EVOLVING DESIGN MODEL SYNCHRONIZATION FOR SYSTEM HEALTH MANAGEMENT USING LAPLACE APPROXIMATION

A Thesis by

Adebayo Opeoluwa Adewunmi

Bachelor of Science, University of Lagos, Nigeria, 2008

Submitted to the Department of Industrial and Manufacturing Engineering
and the faculty of the Graduate School of
Wichita State University
in partial fulfillment of
the requirements for the degree of
Master of Science

May 2015

EVOLVING DESIGN MODEL SYNCHRONIZATION FOR SYSTEM HEALTH MANAGEMENT USING LAPLACE APPROXIMATION

The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Industrial Engineering.

____
Pingfeng Wang, Committee Chair

____
Krishna Krishnan, Committee Member

____
Wilfredo Moscoso, Committee Member

____
Bin Li, Committee Member

DEDICATION

To my dear family and friends

ACKNOWLEDGEMENT

I would like to express my greatest gratitude to my advisor, Dr. Pingfeng Wang. He shared his knowledge and vast experience in research which motivated me to strive for the highest standards. His patience and guidance throughout the duration of my graduate career will forever be greatly appreciated.

I would like to thank my project committee defense members for their time, support and wise feedback during my research development.

I would also like to thank all the brilliant members of my research group at the Reliability Engineering Automation Laboratory at Wichita State University for their support and encouragement. Their gracious acts enabled me to be more productive.

This thesis would not have been possible without the support and wholehearted love I received from my family. I sincerely appreciate their effort in my journey.

ABSTRACT

Lifecycle health management plays an increasingly important role in realizing resilience of aging complex engineered systems since it detects, diagnoses, and predicts system-wide effects of adverse events, therefore enables a proactive approach to deal with system failures. To address an increasing demand to develop high-reliability low-cost systems, this paper presents a new platform for operational stage system health management, referred to as Evolving Design Model Synchronization (EDMS), which enables health management of aging engineered systems by efficiently synchronizing system design models with degrading health conditions of actual physical system in operation. A Laplace approximation approach is employed for the design model updating, which can incorporate heterogeneous operating stage information from multiple sources to update the system design model based on the information theory, thereby increases the updating accuracy compared with traditionally used Bayesian updating methodology. The design models synchronized over time using sensory data acquired from the system in operation can thus reflect system health degradation with evolvingly updated design model parameters, which enables the application of failure prognosis for system health management. Two case studies are used to demonstrate the efficacy of the proposed approach for system health management.

TABLE OF CONTENTS

LIST OF TABLES

Table Page

LIST OF FIGURES

Figure Page

LIST OF NOMENCLATURE

R	=	Reliability
J	=	Jacob matrix
H	=	Hessian matrix
$f_x(x)$	=	Probability density function
G	=	System performance function
$p(.\|.)$	=	Likelihood function
P_f	=	Probability of Failure

CHAPTER 1

INTRODUCTION

1.1 Background

Growing global trends toward increased system complexity and prolonged useful life present unprecedented challenges and opportunities for system designers to create innovative design tools for an emerging need of developing high-reliability low-cost complex engineered systems. The design tools must take into account all potential failure modes that could occur at system operation stage and uncertain future operating condition that may deviate significantly from the one assumed at the system design stage, in order to make engineered systems resilient to functional failures. Resilience defines the ability of the system to sense and withstand adverse events, and to avoid or recover from the effects of the adverse events [1]. In achieving the resilience of engineered systems, awareness of system health condition degradation over time is extremely important, especially for aging complex engineered systems, in order for system health management to effectively address potential system failures through detecting, diagnosing, and predicting the system-wide effects of adverse events [2, 3] and providing valuable information for making failure mitigation/recovery (M/R) decisions.

For many engineered systems, usage monitoring or inspection data are usually available at a regular time interval either via structural health monitoring system or non-destructive inspections. A critical issue becomes how to incorporate the existing knowledge and new sensory information to produce an accurate timely system model for reliability analysis and remaining useful life prognosis. Bayesian updating is the most commonly used approach to incorporate these additional data. Two categories of Bayesian updating approaches have been extensively used for this purpose, namely sampling based methods and posterior approximation-based methods. Markov Chain Monte Carlo (MCMC) [4-9] and Gibbs Sampling (BUGS) [10-12] are commonly used sampling based approaches. Laplace approximation method (LAM), as one approximation based approach, has recently drawn much attention to the engineering research committee due to its high updating efficiency [13-17], which approximates a posterior distribution with a multivariate Gaussian distribution at the maximum value point of the likelihood function. To reduce the computational efficiency and improve the approximation accuracy, hybrid updating methods, such as hybrid MCMC, have been also developed [18]. The Bayesian updating has been widely applied to engineering applications such as structural model updating [18], reliability updating [17], model parameter identification [13-15]. Applying the Bayesian updating approach for real time synchronization of large scale system models (e.g. with over 20 model parameters) requires highly efficient updating methods with guaranteed accuracy. Although the MCMC based approaches preserve good accuracy, the efficiency is quite low due to a slow convergence process of sampling. In order to evolvingly synchronize system models with sensory signals acquired from actual physical systems in operation, developing a novel sensor-based model updating technique that is efficient yet accurate with capability of handling large scale system models is one of paramount research tasks.

1.2 Research Scope and Objectives

The objective of this thesis is to create a new platform for the integration of system design with health management of aging complex engineered systems, which enables post design stage reliability estimation and remaining life prognosis through efficiently synchronizing system design models over time with degrading health conditions of actual physical systems in operation.

Although different methods have been developed for the purpose of prognosis, they are generally dependent on empirical system degradation models without leveraging existing system design models at the design phase. In this thesis, a generic Bayesian framework for evolving system design model updating, referred to as evolving design model synchronization (EDMS), is developed, which enables health management of aging engineered systems by efficiently synchronizing system design models with degrading health conditions of actual physical system in operation. To make the Bayesian updating technique generally applicable to a broad range of engineering problems, a general Bayesian inference mechanism with a Laplace approximation technique is developed. The proposed methodology is able to update system design model parameters so that an accurate model reflecting the health condition of real system in operation can be obtained over time for the reliability estimation and remaining life prognosis. The reliability of the system or components considering time-dependent performance deterioration can be evaluated at an early degradation stage and updated with more sensory data involved over time.

1.3 Organization of the thesis

The rest of this thesis is organized as follows. Chapter 3 introduces time-dependent reliability estimation and remaining life prognosis in system health management. Chapter 4 details the developed EDMS approach. A case study is used to demonstrate the effectiveness of the developed methodology in Chapter 5, and a brief summary of presented work will be given in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

This chapter presents a brief overview of the various journal papers, textbooks, technical publications and conference papers related to this thesis. The

2.1 System Health Management

If the output behavior of an engineering system does not match the intended functionality of the designer, the system is considered to be off-nominal [41]. System Health Management (SHM) is the capability of a system to contain, detect, diagnose, respond to and recover from conditions that may interfere with nominal system operations [42]. This concept plays a vital role in achieving resilience in an engineering system which preserves the ability of a system to function as planned [42].

2.1.1 Health Sensing

Health sensing is performed to ensure high damage detectability by designing an optimal sensor network. Zhang and Srihari [43] studied sensor placement design for SHM under uncertainty. Alsabti, Ranka and Singh [44] considered parametric identification of linear structural systems for optimal location of sensors for efficient health sensing. A Bayesian approach for optimal sensor placement for SHM has been developed by Saxena, and Saad, [45]. Yang, Hwang, Kim, and Chit Tan [46] proposed an optimal sensor location approach for structural damage detection.

2.1.2 Health Reasoning

Health reasoning is employed to extract system health relevant information in real-time and to diagnose system health conditions. In literature, data mining has been utilized of sensor networks for real time applications. Azhar Mahmood, Ke Shi and Shaheen Khatoon [47] proposed a Distributed Data Extraction method which applies clustering and association rule mining techniques. This approach also estimates missing values from available sensory data. Sadagopan and Krishnamachari [48] used a methodology which modeled a maximum data extraction problem as a linear program. The authors then use an iterative approximation algorithm which applies a greedy heuristic based on an exponential metric.

2.1.3 Health Prognosis

It is important for health prognosis techniques to be able to identify the origin of faults, estimate the progression rate and determine the remaining useful life of engineered systems [49]. There are three health prognostic approaches, which are Model-based approach, Data-Driven approach and Hybrid approach.

With the Model-Based approach, Doksum and Hoyland [50] explored the use of Weiner diffusion process to create a degradation model. This approach assumes that one major degradation measure influences failure therefore the process can be modeled using a Weiner diffusion process. Tseng, Hamada and Chiao [51] explored the concept of a degradation path model. This approach fitted the model parameter vector with the measured degradation data. This subsequently allowed for reliability estimation and remaining useful life prediction of components. Meeker and Escobar [52] further explored the lifetime distributions of components by developing an exponential pattern model.

With the Data-Driven approach, Chinnam [53] proposed an artificial neural network model which is utilized for online estimation of component reliability. Another approach by Parker et al [54] utilized polynomial neural network. This enabled the neural network to be trained using vibration sensory data obtained from a helicopter gearbox system.

With the Hybrid approach, Atlas et al [55] proposed a framework for combining the Model-based approach with the Data-driven approach.

2.1.4 Major Challenges

It can be seen from literature that the conjugate and semi-conjugate Bayesian updating models are preferred due to the ease of selecting the prior distributions for the likelihood functions. When this approach is considered, two obvious disadvantages have been identified. Firstly, to guarantee the conjugate property being held, prior distributions for the model parameters have to be specified as particular distribution types. Secondly, the updating mechanism can be applied only once due to the strong correlation of the posterior distribution of the model parameters. This makes it inefficient for real world applications. There are also difficulties that arise due to the application of conjugate and semi-conjugate pairs for large system design models. They include the fact that conjugate and semi-conjugate Bayesian updating are only useful for low dimensional problems while real world system design models are usually high dimensional models. Also, in practical design applications, the design models used are generally implicit without any analytical model formulation which makes it hard to construct the conjugate or semi-conjugate pairs.

CHAPTER 3

RELIABILITY ESTIMATION AND REMAINING LIFE PROGNOSIS

As mentioned previously, the degradation signals can continuously be obtained either through embedded sensor network or online monitoring facilities. To effectively extract the valuable information about the health condition of the monitored components or systems, the generic Bayesian framework employs the Bayesian technique for updating the model parameters with evolving sensory degradation signals. This subsection gives a brief introduction of the Bayesian updating technique.

3.1 Reliability Estimation

For engineered systems, system failure events could occur if system performance function goes beyond its failure thresholds. Consequently, a limit state function, denoted as $G(x) = 0$, can be defined which separates the safe and failure events in the random input space. For static reliability analysis, the probability of failure can be defined as

$$P_f = P(G(\mathbf{x}) > 0) = \int \cdots \int_{G(\mathbf{x})>0} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \quad (1)$$

where $f_x(x)$ is the joint probability density function. However, the performance function is also governed by the time-variant uncertainties such as loading conditions and component deterioration. Thus, a time-variant limit state function can be generally derived as $G(\mathbf{x}, t) = 0$ by taking the time parameter t into account in reliability analysis. Let t_l be the designed system life time of interest, the probability of failure within $[0, t_l]$ can be described as [19-21]

$$P_f\left(0,\ t_l\right) = P\left(\exists t \in \left[0, t_l\right], G\left(\mathrm{x}, t\right) > 0\right) \quad (2)$$

Thus, the task of time-variant reliability analysis is to estimate the P_f in an efficient and accurate manner. Although there are two types of time-variant reliability analysis approaches in the literature, namely extreme performance based approaches [19-23], and the first passage based approaches [24-26], in this study, the time-variant uncertainties can be considered by evolvingly updating the system design model parameters with the post design stage system performance measurements. Thus, existing time independent reliability analysis approaches, such as the Monte Carlo Simulation (MCS) and First- or Second-Order Reliability Method (FORM/SORM) [27-29] can be conveniently used.

3.2 Remaining Life Prognosis

Remaining life prognosis aims to realize prediction of system failures considering system performance degradation over time with the use of operating state sensory signals so that the proactive failure M/R decision making can be made. In the literature, three primary degradation modeling techniques have been developed: (i) a model-based approach [30, 31], (ii) a data-driven approach [32-34], and (iii) a hybrid approach [35-36]. Remaining life prognosis has been successful, in part, in lowering system maintenance costs. However, these modeling techniques are generally dependent on empirical system degradation models without leveraging existing system design models at the design phase.

To employ system design models for remaining life prognosis at the post design stage, Bayesian updating technique can be employed. As shown in Fig. 1, a generic Bayesian framework for system health management generally contains three critical steps: (1) updating system design model parameters based on the prior information and a new sensory degradation signal; (2) updating the lifetime distribution; (3) updating the reliability distribution. The framework is illustrated in Table 1. Note that Step 2 to Step 4 can be repeated until the lifetime distribution can be continuously updated with the evolving degradation signals. Table 1 details the procedure of employing Bayesian inference for remaining life prognosis.

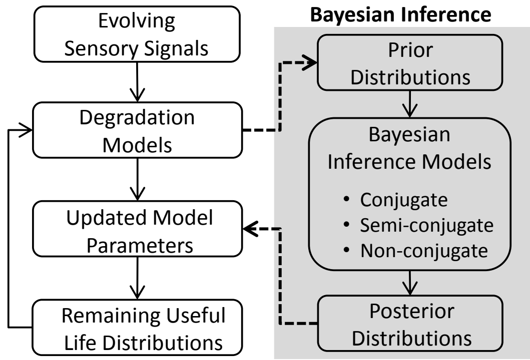

Figure 1: A Generic Bayesian Prognosis Framework

Table 1: Procedure of the Bayesian Prognosis

STEP1	Selecting an appropriate system design models and specifying model parameters considering design uncertainties represented as the prior distributions;
STEP2	Building a likelihood function with the prior distributions and new sensory signal
STEP3	Updating the joint probability distributions of the design model parameters with a prior model information and degradation signals by non-conjugate Bayesian updating technique
STEP4	Estimate the reliability and updating the remaining life distribution based on the updated design model parameters

CHAPTER 4

EVOLVING DESIGN MODEL SYNCHRONIZATION

As mentioned previously, the degradation signals can continuously be obtained either through embedded sensor network or online monitoring facilities. To effectively extract the valuable information about the health condition of the monitored components or systems, the generic Bayesian framework employs the Bayesian technique for updating system design model parameters with evolving sensory degradation signals. This section details the EDMS framework for prognosis employing the Bayesian updating technique.

4.1 A Generic Bayesian Information Model

This subsection gives a brief introduction of the Bayesian updating technique. Let X be design model parameters, a set of random variables with probability density function $f(x, \theta)$, $\theta \in \Omega$. According to the Bayesian point of view, θ is interpreted as a realization of a random variable Θ with a probability density $f_\Theta(\theta)$. The density function expresses what one thinks about the occurring frequency of Θ before any future observation of X is taken, that is, a prior distribution. Based on the Bayes' theorem, the posterior distribution of Θ given a new observation X can be expressed as

$$f_{\Theta|X}(\theta \mid x) = \frac{f_{X,\Theta}(x,\theta)}{f_X(x)} = \frac{f_{X|\Theta}(x \mid \theta) \cdot f_\Theta(\theta)}{f_X(x)} \qquad (3)$$

The Bayesian approach is used for updating information about the parameter θ. First, a prior distribution of Θ must be assigned before any future observation of X is taken. Then, the prior distribution of Θ is updated to the posterior distribution as the new data for X is employed. The posterior distribution is set to a new prior distribution and this process can be repeated with evolution of data sets.

To update system design model parameters using the Bayesian updating technique, the likelihood function $f_{X|\Theta}(x|\theta)$, which combines a new degradation signal with the prior information of model parameters, is quite essential. However, for algebraic convenience, existing researches mostly focus on seeking the conjugate or semi-conjugate models [37, 38]. Conjugate models of Bayesian updating are quite valuable for uncertainty modeling with continuously evolving signals due to its closed form of the posterior distribution. However, only limited conjugate or semi-conjugate models are available and, thus, updating results strongly depend on selection of the models. Although in literature, the conjugate or semi-conjugate Bayesian updating models are preferred and largely studied by carefully selecting the prior distributions for specific likelihood functions. There are two obvious drawbacks of this approach. First, prior distributions for the model parameters must be specified as certain distribution types, for example, lognormal distribution and normal distributions, to guarantee the conjugate property to be held. This specification of prior distributions will substantially limit the application of this approach. Second, since the posterior distribution of the model parameters have strong correlation, the updating mechanism can only be applied once and thus cannot be applied for real-time applications. In addition, it is however almost impossible to apply the conjugate or semi-conjugate pairs for a large system design model. The difficulties are usually because two fundamental reasons: 1) system design models are usually high dimensional models, but conjugate or semi-conjugate pairs in Bayesian updating are only for low dimensional problems only; (2) in practical design applications, the design models used are generally implicit without analytical model formulation, thus constructing the conjugate or semi-conjugate pairs are generally impossible. To make it generic to update the degradation models, different prior distributions may be preferred for different model parameters. In such situations, non-conjugate Bayesian updating framework is more desirable because it can update parameter distributions for any given prior distributions.

To overcome such difficulty, non-conjugate Bayesian updating framework must be developed but its computation is quite complicated. A Laplace approximation method will be developed in this study and used for the non-conjugate Bayesian updating procedure, which will be introduced in the later subsection.

4.2 Laplace Approximation of Bayesian Posterior Distribution

This subsection details the Laplace approximation method for efficient Bayesian posterior distribution approximation. Let **x** be the design model parameter and G represent the given performance data G_1, $G_2 \ldots G_k$, the Bayesian inference model as introduced in the subsection 3.1 can be rewritten as:

$$p(\mathrm{x} \mid L) \propto p(L \mid \mathrm{x}) \cdot p(\mathrm{x}) \quad (4)$$

where $p(\mathbf{x})$ is the prior distribution, $p(L|\mathbf{x})$ is the likelihood function as shown in Eq.(3), and $p(\mathbf{x}|L)$ is the posterior distribution. For the non-conjugate Bayesian inference models, analytical forms for the posterior distributions of unknown variables **x** may not be available, thus different approximation techniques are needed for the posterior distribution estimation. Laplace approximation is one of these methods that can be used to estimate the posterior distribution in Bayesian inference. By using Laplace approximation, the posterior distribution can be considered as a multivariate normal distribution [39, 40]. The following of this subsection details the Laplace approximation method for posterior distribution estimation. Considering the natural logarithm of the posterior distribution, let us expand this natural logarithm to the second order

Taylor series at a point x^* as

$$\begin{aligned} \ln p(\mathrm{x} \mid L) &= \ln p(\mathrm{x}^* \mid L) + \Delta\mathrm{x}^T \cdot J(\mathrm{x}^* \mid L) \\ &\quad + \frac{1}{2}\Delta\mathrm{x}^T \cdot H(\mathrm{x}^* | L) \cdot \Delta\mathrm{x} + O(\mathrm{x}^* \mid L) \\ where\ J(\mathrm{x}^* \mid L) &= \nabla \ln p(\mathrm{x}^* \mid L) \\ H(\mathrm{x}^* \mid L) &= \nabla^2 \ln p(\mathrm{x}^* \mid L) \end{aligned} \tag{5}$$

where J(.) and H (.) are the Jacobian matrix and Hessian matrix of *ln* $p(\mathbf{x}^* \mid L)$ evaluated at x^* respectively. *O* (.) stands for higher order terms in the Taylor series expansion. With the intention of balance the computation cost and approximation accuracy, the higher order terms *O*(.) could be ignored which lead to

$$\begin{aligned} \ln p(\mathrm{x} \mid L) &\approx \ln p(\mathrm{x}^* \mid L) \\ &\quad + \Delta\mathrm{x}^T \cdot J(\mathrm{x}^* \mid L) \\ &\quad + \frac{1}{2!}\Delta\mathrm{x}^T \cdot H(\mathrm{x}^* \mid L) \cdot \Delta\mathrm{x} \end{aligned} \tag{6}$$

If we let the expansion point $\mathbf{x}^*$ to be a local maximum of the ln $p(\mathbf{x} \mid L)$, the Jacobian matrix J(.) will be vanished. Thus, we can obtain a further simplified Taylor series of ln $p(\mathbf{x} \mid L)$ as

$$\ln p(\mathrm{x} \mid L) \approx \ln p(\mathrm{x}^* \mid L) + \frac{1}{2!}\Delta\mathrm{x}^T \cdot H(\mathrm{x}^* \mid L) \cdot \Delta\mathrm{x} \tag{7}$$

After exponentiation of the Taylor series in Eq. (7), the posterior distribution $p\ (x|L)$ can be transformed to

$$\begin{aligned} p(\mathrm{x} \mid L) &= \exp(\ln p(\mathrm{x} \mid L)) \\ &\approx p(x^* \mid L) \cdot \exp\left\{\frac{1}{2}\Delta\mathrm{x}^T \cdot H(\mathrm{x}^* \mid L) \cdot \Delta\mathrm{x}\right\} \\ &\propto \exp\left\{-\frac{1}{2}\Delta\mathrm{x}^T \cdot \Sigma^{-1} \cdot \Delta\mathrm{x}\right\} \end{aligned} \tag{8}$$

where

$$\Sigma = [-H(\mathrm{x}^* \mid L)]^{-1} \qquad (9)$$

From Eq. (8), the posterior distribution can be approximated with a multivariate Gaussian distribution with a mean value vector x^* and a covariance matrix of $[-H\ (x^*|L)]^{-1}$. According to Eq. (8), the normalizing constant for the posterior distribution can be approximated as

$$\begin{aligned} Z &= \int \exp\left\{\ln p(\mathrm{x} \mid L)\right\} d\mathrm{x} \\ &\sim p(\mathrm{x}^* \mid L) \int \exp\left\{-\frac{1}{2} \Delta\mathrm{x}^T \Sigma^{-1} \Delta\mathrm{x}\right\} d\mathrm{x} \end{aligned} \qquad (10)$$

Taking advantage of the fact that integration of a multivariate normal distribution over the entire variable space equals to 1, the approximation of the normalizing constant Z can be further simplified as

$$\begin{aligned} Z &\approx p(\mathrm{x}^* \mid L) \int \exp\left\{-\frac{1}{2} \Delta\mathrm{x}^T \Sigma^{-1} \Delta\mathrm{x}\right\} d\mathrm{x} \\ &= p(\mathrm{x}^* \mid L) \sqrt{(2\pi)^n \mid \Sigma^{-1} \mid} \end{aligned} \qquad (11)$$

where n is the dimension of the unknown variables $\mathbf{x}$, which equals to 3 here. $|\Sigma^{-1}|$ is the determinant of Σ^{-1}. With the normalizing constant Z, the posterior distribution $p(\mathbf{x}^*|L)$ can now be obtained as a multivariate normal distribution

$$p(\mathrm{x} \mid L) \approx \frac{1}{\sqrt{(2\pi)^n \left|\Sigma^{-1}\right|}} \exp\left\{-\frac{1}{2} \Delta\mathrm{x}^T \Sigma^{-1} \Delta\mathrm{x}\right\} \qquad (12)$$

Equation (12) is a multivariate normal distribution, with a mean value vector $\mathbf{x}^*$ and a covariance matrix Σ. Table 2 below summarizes the procedure of using the Laplace approximation method to estimate the posterior distribution in Bayesian updating.

Table 2: Procedure of the EDMS Framework Using Laplace Approximation for Bayesian Updating

STEP 1:	Assign prior distributions of the design model parameters based on initial designs;
STEP 2:	Construct a likelihood function involving the system performance measurements for the joint posterior distributions for the design model parameters;
STEP 3:	Employ Laplace approximation to determine the approximated joint posterior distributions.
STEP 4:	Estimate reliability and remaining life of the system using the updated design model parameters.

CHAPTER 5

CASE STUDIES

In this chapter, a mathematical problem and an implicit design model are used to demonstrate the developed EDMS health management framework for reliability estimation and remaining life prognosis employing system design models.

5.1 Mathematical Problem Description

In this case study, a design model is considered with two model parameters, expressed as:

$$G_m(\mathrm{x}) = 1 - \frac{(x_1 + x_2 - 5)^2}{30} - \frac{(x_1 - x_2 - 12)^2}{120} \qquad (13)$$

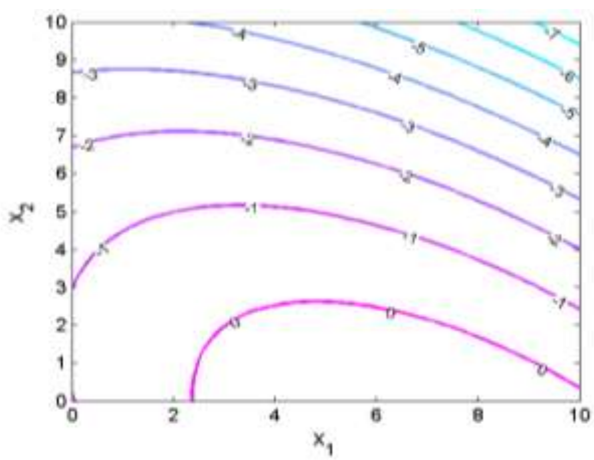

Figure 2: Contour of the Performance Function G_m with the Design Model in System Design Space

where G_m represents system performance function for given design **x** based on the design model while $G_m > 0$ indicates system failure. **x** =[x_1, x_2] is a vector of random parameters for the design model. A contour of system performance function is provided in Fig. 2.

In the rest of this section, this design model will be used to demonstrate the developed

EDMS framework which incorporates the existing knowledge of the design model and new measurements at the operating stage to produce accurate timely system models for reliability analysis and remaining useful life prognosis.

5.2 Procedure of the EDMS Framework

This subsection details the procedure of the EDMS framework with the Laplace approximation technique, as outlined in Table 2, using the design model explained in subsection 4.1. The four steps will be explained.

Step 1: Prior distributions are required to be specified for the design model parameters $\mathbf{x} = [x_1, x_2]$. The prior distributions for the design model parameters can usually be obtained at the system design stage by considering the design uncertainties. In this case study, 10 design points as listed in Table 3 are considered for the demonstration purpose for the design model as given in the Subsection 5.1.

Table 3: The Prior Distributions of the Design Model for 10 Design Points

Des.	Para.	Mean	STD	Des.	Para.	Mean	STD
1	x_1	2.8	0.5	6	x_1	2.8	0.4
	x_2	3.1	0.5		x_2	3.1	0.5
2	x_1	2.9	0.5	7	x_1	2.9	0.5
	x_2	3.1	0.5		x_2	3.1	0.4
3	x_1	3.0	0.5	8	x_1	3.0	0.4
	x_2	3.1	0.5		x_2	3.1	0.4
4	x_1	3.1	0.5	9	x_1	3.1	0.4
	x_2	3.1	0.5		x_2	3.1	0.6
5	x_1	3.2	0.5	10	x_1	3.2	0.6
	x_2	3.2	0.5		x_2	3.2	0.6

Step 2: Likelihood function must be built with the design model and system performance measurements in order to employ the EDMS framework for design model parameter updating. It

is assumed that the system performance measurement G at the operating stage follows the prediction of system design model G_m with the prediction noise. Thus, for the presented case study, the following relationship holds:

$$\begin{aligned} G &= G_m(\mathrm{x},\ t) + \varepsilon \\ &= 1 - \frac{(x_1 + x_2 - 5)^2}{30} - \frac{(x_1 - x_2 - 12)^2}{120} + \varepsilon \end{aligned} \tag{14}$$

where ε represents the random prediction error term which follows normal distribution with zero mean and σ^2 deviation. Suppose a new performance measurement, $G_i = G(t_i)$, be observed as G_1, G_2, …, G_k at times t_1, t_2, …, t_k. Since the error terms, $\varepsilon(t_i)$, i= 1, 2, …, k, are iid normal random variables, the following likelihood function can be obtained for given observations as

$$\begin{aligned} & f\left(G_1, G_2, \cdots, G_k \mid X\right) = \\ & \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^k \cdot e^{-\sum_{i=1}^{k}\left(\frac{\left(G_i - 1 + \frac{(x_1+x_2-5)^2}{30} + \frac{(x_1-x_2-12)^2}{120}\right)^2}{2\sigma^2}\right)} \end{aligned} \tag{15}$$

Step 3: In this step, the Laplace approximation technique will be used to conduct Bayesian updating of the design model parameters **x** based on the likelihood function and the prior distributions. If the prior distributions for **x** are provided, for example, $\pi_0(X)$, the joint posterior distribution for design model parameters $\boldsymbol{X}$ can be expressed as

$$\begin{aligned} & f\left(\mathrm{x} \mid G_1, G_2, \cdots, G_k\right) \sim \\ & \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^k \cdot e^{-\sum_{i=1}^{k}\left(\frac{\left(G_i - 1 + \frac{(x_1+x_2-5)^2}{30} + \frac{(x_1-x_2-12)^2}{120}\right)^2}{2\sigma^2}\right)} \times \pi_0(\mathrm{x}) \end{aligned} \tag{16}$$

As shown in Eq.(16), the posterior joint distribution of the design model parameters **x** depends on the selection of the prior distributions $\pi_0(\mathbf{x})$ and also the post design stage measurements of system performance function G. Following the steps of the Laplace approximation as explained in Subsection 4.2, a multivariate Gaussian distribution will be obtained for the design model parameters by approximating the joint posterior distribution at the local maximum point, as explained early.

Step 4: With the updated model design model parameter distributions, the reliability estimation and the remaining life prognosis can be conducted.

At the post design stage, the reliability of the system with updated design model parameters can be calculated as

$$R = P(G(\mathbf{x}) < 0) = \int \cdots \int_{G(\mathbf{x})<0} f_{\mathbf{x}}^{k}(\mathbf{x}) d\mathbf{x} \qquad (17)$$

In this study, the MCS reliability analysis method is employed to estimate the reliability with the evolvingly updated system design model parameters.

Through continuous updating of system design model parameters, the performance degradation as indicated by the design model parameter changes can be used for failure prognosis of a particular system design. The remaining life distribution information can be obtained when a reliability threshold R_t is provided for the decrease of system reliability over time. By plugging N sets of parameters data extracted from the model parameters' posterior distributions, the reliability degradation over time can be modeled with using a regression model, $R_d(t)$, and the remaining life data can be determined by projecting the reliability degradation model to the critical failure threshold R_t as

$$\begin{aligned} &T_i = \text{roots}\left(R_d(t_i) - R_t = 0\right) - t_i, \\ &where\ i = 1, 2, ..., N. \end{aligned} \qquad (18)$$

where T_i indicates the remaining life data for the i^{th} sample point and t_i represents the current time for the prediction for the i^{th} sample point. Note that the life T_{i} must be the positive real root of the equation as shown in Eq. (18). The sample size, N, can be increased because solving Eq. (18) N times is trivial. The life data, T_1, T_2, …, T_{N}, can be used for creating the life distribution. As system performance measurements can be obtained over time, the remaining life distribution can be updated in real-time by repeating Step 2 to Step 4.

5.3 Mathematical Model Case Study Results

This subsection shows the case study results for the design model as given in Subsection 4.1 using the developed EDMS framework.

Starting with the prior distribution as provided in Table 3, the EDMS employs the Laplace approximation technique to evolvingly update the distribution of the design model parameters $\mathbf{x}$ =[x_1, x_2] based on the system performance measurements at the operating stage. As an example, figure 4 shows the joint distribution of the updated design model parameters for the first design point [2.8, 31] with standard deviation of 0.5 for both x_1 and x_2. In this updating process, a total number of 6 measurements, G = [-0.12, -0.15, -0.2, 0.1, 0.16, 0.2] are used. As can be seen from the figure, the initial uncorrelated random design model parameters become correlated after the updating process. The histograms of the system performance function, G_m, based on the design model before and after the model synchronization process are shown in Figs. (5) and (6), respectively. It can be observed from the figures that the performance function has shifted over time and the model prediction variation has also been reduced with the inclusion of the performance measurement points.

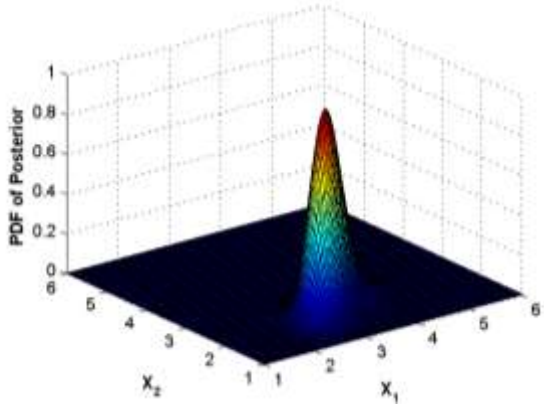

Figure 3: Updated Distribution of the Design Model Parameters **x**

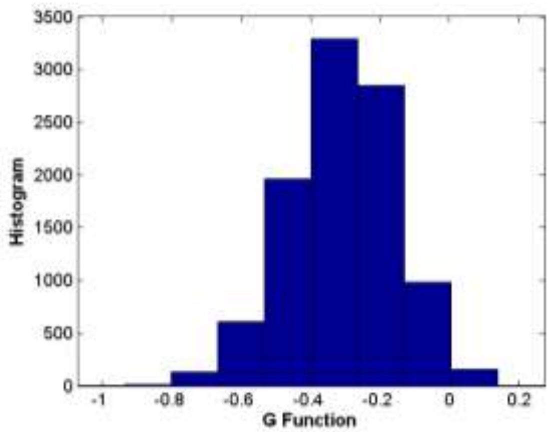

Figure 4: Histogram of G_m before Synchronization

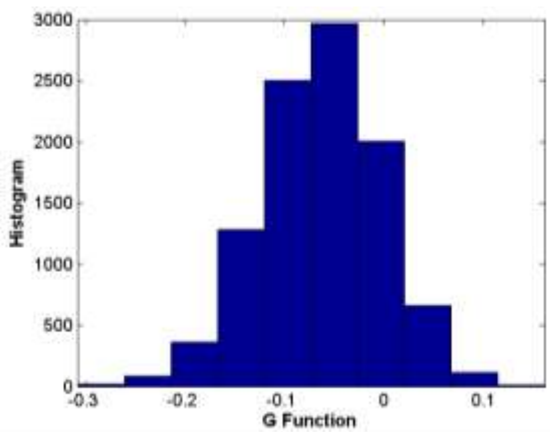

Figure 5: Histogram of G_m after Synchronization

Following the four steps discussed in Subsection 4.2, the reliability estimation for the ten selected design points after employing the EDMS framework with six measurements of the performance function G is listed in Table 4. As can be seen from the table, the measurements of system performance function at the post design stage would bring valuable information of system health conditions and change the initial design model parameters significantly with the evolving performance degradation, as indicated by the joint posterior distribution's mean and the covariance matrix compared with the prior distributions. Consequently, the reliability estimations have been significantly changed from the initial design. It is also shown that the prior distributions of the design model parameters have a notable impact on the reliability estimation after the synchronization process, as the number of measurements G used is small. With the EDMS framework using the Laplace approximation technique, although initial design model parameters are provided differently for different design points, the reliability estimations based on updated design model parameters tend to converge, and with more measurements being available, the impact of design model parameters' prior distributions on the reliability estimation will be reduced.

Table 4: Reliability Estimation Results for the Case Study

Des.	Mean	COV		Reliability	
				Initial	After
1	3.24	0.1415	0.0606	0.9852	0.8546
	2.39	0.0606	0.0797		
2	3.3	0.1481	0.0602	0.9795	0.8453
	2.42	0.0602	0.0772		
3	3.36	0.1552	0.0608	0.9759	0.8315
	2.44	0.0608	0.0764		
4	3.43	0.1605	0.0594	0.9664	0.8171
	2.47	0.0594	0.0722		
5	3.51	0.1718	0.0574	0.9735	0.8418
	2.52	0.0574	0.0702		
6	3.14	0.0946	0.0457	0.986	0.8764
	2.34	0.0457	0.0755		
7	3.41	0.1422	0.0500	0.9904	0.869
	2.47	0.0500	0.0518		
8	3.36	0.1124	0.0407	0.9906	0.8713
	2.44	0.0407	0.0484		

Table 4 (continued)

Des.	Mean	COV		Reliability	
				Initial	After
9	3.29 2.4	0.1122 0.0508	0.0508 0.0925	0.9502	0.8154
10	3.51 2.52	0.2549 0.0827	0.0827 0.0987	0.9497	0.8076

As discussed in the previous subsection, the performance degradation as indicated by the design model parameter changes can be used for failure prognosis, through continuous updating of system design model parameters. The remaining life distribution can be obtained by setting a reliability threshold R_t.

In this case study, R_t = 0.6 has been used and the performance measurements are assumed to obtained over time following an exponential degradation pattern, characterized by

$$G = G_0 + \frac{1}{20} e^{0.02t} + \varepsilon \qquad (19)$$

where e is the measurement noise following standard Gaussian distribution with zero mean and stand deviation of 0.02. For the demonstration purpose, one performance measurement is extracted per unit of time from Eq. (19) and one design model synchronization process will utilize five of these measurements. Take the first design point as an example, Fig. 6 shows five random realizations of the measurements for the first design point, and correspondingly Fig. 7 shows the reliability estimations based on the updated design model over time. Considering the uncertainties involved in the remaining life prognosis process, a total number of 500 independent runs have been executed to obtain important statistical information of the predicted remaining life results. Figure 8 shows the histograms of the predicted remaining lives at two different times for the first design point using the developed EDMS approach with evolvingly updated design model parameters, in which an exponential regression function has been used to fit the reliability decrease over time. As shown in the figure, the predicted remaining life distribution will gradually reduce the prediction variance as the remaining life approaches zero.

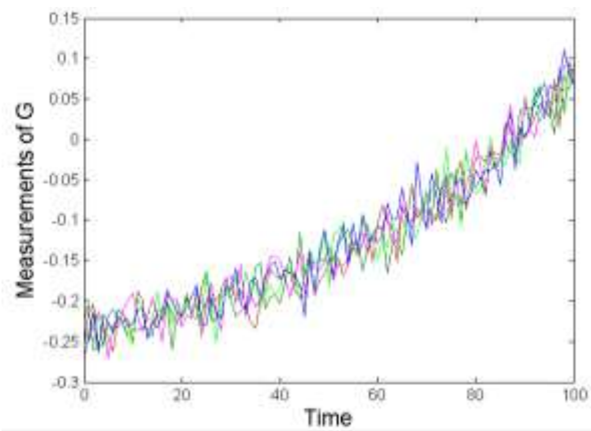

Figure 6: Random Realization of Measurements over Time for the Performance Function

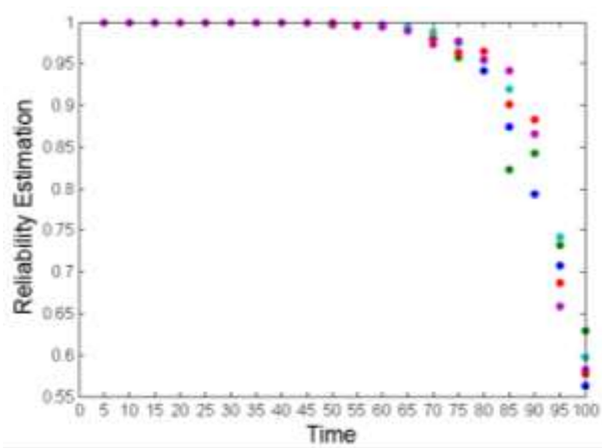

Figure 7: Reliability Estimation Based on Five Measurements over Time

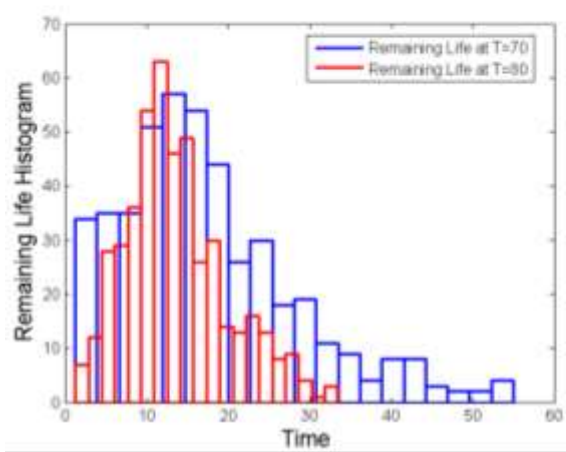

Figure 8: Remaining Life Prognosis at Two Different Times

5.4 Application of EDMS Framework for Wing Panel Displacement Prognosis

This subsection demonstrates the application of the proposed EDMS framework to an implicit design model by predicting the remaining useful life of a rectangular wing panel through finite element analysis (FEA). As shown in Figure 9, the panel is 10m in length and 5m in width and fastened by four rivet joints (L1 – L4). With the joints fixed, there is a harmonic force F with a frequency of 120Hz applied in the middle of the panel. Data from FEA is used as representative sensory information.

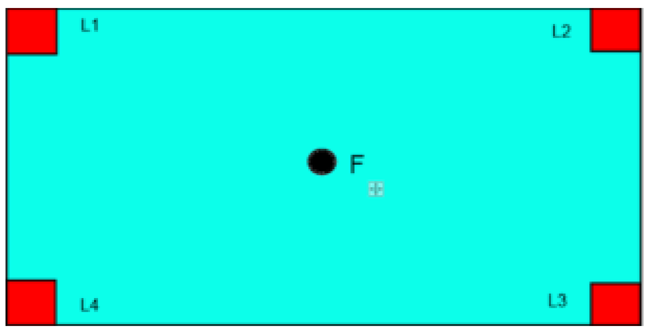

Figure 9: Rectangular Wing Panel with Indicated Rivet Joints

In this study, the joint loosening was realized by reducing the Young's Modulus of the joint itself. The Young's Modulus of the joints are assumed to be obtained from an exponential degrading pattern characterized by

$$YM = 7E10e^{-0.062t} \quad (20)$$

For the purpose of demonstrating the proposed framework, the four joints are considered to loosen at the same rate. The uncertainties in this case study are modeled as random parameters with corresponding statistical distributions listed in Table 5.

Table 5: Random Property of the Wing Panel

Random Variable	Randomness (cm,g,degree)
Young's modulus of wing panel	N(2e11,$1e4^2$)
Young's modulus of loosening joints	N(YM,$4e8^2$)

By analyzing the sensory output, the displacement information of critical areas of the wing panel is extracted to update the finite element model.

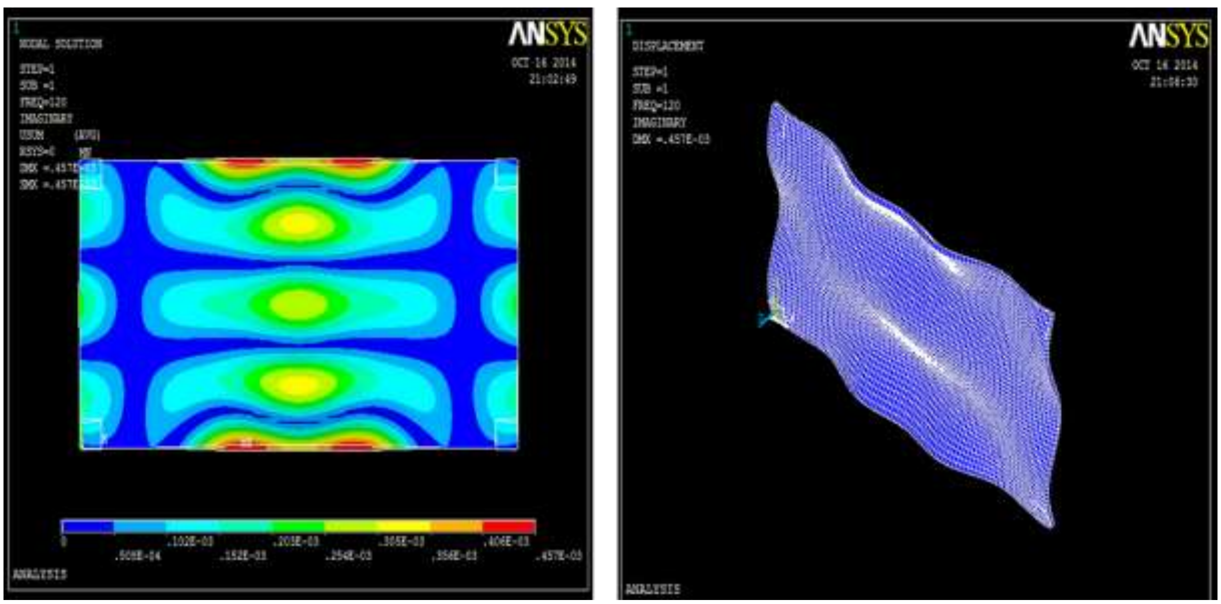

Figure 10: Displacement Contour of the Rectangular Wing Panel

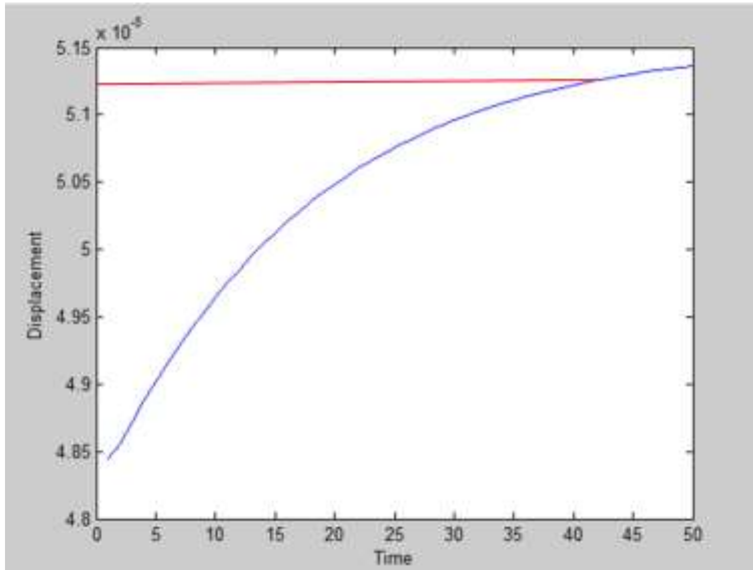

Figure 11: Displacement Degradation over Time

5.5 Implicit Design Model Case Study Results

This subsection shows the case study results for the design model as given in Subsection 4.1 using the developed EDMS framework. The EDMS framework applies the Laplace approximation technique to update the distribution of the design model parameters based on system displacement information obtained from the FEM at the operating stage. For the model updating procedure, the following observed displacement measurements are used, 5.05260E-05, 5.07240E-05, 5.08820E-05, 5.13950E-05, 5.14230E-05, and 5.14450E-05.

For carrying out prognosis of the wing panel, the proposed EDMS framework is applied. The failure event is defined as the displacement of the specified region above 5.1256E-06m as shown in Figure 11. The design model parameters obtained from the updating procedure at time 20 and 30 is listed in Table 6

Table 6: Updated Design Model Parameters

Prognosis Time	Mean	Covariance
20	200000000047.36	0.0486 5.9410
	21641999769.58	5.9140 729.0775
30	199999999999.89	0.924 2.4587
	11633000002.55	2.4587 653.0281

The remaining life distribution is obtained for this case study by setting the reliability threshold to 0.7. Based on the updated design model over time, the reliability estimation is shown in Figure 12. To obtain a statistically significant prediction of the remaining useful life, 500 independent runs were implemented to make up for uncertainties in prediction. Figure 13 shows the histogram of the remaining useful lives at time 20 and 30.

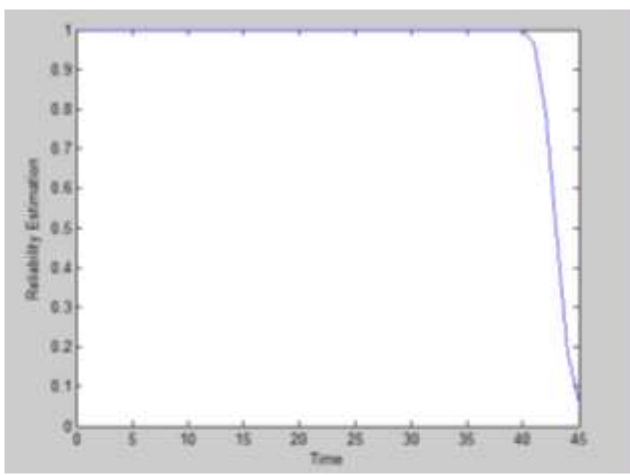

Figure 12: Reliability Degradation over Time

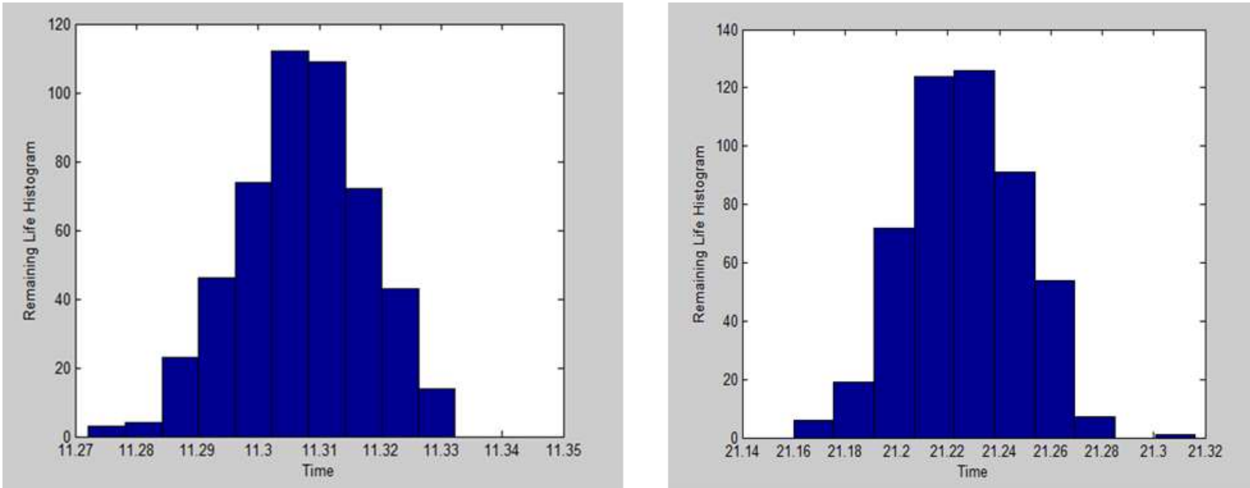

Figure 13: Remaining Life Distribution at Two Different Prognosis Times

From this result, it is noticed that the predicted remaining life distribution for the selected prognosis time reduces the prediction variance as the remaining life approaches zero. This is shown in Table 7.

Table 7: Mean and Standard Deviation of Actual Life

Time	EDMS		True RUL
	Mean	STD	
20	21.0220	0.0220	22
30	11.3071	0.0103	12

CHAPTER 6

CONCLUSION

This thesis presents a new evolving design model synchronization platform for operational stage system health management to estimate reliability and predict the remaining life of a system considering system performance degradation over time. The developed methodology enables health management of aging engineered systems by efficiently synchronizing system design models with degrading health conditions of actual physical system in operation. A Laplace approximation approach has been employed for the design model updating, which can incorporate heterogeneous operating stage performance measurements to update distributions of system design model parameters, thereby increases the updating accuracy compared with traditionally used Bayesian updating methodology. The design models synchronized over time using system performance measurements acquired from the system in operation can thus reflect system health degradation with evolvingly updated design model parameters, which enables the application of failure prognosis for system health management. Case study results demonstrated the feasibility of employing the developed approach for health management by conducting evolving reliability estimation and remaining life prognosis over time.

REFERENCES

REFERENCES

[1]. Resilience, 2009, In Merriam-Webster Online Dictionary. http://www.merriam-webster.com/dictionary/resilience.

[2]. Aven T., 1996, "Condition-based Replacement Policies—a Counting Process Approach," Reliability Engineering and System Safety, v51, p275-281.

[3]. Coolen F.P.A. and Dekker R., 1995, "Analysis of a 2-Phase Model for Optimization of Condition Monitoring Intervals," IEEE Transactions on Reliability, v55, p505-511.

[4]. Christophe Andrieu et al., "An Introduction to MCMC for Machine Learning", 2003

[5]. Bernd A. Berg. "Markov Chain Monte Carlo Simulations and Their Statistical Analysis". Singapore, World Scientific 2004.

[6]. George Casella and Edward I. George. "Explaining the Gibbs sampler". The American Statistician, 46:167–174, 1992. (Basic summary and many references.)

[7]. A.E. Gelfand and A.F.M. Smith. "Sampling-Based Approaches to Calculating Marginal Densities". J. American Statistical Association, 85:398–409, 1990.

[8]. Andrew Gelman, John B. Carlin, Hal S. Stern, and Donald B. Rubin. Bayesian Data Analysis. London: Chapman and Hall. First edition, 1995. (See Chapter 11.)

[9]. S. Geman and D. Geman. "Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images". IEEE Transactions on Pattern Analysis and Machine Intelligence, 6:721–741, 1984.

[10]. Casella, George; George, Edward I. (1992). "Explaining the Gibbs sampler". The American Statistician 46 (3): 167–174. DOI:10.2307/2685208. JSTOR 2685208.

[11]. Gelfand, Alan E.; Smith, Adrian F. M. (1990). "Sampling-Based Approaches to Calculating

[12]. Marginal Densities". Journal of the American Statistical Association 85 (410): 398–409. DOI:10.2307/2289776. JSTOR 2289776. MR 1141740.

[13]. Andrew Gelman, John B. Carlin, Hal S. Stern, and Donald B. Rubin. Bayesian Data Analysis. London: Chapman and Hall. First edition, 1995.

[14]. Adriano Azevedo-Filho and Ross D. Shachter. Laplace's method approximations for probabilistic inference in belief networks with continuous variables. Uncertainty in Arti_cial Intelligence, 1994.

[15]. Christopher Bishop. Pattern Recognition and Machine Learning. Springer, 2007.

[16]. Luke Tierney and Joseph Kadane. Accurate approximations for posterior moments and marginal densities. Journal of the American Statistical Association, 81(393), 1986.

REFERENCES (continued)

[17]. Tierney L, Kadane J. Accurate approximations for posterior moments and marginal densities. Journal of the American Statistical Association 1986; 81:82–6.

[18]. Xuefei Guan , Jingjing He , Ratneshwar Jha, Yongming Liu, "An efficient analytical Bayesian method for reliability and system response updating based on Laplace and inverse first-order reliability computations", Reliability Engineering and System Safety, 97 (2012) 1–13.

[19]. Sai Hung Cheung and James L. Beck , "Bayesian Model Updating Using Hybrid Monte Carlo Simulation with Application to Structural Dynamic Models with Many Uncertain Parameters", DOI: 10.1061/(ASCE)0733-9399(2009)135:4(243).

[20]. Wang, Z., and Wang, P., 2012, "A Nested Extreme Response Surface Approach for Time-Dependent Reliability-Based Design Optimization," *J. Mech. Des.* 134, 121007.

[21]. Wang, Z., and Wang, P., "A New Approach for Reliability Analysis with Time-Variant Performance Characteristics," Reliability Engineering & System Safety, Vol.115, pp. 70-81, 2013.

[22]. Li, J., Chen, J.B., and Fan, W.L., 2007, "The Equivalent Extreme-Value Event and Evaluation of the Structural System Reliability," *Struct. Safety*, 29(2), pp. 112–131.

[23]. Chen, J.-B., and Li, J., 2007, "The Extreme Value Distribution and Dynamic Reliability Analysis of Nonlinear Structures with Uncertain Parameters," *Struct. Safety*, 29(2), pp. 77–93.

[24]. Li, J., and Mourelatos, Z. P., 2009, "Time-Dependent Reliability Estimation for Dynamic Problems Using a Niching Genetic Algorithm," *Journal of Mechanical Design*, 131(7), p. 071009.

[25]. Lutes, L. D., and Sarkani, S., 2009, "Reliability Analysis of Systems Subject to First-Passage Failure," *NASA Technical Report* No. NASA/CR-2009-215782.

[26]. Sudret, B., 2008, "Analytical Derivation of the Outcrossing Rate in Time-Variant Reliability Problems," *Struct. Infrastruct. Eng.*, 4(5), pp. 353–362.

[27]. Hu, Z. and Du, X., 2013, "Time-Dependent Reliability Analysis with Joint Up-crossing Rates," *Structural and Multidisciplinary Optimization,* 2013, DOI: 10.1007/s00158-013-0937-2.

[28]. Youn, B.D., Choi, K. K., and Du, L., 2005, "Enriched Performance Measure Approach (PMA+) for Reliability-Based Design Optimization," AIAA Journal, Vol. 43, No. 4, pp. 874-884.

REFERENCES (continued)

[29]. Youn, Byeng D., Choi, K. K., and Du, L., 2005, “Adaptive Probability Analysis Using An Enhanced Hybrid Mean Value (HMV+) Method,” Journal of Structural and Multidisciplinary Optimization, Vol. 29, No. 2, pp.134-148.

[30]. Wang L.P., Grandhi R.V., 1996, Safety index calculation using intervening variables for structural reliability. Comput. Struct., 59(6):1139−1148.

[31]. Gebraeel N., Lawley M., Li R. and Ryan J.K., 2005, “Residual-Life Distributions from Component Degradation Signals: A Bayesian Approach,” IIE Transactions, v37, p543–557.

[32]. Lu C.J. and Meeker W.Q., 1993, “Using Degradation Measures to Estimate a Time-to-Failure Distribution,” Technometrics, v35, n2.

[33]. Héberger K., Keszler A., and Gude M., 1999. “Principal Component Analysis of Measured Quantities during Degradation of Hydroperoxides in Oxidized Vegetable Oils,” Lipids, Jan., v34, p83-92.

[34]. Yang G., Yang K., 1997, “Performance Degradation Analysis Using Principal Component Method,” Proceedings of Annual Reliability and Maintainability Symposium, p136-141.

[35]. Hu, C., Youn, B.D., and Wang, P., “Ensemble of Data-Driven Prognostic Algorithms for Robust Prediction of Remaining Useful Life”. Reliability Engineering and System Safety, Vol.103, pp. 120–135, 2012.

[36]. Saha, B., Goebel, K., Poll, S., and Christophersen, J., “Prognostics Methods for Battery Health Monitoring Using a Bayesian Framework,” IEEE Trans. on Instrumentation and Measurement, v58, n2, pp291-296, 2009.

[37]. Wang, P., Youn, B.D., and Hu, C., “A Generic Probabilistic Framework for Structural Health Prognostics and Uncertainty Management”, Mechanical Systems and Signal Processing, Vol. 28, pp. 622–637, 2012.

[38]. Doksum, Kjell A., and Arnljot Hbyland., 1992."Models for Variable-Stress Accelerated Life Testing Experiments Based on Wener Processes and the Inverse Gaussian Distribution." Technometrics 34, no. 1: 74-82.

[39]. C. J. Lu., 1995,“Degradation processes and related reliability models,” Ph.D. thesis, McGill University, Montreal, Canada.

[40]. Gregory, Phil., 2005, Bayesian logical data analysis for the physical sciences. Vol. 10. Cambridge, UK: Cambridge University Press.

REFERENCES (continued)

[41]. Guan, Xuefei, Jingjing He, Ratneshwar Jha, and Yongming Liu., 2011 "An efficient analytical Bayesian method for reliability and system response updating based on Laplace and inverse first-order reliability computations." Reliability Engineering & System Safety. Vol. 97, Issue 1, p.1–13, 2012.

[42]. Day, J., Donahue, K., Ingham, M., Kadesch, A., Kennedy, A., & Post, E. (2012, June). Modeling Off-Nominal Behavior in SysML. In Proceedings of the AIAA Infotech@ Aerospace 2012 Conference.

[43]. Johnson, S. B., Gormley, T., Kessler, S., Mott, C., Patterson-Hine, A., Reichard, K., & Scandura Jr, P. (Eds.). (2011). System health management: with aerospace applications. John Wiley & Sons

[44]. Zhang, B., & Srihari, S. N. (2004). Fast k-nearest neighbor classification using cluster-based trees. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 26(4), 525-528.

[45]. Alsabti, K., Ranka, S., & Singh, V. (1997). An efficient k-means clustering algorithm.

[46]. Saxena, A., & Saad, A. (2007). Evolving an artificial neural network classifier for condition monitoring of rotating mechanical systems. Applied Soft Computing,7(1), 441-454.

[47]. Yang, B. S., Hwang, W. W., Kim, D. J., & Chit Tan, A. (2005). Condition classification of small reciprocating compressor for refrigerators using artificial neural networks and support vector machines. Mechanical Systems and Signal Processing, 19(2), 371-390.

[48]. Mahmood, A., Ke, S., & Khatoon, S. (2013). An Efficient Distributed Data Extraction Method for Mining Sensor Networks Data. arXiv preprint arXiv:1306.4069.

[49]. Sadagopan, N., & Krishnamachari, B. (2005). Maximizing data extraction in energy-limited sensor networks. International Journal of Distributed Sensor Networks, 1(1), 123-147.

[50]. Blair, J., & Shirkhodaie, A. (2001, March). Diagnosis and prognosis of bearings using data mining and numerical visualization techniques. In System Theory, 2001. Proceedings of the 33rd Southeastern Symposium on (pp. 395-399). IEEE.

[51]. Doksum, K. A., & Hbyland, A. (1992). Models for Variable-Stress Accelerated Life Testing Experiments Based on Wener Processes and the Inverse Gaussian Distribution. Technometrics, 34(1), 74-82.

[52]. Tseng, S. T., Hamada, M., & Chiao, C. H. (1995). Using degradation data to improve fluorescent lamp reliability. Journal of Quality Technology, 27(4), 363-369.

[53]. Meeker, W. Q., & Escobar, L. A. (1998). Statistical methods for reliability data(Vol. 314). John Wiley & Sons.

REFERENCES (continued)

[54]. Chinnam, R. B. (1999). On-line reliability estimation of individual components, using degradation signals. Reliability, IEEE Transactions on, 48(4), 403-412.

[55]. Parker Jr, B. E., Nigro, T. M., Carley, M. P., Barron, R. L., Ward, D. G., Poor, H. V., ... & DuBois, T. A. (1993, September). Helicopter gearbox diagnostics and prognostics using vibration signature analysis. In Optical Engineering and Photonics in Aerospace Sensing (pp. 531-542). International Society for Optics and Photonics.

[56]. Atlas, L., Bloor, G., Brotherton, T., Howard, L., Jaw, L., Kacprzynski, G., ... & Roemer, M. (2001). An evolvable tri-reasoner ivhm system. In Aerospace Conference, 2001, IEEE Proceedings. (Vol. 6, pp. 3023-3037). IEEE.