

**OPTIMAL DECISION MAKING FOR SPARE PART MANAGEMENT WITH  
TIME VARYING DEMANDS**

A Thesis by

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Master of Science

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I have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science, with a major in Industrial Engineering.

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## **DEDICATION**

To my beloved parents, and my dear friends

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## ABSTRACT

Controlling spare parts inventory of a product is a major effort to many companies. The problem becomes more challenging when the installed base of the product changes over time. In order to cope with the situation, the inventory value needs to be adjusted according to the resulting non-stationary maintenance demand. This problem is usually encountered when a manufacturer starts selling a new product and agrees to provide spare parts for maintenance. In this research, a special case involving a new non-repairable product with a single spare pool is considered. It is assumed that the new sales follow some popular stochastic processes, and the product's failure time follows the Weibull distribution or Exponential distribution. The mathematical model for the resulting maintenance demand is formulated and calculated through simulation. Based on the maintenance demand, a dynamic  $(Q, r)$ - (lotsize/reorder-point) restocking policy is formulated and solved using a multi-resolution approach. Finally, numerical examples with the objective of minimizing the inventory cost under a service level constraint are provided to demonstrate the proposed methodology in practical use.

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# CHAPTER 1

## INTRODUCTION

### 1.1 Motivation and Background

Inventory control is one of the most important tasks faced by product manufacturers. In addition to the raw material, work in process, and finished good inventories, spare parts inventories contribute to large percentage of the overall inventory investment. In this report we limit our discussion to the area of spare parts inventories. Compared to other inventory systems, it is more difficult to control spare parts inventory of a product to meet certain maintenance demand. In most cases, effective maintenance of spare parts plays a key role in improving the manufacturer's reputation. On the other hand, because a significant portion of company's total investment lies in the hands of inventory, it is always important to maintain optimal number of spare parts in the inventory. The three most important things to be satisfied by any inventory system are to provide right quantity with right quality at the right time. This research is expected to provide some important insights that would reduce the spare parts inventory investment.

There are several things to be taken care of while maintaining optimal number of parts in inventory. We concentrate here on the failure time distribution of the part which is a key factor that identifies the number of parts required to be maintained in inventory. The problem we are concentrating on is to investigate the unexpected fluctuations in demand. Some research approaches have been reported in the literature, this research is expected to enrich this area.

Spare parts inventory control is usually implemented to maximize the availability of systems being served. Since the amount of time the machine is down largely depends

on the availability of the part, one has to make sure that the part is available when required. Therefore, the problem lies on an effective balance between the service offered and cost incurred.

### 1.2 Overview of the $(Q, r)$ model:

Regarding the control of inventory systems, two models are widely used: the lot-size/reorder-point  $(Q, r)$  model where  $Q$  represents the order quantity and  $r$  is the reorder point, and the reorder-point/order-up-to-level  $(s, S)$  where  $S$  represents the order-up-to-level and  $s$  is the reorder point. The model developed in our paper tries to link up the assumptions made by the  $(Q, r)$  model with the challenges faced by the company and come up with an optimal solution for the problem. The basic idea behind the  $(Q, r)$  model is to determine how much stock to carry and how many to produce or order at a time.

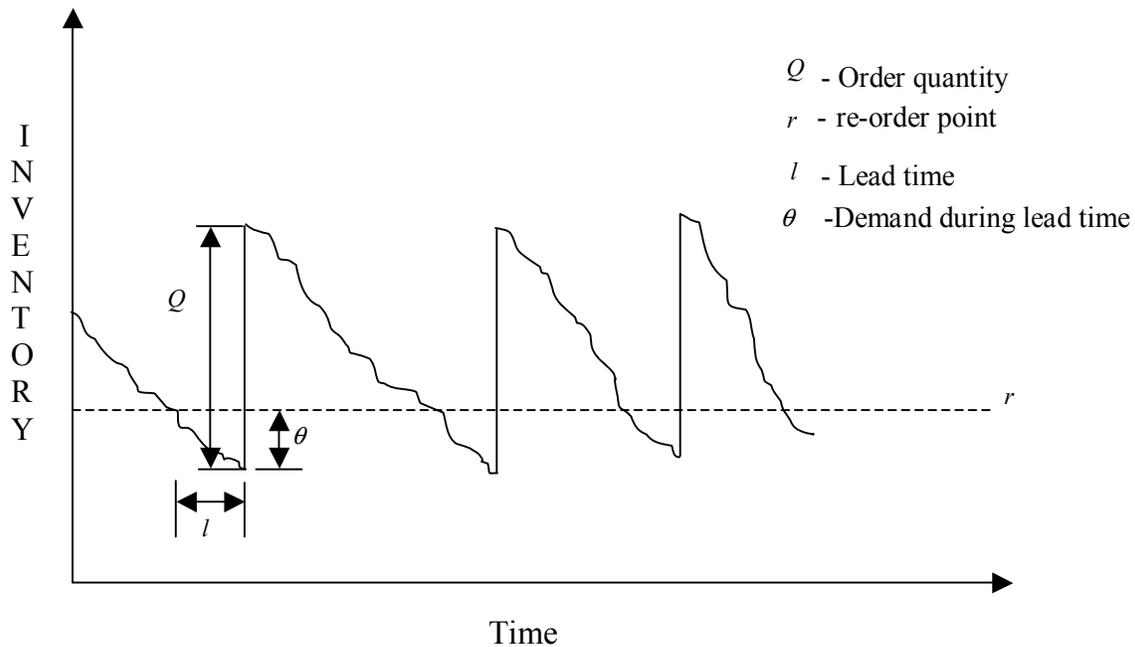


Figure 1.1 Illustration of  $(Q, r)$  inventory control strategy

Assumptions that are made by the  $(Q, r)$  model include

- No product interactions.
- Unfilled demand is backordered.
- Lead times are fixed and known.
- There is a fixed cost associated with each replenishment order.

### **1.3 Research Objective**

The objective of the research presented in this report is to minimize inventory investment of a company by maintaining optimal number of spare parts in inventory. This research addresses a spare parts inventory control problem for a non-repairable product considering new sales. When the product fails, it will be replaced by a new one if spare parts are available. A  $(Q, r)$  inventory control problem is formulated and the operating parameters are optimized based on the resulting maintenance demand. We solve the problem of maintaining optimal number of spare parts using a multi-resolution approach that is expected to have a greater impact on the overall inventory investment. This report would help personnel in the inventory control department while dealing with the issues such as when to order and how much to order.

In order to meet the stated objective effective forecasting of the demand for spare parts is required and it plays a vital role in moving towards the objective. As the demand for the spare parts changes over time, we try to do a periodic review of the demand and come up with a series of decisions regarding the values of optimal order quantity ( $Q$ ) and reorder point ( $r$ ) for each period based on the failure rate of the parts. Since we try to do a periodic review, in addition to the order quantity and reorder point, the other important variable to be determined here is the number of set ups to be made (i.e., the number of times the values of  $Q$  and  $r$  have to be changed) during a certain period of time.

The remainder of this report is organized as follows. In Chapter 2, we look over the previous literature concentrating on the area of spare parts management and gain some important insights that helped us formulate the optimization problem. Chapter 3 addresses the  $(Q, r)$  spare part inventory control problem and provides a multi-resolution approach to solve the problem. In Chapter 4, numerical examples are provided to demonstrate the proposed approach in practical use. Finally, conclusions are given in Chapter 5.

## CHAPTER 2

### LITERATURE REVIEW

Since inventory is a huge topic, there are several papers talking about effective maintenance of inventory. For a good review over the importance of spare parts management, the reader can refer to the paper [1] by Sandeep Kumar and paper [2] by Kennedy et al. The paper by Kennedy et al. gives a good review over the recent literature on spare parts inventories. They discuss about the future areas that are open for research and explain the conditions that have significant impact on maintenance inventory levels. The author divides the paper into different sections each concentrating on various problems (like management issues, age-based replacement, multi-echelon problems, obsolescence, repairable spare parts) to be solved while determining optimal no. of spare parts.

The existing spare parts inventory models can be classified into single-echelon inventory systems and multi-echelon systems. Figure 2.1 depicts the behavior of a single-echelon system and Figure 2.2 shows a typical view of a multi-echelon system without any lateral shipments among regional facilities.

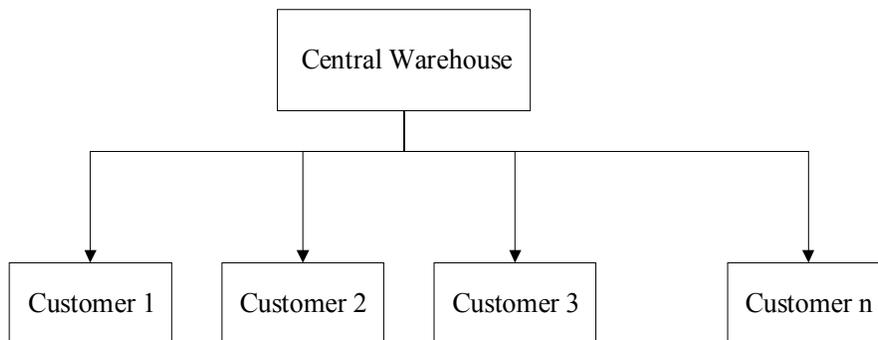
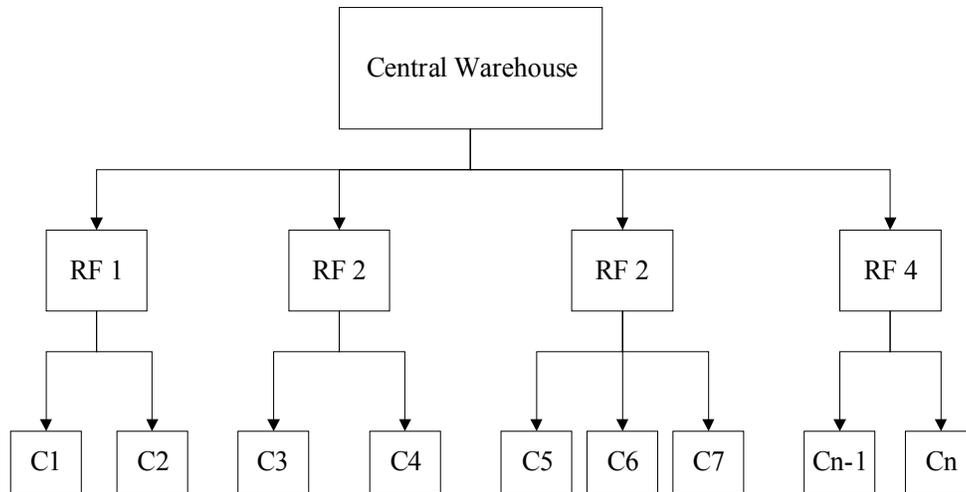


Figure 2.1 Single-echelon system



RF: regional facilities located near to the customers  
 C: customer

Figure 2.2 Multi-echelon system

Based on the operating parameters, these models can be further classified into fixed quantity model and fixed period model. To determine the optimal values of the associated operating parameters, maintenance demands: external vs. internal, deterministic vs. random, stationary vs. non-stationary, need to be characterized first, as the optimal solution is usually sensitive to such characteristics.

## 2.1 Review on Single-echelon Systems

The paper [3] presents an approximate analytical model for comparing system approach with a simpler item approach. System approach reduces inventory investment by showing difference in fill rates between parts with high costs and parts with low costs, while item approach does not vary fill rates by parts and assigns identical fill rates to all kinds of parts. The exact and continuous expressions for these two approaches are developed, to quantify the percentage reduction in inventory investment. In case of exact analysis a continuous-review  $(S_i - 1, S_i)$  inventory policy is used and demand during the lead time is assumed to follow a Poisson distribution. Also, it is assumed that demand for

spare parts have the same lead time. In case of continuous approximations the parameters unit cost and demand rate of the parts are used to represent the skewness of the values across the system. And the demand during the lead time is assumed to follow a Laplace distribution. Finally, they compare the values of inventory investment under Poisson demand and compound Poisson demand and numerical results show that relative improvement in inventory investment depends largely on the skewness of the parameters.

The paper [4] evaluates the performance of two different  $(Q, r)$  models; a simple and an advanced model. The simple model assumes that demand during lead time follows normal distribution and the advanced model assumes that demand follows gamma distribution. The paper investigates the two models using Monte Carlo simulation and shows that the advanced model consistently outperforms the simple model and that the service levels obtained are close to the desired service levels using the advanced approach. The output of the simulation program is given by

$P_2 = 1 - (\text{mean shortage per cycle}/\text{mean demand per cycle})$  [4], where  $P_2$  is the attained service level. It also gives the average inventory level on the basis of simulation and according to the formulae generated. Also, the important insight of this paper is that it shows the significant effect of the variance of the forecast error during lead time when compared to the variance of the demand during lead time.

Paper [5] considers a system where various parts are maintained to support the repairs for a set of products. The objective of their work is to determine an optimal stocking policy for each part while minimizing the expected holding and ordering costs and meeting the service constraints. They develop a heuristic procedure based on the duality theory and show that it is closely related to the well known Greedy heuristic.

They use a periodic review base stock policy and all the parts are stocked to the level  $S_i$  at the beginning of each period (replenishment occurs instantaneously). For the model developed, two forms of service level constraints (chance constraints used by Mitchell (1982, 1983) and part availability constraints used by Beesack (1967)) are considered. In extension to the basic model which does not take into account any priority customers or parts commonality among products (see Figure 2.3); they extend their research to the case where a low and a high priority customer class and also products having similar parts are considered.

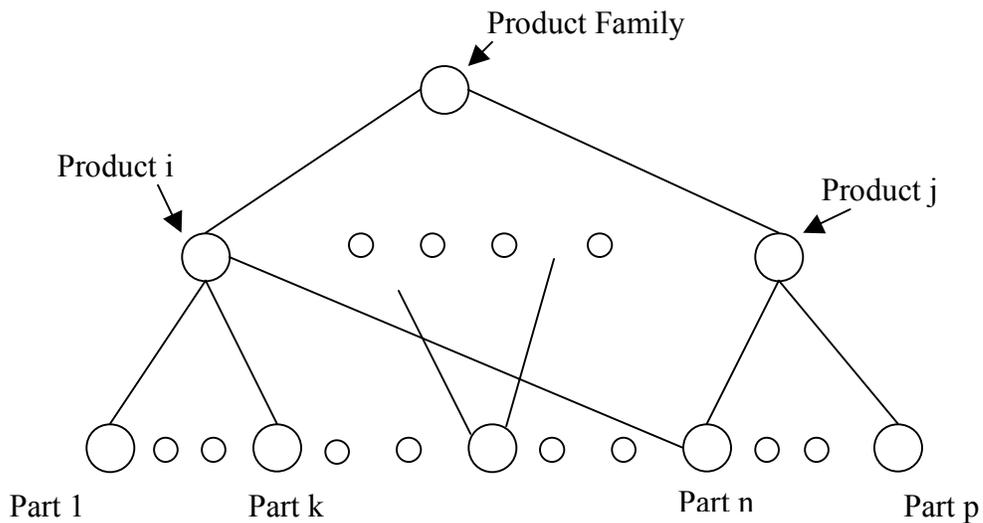


Figure 2.3 Component structure with commonality [5]

In addition to the price and quality of the parts, companies often try to reduce the cycle time in order to gain a competitive advantage. Schultz [6] in his paper tries to address this issue by quantifying the effect of spare parts inventory on the processing time of the machines. They present an existing closed-form approximation to illustrate how the mean and variance of machine repair or downtime affects average cycle time and departure variability at a given workstation and downstream workstations. They initially

consider the case of a single-critical component and then extend their research to the case where there are multiple-critical components in a single machine. The model developed assumes that components fail exponentially and it aims at minimizing the expected backorders subject to an inventory investment constraint. Also, it is assumed that inventory is controlled by a one-for-one replenishment policy and repair time is taken as negligible if a spare part is available. The mean time to repair or downtime of a machine largely depends on the availability of the spare part, so here the author tries to maximize the availability of machine while minimizing the costs associated with inventory. By taking some numerical examples the paper shows how the annual profit can be increased by maintaining optimal number of spare parts.

In process of effective maintenance of spare parts, the first and foremost step is to estimate the demand within a certain period of time. In the paper [7] they present a case study for applying Bayesian approach to forecast demand and to come up with an optimal parameter '  $S$  ' for an  $(S - 1, S)$  inventory system that stocks spare parts for electronic equipment. They consider the case of a company that produces circuit packs as spare parts which are very much essential for continuous running of the electronic equipment. As the demand for spare parts occurs by the random failure of the installed parts, the policy suggested here aims at decreasing the base stock level by a significant amount. They initially present the concepts of Bayesian approach and then show the results of applying Bayesian approach to forecast demand when compared to the current policy used by the firm. The current policy operates on the assumption that time to failure of the parts follow exponential distribution and it calculates the value of '  $S$  ' for a desired service

level  $p$ . Equation 2.1 gives the formula for the calculation of  $S$  while satisfying the service level of the customer.

$$\sum_{k=0}^{S-1} (nL)^k e^{-nL} / k! \geq p. \quad 2.1$$

Here  $L$  is the lead time and  $n$  is the number of circuit packs installed. The equation used above closely resembles to the equation that is used in our report for the calculation of reorder point in our optimization problem. As the current policy takes the parameters as known and constant, the author considers the case where a new part is introduced into the market and no real data is available over the failure rate of the part. In order to account for this, the author assumes a compound Gamma-Poisson probability function for the number of failures  $k$  during the replenishment lead time  $L$  and calculates the critical stock level that satisfies the desired service level. The proposed method significantly lowers the base stock level while achieving the desired service level.

The paper [8] concerns about providing a general and simple algorithm while obtaining an optimal solution for different inventory models in a manufacturing system with a linearly increasing or decreasing trend in demand. Their main focus was to come up with a replenishment or production schedule that minimizes the total cost of the system. The model developed in this paper acts as a general equation and reduces the complexity involved in solving the problem. They conduct the study under a finite time horizon and theoretically prove that total cost is a convex function of the number of production cycles or replenishments done by the company. However, the results obtained from the above approach just determine the optimal number of replenishments to be made and does not talk about other operating parameters that control the stock maintained within the inventory. The model presented in this report presents optimal

operating parameters for each period while controlling the non-stationary maintenance demand and satisfying the service level specified to the customer.

Spare parts are often kept in stock in order to support maintenance operations and to protect against equipment failures. The paper [9] conducts a case study at a large oil refinery for comparing different re-order point policies that optimize the number of parts maintained in inventory. However, it is always difficult to effectively manage spare parts that are very slow moving with highly stochastic and erratic demands. In order to reduce the difficulty of assessing lead time demand distributions for these kinds of parts, the paper [9] tries to test different theoretical models with real demand data taken from a company. The paper studies the case where a company holds 43,000 catalogued materials with a total cost of more than 27 million euros. As controlling a huge number of parts always represent a difficult task due to the variations in criticality and demand for the parts, the company maintains stock by moving to the SAP R/3 information system that operates in terms of min-max levels where the minimum level  $s$  is determined with the help of safety stocks and maximum level  $S$  determined by lot sizing methods.

As the price, demand and criticality of the part varies, the paper defines a combined class “xyz” (where  $x$  represents demand class,  $y$  represents criticality class and  $z$  represents price class) for each item and optimizes the operating parameters for each individual class in process of identifying the best optimization rule for the system. Two types of approaches (ex-ante and ex-post) are developed for optimization of the spare parts. In case of ex-post method whole data set is used for both fitting and testing purposes, whereas in case of ex-ante approach the historical data that is available is divided into two different sets where one is used for fitting the distribution and the other

for performing a simulation and comparing with the performance of the current one. After obtaining the optimal parameters for each distribution and calculating the total cost of the system, the author compares each inventory model with the current system and shows that of all the inventory models considered normal model performs really well achieving total savings of 8.4% (about 1.3 million euros) over the current system.

The paper [10] deals with the case of a firm that offers service contracts on its equipment and stocks a huge variety of parts to support field maintenance of its equipment. Their main objective is to reduce the inventory investment by meeting the service level and order frequency constraints that are set by the user. As the author here concentrates on the maintenance of spare parts that are required in order to increase the availability of the equipment, in our report we try to do it by concentrating on the failure rate (age) of the parts which definitely have a significant impact on the overall inventory investment. Here they present three heuristic algorithms which are easily implementable and insensitive to the addition or subtraction of new parts. They compare the three heuristics developed with the methods previously in use by the firm and show that these methods are more efficient and same customer service level can be attained at 20-25% smaller inventory investment.

In process of developing the model, they make some assumptions that include full backordering; demand follows a Poisson process; lead times are fixed; demand occurs in units of batches, batch size is constant and steady state model irrespective of the age of the parts. In this report we relax the assumptions that demand follows a Poisson process and a steady state model for the parts and take that product's failure time follows Weibull distribution. The author tries to solve the problem using the well known  $(Q, r)$  model and

come up with a good solution. In order to make it simpler he first developed the model for a single product case and then extended their work to a multi product case. The constrained optimization model for the single product case is shown in equation 2.2.

$$\text{Minimize } c(r - \theta + Q/2) \tag{2.2}$$

$$\text{Subject to } \lambda/Q \leq F,$$

$$1 - A(r, Q) \geq S,$$

$$r \geq r-, Q \geq 1, \quad r, Q : \text{integers}$$

where  $c$  is the unit cost of the item, and  $(r - \theta + Q/2)$  is the on hand average inventory at any point of time assuming that safety stock is always greater than or equal to zero. In detail  $\theta$  is the expected lead time demand,  $\lambda$  (mean demand per year),  $F$  and  $S$  are target order frequency and service level specified by the user,  $A(r, Q)$  is the probability of stock out at any point of time.

After the development of heuristics, by taking a numerical example, they compare the effectiveness of heuristics and conclude that type 1 heuristic which calculates the values of order quantity and reorder point works well when the service level is high, hybrid heuristic which is a mixture of type 1 heuristic and type 2 heuristic works well when the order frequency is high and type 2 heuristic which requires both the order quantity and reorder point to be computed consecutively works better than the above two heuristics.

## 2.2 Review on Multi-echelon Systems

Service agreements made by suppliers to the customers play a vital role in maintenance of spare parts in any industry. There are several papers talking about this issue. The paper [11] develops a continuous-review inventory model for a multi-item,

multi-echelon spare parts distribution system with time-based service level requirements. The objective of this paper is to determine base stock levels for all parts at each location while minimizing the inventory investment. The model developed in this paper includes multiple constraints with different time settings across random combinations of items and demand locations. The paper derives exact and continuous expressions for each item and initially develops a greedy algorithm to find near optimal solutions and then it develops a primaldual procedure (Lagrangian based approach) to find near optimal solutions as well as lower bounds at all locations. The author finally compares and shows the effectiveness and scalability of these different procedures by taking three example problems.

The paper [12] in extension to their previous work, considered a two-echelon inventory system that has a distribution centre (DC) and regional facilities that stocks various spare parts required for the system. As the outages due to failures are not supposed to exceed certain number of hours per month, in exchange for the service level constraint that is previously in use by the firm, here they consider an optimization problem where the average total delay due to failures is to be maintained below a specified level. In support for the development of the model, they assume that the DC operates on the basis of continuous review (Q, r) policy and the facilities use the base stock policies for the replenishment of parts from the DC. The model formulated in this problem can be expressed as follows:

$$\text{Minimize } \sum_{i=1}^N c_i \left( h_0^i(r_0^i, Q_0^i) + \sum_{m=1}^M h_m^i(S_m^i) \right) - \text{total inventory investment} \quad 2.3$$

Subject to:

$$\frac{1}{N} \sum_{i=1}^N \frac{\lambda_0^i}{Q_0^i} \leq F;$$

$$\sum_{i=1}^N B_m^i(S_m^i) \leq T_m, m = 1, \dots, M.$$

Here  $N$  = number of items,  $c_i$  = cost of item  $i$ .

$h_0^i(r_0^i, Q_0^i)$  = expected on hand inventory of item  $i$  at DC at any point of  
time

$h_m^i(S_m^i)$  = expected on hand inventory of item  $i$  at facility  $m$  at any point  
of time

$\lambda_0^i$  = expected annual demand for item  $i$  at the DC (parts/yr)

$Q_0^i$  = order quantity for item  $i$  at the DC

$F$  = target order frequency

$B_m^i(S_m^i)$  = expected number of backorders for items  $i$  at facility  $m$  at any  
point of time

$T_m$  = total delay that is allowed per year at facility  $m$

As the model developed represents a large scale non linear discrete optimization problem, they decompose the problem and develop closed form expressions for the control parameters at the facility and the DC.

The paper [13] considers a similar system that has a single central warehouse and several local warehouses to support the failure of the parts occurring at various customer locations. Here they divide the customers into several classes and assign a target aggregate fill rate to each class. They develop a multi-item, single stage spare parts inventory system for this problem. It is assumed that customers have similar machines and failure rate of the part follows a Poisson process. A critical level is specified for all the items per customer class. The satisfaction of demand for part  $i$  from a customer class  $j$

depends on this critical level, the higher the critical level, the lower the service provided for the customer. The optimization problem developed for this case is given by:

$$\text{Min } \sum_{i=1}^I p_i c_{i,j+1} \quad (\text{inventory investment}) \quad 2.4$$

$$\text{Subject to } \sum_{i=1}^I \frac{m_{i,j}}{M_j} \beta_{i,j}(c_i) \geq \beta_{j,obj}, \quad (\text{fill rate constraint})$$

$$0 = c_{i,1} \leq \dots \leq c_{i,J} \leq c_{i,J+1}, [13]$$

Here  $p_i$  denotes the price of the item and  $c_{i,j+1}$  represents the critical level for customer  $j+1$ . As the problem is a nonlinear integer optimization problem, they develop a heuristic solution procedure based on Lagrange relaxation to obtain a good approximate solution that generates both heuristic solution and lower bound for the optimal costs. After the development of the model, expressions for the item fill rates are derived in order to evaluate the effectiveness of the critical level policy followed. Also, the parameters (Lagrange multipliers) are optimized to get the best feasible solution. Computational results show that the error between the costs of lower bound and heuristic solution decreases with the increase in number of items and applying critical level policies reduces the inventory investment in comparison to the general base stock policies with equal aggregate fill rate to all the customers.

The paper [14] deals with maintaining optimal number of spare parts in a multilevel maintenance system. The paper develops computer programs (MASOPS – a FORTRAN program) for calculating the optimal number of spares at two and three maintenance levels. They consider a hierarchical, pyramidal system that has three supply levels of infinite capacity and complete diversity. The optimization procedure developed

aims at minimizing the costs of spare part kits while maximizing the maintenance efficiency. Spare part kits are assigned for each maintenance level and spare parts at lower level are replenished from spare parts at higher level. The paper classifies all the items into different groups and develops an algorithm for calculating the optimum spare part kits. Later, with the help of a numerical example they illustrate the impact of maintenance policy on the efficiency of the maintenance system.

The paper [15] is about effective maintenance of a two-echelon spare parts inventory system. They consider a system that controls equipment failure at various locations with the help of a central warehouse and several field depots (that stocks spare parts) placed at nearest locations to the customer. They develop a continuous review, base stock policy to minimize the overall inventory cost subject to a service constraint at each field depot. The model developed in this paper uses response time of the supplier as the service constraint and aims at keeping it below a threshold level of 4 hours. In process of determining inventory policies at the central warehouse and field depots, they initially develop expressions for finding inventory and backorder levels at all the depots and then present a heuristic algorithm that can be solved for obtaining best solution and best lower bound for the base stock levels at individual depots as well as warehouse. They also show the effectiveness of the algorithm when compared to the heuristic developed in the paper [12].

### **2.3 Review on Criticality Based Approach**

As the management of spares has a big role to play in maximizing the profits for any company, Wang et al. [16] try to optimize the spare part stores by using an improved genetic algorithm in a supply chain kind of system. In process of managing the spare

parts, they classify them into ABC classes based on the capital proportion and usage life of the parts. The author describes some of the key issues (like checking and evaluating the status of equipment, setting up a management information system) that keep update of the requirement of spare parts in time. Based upon the improved genetic algorithm they develop a two-objective optimization model that minimizes the capital cost and maximizes the safety factor. In the simulation phase, the model developed shows that 39% of the overall cost can be saved when compared to the traditional method followed.

Yang et al. [17] proposed four criticality evaluation methods that control initial provisioning for spare parts. Failure of the critical parts often leads to the total shut down of the system. Here, based on the criticality of the parts they estimate the number of parts to be maintained that decreases the total life cycle cost of the system. The four methods proposed in this paper include the RPN (risk priority number), the GRN (grey relation number, the product of RPN and MTAT (maintenance turn around time), and the GRN with MTAT. By considering the RPN and MTAT, the paper [17] determines the criticality of the item and assigns weight for the item. In order to illustrate the methods, the paper studies the case of an ignition system that has several components with varying MTBF, failure rate and unit cost. The optimal spares stocking policy among the four methods is selected based upon the cost effectiveness curves of the system.

## **2.4 Analysis**

From the above literature it can be noted that most research has been focused on either  $(Q, r)$  or  $(s, S)$  inventory control problems assuming stationary maintenance demands, i.e., their distributions do not change over time. The distributions are usually determined by fitting historical demand data or are simply assumed based on experience.

However, this approach is not appropriate when new sales are considered, which expand the installed base over time. In fact, under such circumstance a non-stationary stochastic process is more appropriate for modeling the resulting maintenance demand, and it is desirable to investigate how the product is adopted in order to proactively forecast the upcoming maintenance demand for better spare parts management. This is an important factor that either increases or decreases the number of parts required to be maintained in inventory at any specific point of time. This report studies the case of a new product where the manufacturer agrees to provide spare parts for maintenance. As maintenance demand varies over time, we do a periodic review of the demand and try to reduce the overall inventory investment in a single-echelon system while satisfying the customer.

## **CHAPTER 3**

### **METHOD**

In this chapter, we propose an inventory model that aims at minimizing the inventory investment of a company while meeting the service level specified to the customer. In order to succeed in today's competitive market it became important for companies to see that they deliver their parts at right time with right quality. Therefore companies often try to increase the availability of the system which is possible through effective maintenance of spare parts in inventory. Providing the spare parts when required increase the up-time of the systems, which is crucial for on-time delivery of the products. As increasing or decreasing the number of spare parts maintained highly affects the profits made by the company, they have an important role to play in reducing the operational costs of the company. Here we try to reduce the inventory investment by maintaining optimal number of spare parts in inventory. In order to make this happen it is required to effectively estimate the number of spare parts required at any specific point of time.

The number of spare parts required largely depends on the failure rate of the parts. Effective control of spare parts is a tough task faced by the personnel in the inventory management department today. Systems often start to deteriorate as the age and usage of the system increases. As the failure rate of parts increases with time, the demand for spare parts also increases in proportion with time. Studying the failure rate of the parts helps one to effectively predict the future performance of the system. In regard to these issues as there are different types of technical systems that require spare parts, we consider the case of a new non-repairable product that is service contracted to the

customer [18]. And as the original manufacturer of the product is responsible for providing the spare parts required to maximize the availability of the system, we here come up with a model that deals with this issue while minimizing the maintenance cost of the system. Also as the orders from the clients come without any prior intimation, the inventory control department feels difficult to satisfy the customer when the order is placed for slow moving parts. In this report we initially assume that rate of new sales of the product is constant and the failure time of the product follows Weibull distribution. In process of solving the model we consider a finite period of 12 months with demand for spare parts based on the failure rate of the parts.

We are just concerned about one single component and we do not talk about the issues such as which items to be treated as spare parts and which items to be kept in stock? [18]. The model developed is for a single echelon system where the parts required are shipped directly from the central warehouse to the customer without any lateral transshipment between the warehouses. The critical component for which the optimal stock is determined is considered to be a non-repairable part, and its state is as good as new after each instant replacement. Spare parts are often kept in stock in order to control the lead times that it takes to replace the defective part. As the requirement for spare parts depends on the preventive maintenance activities followed by the clients, we assume that the company makes contract with all the clients that certain rules were followed for performing maintenance activities in order to increase the accuracy of the demand forecasted.

### 3.1 Modeling of Maintenance Demand Considering New Sales [19]

Before formulating the maintenance demand, the following assumptions are made:

1. At the time  $t = 0$ , the first unit is adopted by the customer. New sales occur one at a time following the homogeneous Poisson process with the rate of  $\lambda$ , the probability mass function is given by:

$$\Pr(N(t) = n) = \frac{e^{-\lambda t} (\lambda t)^n}{n!} \quad (3.1)$$

2. The product is non-repairable, and its failure time follows the two-parameter Weibull distribution with pdf:

$$f(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{t}{\eta}\right)^\beta\right) \quad (3.2)$$

where  $\beta > 0$  and  $\eta > 0$  are the shape parameter and scale parameter, respectively.

3. Compared to the total life time of the product, the total service time (e.g. replacement time and time spent waiting for a spare part if any) are ignored (i.e. instant replacement)

It can be noted that, as the product is non-repairable and as good as new after each instant replacement, each unit of product adopted will generate a renewal process. Let  $F(t)$ ,  $t > 0$ , be the cumulative distribution function (Cdf) of the product's failure time, and  $H(t)$ ,  $t > 0$ , be the corresponding renewal function (expected number of renewals).

According to the renewal theory,  $H(t)$  can be expressed as:

$$H(t) = F(t) + \int_0^t H(t-s)dF(s) = \sum_{i=1}^{\infty} F^{(i)}(t), \quad (3.3)$$

This equation gives the expected maintenance demand requested by a single unit installed at time 0. In the above equation  $F^{(i)}(t)$  represents the  $i$ -fold convolution of the underlying failure time distribution  $F(t)$ , which is defined as

$$F^{(i)}(t) = \int_0^t F^{(i-1)}(t-u)dF(u), \quad t \geq u \geq 0, \quad n \geq 2 \quad (3.4)$$

with  $F^{(i)}(t) = F(t)$ , Note that, for the two-parameter Weibull distribution

$$F(t) = 1 - \exp\left(-\left(\frac{t}{\eta}\right)^\beta\right).$$

For the two-parameter Weibull distribution, the analytical solution of this equation does not exist, and the equation has to be solved numerically. In practice, the direct Riemann-Stieltjes integration approach proposed by Xie [20] may be utilized.

Furthermore, let  $S(t)$  be the aggregated expected maintenance demand at time  $t$ . If the product's installed base has multiple units (say  $n$ ) at time 0 and there is no new sale afterwards,  $S(t)$  can be simply expressed as:  $S(t) = nH(t)$ , as each product generates its own renewal process. When new sales are considered, each unit newly adopted will start its renewal process from the time when it is adopted. Figure 3.1 illustrates the maintenance demand under such circumstance.

As a result,  $S(t)$  becomes the aggregation of the expected number of renewals of multiple renewal processes starting from different points in time. The maintenance demand for the case where new sales follow homogeneous Poisson process can be derived as follows:

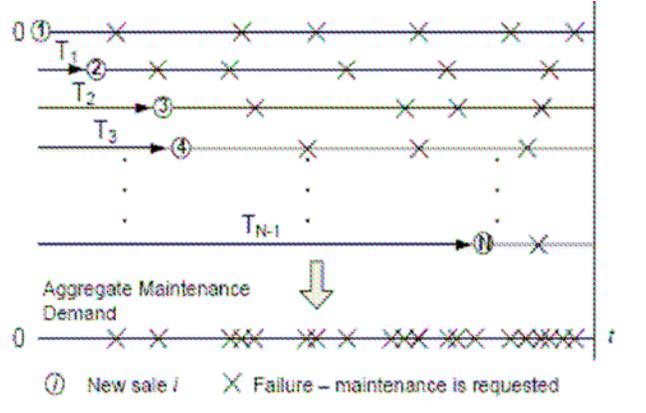


Figure 3.1 Maintenance demand vs. new sales

Case 1: When there is no new sale up to time  $t$  ( $n = 0$  with the probability of  $e^{-\lambda t}$ ), the maintenance demand process is same as the renewal process of the first unit adopted, i.e.:

$$S(t) | \{Case1\} = H(t).$$

Case 2: When there is only one new sale up to time  $t$  ( $n = 1$  with the probability of  $\lambda t e^{-\lambda t}$ ), the conditional expected aggregate maintenance demand is:

$$S(t) | \{Case2\} = H(t) + \int_0^t H(t-t_1) \lambda e^{-\lambda t_1} dt_1.$$

Case 3: When there are two new sales up to time  $t$  ( $n = 2$  with the probability of  $(\lambda t)^2 e^{-\lambda t} / 2!$ ), the conditional expected aggregate maintenance demand becomes:

$$S(t) | \{Case3\} = H(t) + \int_0^{t-t_1} \int_0^{t-t_1-t_2} [H(t-t_1) + H(t-t_1-t_2)] \times \lambda e^{-\lambda t_2} \lambda e^{-\lambda t_1} dt_2 dt_1.$$

Case n: When there are  $i$  new sales up to time  $t$  ( $n = i$  with the probability of  $(\lambda t)^i e^{-\lambda t} / i!$ ), the conditional expected aggregate maintenance demand becomes:

$$H(t) + S(t) \{Casen\} = \int_0^t \int_0^{t-t_1} \int_0^{t-t_1-t_2} \dots \int_0^{t-t_1-t_2-\dots-t_{i-1}} \left[ \sum_{j=1}^i H(t - \sum_{k=1}^j t_k) \right] \times \lambda^i e^{-\lambda \sum_{m=1}^i t_m} dt_i dt_{i-1} \dots dt_1.$$

In summary, considering all the possibilities the expected aggregate maintenance demand at time  $t$  can be expressed as:

$$S(t) = H(t) + \sum_{i=1}^{\infty} \left\{ \int_0^t \int_0^{t-t_1} \int_0^{t-t_1-t_2} \dots \int_0^{t-t_1-t_2-\dots-t_{i-1}} \left[ \sum_{j=1}^i H(t - \sum_{k=1}^j t_k) \right] \times \lambda^i e^{-\lambda \sum_{m=1}^i t_m} dt_i dt_{i-1} \dots dt_1 \right\} \frac{e^{-\lambda t} (\lambda t)^i}{i!} \quad (3.5)$$

Note that the expected number of maintenance demand during the time interval  $[t, t + \Delta t)$  is given by:  $S(t + \Delta t) - S(t)$ . Equation (3.5) gives the exact formulation of the aggregate expected maintenance demand; however it is difficult, if not impossible, to calculate numerically. To overcome this difficulty, a simulation based approach is utilized in this report. The simulation program is developed using Matlab, which generates the time of each new sale as well as corresponding failure times after the sale. While generating demand using Matlab we assume that the failure times of the parts follow Weibull distribution with scale parameter ( $\eta = 1$ ) and shape parameter ( $\beta = 2$ ). Figure 3.2 shows the aggregate maintenance demand (from 100 runs) generated by the simulation program. To implement this simulation approach in estimating the aggregate expected maintenance demand  $S(t)$ , the program will be executed for a large number of runs and  $S(t)$  will be estimated by taking the average from those simulation runs:

$$\hat{S}(t) = \frac{S_i(t)}{M} \quad (3.6)$$

Where  $S_i(t)$  is the aggregate maintenance demand obtained in the  $i^{th}$  simulation run and  $M$  is the total number of runs. After generating the demand we start solving the model to minimize the costs incurred by the company.

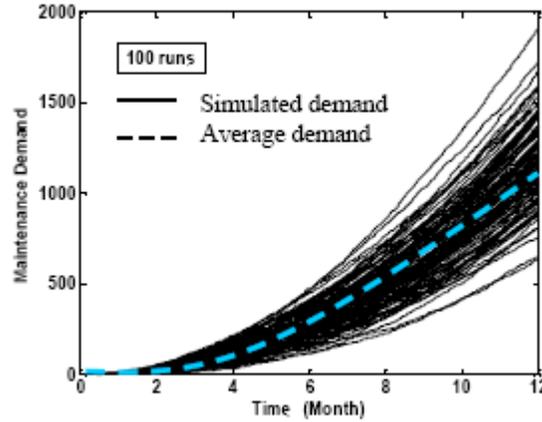


Figure 3.2 Aggregate maintenance demand considering new sales

The total demand for each month can be determined by the summation of demand for new sales and the demand for maintenance.

Total Demand = demand for new sales + demand for maintenance.

### 3.2 Maintenance Demand Approximation [19]

To control the spare parts inventory, one approach is to update the maintenance demand and adjust the spare parts inventory strategy, i.e., the values of  $(Q, r)$ , once a new sale occurs. However, this approach may not be viable in most real world applications because of high cost incurred by frequent setups of the associated operating parameters. To facilitate implementation, a multi-resolution method is proposed, which divides the time period being considered into several smaller time intervals, in which the average aggregate expected maintenance demands are calculated by:

$$\bar{\hat{S}}_i = \frac{\hat{S}(b_i) - \hat{S}(a_i)}{b_i - a_i} \quad (3.7)$$

where  $a_i$  and  $b_i$  are the lower and upper bounds of the  $i$ th time interval, with  $\hat{S}(a_i)$  and  $\hat{S}(b_i)$  estimated from equation (3.6), respectively. In particular, within each interval the maintenance demand is assumed to follow a homogeneous Poisson process. The resolution of this approximation technique depends on the lengths of those time intervals utilized. Figure 3.3 shows an example with a lower resolution and another with a higher resolution. When smaller time intervals (higher resolution) are utilized, the approximation to the maintenance demand will be more accurate. The extreme solution is to predict the maintenance demand whenever a new sale occurs, but its economic performance may become worse due to frequent setups, as we mentioned earlier. In most industry applications, the trade off between the approximation accuracy and the economic efficiency can be made in such spare parts inventory control problem.

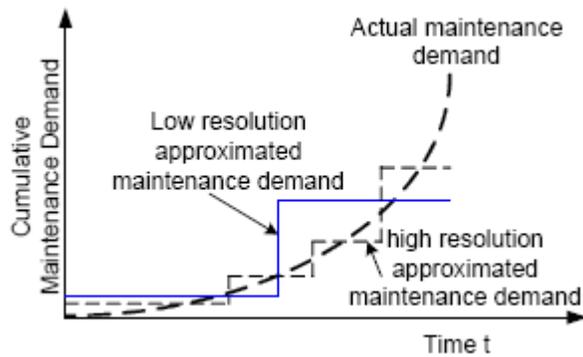


Figure 3.3 Multi-resolution maintenance demand approximation

### 3.3 Model Development [19]

To facilitate the presentation of the proposed inventory control model, the following notations are introduced first.

#### *Notation*

$M_{\max}$  = the maximum number of setups allowed

- $m$  = number of time intervals to be considered ( $\leq M_{\max}$ )
- $c_h$  = holding cost per item per time unit
- $c_s$  = shortage cost per item
- $c_o$  = cost per order
- $c_{sp}$  = cost of setting up the inventory operating parameters for each time interval
- $L_i$  = length of time interval  $i$  [ $c_{hi} = c_h \times L_i$ ]
- $l$  = replenishment lead time
- $X_i$  = random demand during replenishment lead time during time interval  $i$
- $\theta_i$  = expected aggregate demand during replenishment lead time in time interval  $i$
- $r_i$  = reorder point for time interval  $i$
- $Q_i$  = order quantity for time interval  $i$
- $TC$  = total inventory cost over the interested time horizon

The two decision variables that highly influence the stock maintained in any company are the values of  $Q$  (order quantity) – how much to order and  $r$  (reorder point) – when to order. Optimizing these two values significantly increases the profits made by a company. In general reordering has been controlled by systems such as MRP, JIT and statistical inventory control methods that suits the conditions effectively [16]. Higher values of  $Q$  and  $r$  reduce the ordering cost and penalty cost of the company while increases the holding cost of the company. On the flip side, lower values of  $Q$  and  $r$  reduce the holding cost and increase the ordering cost and penalty costs within the company. Therefore a perfect trade off has to be provided in order to minimize the total costs incurred by the company.

In our model, we concentrate on these two parameters and try to come up with a series of decisions regarding the values of  $Q$  and  $r$  while satisfying the constraints of the company. The optimal values for  $Q$  and  $r$  are determined based on the predicted demand for parts during a certain period. As it is always not possible to accurately forecast the demand, in our model we try to reduce the error caused due to this by minimizing the length of the period for which forecasts are made. Hence, the model in this paper comes up with optimal values of  $Q$  and  $r$  for each individual period trying to minimize the operational costs of the company. The values of  $Q$  and  $r$  obtained increases as demand is estimated for each period. By maintaining lower values of  $Q$  and  $r$  in the earlier stage of life cycle of the equipment, we decrease the holding cost of the company which contributes to a significant part of the capital investment made by the company. The inventory system in our model operates on the basis of a continuous review policy with periodic review of demand at certain intervals that helps to revise the values of  $Q$  and  $r$  while minimizing the total cost of the system. Figure 3.4 shows a good view of the approach followed for our model. The entire schedule period is divided into three time intervals, each of which has a set of operating parameters  $(Q_i, r_i)$  to be optimized.

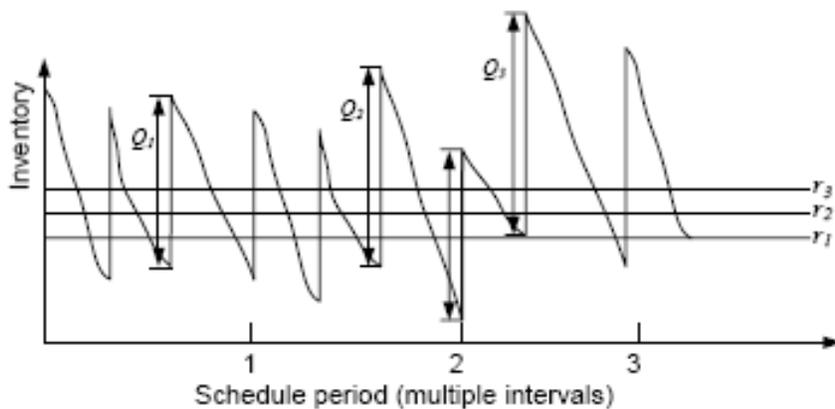


Figure 3.4 Three-interval  $(Q, r)$  spare parts inventory model

From Figure 3.4 we can see that in the initial stages, an order of quantity  $Q_1$  is placed at reorder point  $r_1$  and as the time increases the values of  $Q$  and  $r$  gradually increased. The proposed multi-resolution spare parts inventory control strategy jointly determines the number of intervals  $m$  and all the resulting operating parameters  $(Q_i, r_i)$ ,  $i \in [1, m]$ , in each interval under given constraints. In particular for a given number of intervals  $m$ , the optimal values of  $(Q_i, r_i)$  are determined based on the associated average aggregate expected maintenance demands during those time intervals. In this report, equal-length time intervals are utilized for each resolution strategy, and the average aggregate expected maintenance demand rate in each time interval is calculated using equation (3.7). The total description of the method followed can also be seen from the flowchart pictured in Appendix C.

Also from our spare parts inventory control model, we can see that we are nothing but lowering the values of  $Q$  and  $r$  in the initial stages and expecting to decrease the total cost incurred by the system. This way of decreasing the values of  $Q$  and  $r$  reduces the holding cost but increases the ordering cost of the system. Therefore, in our model we deal with the minimization of holding cost, ordering cost and shortage cost while meeting the service level specified to the customer. Specially, we calculate the shortage cost from the expected number of shortages as follows. The expected number of shortages per cycle can be approximately evaluated by:

$$E[\text{number of shortages/cycle}] = \left[ \sum_{x=r_i+1}^{\infty} \frac{e^{-\theta_i} \theta_i^x}{x!} (x - r_i) \right],$$

where  $\theta_i = l \cdot \bar{S}_i$  is the aggregate expected maintenance demand during the replenishment lead time interval  $i$ . After several algebraic manipulations, the associated expected total shortage cost can be obtained as:

$$E[\text{shortage cost/cycle}] = c_s \left( \theta_i - r_i - \sum_{x=0}^{r_i} \frac{e^{-\theta_i} \theta_i^x}{x!} (x - r_i) \right),$$

where  $c_s$  is the shortage cost per item.

Let  $TC$  be the total inventory cost over the time horizon interested. Considering all cost elements, the multi-resolution optimization model for controlling the spare parts inventory can be formulated as Problem **P<sub>1</sub>**:

$$\begin{aligned} \min_{m, \{Q_i, r_i\}} TC &= E[\text{total inventory cost}] \\ &= \sum_{i=1}^n c_{hi} \left( r_i - \theta_i + \frac{Q_i}{2} \right) + \left[ c_s \left( \theta_i - r_i - \sum_{x=0}^{r_i} \frac{e^{-\theta_i} \theta_i^x}{x!} (x - r_i) \right) + c_o \right] \left( \frac{D_i}{Q_i} \right) + m \cdot c_{sp} \end{aligned}$$

Subject to  $m \leq M_{\max}$

$$\sum_{x=0}^{r_i} \frac{e^{-\theta_i} \theta_i^x}{x!} \geq 0.95; \quad Q_i > r_i \geq 0, \quad Q_i, r_i \in \mathbb{Z}^+, \text{ for } \forall i.$$

The model operates under the constraint that the service level provided by the firm is at least 95% (i.e. probability that the demand is met from the stock maintained). The first part of the objective function calculates the average inventory maintained for each period which when multiplied by the holding cost of the item gives the average inventory investment for that particular period. The second part of the objective function gives the penalty cost associated with the inventory level maintained. We calculate this shortage cost by estimating the expected number of stockouts for that specific period. Since we are reducing the level of inventory maintained we are nothing but increasing the

number of orders placed while satisfying the customer. Therefore, the third part of our objective function aims at minimizing the ordering cost of the inventory system followed. Finally, the last part deals with minimizing the setting cost of the operating parameters for each period. Since we are calculating all the three costs for each period, as the number of intervals for which the demand is predicted increases, the complexity of our model developed also increases.

### 3.4 Solution Technique [19]

The model formulated in the above section is a nonlinear integer programming problem. A bisectional search procedure is utilized to determine the optimal solution. Let  $m_L$  and  $m_H$  be the lower and upper limits of a search interval, respectively. The steps of the algorithm are given as follows:

*Step1.* Let  $j = 1$ ,  $m_L = 1$ ,  $m_H = M_{\max}$ .

*Step2.* For the given  $m_L$  and  $m_H$ , determine  $TC^*(m_L, (Q_i^*, r_i^*)_{all\ i})$  and  $TC^*(m_H, (Q_i^*, r_i^*)_{all\ i})$  by solving the above nonlinear optimization problems and let  $m_m^j = \lfloor (m_L + m_H) / 2 \rfloor$ , where  $\lfloor \cdot \rfloor$  is the floor operator (e.g.,  $\lfloor 1.6 \rfloor = 1$ ).

*Step3.* Let  $j = j + 1$ . If  $TC^*(m_L, (Q_i^*, r_i^*)_{all\ i}) \geq TC^*(m_H, (Q_i^*, r_i^*)_{all\ i})$ , then let  $m_L = m_m^{j-1}$  and  $m_m^j = \lfloor (m_L + m_H) / 2 \rfloor$ ; otherwise let  $m_H = m_m^{j-1}$  and  $m_m^j = \lfloor (m_L + m_H) / 2 \rfloor$ .

*Step4.* If  $|m_m^j - m_m^{j-1}| \leq 1$ , then stop and choose  $\min (TC^*(m_L, (Q_i^*, r_i^*)_{all\ i}), TC^*(m_H, (Q_i^*, r_i^*)_{all\ i}))$  as the overall optimal solution; otherwise go to Step 3.

Note that, in this algorithm, the optimal  $(Q_i^*, r_i^*)_{all i}$  for each given  $m$  are obtained using the Microsoft Excel Solver and other optimization software can also be utilized.

The following chapter deals with the results obtained after implementing the model described above. We study three different cases, one assumes that demand for new sales follow a uniform distribution and the second assumes that demand for new sales follow a homogeneous Poisson process and the third assumes that demand for new sales follow a non-homogeneous Poisson process. The results are expected in a way that decreases the total cost of the system as we try to predict the demand for shorter periods of time and obtain the values of  $Q$  and  $r$  for each individual period. The optimal point that decreases the stock levels maintained is expected. As spare parts that are expensive block huge amounts of capital due to overstocking of parts, this kind of approach is expected to produce more beneficial results.

## CHAPTER 4

### NUMERICAL EXAMPLES

In this chapter, numerical examples are provided to demonstrate the proposed approach in practical use. We initially study the case of demand for new sales following uniform distribution with a mean of 15 units/month and the failure time of the product follows the Weibull distribution with  $\beta = 2$  and  $\eta = 1$  month. To provide maintenance services a spare parts inventory control system is developed. Regarding this system, it is assumed that:  $c_1 = \$2/\text{unit}/\text{month}$ ,  $c_2 = \$20/\text{unit}$ ,  $c_3 = \$15/\text{order}$  regardless of the quantity ordered,  $c_4 = \$10/\text{setup}$ ,  $l = 1$  week, and  $M_{\max} = 12$ . The objective is to minimize the total inventory system during a 1 year period. Using a simulation based approach, the monthly expected maintenance demands are obtained as shown in the Table 4.1

Table 4.1 Expected maintenance demand in a 1 year period for uniform distribution

1	2	3	4	5	6
4.26	20.34	37.66	54.99	72.57	90.22
7	8	9	10	11	12
107.7	125.3	141.9	159.6	177	195.1

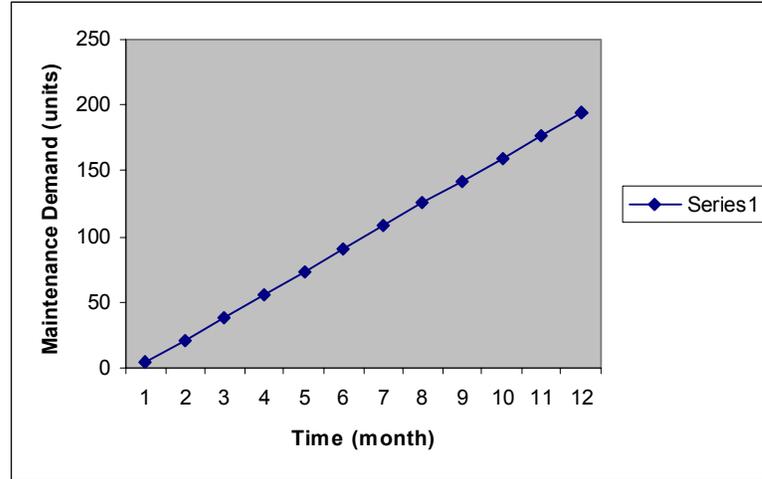


Figure 4.1 Plot of expected maintenance demand in a 1 year period for uniform distribution

Using the estimated maintenance demands, the multi-resolution inventory control systems are constructed and optimized using the bisectional search algorithm. Table 4.2 shows the optimal settings of  $(Q_i, r_i)$  under various resolution levels (i.e. values of  $m$ ). Also, Figure 4.2 illustrates the optimal solution for each value of  $m$ , and the overall optimal solution is obtained when  $m = 3$ , which results in the minimal inventory cost of \$1160.10.

Solving with the help of a Bisectional Algorithm:

Step 1: Let  $j = 1$ ;  $m_L = 1$ ;  $m_H = M_{\max} = 12$

Step 2: For the given  $m_L$  and  $m_H$ ,

$$TC(m_L) = 1198.748 \text{ and } TC(m_H) = 1234.09$$

$$m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = \lfloor 6.5 \rfloor = 6$$

Step 3: Let  $j = j + 1$ . Since  $TC(m_L) < TC(m_H)$

$$m_H = m_m^{j-1} = 6 \text{ and}$$

$$m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = 3$$

Step 4: Since  $|m_m^j - m_m^{j-1}| = |3 - 6| = 3 > 1$

Step 5: Let  $j = j + 1$ . Since  $TC(m_L) > TC(m_H)$

$$m_L = m_m^{j-1} = 3 \text{ and}$$

$$m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = 4$$

Step 6: Since  $|m_m^j - m_m^{j-1}| = |4 - 3| = 1 \leq 1$

We compare  $TC(m_L) = 1160.10$  and  $TC(m_H) = 1178.79$  and choose

$m_L = 3$  as the optimal solution.

Table 4.2 Optimal Settings for  $(Q_i, r_i)$  with various resolution levels in case of demand following uniform distribution

$m$	1	<b>3*</b>	6	12
$(Q_i, r_i)$	(42,31)	<b>(22,11)</b>	(14,6)	(8,3)
		<b>(41,31)</b>	(28,16)	(18,9)
		<b>(54,50)</b>	(38,26)	(25,14)
			(45,36)	(30,19)
			(52,45)	(35,24)
			(57,55)	(39,29)
				(43,34)
				(47,38)
				(50,43)
				(53,48)
				(57,52)
				(59,57)
$TC$ (\$)	1198.75	<b>1160.10</b>	1178.79	1234.09

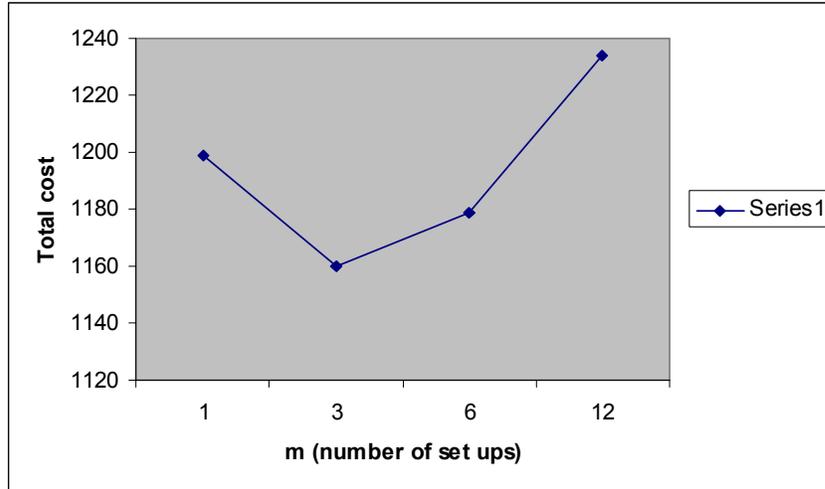


Figure 4.2 TC (Total Cost) vs.  $m$  in searching the optimal solution

After obtaining the optimal solution for initial case, we now assume that the demand for new sales follows homogeneous Poisson process and solve the model again.

Table 4.3 Expected maintenance demand in a 1 year period for homogeneous Poisson process

1	2	3	4	5	6
4.2	20.32	36.6	54.04	70.63	87.05
7	8	9	10	11	12
104.8	121	138.5	154.7	170.4	186.1

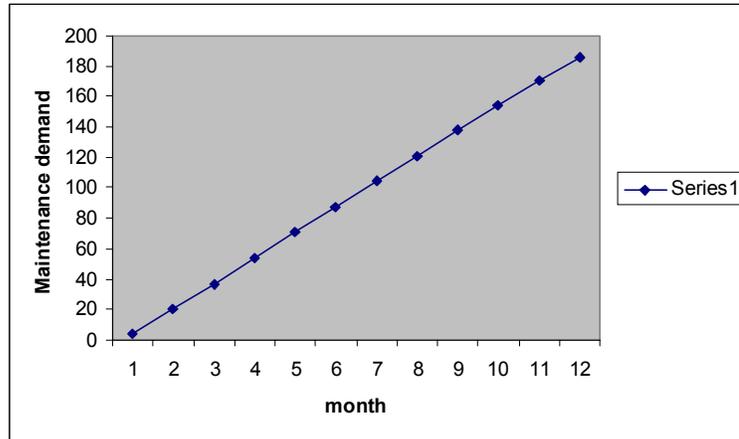


Figure 4.3 Plot of expected maintenance demand in a 1 year period for homogeneous Poisson process

Solving with the help of Bisectional Algorithm:

Step 1: Let  $j = 1$ ;  $m_L = 1$ ;  $m_H = M_{\max} = 12$

Step 2: For the given  $m_L$  and  $m_H$ ,

$$TC(m_L) = 1183.24 \text{ and } TC(m_H) = 1224.99$$

$$m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = \lfloor 6.5 \rfloor = 6$$

Step 3: Let  $j = j + 1$ . Since  $TC(m_L) < TC(m_H)$

$$m_H = m_m^{j-1} = 6 \text{ and } m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = 3$$

Step 4: Since  $|m_m^j - m_m^{j-1}| = |3 - 6| = 3 > 1$

Step 5: Let  $j = j + 1$ . Since  $TC(m_L) < TC(m_H)$

$$m_H = m_m^{j-1} = 3 \text{ and } m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = 2$$

Step 6: Since  $|m_m^j - m_m^{j-1}| = |2 - 3| = 1 \leq 1$

We compare  $TC(m_L) = 1176.53$  and  $TC(m_H) = 1144.18$  and choose

**$m_L = 3$**  as the optimal solution.

Table 4.4 Optimal Settings for  $(Q_i, r_i)$  with various resolution levels in case of demand following homogeneous Poisson Process.

$m$	1	<b>3*</b>	6	12
$(Q_i, r_i)$	(40,31)	<b>(22,11)</b>	(14,6)	(8,3)
		<b>(41,30)</b>	(28,16)	(18,9)
		<b>(54,48)</b>	(36,26)	(24,14)
			(44,35)	(30,19)
			(51,44)	(35,23)
			(63,52)	(39,28)
				(43,33)
				(46,37)
				(49,42)
				(53,46)
				(56,50)
				(57,55)
$TC$ (\$)	1176.53	<b>1144.18</b>	1184.14	1218.39

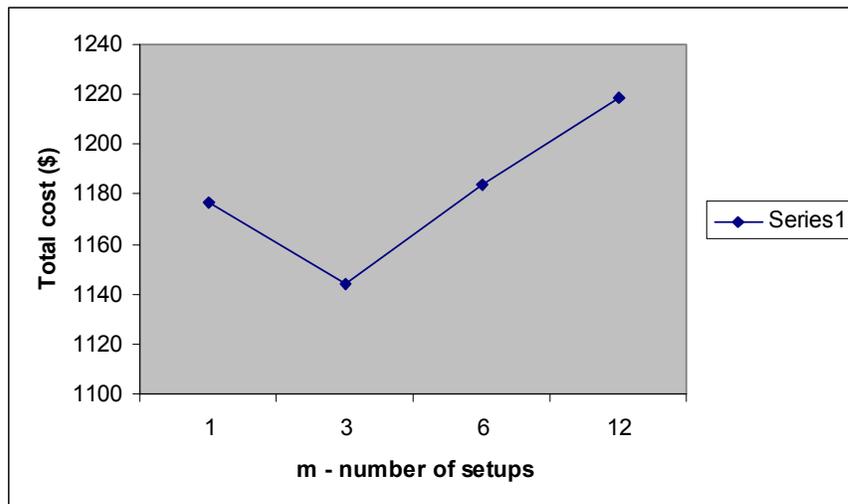


Figure 4.4 TC (Total Cost) vs.  $m$  in searching the optimal solution

Figure 4.4 shows that we get the optimal solution at  $m = 3$  for the case when demand for new sales follows homogeneous Poisson process with a total inventory cost of \$1144.18.

In a similar way, after obtaining the optimal solution for the above two cases, next we study the case of demand for new sales following Non-homogeneous Poisson process

and obtain the optimal parameter settings that reduce the total inventory investment of a company.

Table 4.5 Expected maintenance demand in a 1 year period for Non-homogeneous Poisson process

1	2	3	4	5	6
0.1	1.4	5.66	13.08	24.99	39.3
7	8	9	10	11	12
57.67	79.82	105.7	133.1	171.1	207.8

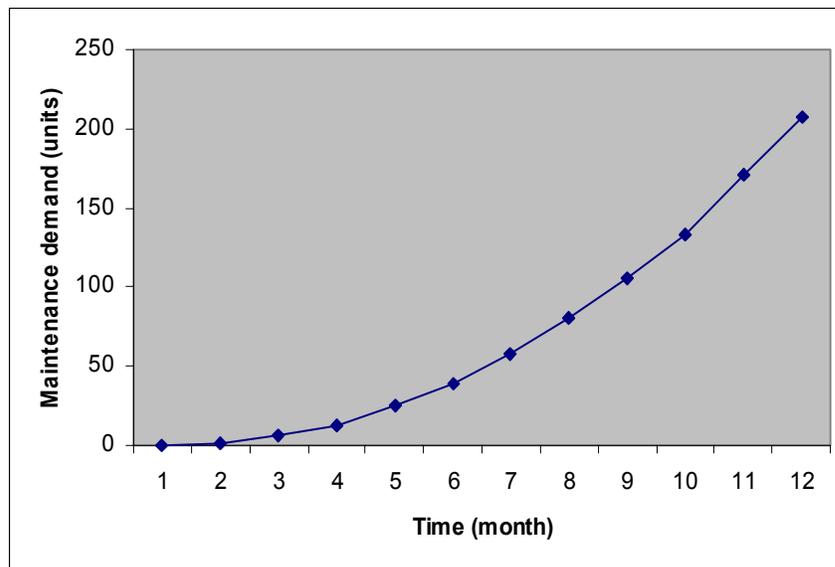


Figure 4.5 Plot of expected maintenance demand in a 1 year period for Non-homogeneous Poisson process

Solving with the help of Bisectional Algorithm:

Step 1: Let  $j = 1$ ;  $m_L = 1$ ;  $m_H = M_{\max} = 12$

Step 2: For the given  $m_L$  and  $m_H$ ,

$$TC(m_L) = 1002.25 \text{ and } TC(m_H) = 960.92$$

$$m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = \lfloor 6.5 \rfloor = 6$$

Step 3: Let  $j = j + 1$ . Since  $TC(m_L) > TC(m_H)$

$$m_L = m_m^{j-1} = 6 \text{ and } m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = 9$$

Step 4: Since  $|m_m^j - m_m^{j-1}| = |9 - 6| = 3 > 1$

Step 5: Let  $j = j + 1$ . Since  $TC(m_L) < TC(m_H)$

$$m_H = m_m^{j-1} = 9 \text{ and } m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = 7$$

Step 6: Since  $|m_m^j - m_m^{j-1}| = |7 - 9| = 2 > 1$

Step 7: Let  $j = j + 1$ . Since  $TC(m_L) < TC(m_H)$

$$m_H = m_m^{j-1} = 7 \text{ and } m_m^j = \lfloor (m_L + m_H) / 2 \rfloor = 6$$

Step 8: Since  $|m_m^j - m_m^{j-1}| = |6 - 7| = 1$

We compare  $TC(m_L) = 909.01$  and  $TC(m_H) = 920.80$  and choose

$m_L = 6$  as the optimal solution.

Table 4.6 Optimal Settings for  $(Q_i, r_i)$  with various resolution levels in case of demand following Non-homogeneous Poisson Process.

$m$	1	<b>6*</b>	7	9	12
$(Q_i, r_i)$	(35,23)	<b>(3,1)</b>	(3,1)	(3,1)	(1,0)
		<b>(12,5)</b>	(10,4)	(7,3)	(5,1)
		<b>(23,12)</b>	(19,9)	(13,6)	(10,3)
		<b>(34,23)</b>	(28,17)	(22,11)	(15,6)
		<b>(45,37)</b>	(38,28)	(28,17)	(20,10)
		<b>(59,55)</b>	(49,40)	(36,24)	(26,14)
			(59,56)	(44,35)	(31,20)
				(51,46)	(37,26)
				(60,58)	(43,33)
					(49,40)
					(55,51)
					(62,60)
$TC(\$)$	1002.25	<b>909.01</b>	920.80	940.44	960.92

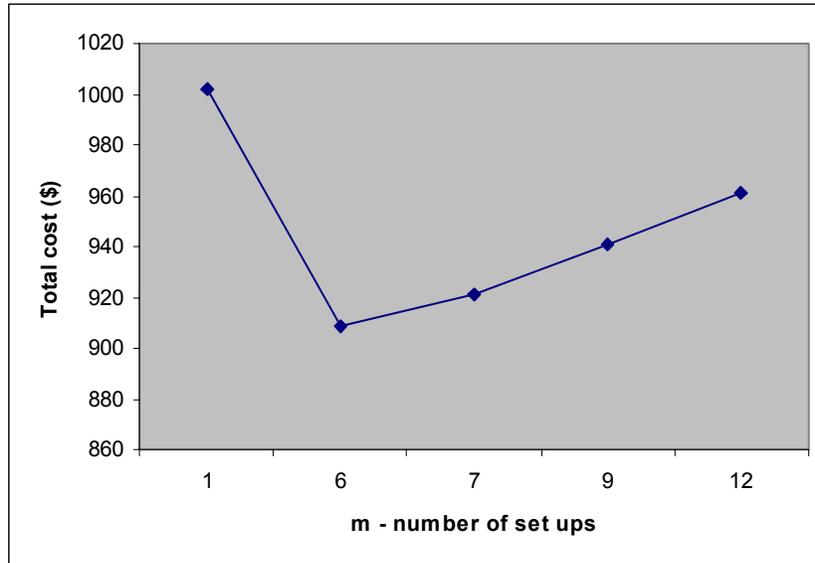


Figure 4.6 TC (Total Cost) vs.  $m$  in searching the optimal solution

#### 4.1 Expected Number of Stock-outs:

After obtaining the optimal parameter settings in each situation, we consider the case of demand for new sales following homogeneous Poisson process and calculate the total number of stock-outs occurring in a period of 1 year. From Figure 4.7, we can see that the number of stock out units decreases with the number of set ups made, i.e. by updating the values of  $Q$  and  $r$  we can reduce the number of stock outs occurred. As the installed base of the product changes with time and the maintenance demand for a product increases, this approach would serve better while reducing the stock out cost of a company. In case of  $m = 1$ , we take the average demand for each month and calculate the values of  $Q$  and  $r$  accordingly. However, this may not be accurate. By doing this we are over estimating the occurrence of demand in the initial stage and under estimating the demand in the later stage. This increases the probability of stock out as we move further thus results in more number of stock outs that increase the stock out cost of the company.

Table 4.7 Expected number of stock-outs in a period of 1 year for different set-ups

m - number of set ups	1	3	6	12
no. of units stocked out	1.32E+02	1.60E+01	5.11E+00	3.15E+00

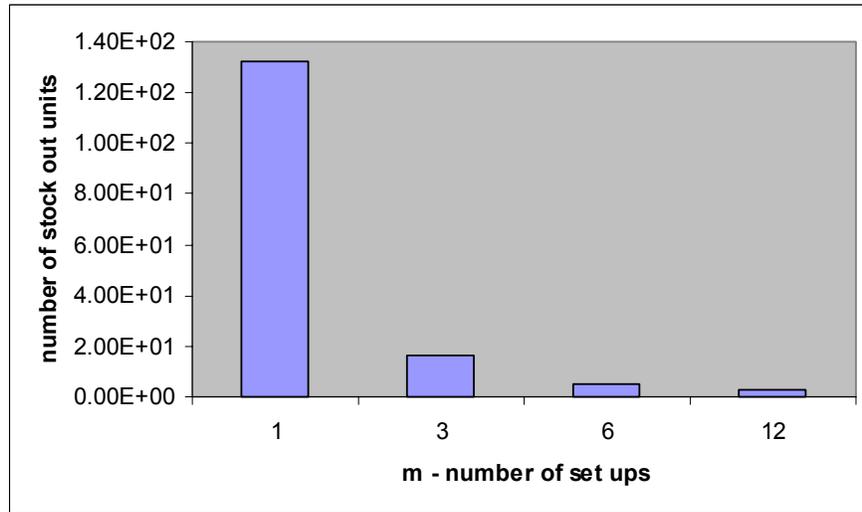


Figure 4.7 Plot of expected number of stock-outs in each case for a period of 1 year

Using the average demand in each case, we obtain the optimal values of  $Q$  and  $r$  that reduce the total cost of the company. As the maintenance demand of the product varies with time, the demand during its replenishment lead time ( $\theta$ ) also varies. In case of  $m = 1$ , we do not vary the value of  $\theta$  and just use a single value obtained from the average demand in process of calculating the values of  $Q$  and  $r$ . This leads to the occurrence of stock outs. Here, we estimate the number of stock outs in each case by varying the value of  $\theta$  and using the values of  $Q$  and  $r$  obtained from the average demand. This helps in calculating the probability of stock out for each month and number of stock outs occurring in each month. Table 4.8 shows the way the number of stock outs are calculated for each month in case of  $m = 1$ .

Table 4.8 Expected number of stock-outs for each month in case of  $m = 1$

month	1	2	3	4	5	6
maintenance demand	4.2	20.32	36.6	54.04	70.63	87.05
actual theta	0.98	4.741	8.54	12.6093	16.48033	20.312
theta used	22.328	22.33	22.32806	22.3281	22.32806	22.328
re-order	31	31	31	31	31	31
order quantity	40	40	40	40	40	40
prob. of stock out	0	0	6.40E-10	3.39E-06	0.000452	0.0099
number of stock outs	0	0	0	0	0	5.07E-02
month	7	8	9	10	11	12
maintenance demand	104.77	121	138.51	155	170.4	186.1
actual theta	24.446	28.233	32.319	36.1	39.77	43.42
theta used	22.328	22.328	22.32806	22.3	22.33	22.33
re-order	31	31	31	31	31	31
order quantity	40	40	40	40	40	40
prob. of stock out	0.0811	0.2629	0.545827	0.77	0.909	0.97
number of stock outs	6.45E-01	3.15	1.03E+01	2.21E+01	3.82E+01	5.81E+01

#### 4.2 Expected Mean and Variance of Stock-outs:

From the Table 4.7, we can see that higher the number of set-ups made lesser the number of stock-outs occurred. Here, we calculate the mean and variance of stock-outs occurring for different set-ups. This gives a clear indication of the importance of varying the values of  $Q$  and  $r$  with time. Figure 4.8 illustrates the way the number of stock-outs decrease with the number of set-ups made. Also, after obtaining the mean and variance of total number of stock-outs in each case, we plot the mean and variance of stock-outs occurring in each individual month for different set-ups made. Figures 4.9, 4.10, 4.11 and 4.12 show the number of stock-outs occurring in each month.

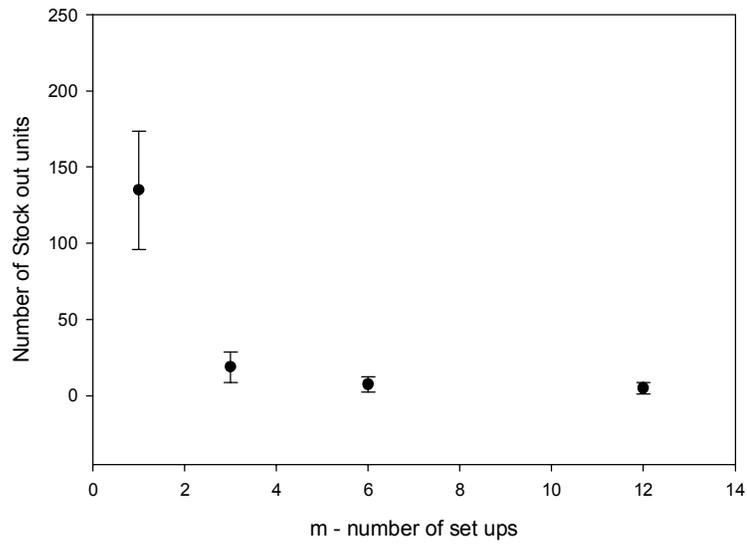


Figure 4.8 Plot of mean and variance of stock-outs for different set-ups

Table 4.9 Mean and Variance of stock-outs for different set-ups

Var	1511.51977	103.889713	24.8688088	13.892124
Stdev	38.8782686	10.1926303	4.98686363	3.72721398
Mean	134.934173	18.7209936	7.37184597	4.88631653

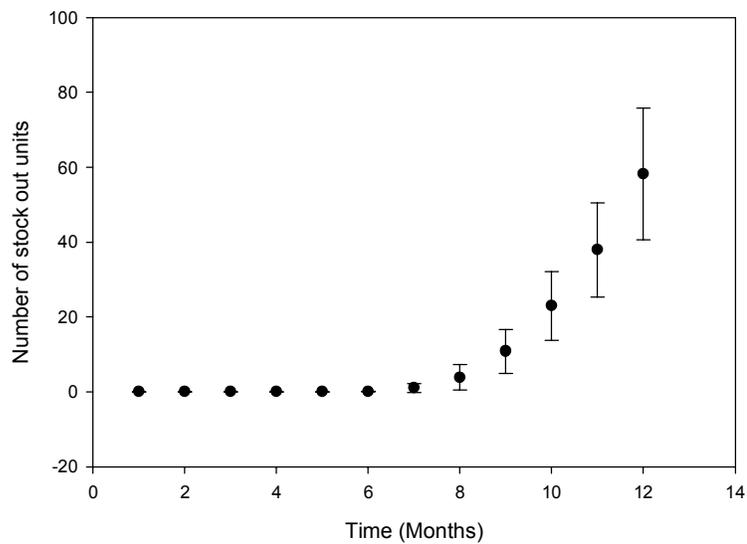


Figure 4.9 Plot of mean and variance of stock-outs for each month @ m = 1

Table 4.10 Mean and Variance of stock-outs for each month @  $m = 1$

Month	1	2	3	4	5	6
Var	0	0	0	0	0	0.022
Stdev	0	0	0	0	0.009	0.148
Mean	0	0	0	0	0.006	0.104
Month	7	8	9	10	11	12
Var	1.493	11.12	34.2	82.97	158.9	309.7
Stdev	1.222	3.335	5.848	9.109	12.61	17.6
Mean	1.056	3.892	10.87	22.98	37.92	58.27

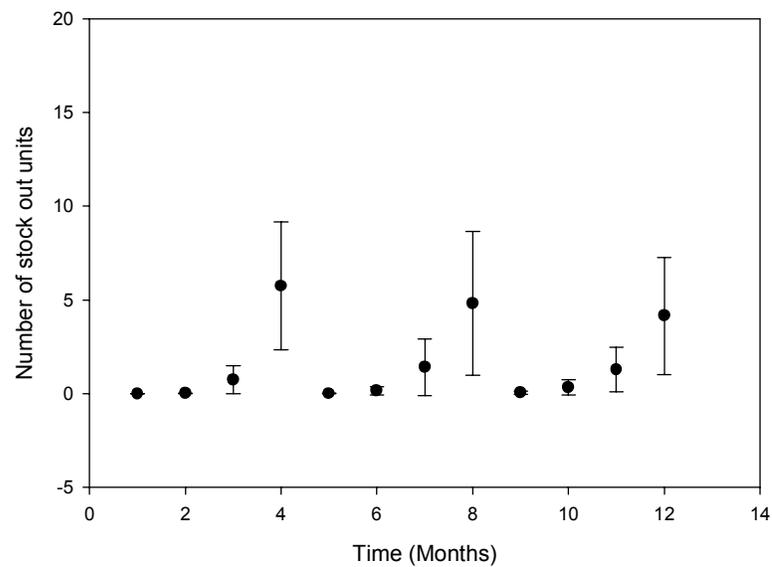


Figure 4.10 Plot of mean and variance of stock-outs for each month @  $m = 3$

Table 4.11 Mean and Variance of stock-outs for each month @ m = 3

Month	1	2	3	4	5	6
Var	0	0	0.544	11.59	0	0.045
Stdev	0	0.031	0.738	3.405	0.016	0.213
Mean	0	0.017	0.751	5.746	0.01	0.158
Month	7	8	9	10	11	12
Var	2.267	14.67	0.007	0.164	1.378	9.715
Stdev	1.506	3.83	0.082	0.405	1.174	3.117
Mean	1.411	4.807	0.051	0.335	1.286	4.151

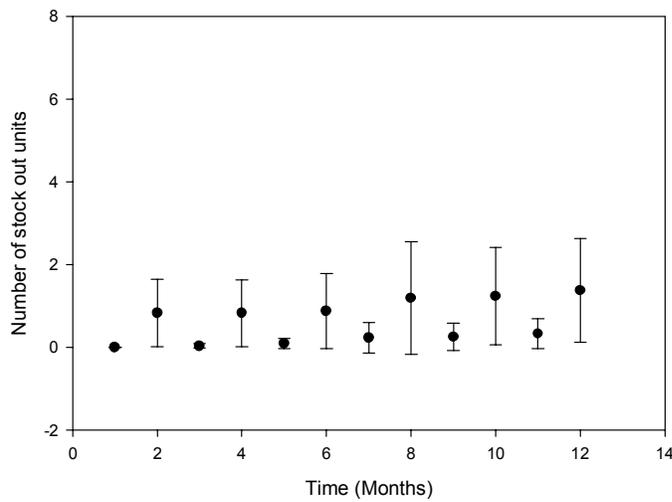


Figure 4.11 Plot of mean and variance of stock-outs for each month @ m = 6

Table 4.12 Mean and Variance of stock-outs for each month @ m = 6

Month	1	2	3	4	5	6
Var	0	0.66	0.003	0.655	0.016	0.834
Stdev	0	0.813	0.053	0.809	0.127	0.913
Mean	0	0.837	0.035	0.826	0.097	0.876
Month	7	8	9	10	11	12
Var	0.136	1.838	0.105	1.38	0.133	1.57
Stdev	0.369	1.356	0.324	1.175	0.365	1.253
Mean	0.232	1.192	0.256	1.238	0.331	1.375

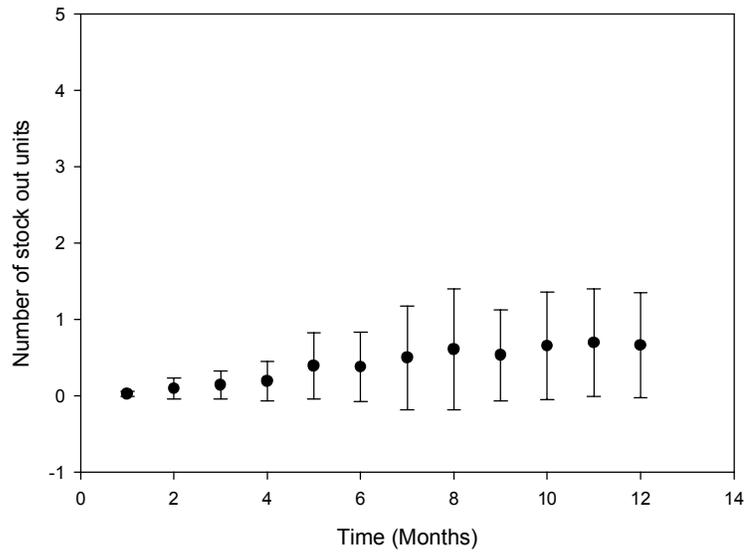


Figure 4.12 Plot of mean and variance of stock-outs for each month @ m = 12

Table 4.13 Mean and Variance of stock-outs for each month @ m = 12

Month	1	2	3	4	5	6
Var	0.001	0.02	0.034	0.065	0.189	0.206
Stdev	0.037	0.141	0.183	0.254	0.434	0.454
Mean	0.026	0.098	0.141	0.194	0.392	0.382
Month	7	8	9	10	11	12
Var	0.459	0.624	0.356	0.493	0.491	0.476
Stdev	0.678	0.79	0.596	0.702	0.701	0.69
Mean	0.499	0.606	0.533	0.655	0.697	0.663

### 4.3 Failure rate of the part following Exponential distribution:

After obtaining the above results, we study the case where the failure rate of the part follows exponential distribution with parameters  $\beta = 1$  and  $\eta = \Gamma(1.5)$ . We start by generating the maintenance demand for these values and obtain the optimal parameter settings that reduce the total inventory investment of a company.

Table 4.14 Expected maintenance demand in a 1 year period with failure rate of the part following exponential distribution

1	2	3	4	5	6
9.59	26.51	43.44	60.37	77.29	94.22
7	8	9	10	11	12
111.15	128.07	145	161.92	178.85	195.77

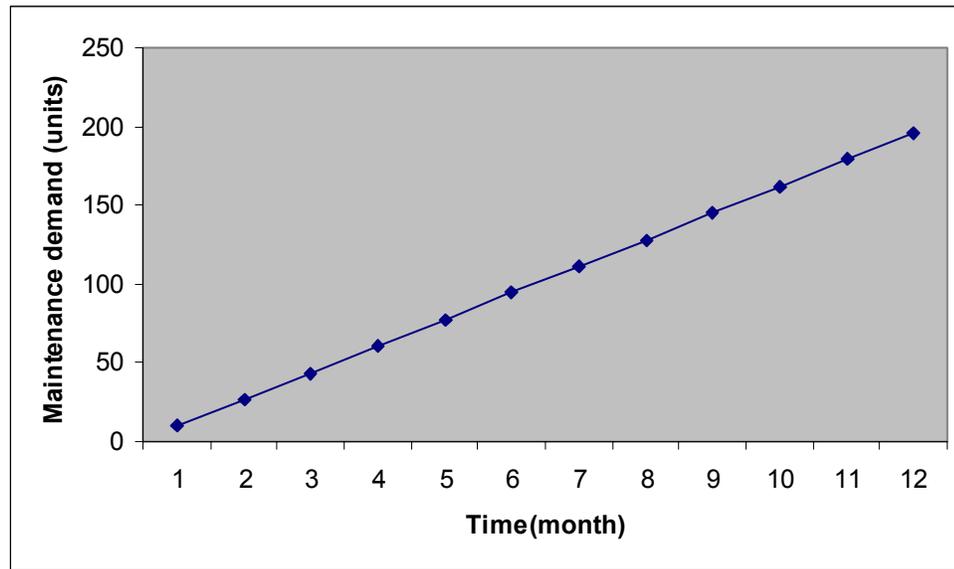


Figure 4.13 Plot of expected maintenance demand in a 1 year period with failure rate of the part following exponential distribution

Table 4.15 Optimal Settings for  $(Q_i, r_i)$  with various resolution levels with failure rate of the part following exponential distribution

$m$	1	2	3	4	5	6
$(Q_i, r_i)$	(42,32)	<b>(30,18)</b>	(24,13)	(21,11)	(19,9)	(17,8)
		<b>(52,46)</b>	(42,32)	(37,25)	(32,21)	(30,18)
			(56,50)	(48,39)	(42,32)	(38,28)
				(56,53)	(51,43)	(46,37)
					(57,54)	(52,46)
						(58,55)
Total cost (\$)	1222.82	<b>1192.19</b>	1194.19	1197.80	1205.65	1212.45
$m$	7	8	9	10	11	12
$(Q_i, r_i)$	(17,7)	(16,7)	(15,6)	(14,6)	(13,5)	(12,5)
	(27,16)	(25,14)	(24,13)	(23,12)	(22,11)	(21,11)
	(35,24)	(34,22)	(31,19)	(30,18)	(28,17)	(27,16)
	(42,32)	(39,28)	(38,26)	(35,24)	(33,22)	(31,21)
	(49,40)	(45,36)	(42,32)	(40,29)	(38,27)	(37,25)
	(53,48)	(50,42)	(47,38)	(45,35)	(43,32)	(40,30)
	(59,55)	(54,50)	(51,45)	(48,41)	(46,37)	(43,35)
		(58,56)	(56,50)	(52,46)	(50,42)	(48,39)
			(59,56)	(56,51)	(53,47)	(50,44)
				(60,56)	(56,52)	(54,48)
					(59,57)	(56,53)
						(60,57)
Total cost (\$)	1223.30	1232.03	1243.07	1252.35	1259.95	1270.08

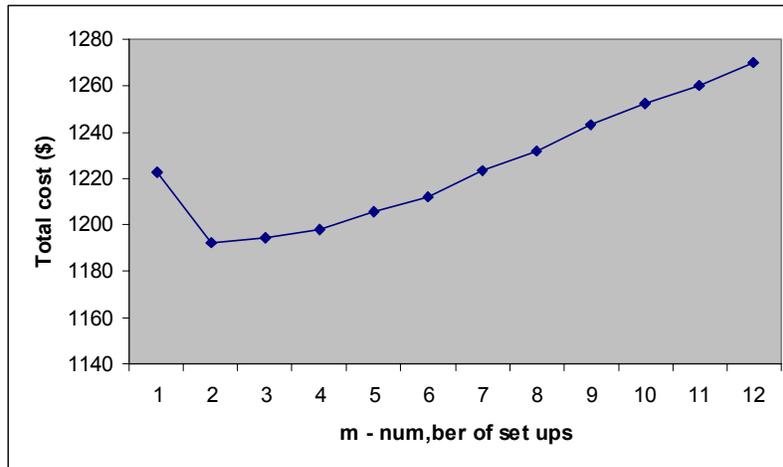


Figure 4.14 TC (Total Cost) vs.  $m$  in searching the optimal solution

From the above calculations, we see that by varying the failure distribution of the part we get a different solution i.e. optimal number of set-ups to be made come up to 2 indicating sensitivity of the model to various distributions followed. Also, as the mean and variance of stock-outs are calculated in the previous case, we repeat the same procedure to see the effect of number of set-ups on the stock-outs occurred.

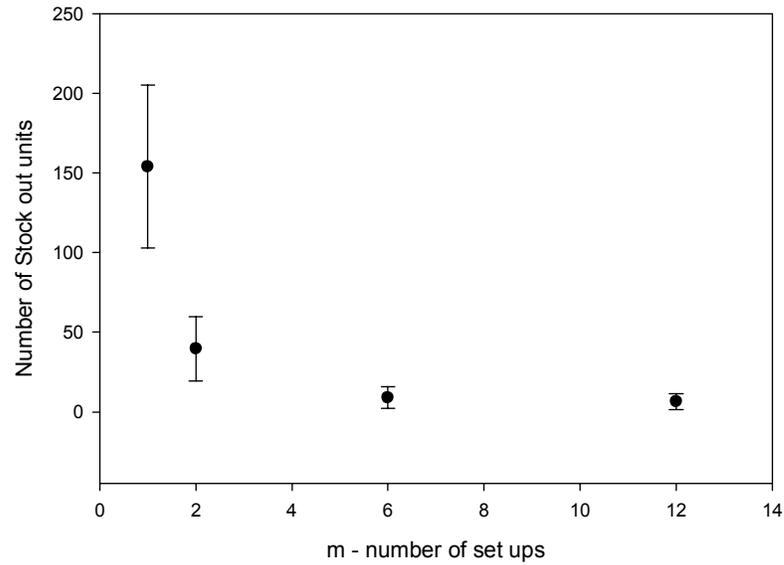


Figure 4.15 Plot of mean and variance of stock-outs for different set-ups in case of exponential distribution

Table 4.16 Mean and Variance of stock-outs for different set-ups in case of exponential distribution

Var	2623.10537	413.269967	47.8370205	26.6683123
Stdev	52.0125455	18.5428833	6.4459872	4.80463803
Mean	155.291327	39.2335469	9.24076738	6.53562309

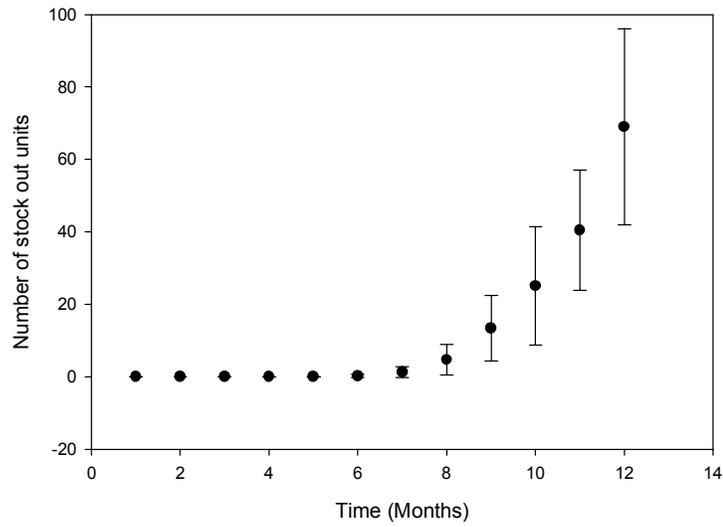


Figure 4.16 Plot of mean and variance of stock-outs for each month @  $m = 1$  in case of exponential distribution

Table 4.17 Mean and Variance of stock-outs for each month @  $m = 1$  in case of exponential distribution

Month	1	2	3	4	5	6
Var	0	0	0	0	0.0006	0.166
Stdev	0	0	0	0	0.0256	0.407
Mean	0	0	0	0	0.0098	0.233
Month	7	8	9	10	11	12
Var	2.1	17.89	81	267.31	277.71	730.43
Stdev	1.45	4.229	9	16.349	16.66	27.02
Mean	1.315	4.655	13.34	25.06	40.466	68.979

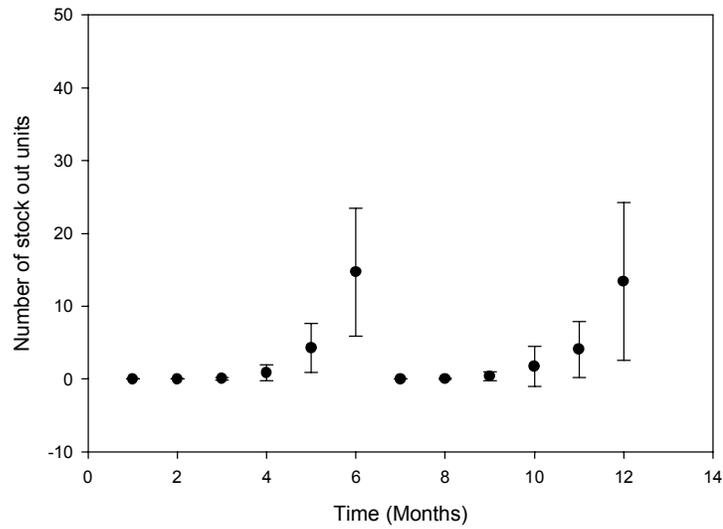


Figure 4.17 Plot of mean and variance of stock-outs for each month @  $m = 2$  in case of exponential distribution

Table 4.18 Mean and Variance of stock-outs for each month @  $m = 2$  in case of exponential distribution

Month	1	2	3	4	5	6
Var	0	0	0.02	1.194	11.19	77.02
Stdev	0	0.002	0.143	1.093	3.346	8.776
Mean	0	0	0.051	0.848	4.278	14.71
Month	7	8	9	10	11	12
Var	0	0.009	0.361	7.707	14.59	116.8
Stdev	0.007	0.095	0.601	2.776	3.82	10.81
Mean	0.003	0.045	0.392	1.754	4.071	13.42

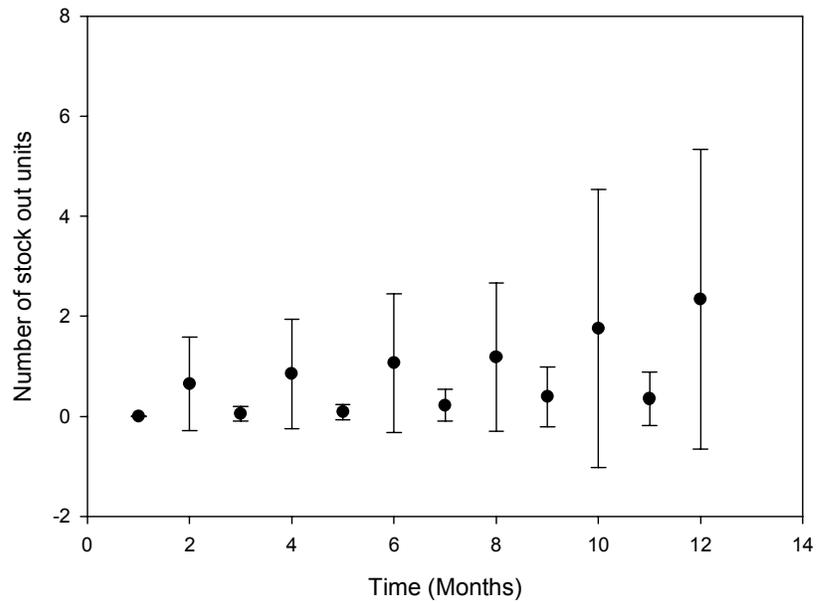


Figure 4.18 Plot of mean and variance of stock-outs for each month @  $m = 6$  in case of exponential distribution

Table 4.19 Mean and Variance of stock-outs for each month @  $m = 6$  in case of exponential distribution

Month	1	2	3	4	5	6
Var	0	0.876	0.02	1.194	0.025	1.917
Stdev	0.008	0.936	0.143	1.093	0.157	1.384
Mean	0.002	0.646	0.051	0.848	0.085	1.065
Month	7	8	9	10	11	12
Var	0.102	2.183	0.361	7.707	0.291	8.968
Stdev	0.319	1.478	0.601	2.776	0.539	2.995
Mean	0.22	1.186	0.392	1.754	0.351	2.343

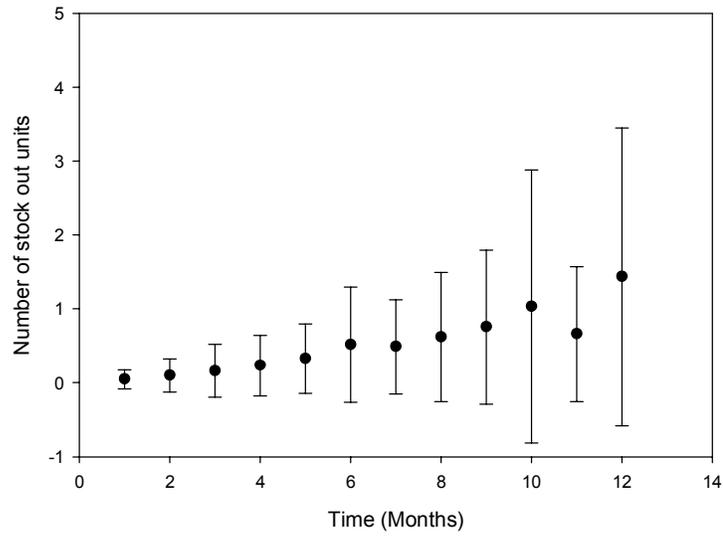


Figure 4.19 Plot of mean and variance of stock-outs for each month @  $m = 12$  in case of exponential distribution

Table 4.20 Mean and Variance of stock-outs for each month @  $m = 12$  in case of exponential distribution

Month	1	2	3	4	5	6
Var	0.016	0.049	0.129	0.168	0.22	0.605
Stdev	0.128	0.222	0.359	0.41	0.469	0.778
Mean	0.049	0.097	0.164	0.235	0.329	0.515
Month	7	8	9	10	11	12
Var	0.404	0.763	1.091	3.416	0.829	4.08
Stdev	0.636	0.874	1.044	1.848	0.91	2.02
Mean	0.486	0.62	0.754	1.034	0.659	1.434

## CHAPTER 5

### CONCLUSIONS AND FUTURE WORK

This report addresses an approach for controlling spare parts inventory considering new sales. The resulting maintenance demand due to new sales has been formulated analytically and calculated using a simulation approach. A new framework for managing the spare parts inventory, referred to as the multi-resolution inventory management model is proposed to deal with the non-stationary maintenance demand. To solve the nonlinear optimization problem, a bisectional search algorithm is developed in searching the optimal settings of the associated operating parameters. In this report, we studied three different cases where the demand for new sales follows uniform distribution, homogeneous Poisson process and non-homogeneous Poisson process and obtained the optimal parameter settings in each case. Also, we tried to identify the number of stock-outs occurring with respect to the number of set-ups made. And lastly, we studied the case where the failure rate of the part follows exponential distribution in process of determining the sensitivity of the model proposed. It will be complex when multi-echelon inventory systems and multiple products sharing the same spare units are considered. Also, the research presented here concentrates on the case where the whole time period is divided into equal time intervals; it might be even more profitable if the length of the time intervals can be varied based upon the failure rate of the parts. These topics will be studied in our future research, and the results will be implemented in solving more complex engineering problems.

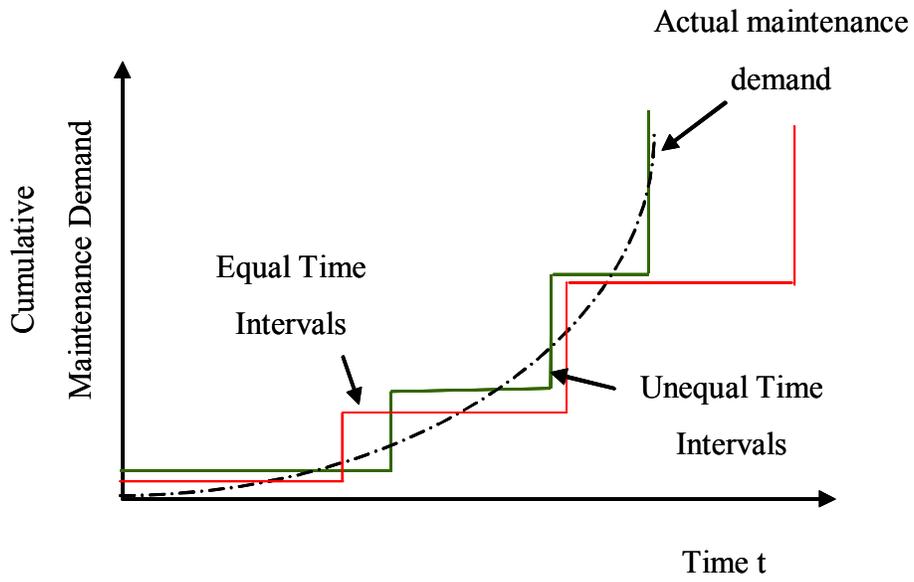


Figure 5.1 Multi-resolution maintenance demand approximation with unequal time intervals

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## **APPENDICES**

## APPENDIX A

### MATLAB CODE FOR DEMAND FOLLOWING UNIFORM DISTRIBUTION

```
for n = 1:100
clear all
S = unidrnd(30,1,12) % 15*ones(1,12) %no. of sales for each month
s1 = S;
for k=1:12
for i = 1:s1(k)
TB2(k,i) = k-1+unidrnd(0,1); %time to buy for each customer in a month.
end
end

for k=1:12
for i=1:s1(k)
for j = 1:25
F(k,i,j) = wblrnd(1,2,1,1);
end
end
end

for k=1:12
for i=1:s1(k)
c(k,i)=0;
Temp=TB2(k,i);
for j=1:25
ActF(k,i,j)=Temp+F(k,i,j);
Temp=ActF(k,i,j);
if ActF(k,i,j)<12
c(k,i)=c(k,i)+1;
end
end
end
end

Temp=1;
for k=1:12
for i=1:s1(k)
MonthlyF(k,Temp:c(k,i)+Temp-1)=ActF(k,i,1:c(k,i));
Temp=Temp+c(k,i);
end
end

YearlyF=0;
for k=1:12
YearlyF=[YearlyF MonthlyF(k,:)];
end
YearlyF=sort(YearlyF,'descend');
YearlyF=YearlyF';
[Num Dum]=size(YearlyF);

YearlyFFinal=0;
```

```

for i=1:Num
    if YearlyF(i) == 0
        break;
    end
    YearlyFFinal(i)=YearlyF(i);
end

YearlyFFinal=sort(YearlyFFinal);

%plot the Times for Request v.s. Number of Requests
[T1 T2]=size(YearlyFFinal);
plot(YearlyFFinal,[1:T2],'k-')
hold on

M(1:12) = zeros(1,12);
for x = 1:T2
    if YearlyFFinal(x)<1
        M(1) = M(1) +1;
    elseif YearlyFFinal(x)>1 && YearlyFFinal(x)<2
        M(2) = M(2) +1;
    elseif YearlyFFinal(x)>2 && YearlyFFinal(x)<3
        M(3) = M(3) +1;
    elseif YearlyFFinal(x)>3 && YearlyFFinal(x)<4
        M(4) = M(4) +1;
    elseif YearlyFFinal(x)>4 && YearlyFFinal(x)<5
        M(5) = M(5) +1;
    elseif YearlyFFinal(x)>5 && YearlyFFinal(x)<6
        M(6) = M(6) +1;
    elseif YearlyFFinal(x)>6 && YearlyFFinal(x)<7
        M(7) = M(7) +1;
    elseif YearlyFFinal(x)>7 && YearlyFFinal(x)<8
        M(8) = M(8) +1;
    elseif YearlyFFinal(x)>8 && YearlyFFinal(x)<9
        M(9) = M(9) +1;
    elseif YearlyFFinal(x)>9 && YearlyFFinal(x)<10
        M(10) = M(10) +1;
    elseif YearlyFFinal(x)>10 && YearlyFFinal(x)<11
        M(11) = M(11) +1;
    else
        M(12) = M(12) +1;
    end
end

M
D = S + M
end

```

## APPENDIX B

### MATLAB CODE FOR DEMAND FOLLOWING HOMOGENEOUS POISSON PROCESS

```
for n = 1:100
clear all
S(1:12) = zeros(1,12);
NS = poissrnd(2,1,1)/30;
while NS <= 1
    S(1) = S(1) + 1;
    NS1(1,S(1)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 1 & NS <= 2
    S(2) = S(2) + 1;
    NS1(2,S(2)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 2 & NS <= 3
    S(3) = S(3) + 1;
    NS1(3,S(3)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 3 & NS <= 4
    S(4) = S(4) + 1;
    NS1(4,S(4)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 4 & NS <= 5
    S(5) = S(5) + 1;
    NS1(5,S(5)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 5 & NS <= 6
    S(6) = S(6) + 1;
    NS1(6,S(6)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 6 & NS <= 7
    S(7) = S(7) + 1;
    NS1(7,S(7)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 7 & NS <= 8
    S(8) = S(8) + 1;
    NS1(8,S(8)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 8 & NS <= 9
    S(9) = S(9) + 1;
    NS1(9,S(9)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 9 & NS <= 10
    S(10) = S(10) + 1;
    NS1(10,S(10)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 10 & NS <= 11
    S(11) = S(11) + 1;
    NS1(11,S(11)) = NS;
    NS = NS + poissrnd(2,1,1)/30;
while NS > 11 & NS <= 12
```



```

YearlyFFinal=0;
for i=1:Num
    if YearlyF(i) == 0
        break;
    end
    YearlyFFinal(i)=YearlyF(i);
end

YearlyFFinal=sort(YearlyFFinal);

%plot the Times for Request v.s. Number of Requests
[T1 T2]=size(YearlyFFinal);
plot(YearlyFFinal,[1:T2],'k-')
hold on

M(1:12) = zeros(1,12);
for x = 1:T2
    if YearlyFFinal(x)<=1
        M(1) = M(1) +1;
    elseif YearlyFFinal(x)>1 && YearlyFFinal(x)<=2
        M(2) = M(2) +1;
    elseif YearlyFFinal(x)>2 && YearlyFFinal(x)<=3
        M(3) = M(3) +1;
    elseif YearlyFFinal(x)>3 && YearlyFFinal(x)<=4
        M(4) = M(4) +1;
    elseif YearlyFFinal(x)>4 && YearlyFFinal(x)<=5
        M(5) = M(5) +1;
    elseif YearlyFFinal(x)>5 && YearlyFFinal(x)<=6
        M(6) = M(6) +1;
    elseif YearlyFFinal(x)>6 && YearlyFFinal(x)<=7
        M(7) = M(7) +1;
    elseif YearlyFFinal(x)>7 && YearlyFFinal(x)<=8
        M(8) = M(8) +1;
    elseif YearlyFFinal(x)>8 && YearlyFFinal(x)<=9
        M(9) = M(9) +1;
    elseif YearlyFFinal(x)>9 && YearlyFFinal(x)<=10
        M(10) = M(10) +1;
    elseif YearlyFFinal(x)>10 && YearlyFFinal(x)<=11
        M(11) = M(11) +1;
    else
        M(12) = M(12) +1;
    end
end

S
M
D = S + M
end

```

## APPENDIX C

### MATLAB CODE FOR DEMAND FOLLOWING NON-HOMOGENEOUS POISSON PROCESS

```
for n = 1:100
clear all
S(1:12) = zeros(1,12);
u = rand;
PS = ((-log(1-u)*2.2)/2)^(1/2.2);           %Point of sales
while PS <= 1
S(1) = S(1) + 1;
PS1(1,S(1)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 + PS^2.2)^(1/2.2);
end

while PS > 1 & PS <= 2
S(2) = S(2) + 1;
PS1(2,S(2)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 + PS^2.2)^(1/2.2);
end

while PS > 2 & PS <= 3
S(3) = S(3) + 1;
PS1(3,S(3)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 + PS^2.2)^(1/2.2);
end

while PS > 3 & PS <= 4
S(4) = S(4) + 1;
PS1(4,S(4)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 + PS^2.2)^(1/2.2);
end

while PS > 4 & PS <= 5
S(5) = S(5) + 1;
PS1(5,S(5)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 + PS^2.2)^(1/2.2);
end

while PS > 5 & PS <= 6
S(6) = S(6) + 1;
PS1(6,S(6)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 + PS^2.2)^(1/2.2);
end

while PS > 6 & PS <= 7
S(7) = S(7) + 1;
```

```

PS1(7,S(7)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 +PS^2.2)^(1/2.2);
end

while PS > 7 & PS <= 8
S(8) = S(8) + 1;
PS1(8,S(8)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 +PS^2.2)^(1/2.2);
end

while PS > 8 & PS <= 9
S(9) = S(9) + 1;
PS1(9,S(9)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 +PS^2.2)^(1/2.2);
end

while PS > 9 & PS <= 10
S(10) = S(10) + 1;
PS1(10,S(10)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 +PS^2.2)^(1/2.2);
end

while PS > 10 & PS <= 11
S(11) = S(11) + 1;
PS1(11,S(11)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 +PS^2.2)^(1/2.2);
end

while PS > 11 & PS <= 12
S(12) = S(12) + 1;
PS1(12,S(12)) = PS;
u = rand;
PS = ((-log(1-u)*2.2)/2 +PS^2.2)^(1/2.2);
end
s1 = S;

for k=1:12
for i=1:s1(k)
for j = 1:25
F(k,i,j) = wblrnd(1,2,1,1);
end
end
end

for k=1:12
for i=1:s1(k)
c(k,i)=0;
Temp=PS1(k,i);
for j=1:25
ActF(k,i,j)=Temp+F(k,i,j);

```

```

Temp=ActF(k,i,j);
if ActF(k,i,j)<12
    c(k,i)=c(k,i)+1;
end
end
end
end

Temp=1;
for k=1:12
    for i=1:s1(k)
        MonthlyF(k,Temp:c(k,i)+Temp-1)=ActF(k,i,1:c(k,i));
        Temp=Temp+c(k,i);
    end
end

YearlyF=0;
for k=1:12
    YearlyF=[YearlyF MonthlyF(k,:)];
end
YearlyF=sort(YearlyF,'descend');
YearlyF=YearlyF';
[Num Dum]=size(YearlyF);

YearlyFFinal=0;
for i=1:Num
    if YearlyF(i) == 0
        break;
    end
    YearlyFFinal(i)=YearlyF(i);
end

YearlyFFinal=sort(YearlyFFinal);

%plot the Times for Request v.s. Number of Requests
[T1 T2]=size(YearlyFFinal);
plot(YearlyFFinal,[1:T2],'k-')
hold on

M(1:12) = zeros(1,12);
for x = 1:T2
    if YearlyFFinal(x)<=1
        M(1) = M(1) +1;
    elseif YearlyFFinal(x)>1 && YearlyFFinal(x)<=2
        M(2) = M(2) +1;
    elseif YearlyFFinal(x)>2 && YearlyFFinal(x)<=3
        M(3) = M(3) +1;
    elseif YearlyFFinal(x)>3 && YearlyFFinal(x)<=4
        M(4) = M(4) +1;
    elseif YearlyFFinal(x)>4 && YearlyFFinal(x)<=5
        M(5) = M(5) +1;
    elseif YearlyFFinal(x)>5 && YearlyFFinal(x)<=6
        M(6) = M(6) +1;
    elseif YearlyFFinal(x)>6 && YearlyFFinal(x)<=7
        M(7) = M(7) +1;
    elseif YearlyFFinal(x)>7 && YearlyFFinal(x)<=8

```

```
M(8) = M(8) + 1;
elseif YearlyFFinal(x) > 8 && YearlyFFinal(x) <= 9
    M(9) = M(9) + 1;
elseif YearlyFFinal(x) > 9 && YearlyFFinal(x) <= 10
    M(10) = M(10) + 1;
elseif YearlyFFinal(x) > 10 && YearlyFFinal(x) <= 11
    M(11) = M(11) + 1;
else
    M(12) = M(12) + 1;
end
end

S
M
D = S + M
end
```

## APPENDIX D

### FLOWCHART OF MODEL IMPLEMENTATION

