OPTIMAL CONTROL OF DISTRIBUTION SYSTEM IN THE PRESENCE OF DISTRIBUTED GENERATORS

A Thesis by

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Bachelor of Science, Azad University of Saveh, 2009

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OPTIMAL CONTROL OF DISTRIBUTION SYSTEM IN THE PRESENCE OF DISTRIBUTED GENERATORS

The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science with a major in Electrical Engineering.

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DEDICATION

To my parents, for their continuous love and support, and my beloved sister
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I would like to express my deepest gratitude to my advisor, Dr. Visvakumar Aravinthan, for his excellent guidance, caring, patience, and support during my master studies.

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ABSTRACT

There have been many studies on large-scale systems to reduce the dimension of power distribution networks to decrease the time of the simulation and analysis. Most of these attempts cannot satisfy the final goal because of some limitations and constraints in their approaches, or they cannot simplify the way of analyzing distribution systems models. The main purpose of this thesis was to develop a new, effective method to reduce the scale of the power distribution system in order to decrease the speed of the simulation and calculations. Also, in this work, a new centralized optimal control method that focuses on the voltage drop correction was applied.

Applications of both methods were tested with the IEEE 13 node test feeder. According to results, the number of nodes and errors between the real and reference values were decreased significantly.
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<tr>
<td>dqo</td>
<td>Direct-quadrature-Zero</td>
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<td>DG</td>
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CHAPTER 1
INTRODUCTION

The smart grid initiative has urged the power system to develop new trends for planning and operation. Smart grid technology requires an infrastructure including communication and control. In the past, data from a network had to be gathered by utility workers who read electric meters and recorded data. Now, the smart grid feature allows the utility to monitor the entire system and adjust or control devices from a central location. It should be noted that, for this purpose, data from the entire network should be processed and analyzed, but the scale of the power network would affect the speed of analysis significantly for large power networks.

Figure 1 shows a typical power system including generator, transmission line, and distribution system. Each component of the system can be studied separately.

![Diagram of a typical power system]

Figure 1. Typical power system

As can be seen in Figure 1, the transmission line delivers generated power to the distribution system, and there are only some power components on the substation side. On the other hand, the distribution system connects the power system to consumers. It is more complex
since many customers are connected to it, and many power components are involved in controlling and protecting the entire power distribution system.

Figure 2, shows a radial power distribution feeder. The radial power distribution system is like a tree, and all customers are connected to one source and controller located at the substation. In a radial distribution system, the amount of voltage drop changes by increasing and decreasing the load, and this can occur at any time. A voltage drop should be measured and then corrected or compensated by voltage control equipment to allow customer’s devices to work normally without interruption. In a radial distribution network, the voltage drop is high for customers located a distance from the substation.

![Figure 2. Typical radial distribution feeder](image)

1.1 **Power Grid and Infrastructure**

Since the power grid has grown quickly due to the increasing number of customers and also the significant evolution of smart devices in the power industry area (like remote control devices), a more reliable power network (power grid) that can respond to systems that change quickly is needed. An effective and quick way to respond to network changes by using twenty-
first century technology is needed, and more studies must be done in this area. Such concerns can be overcome by using smart grid initiatives.

In conventional power distribution systems, some substations supply and deliver power to the rest of the power network. This type of network can feed both small residential or industrial areas with a few nodes and also large areas with many nodes. In the latter network, which is large, considerable power equipment, such as protection devices, voltage control devices, and regulation devices, is needed; therefore, studying the system can be difficult and complicated, and in some cases, the system can exceed thousands of nodes [1]. For this reason, analysis and simulation can take a significantly long period of time [1]. For example, each node has its own characteristics, like the amount of current that enters or leaves it, as well as its voltage, so by increasing the number of nodes, the number of characteristics and order of the system also increases rapidly, thus affecting the speed of calculations.

Since power consumption changes with time, a power system can be modeled as a linear time invariant (LTI) system and can be analyzed using state space analysis [2]. This type of analysis, which is used to incorporate a system’s dynamics into the control architecture, is convenient for multi-input and multi-output systems.

This work focuses on developing a dynamic control to improve distribution-level performance.

1.2 Voltage Control

Electric power utilities try to provide steady and reliable power to consumers to ensure that consumer devices such as motors or electrical equipment work normally and without interruption. “Service drop” happens during the time that current passes through distribution lines to the consumer. Each distribution power line has its own characteristics, such as series
impedance and shunt admittance, which can result in voltage drop and some amount of energy being lost in these elements. In order to accurately model distribution system performance, these characteristics should be considered [9].

Since residential-level electrical appliances operate at a certain voltage range, distribution system operators are required to maintain a voltage level at the service end. ANSI Standard C84.1 [21] governs the consumer-end voltage range that utilities should meet. Figure 3 shows the acceptable voltage range.

![Figure 3. ANSI C84.1 voltage range](image)

Power distribution system characteristics discussed above require a control operation to deal with voltage drop. Customers located at the end of a radial feeder face the highest voltage drop, and when the controller manages to keep this voltage within a range, customers who are
located close to substations may experience an unexpected rise in voltage. Therefore, utility companies need to satisfy both ends of the radial feeder and design an accurate system to model and analyze the system continuously. This should be done by using fast processors, which can have an effect on the cost of electricity [9]. In the United States, all electrical devices use the standard voltage level of 120/240 volts. Because of various distribution line characteristics, some utilities increase this voltage base by 2 volts on the customer side to compensate voltage drop [9].

1.3 Voltage-Drop Correction

There are various ways to control voltage drop: installing regulators in substations, using online transformer tap-changers, employing shunt capacitors, increasing the size of conductors, etc. [9] [10]. With distributed energy resource penetration through the smart grid initiation, the complexity of the power distribution network and control of this system has become more difficult. The amount of loads and the number of distributed generators are increasing, and conventional centralized control is a concern since each node has its own voltage deviation [3]. Figure 4 illustrates the idea of using a smart grid and voltage control. As shown, some sensors, such as smart meters (SMs), measure the voltage and current at each branch and send this recorded data to the control center. In the control center, all received information is processed, and the central controller sends the desired command to the distributed controllers (like distributed generators or DGs) in designated nodes to correct the voltage drop. This means that the controller allows the DGs to change the voltage level at a specified node. All of these operations occur as the result of a combination of the smart grid, smart controller devices, and distributed generators.
Figure 4. Typical distribution feeder in presence of smart grid, smart controllers, and distributed generators

A centralized controller based on a less complex system that can preserve accuracy needs to be developed. Hence, this work aims to address two concerns:

- Reducing the system order to decrease its complexity and decrease the time of simulation and analysis.
- Proposing a new method to optimally control the voltage drop at each node based on a centralized concept by cooperating with distributed generators as distributed controllers.

1.4 Contributions of This Work

The objectives of this work relative to a power distribution system are explained below:

- Develop a new dynamic model for power distribution systems in order to reduce the number of nodes in a power network.
Develop a state space model as a new reduced model.

Design a controller based on the linear quadratic tracking (LQT) method to control voltage in each node of a power network system.

1.5 Outside Scope of This Work

Since this work only focuses on certain principles, such as dynamic modeling and reducing a system’s node and voltage control in power distribution systems, the following tasks are outside the scope of this work:

- Communicating between the control center and the nodes (communication is assumed to be well developed).
- Designing a physical controller in a power system.
- Implementing a controller in a real power system.

1.6 Organization of Thesis

A new method is needed to reduce the number of nodes in a power distribution system in order to decrease the number of calculations and analysis time, thus increasing the speed of communication between the control center and consumers for achieving optimal control. Chapter 1 provides an introduction to this thesis. Chapter 2 explains previous work in this area and reasons for developing the new method. Chapter 3 describes steady-state modeling. Chapter 4 outlines the method for reducing the model’s nodes. Chapter 5 describes the design of the proposed controller. Numerical analysis for the proposed model (reduced) plus the designed controller are given in Chapter 6, and Chapter 7 concludes the thesis. Chapter 8 offers some ideas about possible future research.
CHAPTER 2
LITERATURE REVIEW

2.1 Power Distribution System Reduction Modeling

Modeling transmission system operation and control has matured over the last century. However, due to limited observability, the power distribution system has been operated in a passive mode, with very minimal control. Some of the ideas studied have involved transmission system modeling and control, which can be extended to the distribution system. However, this requires appropriate modeling, because system operating needs are significantly different compared to transmission system needs.

Some methods to reduce system order are direct-quadrature-zero (dqo) transformation [2] and the use of a modified nodal analysis (MNA) equation [1]. One MNA application is in formulating circuit equations to be used in power analysis software, which are easy to use algorithmically on computers [19]-[20].

Baer and Mablekos modeled a balanced three-phase transmission line as a $T$ network, obtained the system state space, converted it to a direct-current (DC) system through dqo transformation, and analyzed the system’s behavior [2]. By doing this, they tried to remove one of the system’s states per each $T$ segment. This method reduced the non-significant states from the entire model. However, one of the limitations of this work is using this model in a larger network. Since this work only analyzed a small part of the power system and did not provide a method to scale it for larger systems, the computation complexity would increase tremendously if applied to a larger system. The dqo method is a mathematical transformation that converts a balanced three-phase transmission line into the DC system in order to simplify the calculation of
the power system. Since a real power system is usually unbalanced, it is difficult to employ the \( -dqo \) transformation.

Other researchers [1] [16] have used the MNA method and applied their proposed method to reduce the network model. They assumed that the general state space framework would not be suitable for a power distribution network and instead used a modified state space framework as the MNA, whereby line segments are divided into separate matrices, and if branches are removed or added, then the size of the matrices will change [1]. In this process, it is possible to obtain a singular matrix, which will result in no feasible solution to the proposed method. Furthermore this method can increase the time of analyzing and simulation since it includes the voltage of all nodes. [1]. In addition, these authors tried to reduce the scale of the system through the state space matrices and not by reducing the power distribution nodes.

2.2 Voltage Drop Correction and Regulation

There are different methods to control voltage in order to regulate it at each node. The conventional method is centralized voltage control, whereby the controller is located at a substation and controls the entire network based on local information using an observer model.

For example, the amount of voltage drop through the system is measured and based on the achieved data, and then the control unit decides to increase or decrease the voltage.

Different methods to control voltage include shunt capacitors (SCs), load tap-changers (LTC), and step voltage regulators (SVRs) [3]-[4]. Capacitor banks can increase the voltage level in a power system. These are installed in parallel to compensate for voltage drop and a correcting power factor [13], and can be controlled online based on the average amount of voltage. An LTC is a voltage control device (autotransformer) that changes the tap of the transformer to keep the voltage in a certain range. It can be controlled manually or automatically. An SVR, a step-type
voltage regulator including an autotransformer with taps in the winding, is primarily located downstream of the network and keeps voltage at a certain level [10]. Voltage/VAr control is another effective way to improve the voltage level by switching capacitors on the load side. These capacitors go on-line during high peak demand.

Some of the above-mentioned methods can be installed in a distribution system downstream near loads that are being called distributed controllers. A combination of centralized and distributed control can improve performance such as service voltage. Senjyu et al. [3] focused on a combination of centralized controllers and distributed controllers to provide optimal control, assuming that the communication infrastructure was well developed.

Another method of voltage control under study is using DGs as distributed controllers. Here, DGs cooperate with other conventional voltage control methods like LTCs or SCs. Malekpour and Pahwa combined DGs as reactive power control with an LTC as a centralized controller [5]. In addition, some researchers have studied a combination of DGs, under load tap changers (ULTCs), and SCs [18].

The original IEEE Standard 1574 limited DGs from supporting voltage control [5]. Due to the large penetration of DGs and the smart grid initiative, this standard was revised in late 2013. There is tremendous motivation for voltage control modeling at the distribution level, such as this one.

Decentralized voltage control is another method of voltage control. Here, distributed controllers use local data to control local customers’ voltage issues. These devices control the voltage independently, and there is no communication between them and the substation.
Recently there have been some studies using DGs as decentralized controllers [22], [23]. But there are some concerns about the reliability of these methods in cooperation with DGs. Also, the effect and reliability of these methods need to be studied.
CHAPTER 3
STATE SPACE MODELING

3.1 Distribution Feeder

A distribution-level feeder usually includes a substation, voltage regulator, distribution transformer, switchgears, lateral lines, and customers. A illustrative diagram of this system is shown in Figure 5. Line segments are connected to each other through network nodes in different configurations such as in a series or junction connections. Also, loads can be at the end or middle of a line or lumped in different parts of the network. A junction configuration means that a single node is connected directly to more than one node.

Figure 5. Power distribution feeder diagram
3.2 Distribution Line Model

A typical distribution line model is illustrated in Figure 6. For example, a phase of a distribution line between bus 4 and bus 5 shown in Figure 5 can be considered as the line model shown in Figure 6. As shown, typically $Z_1$ equals $Z_2$, and their summation is equal to the line impedance. Also, $Z_c$ is the line-to-ground capacitance, which is usually in the middle of the line. $Z$, or line impedance, represents line resistance and inductance ($Z = R + jX_L$). All line elements, such as $R$, $X_L$, and $X_c$, are calculated by using either a data sheet or measurements. The effect of lines on each, such as mutual impedance, needs to be considered in the calculations. In Figure 6, $v_1$ and $v_2$ are input and output voltage, respectively. Also $i_1$, $i_2$, and $i_c$ are the system current that passes through each line element.

![Figure 6. Line segment model as T](image)

3.3 State Space Model

The general state space equation is described as [2]-[6]

$$\dot{x} = Ax + Bu + Gw$$
$$y = Cx + Du$$

(3.1)

where $x$ is the state vector, $u$ is the input vector, $w$ is the system Gaussian noise, $A$ is the state matrix, $B$ is the input matrix, $G$ is the noise matrix, and $C$ is the output matrix.
3.4 System Noise

Noise is part of the power system and is mainly due to measurement errors, communication errors, and processing errors. Typically, power system-level noise can be modeled as Gaussian noise with zero mean [17], which models the change in voltage and current due to small changes in loading conditions, such as Gaussian noise. In order to verify the accuracy of modeling noise, several loading conditions were analyzed.

In this simulation, an unbalanced three-phase load was connected to a one-mile (556 MCM ACSR 26/7) distribution line with load, as shown in Table 1.

TABLE 1
UNBALANCED BASE LOAD

<table>
<thead>
<tr>
<th>Phase-a KVA</th>
<th>Phase-b KVA</th>
<th>Phase-c KVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1124.9 ∠ 26.6</td>
<td>1380.6 ∠ 28.6</td>
<td>1227.7 ∠ 25.3</td>
</tr>
</tbody>
</table>

Loads were increased and decreased by 0.75%, 1.5%, 2%, and 2.5% to test the system’s behavior for different voltage drops (voltage deviations). Also, for each load, different models of load, such as constant power, constant impedance, and constant current, were tested. Then the voltage drop error between the initial and real values for each case was obtained and the data analyzed. Based on these results, shown in Figure 7, the mean of this model was negligible and assumed to be zero. It should be noted that a combination of all system noises was included in the variable part of the controller.
3.5 Single-Phase $T$ Line State Space

As shown previously in Figure 6, using KVL in the right and left loops and KCL in the middle node, equations (3.2), (3.3), and (3.4) can be achieved. Equation (3.2) represents KVL in the left loop of the $T$ line segment:

$$v_1 = \frac{R}{2} i_1 + \frac{L}{2} \frac{di_1}{dt} + v_c + G_1 w$$  \hspace{1cm} (3.2)

Equation (3.3) represents KVL in the right loop of the $T$ line segment:

$$v_c = \frac{R}{2} i_2 + \frac{L}{2} \frac{di_2}{dt} + v_2 + G_2 w$$  \hspace{1cm} (3.3)

By applying KCL in the middle node of the $T$ line model, the result is equation (3.4):

$$i_1 = i_c + i_2$$  \hspace{1cm} (3.4)

Equation (3.4) can be rewritten as

$$C \frac{dv_c}{dt} = i_1 - i_2$$  \hspace{1cm} (3.5)

where the following apply:

$v_1$ and $v_2$ are the voltage of the line to the ground on the left and right side of $T$ circuit, respectively.
$i_1$ and $i_2$ are branch currents, and $i_c$ is the line to ground capacitance current.

$w$ is system Gaussian noise with zero mean

$C$ is the line to ground capacitance

$R$ and $L$ are line resistance and inductance, respectively

$G_1$ and $G_2$ are system Gaussian noise matrix with zero mean

By using equations (3.2), (3.3), and (3.5), the state space matrix can be described as

$$
\frac{d}{dt} \begin{bmatrix}
V_c \\
i_1 \\
i_2
\end{bmatrix} = \begin{bmatrix}
0 & \frac{1}{C} & -\frac{1}{C} \\
-\frac{2}{L} & -\frac{R}{L} & 0 \\
\frac{2}{L} & 0 & -\frac{R}{L}
\end{bmatrix} \times \begin{bmatrix}
V_c \\
i_1 \\
i_2
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & -\frac{2}{L}
\end{bmatrix} \times \begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} + \begin{bmatrix}
0 \\
G_1 \\
G_2
\end{bmatrix} \times [w] 
$$

(3.6)

The line-to-ground capacitance can be neglected in the distribution line. Therefore, equation (3.6) can be modified as

$$
\frac{d}{dt} \begin{bmatrix}
i_1 \\
i_2
\end{bmatrix} = \begin{bmatrix}
-\frac{R}{L} & 0 \\
0 & -\frac{R}{L}
\end{bmatrix} \times \begin{bmatrix}
i_1 \\
i_2
\end{bmatrix} + \begin{bmatrix}
\frac{2}{L} & 0 \\
0 & -\frac{2}{L}
\end{bmatrix} \times \begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} + \begin{bmatrix}
G_1 \\
G_2
\end{bmatrix} \times [w] 
$$

(3.7)

where $i_1$ and $i_2$ represent system states, and $v_1$ and $v_2$ are system inputs.
CHAPTER 4
MODEL REDUCTION METHOD

As shown previously in Figure 5, line segments are connected to each other in different couplings—in parallel, in series, or in a junction configuration—which together form the distribution feeder. One of the objectives in this thesis is to reduce the number of line segments by combining different types of couplings, in other words reducing the model (reducing the number of nodes), which will reduce the number of states. In this work, a method of converting two series $T$ segments (to form a $II$ circuit) is proposed. Also, a new technique of moving the load from the middle of the distribution network to the end side was developed. Both methods are explained in this chapter.

4.1 Line Combination by Converting Two Series $T$ to One $T$

If a distribution feeder is divided into small segments between nodes, then these segments can be combined to make a single line segment. Doing this conversion makes the calculation significantly easier and faster. For example, there are two line segments between bus $B_1$ and $B_5$ shown in Figure 5, and these two line segments are in series, so they can be combined to form a single line segment. To achieve this, the proposed $Pi$-to-$T$ conversion method can be used. Figure 8, shows a line segment that is divided into two identical sections that form the $T$ model.

![T line segment diagram including resistance $R$, inductance $L$, and capacitance $C$](image)

Figure 8. $T$ line segment diagram including resistance $R$, inductance $L$, and capacitance $C$
Figure 9, illustrates two series line segments in the $T$ form. This new formation can be interpreted as a $Pi$ segment. In Figure 9, $R_a$, $L_a$, $R_b$, and $L_b$ represent the characteristics of two separate lines that are in series.

![Diagram of two series T line segments including resistance, inductance, and capacitance](image)

Figure 9. Diagram of two series $T$ line segments including resistance, inductance, and capacitance

The conceptual idea of developing $a$, $b$, $c$, and $d$ parameters of a two-port network [6],[8] is used in this work. The $T$ and $Pi$ circuit and their corresponding two-port network parameters ($a$, $b$, $c$, and $d$) are shown in Figure 10. The matrices in equations (4.1) and (4.2) represent Figures 10(a) and 10(b), respectively.

![Model of distribution line segment: (a) T circuit, (b) Pi circuit](image)

Figure 10. Model of distribution line segment: (a) $T$ circuit, (b) $Pi$ circuit [6]

\[
\begin{bmatrix}
(1 + YZ_1) & (Z_1 + Z_2 + YZ_1Z_2) \\
Y & (1 + YZ_2)
\end{bmatrix} \quad (4.1)
\]

\[
\begin{bmatrix}
(1 + Y_2Z) & Z \\
(Y_1 + Y_2 + Y_1Y_2Z) & (1 + Y_1Z)
\end{bmatrix} \quad (4.2)
\]
where $Z$ and $Y$ are line impedance and admittance, respectively. Also $a$, $b$, $c$, and $d$ in both forms are equal. In other words,

- $a$ equals $(1 + YZ_1)$ and $(1 + YZ_2)$
- $b$ equals $(Z_1 + Z_2 + YZ_1Z_2)$ and $Z$
- $c$ equals $Y$ and $(Y_1 + Y_2 + Y_1Y_2Z)$
- $d$ equals $(1 + YZ_2)$ and $(1 + Y_1Z)$

In the proposed method, two $T$ circuits are in series. By adding the concept of $T$ and $Pi$ matrix equivalents into this method and equating $a$, $b$, $c$, and $d$ parameters of the two circuits, the new converted $T$ circuit parameters can be easily obtained. It should be mentioned that in this case, both line segments are equal in length and material (line characteristics such as $R$, $L$, and $C$). Figure 11 illustrates this concept.

By setting $a$, $b$, $c$, and $d$ equal in equations (4.1) and (4.2), the following relationships can be developed:

$$1 + YZ_1 = 1 + Y_2Z \Rightarrow j\omega C_1(R_1 + j\omega L_1) = j\omega C_2(R + j\omega L)$$

(4.3)

$$1 + YZ_2 = 1 + Y_1Z \Rightarrow j\omega C_2(R_2 + j\omega L_2) = j\omega C_1(R + j\omega L)$$

(4.4)

As mentioned previously, since both line segments are identical, $R_1 = R_2$, then by considering the proposed ideas using equations (4.3) and (4.4), the following relationship can be developed:

$$C(R_1 + R_2 + j\omega(L_1 + L_2)) = (C_1 + C_2)(R) + (C_1 + C_2)(j\omega L)$$

(4.5)
From equation (4.5), it can be concluded that

\[ C_{new} = C = C_1 + C_2 \]  \hspace{1cm} (4.6)

where \( C_{new} \) is the new equivalent line capacitance.

\[ R = R_1 + R_2 \]  \hspace{1cm} (4.7)

\[ R_1 = \frac{R}{2} \quad \text{and} \quad R_2 = \frac{R}{2} \]  \hspace{1cm} (4.8)

\[ L = L_1 + L_2 \]  \hspace{1cm} (4.9)

\[ L_1 = \frac{L}{2} \quad \text{and} \quad L_2 = \frac{L}{2} \]  \hspace{1cm} (4.10)

Based on equations (4.6), (4.8), and (4.10), new equivalent impedances can be written as

\[ R_{1new} = R_1 + \frac{R_1 + R_2}{2} \quad \text{and} \quad R_{2new} = R_2 + \frac{R_1 + R_2}{2} \]  \hspace{1cm} (4.11)

\[ L_{1new} = L_1 + \frac{L_1 + L_2}{2} \quad \text{and} \quad L_{2new} = L_2 + \frac{L_1 + L_2}{2} \]  \hspace{1cm} (4.12)

where \( \omega = 2\pi f \), \( f \) is 60 Hz, \( R_{1new} \) and \( R_{2new} \) are the new equivalent line resistance, and \( L_{1new} \) and \( L_{2new} \) are the new equivalent line inductance.

Accuracy of the proposed model was evaluated. Since the reduced-order model should retain the same voltage at the receiving end as the original model, both models were simulated using a commercial tool, and the percentage error between the steady state voltage of the actual model and the equivalent model was obtained. Table 2 shows the different load amounts. The base load used in this simulation was 102 + j0.16 Ω. Load was increased in steps of 10, 20, and 30 times that of the base load. Results for three different lengths of distribution lines and different loads are shown in Figures 12 to 14. Parameters for the one-mile distribution line are \( R = 0.46\Omega \), \( L = 0.00285 \) H, and \( C = 0.015 \) μF. These results show that the conversion is acceptable since the error is significantly low.
TABLE 2
LOADS AND CORRESPONDING AMOUNTS

<table>
<thead>
<tr>
<th>Load</th>
<th>Ohm (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Load</td>
<td>102 + j0.16</td>
</tr>
<tr>
<td>Load a</td>
<td>1020 + j1.6</td>
</tr>
<tr>
<td>Load b</td>
<td>2040 + j3.2</td>
</tr>
<tr>
<td>Load c</td>
<td>3060 + j4.8</td>
</tr>
</tbody>
</table>

Figure 12. Error in voltage between real and equivalent model for 0.1-mile length

Figure 13. Error in voltage between real and equivalent model for 1-mile length
The accuracy of this method for three different length combinations was examined (Figure 9). In this test, two series $T$ circuits have different lengths (different line characteristics $R$, $L$, and $C$), and the results are shown in Figures 15 and 16. Based on these results, the error in voltage between the actual model and the equivalent model is not significant and therefore validates the proposed model.

Figure 15. Error in voltage between real and equivalent model for combination of 0.1-mile and 1-mile line lengths
Figure 16. Error in voltage between real and equivalent model for combination of 1-mile and 10-mile line lengths

4.2 Junction Nodes

Based on this conversion ($T$ to $Pi$), all possible combinations can result in a single $T$ section until it reaches the junction node, which is connected to other nodes. To preserve the information from each branch, this work does not reduce the order of the junction node.

4.3 Moving Load from One Node to Another

As mentioned previously, in a distribution system, loads can be at the end-user side or in the middle of the distribution system. In some cases, a load exists between two series $T$ circuits, which should be moved to right or left side of this segment.

A new approach to move the load in the middle of the line to the end of the line was developed here. This method creates a new $T$ segment and a new load, and reduces the number of nodes accordingly. Figure 17 shows two different loads as a negative current source connected to a $T$ line model. The objective here is moving the load from the left side of the line to the right side and obtaining a new equivalent line and load (approximate model). It should be noted that by moving the load from the left side, line impedance on both sides will change; this is not the
same as the exact model but rather a ratio of the exact model. In this method, it has been assumed that load voltage on the right side of the line (\(v_3\) in Figure 17) in both cases is constant, which is supported by the argument that the voltage of the furthest consumer is a critical voltage to control.

**Figure 17.** Line segment with two loads (shown as current source \(I_{L1}\) and \(I_{L2}\))

According to Figure 17, the input current \(I\) can be calculated using equations (4.13) to (4.15), by using KCL in the left node (input node) and KVL in the left loop:

\[
I = I_{L1} + I_{L2} + I_c \tag{4.13}
\]

\[
I_c = \frac{V_c}{Z_c} \tag{4.14}
\]

\[
V_c = V - Z_1(I - I_{L1}) \tag{4.15}
\]

Adding equations (4.14) and (4.15) to equation (4.13), yields

\[
\Rightarrow I = I_{L1} + I_{L2} + \frac{V - Z_1(I - I_{L1})}{Z_c} \tag{4.16}
\]

And from equation (4.16), the following can be achieved:

\[
\Rightarrow Z_c I = Z_c I_{L1} + Z_c I_{L2} + V - Z_1(I - I_{L1})
\]

\[
\Rightarrow Z_c I + Z_1 I = Z_c I_{L1} + Z_c I_{L2} + V + Z_1 I_{L1}
\]
\[ V \text{ is the source voltage, } V_c \text{ is the voltage (line to ground) at the capacitance node, } V_3 \text{ is the voltage across load 2 on the right side, } I \text{ is the system's main input current (from source side), } I_{L1} \text{ and } I_{L2} \text{ are load 1 and load 2 current, respectively, } Z_1 \text{ and } Z_2 \text{ are the line impedance, and } Z_c \text{ is the line-to-ground capacitance.} \]

The input current should be equal in both the exact model and the approximate model, which is shown in Figure 18.

![Figure 18. Line segment approximate model](image)

It can be seen that there is one load on the right side, which is the equivalent of both loads in Figure 17. Also, the line impedance on both sides has changed. In the approximate model, both line impedances are represented by the initial impedance multiplied by factor \( x \). By using equations (4.18) and (4.19), \( x \) can be calculated, and based on factor \( x \), modeling of the approximate model can be possible. Equation (4.18) represents the KCL in the input node.

\[
I = I'_{C \text{ new}} + I'_{L \text{ new}} = \frac{V - xZ_1 I}{Z_c} + I'_{L \text{ new}}
\]

\[ \Rightarrow I_{L \text{ new}} = I - \frac{V - xZ_1 I}{Z_c} = \frac{z_{CL} - V + xZ_1 I}{Z_c} \tag{4.18} \]
By using KVL in the right loop and using equation (4.18),

\[ V_3 = V'_C - \chi Z_2 \times I_{new}^L = (V - \chi Z_1 I) - \chi Z_2 \left( \frac{Z \chi l - V + \chi Z_1 I}{Z_c} \right) \]

\[ = \frac{Z_c V - \chi Z_c Z_1 I - \chi Z_2 Z_c I + \chi Z_2 V - \chi^2 Z_2 Z_1 I}{Z_c} \]

\[ \Rightarrow V_3 Z_C = Z_C V - \chi Z_C Z_1 I - \chi Z_2 Z_C I + \chi Z_2 V - \chi^2 Z_2 Z_1 I \]

\[ \Rightarrow \chi^2 Z_2 Z_1 I + \chi (Z_C Z_1 I + Z_2 Z_C I - Z_2 V) + Z_C (V_3 - V) = 0 \] (4.20)

where \( V'_C \) is the voltage on the line to ground capacitance node, \( I_{new}^L \) is the new load’s current, and \( xZ_1 \) and \( xZ_2 \) are line impedance multiplied by factor \( x \). Factor \( x \) can be calculated through the above equations.

### 4.4 Approximate Model Evaluation Results

The voltage error between the exact and the approximate models in the steady state mode obtained through the simulation and results are shown in Figures 19 and 20. In this simulation, the base load on both sides is \((102 + j0.16 \ \Omega)\), and the exact model line’s parameters are \( R = 0.46 \Omega, \ L = 0.00285 \ \text{H}, \) and \( C = 0.015 \ \text{μF} \). According to these results, the model reduction method is feasible and has been applied in this work.

![Figure 19. Error in voltage for different loads in 1-mile distribution line](image-url)
Figure 20. Error in voltage for different loads in 10-mile distribution line
CHAPTER 5
VOLTAGE CONTROL AND CONTROLLER DESIGN

5.1 Voltage Control Purpose and Project Objective

Electric utilities try to provide a steady and reliable voltage to consumers in order to keep consumer devices such as motors and electrical equipment working normally and without interruption [9]. There are different ways to control the voltage drop, such as installing regulators in substations, changing the transformer tap-changer, using shunt capacitors, increasing the size of conductors, etc. [9]-[10]. All of these methods are categorized under the centralized control of the power distribution system.

The goal in this thesis was to optimally control the voltage and current in a radial distribution feeder based on a centralized approach. Therefore, the controller should track a defined reference signal, which is the system’s current for each branch, and keep the real value close to the reference. If \( x \) is the amount of current from the source to the end user and \( r \) is the current as a reference, then the difference between these two is defined as the error. The goal of the controller is to minimize this error \( e = x - r \). Since a dynamic system is being used and the goal is to minimize the error through the optimal control approach, then the linear quadratic tracking method should be used to obtain the desired results. In the LQT method, the voltage of each node is considered the system’s input, the current of each branch is considered the system’s state, and the controller increases or decreases voltage at each node in order to minimize the above-mentioned error. In this thesis, distributed generators are used to control the voltage at each node.
Also in this work, the root mean square (RMS) of a defined sinusoidal current is referenced as $i_{1rms}$, and the system’s real current is $x_{1rms}$. By maintaining currents close to the reference value, it is expected that the voltage can be kept close to the expected value.

### 5.2 Designing the Controller

As mentioned previously, the objective here was to design a controller to track a reference signal, and this was achieved by using the LQT method [11], which is described in the following equations.

The cost function or performance index should be defined as

$$J = \int_0^\infty (e^T(t)Qe(t) + u^T(t)Ru(t)) \, dt$$ \hspace{1cm} (5.1)

$$\mathcal{H} = \frac{1}{2} e^T(t)Qe(t) + \frac{1}{2} u^T(t)Ru(t) + p^T Ax(t) + p^T Bu(t)$$ \hspace{1cm} (5.2)

where $J$ is the cost function, $e$ is the error between the real value and the reference, $Q$ and $R$ are constant and positive matrixes that reflect each signal’s weight, $\mathcal{H}$ is the Hamiltonian, and $p$ is the costate. Also, $A$, $B$, and $C$ are the state, input, and output matrixes, respectively.

First, the Riccati equation must be solved and the result used in equation (5.3) to determine the feedback gain $G$.

$$A^T K + KA - KB R^{-1} B^T K + C^T Q C = 0$$ \hspace{1cm} (5.3)

where $K$ is the solution of the algebraic Riccati solution. Using $K$ in equation (5.4), $G$ can be obtained as

$$G = R^{-1} B^T K$$ \hspace{1cm} (5.4)

Equation (5.5) is one of the optimality conditions:

$$\dot{p} = - \frac{\partial \mathcal{H}}{\partial x}$$ \hspace{1cm} (5.5)

Equation (5.6) can be written based on equation (5.5), where $r(t)$ is the reference. By using this feedback gain, matrix $A$ changes to matrix $\tilde{A}$ such that $\tilde{A} = A - BG$. 

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Equation (5.6) can be written as a simple derivative as

$$\dot{p} = \frac{dp}{dt} = \frac{f(p + \Delta t) - f(p)}{\Delta t} \Rightarrow f(p) = f(p + \Delta t) - \dot{p} \Delta t$$

(5.7)

By plugging equation (5.7) into equation (5.6), the following new relationship can be obtained:

$$p(t) = p(t + \Delta t) + \left((A - BG)^T p + C^T Q r(t)\right) \Delta t$$

(5.8)

It should be noted that the controller is separated into a fixed part and a variable part. Therefore, the fixed part will be obtained by calculating $G$, and then based on $G$, $p$ can be calculated. Also, the system noise is included in the variable part as Gaussian noise with zero mean.

The boundary condition here is $p(\infty) = 0$, because it is necessary to bring the error to zero at steady state. Then the optimal tracking control is

$$u(t) = -Gx(t) + R^{-1}B^T (p(t) + w(t))$$

(5.9)

where $w(t)$ is the system noise. By applying $u$ to the system’s input (voltages at each node), voltage would be the same as the reference voltage range, and the error $e$ becomes very small.
CHAPTER 6

NUMERICAL ANALYSIS AND VERIFICATION

6.1 Reduction of IEEE 13-Node Test Feeder

The IEEE 13-node test feeder is an unbalanced three-phase distribution system. By applying the proposed methods in this test feeder, the number of nodes can be reduced. This work developed a single-phase controller that could be used separately in each phase. Phase-\(a\) is taken as an illustrative example.

A general IEEE 13-node test feeder is illustrated in Figure 21. Figure 22(a) shows the exact model of phase–\(a\), and Figure 22(b) shows a new reduced model for phase-\(a\) by applying the proposed method. It should be noted that for each phase, new load and new line equivalent impedances are obtained.
6.2 Model Reduction Procedure

As can be seen in Figure 22(a), some loads are on nodes 634, 671, and 652, and a distributed load is between nodes 632 and 671. First, by converting \( P_i \) to \( T \) on the line between nodes 671 and 652, node 684 has been eliminated. Then, by moving the loads from node 671 (equivalent of distributed load and spot load) to node 652, node 671 can be removed. Also, integration of the transformer’s equivalent circuit with the line segment between nodes 632 and 634 can remove node 633. Figure 22(b) shows the final reduced circuit for phase-\( a \).

6.2.1 Phase-\( a \) Model Reduction

Reduction of phase-\( a \) can be described in the following steps:

1. A transformer exists between nodes 633 and 634. This transformer is modeled as an equivalent circuit, which is connected in series with a line between nodes 632 and 633 (Figure 23). Transformers \( R \) and \( X \) are calculated based on IEEE 13-node test feeder data, which is shown Table 3.

<table>
<thead>
<tr>
<th>TABLE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANSFORMER DATA</td>
</tr>
<tr>
<td>kVA</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>XFM -1</td>
</tr>
</tbody>
</table>

By using the given information, the equivalent transformer impedance can be calculated as

\[
Z_{\text{base}} = \frac{v_{\text{base}}^2}{s_{\text{base}}} = 34.1\Omega \tag{6.1}
\]

\[
Z = R + jX = Z_{\text{base}} \times Z_{p.u} = 0.38 + j0.69 \tag{6.2}
\]

2. Line impedance can be calculated by using the given information, as shown in the work of Kersting [15]. For example, line characteristics for the circuit between nodes 632 and
633 are $R = 0.75 \ \Omega/\text{mile}$, $X_L = 1.19 \ \Omega/\text{mile}$, and $B = 5.5 \ \mu\text{S/mile}$, where $R$, $X_L$, and $B$ are impedance, reactance, and susceptance, respectively. In Figure 23, $Z_1$ and $Z_2$ are line impedance, $Z_{c1}$ is the line-to-ground capacitance, and $Z_3$ and $Z_4$ are the transformer equivalent impedance. By adding $Z_2$, $Z_3$, and $Z_4$, a new $Z$ (impedance) can be obtained. By doing this, node 632 can be eliminated (Figure 24).

![Figure 23. Line segment between nodes 632 and 633 and transformer equivalent circuit](image)

![Figure 24. Reduced line segment](image)

3. A spot load exists between nodes 632 and 671, and a load also exists at node 671. By using the proposed method, the spot node is moved to node 671, and a new equivalent load and line impedance are calculated. It should be noted that it has been assumed that this spot load is in the middle of the line (between nodes 632 and 671). Therefore, two series $T$ circuits exist between nodes 632 and 671, as shown in Figure 25(a). Figure 25(b) shows the same line segments between nodes 632 and 671 when the spot load is moved to the right side (node 671), and the new two-series line segments are converted from $Pi$ to $T$. 

33
In Figure 25, $Z_1$ and $Z_2$ are line impedance and are divided by 4 since there is a spot load in the middle, $Z_c$ is the line-to-ground impedance ($Z_c = jX_c$), and $Z_{PiT}$ is the impedance after moving the load and converting.

4. Line segments between nodes 671 and 684 and 652 are connected in series. A load exists on node 671 as calculated above. Also, another load exists on node 684, as shown in Figure 26(a). Then, by using the proposed method, the two-series line can be converted from $Pi$ to $T$. After conversion, the load at node 671 is moved to the right side, and the new equivalent load and line impedance are calculated. Figure 26(b) shows the new equivalent circuit.
removed. After removing node 684, the lines between nodes 632 and 671 and 652 are in series. By converting $Pi$ to $T$, node 671 can be removed (Figure 27).

![Figure 27. Equivalent line segment after removing nodes 671 and 684](image)

6.3 State Space of Reduced Model

The state space of phase-$a$ is shown in equation (6.3) and is based on Figure 28:

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{R_{5032}}{L_{5032}} & 0 & 0 \\ 0 & -\frac{R_{3234}}{L_{3234}} & 0 \\ 0 & 0 & -\frac{R_{3252}}{L_{3252}} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{5032}} & -\frac{1}{L_{5032}} & 0 \\ 0 & \frac{1}{L_{3234}} & -\frac{1}{L_{3234}} \\ 0 & 0 & \frac{1}{L_{3252}} \end{bmatrix} \times \begin{bmatrix} v_{650} \\ v_{632} \\ v_{634} \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \times w \quad (6.3)$$

where $x_1$, $x_2$, and $x_3$ are the current of the system, $v$ is the voltage at different nodes, $R$ and $L$ are the distribution line’s resistance and inductance, respectively, $G$ is the system noise matrix, and $w$ is the system Gaussian noise. In this equation, voltage represents the system’s input and the number of inputs is equal to the number of voltages. Also, the current in this model is the system state. It should be mentioned that the state space of the original model has 15 states before reduction and 3 states after reduction. Stability and controllability of all three phases of this system were tested and verified.
Applying the Controller to Reduced Model

Figure 28 shows the reduced model of the IEEE 13-node test feeder for phase-a. The circuit was tested in this work. In this section, the designed controller was applied to different nodes of the reduced IEEE 13-node feeder. This proposed model was tested using MATLAB software for two continuous 15-minute intervals.

In the first interval (minute 0 to 15), the first reference signal’s $Q$ and $R$ matrices are predefined such that the controller significantly minimizes the error. Then, the RMS of the branch currents are measured (system’s state $x$) and compared to the system’s reference signal (RMS value predefined in system), and the controller decreases the difference between these two values by applying voltage into the system nodes (system’s input $u$). In the next interval (minute 15 to 30), load is increased and the new reference signal is applied to the system to observe the effect of changes on the proposed controller. In this simulation, load is increased by 10, 30, and 50 percent, and the controller minimizes the error between two signals as well (reference and measured or real value).

The values of $R$ and $Q$ (signal’s weight) should be predefined to achieve the best optimal performance for the tracking problem. There are two options for adjusting the reference signal compared to the load changes. One option is to set a fixed reference signal despite the change in
load, and another option is to change the reference signal according to load changes. In the second scenario, both signal’s weight matrices \((R \text{ and } Q)\) should be changed to obtain the desired result. In this work, both cases have studied. Results are discussed later on and shown in Figures 29 to 33 for a fixed reference (constant \(R \text{ and } Q\)) and Figures 34 and 35 for changed references (changed by 30% and 50%).

Table 4 shows the value for \(Q\) and \(R\) when the reference for second interval is fixed despite the change in load. Tables 5 and 6 show the \(Q\) and \(R\) values when the reference is changed according to load. It should be mentioned that \(Q\) and \(R\) are diagonal matrixes, and their dimensions are \(3 \times 3\) and \(4 \times 4\), respectively. In Figures 29 to 35, all three RMS currents (state) were studied to validate the controller operations in each node. In all three cases, the controller kept the error between reference signal and real system currents very small for both intervals.

### TABLE 4

**Q AND R VALUES FOR FIXED REFERENCE SIGNAL**

<table>
<thead>
<tr>
<th></th>
<th>(Q)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q)</td>
<td>(Q_{11} = 0.99)</td>
<td>(Q_{22} = 0.31)</td>
</tr>
<tr>
<td>(R)</td>
<td>(R_{11} = 110)</td>
<td>(R_{22} = 1000)</td>
</tr>
</tbody>
</table>

### TABLE 5

**Q AND R VALUES FOR 30% INCREASE IN LOAD AND REFERENCE SIGNAL**

<table>
<thead>
<tr>
<th></th>
<th>(Q)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q)</td>
<td>(Q_{11} = 1.04)</td>
<td>(Q_{22} = 1.5)</td>
</tr>
<tr>
<td>(R)</td>
<td>(R_{11} = 98)</td>
<td>(R_{22} = 50)</td>
</tr>
</tbody>
</table>

### TABLE 6

**Q AND R VALUES FOR 50% INCREASE IN LOAD AND REFERENCE SIGNAL**

<table>
<thead>
<tr>
<th></th>
<th>(Q)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q)</td>
<td>(Q_{11} = 1.06)</td>
<td>(Q_{22} = 1.6)</td>
</tr>
<tr>
<td>(R)</td>
<td>(R_{11} = 88)</td>
<td>(R_{22} = 100)</td>
</tr>
</tbody>
</table>
In Figure 29, all the three branch currents are shown in separate graphs. Here, the solid blue line is the defined reference signal. From minute 15, the amount of increase is based on system need. As can be seen, the amount of the system’s current without using a controller is a fixed value (dashed red line). In the second time interval, the amount of load increased by 10 percent, and the real value after becoming stable reaches a fixed amount. In both time intervals, the controller forced the system’s current (dashed-dotted grin line) to follow a reference signal to make the error between this value and reference very small.

![Figure 29. Current RMS value for x1,x2,x3 of model without and with controller for 10% increase in load with fixed reference signal](image)

In Figure 30, the load is also increased by 30 percent in the second time interval. In this case, the reference is fixed and is the same as referred to in Figure 29. By applying the desired amount of voltage at each node, the controller, kept the error very small between the system’s current and reference signal.
In Figure 31 load increased by 50 percent. The reference signal is the same as in the two previous experiments. As shown, the amount of current when there is no controller in the system is extremely high (dashed red line) compared to the current when a controller is used in the system.

In Figures 32 and 33, the effects of decrease in load are shown. In both cases, the reference signal is fixed. In these two cases, by decreasing the amount of load, the controlled amount of current is close to the reference signal and shows that the designed controller performs normally.
Figure 31. Current RMS value for x1,x2,x3 of model without and with controller for 50% increase in load with fixed reference signal

Figure 32. Current RMS value for x1,x2,x3 of model without and with controller for 10% decrease in load with fixed reference signal
In Figures 34 and 35, the reference signal is fixed for the first interval and by increasing the load in the second time interval, the reference signal is also increased accordingly. Also, with this change, \( Q \) and \( R \) should be changed in order to achieve the best controller performance. According to these figures, the controller minimized the error, thus achieving the desired goal.
In all cases, the controller changed the system node voltage to minimize the error between the reference signal and the real value. As mentioned previously, this change in each node voltage can be done by using new technology such as distributed generators.
CHAPTER 7
CONCLUSION

In this thesis, a new method to reduce the number of nodes in a power distribution system was introduced, and its performance was analyzed and validated. The effectiveness of the $Pi$-to-$T$ conversion method and moving load from one node to another was studied. Based on the achieved results, most $Pi$ segments can be converted easily to $T$. Also, by combining these two methods, the number of nodes decreased as well as the number of states in the state space analysis, so the system analysis can be done quicker than with conventional methods.

Also, a new approach for optimal voltage control in power distribution at each node using the LQT method was presented in this thesis, and its effectiveness was tested. Both methods were examined by the IEEE 13-node test feeder. Based on these results, the proposed method reduced the system order from 15 to 3 for one phase. Moreover, simulation results showed that by changing the load and changing the reference, the controller increases or decreases voltages in each node to minimize error between the reference signal and the measured value.

As mentioned previously, the controller minimizes the error of the reference signal and the real value by increasing and decreasing voltage at each reduced power distribution node. In addition, the main goal is using new advanced technologies such as distributed generators as a voltage controller in power distribution systems, which can be feasible using the proposed control method.
By increasing the number of smart components in power distribution systems, it will be possible for each controller to be more independent of the central controller, which means that each controller can monitor other nodes and can act based on the entire system’s conditions. This process can be categorized as decentralized control, but because it increases system complexity, it needs more study.

Future studies could be undertaken to find a way to convert the junction nodes to a simple $Pi$ or $T$; therefore, analyzing and modeling the power distribution would be much easier and faster since more nodes could be removed.

Since only phase-$a$ was studied in this thesis, for future studies, the controller effect on two other phase (phase-$b$ and phase-$c$) could be studied to determine if they could be used as feasible controllers in power distribution systems.
REFERENCES
REFERENCES


REFERENCES (continued)


APPENDIX
APPENDIX A

LINEAR QUADRATIC TRACKING PROBLEM

The state space equation is given as
\[
\dot{x} = Ax + Bu + Gw
\]  
(A.1)
where \(x\) is the state vector, \(u\) is the input vector, \(w\) is the system Gaussian noise, \(A\) is the state matrix, \(B\) is the input matrix, \(G\) is the noise matrix, and \(C\) is the output matrix.

The work objective of minimizing the error is
\[
(Cx(t) - r(t)) = e(t)
\]  
(A.2)
where \(r(t)\) is the reference signal, and \(e(t)\) is the error.

The performance index equation can be written as
\[
J = \frac{1}{2} e^T(tf)S e(tf) + \frac{1}{2} \int_0^{tf} (e^T(t)Qe(t) + u^T(t)Ru(t)) \, dt
\]  
(A.3)
and the Hamiltonian equation can be written as
\[
H = \frac{1}{2} e^TQe + \frac{1}{2} u^TAx + p^TAx + p^TBu
\]  
(A.4)
where \(J\) is the cost function, \(e\) is the error between the real value and the reference, \(Q\) and \(R\) are the constant and positive matrixes, respectively, that reflect each signal’s weight, \(\mathcal{H}\) is the Hamiltonian, \(p\) is the costate, and \(A, B,\) and \(C\) are state, input, and output matrixes, respectively.

The following mathematical equations introduce the optimality condition:

\[
\dot{p} = -\frac{\partial \mathcal{H}}{\partial x}, \quad \dot{x} = \frac{\partial \mathcal{H}}{\partial p}, \quad \frac{\partial \mathcal{H}}{\partial u} = 0
\]  
(A.5)
Optimality conditions are described in the following equations:

\[
\dot{p} = \frac{\partial}{\partial x} \left( \frac{1}{2} (Cx - r)^TQ(Cx - r) + p^T Ax \right) = -C^TQCx + C^TQr - A^Tp
\]  
(A.5)
\[
\dot{x} = Ax + Bu
\]  
(A.6)
\[
0 = Ru + B^Tp \Rightarrow u = -R^{-1}B^Tp
\]  
(A.7)
For solving $\dot{p}$, $K$ is defined such that

$$p(t) = K(t)x(t) \quad (A.8)$$

Therefore, $u$ can be written as

$$u = -R^{-1}B^T p = -R^{-1}B^T Kx \quad (A.9)$$

The derivation of equation (A.8) is

$$\dot{p} = \dot{K}x + \dot{x}K \quad (A.10)$$

Adding equation (A.6) into equation (A.10) yields

$$\dot{p} = \dot{K}x + K(Ax + Bu) \quad (A.11)$$

Equation (A.11) can be written in a new form by adding equation (A.7):

$$\dot{p} = \dot{K}x + KA - KB\dot{R}^{-1}B^T Kx \quad (A.12)$$

By equating equations (A.5) and (A.12), the Riccati equation is shown as

$$-C^T Qx + C^T Qr - A^T Kx = \dot{K}x + KA - KB\dot{R}^{-1}B^T Kx$$

$$-\dot{K} = A^T K + KA - KB\dot{R}^{-1}B^T K + C^T QC \quad (A.13)$$

**Special Case ($t_f \rightarrow \infty$)**

In this case (infinite), the linear quadratic equation from the finite case can be written as

$$J = \int_0^\infty (e^T(t)Qe(t) + u^T(t)Ru(t)) \, dt \quad (A.14)$$

In equation (A.13), if $\dot{K} = 0$, then

$$A^T K + KA - KB\dot{R}^{-1}B^T K + C^T QC = 0 \quad (A.15)$$

where $K$ is the solution of the algebraic Riccati equation. By using $K$ from equation (A.15), $G$ can be obtained as

$$G = R^{-1}B^T K \quad (A.16)$$

Therefore, the optimal control input can be written as

$$u(t) = -Gx(t) + R^{-1}B^T p(t) \quad (A.17)$$