ANALYSIS OF FIBER WAVINESS IN LAMINATED COMPOSITES SUBJECTED TO COMPRESSIVE LOADS

A Thesis by

Harsha Kantharaju

Bachelor of Engineering, Visvesvaraya Technological University, India, 2007

Submitted to the Department of Mechanical Engineering
and the faculty of Graduate School of
Wichita State University
in partial fulfillment of
the requirements for the degree of
Master of Science

May 2012

ANALYSIS OF FIBER WAVINESS IN LAMINATED COMPOSITES SUBJECTED TO COMPRESSIVE LOADS

The following faculty members have examined the final copy of this thesis for the form and content, and recommend that it can be accepted in partial fulfillment of the requirements for the degree of Master of Science with a major in Mechanical Engineering.

____
Hamid M Lankarani, Committee Chair

____
Michael McCoy, Committee Member

____
Krishna Krishnan, Committee Member

DEDICATION

To
The Hard Work and the Efforts that run in our World

ACKNOWLEDGEMENTS

At the outset, I would like to thank the contributions of every researcher to continuously improve the world in which we live. It would have not been possible to derive an inspiration for this thesis but for your efforts. I would like to thank my thesis advisor, Dr. Hamid M Lankarani, for his encouragement and support during my student life at the Mechanical Engineering Department of Wichita State University. It would have been an impossible task for me to complete my research without his support. Further, I would like to thank Dr. Michael McCoy and Dr. Krishna Krishnan for investing their valuable time to review my research. I would like to thank the enormous support that I received from my parents, my brother, my bhabhi and all my friends. It has given me the much needed strength to wade through even the most difficult times in my life.

ABSTRACT

The competence of composite materials to supersede the use of metals and traditional materials has made it a preferred application in the field of structural engineering, aviation, automobiles, etc. An insightful research into the evaluation mechanical properties and behavior of the material under different loading conditions enables an improved design of the structure. The evaluation of different types of defects in a composite material has been a key area of research in the recent years. In structural analysis, the fiber waviness in laminate layers is often ignored and ideal properties of straight fibers are assumed. These properties are generally lower and non-conservative as the strength and stiffness are lower than that predicted for straight fiber composites.

In this thesis, the compressive behavior of a composite cross ply laminate with fiber waviness is investigated. The reduction in the mechanical properties such as stiffness and strength is studied with the help of analytical equations and finite element modeling. First, an analytical model is developed to predict the reduction in the stiffness for a cross ply laminate. The equations in the model are solved in Maple software. The finite element analysis is then carried out on ABAQUS software to study the effect of waviness on the stress distributions and the loss of strength for a laminate with fiber waviness. The results obtained from various experiments are used to validate the analytical models and the finite element results under similar conditions.

TABLE OF CONTENTS

LIST OF TABLES

Table **Page**

LIST OF FIGURES

Figure **Page**

LIST OF FIGURES (Continued)

Figure **Page**

LIST OF ACRONYMS/NOMENCLATURE

A	Extensional Stiffness
a	Amplitude of Wave
ASTM	American Society for Testing and Materials
B	Coupling Stiffness
C	Ply Stiffness Component
D	Bending Stiffness
E_1	Longitudinal Elastic Modulus
E_2	Transverse Elastic Modulus
FE	Finite Element
G_{12}	In Plane Shear Modulus
h	Laminate Thickness
M	Applied Moment
N	Applied In Plane Load
RVE	Representative Volume Element
ε^0	In Plane Strain
κ	Bending Curvature
ν_{12}	Major Poisson's Ratio
ν_{21}	Minor Poisson's Ratio
θ	Orientation Angle of the Ply
ϕ	Misalignment Angle of the Wavy Ply

LIST OF ACRONYMS/NOMENCLATURE

The subscripts and superscripts indicate the following as applicable

i,j	Principal Material Axis
x,y	Transformed Coordinate Axis
T	Tensile Strength
C	Compressive Strength
S	Shear Strength

CHAPTER 1

INTRODUCTION

1.1 Background

For a very long time, the quest to improve the existing systems has been the driving force for the numerous inventions and discoveries that we have before us today. A lot of efforts dedicated to probe into better processes, materials and usage have led to the birth of several fields of engineering. As the systems evolved, and the processes advanced; it became imperative to research into advanced materials to meet the design requirements. This is typically true in the field of aviation where composite material is the foremost area of technological research. The superior mechanical performance and properties of composite materials and its ability to be optimized provide the designers with a wide range of applicability. However, it also became important to study the effect of defects in composites which may creep in during the manufacturing or fabrication processes. The assessment of composite material behavior and its effect on material integrity, durability and characterization is vital.

1.2 The Requirement to Analyze Defects in a Composite Material

Almost every major airplane manufacturer employs laminated composites in the modern day for the purpose of reduction of weight and the subsequent fuel consumption. While there have been significant gains achieved in terms of high specific properties with composite materials, the challenges to manufacture a defect free composite part are equally high. The manufacturing of cylindrical profiles using composite materials was performed using tape winding process as shown in Figure 1.1. It was with this type of manufacturing that the first instance of fiber waviness was observed as shown in Figure 1.2. Fiber waviness was also observed in flat laminates due to the thermal coefficient mismatch between the composites and the mold material. The problem of waviness became a concern within the industry because there were instances reported where the complete layer(s) of the laminate were found to be undulated due to the aforementioned reasons.

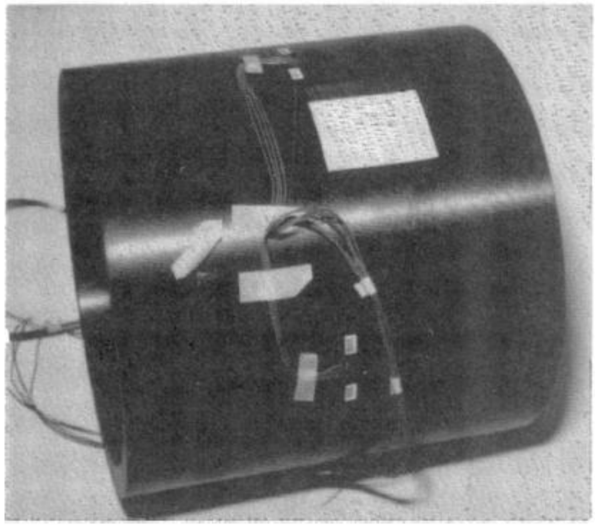

Figure 1.1: Composite Cylinder Fabricated by Tape Winding Process [6]

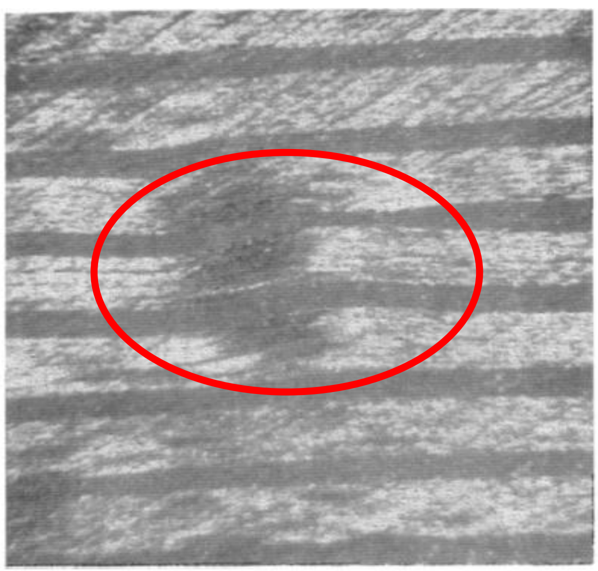

Figure 1.2: Fiber Waviness Observed in the Hoop Layers of the Composite Cylinder [6]

In every manufacturing industry certain amount of defects are allowed in a structure. However, there is no criterion that enables the regulations or guidelines to be laid for a structure with wavy plies. The analytical models provide results that enable to define a performance acceptance (or rejection) for the structure. For defects like delamination, voids, porosity etc., studies have been carried out to determine their effect on the performance of the structures. However, ply waviness in laminates has been studied mostly for unidirectional plies, the data and experiments available for cross ply and quasi laminate configurations are scarce. A study of ply localized waviness in a cross ply laminate composite is carried out in this thesis.

In a composite structure, the waviness in the plies may not have a predefined pattern, orientation or magnitude. In order to understand the effect of waviness of plies, characterization of waviness is essential. The beginning of this research is based on idealization of the defect; wherein a portion of the plies in the laminate midplane are considered as undulated. This isolation or localization of waviness helps to develop a common minimum understanding of response of the structure under various loading conditions. The parameters such as amplitude of the wave, wave length, misalignment angle, etc are discussed considering an idealized environment. The establishment of failure under an idealized condition will create a better understanding through which complex geometries of the wave, loading conditions etc can be researched upon.

In this thesis, analytical and numerical based finite element analyses have been carried out to understand the effects of wavy plies in a composite laminate subjected to compressive loads. In the analytical research, the reductions in elastic properties are studied with the help of analytical models to determine the stiffness values such as Young's modulus, shear modulus, Poisson's ratio, etc. Parametric studies were performed to study the influence of ply waviness on the stiffness in the laminate. The purpose of finite element analysis is to determine the possible regions of failure in the laminate and predict the failure modes and strength. The purpose of the study is to understand the effects of wavy plies in a cross ply composite laminate.

1.3 Literature Review

The complexity of the composite material has been attributed to its heterogeneous nature that has an effect on the microstructure of the material, resulting in defects. The earliest inquests into the defects in composite materials were carried out to study the effects of manufacturing defects such as voids, porosity, fiber misalignment etc. The purpose of studying defects was to measure the deviation from the assumed ideal properties in composites. These defects and their growth in a controlled environment were mainly studied through mechanical testing. This knowledge enabled the establishment of valid acceptance criteria for composites with defects.

There are various causes of fiber waviness in laminated composite. The difference in Coefficient of Thermal Expansion (CTE) between tooling material, fiber and the matrix is of several orders in magnitude. This difference causes development of longitudinal and transverse stress in the part. The temperature gradient experienced by the part while processing and consolidation of matrix is the other waviness inducing mechanism. The uneven cooling results in higher matrix contraction and induces fiber buckling. The lower matrix stiffness at high temperatures is another cause of waviness in composites. Also, the spatial and temporal temperature gradients generated during curing affects the viscoelastic behavior of the matrix, inducing waviness in the fibers. There are two types of modeling approaches that are used for analysis of temperature gradients to determine its effect on fiber waviness. The micromechanical model studies the mismatch of material properties between the fiber and matrix that generally causes residual stresses in a composite laminate. The macro mechanical model study concentrates at the high stress regions within the laminate caused by the temperature gradients.

The processing parameters play a significant role in inducing waviness in laminated composite materials. The parameters that induce fiber waviness during manufacturing are: Hold temperature, hold time during cure stage, pressure, length of the part, width, thickness, tool plate material and cooling rate. Pressure, thickness and width of the part do cause waviness in a laminated composite material. However, other parameters such as fiber waviness amplitude, number of wrinkles formed, wavelength etc., are dependent on the length of the part.

The tooling material used for composite curing must be selected carefully. It is important to make an informed choice because of the differential spatial or temporal gradients between the tool and the composite part that are formed during the cure cycle. The low thermal diffusivity of tooling materials such as steel, brass and copper enhance formation of temperature gradients in the composite material. The effect of these gradients enhance when the temperature during curing reaches the glass transition temperature. Large temperature gradients cause a portion of the laminate to consolidate and transfer of loads while the remaining regions are soft. This effect tends to change the positions of the laminate layers during consolidation. During the consolidation process in the post cure stage, the difference in CTE causes both shrinkage of tooling material and formation of temperature gradients in the composite.

Increase in temperature above the glass transition causes a drop in resin viscosity and hence the load required for fiber motion is also reduced. For some matrix materials the viscoelastic behavior is non linear even at low stress levels. The free volume inside the matrix and mobility of the polymeric chains cause physical aging. This significantly influences viscoelastic properties in the matrix. Accelerated cooling is one of the reasons for insufficient relaxation of viscoelastic stresses in a composite part. This causes fibers to experience elastic deformation leading to waviness in the profile.

Fiber waviness in laminated composites has received attention over the years. Research efforts have been mainly concentrated upon the fiber waviness in unidirectional laminates while the efforts to study waviness in multidirectional/quasi laminates are relatively less. Several analytical models were built to predict the waviness influence on the properties such as strength and stiffness of the laminate subjected to compression. The first of such study was carried out by Rosen [1] to predict the compressive strength based on microbuckling model in the case of straight fibers. Rosen undertook a study to predict the strength based on kinking of fibers under buckling conditions. He predicted that the strength of perfectly aligned fibers with a kink band shown in Figure 1.3 is given by

$$\sigma_c = \frac{G_m}{(1 - v_f)} \tag{1.1}$$

The above equation was approximated such that the strength of the composite in the material would become equivalent to the shear modulus of the material. However, Fleck and Budiansky [7] conducted experiments on laminates with fiber waviness and showed that the compressive strength for the composite material was over predicted by Rosen. The stresses around the kink band were one-fourth of the stresses measured for a laminate with perfectly aligned fibers. It was also argued that to understand the phenomena of kinking, it was necessary to consider the effects of strain hardening, inclination of the kink band and the applied shear stresses.

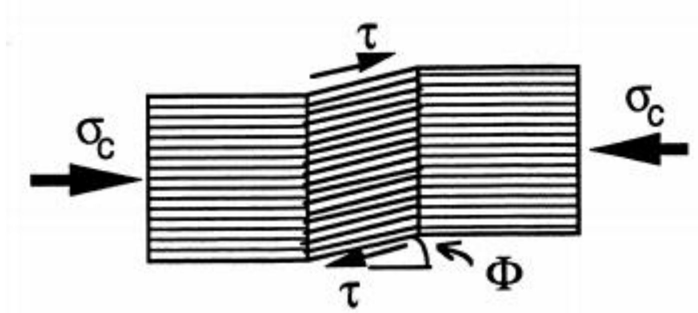

Figure 1.3: Alignment of Kink Band w.r.t Longitudinal Axis [7]

The initial imperfections of a kink band are, however, unable to produce a large knockdown in the composite strength values. The role of plastic and shear nonlinear deformations in reduction of strength of a laminate with kink band was studied by Argon [2]. The author came up with an approximate relation for compressive laminate strength with misaligned fibers as

$$\sigma_c = \frac{\tau_Y}{\Phi} \tag{1.2}$$

where, the shearing yield stress (τ_y) and the initial rotation angle (ϕ) of the fibers were considered as important parameters for predicting strength of the composite with misaligned fibers. The discrepancies with Rosen as well as Argon's equations were put forward by several experiments conducted on laminates with fiber waviness. The formulations largely over predicted the critical stresses in the composite. Though the models were proved incorrect, the idea that the strength of the composite was related to its shear modulus was gained. The yield strength of the material and the fiber waviness parameters such as amplitude, wavelength, orientation angle became important factors that were included in future studies.

As the maturity level with composites application increased, there was a gradual movement from the use of unidirectional fibers towards woven fabric for reinforcement. The fabric materials provide balanced properties for the laminate and a better resistance to impact loading. Ishikawa and Chou [3] studied the effects of fiber undulation in fabric material using a one dimensional model. The fabric can be idealized as a cross ply laminate and the stiffness and compliance of the material can be determined by making use of this idealization. The authors made use of the bound theory and discovered that the fiber undulation causes softening of the in plane stiffness. The authors also researched on the non linear behavior of flexible fiber composites [4] & [5]. The sinusoidal nature of waviness in curved profiles and the relationship between fiber amplitude, wavelength, and fiber lengths were also studied. The authors studied the behavior of waviness in the laminates with different fiber orientations and conducted experiments using an iso phase and a random phase model. The experimental results of the longitudinal and transverse tensile behaviors compared very closely with the predicted values.

Bogetti and Gillespie [9] & [10] developed an analytical model to study the effects of ply waviness in a cross ply laminate shown in Figure 1.4. The model was developed to predict the effects on the stiffness and strength reduction due to ply waviness in a composite laminate. The basis for their research was to study the structural performance of composite cylinders that often employ cross ply laminates in their constructions. The strength reductions and the dominant ply failure mechanisms were analyzed in their investigations. Parametric studies were carried out to determine the effect of amplitude, wavelength, stacking sequence etc on the mechanical properties of the laminate. The strength reduction resulting from inter laminar shear failure was one of their principal findings. The investigations tried to explain why thick laminates with flat autoclave cure cycles had significantly different test results when compared with their coupon properties.

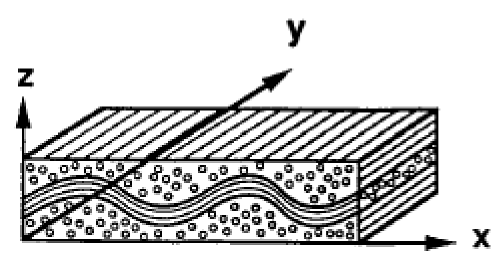

Figure 1.4: Fiber Waviness Depiction in a Cross Ply Laminate [9]

An experiment was carried out to avoid the thermal spiking problem associated with composite laminates during manufacturing that leads to misalignment of plies and voids in a laminate [12]. It was shown that layer waviness induced due to non uniform consolidation and voids causes significant strength reductions. The need for optimizing the processing parameters such as increasing cure pressure, reducing material accretion thickness were proposed as solutions to avoid layer waviness in laminates.

The effects of fiber waviness were further explored analytically by Hsiao and Daniel [13], [14] and [15]. The research was carried on unidirectional laminates with uniform, graded and localized fiber waviness in them. The non linear effects of waviness in laminates under compressive loads were also studied. The analytical models were further expanded to study the waviness effect cross ply laminate. The research discovered that the material anisotropy has an effect on the strength reductions in wavy laminates. These predictions were validated with experimental results and it was found that the results were in good agreement with the analytical predictions. The research also showed that inter laminar shear stresses had a significant influence and was the dominant cause of failure when compared to delamination and buckling of layers under axial compression. An analytical model was built to consider the nonlinear effects based on complimentary strain energy relations. It was found that wavy laminates undergo no shear deformations because their effective response tends to remain specially orthotropic. The research into cross ply laminates was continued by Bradley and Adams [16] where they compared the analytical predictions with finite element and experimental results. Maximum stress theory and Hashim Rotem's failure theories were used to study the behavior of the wavy laminate.

Joyce and Moon [17] were probably the first researchers to have studied the effect of strength reduction in laminates with in plane fiber waviness. The research in itself is remarkable considering the lack of literature related to in plane waviness to compare and verify the results. They found that the strength reductions due to in plane waviness produces similar effects to that of laminates that fail due to stress concentration in the tabbed regions. They also discovered that the compressive strength of T300/P1700 material system is significantly less when compared to AS4/3501-6 material. The Wyoming Test Fixtures (WTF) were used to study the effect of in plane fiber waviness. It was found that the bending correction factors used for D695 test methods are not required while using the WTF test method.

Wisnom and Atkinson [18] induced waviness artificially in the plies of the laminate by preforming the prepreg on a curved plate. A 0.7 mm thick curved aluminum sheet was treated with release agent and sixteen plies were laid on it. The plies were consolidated by application of pressure and heat, and the plate was allowed to spring back to the flat laminate shape. In plane waviness of the fibers was observed on the surface plies along with out of plane waviness in the layers of the laminate. Pin ended buckling rig tests conducted on these laminate coupons showed 17 to 26% reductions in tensile and compressive strains measured. The reductions in the strains were believed to be small in magnitude and were attributed to small thickness of the surface where the waviness in the fibers was observed.

Chun and Daniel [19] investigated the tensile and compressive response of unidirectional laminates with fiber waviness. The objective of the study was to theoretically and experimentally examine the effects of material and geometric nonlinearities on fiber waviness in composite materials. The research was focused on uniform, graded and local fiber waviness patterns commonly found in the laminates. The nonlinearity effects were incorporated in the analytical models based on the complementary energy density function. The analytical model showed a reduction in Young's modulus along the longitudinal direction. It was also shown that the degree of fiber waviness influences the strength of the laminate. Also, it was shown that tensile loads cause stiffening of fibers while compressive loads cause softening of the wavy fibers.

Fleck and Liu [20] used a stress formulation to predict the initiation of microbuckling in a wavy laminate subjected compression and bending loads as shown in Figure 1.5. The authors predicted the microbuckling strength of the laminate using a one dimensional infinite band analysis. With larger values of fiber misalignment, band broadening effects and bending theory were used to predict compressive strength. The bending theory helped in predicting the compressive strength as a function of amplitude and size of the waviness in the laminate.

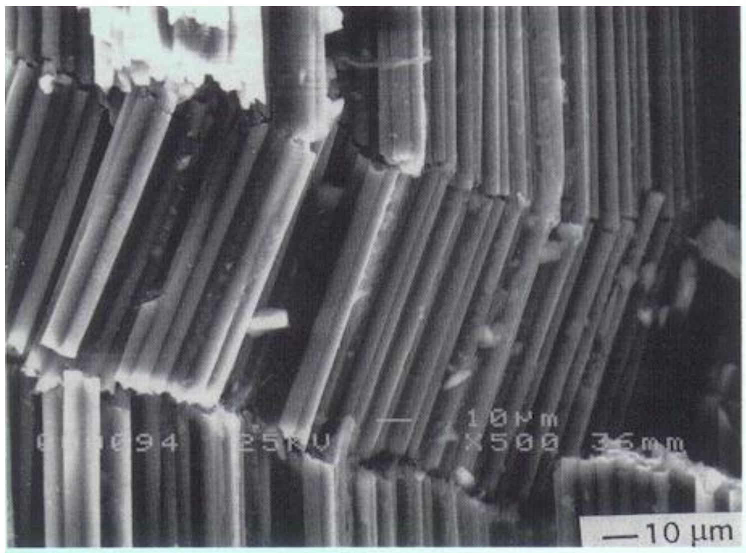

Figure 1.5: Microbuckling Failure of a Composite Laminate Subjected to Compressive Loads [28]

Kugler and Moon [22] identified the material and processing parameters that cause fiber waviness in a laminate as shown in Figure 1.6. They investigated the effect of cure temperature, time, pressure, tooling dimensions, cooling rate and tool plate material which were previously thought to cause waviness due to uneven curing across the laminate. It was found that the rate of cooling affects wavelength and amplitude of the waviness. The thermal expansion coefficient mismatch between tool and the part was found to be significant mechanism causing waviness. The reason for microbuckling was attributed to the inability of the matrix to provide transverse support to the wavy fibers subjected to axial loads.

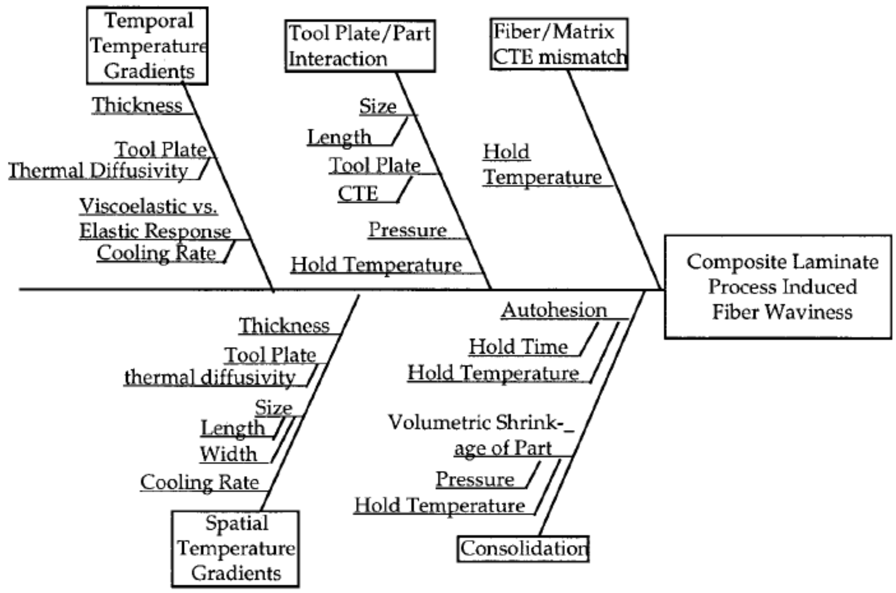

Figure 1.6: Causes of Fiber Waviness during Manufacturing [21]

Garnich and Karami [22] & [23] undertook the finite element study of fiber waviness in laminate at a micromechanical level as shown in Figure 1.7. They built a linear, elastic micromechanical model of a fiber- matrix unit cell as shown in Figure 1.8. The analysis of the unit cell was carried out to characterize the structural stiffness by studying the stress and strain components. The effects of stresses in the wavy laminate on the axial properties cum stiffness and its influence on failure predictions were also studied. The model was also used to study the thermoelastic properties in the composite laminate.

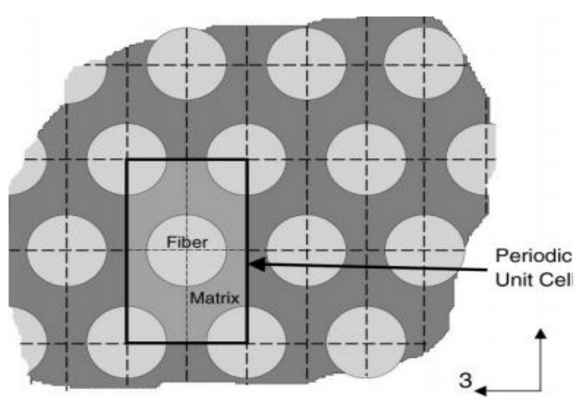

Figure 1.7: Micromechanics Model used to Study Fiber Waviness [22]

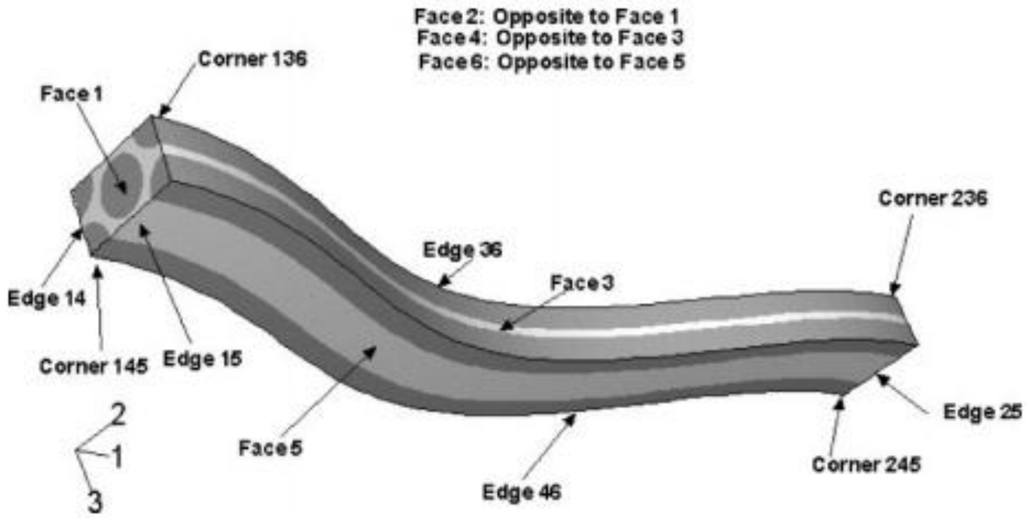

Figure 1.8: FE Analysis of Micromechanical Approach to study Fiber Waviness [22]

1.4 Compression Testing of Composite Laminates

In the last few years, several authors have investigated the effect of compression on a composite laminate. The studies have been carried out on unidirectional [1] and [2], cross ply [11] and multidirectional laminates [29]. The properties of the fibers, matrix and the interface of fiber-matrix regions in the laminate are thought to affect the compressive strength. The focus of the investigations has been to carry out analytical modeling and conducting experimental tests. Micromechanical studies have also been carried out to study the effect of compression in a composite laminate [22].

From various research investigations, it was found that the compression strength for a composite laminate is only about 60% of its tensile strength. The complexity of compression testing arises due to the tendency of the laminates to undergo buckling or inter laminar shear failure. These failures are also accompanied with matrix cracking and splitting which lends the matrix incapable of supporting the fibers, inducing failure in the laminate. The geometric discontinuities such as voids, porosity, fiber waviness, etc compound the issue of microbuckling of fibers within the laminate. The manufacturing defects also cause a non uniform distribution of matrix in laminate. Consequently, under the action of compression loads, the resin rich areas experience a reduction in the in plane load carrying capability inducing matrix dominated failure modes in the laminate. An effort is made in this thesis to analytically study the effect of compression in a cross ply laminate with localized ply waviness.

1.5 Objectives

The main goal of this thesis is to study the influence of fiber waviness on the stiffness and strength of a composite laminate subjected to compression. The individual objectives are as explained

- To study the effects of out of plane waviness (layer/ply waviness) on the reduction of stiffness properties of a composite cross ply laminate.
- To build an analytical model based on the equations of mechanics of composite laminates to calculate the elastic properties by considering the parameters of ply waviness.
- To conduct parametric studies on the waviness of the laminate by considering two types of waves i.e. moderate and severe; which are classified based on their amplitude to wavelength ratio and the misalignment angle in the longitudinal direction.
- To compare the analytical predictions using the available research results obtained in [15].
- To study stress distributions in the composite laminate by building a finite element model.
- To predict failure mode and failure strength of for the laminates with ply waviness using finite element results.

1.6 Methodology

The approach used to investigate the objectives described in Section 1.5 is shown in Figure 1.9. The composite cross ply laminate is built using Carbon/Epoxy T300/1076 unitape prepreg. Local waviness in the plies is considered in the present analysis where the plies in the laminate midplane are assumed to have a sinusoidal wavy profile. The following activities are undertaken in this thesis

- Develop an analytical model based on mechanics of composite laminate to determine the stiffness parameters for a laminate with no wavy layers using Maple software.
- Develop an analytical model which will calculate the stiffness values for a cross ply laminate with wavy plies in the laminate midplane.
- Perform a parametric study and study the effect on stiffness values for the model by varying the waviness parameters such as amplitude, wavelength and misalignment angle of the plies
- Perform a FE analysis using ABAQUS to understand the stress distributions in the wavy layers of the laminate.
- Predict failure mode and failure strength for the laminate by employing Maximum Stress Failure criteria for the laminate

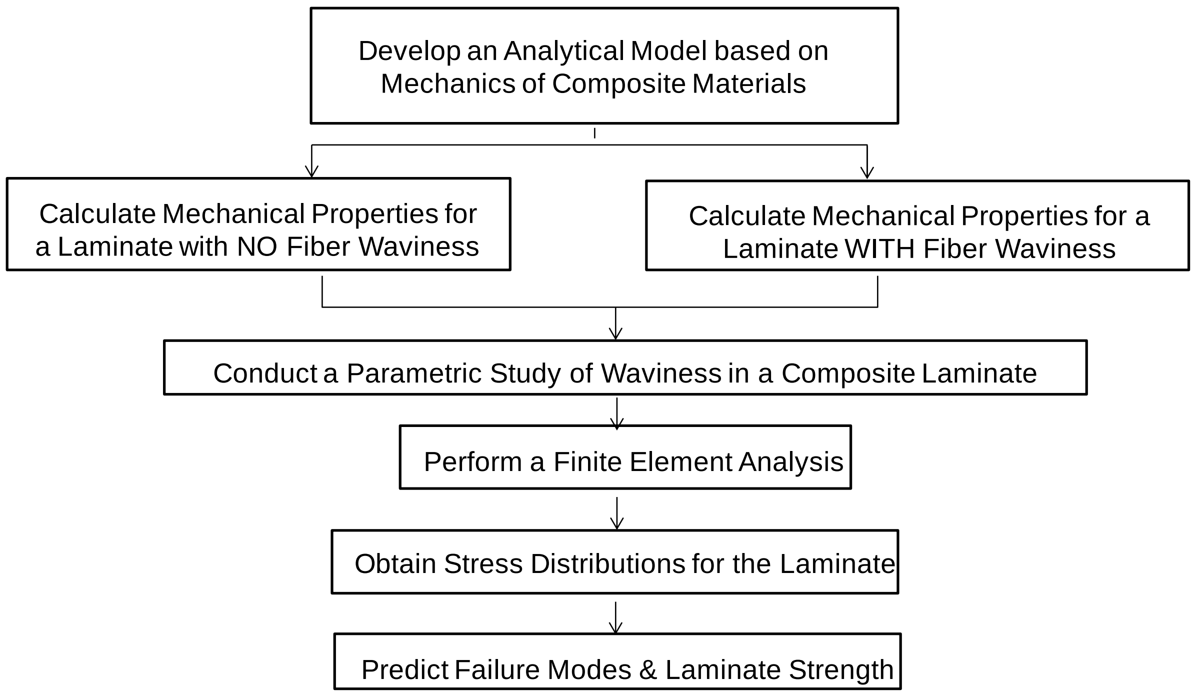

Figure 1.9: Description of Research Methodology

CHAPTER 2

ANALYTICAL MODELING

2.1 Building an Analytical Model

The analytical investigations into fiber waviness require a generic model to be constructed using the stress transformation relations and the parameters of the wave geometry. The localized waviness of the plies can be considered as an out of plane distortion in the orientation of the plies. The analytical model helps to predict the elastic properties of laminates by dividing the laminate into small, finite segments known as Representative Volume Element (RVE) as shown in Figure 2.1. The segments are treated as an off axis lamina and the coefficients of the compliance matrix are calculated using the transformation relations. The strain values for the discrete segments (RVE) are calculated from the compliance matrix. The strain values obtained are then integrated over the wavelength and the average strains are obtained in the RVE. The elastic properties for a composite laminate can be calculated from the average strain values obtained. This chapter deals with the analytical equations that are essential for calculating the laminate properties.

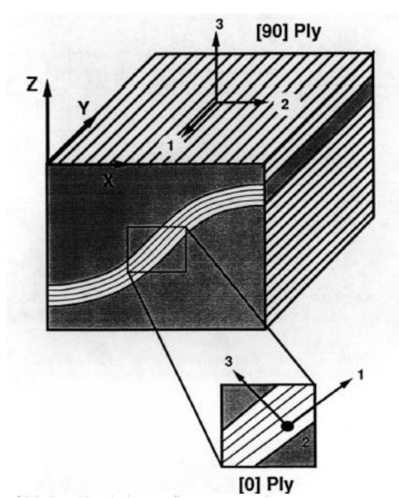

Figure 2.1: RVE Depiction in a Composite Laminate [9]

The classical laminate plate theory is applied to each of the discrete segments and a response is calculated by employing the averaging approach. Along with the prediction of the elastic properties, the analytical model can be developed to predict the stresses in the wavy layers of the laminate. The stresses can be expressed in terms of the in plane loads. The wavy layers tend to develop out of plane stresses due to the undulations. After the calculation of stresses in the layers, maximum stress criteria can be applied to predict the ultimate failure in the laminate. The maximum stress criteria also helps in identification of the failure mode in the laminate. The ability of the analytical model to predict failure enables quantification of strength reduction due to waviness in the laminate.

2.2 Elastic Properties for a Laminate

The first activity in building the analytical model is to construct a model to evaluate the stiffness parameters of a laminate that contains no fiber waviness in its plies. To build such a model it is necessary to evaluate the in plane extensional stiffness matrix [A_{ij}] and its components. The steps to build the analytical model for a laminate with no ply waviness is explained below [28]

Step 1: Obtain the engineering properties of the material that will be used in the analysis. The longitudinal Young's modulus E_1, transverse Young's modulus E_2 and shear modulus G_{12}, Poisson's ratio ν_{12} are the basic properties necessary.

Step 2: The layer stiffness matrix [C_{ij}] for the 0^0 and 90^0 plies are calculated from the equation.

$$[C_{ij}] = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & 0 \\ & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ & & G_{12} \end{bmatrix} \tag{2.1}$$

The stiffness matrix [C_{xy}] for an angled ply is calculated by transforming the stiffness matrix [C_{ij}] using the following relations

$$C_{xx} = m^4 C_{11} + n^4 C_{22} + 2m^2 n^2 C_{12} + 4m^2 n^2 C_{66}$$

$$C_{yy} = n^4 C_{11} + m^4 C_{22} + 2m^2 n^2 C_{12} + 4m^2 n^2 C_{66}$$

$$C_{xy} = m^2 n^2 C_{11} + m^2 n^2 C_{22} + (m^4 + n^4) C_{12} - 4m^2 n^2 C_{66}$$

$$C_{ss} = m^2 n^2 C_{11} - m^2 n^2 C_{22} - 2m^2 n^2 C_{12} + (m^2 - n^2)^2 C_{66} \tag{2.2}$$

Step 3: Obtain the laminate thickness coordinates of the n^{th} layer from the laminate mid plane.

Step 4: The laminate extensional stiffness matrix is calculated using the following equation

$$A_{ij} = \sum_{n=1}^{N} C_{ij}{}^{n} (h_n - h_{n-1}) \tag{2.3}$$

Step 5: The laminate compliance matrix [a] is calculated by inverting the components of [A_{ij}] obtained for the laminate from *Step 4*

Step 6: The mechanical properties for the laminate are obtained by the following relations

$$E_x = \frac{1}{ha_{xx}} \qquad E_y = \frac{1}{ha_{yy}} \qquad G_{xy} = \frac{1}{ha_{ss}} \tag{2.4}$$

$$\nu_{xy} = -\frac{a_{yx}}{a_{xx}} \qquad \nu_{yx} = -\frac{a_{xy}}{a_{yy}} \tag{2.5}$$

2.3 Elastic Properties for a Wavy Laminate

In order to capture the influence of fiber waviness in a composite laminate, it is necessary to mathematically describe waviness. The waviness of a laminate described with an amplitude ‘a’ and wavelength, ‘λ’, is expressed using a wave function as [4]

$$y = a \sin \frac{2\pi x}{\lambda} \tag{2.6}$$

The angle between the tangent of the wavy layer and the longitudinal direction is given by the first derivative of the wave function

$$\frac{dy}{dx} = \tan\theta = \frac{2\pi a}{\lambda}\cos\frac{2\pi x}{\lambda} \tag{2.7}$$

The length of the discretized layer, say *dL*, between any two points along the longitudinal direction, say x and x+dx is given by

$$dL = \sqrt{dx^2 + dy^2} = \sqrt{1+\left(\frac{2\pi a}{\lambda}\right)^2 \cos^2\left(\frac{2\pi a}{\lambda}\right)}dx \tag{2.8}$$

The length of the wavy layer is obtained by integrating the expression for *dL* from 0 to λ

$$L = \int dL = \frac{\lambda}{2\pi}\int_0^{\lambda}\sqrt{1+\left(\frac{2\pi a}{\lambda}\right)\cos^2\beta d\beta} \tag{2.9}$$

The generic classifications of waviness in a laminate are Uniform, Graded and Localized. In this thesis, the elastic properties of localized waviness are researched upon.

2.3.1 Elastic Properties for a Laminate with Localized Central Waviness

For cross ply composites with wavy central layers under the current study, the relation between force and moment with the deformation and curvatures are expressed using the classical laminate plate theory equation as [28]

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}\begin{bmatrix} \varepsilon^o \\ \kappa \end{bmatrix} \tag{2.10}$$

For a balanced and symmetric laminate, the components of coupling matrix do not exist; i.e., $[B_{ij}] = 0$. The laminate under consideration in this thesis is a cross ply laminate. Since the axes of the plies are concurrent with the principal material axes, this laminate is called as a special orthotropic laminate. Further, a uni-axial compressive load is applied to the laminate. No bending moments are applied in our case of analysis. Hence, the above equation reduces to

$$[N]=[A][\varepsilon^{o}] \tag{2.11}$$

The in plane strains are calculated by inversing the in plane extensional stiffness matrix

$$[\varepsilon^{o}]=[a][N] \tag{2.12}$$

where,

$$[\mathrm{a}] = [\mathrm{A}]^{-1} \tag{2.13}$$

The components of the extensional stiffness matrix, $[A_{ij}]$ are calculated using the relation

$$A_{ij}=\sum_{k=1}^{N}(\overline{C}_{ij})_k t_k \tag{2.14}$$

where, $\quad_{ij}$ is the transformed stiffness matrix of the cross ply laminate that includes wavy and unperturbed layers. The average transformed stiffness matrix equations and the calculations are as shown. Under uni-axial loading condition, the resultant in plane load are expressed as

$$\varepsilon_x = a_{xx}N_x = ha_{xx}\sigma_x$$

$$\varepsilon_y = a_{xy}N_x = ha_{xy}\sigma_x$$

$$\varepsilon_z = a_{xz}N_x = ha_{xz}\sigma_x \tag{2.15}$$

The transformed stiffness matrix components in the case of a localized waviness present in a matrix are calculated using the following equations [15]

$$\overline{C}_{11}=C_{xx}=C_{11}m^4+2(C_{12}+2C_{66})m^2n^2+C_{22}n^4$$

$$\overline{C}_{12}=C_{xy}=C_{12}m^2+C_{23}n^4$$

$$\overline{C}_{13}=C_{xz}=(C_{11}+C_{22}-4C_{66})m^2n^2+C_{12}(m^4+n^4)$$

$$\overline{C}_{15}=C_{xr}=C_{11}m^3n-C_{22}mn^3-(C_{12}+2C_{66})(m^2-n^2)mn$$

$$\overline{C}_{22}=C_{yy}=C_{22}$$

$$\overline{C}_{23}=C_{yz}=C_{23}m^2+C_{12}n^2$$

$$\overline{C}_{25} = C_{yr} = (C_{12} - C_{23})mn$$

$$\overline{C}_{33} = C_{zz} = C_{11}n^4 + 2(C_{12} + 2C_{66})m^2n^2 + C_{22}m^4$$

$$\overline{C}_{35} = C_{zr} = C_{11}mn^3 - C_{22}m^3n + (C_{12} + 2C_{66})(m^2 - n^2)mn$$

$$\overline{C}_{44} = C_{qq} = \frac{1}{2}(C_{22} - C_{23})m^2 + C_{66}n^2$$

$$\overline{C}_{46} = C_{qs} = (C_{66} - \frac{1}{2}C_{22} + \frac{1}{2}C_{23})mn$$

$$\overline{C}_{55} = C_{rr} = (C_{11} + C_{22} - 2C_{12})m^2n^2 + C_{66}(m^2 - n^2)^2$$

$$\overline{C}_{66} = C_{ss} = \frac{1}{2}(C_{22} - C_{23})n^2 + C_{66}m^2 \tag{2.16}$$

It is necessary to calculate the entire stiffness matrix coefficients of [$_{ij}$] because the localized waviness in the laminate causes an interaction between the stiffness components.

The overall elastic properties of the laminate with localized waviness are calculated from the equations shown below [15]

$$E_x = \frac{A}{h(A_{22}A_{33} - A_{23}{}^2)}$$

$$E_y = \frac{A}{h(A_{11}A_{33} - A_{13}{}^2)}$$

$$\nu_{xy} = -\frac{A_{13}A_{23} - A_{12}A_{33}}{A_{22}A_{33} - A_{23}{}^2}$$

$$\nu_{yx} = -\frac{A_{13}A_{23} - A_{12}A_{33}}{A_{11}A_{33} - A_{13}{}^2}$$

$$G_{xy} = \frac{A_{66}}{h} \tag{2.17}$$

The components of the transformed stiffness matrix in case of a wavy laminate are calculated using the following relations [15]

$$A_{11} = h[(p + rI_1)C_{11} + (q + rI_5)C_{22} + 2rI_3(C_{12} + 2C_{66})]$$

$$A_{12} = h[(p + q + rI_6)C_{12} + rI_8C_{23}]$$

$$A_{13} = h\{[p + r(I_1 + I_5)]C_{12} + qC_{23} + rI_3(C_{11} + C_{22} - 4C_{66})\}$$

$$A_{22} = h[(p + r)C_{22} + qC_{11}]$$

$$A_{23} = h[(p + rI_6)C_{23} + (q + rI_8)C_{12}]$$

$$A_{33} = h[(p + q + rI_1)C_{22} + rI_5C_{11} + 2rI_3(C_{12} + 2C_{66})]$$

$$A_{22} = h[(p + r)C_{22} + qC_{11}]$$

$$A_{23} = h[(p + rI_6)C_{23} + (q + rI_8)C_{12}]$$

$$A_{33} = h[(p + q + rI_1)C_{22} + rI_5C_{11} + 2rI_3(C_{12} + 2C_{66})]$$

$$A_{44} = h[(\frac{1}{2}p + \frac{1}{2}rI_6)(C_{22} - C_{23}) + (q + rI_8)C_{66}]$$

$$A_{55} = h\{[(p + r(I_5 + I_6)]C_{66} + \frac{1}{2}q(C_{22} - C_{23}) + rI_3(C_{11} - 2C_{12} + C_{22} - 2C_{66})\}$$

$$A_{66} = h[(p + q + rI_6)C_{66} + \frac{1}{2}rI_8(C_{22} - C_{23})]$$

$$A = A_{11}A_{22}A_{33} + 2A_{12}A_{13}A_{23} - A_{13}{}^2A_{22} - A_{12}{}^2A_{33} - A_{23}{}^2A_{11} \tag{2.18}$$

where, the relation between the wavy layers and non wavy layers are

p = Ratio of thickness of non wavy 0^0 layers to total laminate thickness

q = Ratio of thickness of 90^0 layers to total laminate thickness

r = Ratio of thickness of wavy 0^0 layers to total laminate thickness

The components [I_i] are calculated using the following relations. These components assist in capturing the effect of waviness by including the amplitude and the wavelength parameters in their formulations [15].

$$\alpha = 2\pi \frac{a}{L}$$

$$I_1 = \frac{1+\alpha^2/2}{(1+\alpha^2/2)^{3/2}}$$

$$I_3 = \frac{\alpha^2/2}{(1+\alpha^2/2)^{3/2}}$$

$$I_5 = 1 - \frac{1+3\alpha^2/2}{(1+\alpha^2/2)^{3/2}}$$

$$I_6 = \frac{1}{(1+\alpha^2)^{1/2}}$$

$$I_8 = 1 - \frac{1}{(1+\alpha^2)^{1/2}} \tag{2.19}$$

The equations described above were coded in *Maple* software and solved to obtain the required elastic properties and Poisson's ratio for the laminates. The following chapter describes the parametric modeling of fiber waviness in the laminates.

CHAPTER 3

PARAMETRIC STUDY OF FIBER WAVINESS

3.1 Elastic Properties for a No Wave Laminate

The laminate selected for investigation in this thesis is an 18 ply laminate made of carbon/epoxy prepreg T300/1076. The thickness of the carbon/epoxy unitape prepreg used in the research is 0.008”. The layup sequence for the laminate is $[0_2/90_2/0_2/90_2/0]_s$. Carbon/epoxy prepregs are high modulus materials popularly used in primary and secondary structural aerospace applications because of their high strength, toughness and resistance to impact. The material properties for the prepreg shown below were obtained from the research data presented in [6]. The comparison between the properties of the prepreg lamina and the 18 ply laminate without wavy plies is shown in Table 3.1. The elastic property for the laminate without any waviness in the plies is calculated based on the analytical model solved in Maple. The output from the program is shown in the Appendix.

Table 3.1: Comparison of Properties between Lamina and Laminate

Property	**Lamina/Single Ply**	**18 Ply Laminate**
Longitudinal Young’s Modulus, E_1 (Msi)	20.2	11.9
Transverse Young’s Moddulus, E_2 (Msi)	1.5	9.8
In Plane Shear Modulus, G_{12} (Msi)	0.67	0.67
Major Poisson’s Ratio, ν_{12}	0.3	0.04
Minor Poisson’s Ratio, ν_{21}	0.02	0.03

The strength values for the Carbon/Epoxy T300/1076 material is given in Table 3.2.

Table 3.2: Strength Allowable for T300/1076 Unitape [6]

Property	**Value (ksi)**
Longitudinal Tensile Strength, X_T	230.3
Longitudinal Compressive Strength, X_C	191.1
Transverse Tensile Strength, Y_T	11.7
Transverse Compressive Strength, Y_C	32.5
In plane Shear Strength, S_{12}	14.7

3.2 Elastic Properties for a Wavy Laminates

The analysis of fiber waviness in the laminates is investigated by conducting parametric studies on a wavy laminate. The laminate ply sequence is $[0_2/90_2/0_2/90_2/0_w]_s$. The 0^0 ply in the laminate midplane (represented by 0_w in the laminate layup sequence) is assumed to have a localized sinusoidal waviness. Two laminate types are considered for the analyses

(i.) Moderate Wave and,

(ii.) Severe Wave Laminates.

The research conducted by Hyer and Adams [11] and Hsiao and Daniel [13], [14] and [15] concluded that the parameters of waviness i.e. amplitude, a, wavelength, λ, and the misalignment angle, θ, play a significant role in stiffness and strength reductions.

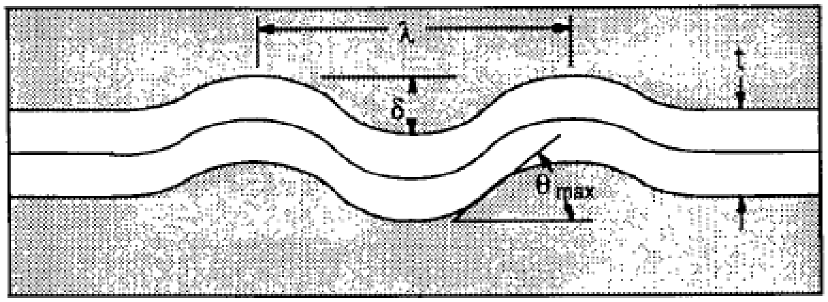

Figure 3.1: Example of Wavy Ply in a Laminate [11]

The observations post static tests showed that the laminates with moderate waviness failed at the tab or the grip regions as shown in Figure 3.2.

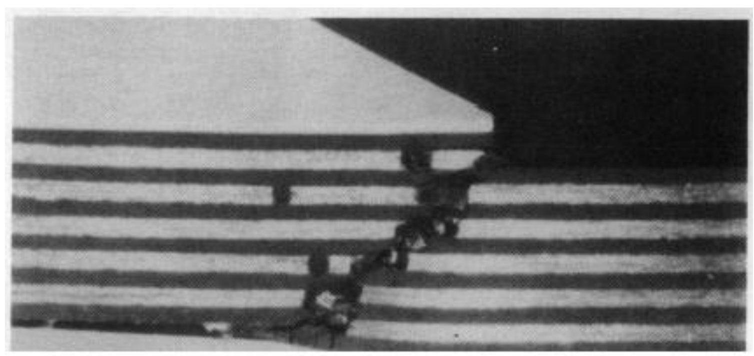

Figure 3.2: Failure at the Grip Region for a Moderate Wave Laminate [11]

The laminate with severe waviness showed a broom type of failure located at the gage sections as shown in Figure 3.3. The orientation of the wavy plies to the longitudinal axis was also decisive in the classification of the laminates by the authors.

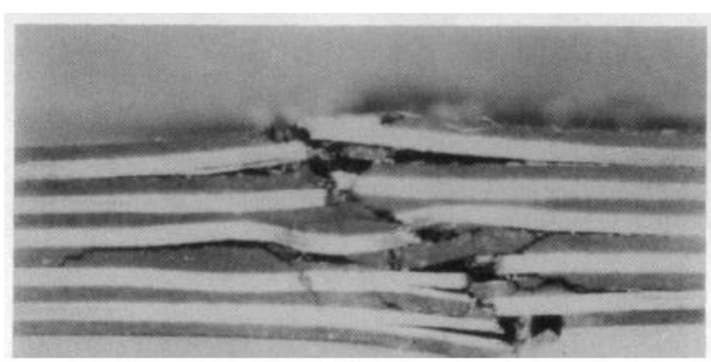

Figure 3.3: Broom Failure Observed at the Gage Section of Severe Wave Laminate [11]

The laminates with higher amplitude to wavelength ratio (a/λ > 0.06) and misalignment angle greater than 12^0 were considered as severe waviness laminates. The parameters of the wavy laminated considered for parametric studies are shown in Table 3.3.

Table 3.3: Wave Parameter Values for the Laminate

Parameter	**Moderate Wave**	**Severe Wave**
Amplitude (inch)	0.0074	0.0148
Wavelength (inch)	0.148	0.237
Misalignment Angle (Degrees)	6	13
Amplitude/Wavelength Ratio	0.05	0.062

The values of the wave parameters for the first laminate considered in parametric study are shown in Table 3.3. The amplitude of the wave is 0.0074" which nearly equal to the thickness of a single ply considered for analysis. The wavelength of 0.148" is close to the thickness of the laminate itself. The amplitude to wavelength ratio is 0.05 and the misalignment angle of 6^0 enables it to be classified as a moderate wave.

The values of the wave parameters given in Table 3.3 for the second laminate are higher than the corresponding values given for moderate wave. The amplitude of the wave is 0.0148" which is equal to thickness of two plies in the laminate. The high amplitude to wavelength ratio of 0.062 and a misalignment angle of 14^0 enables the laminate to be classified as severe wave.

Based on the analytical model discussed in the preceding chapter, the results obtained for laminates with no waviness and moderate waviness are compared in Table 3.4.

.

Table 3.4: Comparison of Properties between No Wave and Moderate Wave Laminate

Property	**E_1 (Msi)**	**E_2 (Msi)**	**G_{12} (Msi)**	**ν_{12}**	**ν_{21}**
No Wave Laminate	11.9	9.8	0.67	0.04	0.03
Moderate Wave Laminate	11.3	9.4	0.66	0.01	0.01
Reduction %	5.0	4.1	1.5	75.0	66.7

It can be observed that the Young's modulus values in the longitudinal and transverse directions have reduced by 5% and 4.1% respectively. There's a slight reduction (of 1.5%) in the in plane shear modulus. The reduction for the shear modulus values will not be significant because the shear modulus value for the lamina does not vary along the principal directions for the material. There is a significant reduction in the values obtained for Poisson's ratio. However, since the overall value of the Poisson's ratio is low (0.01), it indicates that the lateral strains in the laminate are not significant to cause an effect during loading.

Based on the analytical model discussed in the preceding chapter, the results obtained for laminates with no waviness and severe waviness are compared in Table 3.5.

Table 3.5: Comparison of Properties between No Wave and Severe Wave Laminate

Property	**E_1 (Msi)**	**E_2 (Msi)**	**G_{12} (Msi)**	**ν_{12}**	**ν_{21}**
No Wave Laminate	11.9	9.8	0.67	0.04	0.03
Severe Wave Laminate	10.3	8.7	0.65	0.02	0.01
Reduction %	13.4	11.2	3.0	50.0	66.7

From Table 3.5 it is observed that the reductions in the Young's modulus values obtained are higher than the corresponding values obtained for the moderate wave. This verifies the assumption that the stiffness values are dependent on the wave parameters. The reduction for the in plane shear modulus is 3% which indicates a possibility of higher shear stress in the severe wave when compared to moderate wave at the region of waviness. As explained before, the Poisson's ratio will not cause significant lateral strains in the laminate.

3.3 Experimental Data and Validation

Hsiao and Daniel conducted experiments and analytical investigations [15] into the model that were built using the equations described in section 2.3.1. The various laminates containing ply waviness that were built and tested experimentally are shown in Table 3.6. The research was conducted on three grades of waviness namely, Unidirectional Fiber Waviness, Graded Fiber Waviness and Localized Fiber Waviness. The equations to calculate the stiffness parameters for these laminates differ slightly in the three cases because of the variations in the in plane stiffness matrix [A_{ij}] values.

Table 3.6: Experimental Results of Stiffness Properties [15]

Parameter	**Uniform Fiber Waviness**	**Graded Fiber Waviness**	**Localized Fiber Waviness**
No of Plies	150	72	72
Amplitude (inch)	0.047	0.011	0.011
Wavelength (inch)	1.1	0.57	0.57
Tangent (Degrees)	15	7.2	7.2
Amplitude/Wavelength Ratio	0.043	0.02	0.02
Experimental Young's Modulus (Msi)	14	23	13.2
Analytical Young's Modulus (Msi)	13.6	23.1	14.1
Difference (%)	2.8	0.4	6.8

The values shown in Table 3.6 were obtained based on the experimental and analytical studies. The laminates were fabricated using IM6G/3501-6 carbon epoxy composites and static compression tests were conducted by employing NU fixtures to apply the loads.

3.4 Discussion of Results

The cause of reduction in stiffness in the laminate with wavy plies can be ascribed to the rotation of these plies in the out of plane direction. These rotations cause an unstable response within the laminate which results in initiation of microbuckling and an early failure. The misaligned regions are, generally, thought to be weaker in the transverse direction when compared to the longitudinal directions. The role of material anisotropy exhibited by composites leads to an increase in the ultimate strains in the plies, causing a reduction in the modulus properties.

The wavy plies are unable to carry loads in spite of a uniform loading condition imposed on the laminates. This is because of the local stresses generated in these regions in form of inter laminar normal and shear stresses that resist load transfer. The out of plane waviness of the plies also leads to generation of in plane shear coupling stiffness terms [A_{xs}] and [A_{ys}]. These terms cause the development of matrix dominated failure modes in the laminate such as delamination, matrix cracking, splitting, etc which accelerates the effect of microbuckling failure. The aforementioned causes also lead to reduction of stiffness and strength properties for the laminate.

The highlights of the parametric study are

- The reduction in the stiffness parameters obtained for laminates with waviness when compared with laminates with no waviness
- The small values obtained for Poisson's ratio signifies that the lateral strains do not play a decisive role in the laminates.

CHAPTER 4

FINITE ELEMENT MODELING AND ANALYSIS

The objective of undertaking the study of Finite Element Analysis (FEA) in this thesis is to understand the effect of waviness on the stress distributions within the layers of the composite laminate under compression loading. The concerns with the waviness in the laminate are focused to gage the magnitude and effect of stress concentrations in the areas of undulation in the layers. The FEA results can be used in parametric studies where the effect of stress concentration for laminate with similar waviness patterns can be studied. The ability of FEA to predict failure may be used as a parameter while defining the acceptance criteria for employing laminates with wavy layers in structural applications.

A two dimensional FE model is built using ABAQUS software to conduct the finite element analysis. The purpose of the analysis was to study stress distributions for a laminate with ply sequence $[0_2/90_2/0_2/90_2/0]_s$. The analysis is performed on two types of waviness i.e. Moderate and Severe wave as discussed in Section 3.2. The failure stresses and failure modes for the laminate with waviness are predicted from the analysis. The capability of the finite element analysis to predict failure helps to correlate the results obtained from analytical modeling. Any discrepancy in the results obtained will also help in the identification of errors with the FE model.

4.1 Description of Finite Element Model

In general, the finite element models are built with an assumption of symmetry within the layer waviness. However, in actuality, the waviness in the plies may not have a perfectly symmetrical geometry within itself. The waviness in the central 0^0 plies were modeled as sinusoidal waves and the other plies are modeled as perfectly straight lamina. The waviness in the central 0^0 ply causes a slight variation in the thickness of the adjacent 90^0 ply. However, in general, the thickness of the other layers was idealized as constant at 0.008 inch. The length of the modeled region is 5.5 inch which is the standard size of specimens defined per ASTM 6641 standards [27]. The thickness of the model for an 18 ply thick laminate is 0.144 inch as shown in Figure 4.1. To model waviness in the layers, spline feature was used to create the geometry of the wave based upon the wavelength and amplitude parameters defined in Table 3.3. The maximum length of undulations in the wavy laminates was kept at 0.2208 inch and 0.3561 inch respectively for the moderate and severe wavy laminates.

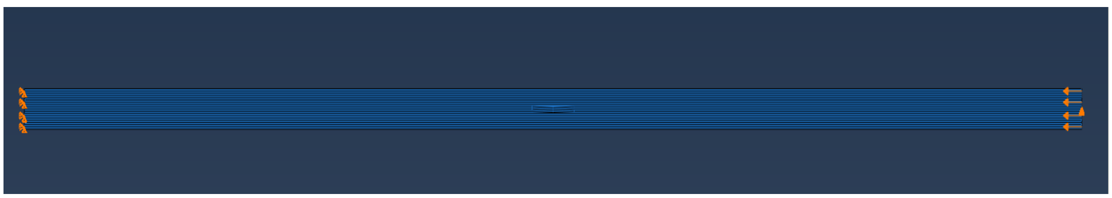

Figure 4.1: Laminate Modeling in ABAQUS

Two dimensional plane strain elements, with quadratic formulations were used to mesh the model. The seeds during meshing were chosen such that two rows of elements were obtained in each of the plies. At the region of waviness in the central 0^0 ply, the seeds were refined further to obtain a fine mesh across the interface of the wavy 0^0 ply and the adjacent 90^0 ply layer as shown in Figure 4.2.

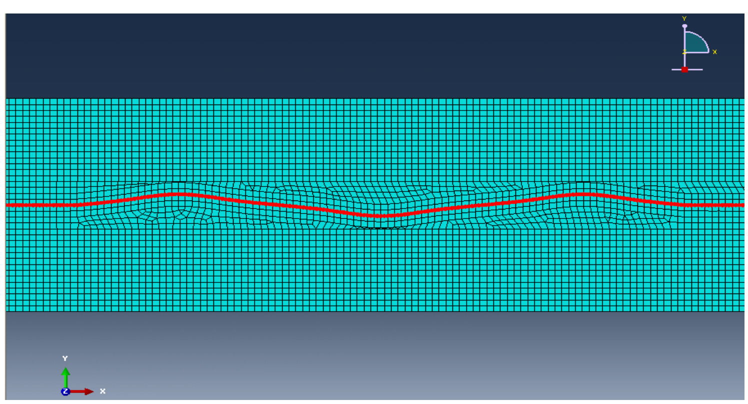

Figure 4.2: Meshed FE Model

The 0^0 and 90^0 plies were modeled using orthotropic material properties. The material properties for the Graphite/Epoxy T300/P1076 prepreg are given in Table 3.2. The material orientation for the layers in ABAQUS was specified such that they were in accordance with the principal material axes of the laminate layers. The fibers in the wavy central 0^0 plies are assumed to be parallel to the elements on the top and bottom of the edge of the ply layers. The left end of the model was fixed and constrained in the X and Y directions ($u_x = u_y = 0$) as shown in Figure 4.3.

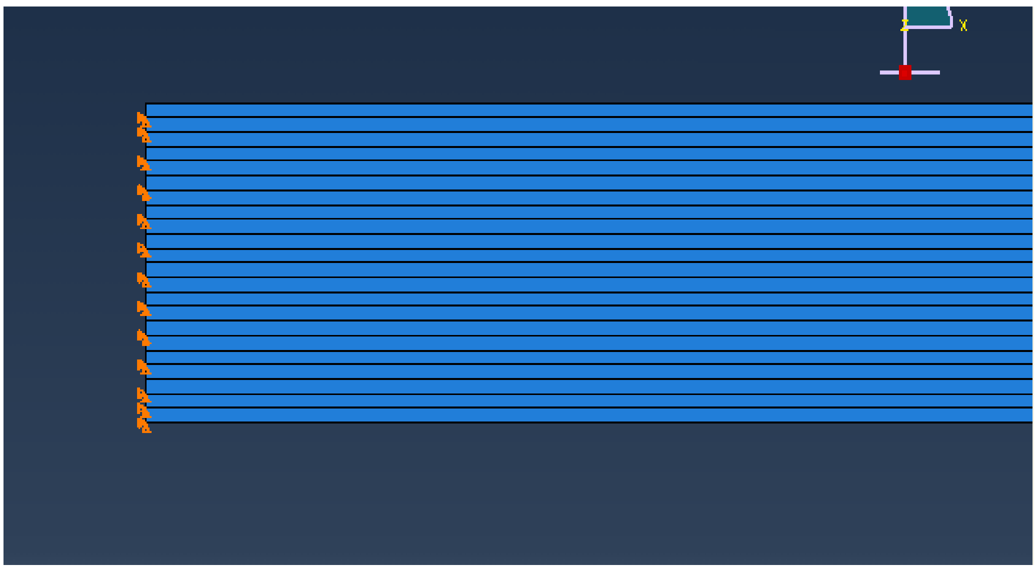

Figure 4.3: Boundary Condition Applied to the Left End of the FE Model

The central 0^0 ply in the laminate mid plane was constrained on the right end of the model to simulate the effect of loading end. A compressive load is applied on the right end of the model as shown in Figure 4.4. The stresses were extracted to study the influence of fiber waviness in the wavy laminate and also to obtain the location of critical high stress regions. Non linear analysis option was chosen while analyzing the FE model.

Figure 4.4: Applied Load and Constraints on the Right End of the FE Model

4.2 Failure Criteria

The failure criteria used in the present analysis is based on the Maximum Stress failure theory. In this criterion, four independent modes of failure are considered. The failure modes are assumed to be non-interacting with each other. The schematic representation of Maximum Stress Failure criteria is shown in Figure 4.5. The fiber compression failure, the first of the four possible failure modes, occurs when the maximum stress in the plies along the length (σ_1) of the laminate direction exceeds the longitudinal strength of the material. Since this criterion is based on the longitudinal strength, it is limited to the 0^0 plies of the laminate. The second mode of failure considered is inter laminar tension. The failure based on this mode is defined to occur when the maximum the normal stresses (σ_2^T) in the interface of the layers exceeds the transverse tensile strength of the material. The third mode of failure considered is inter laminar compression and is similar to the previous failure mode. The failure based on this mode is defined to occur when the maximum the normal stresses (σ_2^C) in the interface of the layers exceeds the transverse compressive strength of the material. The last of the failure modes is inter laminar shear failure which is applicable to the entire laminate. The failure is defined to occur when the shear stresses

(τ_s) in the laminate exceeds the shear strength of the material. The strength allowables used for characterizing the material are shown in Table 3.2.

.

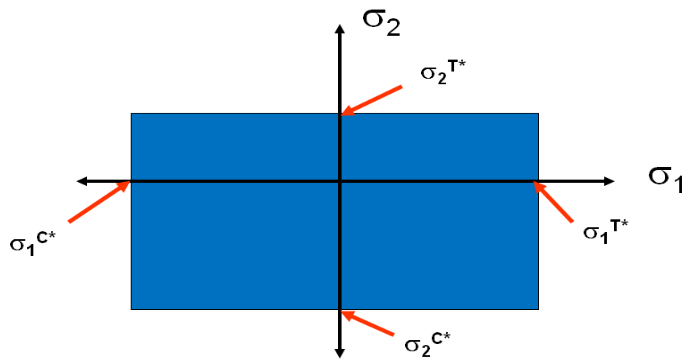

Figure 4.5: Maximum Stress Failure Criteria [28]

4.3 FEM Results – Test Load

In order to obtain the stress distributions in the laminate, a test load was applied to the FE model. A uniform displacement of 0.01 inch/min is applied as a load in the negative X direction. The prediction of failure in the wavy laminate is made by including the material nonlinearity behavior. The stresses are in the laminates are extracted at the center of the wavy 0^0 plies and at the interface of the waviness at the top and bottom of the waves. The stresses in the output are non-dimensionalized by dividing them by their corresponding strength allowable. The X coordinates of the nodes along which the stresses are extracted are normalized by dividing them with the length of the laminate so that they vary from 0 to 1. The stress distributions are compared for the laminates with no waviness, moderate waviness and severe waviness at the laminate midplane.

4.3.1 Stress Distribution along the Laminate Midplane

The longitudinal stresses (σ_1) distributions at the midplane are shown in Figure 4.6.

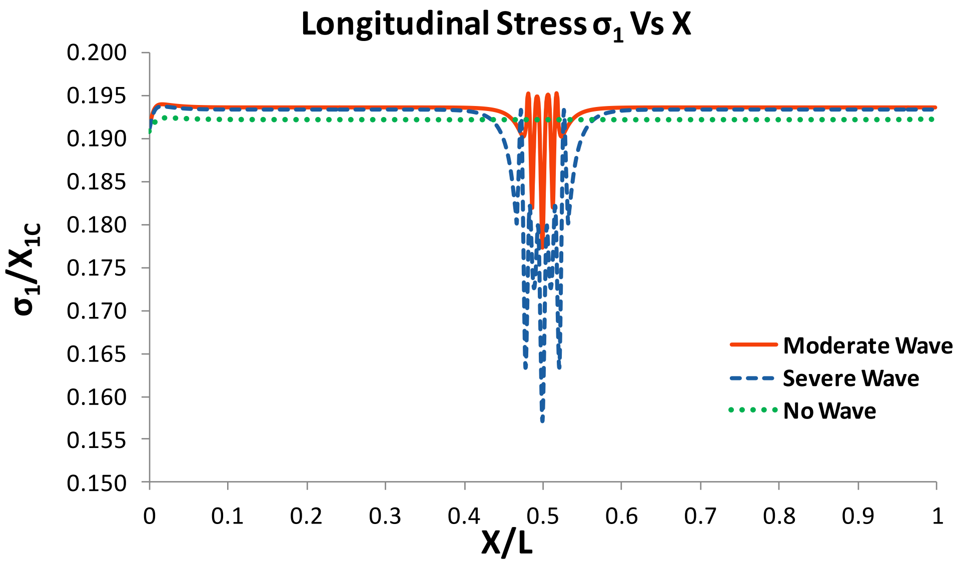

Figure 4.6: Longitudinal Stress (σ_1) Distribution along the Laminate

The greatest magnitude of stress reduction is seen in the severe wavy laminate which indicates the loss of load carrying capability within the laminate due to waviness in the plies. The stress distribution has a uniform trend across the laminate length for the laminate with no waviness. However, a reduction of 12.8% is observed in the peak stress values obtained for the severe wave laminate when compared with moderate wave laminate.

Inter laminar normal stress (σ_2) distributions in the laminate midplane are presented in Figure 4.7. Inter laminar normal stresses are absent for the laminate without waviness because, the applied loads are entirely carried by the fibers which are strained along the longitudinal direction. However, the waviness of the plies as in the case of moderate and severe wave laminates, create inter normal laminar strains in the wavy layers of the laminate. The presence of normal stresses along the central layer of the wavy laminates indicates the inability of the matrix to support the fibers in the transverse direction.

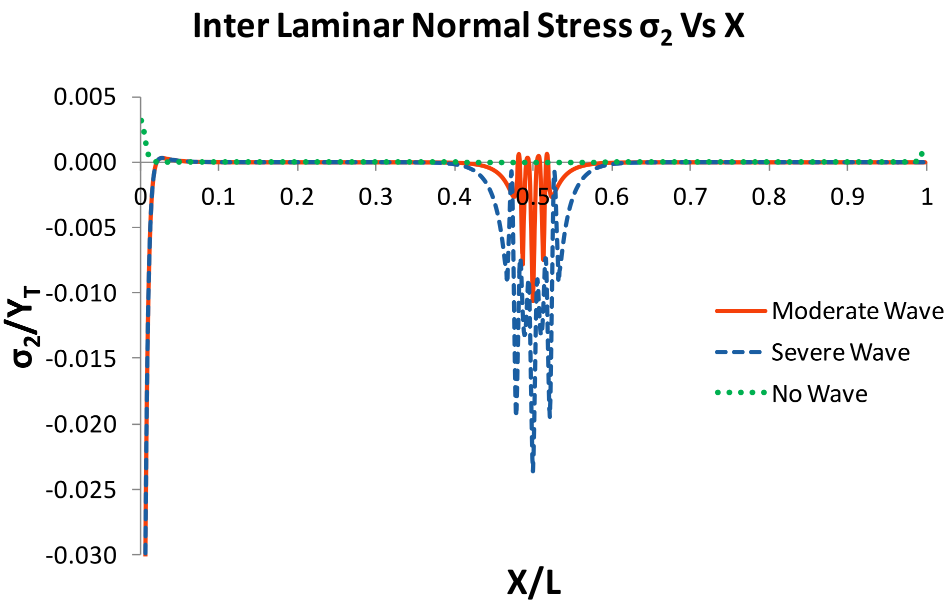

Figure 4.7: Inter Laminar Normal Stress (σ_2) Distribution along the Laminate

Inter laminar shear stress (τ_s) distribution along the wavy layer interface is shown in Figure 4.8. There is no shear stress present in the laminate without wavy layers. The applied load is transferred wholly by the fibers in the longitudinal direction. The absence of shear stresses also indicates no interaction between the layers during load transfer. However, due the misalignment of the plies as in the case of wavy laminates, shear stresses are generated. A 49% increase in the shear stresses is found for severe wave laminate when compared to the corresponding stresses measured for moderate wave laminate.

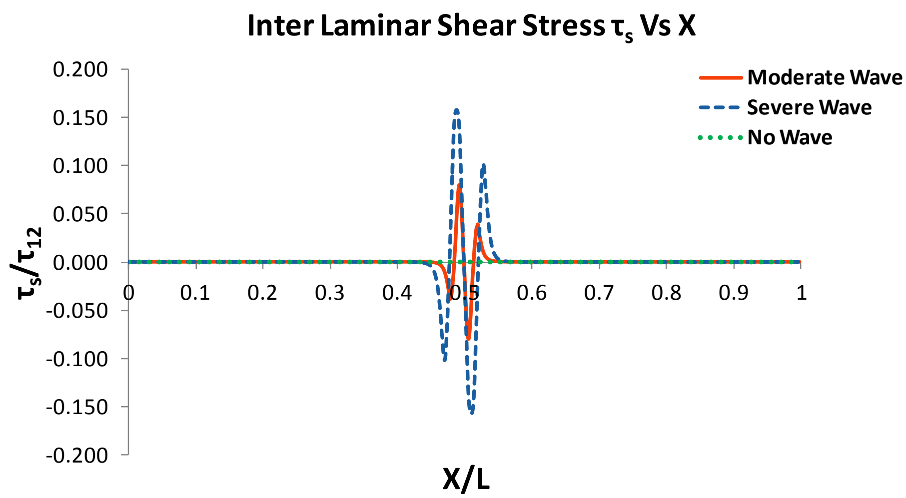

Figure 4.8: Inter Laminar Shear Stress (τ_s) Distribution in the Laminate

4.3.2 Stress Distribution along the Interface of the Wavy Plies

The stresses generated in the nodes along the interface of the wavy plies are extracted separately in the 0^0 and 90^0 plies. However, since 0^0 plies primarily are responsible for carrying loads in the laminate, the stress for only these plies are plotted. The study provides a valuable insight to the stress interactions in the layers for a wavy laminate. The study of stresses at the interface of the wavy plies is limited to inter laminar normal and shear stresses. The stresses are compared at the top and the bottom interface regions in the 0^0 plies to understand the differences in terms of magnitude and location of critical regions. The stress extraction at the interface of the wavy plies is shown in Figure 4.9.

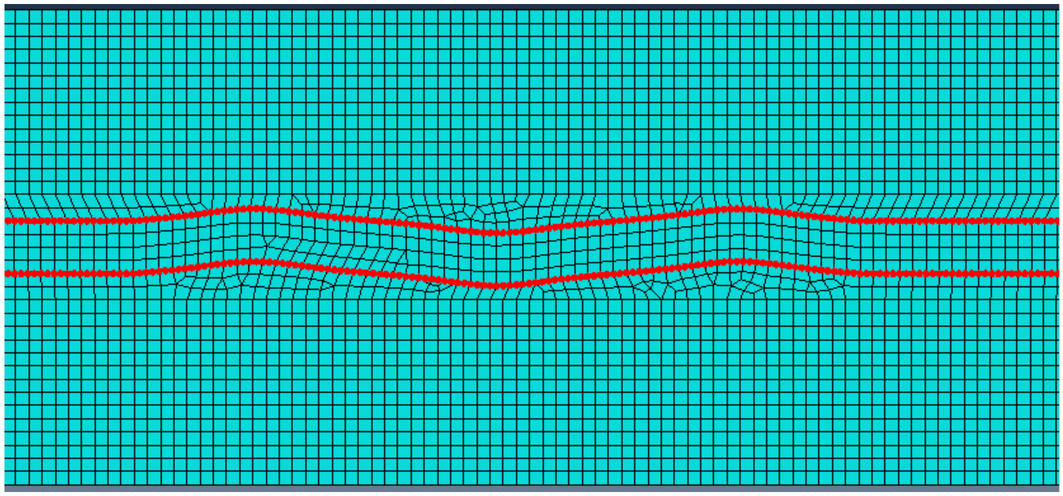

Figure 4.9: Stress Extraction at the Interface of the 0^0 and 90^0 Plies

The stresses shown in Figure 4.10 and Figure 4.11 are similar magnitude wise, but with opposite signs. The changes in the plots for a moderate and severe wavy laminate are seen in the way the stresses diminish.

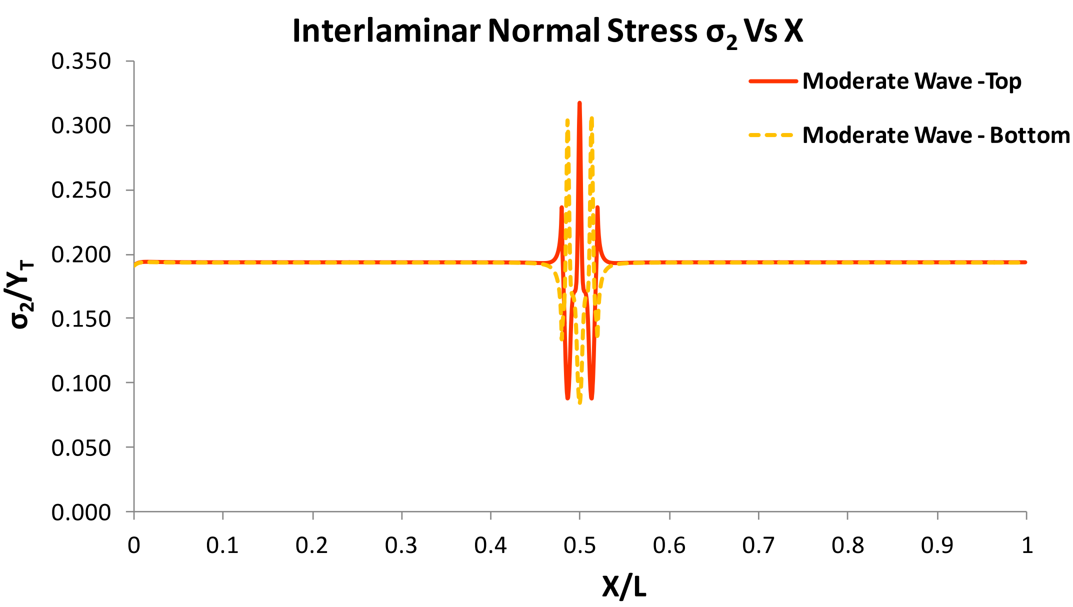

Figure 4.10: Inter Laminar Normal Stress (σ_2) Distribution at the Top and Bottom Interface for Moderate Wave Laminate

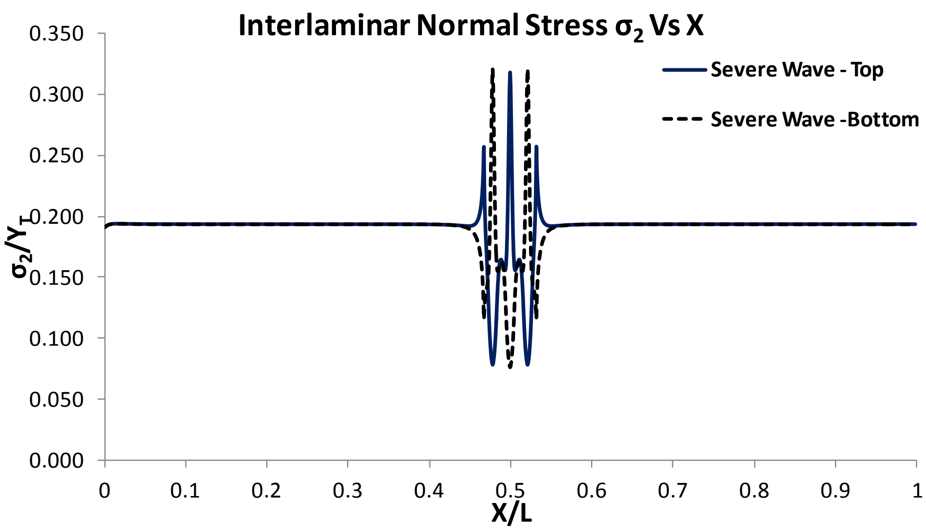

Figure 4.11: Inter Laminar Normal Stress (σ_2) Distribution at the Top and Bottom Interface for Severe Wave Laminate

The plots shown in Figure 4.12 and Figure 4.13 show the difference between the shear stress values for the moderate and severe wave laminate at the interface of the wavy layers. The rapid changes in the magnitude of the shear stresses at the interface show the nonlinear effects due waviness and material behavior. Interestingly, the magnitude of stresses are almost the same in the case of both wavy laminates.

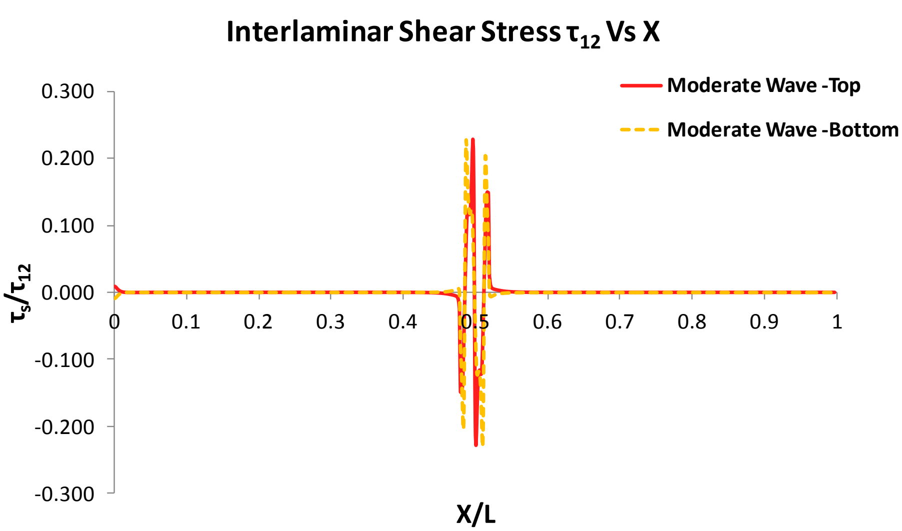

Figure 4.12: Inter Laminar Shear Stress (τ_s) Distribution at the Top and Bottom Interface for Moderate Wave Laminate

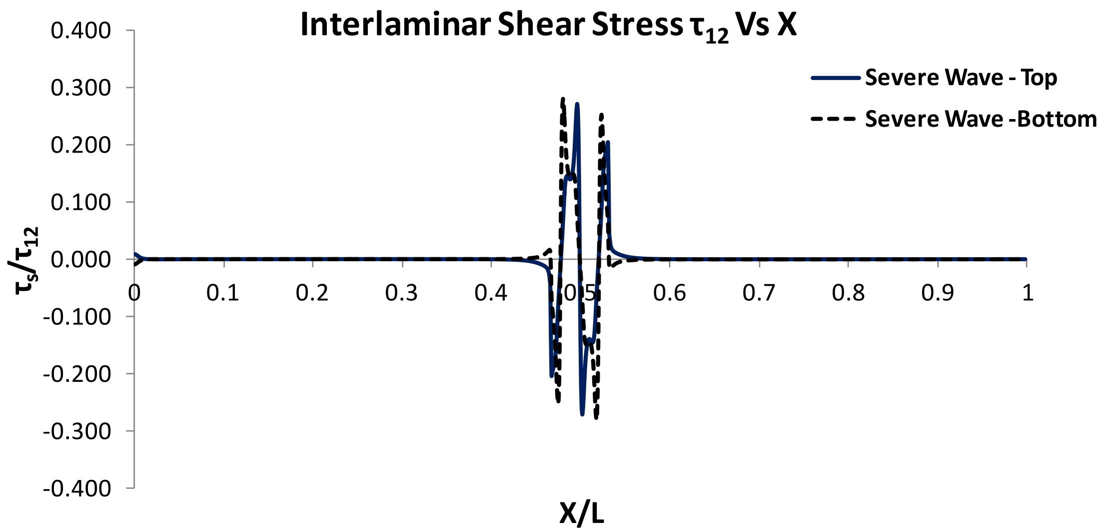

Figure 4.13: Inter Laminar Shear Stress (τ_S) Distribution at the Top and Bottom Interface for Severe Wave Laminate

The greatest magnitude of shear stresses of found on the top interface of the wavy layers when compared to the bottom interface. The fluctuations of the stresses between positive and negative values around the wavy regions are opposite in nature. The magnitude of stress for the at the top of the interface fluctuates from positive to negative before reaching a maximum positive value. The opposite of this behavior is found in the stress distributions in the bottom interface layer waviness. The von mises stress contour plots for the moderate and severe wave laminates are shown in Figure 4.14 and Figure 4.15. It can be observed that at the interface of the wavy plies, the stress distributions are high. Further, it is also observed from Figure 4.10 through Figure 4.13 that the magnitude of stresses at the top interface are higher when compared to the stresses at the bottom interface.

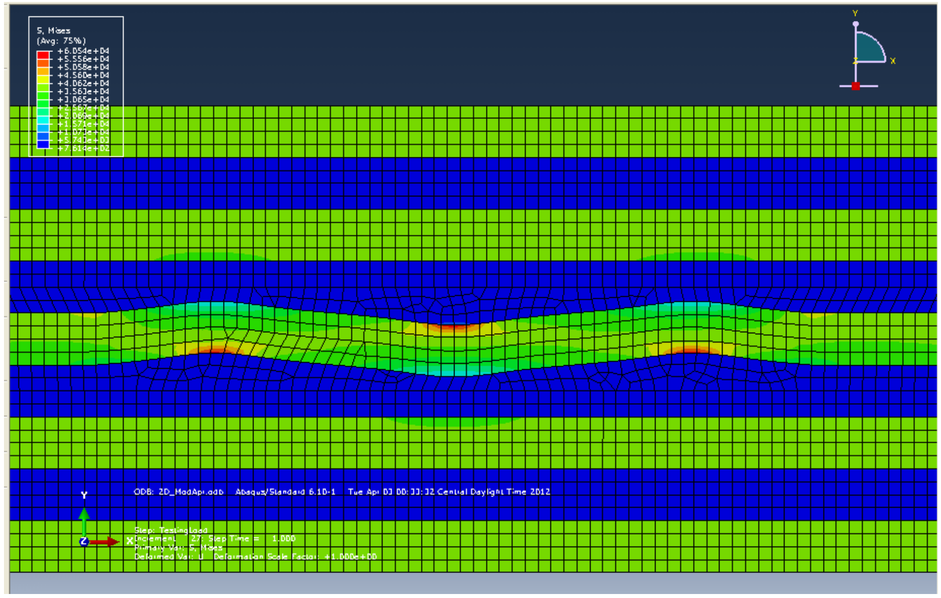

Figure 4.14: Von Mises Stress Contour Plot for Moderate Wave Laminate

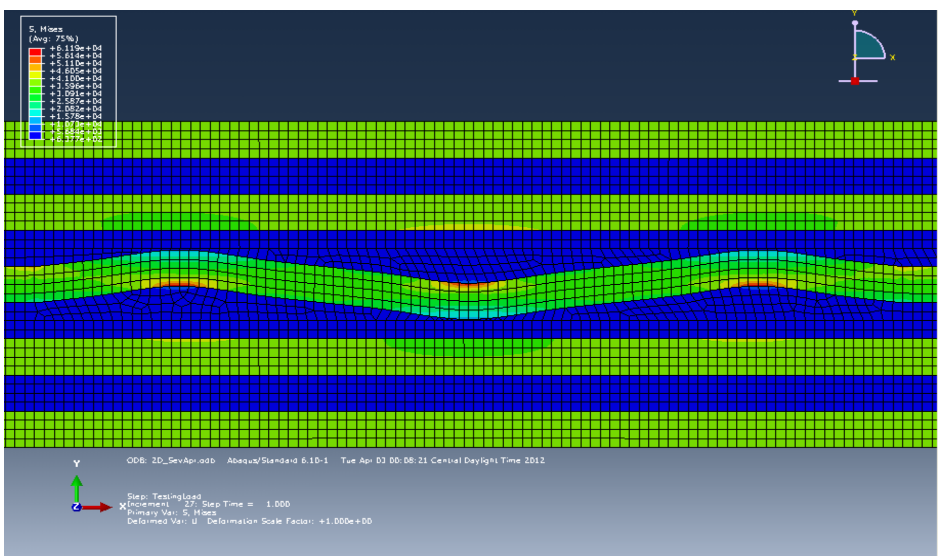

Figure 4.15: Von Mises Stress Contour Plot for Severe Wave Laminate

4.4 FEM Results – Strength Analysis

To obtain the strength of the laminate, an increment method was used to apply the load. The loads were applied in the increments of 20000 psi, till the stresses in the plies reached their strength allowable. The stresses obtained were non-dimensionalized by dividing the values obtained by their corresponding allowables. The location and magnitude of the peak stresses were obtained at the end of each load increment. The peak stresses were found to vary in magnitude but their locations were the same. The failure in transverse compression direction is ignored because the transverse compressive strength of the material is much (~3x) higher than the transverse tensile strength. Therefore, fiber failure, interlaminar shear and tension are the possible modes of failure for the current scenario. Thus, when the stress values are non-dimensionalized, failure in the laminate is said to occur when the value of the component becomes 1. The longitudinal stress, inter laminar normal stress and shear stresses are plotted for the laminates. The stress component that reaches its allowable value is said to cause failure in the laminate.

As observed in section 4.3, the stress values are critical at the interface of the 0^0 and 90^0 plies for the moderate and severe wavy laminates as shown in Figure 4.7 through Figure 4.13. The longitudinal, inter laminar shear and normal stresses are shown in the plots. For the laminate with no wavy plies, the stresses are extraced at the laminate midplane as shown in Figure 4.16. The failure for the laminate without fiber waviness occurs to fiber compression. The predicted failure strength for the laminate is 114.2 ksi.

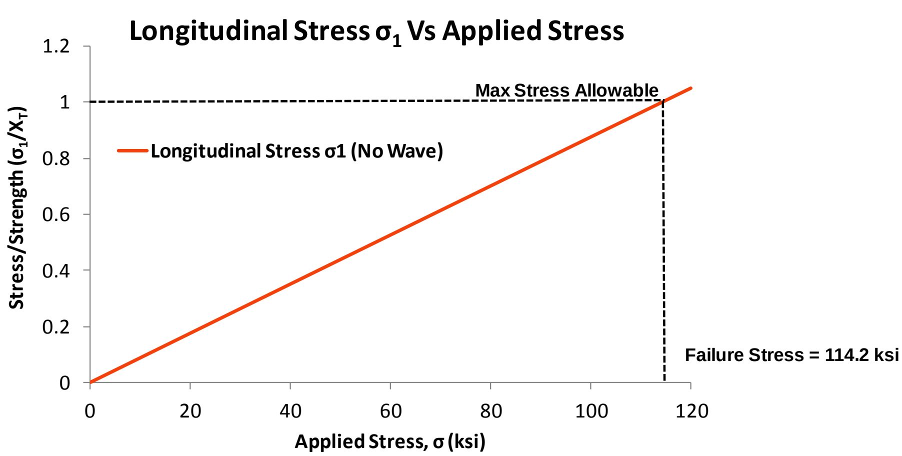

Figure 4.16: Fiber Compression Failure for a No Wave Laminate

The stresses in the wavy laminates were found to be greater at the top than the bottom of the interface of 0^0 and 90^0 plies. The stresses obtained at the way interface region for moderate wave laminate is shown in Figure 4.17. The failure strength predicted for the laminate with moderate waviness is 69.9 ksi. At this predicted failure load, the inter laminar normal stress is 26% of its strength allowable whereas the inter laminar shear stress is 78% of the corresponding shear allowable for the material. Hence, fiber failure the mode of failure for moderate wave laminate. The reduction between the strength values for a laminate with no waviness and for moderate wave laminate is 38%.

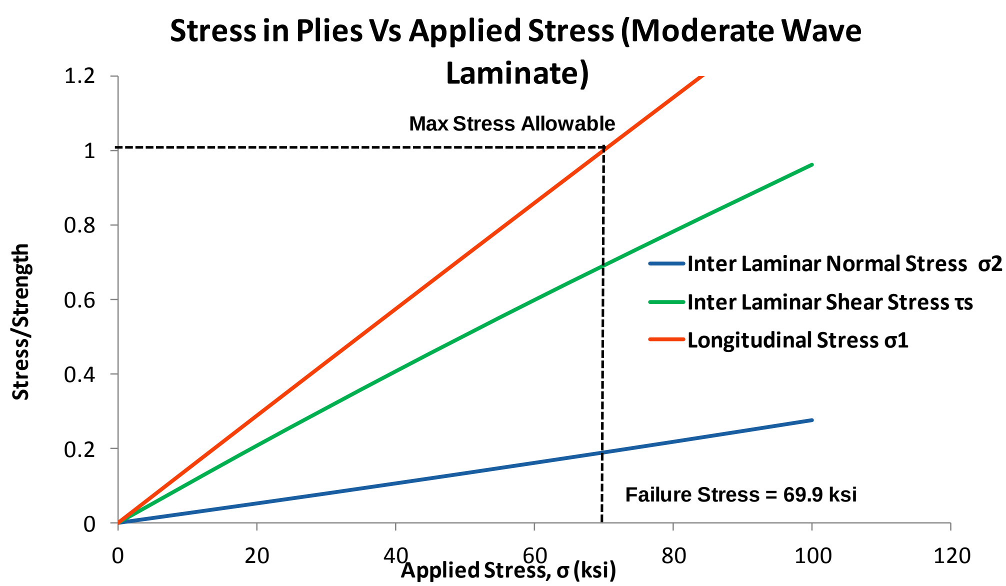

Figure 4.17: Strength of Moderate Wave Laminate

The stresses obtained for severe wave laminate is shown in Figure 4.18. The failure strength predicted for the laminate with severe waviness is 68 ksi. At this predicted failure load, the inter laminar normal stress is 27% of its strength allowable whereas the inter laminar shear stress is 94% of the corresponding shear allowable for the material. Hence, fiber failure the mode of failure for sever wave laminate. The reduction between the strength values for a laminate with no waviness and for moderate wave laminate is 41%.

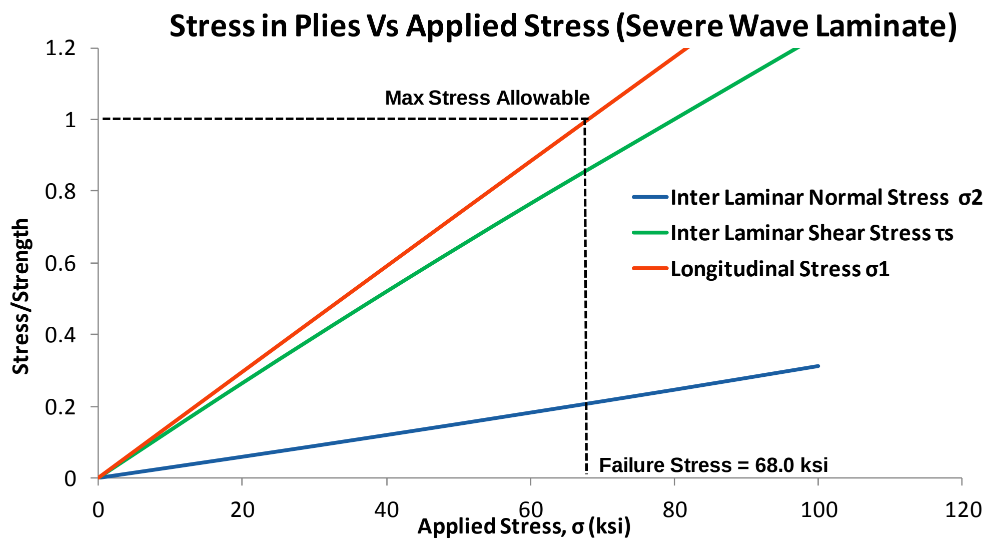

Figure 4.18: Strength of Severe Wave Laminate

FEA predicts a strength reduction of nearly 41% in the laminates with severe wavy geometries. However, these results might not correspond to the static test results because the FE software predicts only the initiation of failure. The strength reduction is predicted due to fiber compression in the laminate layers. The failure stress regions indicate the location of failure to be at the interface of the wavy layers which may be due inability of the matrix to support the fibers along the tranverse direction.

In order to obtain an actual physical understanding of the FE results, conducting static tests on the laminate would help in identifying the failure regions and modes. The interaction between the layer stresses can also be understood with static testing. The fractured surface might offer a better explanation of delamination, microbuckling, matrix cracking etc. It would also make it possible to make consistent predictions between FEA and static testing of the laminates.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The investigations of a laminate with fiber waviness defect in its layers subjected to compression load is discussed. Localized ply waviness at the lamiante midplane was included in a 18 ply laminate. The laminate was built using material properties of Carbon/Epoxy T300/1076 prepreg with a layup sequence of $[0_2/90_2/0_2/90_2/0_w]_s$. The wavy plies were classified per the amplitude to wavelength ratio and the misalignment angle as Moderate and Severe Wave Laminates.

An analytical model was built to calculate the stiffness propeties and the Poisson's ratio for the laminates. The following conclusions were made from analytical models generated for the laminates

- The stiffness values for laminates with waviness were found to be lower than those calculated for laminates without ply waviness.
  - The stiffness in the longitudinal direction reduced by 5.6% and 14.9% respectively for moderate and severe wave laminate when compared with laminates with no waviness
  - The stiffness in the transverse direction reduced by 4.1% and 11.9% respectively for moderate and severe wave laminate when compared with laminates with no waviness
- The low values of Poisson's ratio obtained for the laminates with waviness signify that the Poisson's effects are minimal in the laminate.

A two dimensional plane strain FE model was built to study the stress distributions in the wavy regions of the laminate. The following conclusions were made from the FE analysis

- The absence of inter laminar normal and shear stress distributions for the laminate with no waviness indicate that loads are completely transferred by the fibers
- The strength of the laminate without waviness was found to be 114.2 ksi. The failure mode for such a laminate is fiber compression
- The strength of the laminate with moderate waviness was found to be reduced by 39%. The failure mode in the laminate was determined as fiber compression and the value of laminate strength obtained was 69.9 ksi
- The strength of the laminate with severe waviness was found to be reduced by 41%. The failure mode for the laminate was determined as fiber compression and the value of laminate strength obtained was 68 ksi

5.2 Recommendations for Future Work

Based on the experience gained from studying the waviness of the plies from this thesis, the following areas can be researched upon:

- The laminate can be manufactured and tested to validate the analytical and FE predictions
- In plane fiber waviness (fiber misalignment within a lamina) effects on the mechanical behavior of the laminates can be studied
- The effects of repeating patterns of fiber waviness like multiple waves within a laminate, nested waves as observed in textile fabrics, or opposing wave geometries can be studied
- The thermal effects like CTE of the laminate, thermal stresses etc can be studied using FE models
- Research into materials and processes that prevent fiber waviness during manufacturing

REFERENCES

REFERENCES

[1] Rosen B.W., "Mechanics of Composite Strengthening," Chapter 3, *American Society for Metals*, 1965

[2] Argon A.S., "Fracture of Composites," *Treatise of Material Science and Technology*, Vol. 1, 1972

[3] Ishikawa T., Chou T.W., "One-Dimensional Micromechanical Analysis of Woven Fabric Composites," *American Institute of Aeronautics and Astronautics Journal*, Vol. 21, Issue 1, pp 1714-1721, December 1983.

[4] Chou T.W., Takahashi K., "Non-linear Elastic Behavior of Flexible Fiber Composites," *Composites Journal*, Vol. 18, Issue 1, pp 25-34, January 1987.

[5] Kuo C.M., Takahashi K., Chou T.W., "Effect of Fiber Waviness on the Nonlinear Elastic Behavior of Fiexible Composites," *Journal of Composite Materials*, Vol. 22, pp 1004-1024, November 1988.

[6] Chadhuri R.A., "Prediction of Compressive Strength of Thick-Section Advanced Composite Laminates," Journal of Composite Materials, Vol. 25, pp 1244-1276, August 1990.

[7] Fleck N.A., Budiansky B., "Compressive Failure of Fiber Composites", *Journal of the Mechanics and Physics of the Solids*, Vol. 41, Issue 3, pp 183-211, 1993

[8] Kutlu Z., Chang F.K., "Modeling Compression Failure of Laminated Composites Containing Multiple Through-the-Width Delaminations," *Journal of Composite Materials*, Vol. 26, Issue 3, pp 350-387, January 1992.

[9] Bogetti T.A., Gillespie J.W., Lamontia M.A., "Influence of Ply Waviness on the Stiffness and Strength Reduction on Composite Laminates," *Journal of Thermoplastic Composite Materials*, Vol. 5, pp 344-369, October 1992.

[10] Bogetti T.A., Gillespie J.W., Lamontia M.A., "Influence of Ply Waviness on the Stiffness and Strength Reduction on Composite Laminates," *Army Research Laboratory*, ARL-TR-110, Aberdeen Proving Ground, MD 21005-5066, April 1993.

[11] Adams D.H., Hyer M.W., "Effects of Layer Waviness on the Compression Strength of Thermoplastic Composite Laminates," *Journal of Reinforced Plastics and Composites*, Vol. 12, pp 414-429, April 1993.

[12] Kim C., White S.R., "The Continuous Curing Process for Thermoset Polymer Composites. Part 2: Experimental Results for a Graphite/Epoxy Laminate," *Journal of Composite Materials*, Vol. 30, Issues 5, pp 627-647, January 1996.

[13] Hsiao H.M., Daniel I.M., "Effect of Fiber Waviness on Stiffness and Strength Reduction of Unidirectional Composites under Compressive Loading," *Composites Science and Technology*, Vol. 56, pp 581-593, February 1996.

[14] Hsiao H.M., Daniel I.M., "Nonlinear Elastic Behavior of Unidirectional Composites with Fiber Waviness under Compressive Loading," *Journal of Engineering Materials and Technology*, Vol. 118, pp 561-570, October 1996.

[15] Hsiao H.M., Daniel I.M., "Elastic Properties of Composites with Fiber Waviness," *Composites Part A*, Vol. 27A, pp 931-941, October 1996.

[16] Bradley D.J., Adams D.O., Gascoigne H.E., "Interlaminar Strains and Compressive Strength Reductions Due to Nested Layer Waviness in Composite Laminates," *Journal of Reinforced Plastics and Composites*, Vol. 17, Issue 11, pp 989-1011, 1998.

[17] Joyce P., Moon T.J., "Compression Strength Reduction in Composites with In Plane Fiber Waviness," *Composite Materials: Fatigue and Fracture*, Vol. 7, pp 76-96, 1998.

[18] Wisnom M.R., Atkinson J.W., "Fibre Waviness Generation and Measurement and Its Effect on Compressive Strength," *Journal of Reinforced Plastics and Composites*, Vol. 19, Issue 2, pp 96-110, 2000.

[19] Chun H.J., Shin J.Y., Daniel I.M., "Effects of Material and Geometric Nonlinearities on the Tensile and Compressive behavior of Composite Materials with Fiber Waviness," *Composites Science and Technology*, Vol. 61, pp 125-134, 2001.

[20] Fleck N.A., Liu D., "Microbuckle Initiation from a Large Amplitude Fiber Waviness in a Composite under Compression and Bending," *European Journal of Mechanics –A/Solids*, Vol. 20, Issue 1, pp 23-37, 2001.

[21] Kugler D., Moon T.J, "Identification of the most Significant Processing Parameter on the Development of Fiber Waviness in Thin Laminates," *Journal of Composite Materials*, Vol. 36, Issue 12, pp 1451-1479, 2001

[22] Garnich M.R., Karami G., "Finite Element Micromechanics for Stiffness and Strength of Wavy Fiber Composites," *Journal of Composite Materials*, Vol. 38, Issue 4, pp 273-292, 2004.

[23] Garnich M.R., Karami G., "Localized Fiber Waviness and Implications for Failure in Unidirectional Composites," Journal of Composite Materials, Vol. 39, Issue 14, pp 1225-1245, 2005.

[24] Liu D., Fleck N.A., Sutcliffe M.P.F., "Compressive Strength of Fiber Composites with Random Fiber Waviness," *Journal of the Mechanics and Physics of Solids*, Vol. 52, pp 1481-1505, 2005.

[25] Makeev., Seon G., Lee E., "Failure Predictions for Carbon/Epoxy Tape Laminates with Wavy Plies," *Journal of Composite Materials*, Vol. 44, Issue 1, pp 95-112, 2010.

[26] ASTM D 695-02 a, "Standard Test Method for Compressive Properties of Rigid Plastics," Cited 2002.

[27] ASTM D 6641/D 6641M - 01, "Standard Test Method for Compressive Properties of Polymeric Matrix Composite Laminates using a Combined Loading Compressive (CLC) Test Fixture," Cited 2001.

[28] Daniel I.M., Ishai O., "Engineering Mechanics of Composite Materials", Second Edition, Oxford University Press, 2006

[29] Lee J., Soutis C., "A Study on the Compressive Strength of Thick Carbon Fiber-Epoxy Laminates", Composites Science and Technology, Vol. 67, Issue 10, pp 2015-2026, 2007

APPENDIX

APPENDIX – MAPLE OUTPUT

The elastic property calculation output obtained from Maple for a laminate without waviness is provided in this Appendix

```
restart; with(linalg); Digits := 10
Warning, the protected names norm and trace have been redefined and
unprotected
                          Digits := 10

Enter Material Property for 0 deg Unitape
 E[1] := 20.21
                         E[1] := 20.21
 E[2] := 1.48
                          E[2] := 1.48
 G[12] := .67
                         G[12] := 0.67
 Nu[12] := .3
                         Nu[12] := 0.3
 Nu[21] := 0.2e-1
                         Nu[21] := 0.02
 t := 0.8e-2
                           t := 0.008
 NP := 18
                            NP := 18

 Stiffness Matrix Calculation

 Stiffness Matrix for 0 deg Ply
 Q[11] := E[1]/(1-Nu[12]*Nu[21])
                      Q[11] := 20.33199195
 Q[12] := Nu[21]*E[1]/(1-Nu[12]*Nu[21])
                     Q[12] := 0.4066398390
 Q[22] := E[2]/(1-Nu[12]*Nu[21])
                      Q[22] := 1.488933602
 Q[66] := G[12]
                         Q[66] := 0.67
 Stiffness Matrix for 90 deg Ply
 alpha := 1/2*Pi
                                 1
                         alpha := - Pi
                                 2
 mm := cos(alpha)
                            mm := 0
 m := evalf(mm)
                            m := 0.
 nn := sin(alpha)
                            nn := 1
 n := evalf(nn)
                            n := 1.
 Q[xx] := Q[11]*m^4+(2*Q[12]+2*Q[66])*m^2*n^2+Q[22]*n^4
```

```
                     Q[xx] := 1.488933602
 Q[yy] := Q[11]*n^4+(2*Q[12]+2*Q[66])*m^2*n^2+Q[22]*m^4
                     Q[yy] := 20.33199195
 Q[xy] := (Q[11]+Q[22]-4*Q[66])*m^2*n^2+Q[12]*(m^4+n^4)
                    Q[xy] := 0.4066398390
 Q[ss] := (Q[11]+Q[22]-2*Q[12]-2*Q[66])*m^2*n^2+Q[66]*(m^4+n^4)
                        Q[ss] := 0.67
 Q[xs] := (Q[11]-Q[12]-2*Q[66])*m^3*n+(Q[12]-Q[22]+2*Q[66])*m*n^3
                         Q[xs] := 0.
 Q[ys] := (Q[11]-Q[12]-2*Q[66])*m*n^3+(Q[12]-Q[22]+2*Q[66])*m^3*n
                         Q[ys] := 0.

A Matrix Calculation
 A[xx] := 10*t*Q[11]+8*t*Q[xx]
                     A[xx] := 1.721851107
 A[yy] := 10*t*Q[22]+8*t*Q[yy]
                     A[yy] := 1.420362173
 A[xy] := 10*t*Q[12]+8*t*Q[xy]
                    A[xy] := 0.05855613682
 A[ss] := 10*t*Q[66]+8*t*Q[ss]
                       A[ss] := 0.09648
 Laminate Elastic Properties
 h := t*NP
                          h := 0.144
 E[11] := A[xx]*(1-A[xy]^2/(A[xx]*A[yy]))/h
                     E[11] := 11.94053513
 E[22] := A[yy]*(1-A[xy]^2/(A[xx]*A[yy]))/h
                     E[22] := 9.849797329
 G[12] := A[ss]/h
                    G[12] := 0.6700000000
 Nu[xy] := A[xy]/A[yy]
                   Nu[xy] := 0.04122620127
 Nu[yx] := A[xy]/A[xx]
                   Nu[yx] := 0.03400766569
 E[33] := E[22]
                     E[33] := 9.849797329
```

The elastic property calculation output obtained from Maple for laminate with moderate waviness is provided in this Appendix

```
restart; with(linalg); Digits := 10
Warning, the protected names norm and trace have been redefined and
unprotected
                          Digits := 10
 Laminate Elastic Property Determination
 Material Property for 0 deg Unitape
 E[1] := 20.21
                         E[1] := 20.21
 E[2] := 1.48
                          E[2] := 1.48
 G[12] := .67
                         G[12] := 0.67
 Nu[12] := .3
                         Nu[12] := 0.3
 Nu[21] := 0.2e-1
                         Nu[21] := 0.02
 Nu[23] := .436
                        Nu[23] := 0.436
 Nu[32] := .436
                        Nu[32] := 0.436
 t := .2032
                          t := 0.2032
 NP := 18
                            NP := 18
 ZE := 10
                            ZE := 10
 NI := 8
                            NI := 8
 Stiffness Matrix Calculation
 Q[11] := E[1]/(1-Nu[12]*Nu[21])
                      Q[11] := 20.33199195
 Q[12] := Nu[21]*E[1]/(1-Nu[12]*Nu[21])
                     Q[12] := 0.4066398390
 Q[22] := E[2]/(1-Nu[12]*Nu[21])
                      Q[22] := 1.488933602
 Q[23] := Nu[32]*E[2]/(1-Nu[23]*Nu[32])
                     Q[23] := 0.7967364033
 Q[66] := G[12]
                         Q[66] := 0.67

 Stiffness Matrix Calculation
 h := t*NP
                          h := 3.6576
 beta := 1/2*Pi
                                  1

                          beta := - Pi

                                  2
 m1 := cos(beta)
                            m1 := 0
 m11 := evalf(m1)
```

```
                             m11 := 0.
 n1 := sin(beta)
                              n1 := 1
 n11 := evalf(n1)
                             n11 := 1.
 Q[xx] := Q[11]*m11^4+(2*Q[12]+4*Q[66])*m11^2*n11^2+Q[22]*n11^4
                        Q[xx] := 1.488933602
 Q[yy] := Q[11]*n11^4+(2*Q[12]+4*Q[66])*m11^2*n11^2+Q[22]*m11^4
                        Q[yy] := 20.33199195
 Q[xy] := (Q[11]+Q[22]-4*Q[66])*m11^2*n11^2+Q[12]*(m11^4+n11^4)
                       Q[xy] := 0.4066398390
 Q[ss] := (Q[11]-Q[22]-2*Q[12])*m11^2*n11^2+Q[66]*(m11^2-n11^2)^2
                           Q[ss] := 0.67
 Q[xs]     :=     Q[11]*m11^3*n11-Q[22]*m11*n11^3-m11*n11*(m11^2-n11^2)*Q[12]-
2*m11*n11*(m11^2-n11^2)*Q[66]
                            Q[xs] := 0.
 Q[ys]     :=     Q[11]*n11^3*m11-Q[22]*m11^2*n11^2-2*m11^2*n11^2*Q[12]+(m11^2-
n11^2)^2*Q[66]
                           Q[ys] := 0.67

 A Matrix Calculation
 A[xx] := ZE*t*Q[11]+NI*t*Q[xx]
                        A[xx] := 43.73501810
 A[yy] := ZE*t*Q[22]+NI*t*Q[yy]
                        A[yy] := 36.07719919
 A[xy] := ZE*t*Q[12]+NI*t*Q[xy]
                        A[xy] := 1.487325875
 A[ss] := ZE*t*Q[66]+NI*t*Q[ss]
                         A[ss] := 2.450592

 Crossply Waviness Model
 Enter Fiber Waviness Parameters
 A := 0.736e-2
                            A := 0.00736
 L := .1472
                            L := 0.1472
 theta := 5.71
                           theta := 5.71
 phi := 1/180*theta*Pi
                      phi := 0.03172222222 Pi
 mm := cos(phi)
                    mm := cos(0.03172222222 Pi)
 m := evalf(mm)
                         m := 0.9950382202
 nn := sin(phi)
                    nn := sin(0.03172222222 Pi)
 n := evalf(nn)
                         n := 0.09949341819

Crossply Waviness Property Evaluation
 p := 8*t/(NP*t)
                                    4
                               p := -
                                    9
 q := 8*t/(NP*t)
```

```
                                4
                           q := -
                                9
r := 2*t/(NP*t)
                                1
                           r := -
                                9

Transformed Stiffness Matrix Calculation
TC[xx] := Q[11]*m^4+(2*Q[12]+4*Q[66])*m^2*n^2+Q[22]*n^4
                    TC[xx] := 19.96583727
TC[xy] := Q[12]*m^2+Q[23]*n^2
                    TC[xy] := 0.4105013816
TC[xz] := (Q[11]+Q[22]-4*Q[66])*m^2*n^2+Q[12]*(m^4+n^4)
                    TC[xz] := 0.5862682025
TC[xr] := Q[11]*m^3*n-Q[22]*m*n^3-m*n*(m^2-n^2)*(Q[12]+2*Q[66])
                    TC[xr] := 1.821984324
TC[yy] := Q[22]
                    TC[yy] := 1.488933602
TC[yz] := Q[23]*m^2+Q[12]*n^2
                    TC[yz] := 0.7928748607
TC[yr] := (Q[12]-Q[23])*m*n
                   TC[yr] := -0.03861946380
TC[zz] := Q[11]*n^4+(2*Q[12]+4*Q[66])*m^2*n^2+Q[22]*m^4
                    TC[zz] := 1.495831547
TC[zr] := Q[11]*n^3*m-Q[22]*n*m^3+m*n*(m^2-n^2)*(Q[12]+2*Q[66])
                    TC[zr] := 0.0434738123
TC[yz] := Q[23]*m^2+Q[12]*n^2
                    TC[yz] := 0.7928748607
TC[yr] := (Q[12]-Q[23])*m*n
                   TC[yr] := -0.03861946380
TC[zz] := Q[11]*n^4+(2*Q[12]+4*Q[66])*m^2*n^2+Q[22]*m^4
                    TC[zz] := 1.495831547
TC[zr] := Q[11]*n^3*m-Q[22]*n*m^3+m*n*(m^2-n^2)*(Q[12]+2*Q[66])
                    TC[zr] := 0.0434738123
TC[qq] := 1/2*(Q[22]-Q[23])*m^2+Q[66]*n^2
                    TC[qq] := 0.3493048800
TC[qs] := (Q[66]-1/2*Q[22]+1/2*Q[23])*m*n
                   TC[qs] := 0.03206615890
TC[rr] := (Q[11]+Q[22]-2*Q[12])*m^2*n^2+Q[66]*(m^2-n^2)^2
                    TC[rr] := 0.8496283636
TC[ss] := 1/2*(Q[22]-Q[23])*n^2+Q[66]*m^2
                    TC[ss] := 0.6667937194

Fiber Waviness Relation
W := 2*Pi*A/L
                     W := 0.1000000000 Pi
alpha := evalf(W)
                    alpha := 0.3141592654
P[1] := (1+1/2*alpha^2)/(1+alpha^2)^(3/2)
                     P[1] := 0.9111779621
P[3] := 1/2*alpha^2/(1+alpha^2)^(3/2)
                    P[3] := 0.04285025482
```

```
 P[5] := 1-(1+3/2*alpha^2)/(1+alpha^2)^(3/2)
                      P[5] := 0.0031215283
 P[6] := 1/(1+alpha^2)^(1/2)
                      P[6] := 0.9540282166
 P[8] := 1-1/(1+alpha^2)^(1/2)
                      P[8] := 0.0459717834

 Crossply A Matrix Calculation
 TA[11] := h*((p+r*P[1])*TC[xx]+(q+r*P[5])*TC[yy]+2*r*P[3]*(TC[xy]+2*TC[ss]))
                     TA[11] := 42.33291262
 TA[12] := h*((p+q+r*P[6])*TC[xy]+r*P[8]*TC[yz])
                     TA[12] := 1.508593711
 TA[13]     :=     h*((p+r*(P[1]+P[5]))*TC[xy]+q*TC[yz]+r*P[3]*(TC[xx]+TC[yy]-
4*TC[ss]))
                     TA[13] := 2.435912608
 TA[22] := h*((p+r)*TC[yy]+q*TC[xx])
                     TA[22] := 35.48197815
 TA[23] := h*((p+r*P[6])*TC[yz]+(q+r*P[8])*TC[xy])
                     TA[23] := 2.271288905
 TA[33] := h*((p+q+r*P[1])*TC[yy]+r*P[5]*TC[xx]+2*r*P[3]*(TC[xy]+2*TC[ss]))
                     TA[33] := 5.478249865
 TA[44] := h*((1/2*p+1/2*r*P[6])*(TC[yy]-TC[yz])+(q+r*P[8])*TC[ss])
                     TA[44] := 1.797091004
 TA[55]  :=  h*((p+r*(P[1]+P[5]))*TC[ss]+1/2*q*(TC[yy]-TC[yz])+r*P[3]*(TC[xx]-
2*Q[12]+TC[yy]-2*TC[ss]))
                     TA[55] := 2.233692302
 TA[66] := h*((p+q+r*P[6])*TC[ss]+1/2*r*P[8]*(TC[yy]-TC[yz]))
                     TA[66] := 2.432909255
 TA[0]     :=     TA[11]*TA[22]*TA[33]+2*TA[12]*TA[13]*TA[23]-TA[13]^2*TA[22]-
TA[12]^2*TA[33]-TA[23]^2*TA[11]
                      TA[0] := 7803.937200

Composite with Crossply Fiber Waviness
 TE[C1] := TA[0]/(h*(TA[22]*TA[33]-TA[23]^2))
                     TE[C1] := 11.27585875
 TE[C2] := TA[0]/(h*(TA[11]*TA[33]-TA[13]^2))
                     TE[C2] := 9.441784461
 TE[C3] := TA[0]/(h*(TA[11]*TA[22]-TA[12]^2))
                     TE[C3] := 1.422623923
 TNu[Cxy] := (-TA[13]*TA[23]+TA[12]*TA[33])/(TA[22]*TA[33]-TA[23]^2)
                    TN[Cxy] := 0.01443709123
 TNu[Cyx] := (-TA[13]*TA[23]+TA[12]*TA[33])/(TA[11]*TA[33]-TA[13]^2)
                    TN[Cyx] := 0.01208882682
 TG[Cxy] := TA[66]/h
                    TG[Cxy] := 0.6651654787
```

The elastic property calculation output obtained from Maple for laminate with severe waviness in the plies is provided in this Appendix

```
restart; with(linalg); Digits := 10
Warning, the protected names norm and trace have been redefined and unprotected
                          Digits := 10

Material Property for 0 deg Unitape
 E[1] := 20.21
                         E[1] := 20.21
 E[2] := 1.48
                          E[2] := 1.48
 G[12] := .67
                         G[12] := 0.67
 Nu[12] := .3
                         Nu[12] := 0.3
 Nu[21] := 0.2e-1
                         Nu[21] := 0.02
 Nu[23] := .436
                        Nu[23] := 0.436
 Nu[32] := .436
                        Nu[32] := 0.436
 t := .2032
                          t := 0.2032
 NP := 18
                            NP := 18
 ZE := 10
                            ZE := 10
 NI := 8
                            NI := 8


Stiffness Matrix Calculation
 Q[11] := E[1]/(1-Nu[12]*Nu[21])
                      Q[11] := 20.33199195
 Q[12] := Nu[21]*E[1]/(1-Nu[12]*Nu[21])
                     Q[12] := 0.4066398390
 Q[22] := E[2]/(1-Nu[12]*Nu[21])
                      Q[22] := 1.488933602
 Q[23] := Nu[32]*E[2]/(1-Nu[23]*Nu[32])
                     Q[23] := 0.7967364033
 Q[66] := G[12]
                         Q[66] := 0.67
 Stiffness Matrix Calculation
 h := t*NP
                          h := 3.6576
 beta := 1/2*Pi
                                  1
                          beta := - Pi
                                  2
 m1 := cos(beta)
                            m1 := 0
```

```
 m11 := evalf(m1)
                            m11 := 0.
 n1 := sin(beta)
                             n1 := 1
 n11 := evalf(n1)
                            n11 := 1.
 Q[xx] := Q[11]*m11^4+(2*Q[12]+4*Q[66])*m11^2*n11^2+Q[22]*n11^4
                     Q[xx] := 1.488933602
 Q[yy] := Q[11]*n11^4+(2*Q[12]+4*Q[66])*m11^2*n11^2+Q[22]*m11^4
                     Q[yy] := 20.33199195
 Q[xy] := (Q[11]+Q[22]-4*Q[66])*m11^2*n11^2+Q[12]*(m11^4+n11^4)
                    Q[xy] := 0.4066398390
 Q[ss] := (Q[11]-Q[22]-2*Q[12])*m11^2*n11^2+Q[66]*(m11^2-n11^2)^2
                        Q[ss] := 0.67
 Q[xs]     :=     Q[11]*m11^3*n11-Q[22]*m11*n11^3-m11*n11*(m11^2-n11^2)*Q[12]-
2*m11*n11*(m11^2-n11^2)*Q[66]
                         Q[xs] := 0.
 Q[ys]  :=  Q[11]*n11^3*m11-Q[22]*m11^2*n11^2-2*m11^2*n11^2*Q[12]+Q[66]*(m11^2-
n11^2)^2
                        Q[ys] := 0.67
 A Matrix Calculation
 A[xx] := ZE*t*Q[11]+NI*t*Q[xx]
                     A[xx] := 43.73501810
 A[yy] := ZE*t*Q[22]+NI*t*Q[yy]
                     A[yy] := 36.07719919
 A[xy] := ZE*t*Q[12]+NI*t*Q[xy]
                     A[xy] := 1.487325875
 A[ss] := ZE*t*Q[66]+NI*t*Q[ss]
                      A[ss] := 2.450592
 Elastic Property Calculation
 E[11] := A[xx]*(1-A[xy]^2/(A[xx]*A[yy]))/h
                     E[11] := 11.94053513
 E[22] := A[yy]*(1-A[xy]^2/(A[xx]*A[yy]))/h
                     E[22] := 9.849797328
 G[12] := A[ss]/h
                    G[12] := 0.6700000000
 Nu[xy] := A[xy]/A[yy]
                   Nu[xy] := 0.04122620127
 Nu[yx] := A[xy]/A[xx]
                   Nu[yx] := 0.03400766570
 E[33] := E[22]
                     E[33] := 9.849797328


Crossply Waviness Model
 Enter Fiber Waviness Parameters
 A := 0.1472e-1
                          A := 0.01472
 L := .2374
                          L := 0.2374
 theta := 14
                          theta := 14
 phi := 1/180*theta*Pi
                                        7
                          phi := -- Pi
```

```
                                90
 mm := cos(phi)
                                /7    \

                          mm := cos|-- Pi|

                                \90   /
 m := evalf(mm)
                         m := 0.9702957263
 nn := sin(phi)
                                /7    \

                          nn := sin|-- Pi|

                                \90   /
 n := evalf(nn)
                         n := 0.2419218956
```

Crossply Waviness Property Evaluation

```
 p := 8*t/(NP*t)
                                 4

                             p := -

                                 9
 q := 8*t/(NP*t)
                                 4

                             q := -

                                 9
 r := 2*t/(NP*t)
                                 1

                             r := -

                                 9
```

Transformed Stiffness Matrix Calculation

```
 TC[xx] := Q[11]*m^4+(2*Q[12]+4*Q[66])*m^2*n^2+Q[22]*n^4
                    TC[xx] := 18.21930974
 TC[xy] := Q[12]*m^2+Q[23]*n^2
                    TC[xy] := 0.4294707099
 TC[xz] := (Q[11]+Q[22]-4*Q[66])*m^2*n^2+Q[12]*(m^4+n^4)
                    TC[xz] := 1.416509384
 TC[xr] := Q[11]*m^3*n-Q[22]*m*n^3-m*n*(m^2-n^2)*(Q[12]+2*Q[66])
                    TC[xr] := 4.110858392
 TC[yy] := Q[22]
                    TC[yy] := 1.488933602
 TC[yz] := Q[23]*m^2+Q[12]*n^2
                    TC[yz] := 0.7739055324
 TC[yr] := (Q[12]-Q[23])*m*n
                   TC[yr] := -0.09156962185
 TC[zz] := Q[11]*n^4+(2*Q[12]+4*Q[66])*m^2*n^2+Q[22]*m^4
                    TC[zz] := 1.581876726
 TC[zr] := Q[11]*n^3*m-Q[22]*n*m^3+m*n*(m^2-n^2)*(Q[12]+2*Q[66])
                    TC[zr] := 0.3122816320
```

```
 TC[yz] := Q[23]*m^2+Q[12]*n^2
                     TC[yz] := 0.7739055324
 TC[yr] := (Q[12]-Q[23])*m*n
                    TC[yr] := -0.09156962185
 TC[zz] := Q[11]*n^4+(2*Q[12]+4*Q[66])*m^2*n^2+Q[22]*m^4
                     TC[zz] := 1.581876726
 TC[zr] := Q[11]*n^3*m-Q[22]*n*m^3+m*n*(m^2-n^2)*(Q[12]+2*Q[66])
                     TC[zr] := 0.3122816320
 TC[qq] := 1/2*(Q[22]-Q[23])*m^2+Q[66]*n^2
                     TC[qq] := 0.3650553187
 TC[qs] := (Q[66]-1/2*Q[22]+1/2*Q[23])*m*n
                    TC[qs] := 0.07603124836
 TC[rr] := (Q[11]+Q[22]-2*Q[12])*m^2*n^2+Q[66]*(m^2-n^2)^2
                     TC[rr] := 1.679869546
 TC[ss] := 1/2*(Q[22]-Q[23])*n^2+Q[66]*m^2
                     TC[ss] := 0.6510432808
```

Fiber Waviness Relation

```
 W := 2*Pi*A/L
                      W := 0.1240101095 Pi
 alpha := evalf(W)
                     alpha := 0.3895892490
 P[1] := (1+1/2*alpha^2)/(1+alpha^2)^(3/2)
                      P[1] := 0.8703895133
 P[3] := 1/2*alpha^2/(1+alpha^2)^(3/2)
                     P[3] := 0.06139454070
 P[5] := 1-(1+3/2*alpha^2)/(1+alpha^2)^(3/2)
                      P[5] := 0.0068214052
 P[6] := 1/(1+alpha^2)^(1/2)
                      P[6] := 0.9317840549
 P[8] := 1-1/(1+alpha^2)^(1/2)
                      P[8] := 0.0682159451
```

Crossply A Matrix Calculation

```
 TA[11] := h*((p+r*P[1])*TC[xx]+(q+r*P[5])*TC[yy]+2*r*P[3]*(TC[xy]+2*TC[ss]))
                     TA[11] := 38.57290428
 TA[12] := h*((p+q+r*P[6])*TC[xy]+r*P[8]*TC[yz])
                     TA[12] := 1.580380821
 TA[13]     :=     h*((p+r*(P[1]+P[5]))*TC[xy]+q*TC[yz]+r*P[3]*(TC[xx]+TC[yy]-
4*TC[ss]))
                     TA[13] := 2.536073323
 TA[22] := h*((p+r)*TC[yy]+q*TC[xx])
                     TA[22] := 32.64282299
 TA[23] := h*((p+r*P[6])*TC[yz]+(q+r*P[8])*TC[xy])
                     TA[23] := 2.261174875
 TA[33] := h*((p+q+r*P[1])*TC[yy]+r*P[5]*TC[xx]+2*r*P[3]*(TC[xy]+2*TC[ss]))
                     TA[33] := 5.504411090
 TA[44] := h*((1/2*p+1/2*r*P[6])*(TC[yy]-TC[yz])+(q+r*P[8])*TC[ss])
                     TA[44] := 1.792941976
 TA[55]  :=  h*((p+r*(P[1]+P[5]))*TC[ss]+1/2*q*(TC[yy]-TC[yz])+r*P[3]*(TC[xx]-
2*Q[12]+TC[yy]-2*TC[ss]))
                     TA[55] := 2.310562063
 TA[66] := h*((p+q+r*P[6])*TC[ss]+1/2*r*P[8]*(TC[yy]-TC[yz]))
                     TA[66] := 2.373118404
```

```
 TA[0]     :=     TA[11]*TA[22]*TA[33]+2*TA[12]*TA[13]*TA[23]-TA[13]^2*TA[22]-
TA[12]^2*TA[33]-TA[23]^2*TA[11]
                      TA[0] := 6527.970720
```

Composite with Crossply Fiber Waviness

```
 TE[C1] := TA[0]/(h*(TA[22]*TA[33]-TA[23]^2))
                     TE[C1] := 10.22399958
 TE[C2] := TA[0]/(h*(TA[11]*TA[33]-TA[13]^2))
                     TE[C2] := 8.668578509
 TE[C3] := TA[0]/(h*(TA[11]*TA[22]-TA[12]^2))
                     TE[C3] := 1.420280945
 TNu[Cxy] := (-TA[13]*TA[23]+TA[12]*TA[33])/(TA[22]*TA[33]-TA[23]^2)
                    TNu[Cxy] := 0.01698240300
 TNu[Cyx] := (-TA[13]*TA[23]+TA[12]*TA[33])/(TA[11]*TA[33]-TA[13]^2)
                    TNu[Cyx] := 0.01439879692
TG[Cxy] := TA[66]/h
                    TG[Cxy] := 0.6488184613
```