[ = a(\sin \theta_u^- \cos \pi - \cos \theta_u^- \sin \pi) \mathbf{i} - a(\cos \theta_u^- \cos \pi + \sin \theta_u^- \sin \pi) \mathbf{j} \]

\[ = -a \sin \theta_u^- \mathbf{i} + a \cos \theta_u^- \mathbf{j} \]

\[ \mathbf{v}_u^- = b \dot{\theta}_u^- \cos \theta_w^- \mathbf{i} + b \dot{\theta}_w^- \sin \theta_w^- \mathbf{j} \]
APPENDIX (continued)

\[ \mathbf{r}_{\text{u,lt}}^- \times \mathbf{v}_{\text{u}}^- = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a \sin \theta_{\text{u}}^- & a \cos \theta_{\text{u}}^- & 0 \\ b \dot{\theta}_{\text{u}}^- \cos \theta_{\text{u}}^- & b \dot{\theta}_{\text{u}}^- \sin \theta_{\text{u}}^- & 0 \end{vmatrix} \]

\[ = (-1)^{1+3} \left[ (-a \sin \theta_{\text{u}}^-) (b \dot{\theta}_{\text{u}}^- \sin \theta_{\text{u}}^-) - (a \cos \theta_{\text{u}}^-) (b \dot{\theta}_{\text{u}}^- \cos \theta_{\text{u}}^-) \right] \mathbf{k} \]

\[ = -ab \dot{\theta}_{\text{u}}^- \sin^2 \theta_{\text{u}}^- - ab \dot{\theta}_{\text{u}}^- \cos^2 \theta_{\text{u}}^- \mathbf{k} \]

\[ = -ab \dot{\theta}_{\text{u}}^- \mathbf{k} \]

\[ \mathbf{L}^- = m_L \mathbf{r}_{\text{u,lt}}^- \times \mathbf{v}_{\text{u}}^- = -m_L ab \dot{\theta}_{\text{u}}^- \mathbf{k} \]

e) After impact

\[ \mathbf{L}^+ = m_L \mathbf{r}_{\text{w,lt}}^+ \times \mathbf{v}_{\text{w}}^+ \]

\[ \mathbf{r}_{\text{w,lt}}^+ = a \sin \left( \theta_{\text{w}}^+ - \pi \right) \mathbf{i} - a \cos \left( \theta_{\text{w}}^+ - \pi \right) \mathbf{j} \]

\[ = a \left( \sin \theta_{\text{w}}^+ \cos \pi - \cos \theta_{\text{w}}^+ \sin \pi \right) \mathbf{i} - a \left( \cos \theta_{\text{w}}^+ \cos \pi + \sin \theta_{\text{w}}^+ \sin \pi \right) \mathbf{j} \]

\[ = -a \sin \theta_{\text{w}}^+ \mathbf{i} + a \cos \theta_{\text{w}}^+ \mathbf{j} \]

\[ \mathbf{v}_{\text{w}}^+ = \left( l \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ - a \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ \right) \mathbf{i} + \left( l \dot{\theta}_{\text{w}}^+ \sin \theta_{\text{w}}^+ - a \dot{\theta}_{\text{w}}^+ \sin \theta_{\text{w}}^+ \right) \mathbf{j} \]

\[ \mathbf{r}_{\text{w,lt}}^+ \times \mathbf{v}_{\text{w}}^+ = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a \sin \theta_{\text{w}}^+ & a \cos \theta_{\text{w}}^+ & 0 \\ \left( l \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ - a \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ \right) & \left( l \dot{\theta}_{\text{w}}^+ \sin \theta_{\text{w}}^+ - a \dot{\theta}_{\text{w}}^+ \sin \theta_{\text{w}}^+ \right) & 0 \end{vmatrix} \]

\[ = (-1)^{1+3} \left[ -a \sin \theta_{\text{w}}^+ \left( l \dot{\theta}_{\text{w}}^+ \sin \theta_{\text{w}}^+ - a \dot{\theta}_{\text{w}}^+ \sin \theta_{\text{w}}^+ \right) - (a \cos \theta_{\text{w}}^+) \left( l \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ - a \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ \right) \right] \mathbf{k} \]

\[ = \left[ -a \dot{\theta}_{\text{w}}^+ \sin \theta_{\text{w}}^+ \sin \theta_{\text{w}}^+ + a^2 \dot{\theta}_{\text{w}}^+ \sin^2 \theta_{\text{w}}^+ - l \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ \cos \theta_{\text{w}}^+ + a^2 \dot{\theta}_{\text{w}}^+ \cos^2 \theta_{\text{w}}^+ \right] \mathbf{k} \]

\[ = \left[ -a \dot{\theta}_{\text{w}}^+ \cos \theta_{\text{w}}^+ \sin \theta_{\text{w}}^+ + \sin \theta_{\text{w}}^+ \sin \theta_{\text{w}}^+ + a^2 \dot{\theta}_{\text{w}}^+ \sin^2 \theta_{\text{w}}^+ + \cos^2 \theta_{\text{w}}^+ \right] \mathbf{k} \]
APPENDIX (continued)

\[ \mathbf{L}^+ = m_r \mathbf{r}_{w,t} \times \mathbf{v}_w^+ \]
\[ = m_r \left[ -la \cos(\theta_u^+ - \theta_w^+) \dot{\theta}_u^+ + a^2 \ddot{\theta}_u^+ \right] \mathbf{k} \]
\[ = \left[ -m_r la \cos(\theta_u^+ - \theta_w^+) \dot{\theta}_u^+ + m_r a^2 \ddot{\theta}_u^+ \right] \mathbf{k} \]

3) Equations

With respect to the collision point:

\[ \mathbf{L}^- = \mathbf{L}^+ \]
\[ = \left[ \begin{bmatrix} m_h l^2 \cos(\theta_u^- - \theta_w^-) + 2m_I lb \cos(\theta_u^- - \theta_w^-) - m_r ab \dot{\theta}_u^- - m_r ab \dot{\theta}_w^- \end{bmatrix} \mathbf{k} \right] \]
\[ = \left[ \begin{bmatrix} m_h l^2 + m_I l^2 + m_I b^2 - la \cos(\theta_u^+ - \theta_w^+) \dot{\theta}_u^+ + \left( m_r a^2 - m_r la \cos(\theta_u^+ - \theta_w^+) \right) \dot{\theta}_w^+ \end{bmatrix} \mathbf{k} \right] \]

With respect to the hip:

\[ \mathbf{L}^- = \mathbf{L}^+ \]
\[ -m_r ab \dot{\theta}_u^- \mathbf{k} = \left[ -m_r la \cos(\theta_u^+ - \theta_w^+) \dot{\theta}_u^+ + m_r a^2 \ddot{\theta}_u^+ \right] \mathbf{k} \]

In matrix term,

\[ \begin{bmatrix} m_h l^2 \cos(\theta_u^- - \theta_w^-) + 2m_I lb \cos(\theta_u^- - \theta_w^-) - m_r ab & -m_r ab & \dot{\theta}_u^- \\ -m_I ab & 0 & \dot{\theta}_w^- \end{bmatrix} \mathbf{k} \]
\[ = \begin{bmatrix} m_h l^2 + m_I l^2 + m_I b^2 - la \cos(\theta_u^+ - \theta_w^+) & m_r a^2 - m_r la \cos(\theta_u^+ - \theta_w^+) & \dot{\theta}_u^- \\ -m_r la \cos(\theta_u^+ - \theta_w^+) & m_r a^2 & \dot{\theta}_w^- \end{bmatrix} \mathbf{k} \]
4. Compass gait on a slope

A slope can be introduced by modifying previous configuration as shown in the figure.

With a slope, the new angles with a slope will be

$$\begin{align*}
\dot{\theta}_u(t) &= \dot{\theta}_u(t) - \phi(x) \\
\dot{\theta}_w(t) &= \dot{\theta}_w(t) - \phi(x)
\end{align*}$$

$$\begin{align*}
\ddot{\theta}_u(t) &= \ddot{\theta}_u(t) - \frac{d\phi(x)}{dx} \ddot{x} - \frac{d\phi(x)}{dx} \dddot{x} \\
\ddot{\theta}_w(t) &= \ddot{\theta}_w(t) - \frac{d\phi(x)}{dx} \ddot{x} - \frac{d\phi(x)}{dx} \dddot{x}
\end{align*}$$

$$\begin{align*}
\dot{\theta}_u(t) &= \dot{\theta}_u(t) + \phi(x) \\
\dot{\theta}_w(t) &= \dot{\theta}_w(t) + \phi(x)
\end{align*}$$

$$\begin{align*}
\ddot{\theta}_u(t) &= \ddot{\theta}_u(t) + \frac{d\phi(x)}{dx} \ddot{x} + \frac{d\phi(x)}{dx} \dddot{x} \\
\ddot{\theta}_w(t) &= \ddot{\theta}_w(t) + \frac{d\phi(x)}{dx} \ddot{x} + \frac{d\phi(x)}{dx} \dddot{x}
\end{align*}$$
APPENDIX (continued)

If the slope is unchanged along the walking direction, the new angle will be

\[
\begin{align*}
\theta_u'(t) &= \theta_u(t) - \phi \\
\theta_w'(t) &= \theta_w(t) - \phi
\end{align*}
\]

so

\[
\begin{align*}
\ddot{\theta}_u(t) &= \ddot{\theta}_u(t) \\
\ddot{\theta}_w(t) &= \ddot{\theta}_w(t)
\end{align*}
\]

and

\[
\begin{align*}
\theta_u'(t) &= \theta_u'(t) + \phi \\
\theta_w'(t) &= \theta_w'(t) + \phi
\end{align*}
\]

so

\[
\begin{align*}
\ddot{\theta}_u(t) &= \ddot{\theta}_u(t) \\
\ddot{\theta}_w(t) &= \ddot{\theta}_w(t)
\end{align*}
\]

a. Equation of motion with a slope

The equations of motion without a slope are

\[
\begin{align*}
&\begin{bmatrix}
    m_L l^2 + m_L b^2 + m_L l^2 \\
    -m_L a^2
  \end{bmatrix}
\begin{bmatrix}
    \ddot{\theta}_u \\
    \ddot{\theta}_w
  \end{bmatrix}
\begin{bmatrix}
    0 & -m_L a \sin(\theta_u - \theta_w) \\
    m_L a \sin(\theta_u - \theta_w) & 0
  \end{bmatrix}
\begin{bmatrix}
    \dot{\theta}_u \\
    \dot{\theta}_w
  \end{bmatrix}
\end{align*}
\]

These equations can be simplified as

\[
M\ddot{\theta} + C\dot{\theta} + G = 0
\]

After which, the equations of motion with a slope are

\[
M\dddot{\theta} + C\ddot{\theta} + G = 0
\]

For a constant slope, the equations of motion will be changed to

\[
\begin{align*}
&\begin{bmatrix}
    m_L l^2 + m_L b^2 + m_L l^2 \\
    -m_L a^2
  \end{bmatrix}
\begin{bmatrix}
    \dddot{\theta}_u \\
    \dddot{\theta}_w
  \end{bmatrix}
\begin{bmatrix}
    0 & -m_L a \sin(\theta_u - \theta_w) \\
    m_L a \sin(\theta_u - \theta_w) & 0
  \end{bmatrix}
\begin{bmatrix}
    \ddot{\theta}_u \\
    \ddot{\theta}_w
  \end{bmatrix}
\end{align*}
\]

The slope will effect only the gravity term of the equations of motion.
APPENDIX (continued)

b. Transient period with a slope

The equations for the transient period without a slope are

\[
\begin{bmatrix}
    m_H l^2 \cos(\theta_u^\circ - \theta_w^\circ) + 2m_L l b \cos(\theta_u^\circ - \theta_w^\circ) - m_L a b - m_L a b \\
    - m_L a b \\
    0
\end{bmatrix}
\begin{bmatrix}
    \dot{\theta}_u^\circ \\
    \dot{\theta}_w^\circ
\end{bmatrix}
\]  

They can be simplified as

\[ A A^* = A^* A \]

After which, the equations for the transient period with a slope are

\[ (A^*)^\circ (\theta^\circ) = (A^*)^\circ (\theta^\circ) \]

For a constant slope, there is no change in the equations. Because,

\[ \theta_u^\circ - \theta_w^\circ = \theta_u^\circ - \theta_w^\circ = \theta_u^\circ - \theta_w^\circ = (\theta_u^\circ + \phi) - (\theta_w^\circ + \phi) = \theta_u^\circ - \theta_w^\circ \]