AF MIMO BEAMFORMING RELAY NETWORKS UNDER POWER CONSTRAINTS

A Thesis by

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The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Electrical Engineering.

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Xiaomi Hu, Committee Member
DEDICATION

To my parents—thank you for your love and commitment
To my friends—thank you for keeping my spirits high throughout this process
To Jinyoung—thank you for being my strongest supporter and helper suitable for me
If any of you lacks wisdom, he should ask God, who gives generously to all without finding fault, and it will be given to him (James 1:5).
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ABSTRACT

This thesis studies amplify-and-forward (AF) multiple-input multiple-output (MIMO) beamforming relay networks based on the minimum mean square error (MMSE) criterion under various transmit power restraints (TPRs). The primary contribution of this thesis is the derivation of a set of optimal relay amplifying matrices and source-destination beamforming vectors under diverse conditions of TPRs on the source and the relay for the AF MIMO wireless relay network. By comparing the bit error rate (BER) performance of each case, an efficient design of a half-duplex (HD) AF relay system is presented.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>INTRODUCTION</th>
<th>SYSTEM MODEL</th>
<th>AMPLIFY-AND-FORWARD HALF-DUPLEX MMSE STRATEGIES</th>
<th>SIMULATION RESULTS</th>
<th>CONCLUSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
<td>3.1 Transmit Power Restraints</td>
<td>4.1 Assumptions</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.2 Aggregate Power Restraint</td>
<td>4.2 Iterative Algorithm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.3 Source Power Restraint</td>
<td>4.3 Analysis of Simulations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.4 Relay Power Restraint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td>3.1 Transmit Power Restraints</td>
<td>4.2 Iterative Algorithm</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
<td>3.2 Aggregate Power Restraint</td>
<td>4.3 Analysis of Simulations</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
<td>3.3 Source Power Restraint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td></td>
<td>3.4 Relay Power Restraint</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**LIST OF REFERENCES**

**APPENDICES**

A. Proof: Optimal Relay Amplifying Matrix and Beamforming Vectors

B. MATLAB Code
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>AF HD MIMO wireless relay system consisting of one source of $N_S$ antennas, one relay of $N_R$ antennas, and one destination of $N_D$ antennas</td>
<td>3</td>
</tr>
<tr>
<td>2.</td>
<td>Cost function versus signal-to-noise ratio under transmit power restraint (TPR) in AF MIMO wireless relay network</td>
<td>15</td>
</tr>
<tr>
<td>3.</td>
<td>Bit error rate versus signal-to-noise ratio under transmit power restraint (TPR) at both source and relay, aggregate power restraint (APR), and non-optimum case in AF MIMO wireless relay network</td>
<td>15</td>
</tr>
<tr>
<td>4.</td>
<td>Bit error rate versus signal-to-noise ratio under transmit power restraint (TPR) at both source and relay and aggregate power restraint (APR) in AF MIMO wireless relay network</td>
<td>16</td>
</tr>
<tr>
<td>5.</td>
<td>Bit error rate versus signal-to-noise ratio under source power restraint (SPR) at source and relay power restraint (RPR) at relay in AF MIMO wireless relay network</td>
<td>17</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Iteration Algorithm Procedure</td>
<td>14</td>
</tr>
</tbody>
</table>

x
## LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>APR</td>
<td>Aggregate Power Restraint</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noises</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>CRAN</td>
<td>Cloud Radio Access Network</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>FD</td>
<td>Full-Duplex</td>
</tr>
<tr>
<td>HD</td>
<td>Half-Duplex</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>RPR</td>
<td>Relay Power Restraint</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SPR</td>
<td>Source Power Restraint</td>
</tr>
<tr>
<td>TPR</td>
<td>Transmit Power Restraint</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

\( A \) Matrices
\( a \) Vectors
\( a \) Scalars
\( A^{-1} \) Inverse of Matrix
\( A^{\dagger} \) Pseudo Inverse of Matrix
\( A^* \) Complex Conjugate of Matrix
\( A^* \) Optimal Matrix
\( A^H \) Hermitian of Matrix
\( I_N \) \( N \times N \) Identity Matrix
\( |a| \) Absolute Value of \( a \) for Any Scalar
\( \|a\| \) 2-norm of \( a \)
\( \|A\|_F \) Frobenius-Norm of \( A \)
\( \text{Re}\{A\} \) Real Operator
\( E[\cdot] \) Expectation Operator
CHAPTER 1
INTRODUCTION

Multiple-input multiple-output (MIMO) technology can improve system performance, such as high-speed, high-capacity, and reliability, in a plurality of antennas used to both transmit and receive signals. In addition, promising relay strategies, such as amplify-and-forward (AF) [1]-[4], decode and-forward [5], compress-and-forward [6], and compute-and-forward [7], have been investigated over the past years for the purpose of achieving the gain of spatial diversity order. Due to their simplicity and mathematical tractability, AF MIMO wireless relay systems have recently gained considerable interest in comparison to other systems. Hence, this thesis considers an AF MIMO wireless relay system because the AF relay has lower complexity.

Furthermore, among techniques achieving array processing gain using multiple antennas, beamforming is a simple approach when the channel state information (CSI) is available at the transmitter [8]. Power restraint strategies have gained sufficient attention in an effort to save power and improve system performance in cooperative MIMO wireless relay networks [1]-[4], [9]. Generally, in AF MIMO wireless relay networks, power is constrained at either the sources or the relays [1], [2], [4], individually, or together. Hence, this thesis considers various transmitted power-restraint cases during data transmission for AF MIMO wireless relay networks.

The half-duplex (HD) relay system is popular in pragmatic relay networks because it is practically difficult to set apart a received and a transmitted signal sufficiently at the same time using the same frequency [10]. The work of Lee et al. [11] assumes the AF full-duplex (FD) MIMO wireless relay system with no cross interference at the relay. But it is not practical to implement an FD system with no cross interference. In order to avoid cross interference, HD is
more appropriate than FD in the same system model. In addition, the main difference between a conventional MIMO system and the one in this thesis is the presence of the relay and noise at the relay. Finally, this thesis considers the cooperative AF MIMO HD wireless relay system with beamforming under the assumption that CSI is available at each node. For future generation (5G) wireless communication systems, small cells and cloud radio access networks (CRANs) have been proposed as candidates. Then, all CSI among mobile nodes and relay nodes can be available at all relays because the relay nodes can be connected to a central service station through backhauls (i.e., fiber optics) in small-cell CRANs. Each relay can send itself CSI to the central service station, which can compute optimal relay amplifying matrix and beamforming vectors using full CSI and forward them to all relays. In this thesis, optimal HD relay amplifying matrices and beamforming vectors of the source-destination are analytically derived under power restraints at the source and the relay, separately, and in various combinations, using minimum mean square error (MMSE) criterion.

The performance of the system under different conditions is evaluated and compared in terms of bit error rate (BER) through simulations. In particular, in this thesis, the cost function is defined as the MMSE [12]. As a result, BER performance is better as the cost function value decreases.

The remainder of this thesis is organized into four chapters. Chapter 2 describes an AF MIMO HD wireless relay system model applied. Chapter 3 derives a set of optimal relay amplifying matrices and source-destination beamforming vectors under various power restraints during symbol transmission based on MMSE criterion. Chapter 4 compares BER simulation results under different situations. Section 5 concludes the thesis.
CHAPTER 2
SYSTEM MODEL

In this thesis, an AF HD MIMO wireless relay system is considered. Figure 1 illustrates a two-hop wireless relay system consisting of a source, relay, and destination that have $N_S$, $N_R$, and $N_D$ antennas, respectively. The subscripts S, R, and D refer to source, relay, and destination, respectively. The complex channel matrix between the source and the relay is represented by $F \in \mathbb{C}^{N_R \times N_S}$, and another complex channel matrix between the relay and the destination is denoted by $G \in \mathbb{C}^{N_D \times N_R}$. In addition, the complex channel matrix for the source-destination is represented by $H \in \mathbb{C}^{N_D \times N_S}$. Data symbols, channel elements, and noise are independent and identically distributed. Elements of the $F$, $G$, and $H$ matrices are zero-mean complex Gaussian random variables with variances of $\sigma_H^2$, $\sigma_F^2$, and $\sigma_G^2$, respectively.

Figure 1. AF HD MIMO wireless relay system consisting of one source of $N_S$ antennas, one relay of $N_R$ antennas, and one destination of $N_D$ antennas.
In addition, every channel is assumed to be quasi static, i.e., invariant during a frame data transmission. In other words, the CSI can be almost constant for a static relay. For a wireless relay, the fading is often relatively slow whenever the relative mobility of the relay is low. Furthermore, a destination can know the CSI of $F$, $G$, and $H$ through a backhaul connection to the central service station. In practice, the CSI at the destination can be obtained through standard training methods and fed back to the source and the relay. In fact, the relay destination and source are necessary to compute the desirable amplifying matrices and beamforming vectors. They are computed at a central service station and provided to them.

Data symbols at time $t$ are denoted by $d(t)$ so that the transmitted symbol vector at the source, $s(t) \in \mathbb{C}^{N_s \times 1}$, is given by

$$s(t) = a d(t)$$  

(1)

where $a \in \mathbb{C}^{N_s \times 1}$ and $d \in \mathbb{C}^{1 \times 1}$ are a transmit beamforming vector and a data symbol, respectively. An HD relay needs two-separate phases to receive and retransmit without interference. At the relay, the received signal column vector $r_{R}(t_1) \in \mathbb{C}^{N_R \times 1}$ in the first phase $t_1$ can be expressed as

$$r_{R}(t_1) = F s(t_1) + n_{R}(t_1)$$  

(2)

where $n_{R}(t_1) \in \mathbb{C}^{N_R \times 1}$ is an additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_{n_{R}}^2 I_{N_{R}}$ and zero mean vector. At the same phase, the received signal complex column vector $y(t_1) \in \mathbb{C}^{N_D \times 1}$ at the destination from the source is

$$y = H s(t_1) + n_{x}(t_1)$$  

(3)

where $n_{x}(t_1)$ is a thermal AWGN vector with zero mean vector and covariance matrix $\sigma_{n_{x}}^2 I_{N_{D}}$. In the next phase, $t_2$, the relay multiplies the relay amplifying matrix $W \in \mathbb{C}^{N_R \times N_R}$ to $r(t_2)$ and forwards $x(t_2) \in \mathbb{C}^{N_R \times 1}$ to the destination, which can be written as
\[ x(t_2) = Wr(t_1) \]  \hspace{1cm} (4)

During this time, it is assumed that the source does not transmit a signal, in order to avoid interference problems at the destination between the source and relay signals. Therefore, at the destination, the received signal column vector \( y(t_2) \in \mathbb{C}^{N_D \times 1} \) is

\[ y(t_2) = Gx(t_2) + n_x(t_2) \]  \hspace{1cm} (5)

The destination combines two sequential received signals \( y(t_1) \) and \( y(t_2) \) as follows:

\[ \hat{d} = b^H \begin{bmatrix} y(t_1) \\ y(t_2) \end{bmatrix} = b_1^H y(t_1) + b_2^H y(t_2) \]  \hspace{1cm} (6)

by using a receive beamforming vector \( b^H \triangleq [b_1^H \ b_2^H] \in \mathbb{C}^{1 \times 2N_D} \), where \( b_1^H \in \mathbb{C}^{1 \times N_D} \) and \( b_2^H \in \mathbb{C}^{1 \times N_D} \) combine the direct and relay path signals, respectively. By substituting equations (4) and (5) into equation (6), the expected data symbol \( \hat{d} \) can be rewritten as

\[ \hat{d} = (b_1^H Ha + b_2^H G W F a) d + b_1^H n_{x_1} + b_2^H n_{x_2} + b_2^H G W n_s \]  \hspace{1cm} (7)

where \( n_{x_1} = n_x(t_1) \) and \( n_{x_2} = n_x(t_2) \) are used for simplicity and convenience.
CHAPTER 3
AMPLIFY-AND-FORWARD HALF-DUPLEX MMSE STRATEGIES

The purpose of this section is to design optimal beamforming vectors \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) and a relay amplifying matrix \( \mathbf{W} \) based on the MMSE by individually and separately constraining the transmit power at the source and the relay: (1) transmit power restraints (TPRs), (2) aggregate power restraint (APR), (3) source power restraint (SPR), and (4) relay power restraint (RPR). In detail, in subsection 3.1, beamforming vectors and relay processing matrices are derived in the case of two individual transmit power restraints on the source and relay. Subsection 3.2 shows the case of an aggregate TPR on both the source and the relay. In subsections 3.3 and 3.4, it is assumed that the transmitted power is limited only at the source and only at the relay, respectively.

3.1 Transmit Power Restraints

The optimal beamforming vectors and relay amplifying matrix for the AF MIMO HD wireless relay network based on MMSE criterion are derived under transmit power restraints at the source and the relay. Minimizing the mean square error (MSE) between the transmit signal \( \hat{d} \) from the source and the detected signal \( \hat{d} \) at the destination under the transmit power restraints at the source and relay can be written as

\[
(W^*, a^*, b_1^*, b_2^*) = \arg \min_{(W,a,b_1,b_2)} J(W,a,b_1,b_2) \tag{8}
\]

s.t. \( E[\|s\|^2] = P_S \) and \( E[\|x\|^2] = P_r \) \tag{9}

where the superscript star * in equation (8) means the optimum, \( P_S \) and \( P_r \) in equation (9) are the transmitted aggregate power at the source and the relay, respectively, as

\[
P_S = \|a\|^2 \tag{10}
\]

\[
P_r = \|\mathbf{W}a\|^2 + \sigma_n^2 \|\mathbf{W}\|_F^2 \tag{11}
\]
and the cost function $J(W, a, b_1, b_2) \triangleq E \left[|d - d|^2\right]$ with $E[||d||^2] = 1$ can be written as

$$J(W, a, b_1, b_2) = 1 - 2Re(b_1^H H^H W^H a) - 2Re(b_2^H G W^H F^H a) + 2Re(a^H H^H W^H F^H b_1 b_2^H G W^H a) + |a^H H^H W^H F^H b_1|^2 + \sigma_{n_x}^2 ||b_2^H G W^H b_2||^2 + \sigma_{n_x}^2 (||b_1||^2 + ||b_2||^2) + |b_2^H G W^H F^H a|^2$$  \(12\)

In equation (12), it is assumed that all nodes have the same thermal noise power, i.e., $\sigma_{n_x}^2 = \sigma_{n_x}^2 = \sigma_{n_x}^2$. As shown in equation (12), since the cost function $J(W, a, b_1, b_2)$ is defined as the MSE, the smaller the cost function value, the smaller the MSE [12]. Namely, the system BER performance improves as the cost function value decreases. This will be verified through the simulation in Chapter 4. The optimization problem with the transmit power restraints at both the source and the relay can be written as

$$L(W, a, b_1, b_2, \lambda_s, \lambda_r) = J(W, a, b_1, b_2) + \lambda_s (E[||s||^2] - P_s) + \lambda_r (E[||x||^2] - P_r)$$  \(13\)

using the Lagrangian multipliers $\lambda_s$ and $\lambda_r$ [13]. The restrained Lagrangian optimization $L(W, a, b_1, b_2, \lambda_s, \lambda_r)$ in equation (13) is henceforward stated as $L$. With the cyclic permutation and linearity attributes of the trace function [14] and the linear and nonlinear attributes of the complex derivative vector and matrix in the work of Hjørungnes and Gesbert [15], taking the partial derivative of $L$ with respect to $\{W, a, b_1, b_2, \lambda_s, \lambda_r\}$, respectively, results in

$$\frac{\partial L}{\partial W} = \sigma_{n_a}^2 G^H b_2 b_2^H G W^H + G^H b_2 b_2^H G W^H F^H a + \lambda_r \sigma_{n_s}^2 W - G^H b_2 a^H F^H + \lambda_r W F^H a + \lambda_r b_2 b_2^H H a^H F^H = 0$$  \(14\)

$$\frac{\partial L}{\partial a} = -H^H b_1 - F^H (W^H b_2 H^H b_2 + H^H b_2 H^H b_1 b_2^H G W^H a + F^H b_2 b_2^H W^H G^H a + H^H b_2 b_2^H H a + F^H b_2 b_2^H G W^H a + \lambda_s a + \lambda_r F^H W^H W^H F^H a) = 0$$  \(15\)

$$\frac{\partial L}{\partial b_1} = H a^H H^H b_1 - H a + H a^H F^H W^H G^H b_2 + \sigma_{n_x}^2 b_1 = 0$$  \(16\)

$$\frac{\partial L}{\partial b_2} = \sigma_{n_x}^2 b_2 + G W^H a^H b_1 + \sigma_{n_x}^2 G W^H G^H b_2 + G W^H a^H F^H W^H G^H b_2 - G W^H a = 0$$  \(17\)

$$\frac{\partial L}{\partial \lambda_s} = ||a||^2 - P_s = 0$$  \(18\)
\[ \frac{\partial L}{\partial \lambda_r} = ||WFa||^2 + \sigma_n^2 ||W||^2_F - P_r = 0 \]  

(19)

where \(0_N\) is an \(N \times N\) matrix consisting of all zero entries. Using the matrix inversion identity [16], i.e., \((A + uu^H)^{-1} = A^{-1} - (1 + u^HA^{-1}u)^{-1}A^{-1}uu^HA^{-1}\), when \(A\) and \(u\) are an \(N \times N\) matrix and an \(N \times 1\) column vector, respectively, the optimal relay amplifying matrix \(W^*\), the optimal beamforming vectors \((a^*, b_1^*, \text{and } b_2^*)\), and the Lagrangian multipliers \((\lambda_s^* \text{ and } \lambda_r^*)\) can be obtained, respectively, as

\[
W^* = \frac{(1-b_1^Hb_1)G^Hb_2^H}{\left(\sigma_n^2 + ||Fa||^2\right)(||G^Hb_2||^2 + \lambda_r)} \]

(20)

\[
a^* = \psi(H^Hb_1 + F^HW^HG^Hb_2 + \omega \lambda_r ||WF||_F^2)H^b_1
\]

\[-\omega \lambda_r F^HW^WFH^b_1 + \omega \lambda_r \||WF||_F^2P_rFH^GW^Hb_2 - \omega \lambda_r F^HW^WFH^GW^Hb_2 \]

(21)

\[
b_1^* = \frac{(1-a^Hb_1^Hb_2)Hb_1}{(\sigma_n^2 + ||Hb_1||^2)}
\]

(22)

\[
b_2^* = \frac{(1-a^Hb_1^Hb_2)GWFa}{1+a^Hb_1^Hb_2GWFa}
\]

(23)

\[
\lambda_r^* = \left(\frac{\sqrt{\frac{\|Fa\|^2 + \sigma_n^2 \|G^Hb_2^H \|^2}{\frac{P_r(\sigma_n^2 + \|Fa\|^2)(1-b_1^Hb_1)}}}}{\|G^Hb_2\|^2} - \|G^Hb_2\|^2 \right)^+
\]

(24)

\[
\lambda_s^* = \left(0.5(-\epsilon + (\epsilon^2 - 4P_s\theta)^{1/2})P_s^{-1} \right)^+
\]

(25)

where \((\zeta)^+ = \max(0, \zeta)\),

\[
\psi = (\|H^Hb_1 + F^HW^HG^Hb_2\|^2 + \lambda_r \|WF\|^2_F + \lambda_s)^{-1}
\]

(26)

\[
\omega = (\|H^Hb_1 + F^HW^HG^Hb_2\|^2 + \lambda_s)^{-1}
\]

(27)

\[
\Gamma = (\sigma_n^2 GW^H G^H + \sigma_n^2 I_N)^{-1}
\]

(28)

\[
\epsilon = (2\|H^Hb_1 + F^HW^HG^Hb_2\|^2 + \lambda_r \|WF\|^2_F P_s - a^H(H^Hb_1 + F^HW^HG^Hb_2)
\]

(29)
Due to space limitations, the detailed proofs of equations (20) to (25) are presented in Appendix A. The optimal values $W^*, a^*, b_1^*, b_2^*$, $\lambda_r^*$, and $\lambda_s^*$ are interrelated. Hence, an iterative method used to solve these numerically will be discussed in Chapter 4.

### 3.2 Aggregate Power Restraint

In this subsection, a set of MMSE-based beamforming vectors and a relay amplifying matrix is derived, when an aggregate transmit power restraint is imposed on the source and the relay. This optimization problem can be defined as

$$
\theta = \left( \|H^Hb_1 + F^HW^HG^Hb_2\|^2 (\|H^Hb_1 + F^HW^HG^Hb_2\|^2 + \lambda_r \|WF\|^2_{E}) \right)P_s - a^H(H^Hb_1 + F^HW^HG^Hb_2)(\|H^Hb_1 + F^HW^HG^Hb_2\|^2) - \lambda_r a^H(\|WF\|^2_{E}H^Hb_1 - F^HW^HW^HW^HG^Hb_2). \tag{30}
$$

$$
\text{s.t. } P_t = \text{const.}
$$

where $P_t$ indicates the aggregate transmitted power at the source and the relay. Hence, using the Lagrangian multiplier $\lambda_t$, the constrained Lagrangian optimization problem can be written as

$$
L(W, a, b_1, b_2, \lambda_t) = J(W, a, b_1, b_2) + \lambda_t(E[\|s\|^2] + E[\|x\|^2] - P_t). \tag{33}
$$

Following the previous procedures, the optimal relay amplifying matrix $W^*$, the optimal beamforming vectors $\{a^*, b_1^*, b_2^*\}$, and the Lagrangian multiplier $\lambda_t^*$ can be obtained, respectively, as

$$
W^* = \frac{(1-b_1^HHa)G^Hb_2a^HF^H}{(\sigma_{n_s}^2 + \|Fa\|^2)(\|G^Hb_2\|^2 + \lambda_t^*)}
$$

$$
a^* = \Theta(H^Hb_1 + F^HW^HG^Hb_2 + \lambda_t \xi \|WF\|^2_{E}H^Hb_1
$$

$$
- \lambda_t \xi F^HW^HW^FH^Hb_1 + \lambda_t \xi \|WF\|^2_{E}F^HW^HW^HG^Hb_2 - \lambda_t \xi F^HW^HW^FW^HW^HG^Hb_2)
$$

$$
b_1^* = \frac{(1-a^H F^HW^HG^Hb_2)Ha}{(\sigma_{n_x}^2 + \|Ha\|^2)}
$$

$$
b_2^* = \frac{(1-b_1^HHa)G^Hb_2}{(\sigma_{n_s}^2 + \|Fb_1\|^2)}
$$
where $(\zeta)^+ = \max(0, \zeta)$, $\Gamma$ is the same as equation (28), and

$$\Theta = (\|H^H b_1 + F^H W^H G^H b_2\|^2 + \lambda_t \|WF\|_F^2 + \lambda_t)^{-1}, \quad (39)$$

$$\xi = (\|H^H b_1 + F^H W^H G^H b_2\|^2 + \lambda_t)^{-1}. \quad (40)$$

### 3.3 Source Power Restraint

Similar to the previous subsections, power can be constrained only at the source. Hence, this constrained Lagrangian optimization problem can be transformed as

$$\arg \min_{(W, a, b_1, b_2)} J(W, a, b_1, b_2) \quad (41)$$

subject to $E[\|s\|^2] = P_s \quad (42)$

where $P_s$ indicates the transmitted power only at the source. Therefore, the constrained Lagrangian optimization problem with the Lagrangian multiplier $\lambda_s$ can be written as

$$L(W, a, b_1, b_2, \lambda_s) = J(W, a, b_1, b_2) + \lambda_s (E[\|s\|^2] - P_s) \quad (43)$$

Using the same previous method, the optimal relay amplifying matrix $W^*$, the optimal beamforming vectors $\{a^*, b_1^*, b_2^*\}$, and the optimal Lagrangian multiplier $\lambda_s^*$ can be obtained, respectively, as

$$W^* = \frac{(1 - b_1^H Ha)G^H b_2 a^H F^H}{\left(\sigma_{n_s}^2 + \|Fa\|^2\right)\|G^H b_2\|^2}, \quad (44)$$

$$a^* = \frac{H^H b_1 + F^H W^H G^H b_2}{\|H^H b_1 + F^H W^H G^H b_2\|^2 + \lambda_s^*}, \quad (45)$$

$$b_1^* = \frac{(1 - a^H F^H W^H G^H b_2)Ha}{\|Ha\|^2 + \sigma_{n_s}^2}, \quad (46)$$
\[ b_2^* = \frac{(1-a^H H^H b_1)G W F a}{1+a^H F H W^H G H^* G W F a}, \] (47)

\[ \lambda_s^* = (a^H (H^H b_1 + F^H W^H G^H b_2) P_s^{-1} - \| H^H b_1 + F^H W^H G^H b_2 \|^2)^+. \] (48)

Using the pseudo-inverse property [17], i.e., \((A^H u^H u\ A)^\dagger A^H u = A^H u\|A^H u\|^{-2},\) when \(A\) and \(u\) are an \(N \times N\) matrix and an \(N \times 1\) column vector, respectively. Here, \(\Gamma\) is the same as equation (28).

### 3.4 Relay Power Restraint

Applying the same method as in the case of the source power restraint, when the transmit power is limited to \(P_r\) only in the relay, this problem can be modified as

\[ (W^*, a^*, b_1^*, b_2^*) = \arg \min_{(W, a, b_1, b_2)} J(W, a, b_1, b_2) \] (49)

\[ \text{s.t. } E[\|x\|^2] = P_r \] (50)

Hence, using the Lagrangian multiplier \(\lambda_r\), the optimization problem with the power restraint only at the relay can be written as

\[ L(W, a, b_1, b_2, \lambda_r) = J(W, a, b_1, b_2) + \lambda_r (E[\|x\|^2] - P_r) \] (51)

The optimal solutions of \(L(W, a, b_1, b_2, \lambda_r)\) in (51) can be obtained, respectively, as

\[ W^* = \frac{(1-b_1^H H a)G H^* b_2^* a^H F H}{(\sigma_2^2 + \| F a \|^2)(\| H^H b_2 \|^2 + \lambda_r)} \] (52)

\[ a^* = (\| H^H b_1 + F^H W^H G^H b_2 \|^2 + \lambda_t \| W F \|_F^2)^{-1}(H^H b_1 + F^H W^H G^H b_2)(I_N + \omega \lambda_r \| W F \|_F^2 I_N - \omega \lambda_r W F F^H W^H), \] (53)

\[ b_1^* = \frac{(1-a^H F H W^H G^H b_2) H a}{\| H a \|^2 + \sigma_2^2} \] (54)

\[ b_2^* = \frac{(1-a^H H^H b_1)G W F a}{1+a^H F H W^H G H^* G W F a} \] (55)
\[
\lambda_r^* = \left( \frac{\sqrt{[F_a]^{4\|G^Hb_2\|^2 + \sigma_n^2\|G^Hb_2a^Hf^H\|^2}}}{\sqrt{P_r(\sigma_n^2 + \|F_a\|^2)(1-b_1^Hn_a)^{-1}}} - \|G^Hb_2\|^2 \right)^+
\]  \hspace{1cm} (56)

Here, \( \omega \) and \( \Gamma \) are the same as equations (27) and (28), respectively.
4.1 Assumptions

The Monte-Carlo BER simulation results under various combinations of transmit power restraints are performed to evaluate the proposed AF MIMO HD wireless relay strategy in this thesis. Here, it is assumed that the perfectly known CSI is available and is considered in the cases of $N_S = N_D = 2, 4$ and $N_R = 2, 3, 4$. The signals, which are transmitted from the sources with unit power, are modulated by quadrature phase shift keying (QPSK). The complex channel matrices $F$, $G$, and $H$ are generated from independent Gaussian random variables. During data transmission, the channels are invariant, as stated in Chapter 2. It is also assumed that $P_s = P_r = 1$ and $P_t = 2$. Again, the power restraints are used to find the optimum relay amplifying matrix and beamforming vectors in the Lagrangian optimization formulations. In the end, all nodes are assumed to have the same thermal noise power, i.e., $\sigma_{n_s}^2 = \sigma_{n_k}^2$.

4.2 Iterative Algorithm

When it comes to the derived optimal solutions in Chapter 3, these equations are related to one another. In this condition, an iterative algorithm can provide a way of solving the problem. This algorithm calculates one variable at a time while fixing the others. The cost function of a variable is convex and convergent when the other variables are fixed. As $k$ increases one by one, $L_k$ gradually decreases because the cost function is defined as the MSE in this thesis. Since $L_k$ is positive and the convergence of $L_k$ is guaranteed, the difference between $L_{k-1}$ and $L_k$ can be used as a stopping criterion in the iterative algorithm. For example, Table 1 can be used for optimizing the case of transmit power restraints on both the source and the relay.
In Table 1, initial values are \( k = 0, W_0 = I_{N_R}, \) and \( b_{1_0} = b_{2_0} = [1, \ldots, 1], \lambda_s = \lambda_r = 1, L_0 = 10, \) where \( L \) is \( L(W, a, b_1, b_2, \lambda_s, \lambda_r), \) as defined in Chapter 3.

**TABLE 1**

**ITERATION ALGORITHM PROCEDURE**

| Step 1 | Initialization: \( k = 0 \)
\[
W_0 = I_{N_R}, \quad b_{1_0} = b_{2_0} = [1, \ldots, 1], \quad \lambda_s = \lambda_r = 1, \quad L_0 = 10
\]

| Step 2 | Iteration: \( k \leftarrow k + 1 \)
\[
\begin{align*}
    a_k &= f_a(W_{k-1}, b_{1_{k-1}}, b_{2_{k-1}}, \lambda_s, \lambda_r) \\
    W_k &= f_w(a_k, b_{1_{k-1}}, \lambda_r) \\
    b_{1_k} &= f_{b_1}(W_k, a_k, b_{2_k}) \\
    b_{2_k} &= f_{b_2}(W_k, a_k, b_{1_k}) \\
    \lambda_r &= f_{\lambda_r}(a_k, b_{1_k}, b_{2_k}) \\
    \lambda_s &= f_{\lambda_s}(W_k, a_k, b_{1_k}, b_{2_k}, \lambda_r) \\
    L_k &= f_{L_k}(W_k, a_k, b_{1_k}, b_{2_k}, \lambda_s, \lambda_r)
\end{align*}
\]

| Step 3 | If \( L_{k-1} - L_k \leq \bar{\omega}, \) then go to Step 4, and stop;
otherwise, go back to Step 2 (\( \bar{\omega} = 0.0001 \))

| Step 4 | \( W = W_k, a = a_k, b_1 = b_{1_k}, b_2 = b_{2_k} \)

4.3 **Analysis of Simulations**

Figure 2 shows the cost function versus signal-to-noise (SNR) ratio under transmit power restraints at the source and the relay for \( N_S = N_D = 2 \) and \( N_R = 2, 3, 4, \) respectively, in the AF MIMO wireless relay network. The BER performance is enhanced as the number of relay antennas \( N_R \) increases because the cost function value in Figure 2 decreases as \( N_R \) increases.

Figure 3 shows bit error rate versus SNR under the transmit power restraint at both the source and the relay, aggregate power restraint, and non-optimum case for \( N_S = N_D = 2 \) and \( N_R = 2, 3, 4, \) respectively, in the AF MIMO wireless relay network. At a low input SNR, when the aggregate of the source and relay power is constrained, the better BER performance is observed,
compared to the case where the transmit power at both the source and the relay is constrained, individually and separately, as shown in Figure 3. However, the almost identical BER performance is yielded at a high input SNR. Finally, it can be also seen that the non-optimum relay amplifying matrix (e.g., $W = I$) and equal beamforming vectors ($b_1 = b_2 = [1, \ldots, 1]$) yield significantly worse performance than the proposed case of optimum relay amplifying matrix and optimum beamforming vectors.

Figure 2. Cost function versus signal-to-noise ratio under transmit power restraint (TPR) at source and relay for $N_S = N_D = 2$ and $N_R = 2, 3, 4$, respectively, in AF MIMO wireless relay network.

Figure 3. Bit error rate versus signal-to-noise ratio under transmit power restraint (TPR) at both source and relay, aggregate power restraint (APR), and non-optimum case for $N_S = N_D = 2$ and $N_R = 2, 3, 4$, respectively, in AF MIMO wireless relay network.
Figure 4 shows bit error rate versus signal-to-noise ratio under the transmit power restraint at both the source and the relay and the aggregate power restraint for \( N_S = N_D = 2, 4 \) and only \( N_R = 2 \), respectively in the AF MIMO wireless relay network. The same observation as shown in Figure 3 is investigated in Figure 4. It is observed that more than 1 dB better BER performance is observed when APR is used at the low SNR rather than the TPR.

Figure 5 shows bit error rate versus signal-to-noise ratio under the source power restraint at the source and the relay power restraint at the relay for \( N_S = N_D = 2 \) and \( N_R = 2, 3 \), respectively in the AF MIMO wireless relay network. As in the case of the two previous power restraints, as \( N_R \) increases, the better BER performance is observed. In addition, at a low input SNR, BER performance under the power restraint only at the relay is better than BER performance under the power restraint only at the source. In contrast to this, at a high input SNR, the better BER performance is observed when the power is constrained only at the source.
Figure 5. Bit error rate versus signal-to-noise ratio under source power restraint (SPR) at source and relay power restraint (RPR) at relay for $N_S = N_D = 2$ and $N_R = 2, 3$, respectively, in AF MIMO wireless relay network.
CHAPTER 5
CONCLUSIONS

This thesis presented a set of optimum relay amplifying matrices and source-destination beamforming vectors for the AF HD MIMO wireless relay system under diverse transmit power restraints at the source and the relay using the MMSE criteria. It was observed that BER performance is enhanced as $N_R$ increases, regardless of the locations of the transmit power restraints. In addition, for a given $N_R$, as $N_S$ and $N_D$ increase, the BER performance improves. Through the simulation results, it is suggested that algorithms be used for the case of imposing an aggregate transmit power restraint with the limited power resources. This is because at a low SNR, the APR shows a better BER than the TPR. In addition, it was observed that at a high SNR, the APR shows the same BER as the TPR. Additionally, it is recommended that the transmit power restraint only at the source is imposed when either the source or relay transmit power needs to be allocated. This is because it was observed that the RPR shows a better BER than the SPR for a low SNR, but the RPR shows a worse BER than the SPR for a high SNR under the same environment. The results in this thesis can be useful for designing a power-efficient AF MIMO HD wireless relay network system.
LIST OF REFERENCES


APPENDICES
APPENDIX A

PROOF: OPTIMAL RELAY AMPLIFYING MATRICES AND BEAMFORMING VECTORS

The detail proofs of a set of optimal relay amplifying matrices and source-destination beamforming vectors under transmit power restraints are presented in this appendix. The derivations of the other power restraints are similar to this derivation.

Derivation of Equation (20)

From equation (14),

\[ \frac{\partial L}{\partial W^*} = \sigma_n^2 G^H b_1 b_2^H GW + G^H b_2 b_2^H GWF a^H F^H + \lambda_r \sigma_n^2 W - G^H b_2 a^H F^H + \]

\[ \lambda_r W F a^H F^H + G^H b_2 b_2^H H a a^H F^H = 0_N \]  \hspace{1cm} (A1)

Hence,

\[ G^H b_2 (1 - b_1^H H a) a^H F^H = (G^H b_2 b_2^H G + \lambda_r I_{N_r}) W (F a a^H F^H + \sigma_n^2 I_{N_r}) \]  \hspace{1cm} (A2)

Then,

\[ W = (G^H b_2 b_2^H G + \lambda_r I_{N_r})^{-1} G^H b_2 (1 - b_1^H H a) a^H F^H (F a a^H F^H + \sigma_n^2 I_{N_r})^{-1} \]  \hspace{1cm} (A3)

Apply the matrix inversion lemma: \((A + uu^H)^{-1} = A^{-1} - (1 + u^H A^{-1} u)^{-1} A^{-1} uu^H A^{-1}\)

For \((G^H b_2 b_2^H G + \lambda_r I_{N_r})^{-1} = (\lambda_r I_{N_r} + G^H b_2 b_2^H G)^{-1}\)

Let \(A = \lambda_r I_{N_r}, u = G^H b_2, u^H = b_2^H G\). Then,

\[ (\lambda_r I_{N_r} + G^H b_2 b_2^H G)^{-1} = (\lambda_r I_{N_r})^{-1} - \frac{(\lambda_r I_{N_r})^{-1} G^H b_2 b_2^H G (\lambda_r I_{N_r})^{-1}}{1 + \lambda_r^{-1} b_2^H G G^H b_2} \]

\[ = \lambda_r^{-1} I_{N_r} - \frac{(\lambda_r I_{N_r})^{-1} G^H b_2 b_2^H G (\lambda_r I_{N_r})^{-1}}{1 + \lambda_r^{-1} b_2^H G G^H b_2} = \lambda_r^{-1} I_{N_r} - \frac{\lambda_r^{-2} G^H b_2 b_2^H G}{1 + \lambda_r^{-1} b_2^H G G^H b_2} \]  \hspace{1cm} (A4)
where $\mathbf{G} \in \mathbb{C}^{N_R \times N_D}$ and $\mathbf{b}_2 \in \mathbb{C}^{N_D \times 1}$

For $(\mathbf{F}a^H \mathbf{F}^H + \sigma_{ns}^2 \mathbf{I}_{N_r})^{-1} = (\sigma_{ns}^2 \mathbf{I}_{N_r} + \mathbf{F}a^H \mathbf{F}^H)^{-1}$, let $\mathbf{A} = \sigma_{ns}^2 \mathbf{I}_{N_r}$, $\mathbf{u} = \mathbf{F}a$, $\mathbf{u}^H = a^H \mathbf{F}^H$. Then, applying the matrix inversion lemma,

$$
(\sigma_{ns}^2 \mathbf{I}_{N_r} + \mathbf{F}a^H \mathbf{F}^H)^{-1} = (\sigma_{ns}^2 \mathbf{I}_{N_r})^{-1} - \frac{(\sigma_{ns}^2 \mathbf{I}_{N_r})^{-1} \mathbf{F}a^H \mathbf{F}^H (\sigma_{ns}^2 \mathbf{I}_{N_r})^{-1}}{1 + \sigma_{ns}^2 a^H \mathbf{F}^H \mathbf{F}a}
$$

$$
= \sigma_{ns}^{-2} \mathbf{I}_{N_r} - \frac{\sigma_{ns}^{-4} \mathbf{F}a^H \mathbf{F}^H}{1 + \sigma_{ns}^{-2} \| \mathbf{F}a \|^2}
$$

(A5)

where $\mathbf{F} \in \mathbb{C}^{N_R \times N_S}$ and $\mathbf{a} \in \mathbb{C}^{N_S \times 1}$.

Substituting equations (A4) and (A5) into equation (A3) yields

$$
\mathbf{W} = (\mathbf{G}^H \mathbf{b}_2 \mathbf{b}_2^H \mathbf{G} + \lambda_r I_{N_r})^{-1} \mathbf{G}^H \mathbf{b}_2 (1 - \mathbf{b}_1^H \mathbf{H} \mathbf{a}) a^H \mathbf{F}^H (\mathbf{F}a a^H \mathbf{F}^H + \sigma_{ns}^2 \mathbf{I}_{N_r})^{-1}
$$

$$
= \left( \lambda_r I_{N_r} - \frac{\lambda_r^{-2} \mathbf{G}^H \mathbf{b}_2 \mathbf{b}_2^H \mathbf{G}}{1 + \lambda_r^{-1} \| \mathbf{G}^H \mathbf{b}_2 \|^2} \right) \times \mathbf{G}^H \mathbf{b}_2 (1 - \mathbf{b}_1^H \mathbf{H} \mathbf{a}) a^H \mathbf{F}^H \times \left( \sigma_{ns}^{-2} \mathbf{I}_{N_r} - \frac{\sigma_{ns}^{-4} \mathbf{F}a^H \mathbf{F}^H}{1 + \sigma_{ns}^{-2} \| \mathbf{F}a \|^2} \right)
$$

(A6)

By multiplying $\frac{\sigma_{ns}^2}{\sigma_{ns}^4}$ to the right most term,

$$
\mathbf{W} = \left( \lambda_r^{-1} I_{N_r} + \lambda_r^{-2} I_{N_r} \| \mathbf{G}^H \mathbf{b}_2 \|^2 - \lambda_r^{-2} \mathbf{G}^H \mathbf{b}_2 \mathbf{b}_2^H \mathbf{G} \right) \times \mathbf{G}^H \mathbf{b}_2 (1 - \mathbf{b}_1^H \mathbf{H} \mathbf{a}) a^H \mathbf{F}^H \times \left( \sigma_{ns}^{-2} \mathbf{I}_{N_r} - \frac{\sigma_{ns}^{-4} \mathbf{F}a^H \mathbf{F}^H}{1 + \sigma_{ns}^{-2} \| \mathbf{F}a \|^2} \right)
$$

(A7)

$$
= \left( \lambda_r^{-1} I_{N_r} + \lambda_r^{-2} I_{N_r} \| \mathbf{G}^H \mathbf{b}_2 \|^2 - \lambda_r^{-2} \mathbf{G}^H \mathbf{b}_2 \mathbf{b}_2^H \mathbf{G} \right) \times \mathbf{G}^H \mathbf{b}_2 (1 - \mathbf{b}_1^H \mathbf{H} \mathbf{a}) a^H \mathbf{F}^H \times \left( \sigma_{ns}^{-2} I_{N_r} - \frac{\sigma_{ns}^{-2} \mathbf{F}a a^H \mathbf{F}^H}{\sigma_{ns}^{-2} + \| \mathbf{F}a \|^2} \right)
$$

(A8)

$$
= \left( \lambda_r^{-1} I_{N_r} + \lambda_r^{-2} I_{N_r} \| \mathbf{G}^H \mathbf{b}_2 \|^2 - \lambda_r^{-2} \mathbf{G}^H \mathbf{b}_2 \mathbf{b}_2^H \mathbf{G} \right) \times \mathbf{G}^H \mathbf{b}_2 (1 - \mathbf{b}_1^H \mathbf{H} \mathbf{a}) a^H \mathbf{F}^H \times \left( \frac{\lambda_r^{-1} I_{N_r} + \lambda_r^{-2} I_{N_r} \| \mathbf{G}^H \mathbf{b}_2 \|^2 - \lambda_r^{-2} \mathbf{G}^H \mathbf{b}_2 \mathbf{b}_2^H \mathbf{G}}{\sigma_{ns}^{-2} \| \mathbf{F}a \|^2} \right)
$$

(A9)
$$= \left( \frac{1}{(1 + \lambda_r^{-1}\|G^Hb_2\|^2)(\sigma_n^2 + \|Fa\|^2)} \right) \times \left( \lambda_r^{-1}I_N + \lambda_r^{-2}I_N, \|G^Hb_2\|^2 - \lambda_r^{-2}G^Hb_2b_2^HG \right)$$
$$\times (G^Hb_2(1 - b_1^HHa)a^HF^H) \times \left( I_N + \sigma_n^{-2}I_N, \|Fa\|^2 - \sigma_n^{-2}Fa a^HF^H \right)$$
(A10)

$$= \left( \frac{1}{(1 + \lambda_r^{-1}\|G^Hb_2\|^2)(\sigma_n^2 + \|Fa\|^2)} \right) \times \left( a^HF^H + \sigma_n^{-2}\|Fa\|^2a^HF^H - \sigma_n^{-2}Fa a^HF^H \right)$$
$$\times (\lambda_r^{-1}G^Hb_2 + \lambda_r^{-2}\|G^Hb_2\|^2G^Hb_2 - \lambda_r^{-2}G^Hb_2b_2^HG^HG^Hb_2) \times (1 - b_1^HHa)$$
(A11)

Because $\lambda_r^{-2}\|G^Hb_2\|^2G^Hb_2$ and $\sigma_n^{-2}\|Fa\|^2a^HF^H$ are canceled in (A11),

$$W = \left( \frac{1}{(1 + \lambda_r^{-1}\|G^Hb_2\|^2)(\sigma_n^2 + \|Fa\|^2)} \right)$$

$$\times (\lambda_r^{-1}G^Hb_2) \times (1 - b_1^HHa) \times (a^HF^H)(A12)$$

By multiplying $\frac{\lambda_r}{\lambda_r}$,

$$W = \left( \frac{\lambda_r^{-1}G^Hb_2(1 - b_1^HHa)a^HF^H}{(1 + \lambda_r^{-1}\|G^Hb_2\|^2)(\sigma_n^2 + \|Fa\|^2)} \right) \times \frac{\lambda_r}{\lambda_r}$$
(A13)

Therefore, the optimal $W$ can be written as

$$W^* = \left( \frac{(1 - b_1^HHa)G^Hb_2a^HF^H}{(\|G^Hb_2\|^2 + \lambda_r)(\|Fa\|^2 + \sigma_n^2)} \right)$$
(A14)

**Derivation of Equation (21)**

Here, $a \in \mathbb{C}^{N_S \times 1}$, $b_1 \in \mathbb{C}^{N_D \times 1}$, $W \in \mathbb{C}^{N_R \times N_R}$, $G \in \mathbb{C}^{N_D \times N_R}$, $H \in \mathbb{C}^{N_D \times N_S}$, $F \in \mathbb{C}^{N_R \times N_S}$

From equation (15),

$$\frac{\partial L}{\partial a^H} = -H^Hb_1 - F^HWH^HG^Hb_2 + H^Hb_1b_2^HGWFa + F^HWH^HG^Hb_2b_2^HG^HG^Hb_2 + H^Hb_1b_1^HHa + F^HWH^Hb_2b_2^HGWFa + \lambda_s a + \lambda_r F^HWH^WFa = 0$$
(B1)

$a$ can be written as

$$a = \left( \|H^Hb_1 + F^HWH^HG^Hb_2\|^2I_{N_S} + \lambda_s I_{N_S} + \lambda_r F^HWH^WF \right)^{-1}(H^Hb_1 + F^HWH^Hb_2)$$
(B2)
In order to apply the matrix inversion lemma, substituting as below

\[
\omega = \left( \|H^H b_1 + F^H W^H G^H b_2 \|^2 + \lambda_s \right) I_{N_s} + \lambda_r F^H F b_2 G^H b_2 \left( \beta G^H b_2 a^H F^H F \right)^{-1} (H^H b_1 + F^H W^H G^H b_2) \tag{B3}
\]

\[
= \left( \omega I_{N_s} + (\lambda_r \beta^2 F^H F b_2 G^H b_2) (a^H F^H F) \right)^{-1} (H^H b_1 + F^H W^H G^H b_2) \tag{B4}
\]

Applying the matrix inversion lemma:

\[
\omega = \left( \|H^H b_1 + F^H W^H G^H b_2 \|^2 + \lambda_s \right)
\]

\[
\beta = \frac{(1 - b_1^H Ha)}{(\sigma_n^2 + \|Fa\|^2)(\|G^H b_2\|^2 + \lambda_r)}
\]

\[
WF = \frac{(1 - b_1^H Ha) G^H b_2 a^H F^H F}{(\sigma_n^2 + \|Fa\|^2)(\|G^H b_2\|^2 + \lambda_r)} = \beta G^H b_2 a^H F^H F
\]

\[
(WF)^H = F^H W^H = F^H (F b_2^H G^H) = F^H F b_2^H G^H
\]

Applying the matrix inversion lemma: \((A + uv^H)^{-1} = A^{-1} - (1 + u^H A^{-1} v) A^{-1} u v^H A^{-1}, \)

\[
\left( \omega I_{N_s} + (\lambda_r \beta^2 F^H F b_2 G^H b_2) (a^H F^H F) \right)^{-1}
\]

\[
= \left( \omega I_{N_s} \right)^{-1} - \frac{(\omega I_{N_s})^{-1} (\lambda_r \beta^2 F^H F b_2 G^H b_2) (a^H F^H F) (\omega I_{N_s})^{-1}}{1 + (a^H F^H F) (\omega I_{N_s})^{-1} (\lambda_r \beta^2 F^H F b_2 G^H b_2)} \tag{B5}
\]

\[
= \omega^{-1} I_{N_s} - \frac{\omega^{-2} \lambda_r F^H W^H W F}{1 + \omega^{-1} (a^H F^H F) (\omega I_{N_s})^{-1} \lambda_r \beta^2 F^H F b_2 G^H b_2} \tag{B6}
\]

\[
= \omega^{-1} I_{N_s} - \frac{\omega^{-2} \lambda_r F^H W^H W F}{1 + \omega^{-1} \lambda_r tr(a^H F^H F \beta^2 F^H F b_2 G^H b_2)} \tag{B7}
\]

\[
= \omega^{-1} I_{N_s} - \frac{\omega^{-2} \lambda_r F^H W^H W F}{1 + \omega^{-1} \lambda_r tr(a^H F^H F \beta F F b_2 G^H b_2)} \tag{B8}
\]

The trace is invariant under cyclic permutations, i.e.,

\[
tr(ABCD) = tr(BCDA) = tr(CDAB) = tr(DABC).
\]

Therefore, it can be written as
From \((WF)^H = F^H W^H = F^H (Fab_2^H G \beta) = F^H Fab_2^H G \beta\), it can be written as

\[
\begin{align*}
&= \omega^{-1} I_{Ns} - \frac{\omega^{-2} \lambda_r F^H W^H WF}{1 + \omega^{-1} \lambda_r \|WW^H\|_F} = \omega^{-1} I_{Ns} - \frac{\omega^{-2} \lambda_r F^H W^H WF}{1 + \omega^{-1} \lambda_r \|WF\|_F^2} \\
&= \omega^{-1} I_{Ns} - \frac{\omega^{-2} \lambda_r F^H W^H WF \times \omega}{\omega} = \omega^{-1} I_{Ns} - \frac{\omega^{-1} \lambda_r F^H W^H WF}{\omega + \lambda_r \|WF\|_F^2} \\
&= \frac{I_{Ns} + \omega^{-1} I_{Ns} \lambda_r \|WF\|_F^2 - \omega^{-1} \lambda_r F^H W^H WF}{\omega + \lambda_r \|WF\|_F^2}.
\end{align*}
\] (B11)

Substituting equation (B14) into equation (B4) yields

\[
\begin{align*}
\mathbf{a} &= \frac{I_{Ns} + \omega^{-1} I_{Ns} \lambda_r \|WF\|_F^2 - \omega^{-1} \lambda_r F^H W^H WF}{\Omega} (H^H b_1 + F^H W^H G^H b_2) \\
&= \frac{I_{Ns} + \omega^{-1} I_{Ns} \lambda_r \|WF\|_F^2 - \omega^{-1} \lambda_r F^H W^H WF}{\Omega} (H^H b_1 + F^H W^H G^H b_2) \\
&= \frac{I_{Ns} H^H b_1 + \omega^{-1} I_{Ns} \lambda_r \|WF\|_F^2 H^H b_1 - \omega^{-1} \lambda_r F^H W^H W F H^H b_1}{\Omega} + \\
&= \frac{I_{Ns} F^H W^H G^H b_2 + \omega^{-1} I_{Ns} \lambda_r \|WF\|_F^2 F^H W^H G^H b_2 - \omega^{-1} \lambda_r F^H W^H W F F^H W^H G^H b_2}{\Omega}.
\end{align*}
\] (B15)

where \(\Omega = (\|H^H b_1 + F^H W^H G^H b_2\|^2 + \lambda_s) + \lambda_r \|WF\|_F^2\)

\[
\begin{align*}
&= \frac{H^H b_1 + \omega^{-1} \lambda_r \|WF\|_F^2 H^H b_1 - \omega^{-1} \lambda_r F^H W^H W F H^H b_1}{\Omega} + \\
&= \frac{H^H b_1 + \omega^{-1} \lambda_r \|WF\|_F^2 H^H b_1 - \omega^{-1} \lambda_r F^H W^H W F H^H b_1}{\Omega}.
\end{align*}
\] (B16)
\[
\frac{H^H b_1 + F^H W^H G^H b_2}{\Omega} + \frac{\omega^{-1}\lambda_r\|WF\|^2\|F^H b_1 - \omega^{-1}\lambda_r F^H W^H W F H^H b_1}{\Omega} + \\
\frac{\omega^{-1}\lambda_r\|WF\|^2\|F^H W^H G^H b_2 - \omega^{-1}\lambda_r F^H W^H W F F^H W H^H G^H b_2}{\Omega}.
\]

Therefore, the optimal \( a \) can be written as
\[
a^* = \frac{H^H b_1 + F^H W^H G^H b_2}{(\|H^H b_1 + F^H W^H G^H b_2\|^2 + \lambda_s) + \lambda_r\|WF\|^2} + \frac{\omega^{-1}\lambda_r\|WF\|^2\|F^H b_1 - \omega^{-1}\lambda_r F^H W^H W F H^H b_1}{(\|H^H b_1 + F^H W^H G^H b_2\|^2 + \lambda_s) + \lambda_r\|WF\|^2} + \\
\frac{\omega^{-1}\lambda_r\|WF\|^2\|F^H W^H G^H b_2 - \omega^{-1}\lambda_r F^H W^H W F F^H W H^H G^H b_2}{(\|H^H b_1 + F^H W^H G^H b_2\|^2 + \lambda_s) + \lambda_r\|WF\|^2}.
\]

**Derivation of Equation (22)**

From equation (16),
\[
\frac{\partial L}{\partial b_1^H} = H a a^H H^H b_1 - H a + H a a^H F^H W^H G^H b_2 + \sigma_{nx}^2 b_1 = 0
\]

Hence,

\[
b_1 = (H a a^H H^H + \sigma_{nx}^2 I_{N_S})^{-1} (H a - H a a^H F^H W^H G^H b_2).
\]

Applying the matrix inversion lemma for \((\sigma_{nx}^2 I_{N_S} + H a a^H H^H)^{-1}\), i.e.,

the matrix inversion lemma: \((A + uu^H)^{-1} = A^{-1} - (1 + u^H A^{-1} u)^{-1} A^{-1} uu^H A^{-1}\)

\[
= \left(\sigma_{nx}^2 I_{N_S}\right)^{-1} - \frac{(\sigma_{nx}^2 I_{N_S})^{-1} (H a) (a^H H H) (\sigma_{nx}^2 I_{N_S})^{-1}}{1 + (a^H H H) (\sigma_{nx}^2 I_{N_S})^{-1} (H a)}
\]

\[
= \sigma_{nx}^{-2} I_{N_S} - \frac{\sigma_{nx}^{-4} H a H a^H H H}{1 + \sigma_{nx}^{-2} a^H H H H a} \times \frac{\sigma_{nx}^2}{\sigma_{nx}^2}
\]

\[
= \sigma_{nx}^{-2} I_{N_S} - \frac{\sigma_{nx}^{-2} H a H a^H H H}{\sigma_{nx}^2 + a^H H H H a}
\]

28
because $a^H H^H H a$ is $1 \times 1$, $a^H H^H H a = tr(a^H H^H H a)$.

The trace is invariant under cyclic permutations, i.e.,

$$tr(ABCD) = tr(BCDA) = tr(CDAB) = tr(DABC).$$

Therefore, it can be written as

$$\left(\sigma_{nx}^2 I_{NS} + Haa^H H^H \right)^{-1} = \frac{I_{NS} + \sigma_{nx}^{-2} tr(Haa^H H^H) - \sigma_{nx}^{-2} tr(Haa^H H^H)}{\sigma_{nx}^2 + a^H H^H H a}. \quad (C10)$$

Substituting equation (C11) into equation (C2) yields

$$\frac{I_{NS}}{\sigma_{nx}^2 + a^H H^H H a} (H a - Haa^H F^H W^H G^H b_2). \quad (C12)$$

Therefore, the optimal $b_1$ can be written as

$$b_1^* = \frac{(1-a^H F^H W^H G^H b_2) H a}{(\sigma_{nx}^2 + ||H a||^2)} \quad (C13)$$

**Derivation of Equation (23)**

From equation (17),

$$\frac{\partial L}{\partial b_2} = \sigma_{nx}^2 b_2 + GWFa^H H^H b_1 + \sigma_{ns}^2 GWW^H G^H b_2 + GWFa^H F^H W^H G^H b_2 - GWFa = 0 \quad (D1)$$

$$b_2 = \left(GWFa^H F^H W^H G^H + \sigma_{ns}^2 GWW^H G^H + \sigma_{nx}^2 I_{NS} \right)^{-1} (GWFa - GWFa^H H^H b_1) \quad (D2)$$
\[
\begin{align*}
&= (GWFa^H F^HW^HG^H + \sigma^2_{n_5} GWW^HG^H + \sigma^2_{n_5} I_{N_5})^{-1} GWFa(1 - a^HH^Hb_1) \quad (D3) \\
&= \left( GWFa^H F^HW^HG^H + \sigma^2_{n_5} GWW^HG^H + \sigma^2_{n_5} I_{N_5} \right)^{-1} GWFa(1 - a^HH^Hb_1). \quad (D4)
\end{align*}
\]

Equation (D4) can be written as
\[
b_2 = (A + uu^H)^{-1}u(1 - a^HH^Hb_1). \quad (D5)
\]

Applying the matrix inversion lemma: \((A + uu^H)^{-1} = A^{-1} - (1 + u^HA^{-1}u)^{-1}A^{-1}uu^HA^{-1}\),
equation (D5) can be written as
\[
b_2 = \left( A^{-1} - \frac{A^{-1}uu^HA^{-1}u}{1 + u^HA^{-1}u} \right)u(1 - a^HH^Hb_1) \quad (D6)
\]
\[
= \left( \frac{A^{-1} + A^{-1}u^HA^{-1}u - A^{-1}uu^HA^{-1}u}{1 + u^HA^{-1}u} \right)u(1 - a^HH^Hb_1) \quad (D7)
\]
\[
= \left( \frac{A^{-1} + A^{-1}u^HA^{-1}u - A^{-1}uu^HA^{-1}u}{1 + u^HA^{-1}u} \right)u(1 - a^HH^Hb_1) \quad (D8)
\]
\[
= \left( \frac{A^{-1}u + A^{-1}u^HA^{-1}uu^HA^{-1}u}{1 + u^HA^{-1}u} \right)(1 - a^HH^Hb_1) \quad (D9)
\]

Because of \(u^HA^{-1}u \in \mathbb{C}^{1 \times 1}\), equation (D9) can be written as
\[
b_2 = \left( \frac{A^{-1}u + A^{-1}u^HA^{-1}uu^HA^{-1}u}{1 + u^HA^{-1}u} \right)(1 - a^HH^Hb_1) \quad (D10)
\]
\[
= \left( \frac{A^{-1}u}{1 + u^HA^{-1}u} \right)(1 - a^HH^Hb_1) \quad (D11)
\]

By substituting \(A, u\), and \(u^H\) into equation (D11), therefore, the optimal \(b_2\) can be written as
\[
b_2^* = \left( \frac{1 - a^HH^Hb_1}{1 + a^HH^Hb_1} \right)(1 - a^HH^Hb_1) \quad (D12)
\]

\textbf{Derivation of Equation (24)}

From equation (11),

\[
\]
$$P_r = \|WFa\|^2 + \sigma_{n_s}^2 \|W\|^2_F$$  \hspace{1cm} (E1)$$

Substituting equation (20) into equation (E1) yields

$$P_r = \left(\frac{(1-b_1^H Ha)G^H b_2 a^H F^H}{\sigma_{n_s}^2 + \|Fa\|^2 (\|G^H b_2\|^2 + \lambda_r)} - Fa\right)^2 + \sigma_{n_s}^2 \left(\frac{(1-b_1^H Ha)G^H b_2 a^H F^H}{\sigma_{n_s}^2 + \|Fa\|^2 (\|G^H b_2\|^2 + \lambda_r)}\right)_F^2$$  \hspace{1cm} (E2)$$

$$= \frac{1}{\left(\sigma_{n_s}^2 + \|Fa\|^2\right)^2 (\|G^H b_2\|^2 + \lambda_r^2)} \left[\|\left(1-b_1^H Ha\right)G^H b_2 a^H F^H Fa\|^2_F + \sigma_{n_s}^2 \|\left(1-b_1^H Ha\right)G^H b_2 a^H F^H\|^2_F\right]$$  \hspace{1cm} (E3)$$

Hence,

$$\left(\|G^H b_2\|^2 + \lambda_r\right)^2 = \frac{1}{\left(\sigma_{n_s}^2 + \|Fa\|^2\right)^2 P_r} \left[\|\left(1-b_1^H Ha\right)G^H b_2 \|Fa\|^2_F\right]^2 + \sigma_{n_s}^2 \|\left(1-b_1^H Ha\right)G^H b_2 a^H F^H\|^2_F$$  \hspace{1cm} (E4)$$

$$\left(\|G^H b_2\|^2 + \lambda_r\right) = \pm \frac{1}{\left(\sigma_{n_s}^2 + \|Fa\|^2\right)} \sqrt{\left[\frac{\|\left(1-b_1^H Ha\right)G^H b_2 \|^2_F + \sigma_{n_s}^2 \|\left(1-b_1^H Ha\right)G^H b_2 a^H F^H\|^2_F}{P_r}\right]}$$  \hspace{1cm} (E5)$$

Therefore, the optimal $\lambda_r$ can be written as

$$\lambda_r^* = \left(\sqrt{\frac{\|Fa\|^4 \|G^H b_2\|^2_F + \sigma_{n_s}^2 \|G^H b_2 a^H F^H\|^2_F}{P_r (\sigma_{n_s}^2 + \|Fa\|^2)^{-1}}} - \|G^H b_2\|^2_F\right)^+$$  \hspace{1cm} (E6)$$

where $\left(\zeta^+ = \max(0, \zeta)\right)$.  

**Derivation of Equation (25)**

From equation (10),

$$P_S = \|a\|^2 = a^H a.$$  \hspace{1cm} (F1)$$

Substituting equation (21) into equation (F1),

$$P_S = a^H \left(\frac{H^H b_1 + F^H W^H G^H b_2}{\|H^H b_1 + F^H W^H G^H b_2\|^2_F + \lambda_S} + \lambda_r \|WF\|^2_F\right) + \omega \lambda_r \|WF\|^2_F \left(\frac{H^H b_1 + F^H W^H G^H b_2}{\|H^H b_1 + F^H W^H G^H b_2\|^2_F + \lambda_S} + \lambda_r \|WF\|^2_F\right) \hspace{1cm} (F2)$$
where

\[ \omega = (\|H^b_1 + F^H W^H G^b_2\|^2 + \lambda_s)^{-1}. \]

Let \( K, L, X, \) and \( Y \) be denoted, respectively, as

\[ K = H^b_1 + F^H W^H G^b_2 \]

\[ L = \lambda_r\|WF\|^2 H^b_1 - \lambda_r F^H W^H WF H^b_1 + \lambda_r\|WF\|^2 F^H W^H G^b_2 \]

\[ - \lambda_r F^H W^H W F H^b_1 \]

\[ X = \|H^b_1 + F^H W^H G^b_2\|^2 + \lambda_r\|WF\|^2 \]

\[ Y = \|H^b_1 + F^H W^H G^b_2\|^2 \]

Hence, equation (F2) can be written as

\[ P_S = \frac{a^H (K + \omega^{-1} L)}{X + \lambda_s} \quad (F3) \]

\[ P_S (X + \lambda_s) = a^H K + \frac{a^H L}{Y + \lambda_s} \quad (F4) \]

Multiplying both sides with \( Y + \lambda_s \) yields

\[ P_S (X + \lambda_s)(Y + \lambda_s) = (Y + \lambda_s)a^H K + a^H L \quad (F5) \]

\[ P_S [XY + (X + Y)\lambda_s + \lambda_s^2] = Ya^H K + \lambda_s a^H K + a^H L \quad (F6) \]

\[ P_S \lambda_s^2 + (P_S (X + Y) - a^H K)\lambda_s + P_S XY - Ya^H K - a^H L = 0 \quad (F7) \]

Therefore, \( \lambda_s \) can be written as

\[ \lambda_s = \left( 0.5(-\epsilon + (\epsilon^2 - 4P_S \vartheta)^{1/2})P_s^{-1} \right)^+ \]

where \((\zeta)^+ = \max(0, \zeta)\) and

\[ \epsilon = P_S (X + Y) - a^H K \]

\[ = (2\|H^b_1 + F^H W^H G^b_2\|^2 + \lambda_r\|WF\|^2)P_s - a^H (H^b_1 + F^H W^H G^b_2) \]

\[ \vartheta = P_S XY - Ya^H K - a^H L \]

32
\[
(\|H^Hb_1 + F^HWH^Hb_2\|^2 (\|H^Hb_1 + F^HWH^Hb_2\|^2 + \lambda_r \|WF\|^2) )P_s - a^H(H^Hb_1 + F^HWH^Hb_2)(\|H^Hb_1 + F^HWH^Hb_2\|^2) - \lambda_r a^H(\|WF\|^2 P H^Hb_1 - F^HWHWFH^Hb_1 + \|WF\|^2 F^HWH^Hb_2 - F^HWHWFH^Hb_2)
\]
APPENDIX B
MATLAB CODE

clear all;
clc;
close all;
k=5*10^5;
sigmaF=sqrt(0.5);
sigmaG=sqrt(0.5);
sigmaH=sqrt(0.5);
SNR=6;
p=1;
no=p./10.^(SNR/10);
sigma=sqrt(no/2);
M=4;
N=2;
g0=1;

for m1=0:3
    [a01]=pskmod(m1, M);
    A1(:,g0)=a01;
    g0=g0+1;
end

for v=1:length(SNR)
    Errors1=zeros(1,length(SNR));
    u1=0;
for aa=1:k

% Initialization
namdaS=1;
namdaR=1;
namdaT=1;
J = 10;
W=eye(2);
b1=[1;1];
b2=[1;1];
a=[1;1];

% 2 Source, 2 Destination and 2 Relay

F=sigmaF*(randn(2,2)+sqrt(-1)*randn(2,2));
G=sigmaG*(randn(2,2)+sqrt(-1)*randn(2,2));
H=sigmaH*(randn(2,2)+sqrt(-1)*randn(2,2));
ns=sigma(v)*(randn(2,1)+sqrt(-1)*randn(2,1));
nx1=sigma(v)*(randn(2,1)+sqrt(-1)*randn(2,1));
nx2=sigma(v)*(randn(2,1)+sqrt(-1)*randn(2,1));

%%%%%%%%%%%%%%%%First Iteration with initial variables %%%%%%%%%%%%%%%%%

% Transmit Power restraint
W=((1-b1'*H*a)*G'*b2*a'*F')/((norm(G'*b2)^2+namdaR)*(norm(F*a)^2+(no(v))));

xy = norm(H'*b1+F'*W'*G'*b2)^2+namdaS;
yz = norm(H'*b1+F'*W'*G'*b2)^2+namdaR*norm(W*F,'fro')^2+namdaS;

a = (H'*b1+F'*W'*G'*b2)/yz+(namdaR*norm(W*F,'fro')^2*H'*b1-
   namdaR*F'*W'*W*F*H'*b1)/(xy)*yz+(namdaR*norm(W*F,'fro')^2*F'*W'*G'*b2-
   namdaR*F'*W'*W*F*F'*W'*G'*b2)/xy*yz;

b1=((1-a'*F'*W'*G'*b2)*H*a)/(norm(H*a)^2+(no(v)));

A = (no(v)*G*W*W'*G')+no(v)*eye(2);

b2= ((A\G)*W*F*a*(1-a'*H'*b1))/(1+a'*F'*W'*G'*(A\G)*W*F*a);

namdaR=(((1-
   b1'*H*a)*sqrt(norm(F*a)^4*norm(G'*b2)^2)+(no(v))*norm(G'*b2*a'*F,'fro')^2)))
   /(sqrt(1.5)*(norm(F*a)^2+(no(v))))-norm(G'*b2)^2;
   namdaR = real(namdaR);
   namdaR=max(namdaR,0);

namdaS = -(2*(norm(H'*b1+F'*W'*G'*b2)^2+(namdaR*norm(W*F,'fro')^2))-
   (a'*(H'*b1+F'*W'*G'*b2)))+sqrt((2*(norm(H'*b1+F'*W'*G'*b2)^2+(namdaR*norm(W*F
   ,'fro')^2))-(a'*(H'*b1+F'*W'*G'*b2))^2-
   4*((norm(H'*b1+F'*W'*G'*b2)^2*(norm(H'*b1+F'*W'*G'*b2)^2+(namdaR*norm(W*F,'fr
   o')^2)))-(norm(H'*b1+F'*W'*G'*b2)^2*(a'*(H'*b1+F'*W'*G'*b2))-
   (namdaR*a'*(norm(W*F,'fro')^2*H'*b1-
   F'*W*F*H'*b1+norm(W*F,'fro')^2*F'*W'*G'*b2-
   F'*W*(F)*F'*W'*G'*b2)))))/(2*1.5);
namdaS = real(namdaS);
namdaS = max(namdaS, 0);

\[ J_t = 1 - 2 \times \text{real}(b_1^\dagger H a) - 2 \times \text{real}(b_2^\dagger G W^\dagger F a) + 2 \times \text{real}(a^\dagger H^\dagger b_1 b_2^\dagger G W^\dagger F a) \]
\[ + \text{abs}(a^\dagger H^\dagger b_1)^2 + \text{abs}(a^\dagger F^\dagger W^\dagger G^\dagger b_2)^2 + (\text{no}(v)) \times \text{norm}(W^\dagger G^\dagger b_2)^2 + (\text{no}(v)) \times (\text{norm}(b_1)^2 + \text{norm}(b_2)^2) + \text{norm}(a)^2 - 1 + \text{namdaR} \times (\text{norm}(W^\dagger F a)^2 + (\text{no}(v)) \times \text{norm}(W, '\text{fro}')^2 - 1); \]

\% Aggregate Power restraint

\[ W = \frac{(1 - b_1^\dagger H a) G^\dagger b_2 a^\dagger F'}{(\text{norm}(G^\dagger b_2)^2 + \text{namdaT}) \times (\text{norm}(F a)^2 + (\text{no}(v)))); \]

\[ xy = \text{norm}(H^\dagger b_1 + F^\dagger W^\dagger G^\dagger b_2)^2 + \text{namdaT}; \]

\[ yz = \text{norm}(H^\dagger b_1 + F^\dagger W^\dagger G^\dagger b_2)^2 + \text{namdaT} \times \text{norm}(W F, '\text{fro}')^2 + \text{namdaT}; \]

\[ a = \frac{(H^\dagger b_1 + F^\dagger W^\dagger G^\dagger b_2) / yz + (\text{namdaT} \times \text{norm}(W F, '\text{fro}')^2 H^\dagger b_1 - \text{namdaT} F^\dagger W^\dagger W^\dagger H^\dagger b_1) / (xy) \times yz + (\text{namdaT} \times \text{norm}(W F, '\text{fro}')^2 F^\dagger W^\dagger G^\dagger b_2 - \text{namdaT} F^\dagger W^\dagger F^\dagger W^\dagger G^\dagger b_2) / xy \times yz; \]

\[ b_1 = \frac{(1 - a^\dagger F^\dagger W^\dagger G^\dagger b_2) H a) / (\text{norm}(H a)^2 + (\text{no}(v))); \]

\[ A = (\text{no}(v) \times G W^\dagger W^\dagger G') + (\text{no}(v) \times \text{eye}(2)); \]

\[ b_2 = \frac{((A \backslash G) W F a) / (1 + a^\dagger F^\dagger W^\dagger G^\dagger (A \backslash G)^\dagger W F a)}; \]

\[ \text{namdaT} = \frac{(\text{sqrt}((\text{norm}(G^\dagger b_2 a^\dagger F^\dagger F a)^2 + (\text{no}(v)) \times \text{norm}(G^\dagger b_2 a^\dagger F', '\text{fro}')^2)) \times (2 - \text{norm}(a)^2)) \times ((1 - b_1^\dagger H a) / (\text{norm}(F a)^2 + (\text{no}(v)))) \times \text{norm}(G^\dagger b_2)^2; \]
\[ \text{namdaT} = \text{real(namdaT);} \]
\[ \text{namdaT} = \max(\text{namdaT}, 0); \]

\[ J_t = 1 - 2*\text{real}(b_1' * H * a) - 2*\text{real}(b_2' * G * W * F * a) + 2*\text{real}(a' * H' * b_1 * b_2' * G * W * F * a) + abs(a' * H' * b_1)^2 + abs(a' * F' * W' * G' * b_2)^2 + (\text{no(v)}) \times \text{norm}(W' * G' * b_2)^2 + (\text{no(v)}) \times (\text{norm}(b_1)^2 + \text{norm}(b_2)^2) + (\text{namdaT}) \times (\text{norm}(a)^2 + \text{norm}(W' * F * a)^2 + (\text{no(v)}) \times \text{norm}(W, 'fro')^2) - 2; \]

\% Source Power restraint
\[ W = ((1 - b_1' * H * a) * (G' * b_2 * a' * F')) / (\text{norm}(G' * b_2)^2 * (\text{norm}(F' * a)^2 + (\text{no(v)}))); \]

\[ a = (H' * b_1 + F' * W' * G' * b_2) / (\text{norm}(H' * b_1 + F' * W' * G' * b_2)^2 + \text{namdaS}); \]

\[ b_1 = ((1 - a' * F' * W' * G' * b_2) * H' * a) / (\text{norm}(H' * a)^2 + (\text{no(v)})); \]

\[ A = (\text{no(v)} * G * W * W' * G') + \text{no(v)} * \text{eye(2)}; \]

\[ b_2 = ((A\backslash G) * W' * F' * a * (1 - a' * H' * b_1)) / (1 + a' * F' * W' * G' * (A\backslash G) * W' * F' * a); \]

\[ \text{namdaS} = a' * (H' * b_1 + F' * W' * G' * b_2) - \text{norm}(H' * b_1 + F' * W' * G' * b_2)^2; \]
\[ \text{namdaS} = \text{real(namdaS)}; \]
\[ \text{namdaS} = \max(\text{namdaS}, 0); \]

\[ J_t = 1 - 2*\text{real}(b_1' * H' * a) - 2*\text{real}(b_2' * G' * W' * F' * a) + 2*\text{real}(a' * H' * b_1 * b_2' * G * W * F * a) + abs(a' * H' * b_1)^2 + abs(a' * F' * W' * G' * b_2)^2 + (\text{no(v)}) \times \text{norm}(W' * G' * b_2)^2 + (\text{no(v)}) \times (\text{norm}(b_1)^2 + \text{norm}(b_2)^2) + (\text{namdaT}) \times (\text{norm}(a)^2 + \text{norm}(W * F * a)^2 + (\text{no(v)}) \times \text{norm}(W, 'fro')^2) - 2; \]
\*G'\*b2)^2+(\text{no}(v))\*\text{norm}(W'\*G'\*b2)^2+(\text{no}(v))\*(\text{norm}(b1)^2+\text{norm}(b2)^2)+(\text{namdaS})* (\text{norm}(a)^2-1);

\% Relay Power restraint

\large W=((1-b1'*H*a)*G'\*b2\*a'*F')/((\text{norm}(G'\*b2)^2+\text{namdaR})* (\text{norm}(F*a)^2+(\text{no}(v)))));

\large a = ((\text{eye}(2)+(\text{norm}(H'\*b1+F'\*W'\*G'\*b2)^2)^{-1}\*\text{eye}(2)*\text{namdaR}\*\text{norm}(W\*F,'\text{fro}')^2-(\text{norm}(H'\*b1+F'\*W'\*G'\*b2)^2)^{-1}\*\text{namdaR}\*F'\*W'\*G'\*b2)/((\text{norm}(H'\*b1+F'\*W'\*G'\*b2)^2+\text{namdaR}\*\text{norm}(W\*F,'\text{fro}')^2);)

\large b1=((1-a'*F'*W'\*G'\*b2)*H*a)/(\text{norm}(H*a)^2+(\text{no}(v)));

\large A = (\text{no}(v)*G*W*W'*G')+\text{no}(v)*\text{eye}(2);

\large b2= ((A\*G)*W*F*a*(1-a'*H'*b1))/(1+a'*F'*W'*G'*(A\*G)*W*F*a);

\large \text{namdaR}=(((1-b1'*H*a)*(\text{sqrt}(\text{norm}(F*a)^4*\text{norm}(G'\*b2)^2+(\text{no}(v))*\text{norm}(G'\*b2*a'*F','\text{fro}')^2)))/((\text{norm}(F*a)^2+(\text{no}(v)))))-\text{norm}(G'\*b2)^2;

\large \text{namdaR} = \text{real}(\text{namdaR});

\large \text{namdaR} = \text{max}(\text{namdaR},0);

\large Jt=1-2*\text{real}(b1'*H*a)-2*\text{real}(b2'*G*W*F*a)+2*\text{real}(a'*H'*b1+b2'*G*W*F*a)+\text{abs}(a'*H'*b1)^2+\text{abs}(a'*F'*W'\*G'\*b2)^2+(\text{no}(v))*\text{norm}(W'\*G'\*b2)^2+(\text{no}(v))*(\text{norm}(b1)^2+\text{norm}(b2)^2)+(\text{namdaR})* (\text{norm}(W\*F*a)^2+(\text{no}(v))*\text{norm}(W,'\text{fro}')^2-1);
Jt = real(Jt);

%===============iteration===============================================

t = 1;
while (J - Jt > 0.0001)
    J = Jt;

% Transmit Power restraint
W = ((1 - b1'*H*a)*G'*b2*a'*F')/((norm(G'*b2)^2 + lambdaR) * (norm(F*a)^2 + (no(v))));

xy = norm(H'*b1+F'*W'*G'*b2)^2 + lambdaS;

yz = norm(H'*b1+F'*W'*G'*b2)^2 + lambdaR*norm(W*F,'fro')^2 + lambdaS;

a = (H'*b1+F'*W'*G'*b2)/yz + (lambdaR*norm(W*F,'fro')^2 * H'*b1 - lambdaR*F'*W'*F*H'*b1)/(xy)*yz + (lambdaR*norm(W*F,'fro')^2 * F'*W'*G'*b2 - lambdaR*F'*W'*F'*W'*G'*b2)/xy*yz;

b1 = ((1 - a'*F'*W'*G'*b2)*H*a)/(norm(H*a)^2 + (no(v)));

A = (no(v) * G*W*W'*G') + no(v) * eye(2);

b2 = ((A\G)*W*F*a*(1-a'*H'*b1))/(1+a'*F'*W'*G'*(A\G)*W*F*a);
namdaR = (((1 - b1' * H * a) * (sqrt(norm(F * a)^4 * norm(G' * b2)^2 + (no(v)) * norm(G' * b2 * a' * F', 'fro')^2))) / (sqrt(1.5) * (norm(F * a)^2 + (no(v))))) - norm(G' * b2)^2;

namdaR = real(namdaR);

namdaR = max(namdaR, 0);

namdaS = (-2 * (norm(H' * b1 + F' * W' * G' * b2)^2 + (namdaR * norm(W * F, 'fro')^2)) - (a' * (H' * b1 + F' * W' * G' * b2)) + sqrt((2 * (norm(H' * b1 + F' * W' * G' * b2)^2 + (namdaR * norm(W * F, 'fro')^2)) - (a' * (H' * b1 + F' * W' * G' * b2)))^2 - 4 * (norm(H' * b1 + F' * W' * G' * b2)^2 + (norm(H' * b1 + F' * W' * G' * b2)^2 + (namdaR * norm(W * F, 'fro')^2)) - (namdaR * a' * (norm(W * F, 'fro')^2) * H' * b1 - F' * (W) * F' * H' * b1 + norm(W * F, 'fro')^2 * F' * W' * G' * b2 - F' * (W) * F' * (F) * F' * W' * G' * b2)))) / (2 * 1.5);

namdaS = real(namdaS);

namdaS = max(namdaS, 0);

Jt = 1 - 2 * real(b1' * H * a) - 2 * real(b2' * G * W * F * a) + 2 * real(a' * H' * b1 * b2' * G * W * F * a) + abs(a' * H' * b1)^2 + abs(a' * F' * W' * G' * b2)^2 + (no(v)) * norm(W' * G' * b2)^2 + (no(v)) * (norm(b1)^2 + norm(b2)^2) + (namdaS) * (norm(a)^2 - 1) + (namdaR) * (norm(W' * F' * a)^2 + (no(v)) * norm(W, 'fro')^2 - 1));

% Aggregate Power restraint

W = ((1 - b1' * H * a) * G' * b2 * a' * F') / ((norm(G' * b2)^2 + namdaT) * (norm(F' * a)^2 + (no(v))));

xy = norm(H' * b1 + F' * W' * G' * b2)^2 + namdaT;

yz = norm(H' * b1 + F' * W' * G' * b2)^2 + namdaT * norm(W * F, 'fro')^2 + namdaT;
\[ a = \frac{(H' \ast b1 + F' \ast W' \ast G' \ast b2)}{yz} + \frac{(\text{namdaT} \ast \text{norm}(W \ast F, 'fro')^2 \ast H' \ast b1 - \text{namdaT} \ast F' \ast W' \ast F \ast H' \ast b1)}{(xy) \ast yz} + \frac{(\text{namdaT} \ast \text{norm}(W \ast F, 'fro')^2 \ast F' \ast W' \ast G' \ast b2 - \text{namdaT} \ast F' \ast W' \ast F' \ast W' \ast G' \ast b2)}{xy \ast yz}; \]

\[ b1 = \frac{(1 - a' \ast F' \ast W' \ast G' \ast b2) \ast H' \ast a}{\text{norm}(H' \ast a)^2 + (no(v))}; \]

\[ A = (no(v) \ast G' \ast W' \ast G') + no(v) \ast \text{eye}(2); \]

\[ b2 = \frac{(A \ast G' \ast W' \ast F' \ast a \ast (1 - a' \ast H' \ast b1))}{(1 + a' \ast F' \ast W' \ast G' \ast (A \ast G') \ast W' \ast F' \ast a);}; \]

\[ \text{namdaT} = \frac{\left(\sqrt{\text{norm}(G' \ast b2 \ast a' \ast F' \ast a)^2 + (no(v)) \ast \text{norm}(G' \ast b2 \ast a' \ast F', 'fro')^2}/(2 - \text{norm}(a)^2)\right) \ast \left((1 - b1' \ast H' \ast a)/(\text{norm}(F' \ast a)^2 + (no(v)))) - \text{norm}(G' \ast b2)^2\right)}{\left(\text{sqrt}(\text{norm}(G' \ast b2 \ast a' \ast F' \ast a)^2 + (no(v)) \ast \text{norm}(G' \ast b2 \ast a' \ast F', 'fro')^2)/(2 - \text{norm}(a)^2)\right) \ast \left((1 - b1' \ast H' \ast a)/(\text{norm}(F' \ast a)^2 + (no(v)))) - \text{norm}(G' \ast b2)^2\right)} ; \]

\[ \text{namdaT} = \text{real}(\text{namdaT}); \]

\[ \text{namdaT} = \text{max}(\text{namdaT}, 0); \]

\[ Jt = 1 - 2 \text{real}(b1' \ast H' \ast a) - 2 \text{real}(b2' \ast G' \ast W' \ast F' \ast a) + 2 \text{real}((a' \ast H' \ast b1 \ast b2' \ast G' \ast W' \ast F' \ast a) + \text{abs}(a' \ast H' \ast b1)^2 + \text{abs}(a' \ast F' \ast W' \ast G' \ast b2)^2 + (no(v)) \ast \text{norm}(W' \ast F' \ast a)^2 + (no(v)) \ast (\text{norm}(b1)^2 + \text{norm}(b2)^2) + (\text{namdaT}) \ast (\text{norm}(a)^2 + \text{norm}(W' \ast F' \ast a)^2 + (no(v)) \ast \text{norm}(W, 'fro')^2)^2 - 2); \]

\% Source Power restraint
\[ W = ((1 - b1' \ast H' \ast a) \ast (G' \ast b2 \ast a' \ast F'))/(\text{norm}(G' \ast b2)^2 \ast (\text{norm}(F' \ast a)^2 + (no(v)))); \]

\[ a = \frac{(H' \ast b1 + F' \ast W' \ast G' \ast b2)}{\text{norm}(H' \ast b1 + F' \ast W' \ast G' \ast b2)^2 + \text{namdaS}); \]
\[ b_1 = \frac{(1-a'F'WG'G'b_2)H*a}{(\text{norm}(H*a)^2+(\text{no}(v)))}; \]

\[ A = (\text{no}(v)*GW*WG') + \text{no}(v)*\text{eye}(2); \]

\[ b_2 = \frac{((A\backslash G)*WF*a(1-a'H'b_1))}{(1+a'F'WG'G'G\backslash G)*GW*F*a}; \]

\[ \text{namdaS} = a'((H*b_1+F'WG'G'b_2) - \text{norm}(H*b_1+F'WG'G'b_2)^2); \]

\[ \text{namdaS} = \text{real}(\text{namdaS}); \]

\[ \text{namdaS} = \max(\text{namdaS},0); \]

\[ J_t = 1 - 2*\text{real}(b_1'H*a) - 2*\text{real}(b_2'*GWF*a) + 2*\text{real}(a'H*b_1b_2'*GW*F*a) + \text{abs}(a'H*b_1)^2 + \text{abs}(a'F'WG'G'b_2)^2 + \text{no}(v) * \text{norm}(W'G'b_2)^2; \]

\[ \text{namdaR} = \text{real}(\text{norm}(H'*b_1+F'*WG'G'b_2)^2); \]

\[ \text{namdaR} = \max(\text{namdaR},0); \]

\[ a = \frac{((\text{eye}(2) + \text{norm}(H'*b_1+F'*WG'G'b_2)^2) - \text{eye}(2)*\text{namdaR}*\text{norm}(WF,'fro'))^2}{(\text{norm}(H'*b_1+F'*WG'G'b_2)^2)^2 - 1)*\text{namdaR}*F'*W'*WF*(H'*b_1+F'*WG'G'b_2))}/(\text{norm}(H'*b_1+F'*WG'G'b_2)^2 + \text{norm}(WF,'fro')^2); \]

\[ b_1 = ((1-a'F'WG'G'b_2)*H*a) / (\text{norm}(H*a)^2+(\text{no}(v))); \]

\[ A = (\text{no}(v)*GW*WG') + \text{no}(v)*\text{eye}(2); \]
b2 = ((A\G)*W*F*a*(1-a'*H'*b1))/(1+a'*F'*W'*G'*(A\G)*W*F*a);

namdaR = (((1-b1'*H*a)*(sqrt(norm(F*a)^4*norm(G'*b2)^2+(no(v))*norm(G'*b2*a'*F','fro')^2))}/((norm(F*a)^2+(no(v)))))-norm(G'*b2)^2;

namdaR = real(namdaR);

namdaR = max(namdaR, 0);

Jt = 1-2*real(b1'*H*a) - 2*real(b2'*G*W*F*a) + 2*real(a'*H'*b1*b2'*G*W*F*a) + abs(a'*H'*b1)^2 + abs(a'*F'*W'*G'*b2)^2 + (no(v))*norm(W'*G'*b2)^2 + (no(v))*norm(b1)^2 + norm(b2)^2 + (namdaR)*((norm(W*F*a)^2+(no(v))*norm(W,'fro')^2)-1);

Jt = real(Jt);

end

%%%========iteration end==============================================

ss2 = pskmod(randint(1,1,M),M);
rdl = (b1'*H*a+b2'*G*W*F*a)*ss2+(b1'*nx1+b2'*nx2+b2'*G*W*ns);
Pr = norm(W*F*a)^2+(no(v))*norm(W,'fro')^2;
Ps = norm(a)^2;
Pt = (norm(a)^2+norm(W*F*a)^2+(no(v))*norm(W,'fro')^2);

for i=1:4
distance1(i) = norm(rd1 - (b1'*H*a + b2'*G*F*a)*A1(:,i));
end

[min_distance1, index_distance1] = min(distance1);
x_det1 = A1(1, index_distance1);

Errors1(v) = Errors1(v) + (ss2 ~= x_det1);
end

J1(v) = u1/k
BER1(v) = Errors1(v) / k / log2(M)
end

semilogy(SNR, BER1, '-b', 'Linewidth', 1);
hold on
axis([0 12 10^(-4) 1])
grid on
xlabel('SNR (dB)')
ylabel('BER')
legend('N=2');