OPTIMIZATION OF A FIDUCIAL VOLUME FOR A 10 KILOTON WATER CERENKOV DETECTOR FOR GEO-NEUTRINOS

A Thesis by

Blake John Cicenas

B.A., Marshall University, 2005

Submitted to the Department of Earth, Environmental and Physical Sciences
and the faculty of the Graduate School of
Wichita State University
in partial fulfillment of
the requirements for the degree of
Master of Science

December 2013

OPTIMIZATION OF A FIDUCIAL VOLUME FOR A 10 KILOTON WATER CERENKOV DETECTOR FOR GEO-NEUTRINOS

The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science

____

Nickolas Solomey Committee Chair

____

Tom DeLillo, Committee Member

____

Jason Ferguson, Committee Member

____

Holger Meyer, Committee Member

____

William Parcell, Committee Member

AKNOWLEDGEMENTS

I would like to give a special thanks to Dr. Nickolas Solomey and Dr. Holger Meyer for their constant patience and help. Also thanks to Dr. Ferguson, Dr. Will Parcell and Dr. Tom DeLillo for their time, patience and expertise.

ABSTRACT

A fiducial volume is crucial in particle physics when trying to choose the shape and size of a particle detector. The fiducial volume is defined as the volume at which a specified number of energy events are to be accepted. Fiducial volumes are impacted by aspects such as size and geometry. The fiducial volume in this study is optimized to contain the highest number of events generated from geo-neutrinos for the Hawaii Anti-Neutrino Observatory Project (HANOHANO). Geo-neutrinos are defined as anti-neutrinos coming from the earth either through radioactive decay or from a hypothetical nuclear reactor (geo-reactor). Five different volume types were tested and each type was able to contain at least 98% of anti-neutrino events. This study will demonstrate that an elliptical-cylinder is the best fiducial volume geometry for the HANOHANO project.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (continued)

Figure Page

Introduction

The HANOHANO project intends to build a 10 kT anti-neutrino detector. This detector, unlike other neutrino-detectors, will be portable. This allows the detector to be used for astro-particle research, geo-neutrino studies or other neutrino research.

For this fiducial study, the Cerenkov radiator liquid is water with a density of 1000 kg/m^3. Photomultiplier tubes will be placed inside of the detector and it will be filled with water and placed inside of a typhoon class submarine with dimensions of (at their minimums) 170 m in length, a beam length of 23 m and a draught of 12 m [1]. (See Figure 1.) The HANOHANO project initially intended to place the detector on an oil tanker, however, this vessel presents a more viable option since it is cheaper and is the largest submarine that has ever been deployed. The submarine is capable of deployment around the world to a depth of 400 m but would need to surface in order to travel through the Panama Canal or the locks of the Volgograd Seaway. This is important because it allows access to the Caspian Sea for nuclear proliferation purposes.

Monte Carlo simulations were performed a using C++ program that would simulate an anti-neutrino collision that produces a positron and a neutron. The positron energy spectrum was examined for each fiducial geometric shape to see the impact on the total number of positron path lengths located inside of the volume. While the simulation program Geant was considered for this study, it was not capable of neutrino interactions with Geant 4 version 96, the latest release. All charts and graphs were created using Gnuplot scripts that display the locations of ejected fiducial positrons. Additionally, the surface areas of each of the geometries were approximated for a cost analysis. All of the geometric volumes were designed to maximize free space inside of the submarine. However, unless submarine modifications are made the sphere will need to have a smaller diameter in order to fit inside of the vessel. Through the results of the simulations and calculations, this study will demonstrate than an elliptical-shaped cylinder will provide the best geometry for the detector.

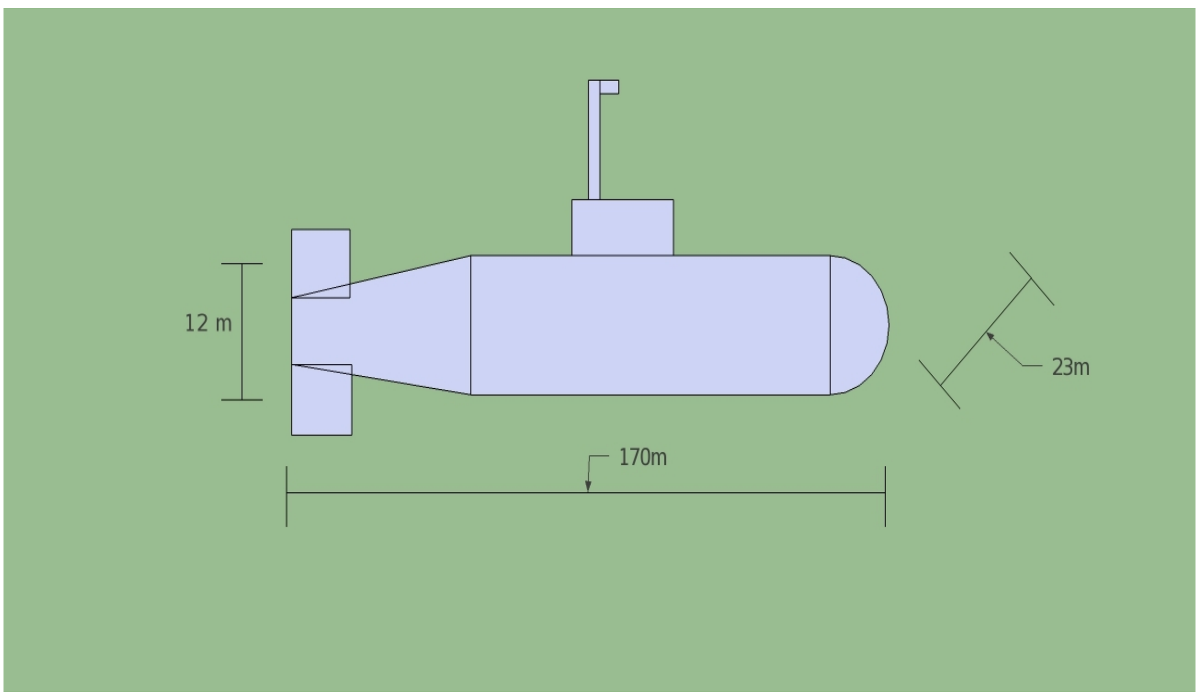

Figure 1: Dimensions of a Typhoon Class Submarine

Geo-neutrinos

Radioactive elements such as Uranium, Thorium and Potassium radiate heat from the mantle and the crust of the earth. Additionally, these radioactive elements emit geo-neutrinos. A geo-neutrino is an anti-neutrino that results from the inverse beta decay of Uranium $U \rightarrow Pb + \alpha + e^{-} + \bar{\nu} + \gamma$, Thorium $Th \rightarrow Pb + \alpha + e^{-} + \bar{\nu} + \gamma$ or Potassium

$K \rightarrow Ca + e^{-} + \bar{\nu} + \gamma$.

Additionally, a geo-neutrino could come from a geo-reactor that is hypothesized to exist at the center of the earth. The current Preliminary Earth Model (PREM) states that the terrestrial heat comes from radioactive decay [3]. Geo-chemists and geophysicists predict that the heat flux from the earth ranges between 14-46 TW [3]. The average measured heat flux for a square meter is 60 mW which indicates that the power output of the earth is 31 TW. While 31 TW falls within the predicted heat flux of 14-46 TW, this does not include gravitational energy, tidal friction or any other forms of thermal radiation. This indicates that more research is needed to better match whole earth heat models to the measured power output.

J. Marvin Herndon proposed a potential solution to this discrepancy in the form of the presence of a geo-reactor at the center of the earth. Scientists are aware that in Earth's past, natural nuclear reactions occurred. In fact, in 1972 a Uranium ore seam in a mine in Gabbon, Africa displayed evidence of past nuclear fission [4]. This is important because it yields proof that nuclear fission can be a natural terrestrial process. Since natural nuclear fission happened in the past, it is also possible that this process could occur now at the earth's core. Additionally, nuclear fission produces ^3He while ^4He is a byproduct of radioactive decay. However, the measured ^3He is much higher than predicted which is a possible indication of a geo-reactor in Earth [5].

The earth's magnetic fields could be explained by the presence of a geo-reactor. The PREM model states that radioactive heat supplies energy to the mantle. The differential heating from the core creates the convection of the conductive mantle and a dynamo is formed. The movement of this large conductive liquid creates the earth's magnetic fields [4]. Herndon proposed that the energy needed to keep the dynamo flowing could be supplied by a 1 TW nuclear reactor located at the core [5]. Additionally, if Earth has a geo-reactor core then other planetary bodies could also have nuclear reactors at their cores which could help explain other planet's magnetic fields [5].

Geo-neutrinos come from many different terrestrial sources. Therefore, a method is needed to isolate those geo-neutrinos resulting from radioactive decay, and those that could be emanating from a geo-reactor. Table 1 shows that the radiogenic heat from the decays of Uranium, Thorium and Potassium have an energy range from 1.31 MeV to 3.26 MeV. A geo-neutrino will need at least 1.8 MeV of energy in order to produce a reaction where a positron and a neutron are the products [6]. The 1.8 MeV threshold is important because the annihilation of the positron and the pairing of the neutron with a proton are the two energy signatures needed to guarantee anti-neutrino detection. Table 1 displays the maximum energies of each of the geo-neutrino reactions. Note that ^{40}K only has a maximum energy of 1.31 MeV. Therefore, the potassium geo-neutrinos will not be detectable with the envisioned HANOHANO detector. Uranium and Thorium both have energies above the 1.8 MeV threshold and will be the only detectable geo-neutrinos that are produced from radioactive decay. This indicates that all other measured events will result from either cosmic anti-neutrinos, commercial reactor neutrinos or potentially from geo-reactor anti-neutrinos.

Table 1. Maximum Energies of Anti-neutrinos (adapted from Fiorentini et al) [4]

Reaction Producing Anti-neutrinos	$T_{1/2}$ (10^9 yr)	E_{max} (MeV)
$^{238}U = {}^{206}Pb + 8\,{}^4He + 6e + 6\bar{\nu}$	4.47	3.26
$^{232}Th = {}^{208}Pb + 6\,{}^4He + 4e + 4\bar{\nu}$	14	2.25
$^{40}K = {}^{40}Ca + e + \bar{\nu}$	1.28	1.31

The first geo-neutrino was recently discovered by the KAMLAND project in Japan [6]. However, KAMLAND is located underground inside the earth's crust and close to many commercial reactors. This is problematic because there is a plethora of background anti-neutrinos that will prove difficult to separate from geo-neutrinos coming from radioactive decay in the earth's crust and reactor geo-neutrinos. Therefore, a detector far away from commercial nuclear reactors and under the surface of the ocean can help minimize the background radiation that could interfere with the optimization of geo-reactor anti-neutrinos. The Russian typhoon class submarine is capable of a depth of 400 m so this should be sufficient to filter out the unwanted background events.

Portability

Every large neutrino detector in the world is located in a stationary position. A unique feature of putting this detector on a submarine is that it can be moved to many different locations on Earth. The detector could be placed offshore near a working commercial nuclear reactor, it could be located in international waters near a rogue nation that is potentially creating nuclear weapons, or it could be located far away from background radiation in the middle of the ocean 400 m below the surface.

Long baseline neutrino oscillations

The portability of the detector can be used for the study of long baseline neutrino oscillations. Neutrinos oscillate between flavors when the pass through a medium (such as Earth). For example, an electron anti-neutrino can change into a muon anti-neutrino over a large distance. All oscillations parameters are well known except for the last mixing angle, θ_{13}. However, θ_{13} was recently found to be non-zero by scientists with the Daya Bay collaboration [7]. Nevertheless, θ_{13} has not been measured precisely and more experimentation is needed. A neutrino or anti-neutrino needs a large distance to oscillate, therefore most scintillation detectors in these experiments are located hundreds of miles away from the source (usually a nuclear reactor). The submarine used in this project is portable which allows the detector to be positioned to receive data at any nuclear reactor that is located within 50 km of the coastline.

Astro-particle research

Many detectors in existence are used for astro-particle research. This detector is different than its previous counterparts due to its size. This 10 kT detector will be bigger than most of the neutrino detectors in use and as a result, will yield more events. Additionally, its portability will allow the detector to read data in the Northern and Southern Hemispheres. This enables one detector to get a complete picture of the night sky, avoiding potential systematic errors if different detectors were used. Furthermore, the depth of the detector can change based on the amount of background radiation that needs to be filtered. For example, low energy physics experiments can have the detector located near the surface and high energy experiments could push the detector down to its 400 m maximum depth.

Nuclear surveillance

The portability of the detector is important with respect to nuclear proliferation. Countries such as Iran and North Korea could pose a nuclear threat to developed nations. Since this detector is located on a submarine, it can be deployed to the various locations of the world to monitor for anti-neutrinos. Reactor anti-neutrinos are extremely energetic (around 6 MeV). When reactor anti-neutrinos are compared to geo-neutrinos, they will create a much larger positron path length. The larger path length of the positron will make it easier to distinguish the reactor anti-neutrinos from the geo-neutrinos. Also of importance is that anti-neutrinos are being continuously produced by nuclear reactions. However, they are difficult to detect because they have a very low energy (in comparison to other particles) and have no charge. Anti-neutrinos are impossible to shield, but are possible to detect with a large neutrino detector. This means that any man-made reactor will produce anti-neutrinos that will be detectable. Furthermore, if the submarine is in motion, the detector could be set at the geo-neutrino maximum energy threshold of 3.26 MeV and a crude anti-neutrino sonogram could be produced. This would offer a clue as to the location of the nuclear point source. In summation, a well placed detector could provide a means to aid developed nations' efforts to patrol their nuclear neighbors.

Background Events

The background noise from man-made and cosmic neutrinos can be minimized if the location of the detector is in close proximity to the earth's mantle. This detector will be placed 400 m below the surface of the ocean off the coast of Hawaii for the HANOHANO project. (Figure 2). While this is still far the mantle, one can note in Figure 3 that the continental crust is much thicker than the oceanic crust. Therefore, the placement of a detector on land has significant background inefficiencies when compared to its oceanic counterpart.

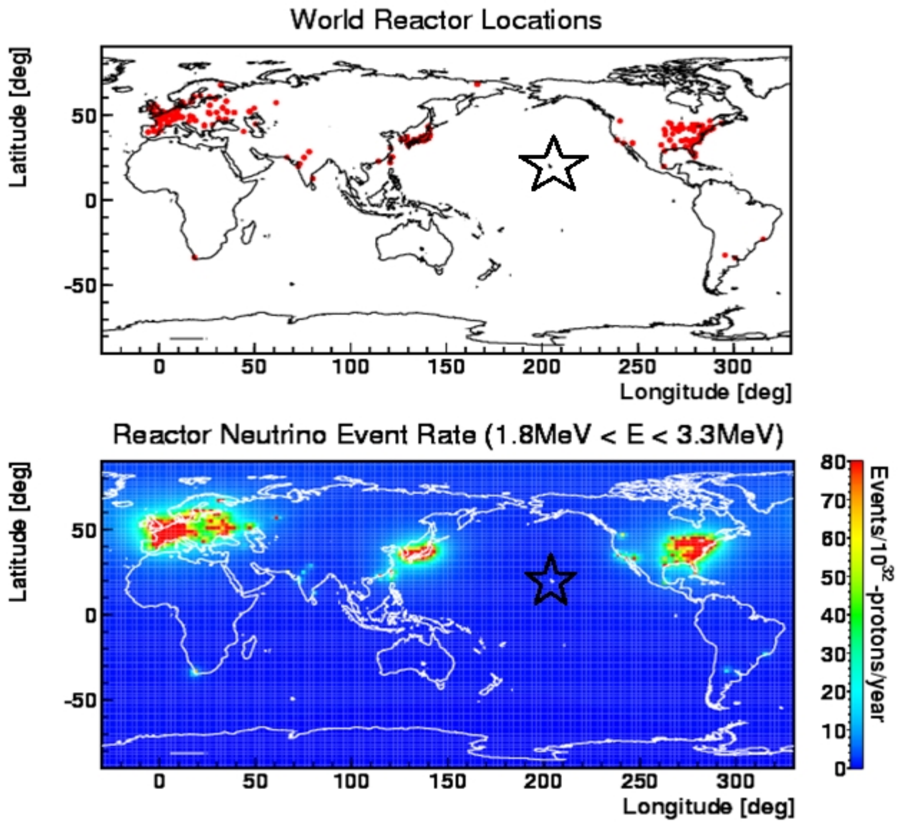

Figure 2: Locations of Reactors Worldwide. The star marks the best location for the detector to search for geo-neutrinos. [9]

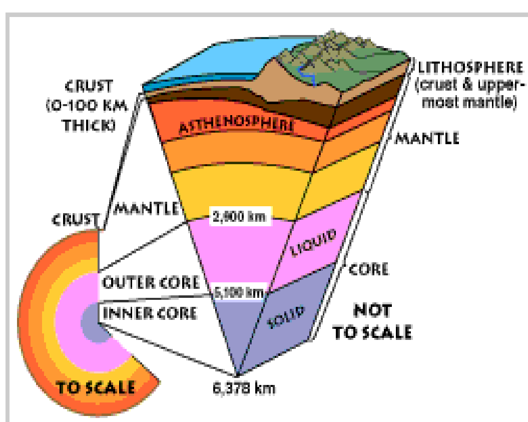

Figure 3 : Earth's Interior [8]

While the abundance of water surrounding the submarine provides protection from background radiation, there will still be non-neutrino events that can play a role in introducing error into experiments. Commercial nuclear reactors are located worldwide but there are some locations that will experience a higher flux of man-made neutrinos. In Figure 2, one can see (noted by the significance of the color red) that areas near Japan, Europe and Eastern North America will experience significantly more background events than locations such as HANOHANO's (marked with a star) [4].

Other background events include astro-particles (such as muons) that can disrupt the detector significantly and could collapse the fiducial volume. The HANOHANO's placement of the detector in the middle of the ocean has many advantages over competing projects when examining background events (Table 2). While the background of Lithium-9, Polonium-210, mantle and accidental events are same, the HANOHANO detector has a much lower number of reactor background events when compared to the land based detectors. Furthermore, the number of crustal geo-neutrinos is much lower due to the thin oceanic crust. One can see that the placement of the detector in the ocean will have a much lower number of overall events. However, Table 2 demonstrates that the events that are detected will be far more likely to be from the mantle.

Table 2: Neutrino Background for 1.8 MeV – 3.4 MeV [5]

		Events (10 kT-yr)$^{-1}$	
	SNO+	Borexino	HANOHANO
^{9}Li	0 ± 0	3 ± 1	3 ± 1
^{210}Po	8 ± 2	8 ± 2	8 ± 2
Accidental	42 ± 1	42 ± 1	42 ± 1
Reactor	528 ± 21	295 ± 12	12 ±1
Crustal Geo-neutrinos	368 ± 74	279 ± 56	31 ± 6
Total Background	946 ± 77	627 ± 57	96 ± 7
Mantle	79	79	79

Why a Fiducial Study?

A fiducial volume is a volume at which a certain fraction (in most cases 90%) of events are able to be measured. When a positron enters the liquid medium it quickly interacts and leaves an energy trail based on the anti-neutrino's initial energy. Most detectors can properly analyze a positron or electron path if 90% of the path is contained within the volume. Therefore, the larger the volume, the more events that can be measured. However, some of the events could be undesirable, (such as a high energy cosmic rays in an anti-neutrino study) so a larger volume also means a larger background to filter. Additionally, the geometry of the volume is significant in determining how many events are able to be measured.

To demonstrate how geometry and size can effect contained events, a simple simulation was created for a cylindrical volume and a rectangular volume each containing the same amount of water and having an identical maximum width.

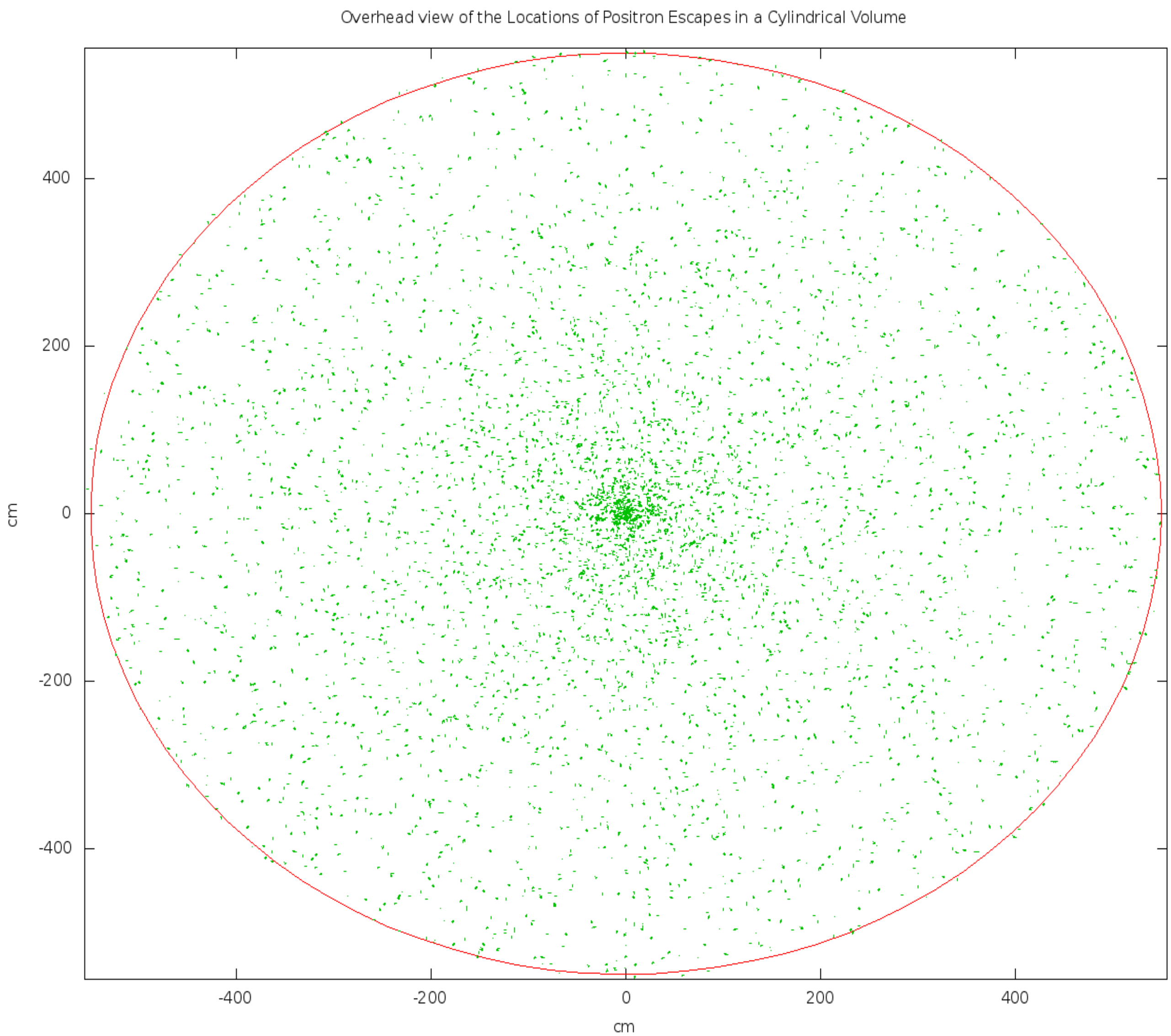

Figure 4: Overhead View of Positron Path Lengths in a Cylindrical Volume

Using C++, a basic simulation was developed that produced random positrons up to 5 cm beyond the volume's edge. Figure 4 shows a two-dimensional overhead view of the positron path lengths in a cylindrical volume. Over 99% of the path lengths of the positrons are contained. However, if the diameter is reduced by a factor of 4, then less than 95% of the positron path lengths are captured as presented in figure 5.

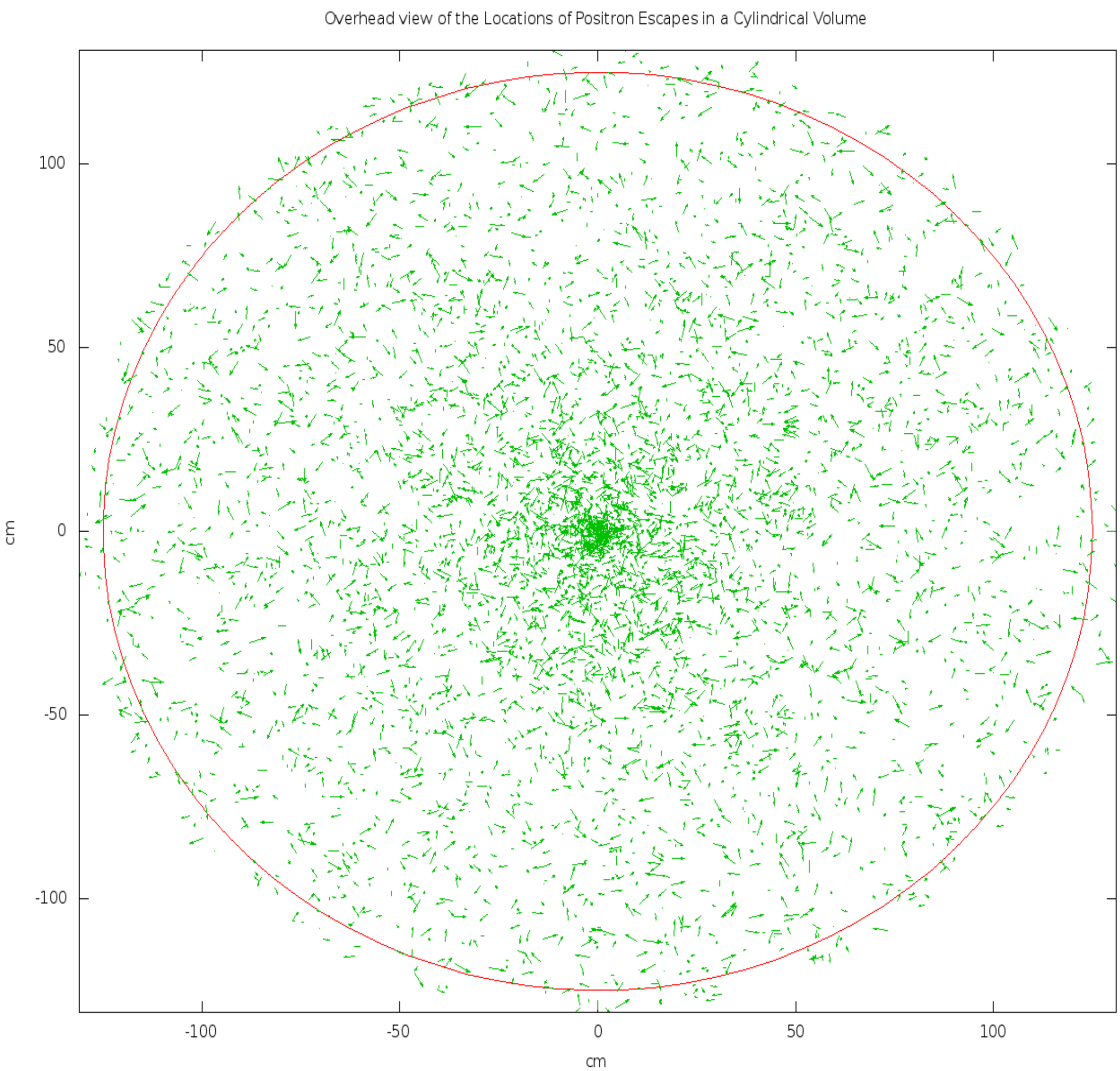

Figure 5: Overhead View of the Positron Path Lengths in a Cylindrical Volume that is ¼ the Diameter of the Original Cylinder. Note that the path lengths become more apparent.

If the volume is further reduced by a factor of 4, then less than 82% of the positron path lengths are captured as shown in Figure 6. The volume is now 1/16th of its original diameter and one can clearly see that while some of the positrons are contained within the volume, others are starting inside the volume and ending outside the volume, or vice versa. Therefore a crucial element to a fiducial study is the size of the detector. The larger the detector, the higher the number of events that can be contained.

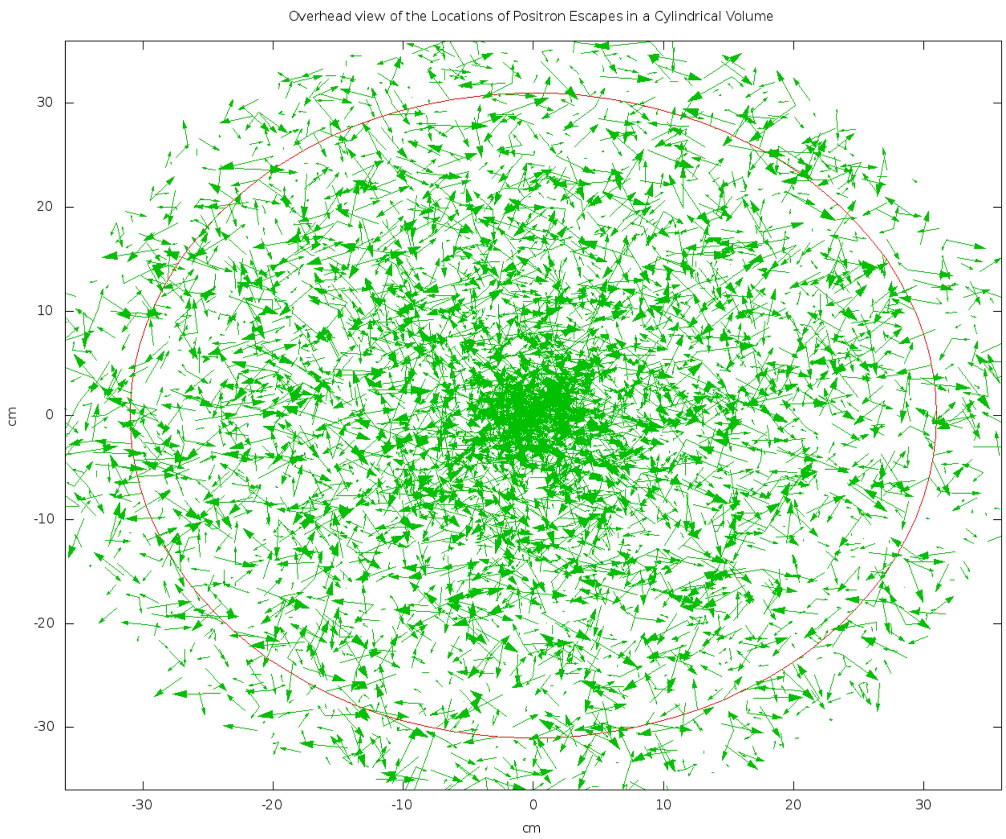

Figure 6: Overhead View of the Positron Path Lengths in a Cylindrical Volume that is 1/16 the Diameter of the Original Volume. Note that even more positrons tails are located in the fiducial region of the volume.

Another important aspect of a fiducial volume is geometry. Using the same volume and the same maximum width as Figure 6, the Monte Carlo simulation is performed for a rectangular volume. With a different shape, only 70% of all possible positron path lengths are able to be kept as valid data points. Figure 7 shows that unlike the circular sections of a cylinder, a rectangle has corners. These edges produce more opportunities for the particles to not be contained within the volume. Therefore, a crucial element in a fiducial study is the presence of curvature.

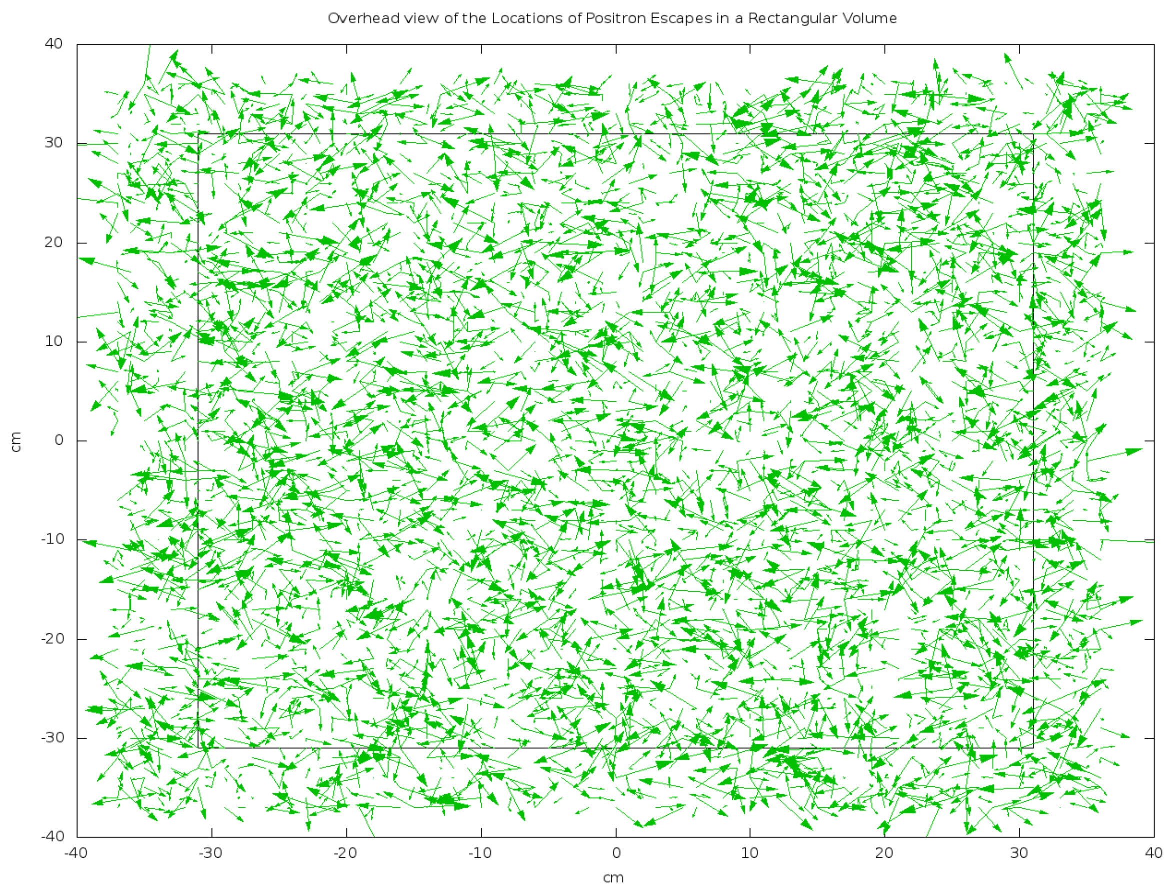

Figure 7: Overhead View of a Rectangular Volume. Note that the corners provide a means for positrons tails to be outside of the volume.

Methodology

There are an enormous amount of geo-neutrinos produced from the Earth. And unlike other particles, they do not interact electromagnetically and they have an extremely small mass. However, given a large enough target and long enough time, they can be tracked by inverse beta decay. An inverse beta decay reaction occurs where an anti-neutrino collides with a proton and a positron and a neutron are produced as products. The positron then travels a specified path length called a continuous slowing down approximation (CSDA) based on the initial anti-neutrino energy. The CSDA ranges were calculated using the ESTAR program from the National Institute of Standards and Technology [10]. While other experiments look for the high energy neutrinos, this study focuses on geo-neutrinos. Geo-neutrinos have a low energy threshold, and the range for detection is between 1.7 MeV and 9.3 MeV. In this simulation, the positron can travel anywhere from 0 to 4.8 cm. These distances are determined by the CSDA ranges of positrons / electrons in water [10].

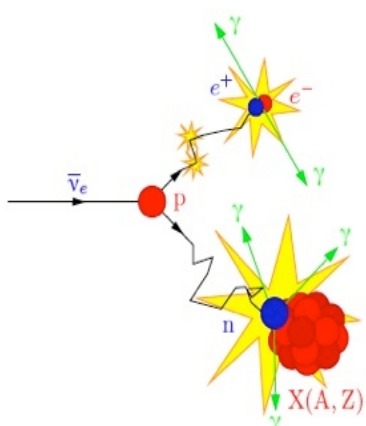

Figure 8: Inverse Beta Decay [11]

The Monte Carlo simulation begins with randomly generating an energy between 1.7 MeV and 9.3 MeV and randomly generating locations up to 5 cm beyond the edge of the volume. This is the energy window for examining geo-neutrinos from the earth [12]. Next, an assumption is made that the geo-neutrinos will collide with a proton to produce a neutron and a positron. The positron will travel a distance based on the initial anti-neutrino energy. Additionally, a random angle is produced to account for the initial hit coming from any direction and coming from any particle. The geo-neutrino can travel up to a maximum distance of 4.8 cm (which corresponds to 9.3 MeV). Next, the geo-neutrino must be matched to the expected energy spectrum (see Figure 8) [12].

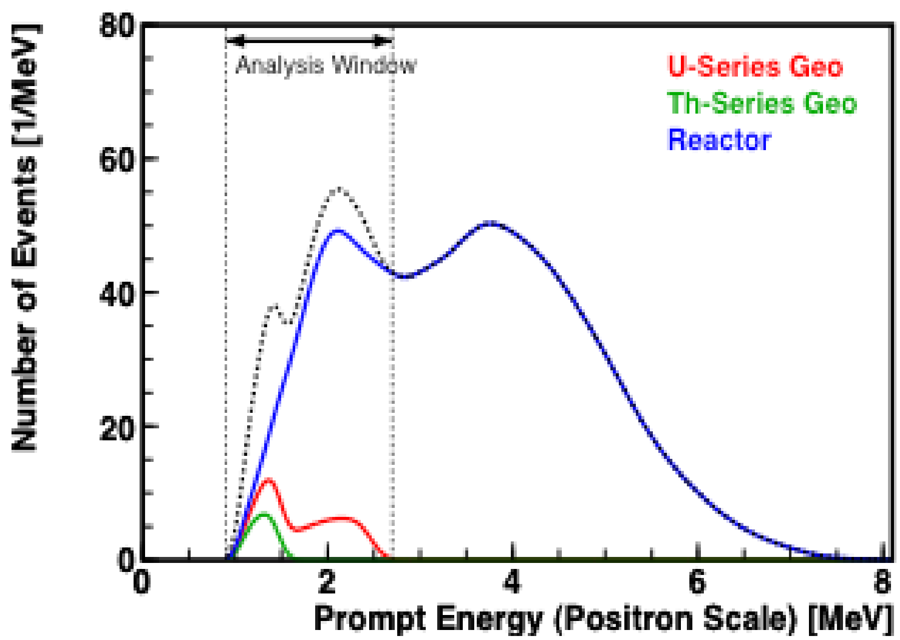

Figure 9: Expected Number of Events/MeV vs Energy of the Positron [12]

Once the geo-neutrino energy was tagged, a positron vector was randomly generated as to simulate a path that the positron would follow. The code for the randomization of a spherical surface is below.

```
mult = ((rand() % 77)*0.1);
double e_energy = (1.7 + mult);
double theta=((rand()%360)*0.01745);
double phi=((rand()%360)*0.01745);
double ptheta=((rand()%360)*0.01745);
double pphi=((rand()%360)*0.01745);
double x = rho*(sin(phi)*cos(theta));
double y = rho*(sin(phi)*sin(theta));
double z = rho*(cos(phi));
```

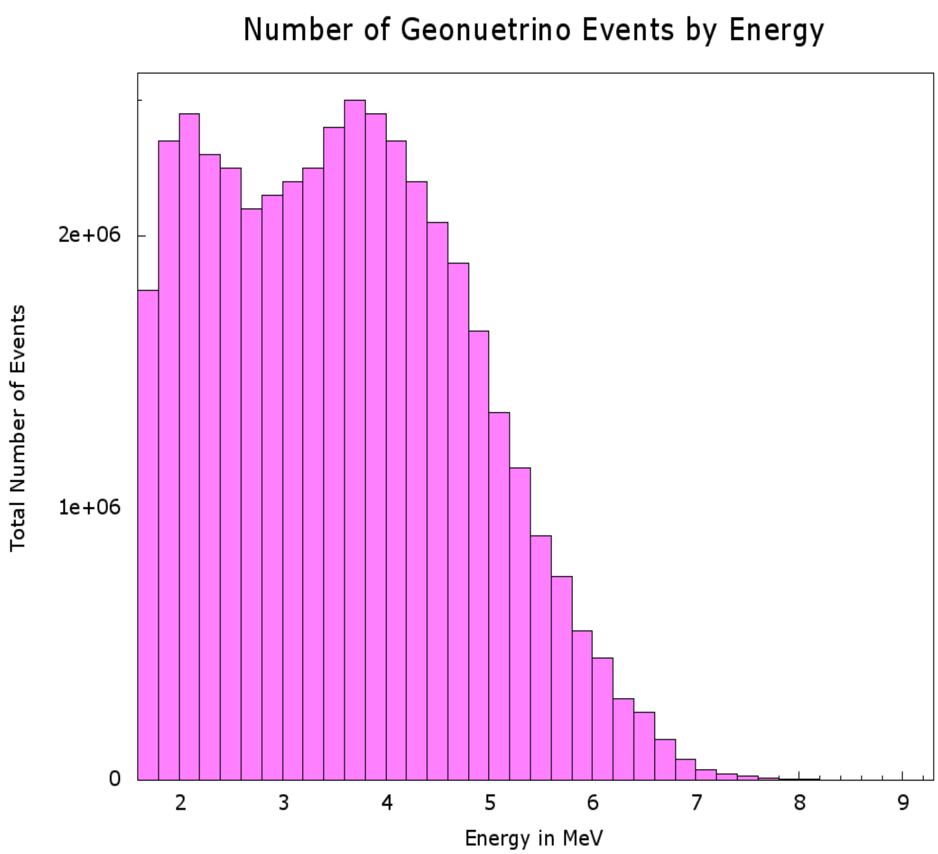

Figure 10: Simulated Number of Events vs the Positron Energy

Once the final position was located, then the track of the positron had to be calculated, particularly where the positron exited the volume. Each volume had their own unique code to locate these positron exits. Next, a subroutine was written to keep all positrons that had 90% of their tails located inside the volume. This left three scenarios; a positron starts inside of the volume but ends up located outside of the volume, a positron starts outside of the volume but ends inside of the volume, and a positron starts outside of the volume, enters into the volume, but then leaves the volume again. Then, the percentage of positrons with 90% of their paths inside the volume was calculated.

Volumes

Five different volumes were tested for a water detector with assumed density of 1000 kg/m^3. The volumes include a rectangular box, a cylinder, a sphere, an elliptical cylinder and an ellipsoid. All 10 kT volumes can fit inside of the submarine vessel with the exception of the sphere. However, the spherical-shaped ellipsoid volume is a good alternative and can be placed inside of the submarine.

A Monte Carlo simulation was specifically created for each of the volumes and 6.25 million events were generated. Using vector calculus, the entering and exiting position of the positron path length was calculated. Then if 90% of the positron tail was inside of the volume it was kept, else it was discarded. The percentage of events that were able to be kept was calculated. Based on the the percentage of the events that could be contained, a recommendation of the shape of the volume can be made.

The five volume shapes will be discussed beginning with the rectangular box. Formulas will be given when pertinent and examples of the Monte Carlo code are displayed to demonstrate the basic methodology.

Rectangular Box

After the events were generated in the code, a check was made to see if the events were permissible to be used as data. The following code keeps all positrons that start and end inside of the volume where *x,y,z* represent the initial position of the geo-neutrino impact on the proton, *a,b,c* represent the final position of the positron and *basex*, *basey*,and *basez* represent the edges of the fiducial volume.

```
if((sqrt(a*a) < basex && sqrt(b*b) < basey && sqrt(c*c) < basez) &&
(sqrt(x*x) < basex && sqrt(y*y) < basey && sqrt(z*z) < basez))
  {
  innerlocations<<a<<" "<<b<<" "<<c<<endl;
  innerct = innerct + 1;
  }
else if......
  {.......
  }
else
  {
  outcount = outcount + 1;
  outlocations<<a<<" "<<b<<" "<<c<<endl;
  }
```

The following fiducial volume is for a 10 kT rectangular detector with dimensions of 10 m by 22 m by 41.24 m. It will have a surface area of 3079 m^2 . Out of 6.25 million positron-tail events, $97.993 \pm 6 \times 10^{-3}$ % were recoverable within the fiducial volume. Figures 9 and 10 show the escapes of the positrons as well as that the rectangular volume can fit inside of generated cylinder that represents the minimum dimensions of the submarine.

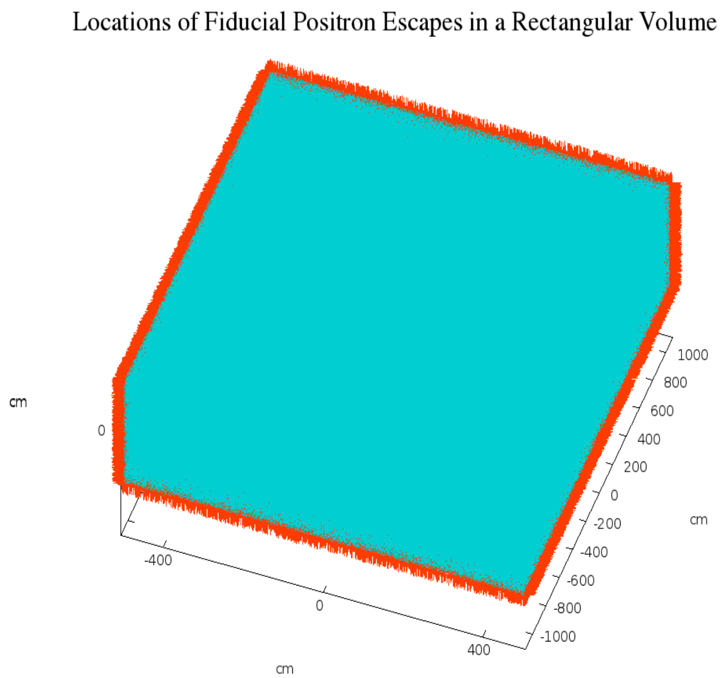

Figure 11: Rectangular Volume

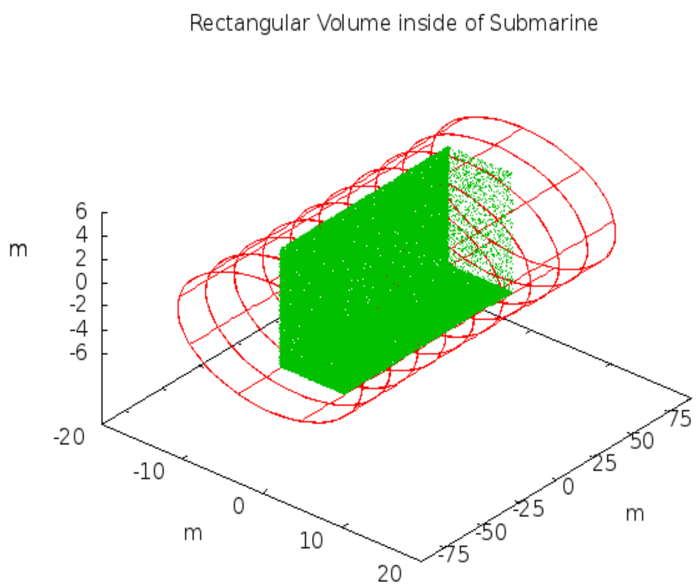

Figure 12: Rectangular Volume inside of Submarine

Cylinder

The Monte Carlo simulation for a 10 kT cylinder detector was able to contain $98.675 \pm 5 \times 10^{-3}$ % of positron path lengths. The radius is 5 m while the cylinder length is 115.5 m. This too, can fit inside of the submarine. While more difficult to build, a cylindrical volume offers a bit more events than its rectangular counterpart. For every 1000 events that could be generated, the cylinder will be able to produce 7 more events than the rectangular box. However, this cylinder requires a surface area of 3786 m^2 . That represents an increase of 22% in building material. Figure 13 shows that the cylinder can fit inside of a submarine. The volume, however, will only leave 0.5 m of space above and below the volume at the submarine's smallest point.

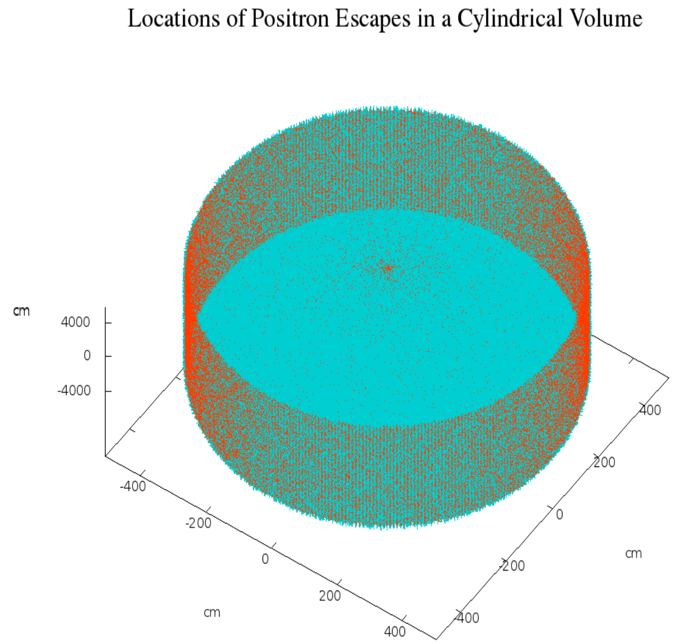

Figure 13: Cylindrical Volume

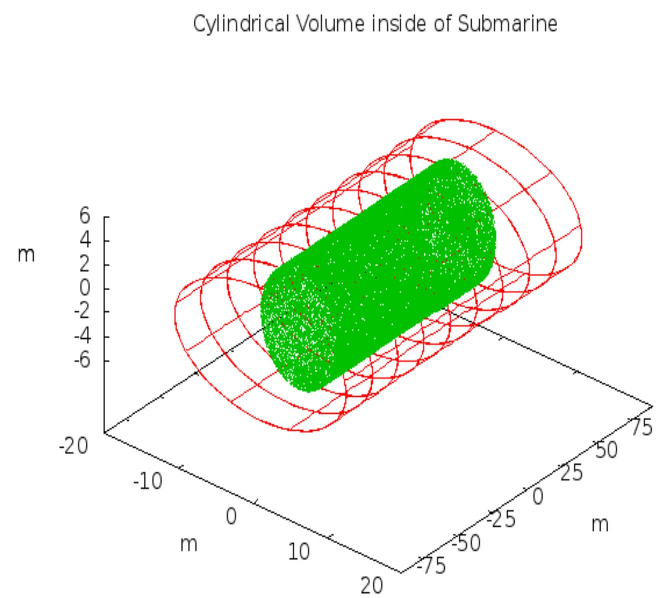

Figure 14: Cylindrical Volume Inside of a submarine.

Sphere

A sphere was included in this study for comparison basis despite that its 12.94 m radius (required for a 10 kT detector) is too large to fit inside of the submarine without extensive vessel modifications (see Figure 14). However, due to the lack of corners,

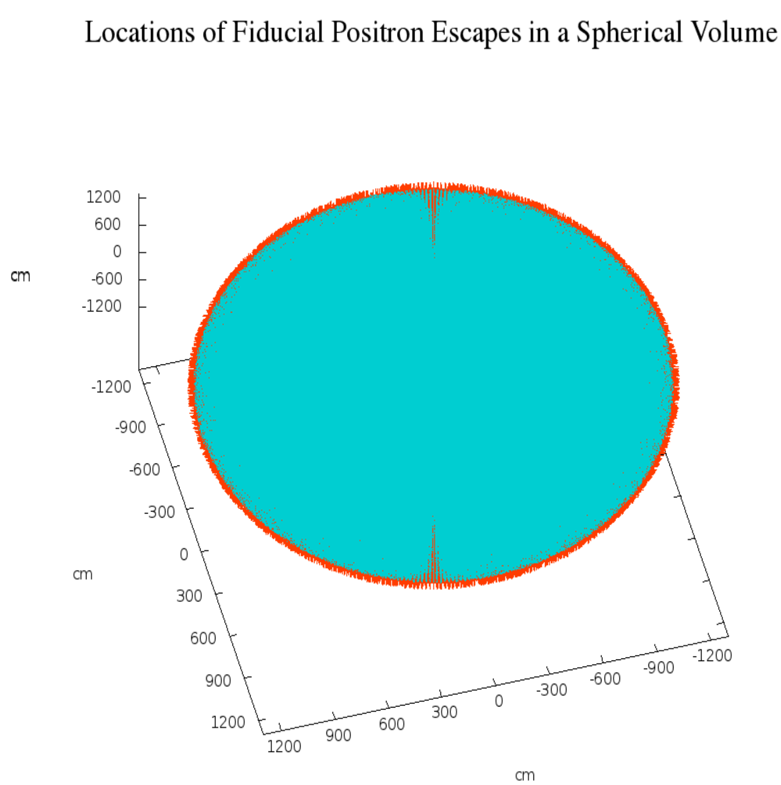

Figure 15: Spherical Volume

a sphere has the highest percentage of measurable events at $99.592 \pm 3 \times 10^{-3}$ %. If this detector were to be used it would be able to produce 16 more events for every 1000 events and would have a fraction of the rectangular box's surface area at 2091 m^2 . That's almost 32% less material required to than the rectangular volume's requirements.

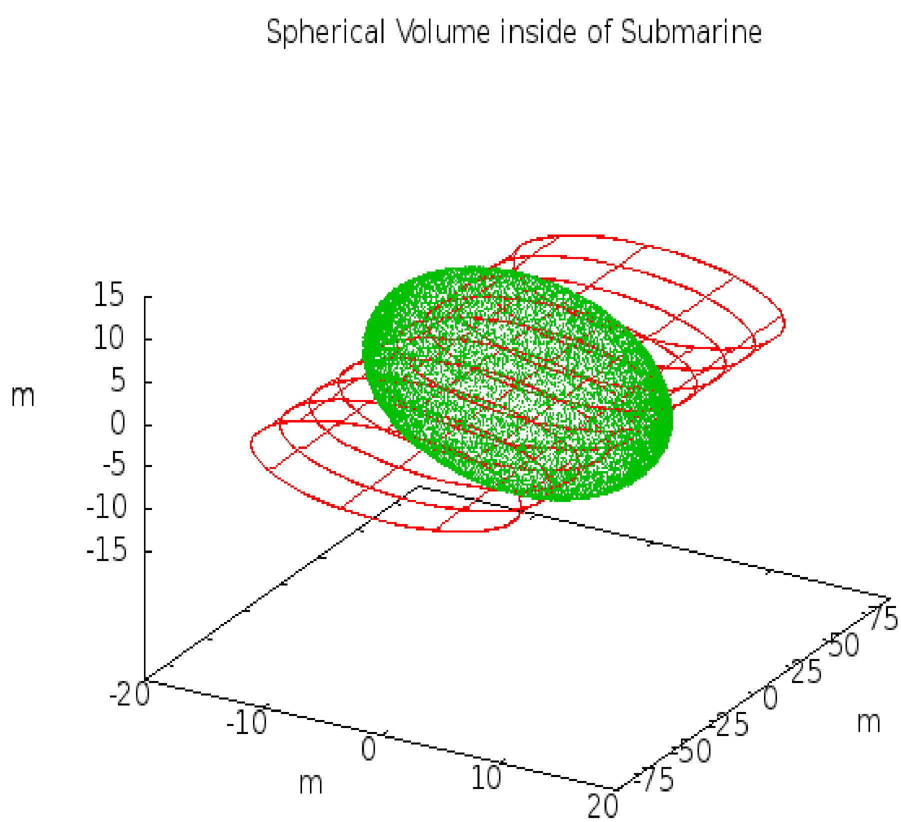

Figure 16: Spherical Volume Inside of Submarine

Elliptical Cylinder

A traditional cylinder does not utilize the space of the submarine well. Therefore an elliptical-shaped cylinder is better suited to the shape of the submarine. A 10 kT elliptical-cylinder was simulated with a length of 57.76 m, a semi-major axis of 10 m and a semi-minor axis of 5 m. Notice the comparison of the shapes in Figure 15. Figure 15 gives a side view comparison of the two types of cylinders when fit inside of the submarine. The cylinder leaves ample space on the sides but nearly touches the edges at the submarine's

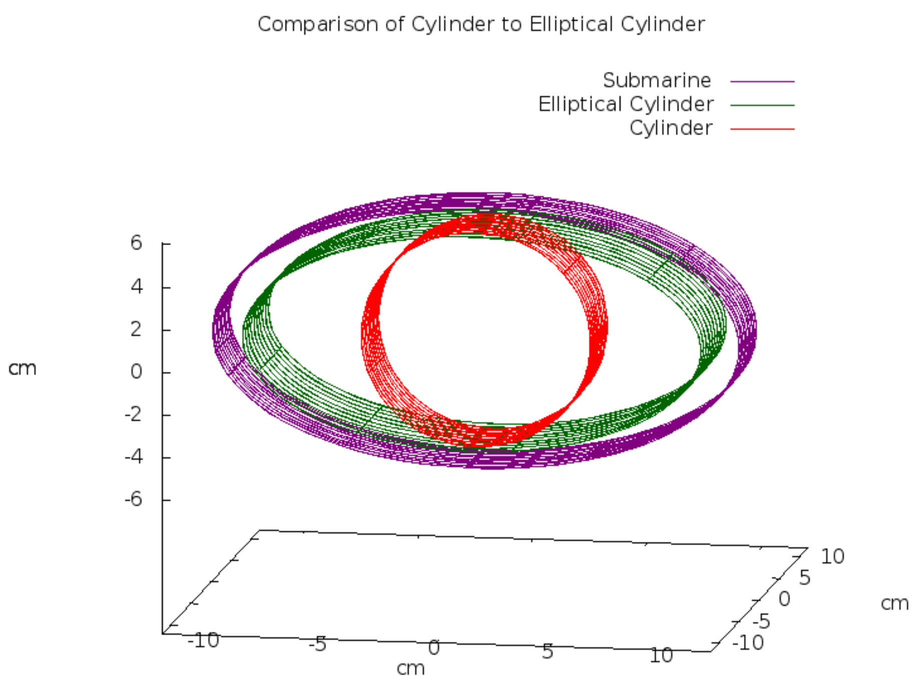

Figure 17: Side by Side Comparison of Cylindrical and Elliptical Volume vs Submarine Outline

top and bottom. Meanwhile the elliptical-cylinder nearly matches the outline of the submarine edge leaving around 1 m of space between the the two surfaces. This also saves up 37.7 m of space on the ends of the submarine for other potential uses. The elliptical cylinder was able to capture $98.966 \pm 3 \times 10^{-3}$ % of positron tails. The surface area was approximated using the Ramanujan's 2nd approximation formula for the perimeter of an ellipse or $\pi(a+b)\frac{1+3h}{10+\sqrt{4-3h}}$ where $h=\frac{(a-b)^2}{(a+b)^2}$ and *a* and *b* are the semi-major and semi-minor axes [13]. The surface area for this volume is quite a bit smaller than the the cylinder at 3026 m^2. When compared to the rectangular volume, this represents 10 more events per 1000 events and 1% less building material will be needed to construct this volume.

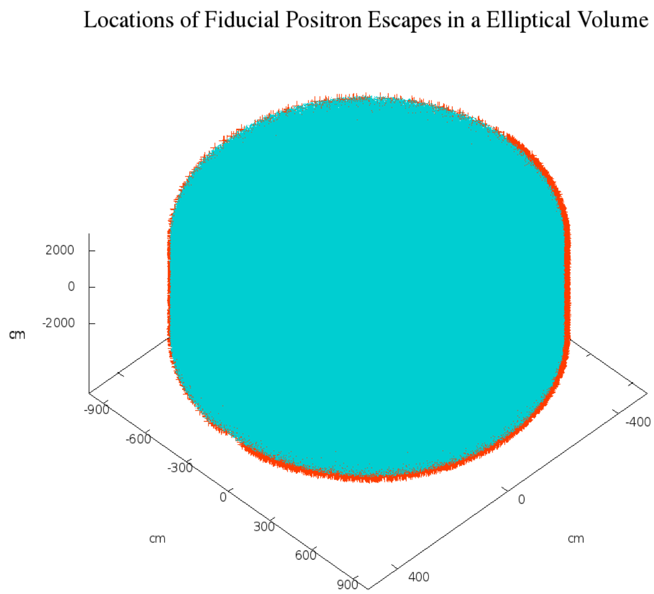

Figure 17: Side by Side Comparison of Cylindrical and Elliptical Volume vs Submarine Outline

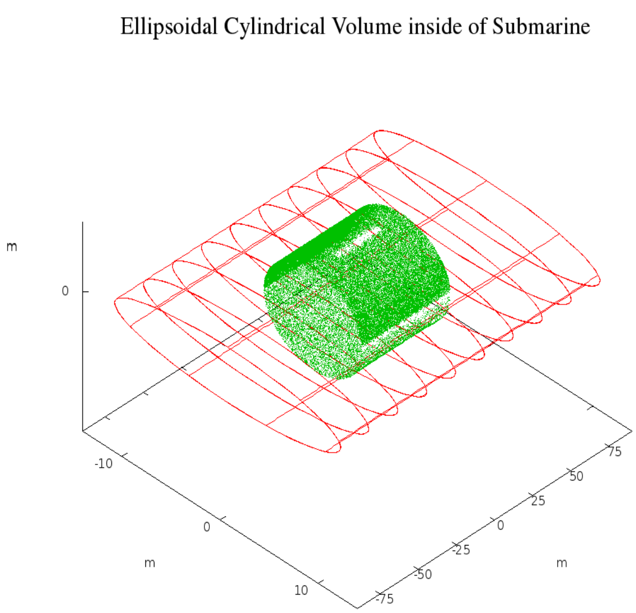

Figure 19: Elliptical Cylinder Volume Inside Submarine

Ellipsoid Volume

An ellipsoid can be viewed as a squashed sphere. This allows for the spherical geometry to be used despite that a traditional sphere does not have the ability to fit inside the submarine. This 10 kT ellipsoid detector has the highest percentage of measurable events next to the (impossible) sphere. It has a semi-major axis of 41.27 m and it has semi-minor axes of 5 m and 10.5 m. $99.015 \pm 4 \times 10^{-3}$ % of events were recoverable which makes this the best detector for maximizing the most events that can fit inside of the submarine. The surface area was found using the following approximation [14].

$$A_{surface} = 2\pi\left(c^2 + abr\left(1 - \frac{r^2(b^2 - c^2)}{6b^2}\left(1 - \frac{r^2(3b^2 + 10c^2)}{56b^2}\right)\right)\right) \quad \text{where} \quad r = \cos^{-1}\left(\frac{c/a}{\sqrt{1 - c^2/a^2}}\right)$$

a, *b* and *c* are the semi-major and semi-minor axes and $a > b > c$. This approximation yields a surface area of 3244 m^2. This is comparable to the surface areas of both the elliptical cylinder and the rectangular box, however the elliptical cylinder has less surface area due to the egg shaped geometry of the ellipsoid.

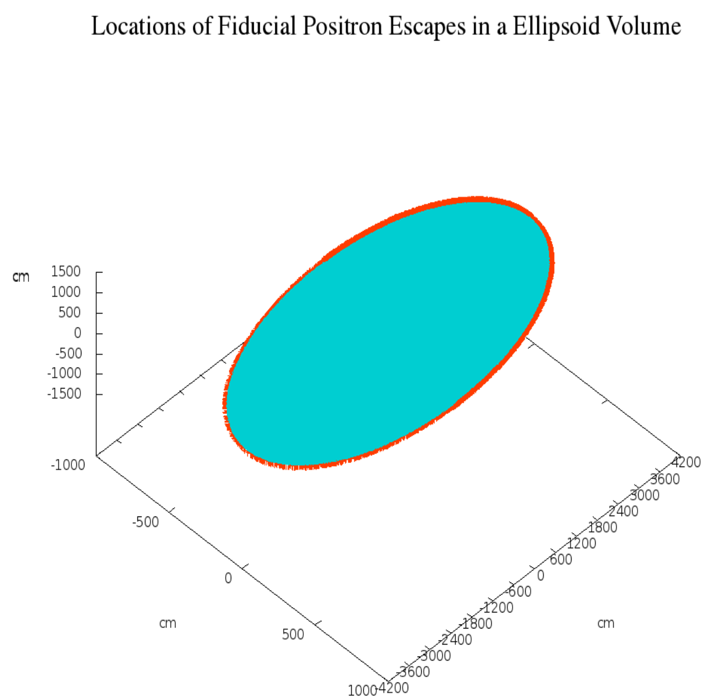

Figure 20: Ellipsoid Volume

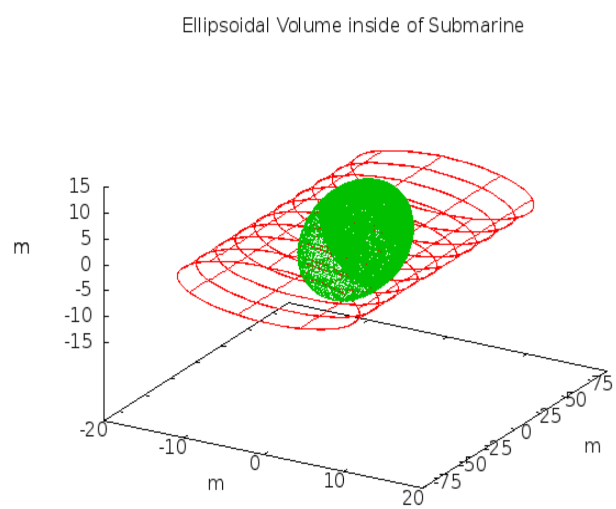

Figure 21: Ellipsoid Volume Inside Submarine

Conclusion

Any neutrino event is difficult to capture. This makes the few data points that an experiment can attain all the more important to the study. While the number of measurable events for each volume is within a fraction of a percent of each other, a few geo-neutrinos lost in an experiment can have a huge impact if the expected value is very low. Table 3 gives a comparison of the advantages and disadvantages of each volume.

Table 3: Comparison of the Volumes

Volume Type	Percent of Events Captured	Surface Area	100 x (% of Events/m^2)
Rectangular	$97.993 \pm 6 \times 10^{-3}$ %	$3079\, m^2$	3.2
Cylindrical	$98.675 \pm 5 \times 10^{-3}$ %	$3786\, m^2$	2.6
Spherical*	$99.592 \pm 3 \times 10^{-3}$ %	$2091\, m^2$	4.8
Elliptical-Cylinder	$98.966 \pm 3 \times 10^{-3}$ %	$3026\, m^2$	3.3
Ellipsoid	$99.015 \pm 4 \times 10^{-3}$ %	$3244\, m^2$	3.1

*Volume is too large for submarine

Table 3 shows that an elliptical cylinder is the best choice of a fiducial volume due to its high number of events per surface area ratio. It has the smallest surface area of all feasible geometries and also has the 2nd highest number of captured events. While an ellipsoid's percent of captured events has a statistical significance over the elliptical cylinder's contained events, the percentage is only larger by 0.049%. Therefore for every 2000 events generated, the ellipsoid will have approximately 1 more event than the elliptical cylinder. Furthermore, a cylindrical geometry is easier to construct and transport due to having two flat sides and it also utilizes the space of the submarine well. Therefore I recommend the HANOHANO project should use an elliptical-cylinder for the shape of their detector.

REFERENCES

REFERENCES

[1] J VanRyzin et al., [Makai Ocean Engineering, Inc], A Deep Ocean Anti-Neutrino Detector near Hawaii - Hanohano" Final Report prepared for The National Defense Center of Excellence for Research in Ocean Sciences (CEROS), 8 (Waimanalo, Hawaii 2006).

[2] Military Today, http://www.military-today.com/navy/typhoon_class.htm accessed on October 28th 2013.

[3] S. T., Dye, Reviews of Geophysics 50, 13 (2012).

[4] G., Fiorentini, et al, Earth Planet Sci. Lett., 238:235-247 (2005) .

[5] S. T., Dye. Measuring U & Th Enrichment of the Silicate Earth Investigating Earth's Origin & Thermal History, talk given at Neutrino Geosciences, Oct 6, 2010.

[6] J. Maričić [KamLAND collaboration], Exploring the Geo-reactor Hypothesis with Neutrinos, talk given at the DOANOW Workshop, University of Hawaii. March 24th 2007.

[7] R. P. Litchfield, Direct Measurement of θ_{13}, talk given at Flavor Physics and CP Violation (FPCP 2012), Hefei, China, May 21-25, 2012.

[8] United States Geological Survey, What on Earth is Plate Tectonics? Accessed from http://geomaps.wr.usgs.gov/parks/pltec/ on October 28th, 2013.

[9] E. Sanhiro,[KamLAND], Geoneutrino Overview, talk from Deep Ocean Anti-Neutrino Observatory Workshop Honolulu, Hawaii March 23-25, 2007, no proceedings.

[10] ESTAR accessed from http://www.physics.nist.gov/PhysRefData/Star/Text/ESTAR.html last accessed on October 28th 2013.

[11] J. Felde, accessed from http://www.quantumdiaries.org/author/jfelde/, accessed on October 28th, 2013.

[12] E. Sanshiro, Neutrino Geophysics and Observation of Geo-Neutrinos at KamLAND, Tohoko University, 2005.

[13] P. Rourke, Circumference of an Ellipse, accessed from http://paulbourke.net/geometry/ellipsecirc/, accessed on October 30th, 2013.

[14] A. Dieckmann, accessed from http://pi.physik.uni-bonn.de/~dieckman/SurfaceEllipsoid/SurfEll.html (2003), accessed on October 28th, 2013.