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Joint Time Delay and Frequency Estimation Without Eigen-Decomposition

M. M. Qasaymeh, Hiren Gami, Nizar Tayem, M. E. Sawan, and Ravi Pendse

Abstract—In this letter, we addressed the problem of estimating the time delay and the frequencies of noisy sinusoidal signals received at two spatially separated sensors. We employ the Propagator Method (PM) in conjunction with the well-known MUSIC/root-MUSIC algorithm; the proposed method would generate estimates of the unknown parameters. Such estimates are based on the observation and/or covariance matrices. Moreover, the PM does not require the eigenvalue decomposition (EVD) or singular value decomposition (SVD) of the cross-spectral matrix (CSM) of received signals; therefore, a significant improvement in computational load is achieved. Computer simulations are also included to demonstrate the effectiveness of the proposed method.

Index Terms—Delay and frequency estimation, MUSIC, propagator method, root-MUSIC.

I. INTRODUCTION

AN accurate time delay estimation (TDE) between two or more noisy versions of the same signal received at spatially separated sensors [1], [2] is an important topic that finds applications in positioning and tracking, speed sensing, direction finding, biomedicine, exploration geophysics, etc. Similarly, frequency estimation [3], [4] has been universally addressed in signal processing literature. Later, these two separated problems were combined as a joint time delay and frequency estimation problem [5]–[7], which appeared in many applications like synchronization in code division multiple access (CDMA) systems, speech enhancement, and pitch estimation using a microphone array.

A Discrete-Time Fourier Transform (DTFT) based method has been derived [8] for estimating the time difference of arrival between sinusoidal signals received at two separated sensors. A subspace algorithm based on state-space realization has been proposed [9] for joint time delay and frequency estimation of sinusoidal signals received at two separated sensors. The frequency estimates are obtained directly from the eigenvalues of the state transition matrix, while the delay is determined using the observation matrix and the estimated frequencies.

In this letter, we addressed the problem of estimating time delay and frequencies of received signal using the Propagator Method (PM) [10] in conjunction with the well-known MUSIC/root-MUSIC algorithm [11]. It is well-known that the computational load of the PM-based method is significantly lower,

as it does not involve eigenvalue decomposition (EVD) or singular value decomposition (SVD) of the cross-spectral matrix (CSM) of received signals. The propagator is a linear operator which only depends on steering vectors and which can be easily extracted from the data set. An estimated propagator from the data set of the first sensor is used to construct an orthogonal projection matrix which represents the noise subspace. The MUSIC/root-MUSIC algorithm is used to estimate the frequencies from the projection matrix. An additional Propagator is estimated from the data sets of both sensors to estimate the delay parameter.

This letter is structured as follows. In Section II, the system model and the problem formulation is presented. The development of the proposed method is presented in Section III. In Section IV, the performance of the PM is illustrated through MATLAB simulations. A comparison with the previous work [5] is made. Finally, some concluding remarks follow in Section V.

II. PROBLEM FORMULATION

Consider the discrete-time sinusoidal signals $x(n)$ and $y(n)$ are the two sensors measurements satisfying

$$\begin{aligned} x(n) &= s(n) + u(n) \\ y(n) &= s(n - D) + z(n), \quad n = 0, 1, \dots, N - 1 \end{aligned} \quad (1)$$

where

$$s(n) = \sum_{i=1}^P a_i e^{j\omega_i n}. \quad (2)$$

The source signal $s(n)$ is modeled by a sum of P complex sinusoids where the amplitudes (a_i) are unknown, complex-valued constants, and the normalized radian frequencies (ω_i) are different. Without a loss of generality, we considered $\omega_1 < \omega_2 \dots < \omega_P$. To simplify the problem we have assumed the number of sources P either known or pre estimated [12]. The two terms $u(n)$ and $z(n)$ are representing the two zero mean, additive white complex gaussian noise processes independent of each other. Also, parameters N represent the number of samples collected at each channel. The variable D is the delay between the received copies of the signal $s(n)$ at the two separated sensors, which is unknown and is to be estimated.

III. DEVELOPMENT OF PROPOSED METHOD

The development of the proposed method is divided into two parts. In the first part, the frequencies are estimated using the received data at the first sensor and by applying the PM method with the MUSIC/root-MUSIC [11] algorithm. In the second part, we used the received data at the two sensors and the estimated frequencies in the first part to extract the time delay information by P eigenvalues of the estimated new propagator.

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A. Frequency Estimation

Using the received data at the first sensor $x(n)$ with N available samples: $x(0), x(1), \dots, x(N-1)$ given by (1), we form the $L \times (N-L+1)$ Hankel matrix

$$\mathbf{X} = \begin{bmatrix} x(0) & x(1) & \dots & x(N-L) \\ x(1) & x(2) & \dots & x(N-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(L-1) & x(L) & \dots & x(N-1) \end{bmatrix} \quad (3)$$

and $\mathbf{X}$ can be rewritten as $\mathbf{X} = [\mathbf{r}(0)\mathbf{r}(1)\dots\mathbf{r}(N-L)]$ where the i th column of $\mathbf{X}$ is given by

$$\begin{aligned} \mathbf{r}(i) &= \mathbf{A}_L(\omega)(\varphi(\omega))^i \mathbf{a} + \mathbf{u}_i, \quad i = 0, 1, \dots, N-L \\ \mathbf{A}_L(\omega) &= \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\omega_1} & e^{j\omega_2} & \dots & e^{j\omega_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(L-1)\omega_1} & e^{j(L-1)\omega_2} & \dots & e^{j(L-1)\omega_P} \end{bmatrix} \\ \varphi(\omega) &= \text{diag}(e^{j\omega_1} \ e^{j\omega_2} \ \dots \ e^{j\omega_P}), \\ \mathbf{a} &= [a_1, a_2, \dots, a_P]^T, \text{ and} \\ \mathbf{u}_i &= [u(i)u(i+1)\dots u(i+L-1)]^T \end{aligned} \quad (4)$$

where $\mathbf{A}_L(\omega)$ is the array response matrix, $\varphi(\omega)$ is a diagonal matrix which contains the information about the frequencies of noisy sinusoidal signals received, $\mathbf{a}$ is an unknown complex amplitude vector, and $\mathbf{u}_i$ is the complex noise matrix. Using (4) we can formulate the received data matrix as

$$\mathbf{X} = [\mathbf{A}_L(\omega)\mathbf{a} \ \mathbf{A}_L(\omega)\varphi(\omega)\mathbf{a} \ \dots \ \mathbf{A}_L(\omega)(\varphi(\omega))^{N-L}\mathbf{a}] + [\mathbf{u}_0 \ \mathbf{u}_1 \ \dots \ \mathbf{u}_{N-L}].$$

In order to employ the Propagator Method we partition $\mathbf{A}_L(\omega)$ into two sub-matrices $\mathbf{A}_{L1}(\omega)$ and $\mathbf{A}_{L2}(\omega)$ with dimensions $P \times P$ and $(L-P) \times P$, respectively. We defined a propagator matrix $\mathbf{P}$ satisfying the following condition

$$\mathbf{P}^H \mathbf{A}_{L1}(\omega) = \mathbf{A}_{L2}(\omega) \quad (5)$$

where $(\cdot)^H$ denotes hermitian transpose and the dimensions of the matrix $\mathbf{P}^H$ are $(L-P) \times P$. Similarly, we partition the received data matrix $\mathbf{X}$ into two sub-matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ with dimensions $P \times (N-L+1)$ and $(L-P) \times (N-L+1)$ respectively. The Propagator matrix $\hat{\mathbf{P}}$ can be estimated by

$$\begin{aligned} \hat{\mathbf{P}} &= \arg \min \|\mathbf{X}_2 - \hat{\mathbf{P}}^H \mathbf{X}_1\|^2 \\ &= (\mathbf{X}_1 \mathbf{X}_1^H)^{-1} \mathbf{X}_1 \mathbf{X}_2^H \end{aligned} \quad (6)$$

where $\|\cdot\|^2$ denotes the Euclidean norm. Matrix $\mathbf{E}$ can be defined as

$$\mathbf{E} = \begin{bmatrix} \mathbf{P} \\ -\mathbf{I} \end{bmatrix} \quad (7)$$

where $\mathbf{I}$ is the identity $(L-P) \times (L-P)$ matrix. Clearly,

$$\mathbf{E}^H \mathbf{A}_L(\omega) = \mathbf{P}^H \mathbf{A}_{L1}(\omega) - \mathbf{A}_{L2}(\omega) = \mathbf{0}. \quad (8)$$

In a noisy channel, the basis of matrix $\mathbf{E}$ is not orthonormal. By introducing an orthogonal projection matrix $\mathbf{Q}$ which represents the noise subspace we have $\mathbf{Q}\mathbf{A}_L(\omega) = \mathbf{0}$, where $\mathbf{Q} =$

$\hat{\mathbf{E}}(\hat{\mathbf{E}}^H \hat{\mathbf{E}})^{-1} \hat{\mathbf{E}}^H$, where $\hat{\mathbf{E}}$ is formulated from the estimated propagator in (6). Apply MUSIC like search algorithm [12] to estimate the frequencies using the following function

$$\hat{P}_{MU}(e^{j\omega}) = \frac{1}{\sum_{i=0}^{L-1} \|\mathbf{e}^H \mathbf{q}_i\|^2} \quad (9)$$

where $\mathbf{q}_i$ is the i th column of the noise subspace matrix $\mathbf{Q}$ while the vector is defined as $\mathbf{e} = [1, e^{j\omega}, \dots, e^{jP\omega}]^T$. Instead of searching for the peaks in (9) an alternative is to use a root-MUSIC. The frequency estimates may be taken to be the angles of the P roots of the polynomial $D(z)$ that are closest to the unit circle

$$D(z) = \sum_{i=0}^{L-1} \mathbf{V}_i(z) \mathbf{V}_i^*(1/z^*) \quad (10)$$

where $\mathbf{V}_i(z)$ is the z -transform [11] of the i th column of the projection matrix $\mathbf{Q}$.

B. Time Delay Estimation

The estimated frequencies obtained in (9) or (10) will be used to estimate the time delay in this section. From the given data record at the second sensor, we can construct a $(\frac{N}{2} \times \frac{N}{2})$ Hankel matrix $\bar{\mathbf{Y}}$

$$\bar{\mathbf{Y}} = \begin{bmatrix} y(0) & y(1) & \dots & y(N/2-1) \\ y(1) & y(2) & \dots & y(N/2) \\ \vdots & \vdots & \ddots & \vdots \\ y(N/2-1) & y(N/2) & \dots & y(N-1) \end{bmatrix}.$$

Let

$$\bar{\mathbf{Y}} = [\xi_0 \ \xi_1 \ \dots \ \xi_{N/2-1}] \quad (11)$$

where

$$\xi_i = \mathbf{B}(\omega) \mathbf{\Omega}(\omega, D) \varphi(\omega)^i \mathbf{a} + \mathbf{z}_i$$

and

$$\begin{aligned} \mathbf{B}(\omega) &= \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\omega_1} & e^{j\omega_2} & \dots & e^{j\omega_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(N/2-1)\omega_1} & e^{j(N/2-1)\omega_2} & \dots & e^{j(N/2-1)\omega_P} \end{bmatrix} \\ \mathbf{\Omega}(\omega, D) &= \text{diag}(e^{-jD\omega_1} \ e^{-jD\omega_2} \ \dots \ e^{-jD\omega_P}), \\ \mathbf{z}_i &= \left[z(i) \ z(i+1) \ \dots \ z\left(i + \frac{N}{2} - 1\right) \right]^T, \\ &\quad i = 0, 1, \dots, \left(\frac{N}{2} - 1\right). \end{aligned}$$

Finally, we can write $\bar{\mathbf{Y}}$ as

$$\bar{\mathbf{Y}} = \left[\mathbf{B} \mathbf{\Omega} \mathbf{a} \ \mathbf{B} \mathbf{\Omega} \varphi \mathbf{a} \ \dots \ \mathbf{B} \mathbf{\Omega} \varphi^{\left(\frac{N}{2}-1\right)} \mathbf{a} \right] + \left[\mathbf{z}_0 \ \mathbf{z}_1 \ \dots \ \mathbf{z}_{\left(\frac{N}{2}-1\right)} \right]. \quad (12)$$

Now we repeat the same steps (11) and (12) with the given data record from the first sensor to obtain the $(\frac{N}{2} \times \frac{N}{2})$ Hankel matrix $\bar{\mathbf{X}}$. We can write the grouped data matrices as

$$\mathbf{Z} = \begin{bmatrix} \bar{\mathbf{X}} \\ \bar{\mathbf{Y}} \end{bmatrix}. \quad (13)$$

Let us define matrix O as follows

$$O = \begin{bmatrix} B & B\varphi & \cdots & B\varphi^{(N/2-1)} \\ B\Omega & B\Omega\varphi & \cdots & B\Omega\varphi^{(N/2-1)} \end{bmatrix}. \quad (14)$$

We partition O into two submatrices O_1 and O_2 with dimensions $(P \times \frac{PN}{2})$ and $((N-P) \times \frac{PN}{2})$ respectively. Similarly, we partition B into B_1 and B_2 with dimensions $P \times P$ and $((N/2) - P) \times P$ respectively. Rewriting O as

$$O = \begin{bmatrix} O_1 \\ O_2 \end{bmatrix} = \begin{bmatrix} B_1 & B_1\varphi \cdots & B_1\varphi^{(N/2-1)} \\ B_2 & B_2\varphi \cdots & B_2\varphi^{(N/2-1)} \\ B_1\Omega & B_1\Omega\varphi \cdots & B_1\Omega\varphi^{(N/2-1)} \\ B_2\Omega & B_2\Omega\varphi \cdots & B_2\Omega\varphi^{(N/2-1)} \end{bmatrix} \quad (15)$$

under the hypothesis that B_1 is a $P \times P$ non singular matrix, the propagator matrix P is a unique linear operator which can be written as $P^H O_1 = O_2$. Similar to part A, the estimated propagator of dimension $(N-P) \times P$ can be derived as

$$\hat{P} = (O_1 O_1^H)^{-1} O_1 O_2^H. \quad (16)$$

The estimated propagation matrix will be partitioned into three sub-matrices as $\hat{P}^T = [P_1 \ P_2 \ P_3]$, where P_2 is a square matrix of size $P \times P$ and both of P_1 and P_3 are of size $((N/2) - P) \times P$. Using (16) we can write

$$P_2 [B_1 \ B_1\varphi \cdots B_1\varphi^{(N/2-1)}] = [B_1\Omega \ B_1\Omega\varphi \cdots B_1\Omega\varphi^{(N/2-1)}]. \quad (17)$$

From (17) the P eigenvalues of the propagator P_2 is corresponding to the P diagonal elements of Ω . This implies that the diagonal matrix Ω can be estimated by finding the P eigenvalues of the estimated propagator P_2 . Therefore, the time delay estimation can be found as

$$D = \frac{\text{trace}(\text{diag}(\langle P_2 \rangle))}{\sum_{i=1}^P \hat{\omega}_i}. \quad (18)$$

IV. SIMULATION RESULTS

In this section, the performance of the proposed method is compared with state-space realization method in [5]. In the first experiment we considered two-sinusoidal signals with amplitudes $a_1 = a_2 = 1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s. We simulated the performance under AWGN environment with different SNRs and 200 independent Monte-Carlo realizations. The number of signal samples was 200 and while L was 25. The MSE is defined as

$$\text{MSE}_{\text{dB}} = 10 \log_{10} \left(\frac{1}{N_t P} \sum_{i=1}^{N_t} \sum_{j=1}^P (\omega_j - \hat{\omega}_j)^2 \right) \quad (19)$$

where $\hat{\omega}_j$ is the estimate of ω_j , and N_t is the number of Monte-Carlo trials. The MSE of the frequencies estimate is compared with the state-space realization method in [5]. Significant improvement in performance was achieved, especially at $\text{SNR} \geq -5$ dB. In order to avoid extensive computation we assumed

Fig. 1. MSE of frequency estimation versus SNR.

Fig. 2. MSE of frequency estimation versus snapshots.

reasonable step size (0.001) for searching frequencies in the MUSIC pseudo-spectrum (9) that causes such dominant behavior of arithmetic errors at very high SNRs. Moreover, to reduce the computational load significantly, we simulated the root-MUSIC algorithm in association with the PM method to evaluate (9) into a closed form polynomial in the z domain (10) along the unit circle.

The performance as a function of number of snapshots is illustrated in Fig. 2 with SNR of 10 dB. The number of snapshots is varied from 40 to 400. It is obvious that our frequencies estimator is better than [5]. For example, in the frequency estimation we need a data record of 40 samples while [5] required 160 samples to achieve the same performance of -40 dB. In third figure we illustrated the estimator behavior with the frequency spacing. The first frequency is varying from 0.2π to 0.9π while the second frequency is assumed to be 0.1π at constant 10 dB SNR. It is clear from the figure that in worst scenario our estimator is showing almost 10 dB improvement in performance with the reference estimator. It is worth to mention that the propagator method is not that sensitive to small frequency spacing [10]. In the fourth figure we tested two algorithms with respect to the number of frequencies. To guarantee fair comparison we assumed constant frequency spacing between the sources. We kept fixed 10 dB SNR, data record of length 200 samples and the number of unknown sources is varying from two to six. The combination of frequencies used to evaluate Fig. 4 is listed in Table I. Since root MUSIC and MUSIC both have almost same performance we compared only root-MUSIC based algorithm

Fig. 3. MSE of Frequency estimation versus frequency spacing.

Fig. 4. MSE of frequency estimation versus number of sources.

TABLE I
FREQUENCY COMBINATION IS USED IN FIG. 4

Un-known sources	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
2	$\pi/10$	$2\pi/10$	--	--	--	--
3	$\pi/10$	$2\pi/10$	$3\pi/10$	--	--	--
4	$\pi/10$	$2\pi/10$	$3\pi/10$	$4\pi/10$	--	--
5	$\pi/10$	$2\pi/10$	$3\pi/10$	$4\pi/10$	$5\pi/10$	--
6	$\pi/10$	$2\pi/10$	$3\pi/10$	$4\pi/10$	$5\pi/10$	$6\pi/10$

with the reference. From the Fig. 4, it is apparent that our algorithm is not much sensitive with respect to number of sources in the system.

To focus on the performance of the delay estimator (18) we used the exact and the estimated frequencies by both (9) and (10). Then we compared with [5]. The time delay was selected to be 0.7 s for sampling interval of 1.0 s while all the other parameters were similar as of experiment one in part A. The delay estimation error as a function of SNR is illustrated in Fig. 5. The reference method is showing better performance than the proposed method by 6 dB. On the other hand, the processing time for the reference method is much more than the proposed one as SVD or EVD and covariance matrices are not used in our estimator. The processing time is growing exponentially with

Fig. 5. MSE of Delay estimation versus SNR.

number of data records for the reference method while it is increasing linearly in our method.

V. CONCLUSION

We proposed a new technique for joint time delay and frequencies estimation of sinusoidal signals received at two separated sensors by applying the PM based method. The frequencies are estimated using the received data matrix. The frequency estimator shows outstanding performance compared with the state-space realization illustrated in [5]. The time delay estimator is showing exceptionally well performance in computational complexity with a tradeoff of 6 dB degradation in MSE with the [5].

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