DISTRIBUTED DETECTION AND DATA FUSION IN RESOURCE CONSTRAINED WIRELESS SENSOR NETWORKS

A Thesis by

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I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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DEDICATION

Dedicated to My Beloved Parents
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I would like to extend my sincere thanks to my graduate advisor Dr. Sudharman K. Jayaweera for all his guidance and motivation not just academically but also morally. I would like to thank Dr. Coskun Cetinkaya and Dr. Krishna Krishnan for being on my committee and taking the pains to go through the thesis.

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ABSTRACT

Wireless sensor networks have received immense attention in recent years due to their possible applications in various fields like battery-field surveillance, disaster recovery etc. Since these networks are mostly resource-constrained there is a need for efficient algorithms in maximizing the network resources. In this thesis, energy and bandwidth-efficient detection and fusion algorithms for such resource constrained wireless sensor systems are developed. A Sequential Probability Ratio Test (SPRT) based detection algorithms for an energy-constrained sensor network is proposed. Performance is evaluated in terms of number of nodes required to achieve a given probability of detection. Simulation results show that a network implementing the SPRT based model outperforms a network having a parallel fusion detector. To implement distributed detection and fusion in energy and bandwidth constrained networks, non-orthogonal communication is considered to be one of the possible solutions. An optimal Bayesian data fusion receiver for a DS-CDMA based distributed wireless sensor network having a parallel architecture is proposed. It is shown that the optimal Bayesian receiver outperforms the partitioned receivers in terms of probability of error. But the complexity of this optimal receiver is exponential in the number of nodes. In order to reduce the complexity, partitioned receivers that perform detection and fusion in two stages are proposed. Several well-known multi-user detectors namely, JML, matched filter, Decorrelator and linear MMSE detectors are considered for the first stage detection and performance is evaluated in terms of probability of error at the fusion center. Conventional detector based fusion receiver has a performance close to that of optimal fusion receiver with quite less complexity under specific channel conditions. Performance and complexity trade-offs should be considered while designing the network.
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Chapter 1

Introduction and Preview

1.1 Wireless Sensor Networks

Recent advancements in wireless communications have enabled the development of low-energy, low-cost sensor networks. These networks consist of sensor nodes that are often equipped with the multiple parameter sensing, programmable computing and communication capabilities. By integrating sensing, signal-processing and communication functions, a sensor network provides a platform for hierarchical information processing.

The untethered nodes in these wireless sensor networks (WSNs) possess the self-organizing capability and maintain a network without a fixed infrastructure [1]. These WSNs are referred as Ad-hoc WSNs or Mobile Ad-hoc Networks (MANET) [2]. Since the nodes self-organize into ad-hoc networks, deployment can be as easy as dropping them by air or sprinkling nodes above the region of interest and setting up a base station for communication with the nodes. These advantages and the cost and size of the nodes make the sensor networks ideal for unreachable or inhospitable locations where deployment is difficult and maintenance impossible.
The potential applications of WSNs are highly varied such as environmental sampling, surveillance, equipment and health monitoring [3], habitat monitoring [4], global positioning systems (GPS) [5], precision agriculture, military applications such as target detection and tracking and so on. Due to ease-of-deployment and the flexibility in operation, sensor networks find their way in various fields.

The applications of sensor networks in various fields’ demands that the protocols and algorithms to be implemented on the WSNs must be designed to achieve fault-tolerance and to provide robust mechanisms [6]. There are several research areas that can be considered in WSNs like hardware aspects, algorithmic approaches, architectural characteristics [7], physical and media-access control (MAC) layers [8], energy and bandwidth efficiencies. Among these aspects, energy and bandwidth efficiencies are of main focus in this thesis.

1.2 Need for Energy Conservation

The energy source provided to the sensors in WSNs is usually battery-operated as application demands, which has not yet reached the stage for the sensors to operate for a long-time without recharging. In some cases these networks may be required to be solely operated on the energy drawn from the environment such as thermal, photovoltaic or seismic conversion. The greatest challenge lies in the need for the unprecedented lifetime of the sensor system. The sluggish progress in the energy density improvements in the battery technologies adds to this challenge.

Moreover, sensor nodes are often intended to be deployed in adverse and remote environment such as lands of extreme desert or arctic climates, surfaces of planets or moons or in surveillance and military applications; it is unfavorable to recharge
or replace the battery power. Hence energy-efficient design without sacrificing the reliability of the system is a vital challenge in the system design.

Energy-efficient design encompasses many areas of research like hardware design, networking, algorithmic design etc. Hardware design requires low-energy component design for maximization of network lifetime within the span of the network [9]. Networking aspect includes designing protocols for efficient communication and routing of information between sensor nodes and to the data gathering node [10,11]. Various algorithmic approaches and design strategies are also considered for energy-efficient design of the networks.

In [12], several optimization and management strategies are proposed at node, link and network levels for significant enhancement in network lifetime by studying the trade-offs between performance, fidelity and energy consumption.

Algorithmic and hardware enablers are implemented in [13], for energy-efficient micro sensor networks consisting of as many as 1000 nodes. The application programming interface introduced by researchers allows the performance to be dynamically adjusted allowing the network to manage energy consumption by trading-off quality for energy.

A novel approach of computing energy-efficient sub-network given a communication network is proposed in [14], also called minimum-energy communication network (MECN). Small MECN (SMECN) is proposed in [15] to provide a smaller network than computed by MECN, provided the broadcast region is circular around a broadcaster for a given power setting.

Threshold sensitive energy-efficient protocol is proposed for wireless sensor networks which act reactively to any changes in the relevant parameter of interest. The
trade-off in this design is that the nodes never communicate if the threshold is not reached [16].

A clustering based protocol called low-energy adaptive clustering hierarchy (LEACH) is proposed in [17], in order to minimize the energy dissipation in sensor networks. The purpose of LEACH is to randomly select the cluster heads from the nodes, so that the energy consumption while communicating with the fusion center is evenly distributed to all the nodes in the network. The protocol is implemented in two phases; setup phase and steady phase. During setup, a sensor node randomly generates a number and if it is less than a pre-computed threshold, it announces to the rest of the network as cluster head. Based on the signal strength received from these cluster heads each node decides to which cluster should it belong to. During the steady phase, the nodes sense and communicate data to the cluster heads and from them to the base station. These phases are repeated for optimal energy utilization.

In [18] a dynamic power management scheme for wireless sensor networks is discussed where different power-saving modes are proposed and inter-transition phases are studied based on threshold times set. The threshold time is found to depend on power consumption of individual nodes and the transition times.

1.3 Need for Distributed Detection

In classical sensor networks, the sensor nodes communicate among each other or with the data aggregation node that performs detection and fusion. These nodes may have to use the common communication channel in the normal circumstances for exchange of information among each other. This process requires huge communication bandwidth and energy consumption between sensor nodes and fusion center. This rises the
need for bandwidth-efficient algorithms to be implemented on the sensor networks. Processing the sensor information as much as possible within the network, so as to avoid large amounts of information communication is the key idea of distributed or decentralized systems [19].

The literature on decentralized detection is vast and provides the insight into various detection problems [20–24]. Pioneering effort of Tenney and Sandell [19] laid the framework for distributed architecture in detection theory. In their architecture each sensor implements a local likelihood ratio test as its optimum decision rule and the threshold computations are coupled among the sensor nodes in order to achieve joint optimization. As the sensor nodes increase, the complexity in the system increases due to coupled threshold computations. Hence [21] considered the optimum fusion rules given individual detector decision rules. Fusion of the local decisions in order to arrive at a final decision at the fusion center can be implemented in several ways such as AND, OR or exclusive-OR combining of the local decisions [25, 26].

Several optimization criteria can be implemented to achieve system level performance for these sensor networks. Criteria like Bayes, where cost function is minimized, Neyman-Pearson (NP), where detection probability can be maximized given fixed false-alarm probability, minimax detection which minimizes the maximum of false alarm and miss probabilities, Shannon’s information, Ali-Silvey distance measures are some to choose from. The former two are the most widely implemented formulations.

Distributed architecture can be applied to several topologies like parallel [21], serial [23], tandem [27], tree [28] etc. Fig. 1.1 depicts a parallel topology network where a data aggregation unit optimally processes all the local decisions from the
individual sensor nodes and yields a final decision [24].

Fig. 1.2 depicts a serial configuration where each sensor in the network sends its quantized decision to the next node in the network and the decision at each node is based on its observation and the quantized decision from the previous sensor. The decision of the last sensor in the network is declared as the decision of the entire network.

Sequential hypothesis testing is another detection technique where just enough data can be collected to achieve a desired level of performance unlike other testing configurations. Data acquisition can be discontinued and an end decision can be declared as soon as enough data has been gathered for decision making. Two different scenarios are possible in sequential hypothesis testing. One, in which each sensor performs a
sequential test and arrives at a local decision and the decisions are passed on to the subsequent sensors until the end criteria is met [29]. In the other case, each sensor sends a sequence of observations or quantized decisions to the fusion center where a sequential test is performed for true hypothesis [30].

In all the detection procedures specified above it is assumed that the probability distribution of the information received is known a-priori. In some real-time detection problems, it might be impractical to assume that the probability distribution of the data is known exactly. Though it can be approximated, it is not safe to assume that the distribution is fixed over the time. For example, sensors collecting acoustic data might experience time-varying background noise due to changing conditions. Non-parametric and robust detection techniques provide solution to these kind of problems. Non-parametric detection addresses the problem of detecting a signal in an unknown noise scenario [31,32]. Usually false-alarm probability is the performance metric that is kept constant which is why this is also referred as constant false-alarm rate (CFAR) detection technique. The formulation of non-parametric parametric model yields the
sign test as the best detection rule [33]. The tests that cannot be modelled with certainty due to minor variations in the design of noise density can be addressed by robust detection techniques [34].

1.4 Thesis Contributions

Our approach to energy-consumption is based on the assumption that the quality of the observations sent by the sensor nodes is based on the amount of energy available at the nodes [35]. We further consider sequential testing which offers the possibility of decision making when enough data has been gathered. An optimal trade-off between quality of decisions and the energy involved in gathering the decisions can be achieved. The performance of the proposed model is compared with that of a parallel fusion architecture. It can be observed that the energy-conservation when combined with sequential testing offers huge energy savings and better error probabilities.

In the case of sensor networks that are energy and bandwidth constrained, non-orthogonal communication among sensor nodes has been proposed to avoid long-wait times. Moreover with non-orthogonal communication like Direct-sequence code division multiple access (DS-CDMA) sensors can simultaneously communicate with all the available bandwidth as opposed to an orthogonal one. The performance of the fusion center for a parallel architecture in the presence of multiple-access interference and noise is investigated. An optimal Bayesian detector for such a DS-CDMA based wireless sensor network has been proposed and it is shown that the complexity of the detector is exponential in the number of sensor nodes in the network. In order to provide a low-complex solution, partitioned multi-sensor detectors are proposed where detection and fusion is divided into two stages. The multi-sensor detectors
considered for detection in the first stage are conventional detector, Joint maximumlikelihood (JML), decorrelator, linear minimum-mean squared error (MMSE) detectors. The second stage of the receiver performs Bayesian fusion based on the output of these detectors. For simplicity sake, binary hypothesis testing problem is considered throughout and the model can be safely extended to a multi-hypothesis testing.

1.5 Thesis Outline

The remainder of this thesis is organized as follows: Chapter 2 outlines some of the sequential detection procedures implemented and approximations like Wald’s identity that are required to establish sequential probability ratio test (SPRT).

Chapter 3 features the system model proposed for low-energy multi-sensor networks followed by the simulation results to validate the improved performance of the proposed model under different channel conditions and performance criteria.

Chapter 4 details the proposed optimal Bayesian detector and low-complex partitioned detectors for resource aware sensor networks.

Conclusions and possible extension for the purpose of the future work are specified in chapter 5.
Chapter 2
Sequential Testing

2.1 Introduction

Some of the distributed detection procedures discussed previously are fixed-sample size detectors like serial, parallel and tree architectures etc. They operate with fixed number of samples predetermined at the time of initial design. In many situations, the observations may arrive sequentially. This problem can be addressed if we fix the desired level of performance and allow variable number of samples to achieve this performance [36, 37]. A detector that uses random number of samples depending on the observation sequence is called a sequential detector. For some realizations of observation sequence it is possible to take a decision with fewer samples and for some other realizations more samples may be required to achieve the desired performance. The decision as to when to discontinue taking the samples is a part of the overall detection process. The average number of nodes required by a sequential detector are quite less compared to that of a fixed-sample size (FSS) detector with the same performance [38]. Since we are considering the average samples for a sequential detector, in certain cases it might take more samples than a FSS detector. But this problem can be overcome by restricting the maximum number of nodes in a network.
implementing sequential detector to be same as that of a network with FSS detector.

Sequential detector is characterized by a pair of procedures \((\phi, \delta)\): a stopping rule \(\phi\) and a terminal decision rule \(\delta\). For an observation sequence \(y_k; k = 1, 2 \cdots\) the stopping rule makes the decision based on the stopping time. Stopping time is a random variable since it depends on the observation sequence. Terminal decision rule is a function that is applied to the observation sequence to arrive at a final decision when the stopping rule is applied.

Stopping rule is given by,

\[
\phi_K(y_1, \cdots y_k) = \begin{cases} 
0 & \text{if } \tau_L < L_K(y_1, \cdots y_k) < \tau_U \\
1 & \text{otherwise}
\end{cases}
\]  

(2.1.1)

where \(L_K(y_1, \cdots y_k) \triangleq \) The likelihood ratio at the \(k\)-th sensor and \(\tau_L\) and \(\tau_U\) are thresholds to choose between null and alternate hypothesis respectively.

When \(\phi_K(y_1, \cdots y_k) = 1\), the test stops and takes a decision based on the terminal decision rule otherwise the test continues to gather more observations.

Terminal decision rule is given by

\[
\delta_K(y_1, \cdots y_k) = \begin{cases} 
1 & \text{if } L_K(y_1, \cdots y_k) \geq \tau_U \\
0 & \text{if } L_K(y_1, \cdots y_k) \leq \tau_L
\end{cases}
\]

where \(L_K(y_1, \cdots y_k) \triangleq \) The likelihood ratio at the \(k\)-th sensor

Fig. 2.1.1 depicts a realization of a sequential test. Under mild conditions, the likelihood ratio, \(L_K(y_1, \cdots y_k)\) converges to 1 under alternative hypothesis and to 0 under null hypothesis. To design optimum test just the thresholds \(\tau_U\) and \(\tau_L\) need to be specified accurately. Unless the observation sequence takes discrete values it is difficult to specify the thresholds exactly. Wald’s identity has been used to compute the approximations for the thresholds specified above.
2.2 Wald’s Identity

Wald’s identity has been a powerful tool in computing expected sample size and error probability performance of sequential tests in case of independent and identically distributed observations [39]. The primary application of Wald’s identity is in the sequential test analysis. It can also be applied to some sub-optimal sequential tests and some other applications like providing Chernoff bound in the analysis of sequential algorithms for decoding trellis codes. Among all the sequential tests with the specified error probabilities sequential probability ratio test is considered to be the most efficient since it jointly minimizes the expected sample size for the statistical hypotheses.
By applying Wald’s approximations [39], the thresholds in the equations 2.1.1 can be given by,

\[
\tau_U \approx \frac{P_d}{P_f} \\
\tau_L \approx \frac{1 - P_d}{1 - P_f}
\]

where \( P_d \) is the probability of detection and \( P_f \) is the probability of false-alarm.

### 2.3 Different Sequential Testing Scenarios

As stated previously, sequential testing may be implemented in several ways based on the system design. For instance, design in which sequential testing is implemented at fusion or the one in which sequential testing is implemented at sensor level. Some of the scenarios possible are briefly reviewed in this section.

#### 2.3.1 Distributed Detection with Fusion Center Performing the Sequential Tests

In this scenario, all the sensor nodes in the distributed architecture communicate their local decisions or intermediate observations to the fusion center where a sequential test is performed based on the information received from the sensors [30, 40–42]. It can be viewed as a network analogous to a parallel network with \( N \) sensors and the observations are independent at each sensor and from sensor to sensor. Though the topology of the network can be compared to that of a parallel network specified in Fig. 1.1, this detector is a variable sample-size detector as opposed to a fixed sample-size detector of a parallel topology. At a given time, each sensor computes...
a summary message and broadcasts that to the fusion center and all other nodes in the network. The summary message is a function of its observation at that time and all the previous summary messages from the other sensors. These summary messages are communicated to the fusion center by all the sensors in the network from time to time. Fusion center performs a sequential test based on these intermediate decisions and decides whether to stop taking the observations and yield a final decision or to continue taking more observations for better decision. Again the decision at the fusion center is based on the stopping and terminal decision rules. Even though our discussion is based on the sensors having access to all the summary messages of all the sensors, there are various formulations which include the summary message from a sensor based solely on its observation [42] or based on all the previous observations at that particular sensor [40].

2.3.2 Distributed Detection with Sensors Performing Sequential Tests

In this scenario, sensor nodes perform the sequential test themselves. There is no involvement of fusion center in this case. This topology can be compared to that of a serial architecture depicted in Fig. 1.2 where the final decision of the network is yielded by the last sensor in the network. But in this case the test gets stopped as soon as enough information is gathered to decide upon a hypothesis. Joint performance index between the sensors is achieved by coupled computations of observations [29,43].

2.3.3 Decentralized Quickest Change Detection

Decentralized quickest change detection involves detection of a brusque change in a system based on the change in the probability distribution of the observations and
communicating it to the central entity with out any delay [44]. The stopping time $t$ is imposed based on the assumption that the observations till time $t$ are independent and identically distributed (i.i.d) with a specific probability distribution and the observations after time $t$ are i.i.d with another distribution [45], [46].

For simplicity’s sake most of the scenarios reviewed above consider binary hypothesis testing for sequential detection. The problem of M-ary hypothesis testing has been visited in [47, 48].

Among these scenarios, distributed detection with sensors performing the sequential tests for a binary hypothesis testing problem has been considered in this thesis.
Chapter 3
Sequential Fusion in Energy-constrained Sensor Networks

3.1 Introduction

In classical multi-sensor systems it is a usual assumption that the data sent by sensor nodes is reliably conveyed to the fusion center. But this might not be a safe assumption considering the density of the network and noisy channels. While error control coding can be used to minimize data corruption it might introduce extra computational complexity over the sensor nodes and may add up to the delay as well. An alternative framework has been introduced in [35] where the quality of the information sent by the sensor nodes depends upon the amount of power available at them. It has also been shown that a network having many low-cost, low-power sensor nodes outperforms the network composed of few high-quality, high-power nodes [49], provided the observations are conditionally independent. We explore this concept of sensor networks to see if we can further reduce the energy consumption of these low-power, low-cost sensor networks.
3.2 System Model

A binary hypothesis testing problem for a $N$-node wireless sensor network performing a sequential detection is considered. Let $H_0$ and $H_1$ be the null and alternate hypothesis respectively. Under the two hypotheses the observation $y_k$ at the $k$-th sensor, for $k = 1, 2, \cdots N$, is assumed to be distributed as,

$$H_0 : \quad y_k \sim \mathcal{N}(-m, \sigma_v^2)$$

$$H_1 : \quad y_k \sim \mathcal{N}(+m, \sigma_v^2)$$

(3.2.1)

where $\mathcal{N}(m, \sigma^2)$ denotes a Gaussian distribution with mean $m$ and variance $\sigma^2$. The local observations are considered to be independent when conditioned on the hypothesis. Each sensor runs a likelihood ratio test (LRT) based on its observation and a relayed decision statistic from the previous sensor. The stopping rule specified previously that is to be applied based on LRT is given by,

$$z_k = \frac{m y_k}{\sigma_v^2} + \frac{m r_{k-1} r_{k-1}}{\sigma_{r_{k-1}}^2} = \begin{cases} 
\tau_U' & \text{declare } H_1 \\
\tau_L' & \text{declare } H_0 \\
& \text{otherwise the test continues}
\end{cases}$$

where $z_k$ is the decision statistic at sensor $k$ and $r_{k-1}$ is the relayed decision statistic from sensor $k-1$ distributed as $\mathcal{N}(m_{k-1}, \sigma_{k-1})$

and $\tau_U' = \frac{1}{2} \log \tau_U$

$\tau_L' = \frac{1}{2} \log \tau_L$

If the test has to continue, the sensor $k$ transmits its amplified decision statistic $z_k$ to the next sensor in the network. The amplification factor $a$ is based on the total
power constraint $A$, the $K$-node sensor network is subject to [35], and is given by,

$$a = \sqrt{\frac{A}{K(m^2 + \sigma_v^2)}}$$

The observation at sensor $k+1$ comprises of its observation, relayed decision statistic from the previous sensor and the noise which can be indicated as, $a z_k + y_{k+1} + n_{k+1}$. Since the sensor 1 doesn’t have a relayed decision statistic $z_0$ is initialized to zero in this case. Hence its LRT is purely based on its observation and noise.

It is known that SPRT is a variable sample size detection system. The performance of the system is compared to that of a parallel architecture which is a fixed sample size detection system. Under Neyman-Pearson (NP) criteria we compare the sample size of both the systems required to achieve a desired level of performance.

### 3.3 Simulation Results

Assuming that a SPRT system requires $K$ out of $N$ nodes ($K \leq N$) to achieve the desired level of performance as a parallel network we explore two different scenarios of energy consumption in the network; one in which energy per node is fixed i.e. each node in the SPRT network has the same energy as each node in a parallel one. The other one in which total energy of both the networks is fixed. Although SPRT network is a $N$-node network it takes only $K$-nodes to achieve the desired performance level. Hence this $K$-node energy is distributed among $N$-node parallel network to see the level of performance it can achieve with the same energy as a SPRT network.

Our performance metric is the number of nodes required to achieve a desired level of probability of detection ($P_d$). We investigate the performance with fixed false-alarm probability and fixed signal-to-noise ratio (SNR). Throughout the simulations
the total energy available to each network is fixed at 10 dB and number of nodes in the parallel network (N) to 50.

Figs. 3.1 and 3.2 show the probability of detection achieved by the network as a function of number of sensor nodes for different false-alarm probabilities. SNR of the observation is fixed at 10 dB in this case. It can be observed that the average number of nodes required to achieve the same level of performance is much less compared to that of a parallel network. And this behavior is consistent with the increasing false-alarm probability.

It is also expected that if more room is given for false-alarm probability i.e. if the false-alarm increases, the number of nodes in the network can achieve greater performance measure ($P_d$).

Number of nodes as a function of probability of detection for different SNRs is
Figure 3.2: Number of Nodes vs Probability of Detection for Different False-alarm Probabilities

Figure 3.3: Number of Nodes vs Probability of Detection for Different SNRs
shown in Figs. 3.3 and 3.4. The probability of false-alarm is fixed to $10^{-4}$ in this case. It can be observed that for different SNRs the number of nodes required, to achieve the same probability of detection as a parallel network, is less for a SPRT network. It can also be observed that given a SPRT network, as the SNR increases the quality of the observations also increase and hence a better probability of detection can be achieved by the network.

Further we explore the performance achieved by the two networks by fixing the total energy utilized by the effective number of nodes in the network. Performance metric in this case is probability of detection as a function of number of nodes in the network for fixed false-alarm probability. It can be observed from the figure that a higher level of performance can be achieved by the SPRT network having the same number of nodes as a parallel one.
Figure 3.5: Probability of Detection vs Number of Nodes for Fixed Total Energy for Different False-alarm probabilities
Chapter 4

Distributed Detection in Energy and Bandwidth Constrained Sensor Networks

4.1 Introduction

We consider dense low-power wireless sensor networks that are energy and bandwidth constrained for distributed detection and data fusion. Current literature in distributed detection and data fusion assumes orthogonal communication of sensors. Due to very large number of sensors in the network, multiple sensors may have to transmit data to the fusion center at the same time. In the case of orthogonal communication, sensors need to wait for a long-time in an "active mode" as in the case of time division multiple access (TDMA) scheme. Moreover, total available bandwidth may be limited in orthogonal communication. This arouse the need to consider non-orthogonal communication so that sensors can simultaneously access the channel at the same time and go to sleep mode after transmission. So we consider Direct-sequence code-division multiple-access (DS-CDMA) channel for sensor communication and propose an optimal Bayesian detector for such a DS-CDMA based distributed wireless sensor
network having a parallel architecture in the presence of additive white gaussian noise (AWGN). Figure 4.1 shows the proposed optimal detector for a DS-CDMA channel.

\[ y = R a b^2 + n_i \]

**Figure 4.1: Optimal Bayesian Data Fusion Receiver**

### 4.2 System Model

A binary hypothesis testing problem in a $K$-node wireless sensor network connected to a data fusion center in a distributed parallel architecture [24] is considered. Let $H_0$ and $H_1$ be the null and alternative hypotheses, respectively having corresponding prior probabilities $P(H_0) = p_0$ and $P(H_1) = p_1$. Under the two hypotheses, the $k$-th sensor observation $z_k$, for $k = 1, \cdots K$, is assumed to be distributed as,

\[
    H_0 : \quad z_k \sim \mathcal{N}(0, \sigma_k^2) \\
    H_1 : \quad z_k \sim \mathcal{N}(\mu_k, \sigma_k^2) \tag{4.2.1}
\]

where $\mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian distribution with mean $\mu$ and variance $\sigma^2$. The local observations are considered to be independent of each other when conditioned on
the hypothesis. Each local sensor processes its observations independently to generate a local decision \( u_k \in \{0, 1\} \). We assume identical likelihood ratio tests and decision rules at all the sensors. Assuming a Bayesian approach, the decision \( u_k \) of the \( k \)-th sensor is computed as

\[
 u_k = \begin{cases} 
 1 & \text{if } L(z_k) \geq \tau_k \\
 0 & \text{if } L(z_k) < \tau_k 
\end{cases}
\]

where \( L(z_k) \) is the local likelihood ratio (llr) defined by

\[
 L(z_k) = \frac{p(z_k|H_1)}{p(z_k|H_0)},
\]

and \( \tau_k \) is the threshold of the likelihood ratio test at the \( k \)-th sensor. Under the Bayesian formulation, these sensor thresholds depend on the prior probabilities and an assumed cost function \([38]\). Assuming independent local sensor decisions

\[
 \tau_k = \frac{p_0(C_{10} - C_{00})}{p_1(C_{01} - C_{11})}
\]

where \( C_{ij} \) is the cost incurred by choosing hypothesis \( H_i \) when hypothesis \( H_j \) is true. For the minimum probability of error detection at the local sensors, the cost function can be chosen to be uniform:

\[
 C_{ij} = \begin{cases} 
 1 & \text{if } i \neq j \\
 0 & \text{if } i = j 
\end{cases}
\]

These local decisions \( u_k \)'s, for \( k = 1, \cdots, K \), are next transmitted to the fusion center over a multiple-access channel using DS-CDMA in which sensor \( k \) employs a normalized signature waveform \( s_k(t) \) of unit energy. It is assumed that local sensors take a series of observations \( z_k(i) \) corresponding to a series of true hypothesis denoted by either \( H_0(i) \) or \( H_1(i) \). For simplicity, a binary phase shift keying (BPSK) system
is assumed, where the binary local decisions $u_k(i)$’s, for $k = 1, \cdots K$, are first symbol mapped to $b_k(i) \in \{+1, -1\}$ and then the resultant symbol stream of each sensor $k$ is modulated using the signature waveform $s_k(t)$ of that sensor. It is clear that by sending the binary local decisions $u_k$’s instead of the local observations $z_k$’s, the distributed detection and fusion system can reduce the transmission requirements leading to considerable energy savings in a wireless sensor network.

Assuming symbol synchronism among the distributed sensors and an AWGN channel, the received signal at the data fusion center can be written as

$$r(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) s_k(t - iT) + n(t)$$

where $M$ is the number of data symbols per sensor per frame, $T$ is the symbol interval, $A_k$ is the received amplitude of sensor $k$, $n(t)$ is the zero-mean AWGN receiver noise with variance $\sigma^2 = \frac{N_0}{2}$ and $\{s_k(t); 0 \leq t \leq T\}$ denotes the normalized signature waveform of the $k$-th sensor.

### 4.3 Optimal Fusion Receiver for a DS-CDMA Wireless Sensor Network

Due to the assumed symbol synchronism, the Bayesian fusion problem can be formulated as one of deciding between $H_0(i)$ and $H_1(i)$ based on the observed receiver waveform $\{r(t) : t \in [iT, (i+1)T]\}$ in order to minimize a cost function. It can easily be shown that a sufficient statistic for this fusion problem is given by the output of a bank of $K$-matched filters each matched to a particular user signature waveform $s_k(t)$ similar to in optimal multi-user detection. The vector of matched filter outputs $\mathbf{y} = [y_1, \cdots y_K]^T$ can then be shown to be given by [50],

$$\mathbf{y} = \mathbf{RA} \mathbf{b} + \mathbf{n} \quad (4.3.1)$$
where \( R \) is the \( K \times K \) normalized cross-correlation matrix of sensor signature waveforms, \( A = \text{diag}(A_1 \cdots A_K) \) is a diagonal matrix consisting of received sensor signal amplitudes and \( n \sim \mathcal{N}(0, \sigma^2 R) \) is the \( K \)-vector of Gaussian receiver noise with mean \( 0 \) and covariance matrix \( \sigma^2 R \). Note that in (4.3.1) we have dropped the time index \( i \) since it is irrelevant due to the assumed symbol synchronism among the sensors.

The data fusion problem for the DS-CDMA wireless sensor network may be interpreted as a binary-hypothesis testing based on the observation vector \( y \). Thus, it can be shown that the Bayesian optimal fusion rule is given by a likelihood ratio-test (LRT) as below:

\[
L(y) = \frac{p(y | H_1)}{p(y | H_0)} P_{H_1} \frac{1}{P_{H_0}} \tau_F
\]

where \( \tau_F \) is the threshold at the fusion center which depends on the prior probabilities \( p_0 \) and \( p_1 \) and the cost function. In the case of minimum probability of error fusion (uniform cost assignment) and equal \textit{a priori} probabilities for two hypotheses, the threshold for the likelihood ratio fusion becomes \( \tau_F = 1 \).

Using the received signal model (4.3.1) the required likelihood ratio at the fusion center can be expressed as,

\[
L(y) = \frac{\sum_{b \in \{+1,-1\}^K} e^{b^T A y - \frac{1}{2} b^T A R A b} p(b | H_1)}{\sum_{b \in \{+1,-1\}^K} e^{b^T A y - \frac{1}{2} b^T A R A b} p(b | H_0)}
\]

where the summations are over all possible \( 2^K \) transmit symbol vectors.

Assuming that the local sensor decisions are independent we can compute the
conditional probabilities $p(b | H_i)$, for $i = 0, 1$ as,

$$p(b | H_i) = \prod_{k=1}^{K} p(b_k | H_i)$$

where

$$p(b_k | H_1) = \begin{cases} 1 - P_{M_k} & \text{if } b_k = +1 \\ P_{M_k} & \text{if } b_k = 0 \end{cases}$$

and

$$p(b_k | H_0) = \begin{cases} P_{F_k} & \text{if } b_k = +1 \\ 1 - P_{F_k} & \text{if } b_k = 0 \end{cases}$$

with $P_{F_k}$ and $P_{M_k}$ representing the false-alarm and miss probabilities of the $k$-th sensor, respectively. It is assumed that fusion center knows these false-alarm and miss probabilities associated with all sensors. Based on the local observation statistics in (4.2.1) we may show that these local probabilities are given by

$$P_{F_k} = Q \left( \tau'_k / \sigma_k \right)$$

(4.3.3)

and

$$P_{M_k} = 1 - Q \left( (\tau'_k - \mu_k) / \sigma_k \right)$$

(4.3.4)

where $Q(x)$ denotes the Gaussian tail distribution (or $Q$-function) and the thresholds $\tau'_k$ are given by,

$$\tau'_k = \frac{\sigma_k^2 log(\tau_k) + \mu_k}{2 \mu_k}.$$ 

Then the optimal fusion receiver decisions are given by

$$\delta_{opt}(y) = \begin{cases} 1 & \text{if } L(y) \geq \tau_F \\ 0 & \text{if } L(y) < \tau_F \end{cases}$$

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where $L(y)$ is given in (4.3.2).

As can be observed from (4.3.2), the computation of the optimal fusion rule can be very high in complexity. For example, for the binary problem at hand, if we define the required number of multiplications (NoM) as the time complexity of a receiver, then the complexity of the optimal fusion scheme can be seen to be of the order of $(K + 2)^2 K$, which is exponential in the number of users $K$.

### 4.4 Low-complexity, Partitioned Fusion Receivers

As noted above the optimal fusion receiver has a very high time complexity which is exponential in the number of users. In order to reduce this complexity, a class of sub-optimal fusion receivers which separate the multi-sensor detection and fusion into two stages as shown in Fig. 4.2 are proposed. In these partitioned receivers, the multi-sensor detection is performed at the first stage as in a traditional multiuser detector in order to estimate the symbols $b_k$’s transmitted by the local sensors. The estimated symbol vector $\hat{b}$ is the input to the second stage of the receiver that performs data fusion. Next, the second stage of the receiver performs data fusion based on these outputs as if they were the true local decisions. These partitioned receivers can be designed so that they provide performance very close to that of the optimal fusion receiver proposed but at a very low computational complexity.

Several well-known multiluser detectors are considered for the first stage of the partitioned receivers like Joint maximum likelihood (JML) detector, conventional single user matched filter and two linear multiuser detectors namely decorrelator and minimum mean square error detectors.
4.4.1 Joint ML First Stage Based Partitioned Fusion Receiver

The joint ML multi-sensor detector at the first stage estimates the symbol vector $\mathbf{b}$ by an estimate that maximizes the joint likelihood function of $\mathbf{b}$. While JML does not necessarily result in minimum probability of error for individual sensors it has been shown that its performance very close to that of the minimum probability of error detector for individual users [50]. The advantage of JML is that it is easier to analyze than the minimum probability of error detector.

The output of the joint maximum likelihood (JML) multi-sensor detector is given by [50]

$$\hat{\mathbf{b}}^{(JML)} = \arg\max_{\mathbf{b} \in \mathbb{R}^K} (2\mathbf{y}^T \mathbf{b} - \mathbf{b}^T \mathbf{A} \mathbf{R}_A \mathbf{b}). \quad (4.4.1)$$

Next, the second stage of the partitioned receiver performs data fusion assuming that the estimated values $\hat{\mathbf{b}}^{(JML)}$ to be the true independent local decisions. It should be observed that first stage decisions are not independent and the above assumption is made to reduce the complexity at the second stage since our goal is to design good low-complexity receivers compared to that of optimal fusion receiver. Thus, the
likelihood ratio employed at the second stage of the receiver is,

\[ L \left( \hat{b}^{(JML)} \right) = \frac{p \left( \hat{b}^{(JML)} | H_1 \right)}{p \left( \hat{b}^{(JML)} | H_0 \right)} \]

\[ = \prod_{k=1}^{K} \frac{p \left( \hat{b}_{k}^{(JML)} | H_1 \right)}{p \left( \hat{b}_{k}^{(JML)} | H_0 \right)}, \tag{4.4.2} \]

where (due to the assumption that the first stage decisions are correct),

\[ p(\hat{b}_{k}^{(JML)} | H_1) = \begin{cases} 1 - P_{M_k} & \text{if } \hat{b}_{k}^{(JML)} = +1 \\ P_{M_k} & \text{if } \hat{b}_{k}^{(JML)} = -1 \end{cases}, \]

and

\[ p(\hat{b}_{k}^{(JML)} | H_0) = \begin{cases} P_{F_k} & \text{if } \hat{b}_{k}^{(JML)} = +1 \\ 1 - P_{F_k} & \text{if } \hat{b}_{k}^{(JML)} = -1 \end{cases}. \]

The required false-alarm and miss probabilities are computed as in (4.3.3) and (4.3.4).

Assuming minimum probability of error Bayesian fusion with uniform costs and equal priors, the LRT at the fusion center is then given by,

\[ \delta_{JML} \left( \hat{b}^{(JML)} \right) = \begin{cases} 1 & \text{if } L \left( \hat{b}^{(JML)} \right) \geq 1 \\ 0 & \text{if } L \left( \hat{b}^{(JML)} \right) < 1 \end{cases} \tag{4.4.3} \]

Due to the exponential time complexity of the JML multiuser detection at the first stage of the partitioned receiver, the time complexity for JML detector can be shown to be of the order of \( K2^K \). Since this is not a significant improvement over the optimal fusion receiver we look to replace the first stage with low-complexity multi-sensor detectors.
4.4.2 Conventional Detector Based Partitioned Fusion Receiver

One of the simplest detectors for the multiple-access channel is the single-user matched filter otherwise called conventional detector which, in the case of BPSK, directly quantizes each component of the matched filter bank output $y$. Thus the output of the single-user matched filter first stage is given by,

$$\hat{b}_{(MF)}^{(MF)} = sgn(y).$$

(4.4.4)

The second stage of the receiver is exactly the same as that described above in Section 4.4.1. i.e. the estimated bits $\hat{b}_{(MF)}^{(MF)}$ are used to compute the likelihood ratio as in (4.4.2) and the global decision is declared using (4.4.3). From (4.4.4) it can be seen that time complexity for a conventional matched filter detector is greatly reduced. It does not require any multiplications (note that likelihood-ratio computation in (4.4.2) can be implemented as a table look-up operation).

It is known that in the presence of severe multiple-access interference the performance of conventional detector can be very poor compared to the best possible performance (however, as we will see in simulation results below in a distributed detection based data fusion problem this may not always be true). Since linear multi-user detectors can provide a balance between performance and complexity, in the following section two partitioned fusion receivers are considered in which the first stage is a linear multi-sensor detector.
4.4.3 Linear Multi-sensor Detector First Stage Based Fusion receivers

The first linear detector we used is the decorrelator:

\[ \hat{b}^{(\text{decor})} = \text{sgn}(R^{-1}y) \]

The decorrelator zero-forces the multiple-access interference for each user at the expense of noise enhancement. In order to minimize the combination of interference and noise we may instead use the minimum mean square error (MMSE) multi-sensor detector:

\[ \hat{b}^{(\text{MMSE})} = \text{sgn}\left((R + \sigma^2 A^{-2})^{-1}y\right) \]

In terms of number of required multiplications both these detectors has a time complexity of \( K \) (note that matrix inverses can be pre-computed since they are fixed for the synchronous channel). The fusion rule at the second stage is again exactly the same as that in (4.4.2) and (4.4.3).

4.4.4 Simulation Results Optimal Fusion and Partitioned Detectors

The performance and complexity trade-offs of the optimal fusion receiver and the partitioned low-complexity receivers are investigated via several representative numerical examples. Although in all cases we limit ourselves to a binary hypothesis testing problem and a BPSK-based synchronous DS-CDMA communications over an AWGN channel, this model can be safely extended to fading channels [51]. The Bayesian cost function considered is the minimum probability of error with equally probable hypotheses. Thus, the performance metric is the probability of error at
the output of the fusion receiver. Considering a 4-node architecture the value of cross-correlation coefficient is set at 0.7 for all the results shown. The performance of different detectors as a function of signal-to-noise (SNR) is investigated. Fig. 4.3 shows the performance of the fusion receivers as a function of average local SNR for fixed channel SNR value of 6 dB for all $k$.

![Figure 4.3: Fusion Probability of Error vs Average Local SNR for Fixed Channel SNR = 6dB for all $k$](image)

It is evident from Fig. 4.3, the performance of JML and conventional detector based partitioned receivers are very close to that of optimal fusion receiver. It can also be observed that the performance of decorrelator based partitioned receiver is poor among all the fusion receivers and its performance is similar to that of individual
sensor's probability of error. Although it is surprising that the performance of conventional detector based partitioned receiver is close to that of optimal fusion receiver in AWGN channel, its performance degrades severely in Rayleigh fading channel [51].

Fig. 4.4 shows the fusion probability of error performance as a function of the average channel SNR for a fixed local sensor SNR value for all \( k \) sensor nodes at 10dB. It can be observed from the figure that the performance of the receivers at the fusion center are highly dependent on the local sensor SNR i.e. even though the channel SNR improves, the fixed local SNR value limits the performance of the fusion receivers and the performance of partitioned receivers converge to that of the optimal fusion.
receiver. Moreover, it can also be observed that there is a clear channel SNR below which single sensor performance is better than that of fusion receivers.
Chapter 5

Conclusions

The main focus of this thesis work is to explore various distributed detection and data fusion strategies for energy and bandwidth constrained sensor networks. In the case of networks which are energy-constrained SPRT based energy-efficient distributed detection model has been proposed. From the numerical results it can be concluded that the proposed network model achieves higher level of performance when compared to that of fixed sample-size detector based networks. With fixed per-node energy the proposed SPRT based detection model results in very less number of nodes required to achieve the probability of detection compared to that of parallel architecture network. With fixed total-energy in the network, the probability of detection achieved by a SPRT based network is improved when compared to a parallel network having same number of nodes.

In the case of energy and bandwidth constrained sensor networks optimal Bayesian fusion receiver operating in non-orthogonal communication channel (DS-CDMA) is proposed and it can observed that its complexity is exponential in the number of local sensor nodes. In order to reduce this complexity, a class of sub-optimal fusion receivers are proposed by partitioning the multi-sensor detection and data fusion
into two consecutive stages. For the first stage multi-sensor detection several well known multiuser detectors such as joint maximum likelihood, conventional single-user matched filter, decorrelator and minimum mean square error detector are investigated. In the case of JML first stage based receiver which was found to perform very close to the optimal fusion receiver in most cases has an exponential complexity. However, the single-user matched filter first stage based fusion receiver also performs remarkably well, and in particular close to the performance of optimal receiver, as long as there is not a large power imbalance in the channel SNR values. Moreover, while fusion performance is not limited by a fixed channel SNR value when the local SNR values are increasing, for a fixed local SNR value we observed that the performance will be limited by it for large channel SNR values. In this case, the performance of all partitioned receivers approaches that of the optimal fusion receiver for large channel SNR values.

As a possible extension for future work, the data dependency in the case of energy-constrained sensor networks based on SPRT based detection model can be explored.
BIBLIOGRAPHY


