ANALYSIS OF MIMO RADAR IMPACT ON MIMO COMMUNICATION SYSTEM USING ZERO-FORCING RECEIVER

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The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Electrical Engineering.

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DEDICATION

To my family, loved ones, and friends

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ABSTRACT

Spectrum sharing is an approach to solving the congestion problem in radio frequency (RF) bands. The 3500–3650 MHz band has been allocated in the United States for spectrum sharing between military radar systems and cellular systems. Hence, this paper considers a multiple-input multiple-output (MIMO) radar system and a MIMO communication system sharing the same spectrum band. The performance of the communications system is analyzed by considering interference from the radar with regard to signals reflected at the radar's target. A majority of papers prefer to omit the effect of the target reflection coefficient, which depends on the scattering characteristic of the target. Therefore, in this paper, the target reflection coefficient is integrated, and the scattering effect to the MIMO communications system is analyzed. Also, the power of the transmitted radar waveforms is varied. The direction of the targets is considered, and their impact on the MIMO communication system is analyzed. Using a zero-forcing (ZF) receiver, the bit error rate (BER) of the communication system is simulated.

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LIST OF ABBREVIATIONS

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
FCC	Federal Communications Commission
i.i.d	Independent and Identically Distributed
LTE	Long Term Evolution
MIMO	Multiple-Input Multiple-Output
NTIA	National Telecommunications and Information Administration
RCS	Radio Cross Section
RF	Radio Frequency
SINR	Signal-to-Interference-Plus-Noise Ratio
UHF	Ultra-High Frequency
VHF	Very-High Frequency
ZF	Zero-Forcing

LIST OF SYMBOLS

$\odot$	Element by Element Product
β	Log Normal Entries
π	Pi
$^{\circ}$	Degree
α	Alpha
λ	Lambda
σ	Sigma
dB	Decibels
$()^H$	Conjugate Transpose

CHAPTER 1

INTRODUCTION

With increased bandwidth demands, spectrum scarcity has become a serious challenge, particularly regarding commercial utilization. In order to accommodate these growing bandwidth demands, many regulators and operators have taken the initiative of exploring a secondary access to very-high frequency/ultra-high frequency (VHF/UHF) bands occupied by radar. Spectrum sharing between the commercial cellular system and radar systems is a proposed solution to the spectrum-scarcity problem [1]. This approach has been highly promoted by the National Telecommunications and Information Administration (NTIA) and Federal Communications Commission (FCC) in order to make use of the underutilized spectrum.

As a solution to the increased bandwidth demand problem, this approach promises enormous economic and social prospects but also brings in new challenges for the optimal operation of the incumbent radar systems and commercial cellular users. The spectrum-sharing approach also brings new challenges such as electromagnetic interference to the radar system or communication system. Traditionally, radar systems have high transmit power and high-peak side lobes, which have a negative effect on the communication system receivers that typically operate at lower power levels. This work considers multiple-input multiple-output (MIMO) radar waveforms as an interference to a MIMO communication system and the impact such interference has on the communication system with respect to the transmit power of radar waveforms, target reflection coefficient, and target direction [2, 3].

Interference mitigation is very important, and detailed studies have been performed in this regard. In the work of Babaei et al. [2], it was shown that radar performance can be improved at the cost of non-zero interference at the communication users' end by projecting the radar signal onto a subspace. Also, Khawar et al. [3] analyzed radar performance under cellular interference. They studied the impact on radar performance due to cellular interference by deriving bounds on the probability of detection and the probability of missed detection.

In the work of Khawar et al. [4], projecting the radar signal onto a null space of the interference channel between the MIMO radar system and the long-term evolution (LTE) MIMO system using an interference-channel-selection algorithm was proposed to have zero interference from the MIMO radar system. By using a channel-selection-algorithm, it was shown that the loss in radar performance is minimal when the proposed algorithm is used to select the channel onto which the radar signals are projected.

Sodagari et al. [5] showed that the waveform of radar was projected onto a null space of an interference channel between the radar and communication systems. This null space projection of the radar waveform mitigates radar interference but in turn causes slight degradation in radar performance.

With so much work having been done on analyzing the performance of radar with interference from communication systems, the goal of this thesis is to analyze the performance of the MIMO communications system considering MIMO radar signals reflected at the target as interference. The bit error rate (BER) of the communications system is analyzed. It will be shown that decreasing the radar waveform transmit power improves the performance of the communication system. As the variance of the target reflection coefficient increases, the performance at the communication system decreases.

Fishler et al. [6] observed that targets are complex bodies and are generally composed of many small elemental scatterers. Also, the range and orientation (target direction) of the target determines the amount of energy that is reflected from these scatterers. In addition, small changes in the target's orientation can result in a large increase or decrease in the amount of energy reflected from the target. This thesis concentrates on analyzing the impact of the target-reflected signals on the MIMO communication system during spectrum sharing.

CHAPTER 2

SYSTEM MODEL

This section describes both the MIMO communication system and the MIMO radar system. Figure 1 shows the coexistence of the two.

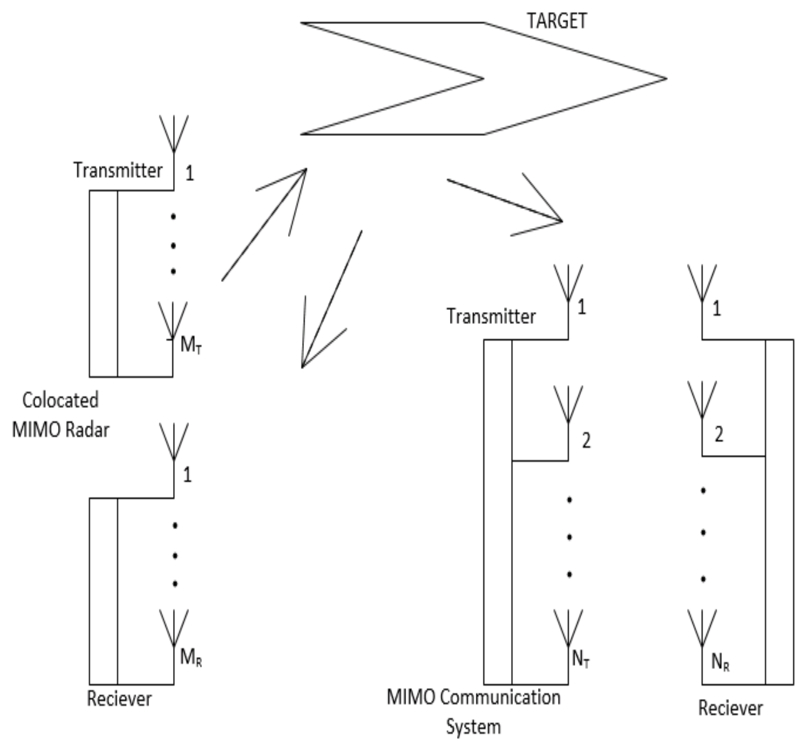

Figure 1. Spectrum Sharing between MIMO Radar and MIMO Communication System.

2.1 MIMO Communication System

The MIMO communication system consists of N_T transmit antennas and N_R receive antennas. The received signal vector $\boldsymbol{y}(t)$ of dimension $N_R \times 1$ at the receive antennas of the communication system is

$$\boldsymbol{y}(t) = H\boldsymbol{x}(t) + \boldsymbol{n}(t) \tag{1}$$

where H is the $N_R \times N_T$ communications channel matrix, $\boldsymbol{x}(t)$ is the $N_T \times 1$ transmit signal vector from the N_T transmit antennas, and $\boldsymbol{n}(t)$ is the $N_R \times 1$ additive white Gaussian noise (AWGN) vector of mean zero and variance one [7]. The signals received on the N_R receive antennas from the N_T transmit antennas are desirable, and any other signals act as interference.

2.2 MIMO Radar System

MIMO radar makes use of multiple antennas at the transmitter to emit several orthogonal waveforms and multiple antennas at the receiver to receive the echoes reflected by the target. Relative to array configuration, there exist widely separated and colocated MIMO radar.

The widely separated MIMO radar uses widely separated transmit and receive antennas to capture the spatial diversity of the target's radar cross section (RCS), whereas the colocated MIMO radar with colocated antennas improves angular resolution, increases the upper limit on the number of detectable targets, improves parameter identifiability, and enhances flexibility of the transmit/receive beam-pattern design [13].

The radar type considered in this research is a MIMO colocated radar with M_T transmit antennas and M_R receive antennas. The spacing between the antennas is half the wavelength. This is different from the widely spaced MIMO radar where the antenna elements are spaced widely apart. Also, the collocated MIMO radar has good spatial resolution and target parameter identification compared to the widely spaced radar [9].

The M_T antennas transmit a finite-alphabet constant-envelop BPSK waveform. The radar waveform $x_r(t)$ can be expressed as

$$\boldsymbol{x}_r(t) = [x_1(t)e^{jw_c t}\ x_2(t)e^{jw_c t} \dots x_{M_T}(t)e^{jw_c t}]^T \tag{2}$$

where $x_k(t)$ is the baseband signal from the k^{th} transmit elements, w_c is the carrier angular frequency, and $t \in [0, T_0]$ with T_0 being the observation time [7]. The receive and transmit steering vectors are defined as [13]

$$\boldsymbol{a}_t(\theta) = [1 \ \mathrm{e}^{-j2\pi d_t \sin\theta/\lambda} \ldots e^{-j(M_T-1)2\pi d_t \sin\theta/\lambda}]^T \tag{3}$$

$$\boldsymbol{a}_r(\theta) = [1 \ \mathrm{e}^{-j2\pi d_r \sin\theta/\lambda} \ldots e^{-j(M_R-1)2\pi d_r \sin\theta/\lambda}]^T \tag{4}$$

where λ is the wavelength, d_t and d_r are the spacing of the transmitting and receiving elements, respectively, and θ is the target direction [9]. Both the transmitting and receiving arrays are assumed to be close to each other in space so they see targets in the same direction [13].

The transmit-receive steering matrix is

$$A(\theta) = \boldsymbol{a}_t(\theta)\boldsymbol{a}_r^T(\theta) \tag{5}$$

$$A(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi d_t \sin\theta/\lambda} \\ . \\ . \\ . \\ e^{-j(M_T-1)2\pi d_t \sin\theta/\lambda} \end{bmatrix} \begin{bmatrix} 1 & e^{-j2\pi d_r \sin\theta/\lambda} & . & . & . & e^{-j(M_R-1)2\pi d_r \sin\theta/\lambda} \end{bmatrix} \tag{6}$$

$$A(\theta) = \begin{bmatrix} 1 & e^{-j2\pi d_r \sin\theta/\lambda} & . \ . \ . & e^{-j(M_r-1)2\pi d_r \sin\theta/\lambda} \\ e^{-j2\pi d_t \sin\theta/\lambda} & e^{-j2\pi \sin\theta/\lambda(d_t+d_r)} & . \ . \ . & e^{-j2\pi \sin\theta/\lambda(d_t+d_r(M_R-1))} \\ . & . & . \ . \ . & . \\ . & . & . \ . \ . & . \\ . & . & . \ . \ . & . \\ e^{-jd_t(M_t-1)2\pi \sin\theta/\lambda} & e^{-j2\pi \sin\theta/\lambda(d_t+d_r)} & . \ . \ . & e^{-j2\pi \sin\theta/\lambda(d_t(M_T-1)+d_r(M_R-1))} \end{bmatrix} \tag{7}$$

Since M_T and N_R are the same, $\mathbf{a}_T(\theta) \approx \mathbf{a}_R(\theta)$. The signal received from a target in the far field at an angle θ can be written as [9]

$$\boldsymbol{y}_r(t) = \alpha A(\theta)\boldsymbol{x}_r(t - \tau(t)) + \boldsymbol{n}(t) \tag{8}$$

where $\tau(t)$ is the sum of propagation delays between the target and the transmit/receive elements, α is the complex path loss matrix including the propagation loss and coefficient of reflection. In order to keep the analysis traceable, the following assumptions are considered: α is the same for all transmit and receive elements due to the far field assumption; angle θ is the azimuth angle of the target [9]. By compensating for the range-Doppler parameters, equation (8) becomes

$$\boldsymbol{y}_r(t) = aA(\theta)\boldsymbol{x}_r(t) + \boldsymbol{n}(t) \tag{9}$$

CHAPTER 3

SPECTRUM SHARING

With regards to spectrum sharing, the MIMO radar and the MIMO communication system are co-primary users of the same spectrum band. Relative to the coexistence illustrated in Figure 1, the signals transmitted from the radar transmit antennas are reflected at the target and received at the communication system receive antennas. The received signal at the communication system is given by

$$\boldsymbol{r}(t) = \alpha A(\theta)\boldsymbol{x}_r(t) \ + H\boldsymbol{x}(t) + \boldsymbol{n}(t) \tag{10}$$

Hence, considering the i^{th} receive antenna, j^{th} transmit antenna at the communication system and the k^{th} radar transmit antenna, the received signal can be written as

$$r_{ij} = \alpha_{ik} a_{ik} x_{ikr} + h_{ij} x_{ij} + \sum_{l \neq j}^{N_T} h_{il} x_{il} + n_{ij} \tag{11}$$

. The radar component acts as interference to the communication system in addition to the interference signals from the remaining l^{th} transmit antennas at the communication system and the AWGN component. Hence, it is neglected. The propagation path traversed by the signal transmitted from the radar is usually characterized by the propagation loss factor and the target reflection coefficient.

3.1 Propagation Path

In practice, each propagation path between a set of transmitters, targets, and receivers has different characteristics, depending on path loss, target reflectivity, and phase errors. This paper is interested in the target reflection coefficient and how its variation impacts the MIMO communication system when sharing a spectrum with MIMO radar. Other factors that could possibly impact the MIMO communication system are also analyzed.

3.1.1 Target Reflection Coefficient

Suppose that the transmitters of the MIMO radar and the receivers of the MIMO communication system are separated far enough apart; then the reflection gains are independent. Also, if the target has a large number of small independent and identically distributed (i.i.d) random scatterers, then g_i would be i.i.d complex Gaussian vectors with a probability density function of mean 0 and variance $\sigma_g^2 \boldsymbol{I}_{M_T}$. Hence, the target reflection gain vector for the i^{th} receive antenna at the communication system is $g_i \approx [\mathrm{g}_{i,1}, \mathrm{g}_{i,2}, \cdots, \mathrm{g}_{i,M_T}]^T$ [10, 11]. The target scatterer matrix is given by

$$G = \begin{bmatrix} \boldsymbol{g}_1 & \boldsymbol{g}_2 & \cdot & \cdot & \cdot & \boldsymbol{g}_{M_R} \end{bmatrix} \tag{12}$$

$$G = \begin{bmatrix} g_{11} & g_{21} & \cdot & \cdot & \cdot & g_{M_R 1} \\ g_{12} & g_{22} & \cdot & \cdot & \cdot & g_{M_R 2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ g_{1M_T} & g_{2M_T} & \cdot & \cdot & \cdot & g_{M_R M_T} \end{bmatrix} \tag{13}$$

3.1.2. Propagation Loss Factor

The propagation loss depends on the target proximity (that is, distance from the transmit antenna to the target and also distance from the target to the receive antenna) and antenna properties. The propagation loss of the radar waveform from the k^{th} transmit antenna received at the i^{th} receive antenna of the communication system is given by [10, 11]

$$p_{i,k} = \frac{c}{d_t^k d_r^i \sqrt{A_t^k A_r^i}} \tag{14}$$

where c is a constant, d_t^k and d_r^i denote the distance between the target and k^{th} transmitter and between the target and the i^{th} receiver, respectively, while A_t^k and A_r^i denote the transmission and receiving antenna gains, respectively. For the i^{th} receiver, the propagation loss vector is given by

$$p_i = \begin{bmatrix} p_{i1} & p_{i2} & . & . & . & p_{iM_T} \end{bmatrix}^T \tag{15}$$

Hence, the propagation loss matrix is denoted by P as shown below

$$P = \begin{bmatrix} \boldsymbol{p}_1 & \boldsymbol{p}_2 & . & . & . & \boldsymbol{p}_{M_R} \end{bmatrix} \tag{16}$$

$$P = \begin{bmatrix} p_{11} & p_{12} & . & . & . & p_{M_R1} \\ p_{12} & p_{22} & . & . & . & p_{M_R2} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ p_{1M_T} & p_{2M_T} & . & . & . & p_{M_RM_T} \end{bmatrix} \tag{17}$$

Combining both the propagation loss and the target reflection coefficient, the complex path loss in equation (6) is α which is the product ($\odot$) between G and P, where G is the target scatterer matrix, $\odot$ denotes the element-by-element product, and P denotes the propagation loss matrix. Hence, the complex path loss matrix is given by

$$\alpha = \begin{bmatrix} g_{11} & g_{21} & . & . & . & g_{M_R1} \\ g_{12} & g_{22} & . & . & . & g_{M_R2} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ g_{1M_T} & g_{2M_T} & . & . & . & g_{M_RM_T} \end{bmatrix} \odot \begin{bmatrix} p_{11} & p_{21} & . & . & . & p_{M_R1} \\ p_2 & p_{22} & . & . & . & p_{M_R2} \\ & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ p_{1M_T} & p_{2M_T} & . & . & . & p_{M_RM_T} \end{bmatrix} \tag{18}$$

$$\alpha = \begin{bmatrix} g_{11}p_{11} & g_{21}p_{21} & \cdot & \cdot & \cdot & g_{M_R 1}p_{M_R 1} \\ g_{12}p_{12} & g_{22}p_{22} & \cdot & \cdot & \cdot & g_{M_R 2}p_{M_R 2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ g_{1M_T}p_{1M_T} & g_{2M_T}p_{2M_T} & \cdot & \cdot & \cdot & g_{M_R M_T}p_{M_R M_T} \end{bmatrix} \tag{19}$$

CHAPTER 4

MATHEMATICAL ANALYSIS

In this section, considering the work performed by Quoc et al. [8], the signal in equation (9) received by the communication system is analyzed using a zero-forcing (ZF) equalizer at the receiver. The ZF equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. From the work of Quoc et al. [8], the ZF component is given by

$$r_{ZF} = \left(H^H H\right)^{-1} H^H \tag{20}$$

where H is the $N_R \times N_T$ channel matrix for the MIMO communication system and can be written as

$$H = \begin{bmatrix} h_{11} & h_{12} & . & . & . & h_{1M_T} \\ h_{21} & h_{22} & . & . & . & h_{2M_T} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ h_{M_R 1} & h_{M_R 2} & . & . & . & h_{M_R M_T} \end{bmatrix} \tag{21}$$

Considering equations (11) and (20),

$$r_{ZF}^{ij} = \left(h_{ij}^H h_{ij}\right)^{-1} h_{ij}{}^H r_{ij} \tag{22}$$

$$r_{ZF}^{ij} = (h_{ij}^H h_{ij})^{-1} h_{ij}^H [\alpha_{ik} a_{ik} x_{ikr} + h_{ij} x_{ij} + \sum_{l \neq j}^{N_T} h_{il} x_{il} + n_{ij}] \tag{23}$$

$$r_{ZF}^{ij} = (h_{ij}^H h_{ij})^{-1} h_{ij}^H \alpha_{ik} a_{ik} x_{ikr} + (h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} x_{ij} + (h_{ij}^H h_{ij})^{-1} h_{ij}^H \sum_{l \neq j}^{N_T} h_{il} x_{il} + (h_{ij}^H h_{ij})^{-1} h_{ij}^H n_{ij} \tag{24}$$

where $(h_{ij}^{H}h_{ij})^{-1}h_{ij}^{H}h_{ij}x_{ij}$ represents the desired signal component, $(h_{ij}^{H}h_{ij})^{-1}h_{ij}^{H}\alpha_{ik}a_{ik}x_{ikr}$, $(h_{ij}^{H}h_{ij})^{-1}h_{ij}^{H}\sum_{l\neq j}^{N_T}h_{il}x_{il}$ and $(h_{ij}^{H}h_{ij})^{-1}h_{ij}^{H}n_{ij}$ represent the represent the interference-plus-noise component. Applying the expectation for each component yields the power for both the desired signal and the interference plus noise (see Appendix A).

Hence, from equation (22) the signal-plus-interference-to-noise ratio (SINR) is

$$SINR = \frac{1}{\frac{1}{h_{ij}^{H}h_{ij}}[\alpha_{ij}^{2}a_{ij}^{2} + N_T\sum_{l\neq j}^{N_T}h_{il}h_{ij}{}^{H} + 1]} \tag{25}$$

Furthermore, considering the work of Quoc et al. [8], log-normal shadowing is introduced to the MIMO communication system. Here

$$G = HD^{\frac{1}{2}} \tag{26}$$

where H is the $N_R \times N_T$ communications channel matrix and D is an $N_T \times N_R$ diagonal matrix with a log-normal entry β on its leading diagonal [8]. Hence, equation (11), becomes

$$r_{ij} = \alpha_{ik}a_{ik}x_{ikr} + g_{ij}x_{ij} + \sum_{l\neq j}^{N_T}g_{il}x_{il} + n_{ij} \tag{27}$$

Also, equation (20) becomes

$$r_{ZF} = \left(G^{H}G\right)^{-1}G^{H} \tag{28}$$

Considering equations (27) and (28),

$$r_{ZF}^{ij} = \left(g_{ij}^{H}g_{ij}\right)^{-1}g_{ij}^{H}r_{ij} \tag{29}$$

$$r_{ZF}^{ij} = (g_{ij}^{H}g_{ij})^{-1}h_{ij}^{H}[\alpha_{ik}a_{ik}x_{ikr} + g_{ij}x_{ij} + \sum_{l\neq j}^{N_T}g_{il}x_{il} + n_{ij}] \tag{30}$$

$$r_{ZF}^{ij} = (g_{ij}^H g_{ij})^{-1} g_{ij}^H \alpha_{ik} a_{ik} x_{ikr} + (g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij} + (g_{ij}^H g_{ij})^{-1} g_{ij}^H \sum_{l \neq j}^{N_T} g_{il} x_{il} + (g_{ij}^H g_{ij})^{-1} g_{ij}^H n_{ij} \tag{31}$$

where $(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij}$ represents the desired signal component, $(g_{ij}^H g_{ij})^{-1} g_{ij}^H \alpha_{ik} a_{ik} x_{ikr}$ $(g_{ij}^H g_{ij})^{-1} g_{ij}^H \sum_{l \neq j}^{N_T} g_{il} x_{il}$ and $(g_{ij}^H g_{ij})^{-1} g_{ij}^H n_{ij}$ represent the interference-plus-noise component. Applying the expectation for each component yields the power for both the desired signal and for the interference plus noise (see Appendix B).

Hence, from equation (29) the signal-plus-interference-to-noise ratio becomes

$$SINR = \frac{1}{\frac{1}{g_{ij}^H g_{ij}} [\alpha_{ij}^2 a_{ij}^2 + N_T \sum_{l \neq j}^{N_T} g_{il} g_{ij}{}^H + 1]} \tag{32}$$

From equation (26), $g_{ij} = \sqrt{\beta_i} h_{ij}$. Hence, the SINR becomes

$$SINR = \frac{1}{\frac{1}{\beta_{ij}^2 h_{ij}^H h_{ij}} [\alpha_{ij}^2 a_{ij}^2 + N_T \beta_{il} \beta_{ij} \sum_{l \neq j}^{N_T} h_{il} h_{ij}{}^H + 1]} \tag{33}$$

CHAPTER 5

SIMULATION AND RESULTS

In modeling the simulation of the previous analysis, the following parameters were considered: $N_T = N_R = M_T = M_R = 10$, carrier frequency is 3.55 GHz, speed of light c is $3\times10^8 m/s$, $d_t^k = d_r^i = 5km$, and antenna spacing is half the wavelength. In modelling the propagation loss factor, all antennas are assumed to be isotropic; therefore, equation (14) becomes $p_{i,k} = c/d_t^k d_r^i$.

In the simulation, first all MIMO radar antennas transmit waveforms at the same power level, which is fixed to unity (i.e., 0 dB), the target direction is $\theta = 15^o$, and the noise variance and variance of target reflectivity is set to 1, i.e., $\sigma_n^2 = \sigma_g^2 = 1$. Also, the antennas of the communication system transmit at a unity power level (0 dB). From Figure 2, simulation results show a very high bit error rate at the MIMO communication system (i.e., a complete straight line). By reducing the radar transmit power by 20 dB, 30 dB, and 40 dB (i.e. –20 dB, –30 dB, –40 dB), a significant performance improvement is achieved at the communication system.

Second, in order to simulate the impact of the target reflection coefficient on the communication system σ_g^2 is now increased to 2. Examining the results shown in Figure 3, it can be seen that the BER is further increased at the communication system, leading to a decrease in performance at the communication system. Comparing Figures 2 and 3, it is clear that as the σ_g^2 increases, the BER of the MIMO communication system increases.

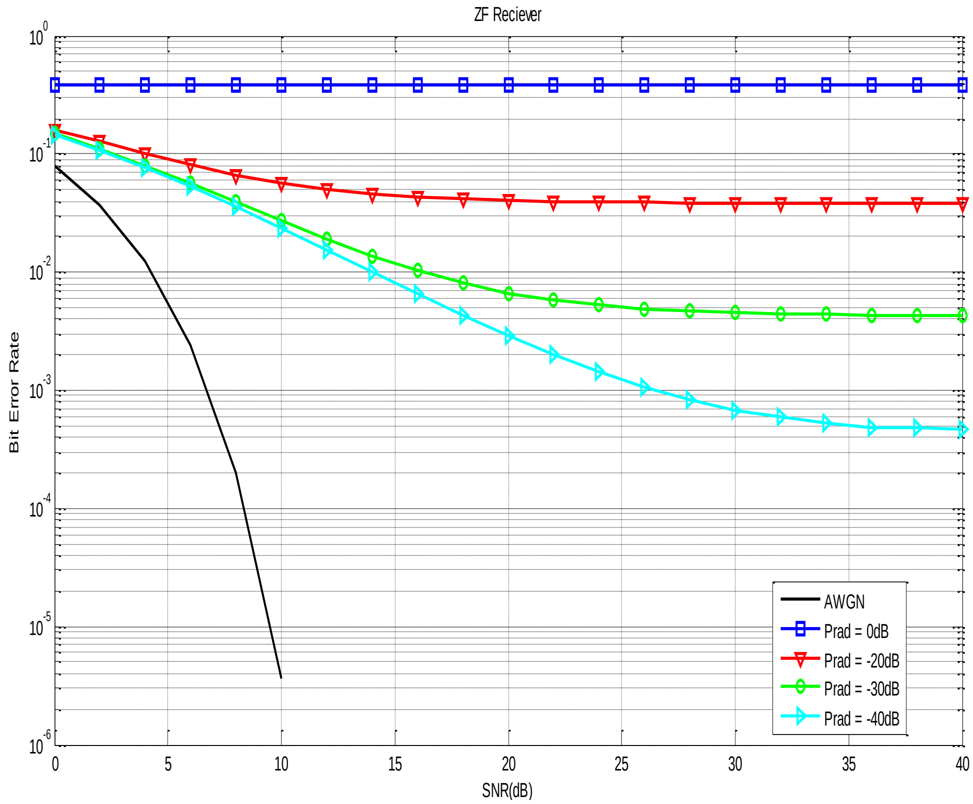

Figure 2. Plot of Radar Transmit Power Variation.

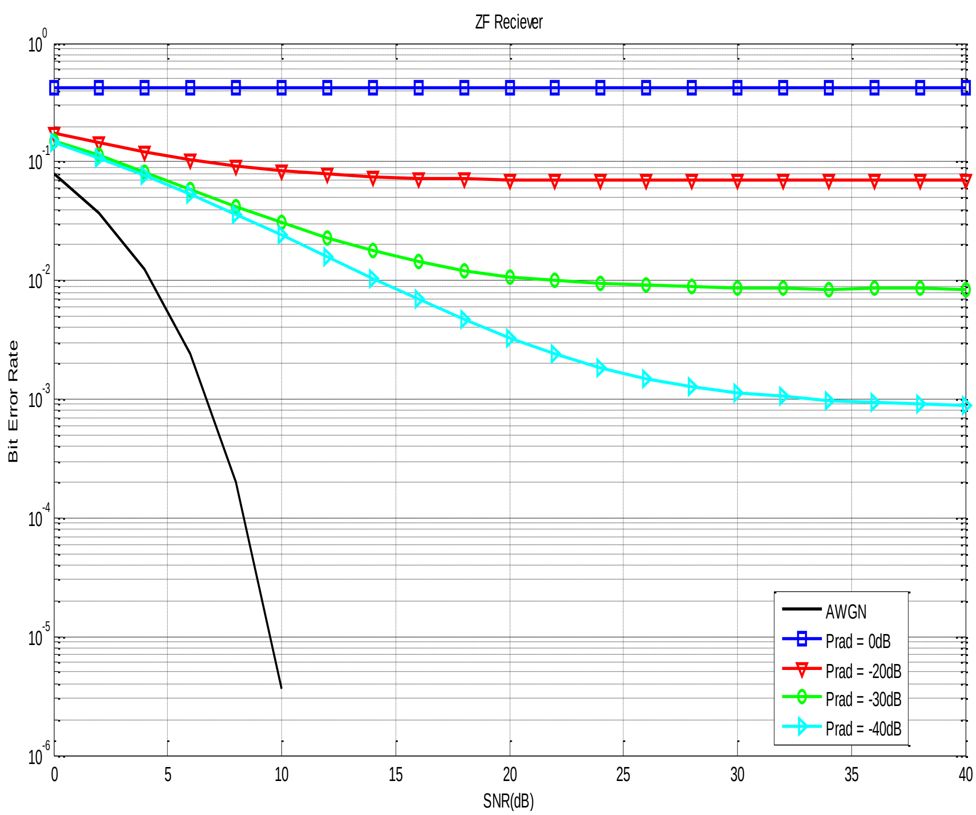

Figure 3. Plot of Target Reflectivity Variance Variation.

Furthermore, by subjecting the communication system to a real practical environment, the concept of log-normal shadowing with standard deviations (sigma) of 1 dB, 3 dB, and 6 dB are introduced for radar transmitting at the –30 dB power level, a target direction of $\theta = 15^o$, and $\sigma_n^2 = \sigma_g^2 = 1$. Simulation results shown in Figure 4 indicate further performance degradation by the communication system in such an environment. Therefore, when the standard deviation increases, the BER increases, and hence there is a decrease in performance.

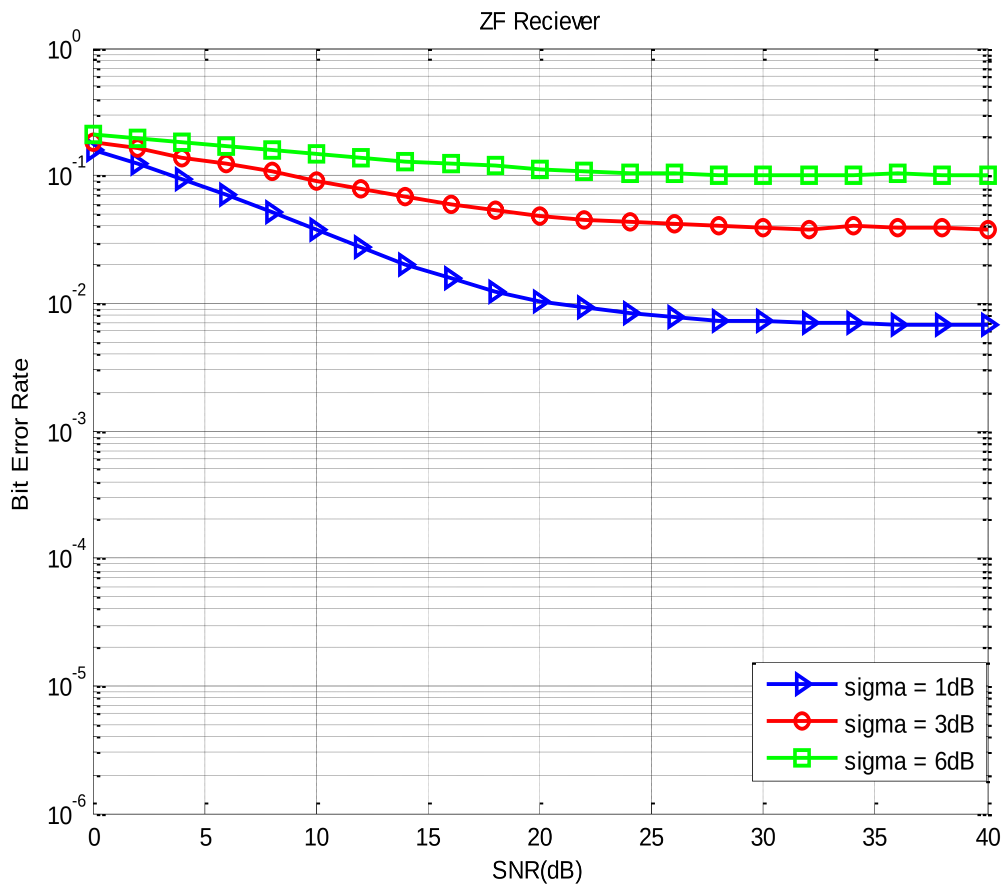

Figure 4. Plot of Log-Normal Standard Deviation Variation.

Also, the results shown in Figure 5 show that the simulation was carried out with a variation in the target direction, which is when $\theta = 0^o$, $\theta = 10^o$, $\theta = 30^o$. Radar transmit power was set at the –30 dB level and $\sigma_g^2 = 1$. From Figure 5, the curves show how the target orientation affects the return signal and thereby degrades the communication system performance.

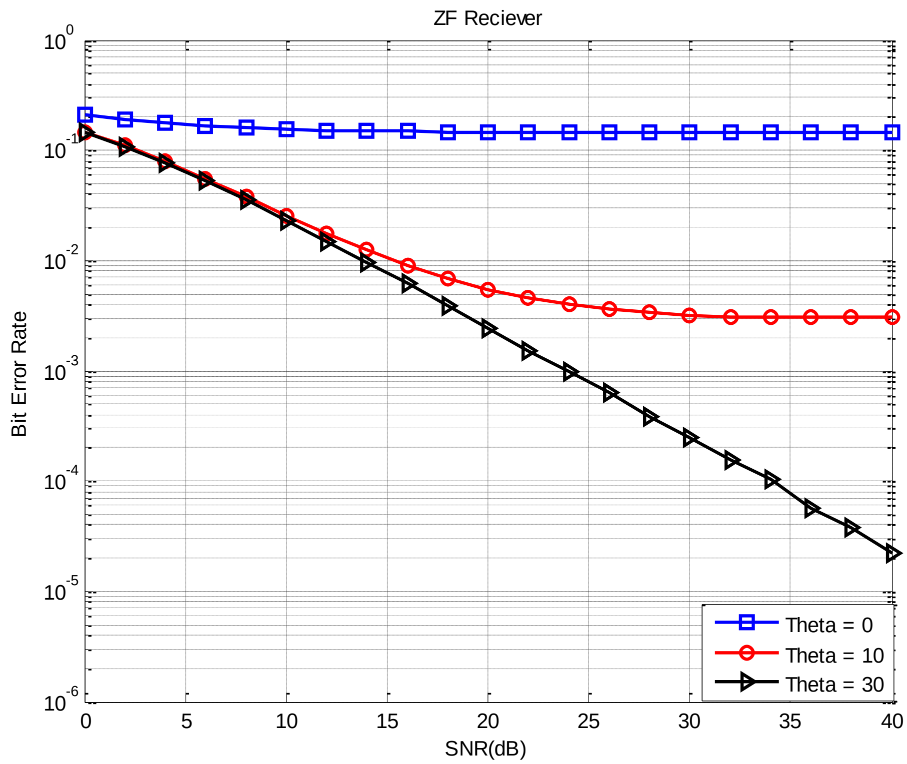

Figure 5. Plot of Target Direction Variation and $\theta = 0^o$, $\theta = 10^o$, $\theta = 30^o$.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH

6.1 Conclusion

Spectrum sharing between the MIMO radar system and the MIMO communication system is promising in terms of mitigating spectrum scarcity; however, the effects of interference from the MIMO radar system to the MIMO communications system must be examined carefully. The target and target-scattering characteristic is of importance and should be look at closely.

From the analysis and results, it can be concluded that the variation of transmitted power from the MIMO radar transmit antennas affects performance of the MIMO communications system. It is shown that when the radar transmit antennas transmit at unit power, the BER of the communication system is very high implying that performance at the MIMO communication system will be drastically reduced. By reducing the transmit power by 20 dB, 30 dB, and 40 dB, significant performance improvement is achieved. The scattering characteristic of the target can impact the communication system, and from the simulation results, increasing the variance σ_g^2 negatively impacts the communication system.

6.2 Future Research

In this thesis, the system model considered a MIMO radar and a MIMO communication system with both ten receive and transmit antennas. Also, only a single point target was considered.

With regards to the future research, the following points below will be taken into considerations: Firstly, instead of a single point target, the future research will consider multiple targets at different orientations (that is target range and target direction) with varying scattering power for each elemental scatterer on the targets. Secondly, the future work will research advanced mitigation techniques to reduce the impact of the interference from the increasing number of targets and improve the performance at the communication system.

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APPENDICES

APPENDIX A

SIGNAL-TO-INTERFERENCE-PLUS-NOISE RATIO DERIVATION

From equation (24), consider the desired signal component:

$$E\{[(h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} x_{ij}]^2\} = E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} x_{ij} x_{ij}^H h_{ij}^H h_{ij} (h_{ij}^H h_{ij})^{-1}\} \quad \text{(A-1)}$$

However, $E\{x_{ij} x_{ij}^H\} = 1$. Hence,

$$E\{[(h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} x_{ij}]^2\} = E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} h_{ij}^H h_{ij} (h_{ij}^H h_{ij})^{-1}\} \quad \text{(A-2)}$$

$$E\{[(h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} x_{ij}]^2\} = (h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} \quad \text{(A-3)}$$

$$E\{[(h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} x_{ij}]^2\} = \frac{h_{ij}^H h_{ij}}{h_{ij}^H h_{ij}} \quad \text{(A-4)}$$

$$E\{[(h_{ij}^H h_{ij})^{-1} h_{ij}^H h_{ij} x_{ij}]^2\} = 1 \quad \text{(A-5)}$$

From equation (A-5), the power of the desired signal component equals 1. Also, from equation (24), consider the interference-plus-noise component:

$$\begin{aligned} &E\{[(h_{ij}^H h_{ij})^{-1} h_{ij}^H \alpha_{ik} a_{ik} x_{ikr} + (h_{ij}^H h_{ij})^{-1} h_{ij}^H \sum_{l\neq j}^{N_T} h_{il} x_{il} + (h_{ij}^H h_{ij})^{-1} h_{ij}^H n_{ij}]^2\} = \\ &E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H \alpha_{ik} a_{ik} x_{ikr}\} + E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H \sum_{l\neq j}^{N_T} h_{il} x_{il}\} + E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H n_{ij}\} \end{aligned} \quad \text{(A-6)}$$

$$\begin{aligned} &E\{[(h_{ij}^H h_{ij})^{-1} h_{ij}^H \alpha_{ik} a_{ik} x_{ikr} + (h_{ij}^H h_{ij})^{-1} h_{ij}^H \sum_{l\neq j}^{N_T} h_{il} x_{il} + (h_{ij}^H h_{ij})^{-1} h_{ij}^H n_{ij}]^2\} = \\ &E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H \alpha_{ik} a_{ik} x_{ikr} x_{ikr}^H a_{ik} \alpha_{ik} h_{ij} (h_{ij}^H h_{ij})^{-1}\} + \\ &E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H \sum_{l\neq j}^{N_T} (h_{il} x_{il} x_{il}^H h_{il}^H) h_{ij} (h_{ij}^H h_{ij})^{-1}\} + E\{(h_{ij}^H h_{ij})^{-1} h_{ij}^H n_{ij} n_{ij} h_{ij} (h_{ij}^H h_{ij})^{-1}\} \end{aligned} \quad \text{(A-7)}$$

From equation (A-7), $E\{x_{i\,kr}x_{i\,k\,r}^H\}=1$, $E\{x_{i\,l}x_{i\,l}^H\}=1$ and $E\{n_i n_i^H\}=1$. Hence equation (A-7) becomes

$$E\{[(h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\sum_{l\neq j}^{N_T}h_{i\,l}x_{i\,l} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H n_{i\,j}]^2\} =$$
$$E\{(h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\alpha_{i\,k}a_{i\,k}a_{i\,k}\alpha_{i\,k}h_{i\,j}(h_{i\,j}^H h_{i\,j})^{-1}\} + E\{(h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\sum_{l\neq j}^{N_T}(h_{i\,l}h_{i\,l}^H)h_{i\,j}(h_{i\,j}^H h_{i\,j})^{-1}\} + E\{(h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H h_{i\,j}(h_{i\,j}^H h_{i\,j})^{-1}\} \quad \text{(A-8)}$$

$$E\{[(h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\sum_{l\neq j}^{N_T}h_{i\,l}x_{i\,l} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H n_{i\,j}]^2\} =$$
$$\alpha_{i\,j}^2 a_{i\,j}^2\frac{1}{h_{i\,j}^H h_{i\,j}} + N_T(\frac{1}{h_{i\,j}^H h_{i\,j}})\sum_{l\neq j}^{N_T}(h_{i\,l}h_{i\,l}^H) + \frac{1}{h_{i\,j}^H h_{i\,j}} \quad \text{(A-9)}$$

$$E\{[(h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\sum_{l\neq j}^{N_T}h_{i\,l}x_{i\,l} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H n_{i\,j}]^2\} =$$
$$\frac{\alpha_{i\,j}^2 a_{i\,j}^2}{h_{i\,j}^H h_{i\,j}} + \frac{N_T}{h_{i\,j}^H h_{i\,j}}\sum_{l\neq j}^{N_T}(h_{i\,l}h_{i\,l}^H) + \frac{1}{h_{i\,j}^H h_{i\,j}} \quad \text{(A-10)}$$

$$E\{[(h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H\sum_{l\neq j}^{N_T}h_{i\,l}x_{i\,l} + (h_{i\,j}^H h_{i\,j})^{-1}h_{i\,j}^H n_{i\,j}]^2\} =$$
$$\frac{1}{h_{i\,j}^H h_{i\,j}}[\alpha_{i\,j}^2 a_{i\,j}^2 + N_T\sum_{l\neq j}^{N_T}(h_{i\,l}h_{i\,l}^H) + 1] \quad \text{(A-11)}$$

From equation (A-11), the power of the interference-plus-noise component is $\frac{1}{h_{i\,j}^H h_{i\,j}}[\alpha_{i\,j}^2 a_{i\,j}^2 + N_T\sum_{l\neq j}^{N_T}(h_{i\,l}h_{i\,l}^H) + 1]$.

Hence, equation (25) is written as

$$SINR = \frac{1}{\frac{1}{h_{i\,j}^H h_{i\,j}}[\alpha_{i\,j}^2 a_{i\,j}^2 + N_T\sum_{l\neq j}^{N_T}(h_{i\,l}h_{i\,l}^H) + 1]} \quad \text{(A-12)}$$

APPENDIX B

SIGNAL-TO-INTERFERENCE-PLUS-NOISE RATIO DERIVATION WITH LOG-NORMAL COMPONENT

From equation (31), consider the desired signal component:

$$E\{[(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij}]^2\} = E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij} x_{ij}^H g_{ij}^H g_{ij} (g_{ij}^H g_{ij})^{-1}\} \tag{B-1}$$

However, $E\{x_{ij} x_{ij}^H\} = 1$. Hence,

$$E\{[(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij}]^2\} = E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} g_{ij}^H g_{ij} (g_{ij}^H g_{ij})^{-1}\} \tag{B-2}$$

$$E\{[(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij}]^2\} = (g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} \tag{B-3}$$

$$E\{[(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij}]^2\} = \frac{g_{ij}^H g_{ij}}{g_{ij}^H g_{ij}} \tag{B-4}$$

$$E\{[(g_{ij}^H g_{ij})^{-1} g_{ij}^H g_{ij} x_{ij}]^2\} = 1 \tag{B-5}$$

From equation (B-5), the power of the desired signal component equals 1. Also, from equation (31), consider the interference-plus-noise component:

$$\begin{aligned} &E\{[(g_{ij}^H g_{ij})^{-1} g_{ij}^H \alpha_{ik} a_{ik} x_{ikr} + (g_{ij}^H g_{ij})^{-1} g_{ij}^H \sum_{l \neq j}^{N_T} g_{il} x_{il} + (g_{ij}^H g_{ij})^{-1} g_{ij}^H n_{ij}]^2\} = \\ &E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H \alpha_{ik} a_{ik} x_{ikr}\} + \quad E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H \sum_{l \neq j}^{N_T} g_{il} x_{il}\} + E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H n_{ij}\} \end{aligned} \tag{B-6}$$

$$\begin{aligned} &E\{[(g_{ij}^H g_{ij})^{-1} g_{ij}^H \alpha_{ik} a_{ik} x_{ikr} + (g_{ij}^H g_{ij})^{-1} g_{ij}^H \sum_{l \neq j}^{N_T} g_{il} x_{il} + (g_{ij}^H g_{ij})^{-1} g_{ij}^H n_{ij}]^2\} = \\ &E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H \alpha_{ik} a_{ik} x_{ikr} x_{ikr}^H a_{ik} \alpha_{ik} g_{ij} (g_{ij}^H g_{ij})^{-1}\} + \\ &E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H \sum_{l \neq j}^{N_T} (g_{il} x_{il} x_{il}^H g_{il}^H) g_{ij} (g_{ij}^H g_{ij})^{-1}\} + E\{(g_{ij}^H g_{ij})^{-1} g_{ij}^H n_{ij} n_{ij} h_{ij} (g_{ij}^H g_{ij})^{-1}\} \end{aligned} \tag{B-7}$$

From equation (B-7), $E\{x_{i\,kr}x_{i\,k\,r}^{H}\}=1$, $E\{x_{i\,l}x_{i\,l}^{H}\}=1$ and $E\{n_{i\,j}n_{i\,j}^{\;H}\}$=1. Hence equation (B-7) becomes

$$\begin{aligned}E\{[(g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\sum_{l\neq j}^{N_T}g_{i\,l}x_{i\,l} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}n_{ij}]^2\} = \\ E\{(g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\alpha_{i\,k}a_{i\,k}a_{i\,k}\alpha_{i\,k}g_{ij}(g_{ij}^{H}g_{ij})^{-1}\}+ \\ E\{(g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\sum_{l\neq j}^{N_T}(g_{i\,l}g_{i\,l}^{H})g_{ij}(g_{ij}^{H}g_{ij})^{-1}\}+E\{(g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}g_{ij}(g_{ij}^{H}g_{ij})^{-1}\}\end{aligned} \tag{B-8}$$

$$\begin{aligned}E\{[(g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\sum_{l\neq j}^{N_T}g_{i\,l}x_{i\,l} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}n_{ij}]^2\} = \\ \alpha_{ij}^2a_{ij}^2\frac{1}{g_{ij}^{H}g_{ij}} + \quad N_T(\frac{1}{g_{ij}^{H}g_{ij}})\sum_{l\neq j}^{N_T}(g_{i\,l}g_{i\,l}^{H}) \ + \frac{1}{g_{ij}^{H}g_{ij}}\end{aligned} \tag{B-9}$$

$$\begin{aligned}E\{[(g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\sum_{l\neq j}^{N_T}g_{i\,l}x_{i\,l} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}n_{ij}]^2\} = \\ \frac{\alpha_{ij}^2a_{ij}^2}{g_{ij}^{H}g_{ij}} + \quad \frac{N_T}{g_{ij}^{H}g_{ij}}\sum_{l\neq j}^{N_T}(g_{i\,l}g_{i\,l}^{H}) \ + \frac{1}{g_{ij}^{H}g_{ij}}\end{aligned} \tag{B-10}$$

$$\begin{aligned}E\{[(g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}\sum_{l\neq j}^{N_T}g_{i\,l}x_{i\,l} + (g_{ij}^{H}g_{ij})^{-1}g_{ij}^{H}n_{ij}]^2\} = \\ \frac{1}{g_{ij}^{H}g_{ij}}[\alpha_{ij}^2a_{ij}^2 + N_T\sum_{l\neq j}^{N_T}(g_{i\,l}g_{i\,l}^{H}) + 1]\end{aligned} \tag{B-11}$$

$$\begin{aligned}E\{[(h_{ij}^{H}h_{ij})^{-1}h_{ij}^{H}\alpha_{i\,k}a_{i\,k}x_{i\,kr} + (h_{ij}^{H}h_{ij})^{-1}h_{ij}^{H}\sum_{l\neq j}^{N_T}h_{i\,l}x_{i\,l} + (h_{ij}^{H}h_{ij})^{-1}h_{ij}^{H}n_{ij}]^2\} = \\ \frac{1}{\beta_{ij}^2h_{ij}^{H}h_{ij}}[\alpha_{ij}^2a_{ij}^2 + N_T\beta_{i\,l}\beta_{ij}\sum_{l\neq j}^{N_T}h_{i\,l}h_{ij}^{\;H} + 1]\end{aligned} \tag{B-12}$$

From equation (B-12), the power of the interference-plus-noise component with the log-normal component is $\frac{1}{\beta_{ij}^2h_{ij}^{H}h_{ij}}[\alpha_{ij}^2a_{ij}^2 + N_T\beta_{i\,l}\beta_{ij}\sum_{l\neq j}^{N_T}h_{i\,l}h_{ij}^{\;H} + 1]$.

Hence, equation (33) is written as

$$SINR = \frac{1}{\frac{1}{\beta_{ij}^{2} h_{ij}^{H} h_{ij}} [\alpha_{ij}^{2} a_{ij}^{2} + N_T \beta_{il} \beta_{ij} \sum_{l \neq j}^{N_T} h_{il} h_{ij}{}^{H} + 1]} \tag{B-13}$$

APPENDIX C

MATLAB CODE

```
% Matlab code for MIMO radar and MIMO communication system
% spectrumn sharing
% BER
% BPSK modulation
clear all;
clc;
NoOfBlock = 10000;
Nt=10;
Nr=10;
SNRindB=0:2:40;
Mt = 10; % Radar transmit antennas
Mr = 10; % Radar receive antennas
fc = 3.55 * 10^9; % Carrier frequency 3.55GHz
B = 5 * 10^6; % Bandwidth 5Hz
T = 300; % Temperature 300k
dtt = 5000; % distance from transmitter to target in m
dtr = 5000; % distance from target to reciever in m
c = 3 * 10^8; % in m/s
lambda = c/fc; % Wavelength in meters
dr= 0.5* (lambda); % Inter-element antenna spacing
dt = dr;
theta_tar = 15*pi/180;  %Vary the theta_tar here % Direction of target is 15
degress to the broadside of the array
a = exp(-1i*2*pi*dr*(0:Mt-1)*sin(theta_tar)/lambda);  % steering vector
%a_tar = exp(-1i*2*dt*pi*fc*(0:Mr-1)'*sin(theta_tar));  % Uplink steering
vector
%b_tar = exp(-1i*2*dr*pi*fc*(0:Mt-1)'*sin(theta_tar));% Downlink steering
vector;
for  i=1:length(SNRindB)
    i
    errInOneBlock =0;
    %Communication System transmit
    m = rand(Nt,1)>0.5;
    x = (2 * m - 1);   %*(10^(5/20));
    %Radar transmit
    mr= rand(Mt,1)>0.5;
    xr = (2*mr -1);
    T=1; % Bit duration
    Eb=T/2; % This will result in unit amplitude waveforms
    w=sqrt(2*Eb/T)*cos(2*pi*fc);  % carrier waveform
    Xrt = (xr* w)*(10^(-30/20));
    for IndexOfBlock=1:NoOfBlock
        H = 1/sqrt(2) * (randn(Nr,Nt) + randn(Nr,Nt)*1i);
        G = 1/sqrt(2) * (randn(Nr,Mt) + randn(Nr,Mt)*1i);
        C = 1;
        P = (ones(10,10)).*(C/dtt*dtr);
        alpha = G.*P;
        n = 1/sqrt(2) * (randn(Nr, 1) + randn(Nr, 1)*1i);
```

```
        S = 10;
        z = 10;
        Ac = zeros(S,S);
        for q=1:10
           elec = ((lognrnd(0,1)));  % Log-normal shadowing
           Ac(q,q) = (sqrt(elec));
        end
       %Aint = zeros(z,z);
       %for t=1:10
            %eleint = (lognrnd(0,3));
           % Aint(t,t) = (sqrt(eleint));
        %end
        k = (H*Ac);
         %Gint = (Hint*Aint);
         %II = (a.*Xrt');
        y = alpha*(a*transpose(a))*Xrt + k*x + 10^(-SNRindB(i)/20)*n;
        %y = x + 10^(-SNRindB(i)/20)*n;
        hy = H' * y;
        hhhy= (H' * H)^(-1) * hy;
        Hat = real(hhhy)>0;
        errInOneBlock = errInOneBlock + nnz(m-Hat);
    end
    err(i) = errInOneBlock;
enD
ber = err./(NoOfBlock*Nt);
hold all;
semilogy(SNRindB,ber,'b','LineWidth',2);
axis([0 40 10^-6 1]);
xlabel('SNR(dB)');
ylabel('Bit Error Rate');
title('ZF Reciever');
grid on;
```